

INDEX FUTURES ARBITRAGE OPPORTUNITIES IN SOUTH AFRICA

Kingsley Kendall Williams

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ABSTRACT

Persistent arbitrage opportunities are indicative of inefficiencies within a market. While South Africa's financial markets are generally considered advanced in relation to the country's emerging market status, the degree of arbitrage opportunities and efficiency among its Top 40 index futures and Satrix exchange-traded funds has yet to be tested.

Using high-frequency intraday data over the first quarter of 2006, and factoring in various costs and constraints in executing arbitrage opportunities, this study revealed that South Africa's markets do indeed live up to their claim. However, the Satrix Top 40 exchange-traded fund was priced at an intraday premium relative to the index, but generally had insufficient liquidity to yield any economically significant arbitrage opportunities, except between 14:00 and 14:30, where an additional, almost risk-free profit of 0.1485% was found. The lack of pervasive arbitrage opportunities and low levels of friction are indicative of market efficiency and have important implications for those seeking foreign investment into South Africa's financial markets.

THE DECLARATION

I, Kingsley Kendall Williams, declare that this research report is my own, unaided work. It is submitted in partial fulfilment of the requirements for the degree of Master of Business Administration at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in this or any other university.

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Kingsley Kendall Williams

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1 INTRODUCTION

1.1 Context of the Study

The South African financial market is considered well developed among emerging markets (World Bank Group, 2006). While this may be a prerequisite for the existence of market efficiency, factors such as costs, taxation, regulation and other frictions also play a role in determining market efficiency. Since market efficiency is a broad field of study, it is typically confined to a narrower context in order to quantify the degree of inefficiency within that context in conjunction with identifying the underlying causes of apparent inefficiencies. The efficiency of index futures has had a considerable body of literature devoted to it since index futures were introduced in the United States in 1982. South African index futures contracts first appeared in 1987 followed closely by the formation of the South African Futures Exchange in 1988 (SAFEX, 2007). Despite South Africa's long history with this financial instrument, a formal study on its efficiency relative to its underlying index has yet to be conducted.

Equity index futures are standardised contracts for a deferred transaction in an equity index at a predetermined price. They offer a cost effective means with which to hedge an equity market exposure, or to speculate and take advantage of anticipated price movements in the equity market (Fung, Cheng & Chan, 1997). However, since futures contracts and their corresponding equity indices are equivalent assets whose prices are established in separate markets, opportunities for arbitrage may exist when index and futures prices diverge sufficiently (Chung, 1991). Index futures arbitrage is the process of capitalising these pricing differences in order to generate a risk-free profit, and by so doing serves in keeping the index futures market efficient (Fremault, 1991).

Exchange-traded funds are a more recent addition to the financial instrument landscape, having first traded on the American Stock Exchange in 1993 (Porteba & Shoven, 2002), and provide investors with a unitised investment that replicates the performance of an index. South Africa has a number of exchange-traded funds listed on the Johannesburg Securities Exchange (JSE), known as Satrix securities. Since Satrix securities have their own supply and demand characteristics and proxy for the index, the pricing efficiency of these exchange-traded funds and their equivalent index futures contracts can also be tested.

1.2 Problem Definition

Studies conducted on the U.S. by Figlewski, 1984, Chung, 1991 and Chu & Hsieh, 2002, as well as in other markets such as Hong Kong (Fung, Chen & Chan, 1997), Australia (Twite, 1998) and Korea (Gay & Jung, 1999) indicate very few index futures arbitrage opportunities, and as such a high level of market efficiency. This indicates that arbitrageurs and market makers are actively engaged in keeping their equity and associated futures markets correctly priced. However, a similar study has not been conducted on South Africa. Research conducted by Wang & Hsu (2005) on the Taiwan futures market showed that arbitrage opportunities were more likely in emerging markets than they were in developed markets. Since arbitrage performs a critical role in keeping markets efficient (Shleifer & Vishny, 1997), this research examines whether futures arbitrage opportunities exist against the equivalent index and exchange-traded funds within the South African market to determine in part whether South Africa's equity and associated futures markets are efficient.

The following sub-problems are defined in this study:

- Determine the intraday fair-value of index futures using parameters such as expected dividend yields and interest rates. The fair-value relies heavily on quantifying the costs involved in performing an arbitrage trade, including the impact cost of executing such a trade;
- Determine whether Satrix exchange-traded funds representing the index offer arbitrage opportunities against the index futures contract;
- Analyse the liquidity of the underlying securities representing the index as well as the liquidity of index futures contracts in order to ascertain the feasibility of executing arbitrage trades, and to determine whether any relationship exists between the magnitude of arbitrage opportunities and the available liquidity;
- Determine whether any applicable behavioural finance anomalies exist which could yield predictable arbitrage opportunities.

1.3 Assumptions

Future dividend payments are uncertain, so the most reasonable forecast of dividends is the market consensus of dividends as supplied by Inet Bridge. This is an average of the expected dividends on underlying shares within the index as published by research analysts. While the actual dividends paid out on each share could have been used for this research, doing so would provide an *ex post* analysis of arbitrage opportunities, which is not the way arbitrage

opportunities are evaluated in practise. Since the consensus dividend forecast may differ substantially from the actual dividend, this further indicates why index futures arbitrage is not entirely risk-free.

The costs incurred in executing trades in the underlying securities representing the index are those incurred by the lowest-cost market participants (stockbrokers or exchange members), since theoretically no arbitrage should exist for the second lowest-cost market participants, such as institutional investors (Gay & Jung, 1999).

The risk-free rate used is the three-month Johannesburg Interbank Agreed Rate (JIBAR), which is supplied by the South African Futures Exchange (SAFEX) on a daily basis. The three-month JIBAR is considered a better proxy for the risk-free rate than one of the Reserve Bank treasury rates in the same way that the London Interbank Offer Rate (LIBOR) is preferred over treasury rates (Hull, 2006). This is due to taxation and other issues that serve to lower the treasury rate artificially.

While numerous models have been developed to determine index futures prices, this research will assume that forward and futures prices are equal, thereby restricting the arbitrage test to the well-known cost-of-carry or spot-futures parity model. Furthermore, for ease of use, continuous compounding will be employed within this model, which is equivalent to compounding daily, even though in practise, discrete simple interest rates are used (Collins & Fabozzi, 1999 and Hull, 2006).

Initial tests considered an arbitrageur to have a book size worth R100 million with which to settle any variation margin calls once an arbitrage position had been established. This exposure was not regarded as excessive by participants within the market (Shannon, G., personal communication, April 11, 2006).

1.4 Delimitations

Index futures arbitrage was limited to the JSE Top 40 index and its associated SAFEX futures contract (hereinafter referred to as the TOP40¹) over the first quarter of 2006. This index was chosen because it represents the majority of South Africa's equity market as well as having both liquid constituent shares and futures contracts.

The instruments used to represent the index (often referred to as the underlying) include the index constituent shares, as well as the Satrix Top 40 exchange-traded fund, which will be explored as a more cost effective alternative with which to represent the Top 40 index.

The Shareholder Weighted Top 40 (SWIX40) index and its futures contract were not considered due to limited liquidity on the futures contract and the lack of an equivalent Satrix security with which to represent the index.²

1.5 Definitions

Using terminology employed by Brennan & Schwartz (1990), long arbitrage refers to buying the underlying instrument and selling the futures contract. This strategy would be employed if the index futures contract were priced at a premium to fair value. Conversely, short arbitrage refers to selling the underlying instrument and buying the futures contract. A short arbitrage trade would be employed if the index futures contract were priced at a discount to fair value.

Ex ante is a Latin term meaning “before the event” (Investopedia, 2005). In practise, arbitrage opportunities are evaluated in this manner, meaning the price at which an opportunity is identified is likely to differ from the realised execution price.

Ex post is a Latin term meaning “after the fact” (Investopedia, 2005). Research has the benefit of looking at historical data to determine whether arbitrage opportunities existed. However, care should be taken when inferring the profitability of historical arbitrage opportunities, since in practise a lag exists between identifying and executing the opportunity (Chung, 1991).

¹ This term is used in an attempt to remove ambiguity when referring to the Top 40 index and its associated futures contract. The TOP40 futures contract is commonly referred to as the ALSI.

² A Satrix security based on the Shareholder Weighted Top 40 index has subsequently been launched.

The TOP40 index futures contract is quoted in the same units as the index, with each contract valued at R10 per index point.

The Satrix TOP40 security is an exchange-traded fund, which tracks the performance of the TOP40 index, and is quoted in cents at $1/10^{\text{th}}$, or in Rands at $1/1000^{\text{th}}$ of the value of the index. These ratios are important when determining whether an arbitrage opportunity exists, and in particular when calculating the liquidity of each instrument.

Values in currency are expressed in South African Rands, which over the period under consideration were valued between R6.00 and R6.40 to the U.S. Dollar.

2 LITERATURE REVIEW

2.1 Arbitrage and Efficiency

In theory, two assets that are equivalent in all economic respects, such as an index and its futures contract, should have the same price (Bodie, Kane & Marcus, 2005). This is known as the Law of One Price, which exists to preclude risk-free arbitrage opportunities. In the event this law is violated, arbitrageurs will enter the market to capitalise on the pricing differences with little to no risk, and in so doing apply pressure to restore the Law of One Price. They accomplish this by purchasing the under priced asset and thereby increasing its level of demand (price), while at the same time selling the overpriced asset, which reduces its level of demand (price).

An index futures contract and its underlying index are economically equivalent in most respects and should therefore have the same price, as per the Law of One Price. However, certain features of the respective instruments mean their prices will differ by at least the holding cost incurred by purchasing an underlying equity index. This holding cost is equivalent to the risk-free interest that could have been earned on the initial outlay required to purchase the index. The holding cost is offset by the dividends earned by owning the index (Hull, 2006). When the price of futures contracts do not correspond to this basic formula, commonly referred to as the cost-of-carry, there is opportunity for arbitrageurs to exploit these price differences to generate risk-free profits. When arbitrage opportunities are persistent and offer large profit opportunities the efficiency of the futures markets may be called into question (Figlewski, 1984).

However, as Shleifer & Vishny (1997) explain, arbitrage is seldom entirely risk-free, since among other reasons sufficient capital is required to fund the variation margin required over the duration of the arbitrage position. This is no less the case with index futures arbitrage (Fremault, 1991), since the time between identifying an arbitrage opportunity and establishing a position contributes to execution risk (Twite, 1998 and Chung, 1991). Furthermore, uncertain dividend yields and interest rates, coupled with transaction costs, liquidity constraints, regulatory restrictions and taxation, render the holding cost uncertain. This allows futures price variation within a band on either side of its fair-value, without being indicative of arbitrage opportunities. However, once futures contracts are priced outside these bands, they do offer the potential for profitable arbitrage (MacKinlay & Ramaswamy, 1988).

2.2 Exchange-Traded Fund Efficiency

So & Tse (2004) explain that the most efficient market or asset should be predominant in determining the underlying asset price. They considered the Hang-Seng index futures and exchange-traded tracker fund to determine which asset was predominant in index price determination. Consistent with the trading costs hypothesis, they found that the index futures market contains more information than the underlying index due to lower costs, thereby leading the index price. This was the case, even though the Hang Seng index could easily be replicated with only a few of its larger constituent shares – a similar situation to the JSE's TOP40 index.

Since an exchange-traded fund is an equivalent asset to the collection of underlying shares making up the index it represents, it may be used as a substitute for the index. There is therefore scope to test the efficiency of the Satrix TOP40 exchange-traded fund relative to the TOP40 index futures contract, by simply substituting the Satrix TOP40 for the underlying index in an index futures arbitrage transaction.

2.3 Theoretical Fair-Value

Modest & Sundaresan (1983) conducted some of the first research into the efficiency of the newly introduced index futures market. They found that after accounting for assumed transaction costs, pricing anomalies did exist for arbitrageurs able to use more than half of their short-sale proceeds.

Cornell & French (1983) extended the work by Modest & Sundaresan (1983) on the efficiency of the S&P500 futures market. Among their findings, they clearly detailed the assumptions under which the basic cost-of-carry formula holds true. The first assumption is that capital markets are perfect (frictionless) and therefore have no taxes, transactions costs or restrictions on short sales, and that securities are perfectly divisible. The second assumption is that risk-free lending and borrowing rates are equal and constant. The final assumption is that dividends are paid continuously at a constant rate.

Since these assumptions do not reflect reality, Cornell & French (1983) extended their model to include the effect of taxes as well as stochastic dividend yields and interest rates in an attempt to represent more accurately the fair-value of index futures. They showed that by accounting for the option to defer the effect of taxes until gains or losses are realised – an option not available with

index futures – provided a better model in determining the fair-value of S&P500 index futures. However, their results still indicated valuations in excess of the observed futures prices.

Since the Cornell & French (1983) model catered for stochastic dividend yields and interest rates as well as the option to defer taxes, the implication was that other factors in their first assumption of frictionless markets caused the apparent under pricing. They argued this was not caused by short sale constraints on underlying securities representing the index, since existing shareholders were not bound by this restriction. Existing shareholders could sell their holdings and instead purchase the cheaper index futures, thereby increasing the price of the index futures. Neal (1996) later confirmed this explanation empirically.

Figlewski (1984) found in a subsequent study that much of the index futures price discount could be attributed to the transitory phenomenon of the infancy of the index futures market, and not the absence of certain explanatory factors. Figlewski (1984) confirmed this with other findings, which pointed to changes in the character of the futures market, such as the speed at which the futures market returned to fair-value. Among these findings, studies by Hill, Schneeweis & Mayerson (1982) and Elton, Gruber & Rentzler (1984) also indicate that the marking to market characteristic of futures contracts is economically *insignificant* in the determination of the futures fair-value price. The conclusion was that the equity index futures market in the United States was mostly efficient and becoming more efficient with time.

However, studies following those of Modest & Sundaresan (1983), Cornell & French (1983) and Figlewski (1984) found that by considering many of the practical nuances of index futures arbitrage as well as other factors in the determination of the fair-value of index futures contracts resulted in fewer apparent arbitrage opportunities, and a far more efficient market.

In particular, Chung (1991) articulated some of the weaknesses of previous research into index futures arbitrage, such as not measuring the size and frequency of arbitrage profit opportunities, and inferring *ex post* findings as *ex ante* results. Chung (1991) furthermore illustrated the weaknesses of only using closing prices, which ignore all the intraday behaviour between an index and its futures contracts. Even though MacKinlay & Ramaswamy (1988) conducted an intraday test of arbitrage opportunities, they used the published index value, which can lag the actual index and therefore the index futures price. This can create the illusion of arbitrage opportunities, particular in a rapidly moving market. After accounting for these anomalies, the

conclusion was that the futures market was far more efficient than had previously been stated. This suggests that other factors may be responsible in the determination of the futures fair-value price, and that a more thorough analysis in the determination of arbitrage opportunities is required. The following Section is therefore directed at costs; a large source of market friction involved in index futures arbitrage.

2.4 Costs

Research conducted by Aitken & Siow (2004) into the relevant efficiency of global equity markets defined efficiency by the degree of costs associated with converting securities into cash and vice versa. Their assertion was that the cheaper it is to trade within a market, the more efficient that market is likely to be. Chung (1991), Heaney (1995) and Aitken & Siow (2004) all note the non-trivial nature of costs. As such, numerous studies have been conducted to understand further the effect costs have on market behaviour (Berkowitz, Logue & Noser, 1988; Chan & Lakonishok, 1995 and Stoll, 2000).

In particular, MacKinlay & Ramaswamy (1988) focussed their research on the effect transaction costs have on arbitrage opportunities between S&P500 index futures and the spot price of its underlying securities. Total costs included the commission on both underlying securities and index futures contracts as well as the impact on both the underlying and futures markets resulting from the trade. They found futures pricing breaches of their assumed total transaction cost bounds, indicating that arbitrage opportunities were still present five years after the introduction of index futures contracts in the United States.

Gay & Jung (1999) extended the basic cost-of-carry model on the assumption of realizing a profit on the futures maturity date to derive the bounds within which index futures arbitrage was not profitable. Using the first two years of index futures trading on the Korea Stock Exchange, between 1996 and 1998, they found that the lowest cost participants could generate arbitrage profits from under priced futures contracts, even after factoring in transaction costs. However, their model only expressed cost as a function of the trade value (variable costs). Since transactions on the JSE and SAFEX exchanges incur holding costs, which Tuckman and Vila (1992) found to have a significant effect on arbitrageur behaviour, as well as fixed costs, the following Subsections describe in detail the costs incurred in executing an index futures arbitrage trade on the JSE and SAFEX exchanges.

2.4.1 Interest Rates

The risk-free rate used was the three-month JIBAR, as supplied by SAFEX on a daily basis. In order to further relax the assumptions articulated by Cornell & French (1983) of equal lending and borrowing rates, 0.05% was added to JIBAR when borrowing and similarly subtracted from JIBAR when lending, creating a bid-ask spread between these rates of 0.10%. This was considered reasonable by derivatives market practitioners (Shannon, G., personal communication, 13 April 2006), and has important implications for determining the arbitrage bounds.

2.4.2 Brokerage

Since arbitrage is most likely exploited by the lowest cost market participants, as per the findings of Gay & Jung (1999), brokerage need not be included in the costs of arbitrage. This is because arbitrageurs are likely to be members of the JSE and SAFEX exchanges, and therefore able to deal directly with each exchange.

2.4.3 Financial Services Board Levy

The Financial Services Board (FSB) imposes an insider-trading levy on all JSE transactions to cover the costs of its regulatory function. The cost is 0.0003% of the value of the trade in both equities and index futures contracts.

2.4.4 STRATE Fees

Securities Transactions Totally Electronic (STRATE) is the Central Securities Depository for settling the JSE's transactions. Every contract note (trade settlement) incurs STRATE settlement fees according to Table 1 (STRATE, 2005). As indicated, the size of each trade will determine whether the cost is fixed or variable. This has important implications when considering the liquidity available for executing index futures arbitrage transactions.

Trade Value	Contract Note Fee
< R200,000	R10.92
R200,000 – R1,000,000	0.005459%
> R1,000,000	R54.59

Table 1: STRATE Fees Scale

2.4.5 Clearing Fees

Clearing fees are charged by SAFEX clearing members, at a rate of 0.30% of the value of an index futures trade (Wilks, D., personal communication, 9 November 2005).

2.4.6 Value Added Tax

The FSB levy in conjunction with STRATE and clearing fees are all quoted exclusive of Value Added Tax (VAT), which needs to be added at a rate of 14%.

2.4.7 Uncertified Securities Tax

Uncertified Securities Tax (UST) is a tax on the transfer of beneficial ownership, payable by the buyer in a transaction, at a rate of 0.25% of the sale price.

2.4.8 Custodian Fees

Various categories of custodian fees exist (Wilks, D., personal communication, 9 November 2005). In addition to STRATE fees, each contract note incurs an on-market settlement fee of approximately R65. These costs apply to both purchase and sale settlements on each underlying share.

A safe custody fee for holding securities in dematerialized form is charged at 0.0035% of the market value of the shares. This fee is an annual rate, billed for each month the securities are held in safe custody.

In the event a short position is taken in the underlying securities representing an index, scrip-lending fees will be incurred by the custodian at a rate of 0.55% of the value of the short sale per annum.

Finally, when selling the underlying securities short, 105% of the value of the sale is required as margin, to be deposited in an interested bearing account. The proceeds of the sale would form part of this margin, requiring an additional 5% to make up the required amount.

2.4.9 Futures Contract Margin

SAFEX imposes the initial margin requirement as a fixed cost per contract, in order to simplify the administration of initial margin requirements. The TOP40 index futures contract had an initial margin requirement of R9000 during the period under review (SAFEX, 2005).

While margin requirements are independent of the futures price, they do depend on the number of contracts bought or sold. However, given the futures price, margin can easily be converted into a variable rate in order to calculate the futures fair value upper and lower bounds independently of the number of futures contracts. The margin rate equated to approximately 5% of the value of the index futures trade.

2.5 Realising Profits

Thus far, determining whether an arbitrage opportunity exists has been premised on closing out the arbitrage position at futures contract maturity. However, an arbitrage position may be rolled forward by selling (buying) the near-dated futures contract and buying (selling) a longer-dated futures contract, depending on the type of arbitrage position in place. Conversely, an arbitrage position may be closed out prior to maturity, in the event it is profitable to do so (Chung, 1991). As MacKinlay & Ramaswamy (1988) and Twite (1998) explain, both these options have value, and therefore serve to narrow the upper and lower bounds within which arbitrage is not possible, thereby increasing the likelihood of profitable arbitrage opportunities. Only realising profits at maturity may therefore be considered a worst-case arbitrage scenario, as it excludes the option of realising profits at any other time. Therefore, if profits are readily achievable outside the arbitrage bounds, the pricing efficiency of the South African futures market may be disputed. Conversely, if no significant arbitrage opportunities are uncovered, this is indicative of futures market efficiency. This is the case even if profitable arbitrage opportunities do exist with rolling forward or early closeout, as these strategies entail additional risk.

Assuming profits are indeed realised at futures contract maturity, MacKinlay & Ramaswamy, (1988) and Gay & Jung (1999), found the magnitude of arbitrage opportunities reduced as time approached maturity. They attributed this to a risk premium associated with greater uncertainty in longer dated futures contracts.

2.6 Predicting Arbitrage Magnitude

Prior research into index futures arbitrage has also embarked on trying to identify patterns and relationships so that arbitrage opportunities can be predicted and exploited. For example, Gay & Jung (1999) utilised a regression to test a number of variables for significant explanatory power to show a causal relationship in the mispricing of index futures contracts. They found, amongst others, that time to maturity was a predictor of the magnitude of arbitrage opportunities, confirming the findings of MacKinaly & Ramaswamy (1988). Furthermore, Heaney (1995) utilised a regression to test whether the risk free rate had any explanatory power for the cost of carry model, and found that for futures based on interest bearing securities, the risk free rate was significant, but not for equity index futures.

The field of behavioural finance has lodged a number of disputes at the door of the efficient market hypothesis (Malkiel, 2003 and Bodie, Kane & Marcus, 2005). Through numerous studies that have identified patterns and characteristics that predict above average market returns without incurring above average risks, various market anomalies have been uncovered, making the case for an efficient market more difficult to defend. One such anomaly, which was uncovered by Keim (1983), Reinganum (1983) and Blume & Stambaugh (1983) is the “small-firm-in-January” effect, which results in above average returns for investments made in small firms during the first two weeks of January. Uncovering similar effects in the behaviour of index futures arbitrage may suggest market inefficiency.

Another anomaly uncovered by Amihud & Mendelson (1986), is that of liquidity. Since investors demand liquidity to execute their trades, a liquidity shortage will demand a premium to the market return in order to compensate for the additional cost and risk incurred. Research conducted on arbitrage and the factors influencing its behaviour has found that liquidity plays an important role in determining whether arbitrage opportunities can be exploited in practise (Sadka, 2003 and Roll, Schwartz & Subrahmanyam, 2005). This is hardly surprising since liquidity, or the lack thereof, represents a further market friction. As such, the lack of liquidity is not a true anomaly since it increases risk, and therefore does not dispute the efficiency of a market, but serves further to highlight the complexity involved in exploiting apparent arbitrage opportunities (Sadka, 2003). By controlling for liquidity in the determination of arbitrage opportunities makes the test for efficiency more robust, as it presents another hurdle to overcome before real arbitrage opportunities may be claimed.

3 HYPOTHESES

The following hypotheses are based on the sub-problems defined in Subsection 1.2, in order to test the pricing efficiency of the South African equity index and futures markets.

Hypothesis 1

H_0 : The Satrix and TOP40 index closing prices are equivalent;

H_A : The Satrix and TOP40 index closing prices are different.

$$H_0: \mu_{\text{Satrix}} - \mu_{\text{TOP40}} = 0$$

$$H_A: \mu_{\text{Satrix}} - \mu_{\text{TOP40}} \neq 0$$

Hypothesis 2

H_0 : The magnitudes of all arbitrage opportunities are equal;

H_A : The magnitudes of all arbitrage opportunities are not all equal.

$$H_0: \mu_i = \mu_j \quad \forall i, j$$

$$H_A: \mu_i \neq \mu_j \text{ for at least one } i, j$$

Hypothesis 3

H_0 : The liquidity coefficients are insignificant in determining the magnitude of significant arbitrage opportunities;

H_A : The liquidity coefficients are significant in determining the magnitude of significant arbitrage opportunities.

$$H_0: \beta_{\text{liquidity}} = 0$$

$$H_A: \beta_{\text{liquidity}} \neq 0$$

Hypothesis 4

H_0 : The magnitudes of significant arbitrage opportunities during different time slots of the day are all equal;

H_A : The magnitudes of significant arbitrage opportunities during different time slots of the day are not all equal.

$$H_0: \mu_i = \mu_j \quad \forall i, j$$

$$H_A: \mu_i \neq \mu_j \text{ for at least one } i, j$$

4 RESEARCH METHODOLOGY

4.1 Data

4.1.1 Research Population

The population from which the data were sampled is every transaction of the TOP40 index futures contracts and its 41 underlying instruments representing the index, as well as the Satrix TOP40³. The population includes transactions prior to and after the period under consideration, as well as transactions within the period not captured by the sampling methodology.

4.1.2 Sampling Methodology

In order to gain the necessary insights into the intraday behaviour of index futures arbitrage, high frequency data were required. Therefore, the sampling approach captured a snapshot of both the JSE and SAFEX markets every five seconds. By considering all prices at an instant in time, rather than non-aligned tick data, is how arbitrage opportunities are identified in practise. This form of sampling leaves little room for additional transactions to occur within the five second snapshot window, and is therefore highly representative of intraday activity over the period under review. Sampling took place over the first quarter of 2006, providing a substantial amount of data, which also covers a full futures cycle from 19 December 2005 – 16 March 2006. Capturing data over a full cycle allows the characteristics of arbitrage opportunities to be better understood as time to maturity elapses. However, intraday prices on the futures closeout day (16 March 2006) were not considered due to anomalous pricing behaviour and congestion effects (Stoll & Whaley, 1996).

The data are comprised of the bid price, bid volume, ask price, ask volume, last price, last volume and daily cumulative volume of each instrument. In addition, each underlying instrument contains the consensus dividend forecast as supplied by Inet-Bridge.

³ Investec has two shares listed on the JSE, hence the reason there are 41 TOP40 index constituents.

4.1.3 Data Collection and Processing

Cadiz Financial Strategy Group collected and supplied the high-frequency intraday data necessary for this research. Along with intraday data, daily index constituent data was sourced from the JSE to calculate the proportional weights of each index constituent to recalculate an intraday index value. In addition to the fields mentioned above, the JIBAR risk free rate for each day was sourced from SAFEX, which in conjunction with the costs detailed in Section 2.3 provide sufficient data to determine the fair value of an index futures contract, and whether a profitable arbitrage opportunity exists.

Due to the vast quantity of data utilised in this research, all data were stored in a relational database, which allows for simple yet powerful extraction techniques utilising the structured query language (SQL). Data were stored using the MySQL database management system and were extracted and processed by various PHP programmes. This facilitated a simple web interface to specify run-time parameters and display results for further statistical analysis using the NCSS statistical software package.

4.1.4 Validity & Reliability

The sampled data is assumed both reliable and valid as it will be sourced from multiple commercial data vendors and crosschecked. The upper and lower bound fair value calculations detailed in Section 4.2, are determined using the formulae in Equations 3 and 4 and were validated by ensuring they produce results on the appropriate side of the cost-of-carry fair-value formula specified in Equation 2. Furthermore, arbitrage opportunities yielding large profits and with sufficient available liquidity, were analysed by performing a cash flow analysis from the entry trade through to futures closeout, in order to confirm the validity of the claimed arbitrage magnitude. Finally, the following filters were placed on the extracted results to ensure only realistic arbitrage opportunities were considered:

- No market trades occur on the JSE during an auction period, except for the closing trade, so intraday prices were only considered between the market open (09:00) and the closing auction (16:50);
- To ensure the correct index value was being calculated, only records with a calculated last index price within a 0.10% tolerance of the published index value were considered;
- Only arbitrage profits up to a maximum of 5% were considered.

4.2 Index Futures Fair-Value

This Section details the calculations involved to determine accurately whether an index futures arbitrage opportunity is present at any moment in time. This involves the calculation of the index value, as well as the index futures fair-value. Using the basic cost-of-carry formula as a base, upper and lower bounds are derived, within which a futures price may exist without yielding arbitrage opportunities with its underlying instrument. By so doing, the assumptions in the basic cost-of-carry formula are addressed to provide a better approximation of actual market conditions.

4.2.1 Index Calculation

Consistent with the methodology employed by Chung (1991), the value of the index is calculated using the appropriate price of its underlying shares, rather than using the published index price. This provides a more accurate measure of the true value of the index, since the published index is only updated every 15 seconds and therefore lags the price of the futures contract. Furthermore, this allows bid and ask index values to be calculated in order to determine the feasibility of arbitrage after considering the impact cost of crossing the spreads. As per FTSE (2005), this is done using the formula in Equation 1.

$$P(t) = \sum_{i=1}^n p_i \times s_i \times f_i / d \quad (1)$$

$P(t)$ = index value

p_i = price (last, bid or ask) of share i within index of n constituents

s_i = number of shares in issue for share i within index of n constituents

f_i = free float factor of share i within index of n constituents

d = index divisor

4.2.2 Cost-of-Carry

The most widely used form of the index futures cost-of-carry formula was first specified by Figlewski (1984), which holds true under the perfect market assumptions detailed in Section 2.2, but relaxes the assumption of a continuous and constant dividend yield. Even though Equation 2 is derived using a no-arbitrage argument between an index and its *forward* contract, it is commonly applied in the determination of *futures* prices as well (Figlewski, 1994; Collins & Fabozzi, 1999 and Hull, 2006).

$$F(t, T) = P(t)e^{r(T-t)} - D(t, T) \quad (2)$$

$F(t, T)$ = index futures fair-value price at time t ; maturing at time T

r = risk-free rate

$D(t, T)$ = sum of future value of dividends at time T , where D_i is paid on dividend pay

date i , between time t and T , such that $D(t, T) = \sum_{i=t}^T D_i e^{r(T-i)}$

4.2.3 Upper and Lower Bound Derivation

Using the methodology employed by Gay & Jung (1999) the following cost parameters are abstracted from the costs detailed in Section 2.3, to model the different values and characteristics of costs involved in index futures arbitrage, and by so doing relaxes the assumptions of no costs:

k_{PL} = variable cost of buying underlying securities representing the index;

k_{PS} = variable cost of selling underlying securities representing the index;

h_{PL} = holding cost for buying underlying securities representing the index;

h_{PS} = holding cost for selling underlying securities representing the index.

f_{PL} = fixed cost of buying underlying securities representing the index;

f_{PS} = fixed cost of selling underlying securities representing the index;

k_{FL} = variable cost of buying the index futures contract;

k_{FS} = variable cost of selling the index futures contract;

m_P = underlying short sale margin;

m_F = futures contract margin.

Assuming no restrictions on short sales (Cornell & French, 1983; Modest & Sundaresan, 1983 and Neal, 1996) and the full use of the proceeds of such sales, the upper and lower bounds are derived in Tables 2 and 3 respectively, using the cost parameters defined above in conjunction with the following parameters:

s = cost applied to r , such that $2s$ is the bid-ask spread between the borrowing and lending rate, thereby relaxing the equal borrowing and lending rates assumption;

n = number of futures contracts under consideration⁴.

⁴ Fixed costs are scaled to the value of the index, by dividing them by the futures contract gearing of R10 per index point. Furthermore, fixed costs become proportionally smaller the greater the number of futures contracts considered and are therefore divided by n .

Transaction	T	T
Long Underlying	$-P(t)$	$P(T)$
Variable Cost	$-k_{PL}P(t)$	$-k_{PS}P(t)$
Fixed Cost	$-f_{PL}/n$	$-f_{PS}/n$
Holding Cost	$-h_{PL}(T-t)P(t)$	
Dividends		$D(t,T)$
Short Futures		$F(t) - P(T)$
Variable Cost	$-k_{FS}F(t)$	
Margin	$-m_F F(t)$	$m_F F(t)e^{(r-s)(T-t)}$
Borrow @ t	$[P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n + F(t)(k_{FS} + m_F)]$	
Repay @ T	$-e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n + F(t)(k_{FS} + m_F)]$	
Total @ T	$F(t) + m_F F(t)e^{(r-s)(T-t)} - k_{PS}P(t) - f_{PS}/n + D(t,T)$	

Table 2: Cash Flows of Long Arbitrage

The cash flows at time T , are set to preclude positive payoffs from a long arbitrage transaction.

Solving for $F(t)$ yields the upper bound F_t^U , as specified in Equation 3.

$$F(t) + m_F F(t)e^{(r-s)(T-t)} - e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n + F(t)(k_{FS} + m_F)] - k_{PS}P(t) - f_{PS}/n + D(t,T) \leq 0$$

$$F(t) + m_F F(t)e^{(r-s)(T-t)} - F(t)(k_{FS} + m_F)e^{(r+s)(T-t)} \leq e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n] + k_{PS}P(t) + f_{PS}/n - D(t,T)$$

$$F(t)[1 + e^{(r+s)(T-t)}(m_F(e^{-2s(T-t)} - 1) - k_{FS})] \leq e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n] + k_{PS}P(t) + f_{PS}/n - D(t,T)$$

$$F(t) \leq \frac{e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n] + k_{PS}P(t) + f_{PS}/n - D(t,T)}{1 + e^{(r+s)(T-t)}(m_F(e^{-2s(T-t)} - 1) - k_{FS})} = F_t^U$$

$$F_t^U = \frac{e^{(r+s)(T-t)} [P(t)(1 + k_{PL} + h_{PL}(T-t)) + f_{PL}/n] + k_{PS}P(t) + f_{PS}/n - D(t,T)}{1 + e^{(r+s)(T-t)}(m_F(e^{-2s(T-t)} - 1) - k_{FS})} \quad (3)$$

Transaction	t	T
Short Underlying	$P(t)$	$-P(T)$
Variable Cost	$-k_{PS}P(t)$	$-k_{PL}P(t)$
Fixed Cost	$-f_{PS}/n$	$-f_{PL}/n$
Holding Cost	$-h_{PS}(T-t)P(t)$	
Margin	$-m_P P(t)$	$m_P P(t)e^{(r-s)(T-t)}$
Dividends		$-D(t, T)$
Long Futures		$P(T) - F(t)$
Variable Cost	$-k_{FL}F(t)$	
Margin	$-m_F F(t)$	$m_F F(t)e^{(r-s)(T-t)}$
Borrow ⁵ @ t	$[P(t)(-1 + k_{PS} + h_{PS}(T-t) + m_P) + f_{PS}/n + F(t)(k_{FL} + m_F)]$	
Repay @ T	$-e^{(r+s)(T-t)} [P(t)(-1 + k_{PS} + h_{PS}(T-t) + m_P) + f_{PS}/n + F(t)(k_{FL} + m_F)]$	
Total @ T	$-F(t) + m_F F(t)e^{(r-s)(T-t)} + P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) - f_{PL}/n - D(t, T)$	

Table 3: Cash Flows of Short Arbitrage

The cash flows at time T , are set to preclude positive payoffs from a short arbitrage transaction. Solving for $F(t)$ yields the lower bound F_t^L , as specified in Equation 4.

$$\begin{aligned}
& -F(t) + m_F F(t)e^{(r-s)(T-t)} - e^{(r+s)(T-t)} [P(t)(-1 + k_{PS} + h_{PS}(T-t) + m_P) + f_{PS}/n + F(t)(k_{FL} + m_F)] \\
& + P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) - f_{PL}/n - D(t, T) \leq 0 \\
& -F(t) + m_F F(t)e^{(r-s)(T-t)} - F(t)(k_{FL} + m_F)e^{(r+s)(T-t)} \\
& \leq e^{(r+s)(T-t)} [P(t)(-1 + k_{PS} + h_{PS}(T-t) + m_P) + f_{PS}/n] - P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) + f_{PL}/n + D(t, T) \\
& -F(t) [1 + e^{(r+s)(T-t)} (m_F (1 - e^{-2s(T-t)}) + k_{FL})] \\
& \leq -e^{(r+s)(T-t)} [P(t)(1 - k_{PS} - h_{PS}(T-t) - m_P) - f_{PS}/n] - P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) + f_{PL}/n + D(t, T) \\
& F(t) \geq \frac{e^{(r+s)(T-t)} [P(t)(1 - k_{PS} - h_{PS}(T-t) - m_P) - f_{PS}/n] + P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) - f_{PL}/n - D(t, T)}{1 + e^{(r+s)(T-t)} (m_F (1 - e^{-2s(T-t)}) + k_{FL})} = F_t^L
\end{aligned}$$

⁵ The margin required for short sales is 105% of the value of the trade. Therefore, this shortfall has to be borrowed.

$$F_t^L = \frac{e^{(r+s)(T-t)} [P(t)(1 - k_{PS} - h_{PS}(T-t) - m_P) - f_{PS}/n] + P(t)(m_P e^{(r-s)(T-t)} - k_{PL}) - f_{PL}/n - D(t, T)}{1 + e^{(r+s)(T-t)} (m_F(1 - e^{-2s(T-t)}) + k_{FL})} \quad (4)$$

A few points in the above derivation are worth noting. The short arbitrage lower bound is determined by both borrowing ($r + s$) and lending ($r - s$) rates, whereas without costs only the risk-free rate would be considered. This is caused by the margin requirement for selling the underlying shares short, which amounts to 105% of the sale proceeds. STRATE settlement fees are payable on transactions in the underlying shares, and are included as part of k_{PL} and k_{PS} between trade values of R200,000 and R1,000,000, otherwise they form part of f_{PL} and f_{PS} for long and short transactions respectively. The values substituted into the parameters will be based on the size of arbitrage opportunities available.

Every valid five-second price snapshot is used to calculate the relevant index value, $P(t)$. The TOP40 Satrix is also substituted into $P(t)$ to determine its degree of efficiency relative to the TOP40 index futures contract. $P(t)$ and the other parameters detailed in this section are then substituted into Equations 3 and 4, to determine whether an arbitrage opportunity exists. Section 4.3 details each of the arbitrage transactions types under consideration, while Section 4.4 describes the statistics collected once an arbitrage opportunity is discovered.

4.3 Arbitrage Transaction Types

In order to better approximate *ex ante* results in analysing an *ex post* dataset and to cater for the hidden impact cost of crossing the spreads, the scope of arbitrage transaction types is extended to consider the bid and ask price of the respective securities. Recall that a long arbitrage trade refers to going long on the underlying, thereby requiring a short sale on the futures contract, and vice versa for a short arbitrage trade. The last price and the appropriate bid and offer prices are therefore considered for both long and short arbitrage trades. The types of arbitrage transactions considered on the TOP40 futures contract against the index and Satrix, and the prices used to determine the opportunities are listed in Table 4.

Arbitrage Transaction	Index Futures Contract Price	Underlying Price
Index Long Last	TOP40 Last	Index Last
Index Short Last	TOP40 Last	Index Last
Index Long	TOP40 Bid	Index Ask
Index Short	TOP40 Ask	Index Bid
Satrix Long Last	TOP40 Last	Satrix Last
Satrix Short Last	TOP40 Last	Satrix Last
Satrix Long	TOP40 Bid	Satrix Ask
Satrix Short	TOP40 Ask	Satrix Bid

Table 4: Arbitrage Transaction Types

4.4 Arbitrage Statistics

For each of the arbitrage transaction types in Table 4, the statistics calculated are the proportion, instance, persistence, magnitude and liquidity of arbitrage opportunities. The statistics are all based on index futures price breaches of the upper and lower bounds as specified in Equations 3 and 4, and therefore include only positive arbitrage payoffs.

The proportion is a measure of the number of arbitrage samples divided by the entire sample of price snapshots. The instance is a count of the number of arbitrage periods. Any break in the continuity of an arbitrage opportunity results in a new instance when the next arbitrage opportunity is encountered. The time between the start and end of an arbitrage instance or period is measured in seconds as the persistence of that arbitrage opportunity. The magnitude of an arbitrage opportunity is the independent-of-price-level, normalised spread of the index futures prices above the upper bound or below the lower bound, as defined by MacKinlay & Ramaswamy (1988) in Equations 5 and 6.

$$X_t^L = (F - F_t^U) / P(t) \quad (5)$$

$$X_t^S = (F_t^L - F) / P(t) \quad (6)$$

F = actual futures contract price

X_t^L = normalised payoff arising from a long arbitrage transaction

X_t^S = normalised payoff arising from a short arbitrage transaction

The standard deviation of the magnitude of arbitrage opportunities is also supplied to give an indication of the distribution of the payoffs available. As mentioned in Chapter 2, the liquidity of arbitrage opportunities is defined as either a measure of the change in cumulative volume between the start and end of an arbitrage period, or the bid and ask volume of specified in terms of number of futures contracts.

Liquidity is defined as, “the speed and ease with which an asset can be converted into cash” (Bodie, Kane & Marcus, 2005: 1053), and as such is not always trivial to measure directly. One of the common ways to measure liquidity is to use the bid-ask spread of the market orders or a combination of the last trade price and the current bid-ask mid-price (Sadka, 2003 and Roll, Schwartz & Subrahmanyam, 2005). Since the bid and ask prices are already factored into the determination of various types of arbitrage, a simpler measure of liquidity is used, which is the volume available on the respective instruments’ bid or ask prices, or the actual change in traded volume over an arbitrage period. These measures will be used to determine whether sufficient liquidity exists to fulfil the claimed arbitrage opportunities and whether the magnitude of arbitrage opportunities is determined by liquidity.

4.5 Data Analysis

Each of the following Subsections outline the analysis used to test the hypotheses listed in Chapter 3.

4.5.1 Satrix and Index Closing Price Equivalence

The rationale for including Satrix in tests of index futures arbitrage is that holdings in Satrix may be converted into the underlying index constituents and vice-versa, thereby providing a mechanism for the Satrix price to remain consistent with the index (Poterba & Shoven, 2002). The closing prices of the underlying shares and Satrix are usually used to do this conversion (Brown, M., personal communication, April 18, 2006). Therefore, in order to ensure that Satrix was equivalent to the index over the period under consideration, the paired mean difference between the Satrix TOP40 and the closing price of the TOP40 index is tested for significance. Finding the difference to be *insignificant* allows Satrix to be used as a proxy for the underlying instrument and for its efficiency relative to the index futures contract to be tested.

4.5.2 Significant Arbitrage Transaction Types

The object of this test is to identify which of the arbitrage transaction types detailed in Table 4 provide significant opportunities, thereby justifying further analysis. Due to the large sample size under consideration and the relative infrequency of arbitrage opportunities on offer, testing for statistical significance of magnitudes greater than zero is unlikely to add further insight. Instead, an ANOVA F-test could be used to determine whether the magnitude of any arbitrage transaction types are significantly different, in which case, confidence intervals will be used to identify the arbitrage transaction types yielding the most profitable opportunities. The F-test will include *all* price snapshots (including negative magnitudes) to calculate a total arbitrage magnitude. This is so the proportion of arbitrage opportunities is factored into the mean and to compare means based on equivalent samples sizes.

However, the F-test is likely to suffer from the same shortcomings as the test for magnitudes greater than zero. A simpler measure will therefore also be used to determine the economically significant arbitrage transaction types worthy of further analysis, which is the proportion multiplied by the magnitude of each arbitrage transaction type.

4.5.3 Liquidity and Arbitrage Magnitude

Once significant arbitrage transaction types have been identified, these are further analysed to determine whether sufficient liquidity exists to fulfil the apparent arbitrage opportunities. This will be done by performing a regression of the various liquidity measures described in Section 4.7 against the magnitude of the arbitrage opportunity, similar to the methodology employed by Merrick (1987). If the liquidity coefficient is significant in determining the arbitrage magnitude, the regression equation will be used to understand the liquidity available for various levels of arbitrage profit.

The first liquidity measure uses the volume available at the bid or ask price at a particular point in time and compares it to the magnitude of the arbitrage opportunity available at that same point in time. This gives an indication of the *ex ante* liquidity available, providing an indication of what an arbitrageur is likely to achieve in practise. However, this measure is not entirely accurate since each price snapshot considers the available bid and offer volume independently of any other price snapshots. This has the effect of counting the same available liquidity multiple times thereby ignoring the effect an arbitrage transaction would have on liquidity. This is important,

since the purpose of arbitrage is to remove any liquidity available at an arbitrage price. As such, this measure is likely to overstate the available liquidity. However, this measure of liquidity is useful when calculating arbitrage opportunities using the bid or ask price. The regression used to test the significance of the futures and underlying index liquidity factors in relation to the spot arbitrage magnitude, is defined in Equation 7.

$$X_{spot} = \alpha + \beta_{liquidity_{index}} liquidity_{index} + \beta_{liquidity_{futures}} liquidity_{futures} + \varepsilon \quad (7)$$

The second liquidity measure uses the change in cumulative volume over an arbitrage instance or period and compares it to the average magnitude of the arbitrage opportunity over the same period. This is an *ex post* measure of liquidity and ignores the bid and ask volume available at the price at which arbitrage opportunities are being determined. This has the advantage that it considers actual volume traded over a period, thereby overcoming the limitation of the former liquidity measure. Furthermore, this measure may be used for arbitrage opportunities determined with the last as well as the bid and ask prices. However, this measure's strength is also its weakness in that it may understate the liquidity available, since it ignores the available volume and any increases in the actual volume due to arbitrage transactions. The regression used to test the significance of each factor in relation to the period arbitrage magnitude, is defined in Equation 8.

$$X_{period} = \alpha + \beta_{liquidity_{index}} liquidity_{index} + \beta_{liquidity_{futures}} liquidity_{futures} + \varepsilon \quad (8)$$

4.5.4 Predicting Arbitrage Magnitude

In order to determine whether the magnitude of arbitrage opportunities may be predicted by certain factors, the time to maturity, risk-free rate, persistence and liquidity of each arbitrage opportunity under consideration is incorporated into a stepwise regression. Factors with a significance level of 5% or less will be included in the regression, while factors with a significance level greater than 10% will be excluded. The results of this test will provide insights into whether the magnitude of arbitrage opportunities may be predetermined by other factors. However, this test will not indicate when arbitrage opportunities are most likely.

In order to test whether anomalies exist in the occurrence of intraday index futures arbitrage, each arbitrage statistic for the period under review is categorised according to a 30-minute time slot over the trading day, to determine whether any pattern emerges in the intraday behaviour of arbitrage opportunities. This is analogous to testing for the January effect in the returns of small

firms. Specifically, an ANOVA F-test is conducted to establish whether significant differences exist between the magnitudes of arbitrage opportunities during the course of the day. Due to the large sample and low standard error of each mean, a significance level of 1 % will be used to test for differences between the 30-minute time slot means.

5 PRESENTATION OF RESULTS

5.1 Satrix and Index Closing Price Equivalence

Before considering the arbitrage results of the index futures with Satrix, the difference in closing price between Satrix and its index was tested for statistical significance. Finding the difference to be *insignificant* is indicative that the Satrix closing price is equivalent to the index. This was indeed found to be the case at the 1%, 5% and 10% level of significance as presented in this Section. As such, both the index level and the Satrix price were substituted into $P(t)$ in Equations 3 and 4, allowing all of the arbitrage transaction types presented in Table 4 to be considered.

$$n = 51$$

$$\bar{X}_{\text{Satrix-TOP40}} = 3.28$$

$$S_{\text{Satrix-TOP40}} = 65.76$$

$$p\text{-value} = 0.28$$

5.2 Significant Arbitrage Transactions Types

5.2.1 R100 million Assumption

The first set of summary statistics results are presented using the initial assumption of an arbitrage book worth R100 million. A futures contract gearing of R10 per index point results in excess of 500 futures contracts being considered, which has the effect of largely eliminating the fixed cost components of the upper and lower bounds in Equations 1 and 2. Under this assumption and using 249 260 intraday price snapshots over the period under review, produced the summary statistics presented in Table 5, for each of the arbitrage transaction types.

The liquidity statistics supplied in Table 5 represent the average change in cumulative volume for identified arbitrage opportunities, in terms of the number of futures contracts. Each futures contract is worth 10 000 Satrix shares. However, since there is no easy way to determine the change in cumulative volume for the TOP40 index in terms of futures contracts, these measures are not calculated. As can be seen from the results, this should not hamper further analysis as only arbitrage opportunities involving Satrix occur with any substantial degree of *economic significance*.

Statistic	Index Long Last	Index Short Last	Index Long Index	Index Short Index	Satrix Long Last	Satrix Short Last	Satrix Long	Satrix Short
Count	1 255	555	5	2	696	156 739	3	69 996
Proportion	0.01	0.00	0.00	0.00	0.00	0.63	0.00	0.28
Instance	34	39	3	1	17	900	1	2 719
Persistence	182	71	3	5	221	896	10	131
Magnitude	0.11%	0.10%	0.69%	0.05%	0.33%	0.26%	0.17%	0.13%
Standard Deviation	0.09%	0.10%	0.70%	0.00%	0.39%	0.21%	0.01%	0.13%
Satrix Liquidity					0.00	5.33	0.00	0.53
Futures Liquidity	24.24	53.1	0.33	0.00	154.65	409.45	7.00	57.15
Economic Significance	0.00	0.00	0.00	0.00	0.00	0.16	0.00	0.04

Table 5: Arbitrage Statistics at R100 million Exposure

The ANOVA F-test results in Tables 6 and 7 indicate that the arbitrage magnitudes for *all* price snapshots (not just positive arbitrage price snapshots) of the various arbitrage transaction types are not all equal, providing sufficient evidence for the second null hypothesis to be rejected. Furthermore, the 99.5%⁶ confidence intervals for each of the transaction type magnitudes are consistent with the results in Table 5, in that the Satrix Short Last and Satrix Short arbitrage transaction types have the largest of the total magnitudes. Subsequent analysis will therefore be restricted to these arbitrage transaction types. For further details on this result, see the Tukey-Kramer's Simultaneous Confidence Intervals for Multiple Comparisons of All Arbitrage Transaction Type Pairs in Appendix 1.

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Probability Level	Power ($\alpha=0.005$)
A: Type	7	546103	78015	978584	0.000000	1.000000
S	1957746	156076	0.07972203			
Total (Adjusted)	1957753	702178				
Total	1957754					

Table 6: Analysis of Variance of Arbitrage Transaction Type Magnitudes

⁶ This high degree of confidence is chosen due to the large sample size and resultant small standard error.

Statistic	Index Long Last	Index Short Last	Index Long	Index Short	Satrix Long Last	Satrix Short Last	Satrix Long	Satrix Short
Count	247581	247581	241656	241656	247581	247581	242059	242059
Mean	-0.751%	-0.511%	-0.940%	-1.562%	-1.357%	0.093%	-0.667%	-0.119%
Std Error	0.000567	0.000567	0.000574	0.000574	0.000567	0.000567	0.000574	0.000574
Lower CI	-0.753%	-0.512%	-0.942%	-1.564%	-1.358%	0.091%	-0.668%	-0.121%
Upper CI	-0.750%	-0.509%	-0.939%	-1.561%	-1.355%	0.094%	-0.665%	-0.117%

Table 7: Confidence Intervals of Arbitrage Transaction Type Magnitudes

5.2.2 Single Contract Assumption

While substantial liquidity is available on the futures contract, there is nowhere near the level of liquidity implied by the R100 million arbitrage book size assumption used to determine the Satrix arbitrage upper and lower bounds (see highlighted columns in Table 5). Once crossing the bid and ask spreads is taken into account (Short Satrix arbitrage transaction), there appears to be insufficient liquidity to execute these trades at all. For this reason, a more conservative approach is taken in determining whether an arbitrage opportunity exists, by recalculating the same statistics under the assumption of fulfilling only a single TOP40 futures contract worth of volume.

Under these assumptions, the fixed cost components in the upper and lower bound equations were affected in two ways. Firstly, by only considering a single contract, the full value of the fixed costs were included in the bound calculations, as they are no longer divided by the number of contracts. Secondly, the STRATE component of the fixed costs was reduced to R10.92 (as per Table 1), as the total value of the trade was less than R200 000.

The results from this test are presented in Table 8, which show that any possibility of arbitrage on the TOP40 index of constituent stocks was eliminated. This is due to the large rise in fixed costs being levied against each underlying stock representing the index. Since Satrix is a single security, it is not penalised by a large increase in fixed costs. As such, the overall results for Satrix arbitrage opportunities are largely unchanged, except for an increase in the average Satrix liquidity of the Short Satrix arbitrage transactions.

Statistic	Index Long Last	Index Short Last	Index Long Index	Index Short Index	Satrix Long Last	Satrix Short Last	Satrix Long	Satrix Short
Count	0	0	0	0	481	127 647	0	35 334
Proportion	0.00	0.00	0.00	0.00	0.00	0.51	0.00	0.14
Instance	0	0	0	0	13	1 005	0	1 366
Persistence	0	0	0	0	195	646	0	126
Magnitude	0.00%	0.00%	0.00%	0.00%	0.36%	0.22%	0.00%	0.12%
Standard Deviation	0.00%	0.00%	0.00%	0.00%	0.42%	0.21%	0.00%	0.15%
Satrix Liquidity					0.00	4.36	0.00	0.73
Futures Liquidity	0.00	0.00	0.00	0.00	159.08	293.79	0.00	62.88
Economic Significance	0.00	0.00	0.00	0.00	0.00	0.11	0.00	0.02

Table 8: Arbitrage Statistics at Single Futures Contract Exposure

5.3 Liquidity and Arbitrage Magnitude

The regression results of each of the liquidity measures against the magnitude of the Satrix Short Last and Satrix Short arbitrage transaction types are presented in this Section. Since the object of this test is to determine the level of liquidity available at various arbitrage magnitudes, the initial assumption of an arbitrage book of R100 million is used.

5.3.1 *Ex Ante* Liquidity

The *ex ante* liquidity measure uses the volume available at the bid or ask price at a particular point in time and compares it to the magnitude of the arbitrage opportunity available at that same point in time. It is therefore only suitable as a liquidity measure for the Satrix Short arbitrage transaction type, as these opportunities are derived using the bid and ask price. The regression resulted in an adjusted R^2 of only 0.0074 with the coefficients detailed in Table 9. All coefficients were found to be significant, with a small inverse relationship on the liquidity of the Satrix, and an even smaller positive relationship on the liquidity of the futures contract in relation to the magnitude of the arbitrage opportunities. Substituting the mean liquidity values into the regression equation yields an arbitrage magnitude of 0.1257%; essentially unchanged from the average magnitude in Table 5.

Independent Variable	Regression Coefficient	Standard Error	t-value	p-value	Mean	Standard Deviation
Intercept	0.1287	0.0007	184.803	0.0000		
liquidity _{Satrix bid}	-0.0022	0.0001	-22.665	0.0000	1.965798	5.340774
liquidity _{futures ask}	0.0002	0.0001	2.993	0.0028	6.691639	7.705352

Table 9: Satrix Short *Ex Ante* Liquidity Regression

5.3.2 *Ex Post* Liquidity

The *ex post* liquidity measure uses the change in cumulative volume over an arbitrage instance or period and compares it to the average magnitude of the arbitrage opportunity over the same period. Since this is a measure of actual volume, it is independent of the price used to determine the arbitrage opportunity and can be used for both the Satrix Short Last and Satrix Short arbitrage transaction types.

The Satrix Short Last regression resulted in an adjusted R^2 of 0.3343, 0.3147 of which was contributed by the futures liquidity independent variable. The coefficients are detailed in Table 10 and were all found to be significant at the 5% level. The Satrix liquidity measure was the least significant of the regression coefficients with a p-value of 0.0404. Substituting the mean liquidity values into the regression equation yields an arbitrage magnitude of 0.1012%, which is less than half the average magnitude specified in Table 5.

Independent Variable	Regression Coefficient	Standard Error	t-value	p-value	Mean	Standard Deviation
Intercept	0.0656	0.0041	16.136	0.0000		
liquidity _{Satrix}	0.0001457	0.0001	2.052	0.0404	5.319079	52.84537
liquidity _{futures}	0.00008497	0.0000	20.650	0.0000	409.7054	911.999

Table 10: Satrix Short Last *Ex Post* Liquidity Regression

The Satrix Short regression resulted in an adjusted R^2 of only 0.0541. The coefficients are detailed in Table 11, with only the intercept and futures liquidity coefficients found to be significant. As a result, the regression equation was adjusted to exclude the Satrix liquidity measure. By substituting the mean futures liquidity into the regression equation yields an arbitrage magnitude of 0.0748%, which is approximately half the average magnitude specified in Table 5.

Independent Variable	Regression Coefficient	Standard Error	t-value	p-value	Mean	Standard Deviation
Intercept	0.0649	0.0021	30.312	0.0000		
liquidity _{Satrix}	0.0001291	0.0002	0.662	0.5078	0.527171	10.38555
liquidity _{futures}	0.0001589	0.0000	12.514	0.0000	56.92523	159.3956

Table 11: Satrix Short *Ex Post* Liquidity Regression

5.4 Predicting Arbitrage Magnitude

5.4.1 Arbitrage Magnitude Factors

Performing a stepwise regression on the independent variables of time to maturity, risk-free rate, persistence and the Satrix and futures ex-post liquidity for both the Satrix Short Last and Satrix Short arbitrage transaction types, confirmed the liquidity factors (on both the futures contract and underlying Satrix instrument) as the only significant factors for determining the Satrix Short Last magnitude. As such, the regression coefficients and statistics are identical to those already presented in Table 10.

However, after crossing the bid and ask price for the Satrix Short arbitrage transaction type, the Satrix liquidity factor falls out of the regression as per the previous result in Table 11, and is replaced by the persistence factor, with an adjusted R^2 of 0.0557. Table 12 details the significant regression coefficients and by substituting the mean futures liquidity and persistence into the regression equation, produces an arbitrage magnitude of 0.0740%, which is fractionally less than Satrix Short *Ex Post* Liquidity Regression result.

Independent Variable	Regression Coefficient	Standard Error	t-value	p-value	Mean	Standard Deviation
Intercept	0.0655	0.0022	30.430	0.0000		
persistence	-0.000008798	0.0000	-2.273	0.0231	161.7673	598.1928
liquidity _{futures}	0.0001759	0.0000	12.108	0.0000	56.92523	159.3956

Table 12: Satrix Short Regression

5.4.2 Intraday Behaviour

The results in the previous sections indicate liquidity is a significant factor with the mean underlying Satrix liquidity restricting the magnitude of potential arbitrage opportunities. As such, the more conservative assumption of executing only a single futures contract worth of

volume, rather than the initial assumption of R100 million worth of volume, will be used in the analysis of any intraday anomalies. The null hypothesis is that the intraday magnitudes of arbitrage opportunities are independent of the time at which they occur. More generally, the arbitrage statistics described in Section 4.7 are presented in Figures 1 through 8 for each 30-minute time slot over the period under review for the Satrix Long Last, Satrix Short Last and Satrix Short transaction types.

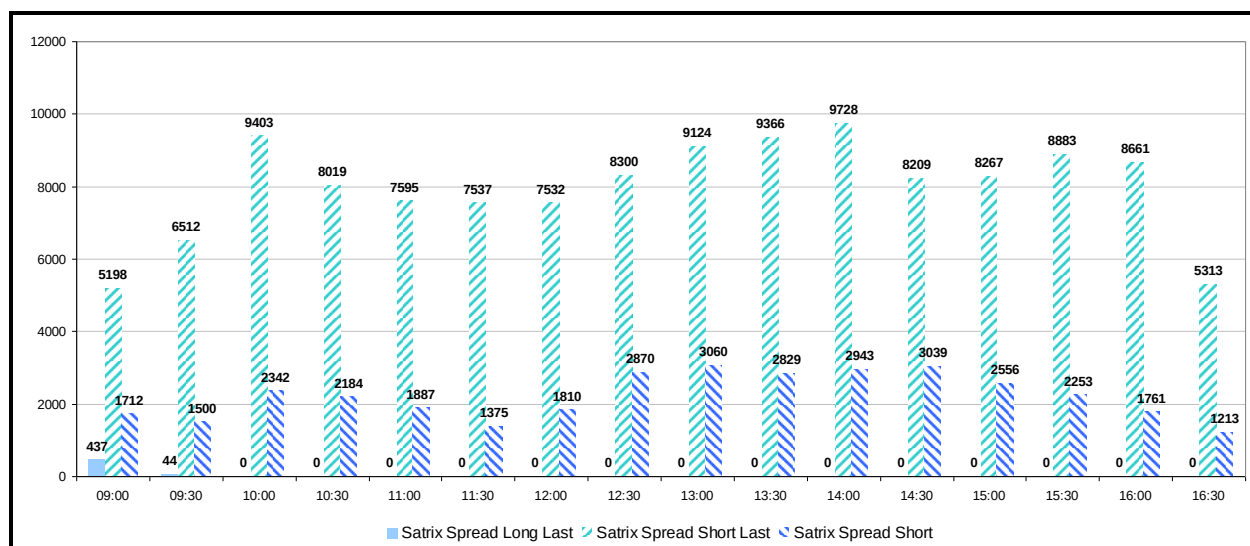


Figure 1: Intraday Arbitrage Count

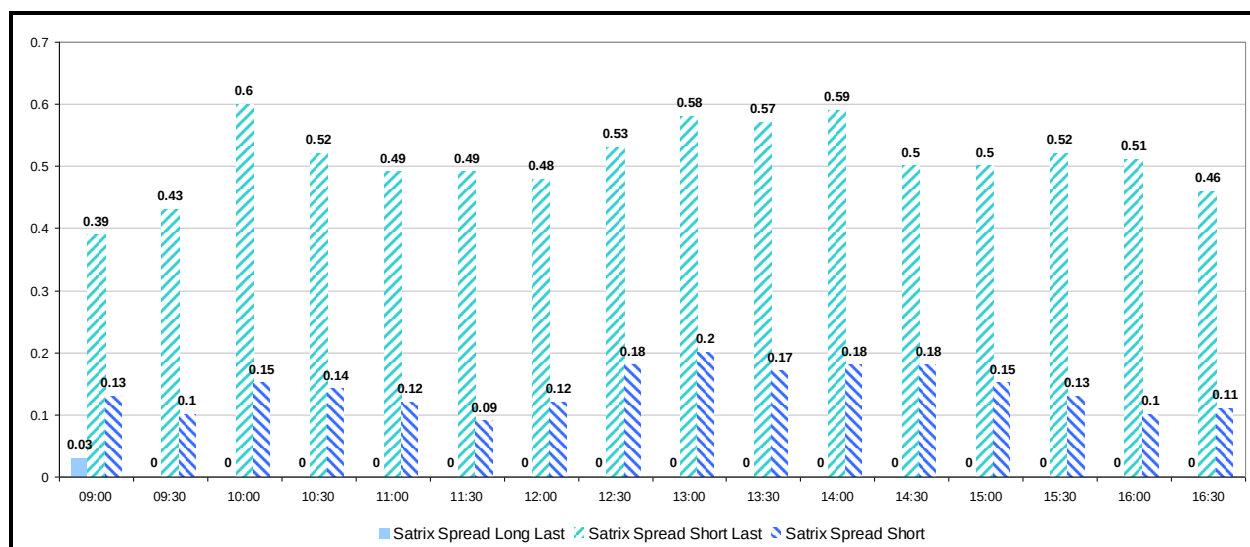


Figure 2: Intraday Arbitrage Proportion

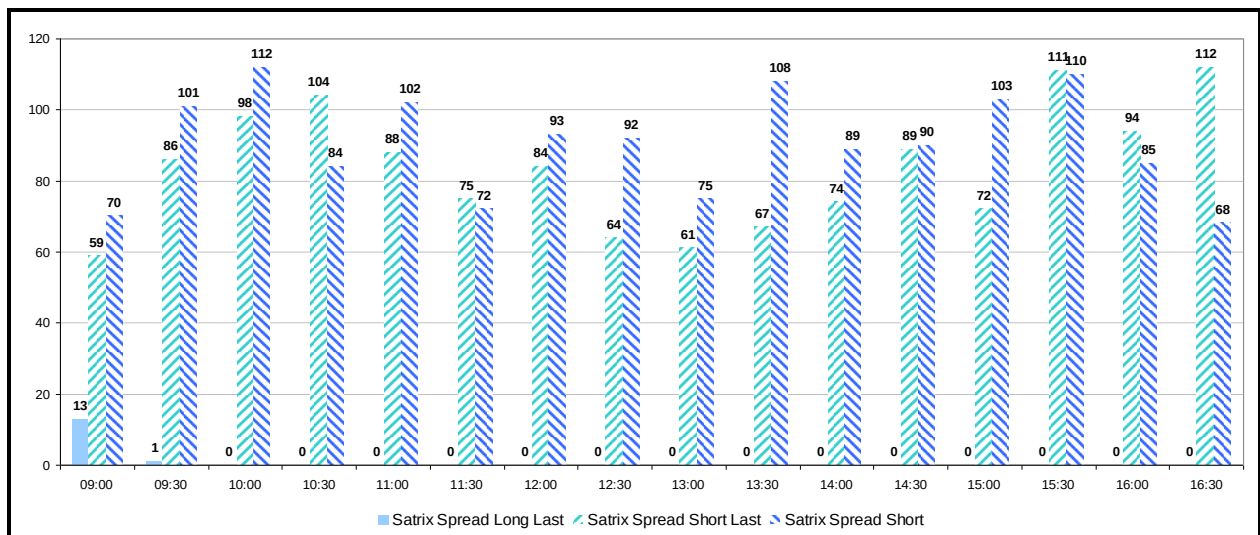


Figure 3: Intraday Arbitrage Instance

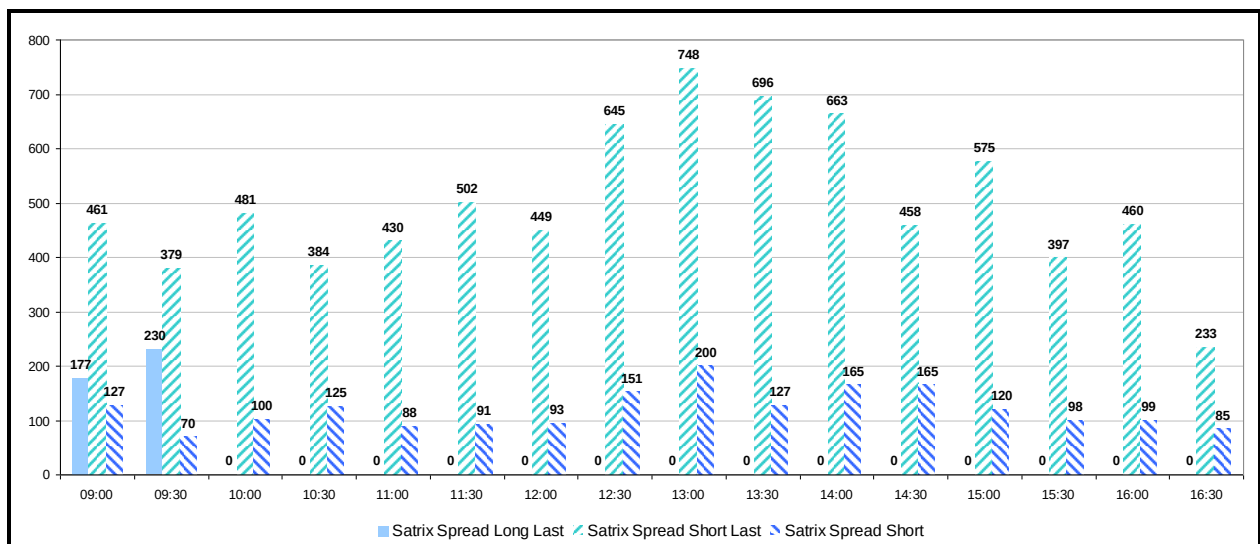


Figure 4: Intraday Arbitrage Persistence

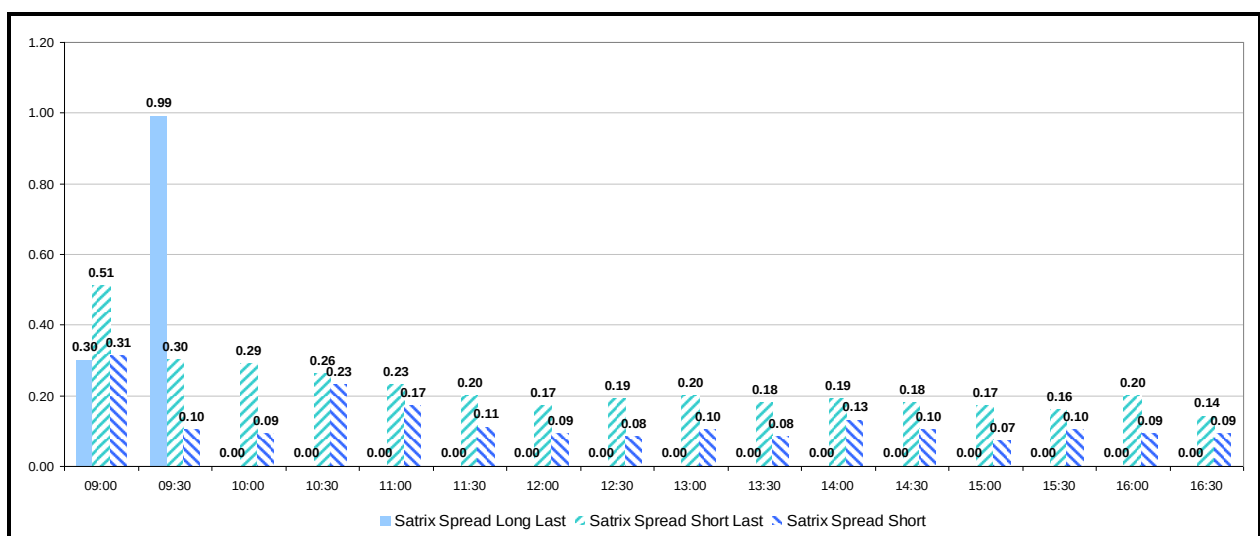


Figure 5: Intraday Arbitrage Magnitude

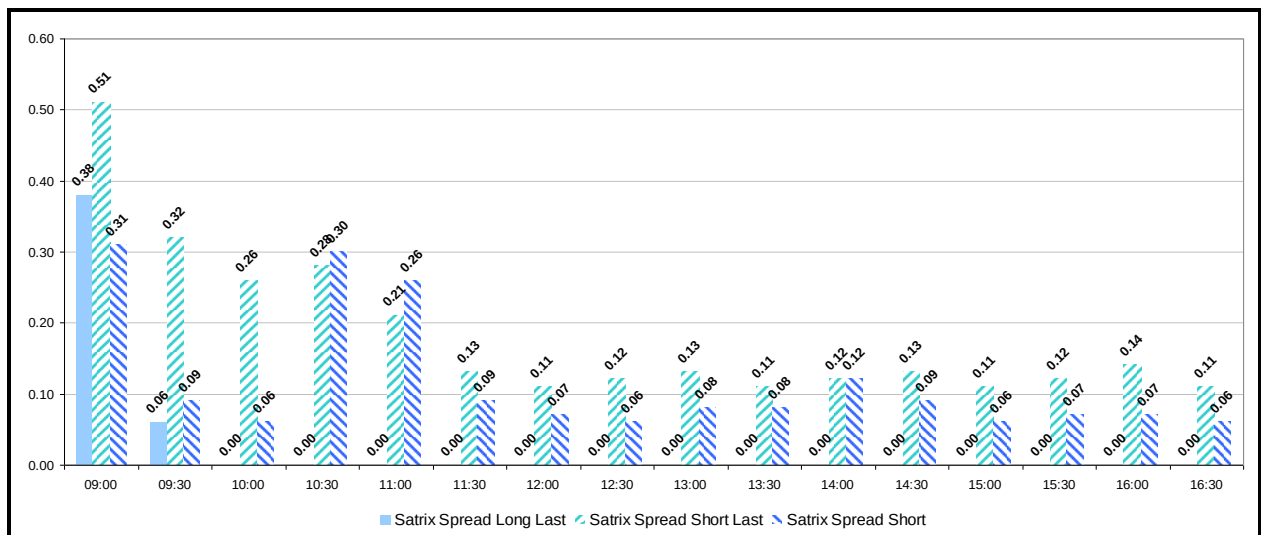


Figure 6: Intraday Arbitrage Standard Deviation

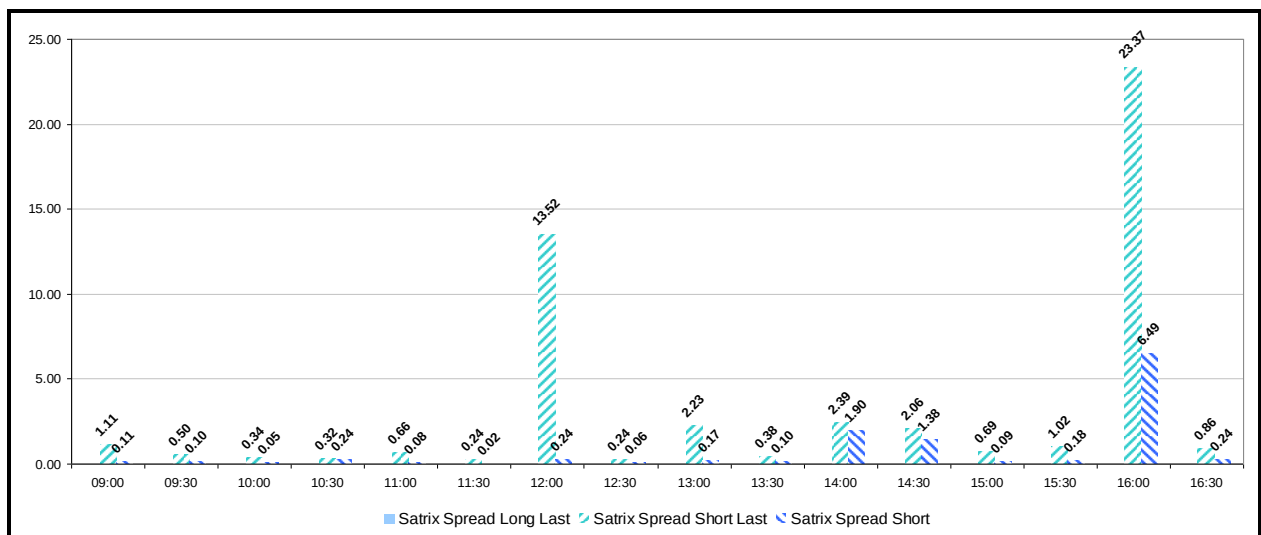


Figure 7: Intraday Arbitrage Satrix Liquidity

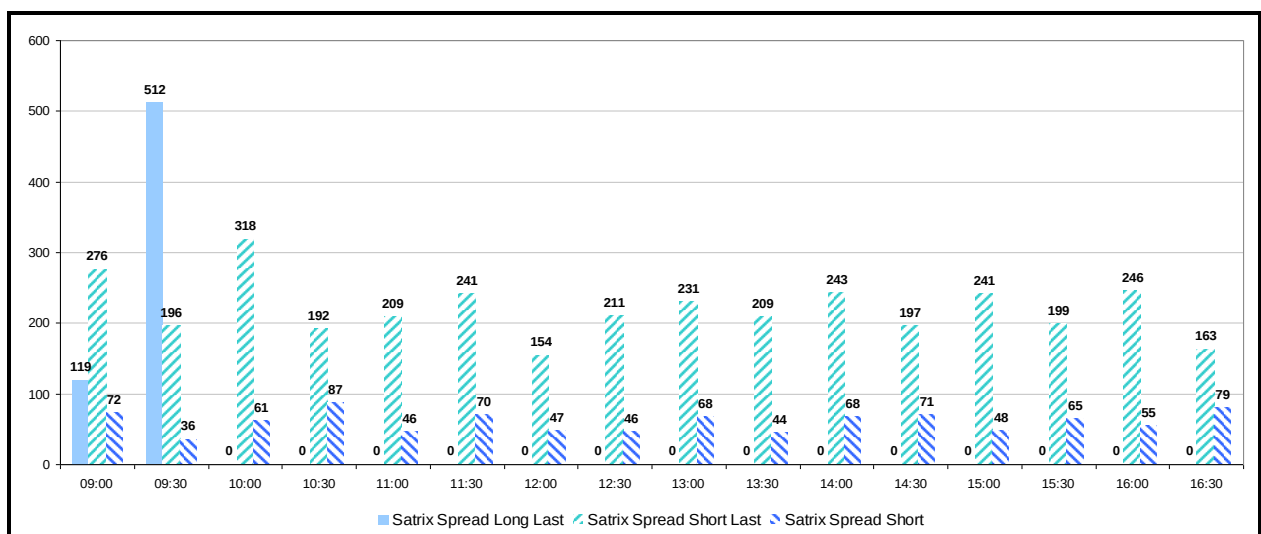


Figure 8: Intraday Arbitrage Futures Liquidity

The reason for including the Satrix Long Last intraday results becomes clear in the figures, with all the opportunities occurring before 10:00 over the entire period under review. While this transaction type was excluded from further analysis due to its overall low proportion of occurrence, it does appear to offer opportunities worth considering during the first hour of trade. However, since no arbitrage opportunities are present once the impact of crossing the bid-ask spreads is included (as per Table 8), any possibility of implementing a Satrix Short arbitrage transaction disappears. For the same reason, the only arbitrage transaction type of real interest is the Satrix Short, which is analysed using the ANOVA F-test and Tukey-Kramer's Simultaneous Confidence Intervals for Multiple Comparisons of All 30-minute Pairs. Table 13 details the ANOVA F-test results and confirms that *not* all 30-minute intraday magnitudes are equal, providing sufficient evidence to reject the fourth null hypothesis.

Source Term	DF	Sum of Squares	Mean Square	F-Ratio	Probability Level	Power ($\alpha=0.005$)
A: Bin30	15	119.669	7.977919	485.19	0.000000	1.000000
S	69880	1150.67	0.0164428			
Total (Adjusted)	69995	1270.34				
Total	69996					

Table 13: Analysis of Variance of Intraday Satrix Short Magnitude

Table 14 specifies the 99.5% confidence intervals for each of the 30-minute magnitudes, with the 09:00, 10:30, 11:30 and 14:00 time periods offering the most lucrative opportunities, while the last 30-minute period of the day offers the least lucrative arbitrage opportunities. The full results for the Tukey-Kramer's Simultaneous Confidence Intervals for Multiple Comparisons of All Satrix Short 30-minute Pairs are detailed in Appendix 2.

Statistic	Count	Mean	Standard Error	Lower CI	Upper CI
09:00	2329	0.3068%	0.002657	0.2993%	0.3142%
09:30	3003	0.1189%	0.002340	0.1123%	0.1254%
10:00	4143	0.1184%	0.001992	0.1128%	0.1240%
10:30	4042	0.1919%	0.002017	0.1862%	0.1975%
11:00	3823	0.1502%	0.002074	0.1444%	0.1560%
11:30	3788	0.0955%	0.002083	0.0896%	0.1013%
12:00	3866	0.1075%	0.002062	0.1017%	0.1133%
12:30	5210	0.1113%	0.001777	0.1063%	0.1163%
13:00	5354	0.1272%	0.001752	0.1222%	0.1321%
13:30	5497	0.1131%	0.001730	0.1083%	0.1180%
14:00	4975	0.1485%	0.001818	0.1434%	0.1536%
14:30	5763	0.1196%	0.001689	0.1149%	0.1244%
15:00	5759	0.0964%	0.001690	0.0917%	0.1012%
15:30	5011	0.1070%	0.001811	0.1019%	0.1121%
16:00	4465	0.0966%	0.001919	0.0912%	0.1019%
16:30	2968	0.0942%	0.002354	0.0876%	0.1008%

Table 14: Confidence Intervals of Intraday Satrix Short Magnitude

6 INTERPRETATION OF RESULTS

The results from Chapter 5 indicate that South Africa's TOP40 equity index, as determined by its constituent shares, and the associated TOP40 futures market are priced correctly relative to each other, and that the fundamental Law of One Price is actively enforced by arbitrageurs. As such, these markets may be deemed efficient. However, when considering the Satrix TOP40 product as a proxy for holding each of the TOP40 index constituents in their appropriate weights, the case for an efficient market between Satrix and the TOP40 futures requires further analysis in order to reach a conclusion.

6.1 Satrix Intraday Premium

The TOP40 index and its futures contract are priced correctly relative to each other, since no economically significant results were found in the period under review. As such, the economically significant arbitrage opportunities found between Satrix and the TOP40 index futures contract are indicative of the premium at which Satrix trades relative to the index it represents. This premium may be indicative of market inefficiency, particularly as there are no obvious regulatory restrictions such as short sale restrictions, which could otherwise have been responsible for keeping the Satrix price artificially high.

However, there are other frictions around the Satrix product that may be responsible for its premium, such as the minimum consideration of R1 million required to redeem Satrix shares for TOP40 index constituents (Satrix, 2006). This equates to just over five futures contracts, which corresponds closely to the Satrix Short Last liquidity statistics in Tables 5 and 8. This suggests that inefficiency is restricted only to low levels of volume.

This is supportive of another reason for the premium at which Satrix trades relative to its index; namely a *convenience premium* (Fama, E., personal communication, 25 May 2006). One of Satrix's main benefits is for retail investors who desire to own the performance and diversification benefits of an index without having to incur the large transaction costs from buying small quantities in each index constituent, nor needing to rebalance their portfolios to track the index perfectly. These benefits among others like scrip lending, which exchange-traded funds do to lower costs, provide a number of reasons for why a convenience premium may exist at low levels of volume.

6.2 Liquidity

Liquidity is a vast topic with large amounts of research devoted to understanding its effect on financial markets (Amihud & Mendelson, 1986; Shleifer & Vishny, 1997; Stoll, 2000; Sadka, 2003; Aitken & Siow, 2004 and Roll, Schwartz & Subrahmanyam, 2005). It is therefore hardly surprising that it has played such a large role in understanding the behaviour of arbitrage opportunities between the Satrix TOP40 exchange-traded fund and its futures contract equivalent.

Liquidity was measured explicitly in two ways and catered for implicitly by factoring in the spread between the bid and ask prices of the respective instruments. The implicit measure provides a quantifiable difference in the expected magnitude of arbitrage opportunities by comparing the Satrix Short Last and Satrix Short magnitudes. Under the R100 million assumption the difference was 0.13% and under the single futures contract assumption it was 0.10%. These measures, while similar but not identical to those calculated by Stoll (2000) and Aitken & Siow (2004), compare very favourably with the most efficient of international markets; namely the New York Stock Exchange and Euronext Paris, confirming South Africa's reputation as an advanced financial market.

The explicit measures of liquidity on the Satrix Short arbitrage transaction between the R100 million and single futures contract assumptions increased, when the trend for the other statistics for both Satrix Short Last and Satrix Short transaction types was to decrease under the more conservative assumption. This is explained by previously marginal arbitrage opportunities with low liquidity being excluded from the arbitrage results, thereby increasing the average liquidity of the remaining arbitrage opportunities.

The explicit liquidity measures that were regressed against the magnitudes of various arbitrage transaction types yielded some interesting results. Except for the *ex post* liquidity against the Satrix Short Last magnitude, the adjusted R^2 statistics were too small to accurately determine the magnitude of an arbitrage opportunity from either the *ex ante* or *ex post* liquidity measures, even though the regression coefficients were found to be significant. The significant regression coefficients are attributed to the large sample size and consequential small standard error. The order of magnitudes larger adjusted R^2 on the Satrix Short Last arbitrage transaction type is attributed to the fact that the implicit measure of liquidity was not isolated as an independent variable or factored into the determination of arbitrage magnitude. As such, the explicit liquidity

measures were only able to explain arbitrage magnitude before implicit liquidity had been taken into account. This indicates that the implicit measure of liquidity is a better way to measure liquidity than the explicit *ex ante* and *ex post* measures of liquidity.

The liquidity regression also produced a somewhat counter-intuitive positive relationship between the futures liquidity and arbitrage magnitude, which occurred in all three of the liquidity regressions. As discussed in Section 2.5, the lack of liquidity is a market friction, thereby providing opportunities for larger pricing discrepancies. As a result, a negative relationship on the futures liquidity coefficient could be expected, as with the Satrix liquidity coefficient, indicating that as liquidity decreased, increasingly lucrative arbitrage opportunities would become available. Since the opposite was found to be the case, an alternative explanation is put forth. Because the futures liquidity is much larger than the Satrix liquidity as detailed in Tables 5 and 8 to 11, a friction would occur on the Satrix liquidity long before futures liquidity could affect the fair price determination process. However, this does not explain the more significant positive futures liquidity coefficient. Increases in futures liquidity may coincide with larger Satrix arbitrage opportunities due to arbitrage activity between the TOP40 index of constituent shares and the futures contract. This activity would increase futures volume and adjust the futures and index prices without an automatic adjustment in the Satrix price, thereby causing a price discrepancy and an arbitrage opportunity. This result is supported by research conducted by Merrick (1987) into index futures arbitrage trades causing additional volume on the NYSE.

6.3 Intraday Behaviour

The stepwise regression performed on all independent variables only found the persistence to be significant along with the already significant futures liquidity. The time-to-maturity factor, which was found to be a significant factor by MacKinlay & Ramaswamy (1988) and Gay & Jung (1999), was not found to be significant in the sample under consideration. This is probably due to considering only the near dated futures contract over a single period, whereas previous research considered futures contracts with multiple expiry dates over multiple periods. The positive loading on the persistence factor seems consistent with rational market behaviour, since the longer an arbitrage opportunity lasts the less risk-free profit it should yield.

The periods during the day yielding the most profitable arbitrage opportunities are the first half and hour from 09:00 to 09:30, then again for an hour from 10:30 to 11:30 and finally just after

lunch from 14:00 to 14:30. The early morning slot seems to make sense, as it may take a while for market participants to agree on price and for investors' orders to start reaching the market. However, the other time slots do not seem to have any particular reason for presenting greater opportunities than any other time slot. What is consistent, except for the 14:00 period, is that each of the high-magnitude periods corresponds to a low Satrix liquidity period. Therefore, for arbitrageurs wishing to exploit the Satrix intraday premium, the best opportunity would be from 14:00 to 14:30, where an additional, almost risk-free return of 0.1485%, with almost two futures contracts worth of liquidity lasting three minutes can be expected.

The implication for pure long-only investors in the Satrix TOP40 is to buy these securities during their lowest premium period, which is the last half an hour of the day. For investors wishing to sell their Satrix TOP40 securities, it makes sense to do this between 09:00 and 09:30, to take advantage of the largest bid premium on offer. However, as with many previously uncovered behavioural anomalies in the market, *caveat emptor* should prevail, as the trends identified in this research are restricted to a single quarter's prices, and may either be an anomaly in themselves and be restricted to this period, or disappear now that they have been uncovered. Investors concerned about the premium they may be paying for Satrix TOP40 securities should perform a similar analysis to check whether they are receiving a fair price, rather than assuming the best price is available in the evening when buying or the morning when the selling.

6.4 Index Futures Efficiency

As already concluded, the TOP40 index of constituent shares and the futures contract are priced correctly relative to each other for the lowest cost market participants after factoring in their transaction and holding costs and the impact cost of crossing the bid-ask spreads. As such, these instruments are priced efficiently, since there appear to be no arbitrage opportunities for the lowest cost market participants.

However, the TOP40 Satrix and futures contract appear to present arbitrage opportunities at low levels of liquidity (volume), during certain times of the day. Since the magnitudes of these opportunities at the available levels of liquidity translate into profits in the tens of thousands of Rands, this provides sufficient evidence that inefficiencies in the Satrix TOP40 exchange-trade fund exist. However, as discussed in the introduction, Shleifer & Vishny (1997) and Twite (1998) point out that these profits are not entirely risk-free. In this particular case, risk is

introduced by using the TOP40 Satrix as a proxy for the TOP40 index, where the same guarantee of price convergence at futures contract maturity does not exist. The mechanics of implementing such an arbitrage trade means that an identified opportunity may be quite different from what is actually realised. Whether the potential opportunities on offer account for the risks involved is at the very heart of the market and those who position themselves to exploit these gains with the least risk will reap the rewards, while at the same time benefiting the remainder of the market by making it more efficient. Therefore, it is inconclusive as to whether the TOP40 Satrix is priced efficiently, particularly in light of other conveniences, which may account for the premium at which it trades.

7 CONCLUSION

7.1 Summary

After considering various frictions within the South African index and futures market and thereby relaxing the frictionless market assumptions, no economically significant arbitrage opportunities between the TOP40 index of constituent shares and its futures contract were found. However, the TOP40 Satrix exchange traded-fund seemed to trade at an intraday premium to its index, allowing arbitrage transactions between the TOP40 Satrix instrument and the TOP40 index futures contract. However, the liquidity of the Satrix product allowed only between one and two futures contracts worth of volume to be executed, between 14:00 and 14:30. More lucrative arbitrage opportunities were present at other times during the day, but had insufficient liquidity to fill even one futures contract worth of volume.

The purer form of arbitrage between the TOP40 index and its futures contract, which enforces the futures and index price to converge at maturity, presented no economically significant opportunities, resulting in an efficiently priced TOP40 futures contract relative to its index. However, the TOP40 Satrix instrument, which proxies for the TOP40 index and trades at an intraday premium over the period under review, does present arbitrage opportunities against the TOP40 futures contract. However, this strategy entails additional risk because the TOP40 Satrix price is not enforced to converge to the TOP40 index futures price. Therefore, these arbitrage opportunities are not necessarily an indication of market inefficiency, particularly as the Satrix TOP40 presents other conveniences, which may account for the premium at which it trades.

7.2 Limitations

By only considering a single quarter's worth of data, even though the sample size was large, means the results have not been tested over longer periods.

In practise, arbitrage opportunities are evaluated using discrete simple interest rates, rather than the more academic continuous compounding interest rates. These may marginally effect the frequency and magnitudes of opportunities reported.

7.3 Recommendations

There is always scope to make a market more efficient, as long as frictions prevail. This is typically done through regulatory changes and increased competition, which serve to lower costs. This results in narrower no-arbitrage bounds, leaving less room for prices to diverge before arbitrageurs exploit these opportunities.

The relative scarcity of arbitrage opportunities and the low levels of friction within the South African financial market should be more aggressively marketed by those seeking foreign investment into our financial markets.

7.4 Suggestions for Further Research

Since only a single quarter of data were considered, there is scope to extend the period under review to determine whether these results are persistent over longer periods. Furthermore, using multiple futures closeout periods allows the possibility of entering arbitrage opportunities for longer periods to be considered, and thereby testing whether the option to close a position prematurely or roll it forward leads to further arbitrage opportunities (Brennan & Schwartz, 1990), as well as testing whether the time to maturity factor comes into effect (MacKinlay & Ramaswamy, 1988).

The cash flow methodology used to validate some of the results produced by the upper and lower bound formulae could be employed to determine the magnitude of arbitrage opportunities. This methodology provides the opportunity to evaluate the early closeout option as well as the mark-to-market effect of arbitrage positions. Furthermore, this would facilitate the determination of arbitrage opportunities using simple discrete interest rates, rather than compounded interest rates, thereby providing a more realistic analysis of how arbitrage opportunities are evaluated in practise.

Finally, due to insufficient liquidity on the Shareholder Weighted Top 40 (SWIX40) index futures for the period under review, there were insufficient results to analyse arbitrage opportunities. However, the subsequent quarter saw the introduction of a Satrix SWIX40 security, which may improve the liquidity on the SWIX40 index futures contract (Merrick, 1987). Research could be conducted to determine the effect exchange-traded funds have on the liquidity of their respective futures contracts.

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APPENDICES

Appendix 1: Tukey-Kramer's Simultaneous Confidence Intervals for Multiple Comparisons of All Arbitrage Transaction Type Pairs

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Satrix Long Last	247581	-1.35673				
- Satrix Short Last	247581	0.0925056	-1.452217	-1.449236	-1.446254	L
- Satrix Long	242059	-0.6665225	-0.6932061	-0.6902074	-0.6872088	L
- Satrix Short	242059	-0.1190837	-1.240645	-1.237646	-1.234648	L
- Index Long Last	247581	-0.7511273	-0.6085843	-0.6056026	-0.6026209	L
- Index Short Last	247581	-0.5106384	-0.8490732	-0.8460916	-0.8431098	L
- Index Long	241656	-0.9404566	-0.4192733	-0.4162734	-0.4132734	L
- Index Short	241656	-1.562196	0.2024666	0.2054666	0.2084665	U

Table 15: Satrix Long Last Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Satrix Short Last	247581	0.0925056				
- Satrix Long Last	247581	-1.35673	1.446254	1.449236	1.452217	U
- Satrix Long	242059	-0.6665225	0.7560295	0.7590281	0.7620268	U
- Satrix Short	242059	-0.1190837	0.2085907	0.2115894	0.214588	U
- Index Long Last	247581	-0.7511273	0.8406512	0.8436329	0.8466147	U
- Index Short Last	247581	-0.5106384	0.6001623	0.603144	0.6061257	U
- Index Long	241656	-0.9404566	1.029962	1.032962	1.035962	U
- Index Short	241656	-1.562196	1.651702	1.654702	1.657702	U

Table 16: Satrix Short Last Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Satrix Long	242059	-0.6665225				
- Satrix Long Last	247581	-1.35673	0.6872088	0.6902074	0.6932061	U
- Satrix Short Last	247581	0.0925056	-0.7620268	-0.7590281	-0.7560295	L
- Satrix Short	242059	-0.1190837	-0.5504543	-0.5474387	-0.5444232	L
- Index Long Last	247581	-0.7511273	0.08160613	0.08460481	0.08760348	U
- Index Short Last	247581	-0.5106384	-0.1588828	-0.1558841	-0.1528855	L
- Index Long	241656	-0.9404566	0.2709173	0.2739341	0.2769508	U
- Index Short	241656	-1.562196	0.8926572	0.895674	0.8986908	U

Table 17: Satrix Long Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Satrix Short	242059	-0.1190837				
- Satrix Long Last	247581	-1.35673	1.234648	1.237646	1.240645	U
- Satrix Short Last	247581	0.0925056	-0.214588	-0.2115894	-0.2085907	L
- Satrix Long	242059	-0.6665225	0.5444232	0.5474387	0.5504543	U
- Index Long Last	247581	-0.7511273	0.6290449	0.6320436	0.6350423	U
- Index Short Last	247581	-0.5106384	0.388556	0.3915547	0.3945533	U
- Index Long	241656	-0.9404566	0.818356	0.8213728	0.8243896	U
- Index Short	241656	-1.562196	1.440096	1.443113	1.44613	U

Table 18: Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Index Long Last	247581	-0.7511273				
- Satrix Long Last	247581	-1.35673	0.6026209	0.6056026	0.6085843	U
- Satrix Short Last	247581	0.0925056	-0.8466147	-0.8436329	-0.8406512	L
- Satrix Long	242059	-0.6665225	-0.08760348	-0.08460481	-0.08160613	L
- Satrix Short	242059	-0.1190837	-0.6350423	-0.6320436	-0.6290449	L
- Index Short Last	247581	-0.5106384	-0.2434707	-0.2404889	-0.2375072	L
- Index Long	241656	-0.9404566	0.1863293	0.1893293	0.1923292	U
- Index Short	241656	-1.562196	0.8080692	0.8110692	0.8140691	U

Table 19: Index Long Last Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Index Short Last	247581	-0.5106384				
- Satrix Long Last	247581	-1.35673	0.8431098	0.8460916	0.8490732	U
- Satrix Short Last	247581	0.0925056	-0.6061257	-0.603144	-0.6001623	L
- Satrix Long	242059	-0.6665225	0.1528855	0.1558841	0.1588828	U
- Satrix Short	242059	-0.1190837	-0.3945533	-0.3915547	-0.388556	L
- Index Long Last	247581	-0.7511273	0.2375072	0.2404889	0.2434707	U
- Index Long	241656	-0.9404566	0.4268183	0.4298182	0.4328181	U
- Index Short	241656	-1.562196	1.048558	1.051558	1.054558	U

Table 20: Index Short Last Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Index Long	241656	-0.9404566				
- Satrix Long Last	247581	-1.35673	0.4132734	0.4162734	0.4192733	U
- Satrix Short Last	247581	0.0925056	-1.035962	-1.032962	-1.029962	L
- Satrix Long	242059	-0.6665225	-0.2769508	-0.2739341	-0.2709173	L
- Satrix Short	242059	-0.1190837	-0.8243896	-0.8213728	-0.818356	L
- Index Long Last	247581	-0.7511273	-0.1923292	-0.1893293	-0.1863293	L
- Index Short Last	247581	-0.5106384	-0.4328181	-0.4298182	-0.4268183	L
- Index Short	241656	-1.562196	0.6187218	0.6217399	0.6247579	U

Table 21: Index Long Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
Index Short	241656	-1.562196				
- Satrix Long Last	247581	-1.35673	-0.2084665	-0.2054666	-0.2024666	L
- Satrix Short Last	247581	0.0925056	-1.657702	-1.654702	-1.651702	L
- Satrix Long	242059	-0.6665225	-0.8986908	-0.895674	-0.8926572	L
- Satrix Short	242059	-0.1190837	-1.44613	-1.443113	-1.440096	L
- Index Long Last	247581	-0.7511273	-0.8140691	-0.8110692	-0.8080692	L
- Index Short Last	247581	-0.5106384	-1.054558	-1.051558	-1.048558	L
- Index Long	241656	-0.9404566	-0.6247579	-0.6217399	-0.6187218	L

Table 22: Index Short Magnitude Simultaneous Confidence Intervals

Appendix 2: Tukey-Kramer's Simultaneous Confidence Intervals for Multiple Comparisons of All Satrix Short 30-minute Pairs

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
09:00	2329	0.3067840				
- 09:30	3003	0.1188545	0.1735525	0.1879296	0.2023066	U
- 10:00	4143	0.1184238	0.1748748	0.1883602	0.2018456	U
- 10:30	4042	0.1918654	0.1013727	0.1149186	0.1284645	U
- 11:00	3823	0.1502145	0.1428826	0.1565695	0.1702565	U
- 11:30	3788	0.0954567	0.1976164	0.2113273	0.2250382	U
- 12:00	3866	0.1074935	0.1856323	0.1992905	0.2129486	U
- 12:30	5210	0.1112841	0.1825210	0.1955000	0.2084789	U
- 13:00	5354	0.1271685	0.1666906	0.1796156	0.1925405	U
- 13:30	5497	0.1131417	0.1807684	0.1936423	0.2065162	U
- 14:00	4975	0.1485447	0.1451660	0.1582393	0.1713126	U
- 14:30	5763	0.1196113	0.1743876	0.1871727	0.1999578	U
- 15:00	5759	0.0964143	0.1975833	0.2103697	0.2231561	U
- 15:30	5011	0.1069767	0.1867490	0.1998074	0.2128657	U
- 16:00	4465	0.0965510	0.1969238	0.2102331	0.2235423	U
- 16:30	2968	0.0942385	0.1981315	0.2125455	0.2269595	U

Table 23: 09:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
09:30	3003	0.1188545				
- 09:00	2329	0.3067840	-0.2023066	-0.1879296	-0.1735525	L
- 10:00	4143	0.1184238	-0.0120485	0.0004306	0.0129097	
- 10:30	4042	0.1918654	-0.0855554	-0.0730109	-0.0604665	L
- 11:00	3823	0.1502145	-0.0440567	-0.0313600	-0.0186633	L
- 11:30	3788	0.0954567	0.0106753	0.0233978	0.0361202	U
- 12:00	3866	0.1074935	-0.0013046	0.0113610	0.0240265	
- 12:30	5210	0.1112841	-0.0043596	0.0075704	0.0195004	
- 13:00	5354	0.1271685	-0.0201852	-0.0083140	0.0035572	
- 13:30	5497	0.1131417	-0.0061028	0.0057128	0.0175284	
- 14:00	4975	0.1485447	-0.0417228	-0.0296903	-0.0176577	L
- 14:30	5763	0.1196113	-0.0124757	-0.0007568	0.0109620	
- 15:00	5759	0.0964143	0.0107199	0.0224402	0.0341604	U
- 15:30	5011	0.1069767	-0.0001385	0.0118778	0.0238941	
- 16:00	4465	0.0965510	0.0100150	0.0223035	0.0345921	U
- 16:30	2968	0.0942385	0.0111387	0.0246159	0.0380932	U

Table 24: 09:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
10:00	4143	0.1184238				
- 09:00	2329	0.3067840	-0.2018456	-0.1883602	-0.1748748	L
- 09:30	3003	0.1188545	-0.0129097	-0.0004306	0.0120485	
- 10:30	4042	0.1918654	-0.0849533	-0.0734416	-0.0619298	L
- 11:00	3823	0.1502145	-0.0434681	-0.0317906	-0.0201132	L
- 11:30	3788	0.0954567	0.0112617	0.0229671	0.0346726	U
- 12:00	3866	0.1074935	-0.0007133	0.0109303	0.0225739	
- 12:30	5210	0.1112841	-0.0036991	0.0071398	0.0179787	
- 13:00	5354	0.1271685	-0.0195188	-0.0087446	0.0020295	
- 13:30	5497	0.1131417	-0.0054307	0.0052821	0.0159950	
- 14:00	4975	0.1485447	-0.0410726	-0.0301209	-0.0191692	L
- 14:30	5763	0.1196113	-0.0117935	-0.0011875	0.0094186	
- 15:00	5759	0.0964143	0.0114019	0.0220095	0.0326171	U
- 15:30	5011	0.1069767	0.0005134	0.0114472	0.0223810	U
- 16:00	4465	0.0965510	0.0106406	0.0218729	0.0331052	U
- 16:30	2968	0.0942385	0.0116636	0.0241853	0.0367070	U

Table 25: 10:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
10:30	4042	0.1918654				
- 09:00	2329	0.3067840	-0.1284645	-0.1149186	-0.1013727	L
- 09:30	3003	0.1188545	0.0604665	0.0730109	0.0855554	U
- 10:00	4143	0.1184238	0.0619298	0.0734416	0.0849533	U
- 11:00	3823	0.1502145	0.0299037	0.0416509	0.0533982	U
- 11:30	3788	0.0954567	0.0846336	0.0964087	0.1081838	U
- 12:00	3866	0.1074935	0.0726583	0.0843719	0.0960855	U
- 12:30	5210	0.1112841	0.0696673	0.0805813	0.0914954	U
- 13:00	5354	0.1271685	0.0538472	0.0646969	0.0755467	U
- 13:30	5497	0.1131417	0.0679348	0.0787237	0.0895126	U
- 14:00	4975	0.1485447	0.0322946	0.0433207	0.0543468	U
- 14:30	5763	0.1196113	0.0615712	0.0722541	0.0829370	U
- 15:00	5759	0.0964143	0.0847667	0.0954511	0.1061355	U
- 15:30	5011	0.1069767	0.0738804	0.0848888	0.0958971	U
- 16:00	4465	0.0965510	0.0840096	0.0953145	0.1066193	U
- 16:30	2968	0.0942385	0.0850401	0.0976269	0.1102137	U

Table 26: 10:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
11:00	3823	0.1502145				
- 09:00	2329	0.3067840	-0.1702565	-0.1565695	-0.1428826	L
- 09:30	3003	0.1188545	0.0186633	0.0313600	0.0440567	U
- 10:00	4143	0.1184238	0.0201132	0.0317906	0.0434681	U
- 10:30	4042	0.1918654	-0.0533982	-0.0416509	-0.0299037	L
- 11:30	3788	0.0954567	0.0428207	0.0547578	0.0666949	U
- 12:00	3866	0.1074935	0.0308445	0.0427210	0.0545975	U
- 12:30	5210	0.1112841	0.0278417	0.0389304	0.0500192	U
- 13:00	5354	0.1271685	0.0120206	0.0230460	0.0340715	U
- 13:30	5497	0.1131417	0.0261073	0.0370728	0.0480383	U
- 14:00	4975	0.1485447	-0.0095292	0.0016698	0.0128688	
- 14:30	5763	0.1196113	0.0197420	0.0306032	0.0414644	U
- 15:00	5759	0.0964143	0.0429375	0.0538002	0.0646629	U
- 15:30	5011	0.1069767	0.0320563	0.0432378	0.0544194	U
- 16:00	4465	0.0965510	0.0421900	0.0536635	0.0651371	U
- 16:30	2968	0.0942385	0.0432374	0.0559760	0.0687145	U

Table 27: 11:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
11:30	3788	0.0954567				
- 09:00	2329	0.3067840	-0.2250382	-0.2113273	-0.1976164	L
- 09:30	3003	0.1188545	-0.0361202	-0.0233978	-0.0106753	L
- 10:00	4143	0.1184238	-0.0346726	-0.0229671	-0.0112617	L
- 10:30	4042	0.1918654	-0.1081838	-0.0964087	-0.0846336	L
- 11:00	3823	0.1502145	-0.0666949	-0.0547578	-0.0428207	L
- 12:00	3866	0.1074935	-0.0239409	-0.0120368	-0.0001328	L
- 12:30	5210	0.1112841	-0.0269456	-0.0158274	-0.0047091	L
- 13:00	5354	0.1271685	-0.0427669	-0.0317118	-0.0206567	L
- 13:30	5497	0.1131417	-0.0286804	-0.0176850	-0.0066896	L
- 14:00	4975	0.1485447	-0.0643163	-0.0530880	-0.0418598	L
- 14:30	5763	0.1196113	-0.0350460	-0.0241546	-0.0132633	L
- 15:00	5759	0.0964143	-0.0118505	-0.0009576	0.0099352	
- 15:30	5011	0.1069767	-0.0227307	-0.0115200	-0.0003092	L
- 16:00	4465	0.0965510	-0.0125963	-0.0010942	0.0104078	
- 16:30	2968	0.0942385	-0.0115461	0.0012182	0.0139824	

Table 28: 11:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
12:00	3866	0.1074935				
- 09:00	2329	0.3067840	-0.2129486	-0.1992905	-0.1856323	L
- 09:30	3003	0.1188545	-0.0240265	-0.0113610	0.0013046	
- 10:00	4143	0.1184238	-0.0225739	-0.0109303	0.0007133	
- 10:30	4042	0.1918654	-0.0960855	-0.0843719	-0.0726583	L
- 11:00	3823	0.1502145	-0.0545975	-0.0427210	-0.0308445	L
- 11:30	3788	0.0954567	0.0001328	0.0120368	0.0239409	U
- 12:30	5210	0.1112841	-0.0148436	-0.0037905	0.0072626	
- 13:00	5354	0.1271685	-0.0306645	-0.0196749	-0.0086853	L
- 13:30	5497	0.1131417	-0.0165777	-0.0056482	0.0052813	
- 14:00	4975	0.1485447	-0.0522149	-0.0410512	-0.0298875	L
- 14:30	5763	0.1196113	-0.0229426	-0.0121178	-0.0012929	L
- 15:00	5759	0.0964143	0.0002529	0.0110792	0.0219056	U
- 15:30	5011	0.1069767	-0.0106293	0.0005169	0.0116631	
- 16:00	4465	0.0965510	-0.0004966	0.0109426	0.0223817	
- 16:30	2968	0.0942385	0.0005475	0.0132550	0.0259625	U

Table 29: 12:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
12:30	5210	0.1112841				
- 09:00	2329	0.3067840	-0.2084789	-0.1955000	-0.1825210	L
- 09:30	3003	0.1188545	-0.0195004	-0.0075704	0.0043596	
- 10:00	4143	0.1184238	-0.0179787	-0.0071398	0.0036991	
- 10:30	4042	0.1918654	-0.0914954	-0.0805813	-0.0696673	L
- 11:00	3823	0.1502145	-0.0500192	-0.0389304	-0.0278417	L
- 11:30	3788	0.0954567	0.0047091	0.0158274	0.0269456	U
- 12:00	3866	0.1074935	-0.0072626	0.0037905	0.0148436	
- 13:00	5354	0.1271685	-0.0260175	-0.0158844	-0.0057513	L
- 13:30	5497	0.1131417	-0.0119255	-0.0018576	0.0082103	
- 14:00	4975	0.1485447	-0.0475824	-0.0372607	-0.0269389	L
- 14:30	5763	0.1196113	-0.0182814	-0.0083272	0.0016269	
- 15:00	5759	0.0964143	0.0049139	0.0148698	0.0248256	U
- 15:30	5011	0.1069767	-0.0059953	0.0043074	0.0146101	
- 16:00	4465	0.0965510	0.0041141	0.0147331	0.0253521	U
- 16:30	2968	0.0942385	0.0050710	0.0170455	0.0290201	U

Table 30: 12:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
13:00	5354	0.1271685				
- 09:00	2329	0.3067840	-0.1925405	-0.1796156	-0.1666906	L
- 09:30	3003	0.1188545	-0.0035572	0.0083140	0.0201852	
- 10:00	4143	0.1184238	-0.0020295	0.0087446	0.0195188	
- 10:30	4042	0.1918654	-0.0755467	-0.0646969	-0.0538472	L
- 11:00	3823	0.1502145	-0.0340715	-0.0230460	-0.0120206	L
- 11:30	3788	0.0954567	0.0206567	0.0317118	0.0427669	U
- 12:00	3866	0.1074935	0.0086853	0.0196749	0.0306645	U
- 12:30	5210	0.1112841	0.0057513	0.0158844	0.0260175	U
- 13:30	5497	0.1131417	0.0040286	0.0140268	0.0240249	U
- 14:00	4975	0.1485447	-0.0316299	-0.0213763	-0.0111226	L
- 14:30	5763	0.1196113	-0.0023265	0.0075572	0.0174408	
- 15:00	5759	0.0964143	0.0208689	0.0307542	0.0406395	U
- 15:30	5011	0.1069767	0.0099572	0.0201918	0.0304264	U
- 16:00	4465	0.0965510	0.0200647	0.0306175	0.0411704	U
- 16:30	2968	0.0942385	0.0210140	0.0329299	0.0448459	U

Table 31: 13:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
13:30	5497	0.1131417				
- 09:00	2329	0.3067840	-0.2065162	-0.1936423	-0.1807684	L
- 09:30	3003	0.1188545	-0.0175284	-0.0057128	0.0061028	
- 10:00	4143	0.1184238	-0.0159950	-0.0052821	0.0054307	
- 10:30	4042	0.1918654	-0.0895126	-0.0787237	-0.0679348	L
- 11:00	3823	0.1502145	-0.0480383	-0.0370728	-0.0261073	L
- 11:30	3788	0.0954567	0.0066896	0.0176850	0.0286804	U
- 12:00	3866	0.1074935	-0.0052813	0.0056482	0.0165777	
- 12:30	5210	0.1112841	-0.0082103	0.0018576	0.0119255	
- 13:00	5354	0.1271685	-0.0240249	-0.0140268	-0.0040286	L
- 14:00	4975	0.1485447	-0.0455923	-0.0354030	-0.0252138	L
- 14:30	5763	0.1196113	-0.0162864	-0.0064696	0.0033472	
- 15:00	5759	0.0964143	0.0069090	0.0167274	0.0265458	U
- 15:30	5011	0.1069767	-0.0040050	0.0061651	0.0163351	
- 16:00	4465	0.0965510	0.0061005	0.0165908	0.0270810	U
- 16:30	2968	0.0942385	0.0070426	0.0189032	0.0307637	U

Table 32: 13:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
14:00	4975	0.1485447				
- 09:00	2329	0.3067840	-0.1713126	-0.1582393	-0.1451660	L
- 09:30	3003	0.1188545	0.0176577	0.0296903	0.0417228	U
- 10:00	4143	0.1184238	0.0191692	0.0301209	0.0410726	U
- 10:30	4042	0.1918654	-0.0543468	-0.0433207	-0.0322946	L
- 11:00	3823	0.1502145	-0.0128688	-0.0016698	0.0095292	
- 11:30	3788	0.0954567	0.0418598	0.0530880	0.0643163	U
- 12:00	3866	0.1074935	0.0298875	0.0410512	0.0522149	U
- 12:30	5210	0.1112841	0.0269389	0.0372607	0.0475824	U
- 13:00	5354	0.1271685	0.0111226	0.0213763	0.0316299	U
- 13:30	5497	0.1131417	0.0252138	0.0354030	0.0455923	U
- 14:30	5763	0.1196113	0.0188565	0.0289334	0.0390103	U
- 15:00	5759	0.0964143	0.0420519	0.0521304	0.0622090	U
- 15:30	5011	0.1069767	0.0311467	0.0415681	0.0519894	U
- 16:00	4465	0.0965510	0.0412597	0.0519938	0.0627279	U
- 16:30	2968	0.0942385	0.0422294	0.0543062	0.0663829	U

Table 33: 14:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
14:30	5763	0.1196113				
- 09:00	2329	0.3067840	-0.1999578	-0.1871727	-0.1743876	L
- 09:30	3003	0.1188545	-0.0109620	0.0007568	0.0124757	
- 10:00	4143	0.1184238	-0.0094186	0.0011875	0.0117935	
- 10:30	4042	0.1918654	-0.0829370	-0.0722541	-0.0615712	L
- 11:00	3823	0.1502145	-0.0414644	-0.0306032	-0.0197420	L
- 11:30	3788	0.0954567	0.0132633	0.0241546	0.0350460	U
- 12:00	3866	0.1074935	0.0012929	0.0121178	0.0229426	U
- 12:30	5210	0.1112841	-0.0016269	0.0083272	0.0182814	
- 13:00	5354	0.1271685	-0.0174408	-0.0075572	0.0023265	
- 13:30	5497	0.1131417	-0.0033472	0.0064696	0.0162864	
- 14:00	4975	0.1485447	-0.0390103	-0.0289334	-0.0188565	L
- 15:00	5759	0.0964143	0.0134952	0.0231970	0.0328988	U
- 15:30	5011	0.1069767	0.0025772	0.0126347	0.0226921	U
- 16:00	4465	0.0965510	0.0126792	0.0230604	0.0334416	U
- 16:30	2968	0.0942385	0.0136086	0.0253728	0.0371370	U

Table 34: 14:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
15:00	5759	0.0964143				
- 09:00	2329	0.3067840	-0.2231561	-0.2103697	-0.1975833	L
- 09:30	3003	0.1188545	-0.0341604	-0.0224402	-0.0107199	L
- 10:00	4143	0.1184238	-0.0326171	-0.0220095	-0.0114019	L
- 10:30	4042	0.1918654	-0.1061355	-0.0954511	-0.0847667	L
- 11:00	3823	0.1502145	-0.0646629	-0.0538002	-0.0429375	L
- 11:30	3788	0.0954567	-0.0099352	0.0009576	0.0118505	
- 12:00	3866	0.1074935	-0.0219056	-0.0110792	-0.0002529	L
- 12:30	5210	0.1112841	-0.0248256	-0.0148698	-0.0049139	L
- 13:00	5354	0.1271685	-0.0406395	-0.0307542	-0.0208689	L
- 13:30	5497	0.1131417	-0.0265458	-0.0167274	-0.0069090	L
- 14:00	4975	0.1485447	-0.0622090	-0.0521304	-0.0420519	L
- 14:30	5763	0.1196113	-0.0328988	-0.0231970	-0.0134952	L
- 15:30	5011	0.1069767	-0.0206214	-0.0105623	-0.0005033	L
- 16:00	4465	0.0965510	-0.0105194	-0.0001366	0.0102461	
- 16:30	2968	0.0942385	-0.0095898	0.0021758	0.0139414	

Table 35: 15:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
15:30	5011	0.1069767				
- 09:00	2329	0.3067840	-0.2128657	-0.1998074	-0.1867490	L
- 09:30	3003	0.1188545	-0.0238941	-0.0118778	0.0001385	
- 10:00	4143	0.1184238	-0.0223810	-0.0114472	-0.0005134	L
- 10:30	4042	0.1918654	-0.0958971	-0.0848888	-0.0738804	L
- 11:00	3823	0.1502145	-0.0544194	-0.0432378	-0.0320563	L
- 11:30	3788	0.0954567	0.0003092	0.0115200	0.0227307	U
- 12:00	3866	0.1074935	-0.0116631	-0.0005169	0.0106293	
- 12:30	5210	0.1112841	-0.0146101	-0.0043074	0.0059953	
- 13:00	5354	0.1271685	-0.0304264	-0.0201918	-0.0099572	L
- 13:30	5497	0.1131417	-0.0163351	-0.0061651	0.0040050	
- 14:00	4975	0.1485447	-0.0519894	-0.0415681	-0.0311467	L
- 14:30	5763	0.1196113	-0.0226921	-0.0126347	-0.0025772	L
- 15:00	5759	0.0964143	0.0005033	0.0105623	0.0206214	U
- 16:00	4465	0.0965510	-0.0002901	0.0104257	0.0211415	
- 16:30	2968	0.0942385	0.0006776	0.0127381	0.0247986	U

Table 36: 15:30 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
16:00	4465	0.0965510				
- 09:00	2329	0.3067840	-0.2235423	-0.2102331	-0.1969238	L
- 09:30	3003	0.1188545	-0.0345921	-0.0223035	-0.0100150	L
- 10:00	4143	0.1184238	-0.0331052	-0.0218729	-0.0106406	L
- 10:30	4042	0.1918654	-0.1066193	-0.0953145	-0.0840096	L
- 11:00	3823	0.1502145	-0.0651371	-0.0536635	-0.0421900	L
- 11:30	3788	0.0954567	-0.0104078	0.0010942	0.0125963	
- 12:00	3866	0.1074935	-0.0223817	-0.0109426	0.0004966	
- 12:30	5210	0.1112841	-0.0253521	-0.0147331	-0.0041141	L
- 13:00	5354	0.1271685	-0.0411704	-0.0306175	-0.0200647	L
- 13:30	5497	0.1131417	-0.0270810	-0.0165908	-0.0061005	L
- 14:00	4975	0.1485447	-0.0627279	-0.0519938	-0.0412597	L
- 14:30	5763	0.1196113	-0.0334416	-0.0230604	-0.0126792	L
- 15:00	5759	0.0964143	-0.0102461	0.0001366	0.0105194	
- 15:30	5011	0.1069767	-0.0211415	-0.0104257	0.0002901	
- 16:30	2968	0.0942385	-0.0100194	0.0023124	0.0146442	

Table 37: 16:00 Satrix Short Magnitude Simultaneous Confidence Intervals

Comparison Groups	Count	Mean	Lower 99.5% Simult.C.I.	Mean Difference	Upper 99.5% Simult.C.I.	Test Result
16:30	2968	0.0942385				
- 09:00	2329	0.3067840	-0.2269595	-0.2125455	-0.1981315	L
- 09:30	3003	0.1188545	-0.0380932	-0.0246159	-0.0111387	L
- 10:00	4143	0.1184238	-0.0367070	-0.0241853	-0.0116636	L
- 10:30	4042	0.1918654	-0.1102137	-0.0976269	-0.0850401	L
- 11:00	3823	0.1502145	-0.0687145	-0.0559760	-0.0432374	L
- 11:30	3788	0.0954567	-0.0139824	-0.0012182	0.0115461	
- 12:00	3866	0.1074935	-0.0259625	-0.0132550	-0.0005475	L
- 12:30	5210	0.1112841	-0.0290201	-0.0170455	-0.0050710	L
- 13:00	5354	0.1271685	-0.0448459	-0.0329299	-0.0210140	L
- 13:30	5497	0.1131417	-0.0307637	-0.0189032	-0.0070426	L
- 14:00	4975	0.1485447	-0.0663829	-0.0543062	-0.0422294	L
- 14:30	5763	0.1196113	-0.0371370	-0.0253728	-0.0136086	L
- 15:00	5759	0.0964143	-0.0139414	-0.0021758	0.0095898	
- 15:30	5011	0.1069767	-0.0247986	-0.0127381	-0.0006776	L
- 16:00	4465	0.0965510	-0.0146442	-0.0023124	0.0100194	

Table 38: 16:30 Satrix Short Magnitude Simultaneous Confidence Intervals