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Masters Research Report

Exploring grade 10 learners' errors and misconceptions involved in solving probability problems using different representations

LYDIA MUTARA
Student No: 579347

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Supervisor: Dr Judah. P. Makonye

Abstract

The Curriculum Assessment Policy Statements (CAPS) re-introduced some mathematics topics such as probability. An immediate effect of this re-introduction is that most teachers and learners were not well equipped to deal with this topic. To at least begin addressing this problem, this research explored the errors and misconceptions that learners have when solving probability problems using different representations. The study draws from Nesher's (1987) theory of errors and misconceptions as well as Sfard's (2007) theory of commognition in explaining representations and prevalence of errors in learning mathematics. Twenty two Grade 10 learners wrote probability tasks after which their scripts were analysed for errors. Six of those learners were interviewed on the errors they made in solving probability problems with different representations. The findings reveal five main categories of errors and misconceptions. These are: (1) difficulty with construction of visual representations; (2) improper distinction between simple and compound events; (3) application of inappropriate routines; (4) errors associated with familiarity; and, (5) misinterpreted language. The findings also showed that inappropriate choice of representations was caused by misinterpretation of probability terminology. Concurring with Zahner and Corter (2010) the researcher found that learners made a multitude of errors if they constructed and used their own probability representations. Further, learners committed fewer errors where the task provided representations. Results also show that learners were most confident in using tree diagram representations even though they struggled to construct them from scratch. Most learners avoided Venn diagrams, outcome listings and matrix representations even though they would be the most useful in answering the questions. As a result many errors and misconceptions resulted when learners tried to use these representations. The study recommends that teachers take time to discuss probability terminology and the use of different representations with their learners. This promotes both the conceptual and procedural knowledge of probability. Also, to reduce learners' errors and misconceptions on the topic, teachers need to scaffold the construction of representations by providing partially constructed representations and gradually encourage learners to construct their own probability representations.

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Declaration

I declare that this project is my own work and no part of it has been copied from another source (unless indicated as quote). All phrases, sentences and paragraphs taken directly from other work have been cited and the reference recorded in full in the reference list. The project is being submitted for the Degree of Masters in Science Education with the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

Signature: ----------

Date: ----------

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List of Abbreviations

CAPS ……..Curriculum Assessments Policy Statements

DBE…….. Department of Basic Education

FET……Further Education and Training

GDE……...Gauteng Department of Education

JBC……... Pseudonym for the school where data was collected for the current study

PCK……...Pedagogical Content Knowledge

TSPCK……..Topic Specific Pedagogical Content Knowledge

TIMSS…….. Trends in Mathematics and Science Study

RNCS…… Revised National Curriculum Statement

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CHAPTER 1: An overview of the study

1.1 Introduction

This chapter presents the research problem of this study. It also locates the study in the South African curriculum, focussing particularly on learner errors and misconceptions on the mathematics topic of probability. Then the research problem is articulated, as are the aims and research questions. In addition, the significance of the study is discussed. The chapter concludes by describing how the rest of the study is structured.

1.2 Contextual background of the study

A number of curriculum reforms have occurred in South Africa since the dawn of democracy in 1994[1], particularly in the mathematics and science education learning areas (see, Department of Basic Education (DBE), 1998; 2005, 2011). These reform efforts have been triggered by the country's very poor performance in national school assessments and Trends in Mathematics and Science Study (TIMSS) (Howie, 2001, 2003, 2004; Moloi & Chetty, 2010; Reddy, 2006; Taylor & Taylor, 2013) which is an international performance comparison test. According to the DBE (2011), the curriculum changes have been necessary to make the current system run better and to bring about "equality within a public school system" (p.17). There have been three curriculum changes from 1998 to 2012. These are Curriculum 2005, Revised National Curriculum Statement (RNCS) and now Curriculum and Assessment Policy Statements (CAPS).

Curriculum 2005 was outcomes-based, resource intensive and not very directive. It was aimed at empowering teachers but proved to be too complex to implement in most schools (Zenex Foundation, 2013). The RNCS focused on addressing the complexity of Curriculum 2005 by stressing on basic skills, content knowledge and grade progression as well as simplifying the outcomes statements. Implementation of the RNCS did not seem to give the intended outcomes, resulting in it being replaced by CAPS. According to the Zenex Foundation (2013), CAPS is a more regulated learning programme in that it has workbooks which sequence and pace the work on a daily basis. In this regard, CAPS reduces the teacher's responsibility to interpret the curriculum outcomes, but the teacher still has to ensure that learning takes place in the classroom.

One major aim of CAPS is to produce learners with a deeper conceptual understanding of mathematical ideas (DBE, 2011). As a result, CAPS re-introduced the topics of geometry and probability into the main mathematics curriculum. Solving geometrical and probability problems involve logical thinking, justification and reflection which are parts of mathematical reasoning (Kilpatrick, Swafford & Findell, 2001). According to Kilpatrick et al. (2001), mathematical reasoning refers to logical thinking, explanation and justification of

[1] 1994 was the year South Africa changed from apartheid government to democracy.

strategies chosen for solving particular problems, as well as reflecting on such strategies with an aim of making them more effective. The crucial role of mathematical reasoning in problem solving has been acknowledged by other researchers such as Hiebert and Lefevre (1986). According to Hiebert and Lefevre (1986), reflection, which is a part of mathematical reasoning, is a critical requirement for improving one's practice and making connections between the two pieces of knowledge which promote conceptual understanding. These authors argue that if a person has a good understanding of a concept, she/he can use it flexibly. In this regard, the re-introduction of geometry and probability into the main mathematics curriculum has underlined the commitment of the DBE to promote acquisition of high skills and knowledge in mathematics.

One of the problems that arose with the re-introduction of geometry and probability was that teachers and learners did not have sufficient learning and teaching support material (LTSM) for the new topics. For example, there are limited text books and past examination questions that deal with these topics. Given this scenario, I became interested in finding out how teaching and learning took place in these topics, focusing particularly on probability. In an attempt to understand instructional challenges embedded in the teaching and learning of Grade 10 probability, I explored learners' errors and misconceptions. The researcher's assumption was that South African learners performed poorly in mathematics because they held many unresolved errors and misconceptions in many mathematics topics, including probability.

1.3 Probability as a school mathematics topic

Probability is an important topic in school mathematics. Hirsch and O'Donnell (2001) describe probability as "the study of likelihood and uncertainty" (p. 1). These authors argue that understanding probability is necessary for learners as most things in commerce, industry and daily life are probability-based. Practical concerns such as product marketing, deciding which product to purchase, determining car insurance rates, deciding what and when to plant in farming, determining risks of child-birth defects and interpreting weather reports, among others, often rely on reasoning about probabilities. In South Africa, just like any other part of the world, probability is often used in marketing. Consumers can be convinced to buy a product through use of its supposed success rate or percentage. For example, a sliming cream which promises that 90% of the people who use it will lose weight in a month is likely to sell quickly and in large quantities due to its supposed success rate. The application of probability in these contexts demonstrates that it is necessary to include the topic in the school mathematics curriculum.

At high school level, instruction on probability could involve learning how to calculate the chances of events happening or not happening (see, Appendix 1). In the CAPS curriculum at Grade 10 level, section 3 of DBE (2011) document, the topic occupies number six on the list

of main topics in further education and training (FET) mathematics curriculum. The content to be covered includes theoretical and experimental probability, dependent and independent events, simple and compound events and generalisation of the fundamental counting principle across the grades.

The weighting for probability is 15% in Grade 10, 20% in Grade 11 and 15% in Grade 12, with a free play of plus or minus 3% in each case. The weighting of mathematics content areas gives guidance on the amount of time needed to address the content adequately as well as the spread of content in examinations. As a result, the topic is allocated about two weeks contact time in each grade. The curriculum expects learners to be able to "identify and solve problems and make decisions using critical and creative thinking" (DBE, 2011; p. 5). They are also expected to communicate effectively using language, visual and symbolic skills in various modes. This involves the use of Venn diagrams, contingency tables, tree diagrams, matrices and outcome listings.

The teaching and learning of probability is particularly challenging because it is an abstract concept. For this reason, it is important to employ different representations to mediate learning. Such representations could be tree diagrams, Venn diagrams, contingency tables, matrices or outcome listings. Although there have been studies involving use of representations in solving probability problems (for example, Zahner & Corter, 2010), none of these were focused on learners' errors involving representations. Therefore this study is focusing on a relatively new area of research in probability.

Earlier researchers such as Piaget and Inhelder (1951), argue that modelling probabilistic situations are complex for learners. Later, the community of mathematics and statistics educators (see, Fischbein, 1999; Freudenthal, 1973; Shaughnessy, 1992) also concluded that modelling probabilistic situations is complex for learners and is often hindered by the learners' wrong intuitions, biases and primitive conceptions to mention a few. It is therefore clear that probability as a section of mathematical reasoning is quite sensitive to the presence of misconceptions. The known misconceptions in probability are also well documented by research (for example, Fischbein & Schnarch, 1997; Fischbein, 1999; Shaughnessy, 1992). For this reason, I decided to explore the learners' errors and misconceptions associated with solving probability problems involving representations in order to support learners' development of mathematical proficiency in the topic. It is particularly interesting to investigate whether the documented misconceptions will have the same nature in a South African context as was seen by the earlier researchers.

Understanding learners' errors is important in the teaching and learning of any subject matter. According to Riccomini (2005), teachers' understanding of learners' errors and misconceptions enriches their instructional effectiveness. Makonye (2011) argues that

establishing the extent of the learners' understanding of mathematical concepts by way of error diagnosis and analysis helps teachers to prepare lessons that cater for learners' conceptual levels. Additionally, "when teachers are aware of likely misconceptions and errors from a specific topic in mathematics, their lesson preparation as well as their lesson evaluation strategies are sharper and address the learners' likely errors and misconceptions adequately" (Makonye & Luneta, 2014; p. 119). Hence, the question arises; what are learners' errors and misconceptions?

1.4 What are learners' errors and misconceptions?

Errors are an indication that something is not quite right. Olivier (1989) regards an error as an unintended or intended deviation from accuracy. There are two types of errors made by learners. These are systematic and non-systematic errors.

Systematic errors are "recurrent wrong answers methodically reproduced across space and time" (Makonye & Luneta, 2014; p. 120). These kinds of errors result from an underlying incorrect premise. Nesher (1987) calls these incorrect premises, misconceptions. A *misconception* is therefore a faulty hypothesis that causes systematic errors. As a result, learners do not recognise systematic errors as wrong because they are intuitively sensible to them (Brodie & Berger, 2010; Green, Piel & Flowers, 2008; Nesher, 1987; Olivier, 1996; Riccomini, 2005; Smith, DiSessa & Roschelle, 1993). Systematic errors may also stem from misuse or overstretching of some conceptual structures. They can go undetected for a long time as they can produce correct answers in some occasions (Nesher, 1987).

Non-systematic errors are superficial, non-recurring and unintended wrong answers (Khazanov, 2008). Olivier (1996) considers them to be slips because they are unconnected and do not necessarily result from misconceptions. Slips are errors which can be corrected easily by the learners themselves, when pointed out, through normal classroom instruction or self-checking in the process of verifying solutions (Brodie & Berger, 2010). Although non-systematic errors are superficial and may seem unimportant, they can have undesirable consequences in mathematics learning, especially when they are carried over in multi-step mathematics tasks (Makonye & Luneta, 2014).

In this study, my focus is on systematic errors because they can be detected by particular activities (Nesher, 1987). The discussion on types of errors sheds light on the problem statement of the study.

1.5 The problem statement

The re-introduction of new topics in the CAPS curriculum as from 2012 has resulted in learners facing new challenges in learning mathematics. In my teaching, for example, I have noted that learners struggle to grasp probability concepts. Initially, I doubted the effectiveness

of my teaching strategies. However, upon comparing my learners' performance on probability questions in common examinations with similar classes at my school and neighbouring schools, I did not find the results any different. Nesher (1987) says, "it is only when doubts about our beliefs are raised that we stop to examine them and start an inquiry in order to appease our doubts and settle our opinions" (p.33). I began to raise questions as to what could be causing poor learners' understanding of probability.

According to the 2013 and 2014 matric roadshow feedback, many learners struggle to grasp probability concepts. This is evidenced by their failure to solve probability problems in tests and examinations. Evidence also shows that many learners do not attempt probability questions, particularly those that involve representations (see, section 2.4.6). The few who attempt the questions usually give inappropriate responses. My assumption is that the use of different probability representations in representing or interpreting the probability problems is associated with these challenges. As a result, the study focuses on solving probability problems using different representations.

It is not quite clear what causes learners to perform poorly in probability. Hirsch and O'Donnell (2001) suggest that the poor performance could be due to learners not understanding laws of probability or errors resulting from violations in the application of the laws. The fact that learners try to make sense of a concept implies that mathematical knowledge cannot be transmitted or transferred from the teacher to the students without their understanding (Makonye & Luneta, 2014). It follows that there must be some dynamic re-interpretation, re-organisation and reconstruction in each learner's mind (Bauersfeld, 1995; Hatano, 1996). It is due to some of these *understandings* that the learners construct, which are misconceptions in most cases, that their progress and achievement in learning mathematics is hindered. These kinds of understandings often result in errors (Makonye, 2011). A question arises. What can the teacher do to promote conceptual understanding of probability in learners?

In a way to at least begin to address my doubts about my teaching practice, I asked my colleagues at the school and schools in our cluster how they are dealing with the new topics. The preliminary survey revealed three scenarios in the South African classroom pertaining to teachers' preparedness to handle the topic probability:

1. Some of the teachers have learnt and taught probability at school but have not been teaching it for some time.
2. Some of the teachers did probability at school a long time ago and have not taught it at school until now.
3. Some of the teachers did not do probability at school or as part of their training but now have to teach it.

These scenarios show that many teachers are learning probability as they teach it. Teacher competence in handling the topic is therefore questionable. In the face of content and pedagogical deficiencies, the temptation to just explain concepts rather than pay attention to learners' errors and misconceptions becomes more overwhelming. Sasman, Linchevski, Oliver and Liebenberg (1998) argue that the teacher's role in teaching and learning must be more than just explaining concepts and ideas to learners.

The new CAPS curriculum does not provide teachers with examples of how they can promote mathematical thinking in learners so that they can develop conceptual understanding. Several researches have proposed ways in which teachers can teach for conceptual understanding but it should be noted that implementing recommendations from research in a class is a complex task for most teachers (Kazemi & Stipek, 2001). Hence teachers find themselves struggling to implement the new ideas especially those which are not topic specific. Bolyard and Moyer-Packenham (2008) argue that teaching depends mainly on the richness of a teacher's pedagogical content knowledge even though content knowledge is also very important. It is also noted that there have been few studies focusing on topic specific pedagogical content, even more so in high school mathematics (Cankoy, 2010). It is in this light that this study aims to contribute towards the development of topic specific pedagogical content knowledge In particular the topic probability. This will be accomplished by highlighting the errors and misconceptions, establishing the reasons for the errors associated with using different representations in solving probability problems.

My definition of Topic Specific Pedagogical Content Knowledge (TSPCK) resonates from Cankoy's (2010) definition of pedagogical content knowledge (PCK). Cankoy (2010) defines PCK as a set of special attributes that help a teacher to transfer knowledge of content to others. In this study, TSPCK is a set of instructional attributes that are specific to a topic. Identifying, assessing and analysing errors and the reasons for the errors (which are possibly misconceptions) from learners work will lead to teachers gaining knowledge of learners' thinking which enhances the teachers' PCK.

I consider it important for a mathematics teacher to develop an interest in the errors and misconceptions made by learners. An awareness of such errors and misconceptions helps her/him to design effective intervention strategies (Riccomini, 2005). This can help learners to achieve higher scores and develop greater interest in mathematics. This explains why I intend to investigate the causes of the errors and misconceptions that learners make in the teaching and learning of probability problems using different representations.

1.6 Aims of the study

This study aims to investigate the errors and misconceptions that grade 10 learners have when answering probability questions using different representations. In addition it intends to

explain the errors and misconceptions that learners have when they use particular representations in solving probability problems. In view of the research aim, the study was guided by the following research questions.

1.7 Research questions

1.7.1 What errors do grade 10 learners make when using different representations to solve probability problems?

1.7.2 How can the relationship between grade 10 learners' solution representations and their errors and misconceptions in solving probability problems be explained?

1.8 Rationale and significance

Several factors could contribute to what is eventually learnt in a given subject matter lesson. Vygotsky (1978) points out that learning occurs in a learner's Zone of Proximal Development when a learner is scaffolded to learn by a more knowledgeable other. In the classroom context, the teacher is considered to be the more knowledgeable other. Modiba (2011) and Carnoy, Chilisa and Chisholm (2012) observe that even the best teachers need adequate subject matter knowledge for them to provide effective instruction. Nesher (1987) notes that, while the teacher provides the subject content knowledge and pedagogical skills, the learner provides challenges in the form of conceptual errors. The extent to which the teacher uses her/his pedagogical skills to communicate the subject content and manage learners' misconceptions determines the effectiveness of learning. Effective management of learners' misconceptions can only take place if the teacher is aware of the learning weaknesses and how they arise. In other words, she/he can only craft effective pedagogical strategies if she/he is aware of learners' conceptual challenges.

In the classroom, the teacher has to encourage "newcomers to become old-timers" through what Lave (1993) calls "legitimate peripheral participation" (p.68). Hanks (1991) adds by saying that effective learning is a function of more active involvement. In other words, the learner's capabilities improve as he/she participates more and constructs knowledge for herself/himself. According to Lave (1993, p.68), learners gradually move from being 'novices' to 'experts' if they participate more actively in learning.

Making learners participate fully in classroom activities means that the teacher should allow them to make mistakes. The benefit of this is that the teacher becomes knowledgeable of learners' errors and misconceptions and can therefore craft relevant and more effective pedagogical strategies. Nesher (1987) argues that mistakes are learners' contribution to the lesson which the teacher can use as a feedback mechanism for real learning. Therefore a study into the errors and misconceptions made by learners in the teaching and learning of probability is important because it can result in improved understanding of how the topic can

be taught. It can also conscientise mathematics teachers on the need to reflect on their practice constantly so that they develop a more sound understanding of how it aids or retards learning. Teacher educators also stand to benefit because they develop a better understanding of learners' conceptual inadequacies and learning needs in the topic probability. As a result, they can properly skill student teachers on how to handle the topic in the classroom.

Given the current content and pedagogical inadequacies obtaining in the classroom on the topic probability, the study can go a long way towards developing the practice of practising teachers. A culture of the need for teachers to constantly interrogate their practice also takes root. There is a tendency among teachers to shift blame to learners or some other factors when teaching and learning do not produce the expected results. As an individual, I will also benefit immensely from this study. It will sharpen my research and analytical skills as well as my teaching practice.

Another significance of this research is that it can deepen teachers' understanding of well documented errors and misconceptions in the domain of probability by looking at them in a different perspective. Several researches on the use of representations in solving problems have been done. Suh (2007) argues that learners make more meaningful connections between new information and previously acquired knowledge when representing a mathematical idea in multiple modes. Thus, the practice of multiple representation of a mathematical problem promotes the development of mathematical proficiency. On the other hand, Zahner and Corter (2010) argue that external visual representations can facilitate probability problem solving at the stage of finding a solution strategy if an appropriate representation is chosen. Hence, this exploration will give an insight into the learners' wrong choices of solution strategies leading to inappropriate answers in solving probability problems.

1.9 Definition of terms used in the study

1.9.1 *Probability*

According to Hirsch & O'Donnell (2001) probability is the study of likelihood and uncertainty.

1.9.2 *Errors*

Errors are unintended or intended deviations from accuracy (Olivier, 1989). The errors are said to be systematic if they are recurring basing on a faulty hypothesis and non-systematic if they are non-recurring and random errors. This study is focusing on systematic errors.

1.9.3 *Misconceptions*

Nesher (1987) refers to misconception as an underlying incorrect premise or a faulty hypothesis that a learner refers to and generates a series of errors. In the context of this study, misconceptions mean an underlying premise that learners refer to in construction or interpretation of representations and generates a series of errors.

1.9.4 *Internal visual representations*

Internal visual representations are mental images or models of a word problem (Polya, 1957). According to Polya (1957), these images are mentally manipulated in the process of solving problems.

1.9.5 *External visual representations*

External visual representations are external inscriptions of mental images or models of a problem (Zahner & Corter, 2010). Zahner and Corter (2010) propose that problem solvers may externalise their mental inscriptions to aid in understanding of the text or when they have to share their solutions with others. These external inscriptions include pictures, drawings, outcome listings, tables, graphs and spatial re-organisation of given information.

1.10 Conclusion

This chapter outlined the context of the study and why it is relevant to mathematics education in South Africa at this time. The chapter also discusses how the study relates to the mathematics education curriculum in South Africa. The research problem was then deliberated. Also conversed were the purpose of the research, the research questions, significance and definitions of key constructs in the study. I outline the theoretical framework that guided this study and the literature that relates to the study in chapter two. The analytical framework and the categories used to analyse the data are also discussed in chapter 2. Chapter 3 describes the methodology including sample selection, data collection, how issues and how issues of validity and reliability were dealt with. Chapter 4 is on data analysis and discussions. Conclusions, findings of the study and recommendation end the research report with chapter 5.

CHAPTER 2: Theoretical framework and literature review

2.1 Introduction

In this chapter, I present the theoretical framework for this study, which is informed by Nesher (1987) and Sfard's (2007). The conceptual framework of this study is enlightened by Brodie and Berger (2010). I then review literature on errors and misconceptions in learning probability, types of errors, known misconceptions, use of representations and types of representations in probability.

2.2 Theoretical Framework

This study aims to investigate the errors and misconceptions that Grade 10 learners make in solving probability problems using different representations. Learning theories provide useful lens to explain and understand teaching and learning in a greater depth. However, characteristically lenses draw certain areas closer to the eye while ignoring other aspects and hence it is not likely possible to have a theory that encompasses every aspect about learning. For this reason, my study draws from Nesher's (1987) theory of errors and misconceptions to inform the process of error detection. Sfard's (2007) theory of commognition was also considered as it relates to representations and prevalence of error in learning. Brodie and Berger (2010) used both the constructivists' perspectives and Sfard's view of mathematics as a discourse to develop a discursive framework for learners' errors which include representations. Since my study draws from the constructivists and discursive perspectives, the discursive framework becomes a suitable and useful tool to classify and analyse the learners' errors and misconceptions in Grade 10 probability.

2.2.1 Nesher's (1987) theory of errors and misconceptions

Nesher's (1987) theory of errors and misconceptions is based on the idea that learners' contribution to their learning is their "expertise in making errors" (p. 33). In Nesher's terms, committing an error reveals the incompleteness of the learner's knowledge and enables the teacher to contribute additional knowledge to complete it or guide the learner to realise where s/he is wrong. The teacher therefore needs to tap into the learner's misconceptions rather than shy away from them. Hence, the learners' erroneous performance informs instruction. It is for this reason that the current study aims to investigate learners' errors in solving probability problems and the misconceptions from which the errors result.

Over and above anticipating the errors, Nesher (1987) suggests that it is important for a teacher to be "aware of the cases that discriminate between various types of misconceptions and those that do not discriminate misconceptions at all" (p. 36) as some misconceptions can give correct answers. For example, a learner may have a misconception that a decimal with more digits on the right of the comma is greater than the one with fewer digits. Consider the question; which decimal is greater between 0.5 and 0.456? If the learner answers 0.456 we

might suspect that s/he has this misconception. If the learner answers 0.5, we cannot know whether s/he understands order in decimals. Therefore, the teacher should intentionally look for discriminating items for particular misconceptions. However, Nesher's suggestions are dependent on the teacher being aware of learners' possible misconceptions. Nesher (1987) recommended that for teachers to know possible misconceptions and errors they produce, the teachers should:

a) Know how the new knowledge is integrated into larger knowledge system that the learner have;

b) Be aware how the previously learnt procedures may interfere with the new knowledge; and,

c) Clearly discriminate new elements from the old ones.

Figure 2.1 summarises the process of error and misconception detection as outlined by Nesher (1987).

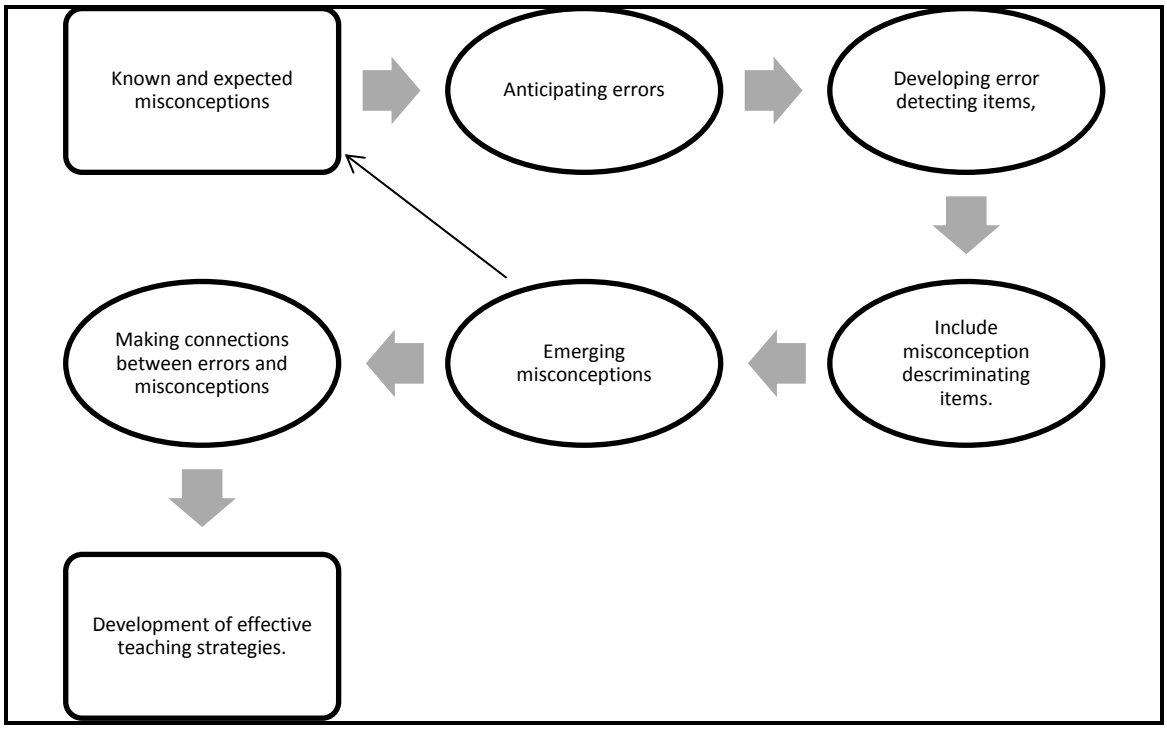


Figure 2.1: The process of error and misconception detection

Nesher's theory of errors and misconceptions focuses on error detection but it is silent on the use and effect of representations. It is also not crystal clear from which part of the subject content the teacher should look to implement Nesher's (1987) recommendations mentioned in Figure 2.1. Bearing in mind that many teachers do not find it easy to implement research

results especially if they are not very specific (Kazemi & Stipek, 2001) I turn to Sfard's (2007) theory of commognition for insight in relation to representations and prevalence of errors.

2.2.2 Discourse and the prevalence of errors (Sfard, 2007)

Thinking is dialogical in nature as it involves "informing ourselves, arguing, asking questions and waiting for our own responses" (Sfard, 2007; p. 572). Therefore human thinking can be defined as "communicating with oneself" and thus regarded as a form of discourse (Sfard, 2007). Sfard (2007) argues that, if mathematical thinking can be defined as "the activity of communicating with oneself" (p. 571) then mathematics can be seen as a discourse. For Sfard (2007), a discourse is considered mathematical if it consists of mathematical words, visual mediators, narratives and routines, and these are discussed in the next sections.

2.2.2.1 ***Mathematical words*** include words relating to shape and quantities. Cubic, rectangle and negative 4 (minus 4), among others, are examples of mathematical words. These words have a peculiar meaning in mathematics which may be different in everyday language. Learners need to comprehend the mathematical words correctly for them to be able to relate them to pre-learnt procedures and select the correct one to solve a mathematical problem.

2.2.2.2 ***Visual mediators*** can be symbolic artifacts like formulae or images of concrete objects like graphs, diagrams and drawings. Visual mediators are means by which "participants identify the object of their talk and coordinate their communication" (Sfard, 2007, p. 573).

2.2.2.3 ***Narratives*** are written or spoken texts which are a description of objects or relationships between objects. Narratives can be at object-level (stories about objects, eg. $y = 2x + 1$) or at meta-level (stories about the discourse itself, eg. When calculating for an unknown, make the unknown the subject of the formula). Narratives are therefore subject to endorsement or rejection. The endorsement criteria are different from one discourse to another. It is from these narratives that misconceptions are most likely to originate.

2.2.2.4 ***Routines*** are well-defined repetitive patterns involving word use, mediator use and endorsing narratives in interlocutors' actions (Sfard, 2007). A routine's general direction is regulated by object-level and meta rules of the discourse in which it is applied. Therefore a learner needs to have a good understanding of the words, mediators and narratives for him/her to execute routines appropriately.

Sfard's description of mathematics as a discourse provide teachers with direction to which part of the subject content they should look to implement Nesher's (1987) recommendations. Hence, to determine possible misconceptions which may result into errors, the teacher needs

to analyse the words, visual mediators, narratives and routines of the learners' previously learned knowledge and the new work to be learnt.

According to Sfard (2008) misconceptions result from applying meta-rules of one discourse to another discourse where they no longer hold. This happens when the discourse changes and the learner is not aware of the change. However, in a different study, Brodie and Berger (2010) discovered that some errors are not necessarily caused by misconceptions. These authors developed a discursive framework to account for learners' errors resonating from both the constructivists and participations notions of errors and misconceptions which I will discuss below.

2.3 Conceptual framework

In order to describe and account for learners' errors in the current study, I draw from Brodie and Berger's (2010) discursive framework for learner errors in mathematics. Brodie and Berger (2010) locate errors as rooted in narratives and are manifested in the words, visual mediators and routines. The discursive framework consists of three basic categories of errors which are; errors of mediators, errors of signifiers and errors of routines. In the next sections I describe what each category of error entails according to Brodie and Berger (2010).

2.3.1 Errors of visual mediators

Errors of visual mediators can be divided into three categories. These categories are inappropriate visual scan, inappropriate use of visual detail and difficulty with visual construction. These are discussed in detail below.

2.3.1.1 *Inappropriate visual scan*

According Sfard (2008), written symbols, like formulae, tables and numerals, visually mediate mathematical thinking. Therefore if a learner visually scans inappropriately, it may result in the learner inferring relationships between symbols without considering the underpinning mathematics (Brodie & Berger, 2010). Therefore there are variations between the data as given in the item and how the learner denotes it. This type of errors concurs with misused data errors (Movshovitz-Hadar, Zaslavsky & Inbar, 1987) which may occur due to misreading at the beginning or during a problem solving process. The choice of procedure carried out by the learner is not because of the mathematical relationship between the different signifiers involved. Hence, errors of this kind do not necessarily result from misconceptions.

2.3.1.2 *Inappropriate use of visual detail*

In questions involving interpreting diagrams or graphs, an error is regarded inappropriate use of visual detail if some information is ignored when interpreting the question (Brodie & Berger, 2010). Brodie and Berger (2010) further noted that, if other information which is not

given in the question is assumed and used by the learner in interpreting the question it is also referred to as inappropriate use of visual detail.

2.3.1.3 *Difficulty with visual construction*

Certain questions require the learner to construct the visual mediator (concretely or imagination), which can be pictures, spatial re-organisation of given information, listing outcomes, contingency tables, Venn diagram, tree diagram or probability matrix. In the current study all of these notions constitute mathematical representations. An external visual representation can facilitate probability problem solving when it is appropriate and correctly constructed (Zahner & Corter, 2010). However, some learners are not eloquent in visual construction or drawing (Brodie & Berger, 2010). In this case, a faulty visual construction will lead to a wrong answer signifying the error due to difficulty with visual construction.

2.3.2 Errors of Signifiers

Errors of signifiers are sub-divided into two groups. These include errors due to familiarity as well as difficulty with visual construction.

2.3.2.1 *Difficulty with visual construction*

Learners are often guided by previously known signifiers in familiar discourses on how to use new and similar signifiers. Thus the learner "inserts a new signifier into a familiar discursive template" (Brodie & Berger, 2010, p. 175). For example, due to the amount of time spend on linear relations like proportion, learners become very familiar with proportion in such a way that they will attempt using proportion to solve problems even when it is not appropriate (Van Dooren, De Bock, Depaepe, Janssens & Verschaffel, 2003).

2.3.2.2 *Familiarity*

Learners may choose a certain familiar routine or representation to solve a probability problem because s/he recognises it or has prior meaning to him/her (Brodie & Berger, 2010). Zahner and Corter (2010) argue that certain choices of representations are associated with higher rates of solution success. Therefore learners might inappropriately choose a representation due to its previous success rate. For example, a learner who is familiar with a tree diagram and believe that it usually leads to correct solutions would use it even in a situation where it is not appropriate.

2.3.3 Errors of Routines

Errors of routines include errors due to keyword triggers as well as halting signals. These are discussed in the next sections.

2.3.3.1 *Keyword trigger*

Keywords maybe words which appear in both the everyday and in the mathematical register resulting in the keywords having ambiguous meaning to the learner (Brodie & Berger, 2010).

A certain word or term in the question may signal to the learner that s/he should apply a particular routine to solve the problem. Consider the question: *what is the probability of scoring a mark less than 10 in the test?* The word less is known to trigger the use of subtraction even when it is not mathematically correct (Kilpatrick, Swafford & Findell, 2001). Therefore, if a word causes an incorrect response, it becomes a keyword trigger.

2.3.3.2 *Halting signal*

A learner expecting a proper fraction as an answer may accept a routine to be complete when s/he derives a proper fraction. A multiple choice distractor which causes the learner to prematurely stop the routine becomes a halting signal. Brodie and Berger (2010, p.174) describes a halting signal as a "trigger for premature closure of a routine".

Figure 2.2 summarises the classification of errors and misconceptions as outlined by Brodie and Berger (2010).

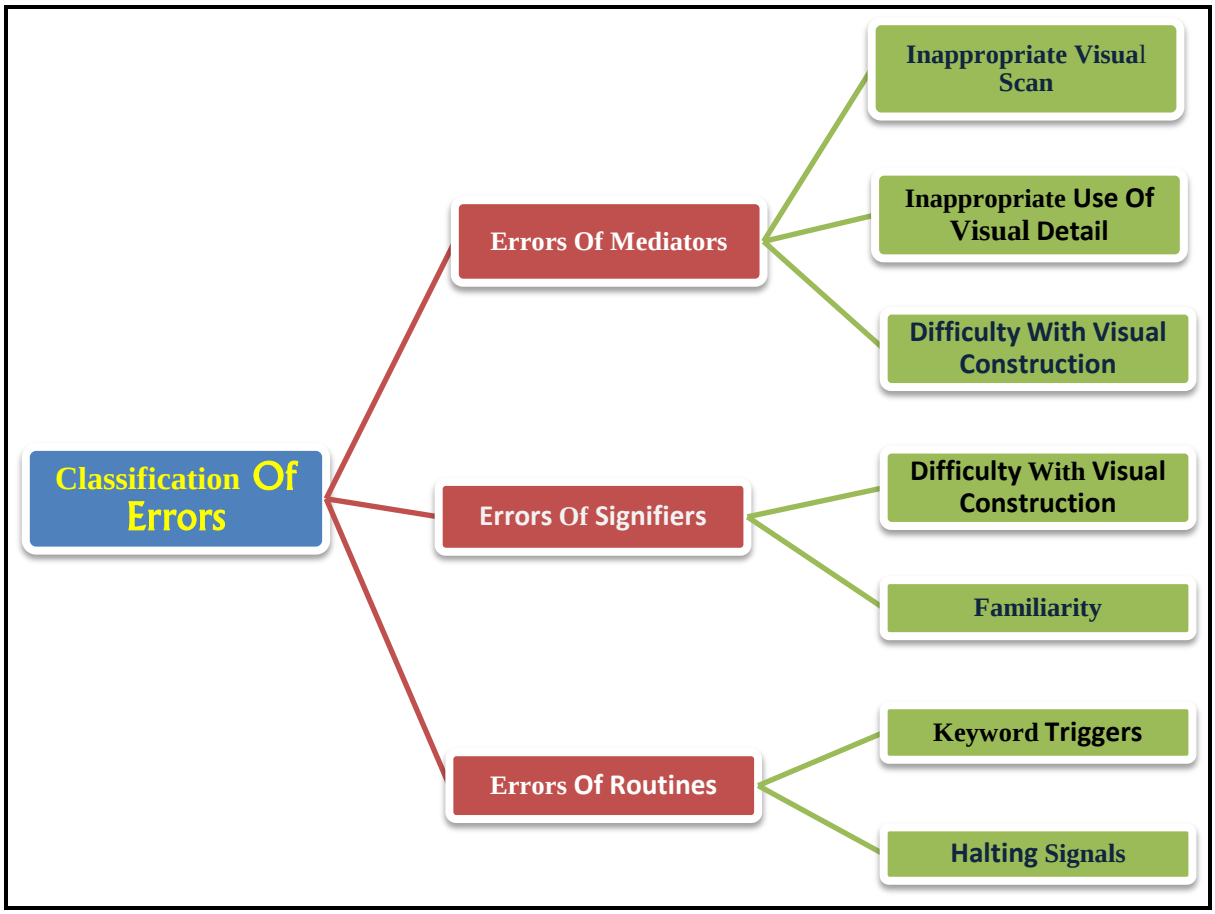


Figure 1.2: Classification of learner errors according to Brodie and Berger (2010)

For the purposes of this study, the categories mentioned in Figure 2.2 are the main tool, but not limited to them, of classification and analysis of the learner errors as well as determining the reasons for the errors. Unlike Brodie and Berger (2010) who only analysed distractors which were chosen by more than 25% of the learners, errors as pointed to by discriminating items in the test were analysed. In some cases, learners whose errors cannot be accounted for in relation to the discriminating items in place were interviewed.

2.4 Literature Review

This study intended to investigate the errors and misconceptions the learners make when solving probability problems. In order to be able to answer my research questions, I draw from four sources of information: theories on errors and misconceptions; types of errors; known misconceptions in probability; visual representations in probability.

2.4.1 Errors and misconceptions in mathematics education

Cognitivists like Piaget (1964) and Vygotsky (1986, 1978) view learning as the acquisition of more sophisticated conceptual structures in the mind. The mind is where the biological factors and social factors interact, creating cognitive conflict (Brodie & Berger, 2010). This happens through the process of equilibration, which consists of assimilation and accommodation (Piaget, 1964). Assimilation can be described as a process of acquiring new knowledge by associating it with existing knowledge (Hatano, 1996). For example in learning algebraic fractions, learners may assimilate the concept of adding algebraic fractions by linking it to ordinary fractions. Thus, new knowledge must be familiar to the existing knowledge structures, schema or conceptual framework for assimilation to be possible (Nesher, 1987; Olivier, 1989; Piaget, 1964). Accommodation occurs when learners encounter a concept which is different from what they know (Hatano, 1996). Re-organisation of existing structures or creation of a new structure becomes necessary (Hatano, 1996) because learners do not have any structures to interact with the new concept as it is very different to the existing concepts in place. It can then be said that accommodation begins with the state of uneasiness and ends with a shift of position which may be comfortable or uncomfortable.

For constructivists, it is through equilibration that misconceptions may result. As learners assimilate or accommodate new concepts better understanding might be enhanced or misconceptions might occur (Hatano, 1996; Nesher, 1987; Olivier, 1989). According to Olivier (1989) misconceptions arise from attempts to connect or integrate new knowledge with existing knowledge. This implies that the misconceptions, and the errors resulting from them, are quite normal in the process of acquiring knowledge (Brodie & Berger, 2010; Smith, DiSessa & Roschelle, 1993). In other words learning includes dealing with errors and misconceptions. Thus the occurrence of errors, which create difficulties in understanding, promotes learning.

Teachers may find learners' misconceptions difficult to understand but they actually make sense to learners (Erlwanger, 1973). Misconceptions make sense to the learners because they are derived from previous instruction (Nesher, 1987) although they are not explicitly taught (Brodie & Berger, 2010). Smith et al. (1993) also propose that misconceptions result from overgeneralisation of concepts from one domain to another. However, in their study, Brodie and Berger (2010) found out that some errors are not necessarily caused by misconceptions. Brodie and Berger (2010) suggest that errors occur when certain rules are used in different discourses which are incommensurable. This idea concurs with Sfard's (2007, 2008) idea that misconceptions result from applying meta-rules of one discourse to another discourse where they no longer hold. Sfard (2007) argues that misconceptions occur when the rules of the discourse change but nobody tells you. As a result the rules are applied inappropriately to situations they no longer hold. It was also reported by Smith et al. (1993) that errors and misconceptions they result from cannot be corrected through normal classroom instruction. Hatano (1996) also suggested that since misconceptions are mental constructions, they need to be restructured by the learners into mathematically correct and acceptable structures. This raises the question; what is the nature of errors?

2.4.2 Types of learner errors

According to Makonye (2011), errors can be classified into domain general errors and domain specific errors. Domain general errors refer to errors that are not subject specific while domain specific errors are subject or topic specific. Davis (1984) also noted that there are error patterns which are common across different learners and those that are peculiar to individual learners. In this study both types will be considered.

Donaldson (1963) suggested that both the domain general errors and domain specific errors can be classified into three categories. These categories are arbitrary, structural and executive errors. These categories still hold today. As a result, Makonye (2011) referred to these categories as the most important classification of errors in the teaching and learning of mathematics.

2.4.3 Donaldson's (1963) classification of errors

2.4.3.1 ***Arbitrary errors*** are errors resulting from selective processing of information and ignoring other attributes. For example $\frac{1}{8}$ is considered the same as **8**. These kinds of errors occur when learners ignore part of the available information while acting on the rest of the information (Donaldson, 1963). In some cases, when learners fail to take account of the given information, they choose to forcibly fit the question to what they know or are familiar with (Makonye, 2011) resulting in arbitrary errors.

2.4.3.2 ***Structural errors*** are errors due to "failure to appreciate the relationships involved in the problem or to grasp some principle essential to solution" (Donaldson, 1963, p. 41). These errors emanate from a lack of understanding of ideas inherent in a mathematical problem. Therefore structural errors are conceptual in nature. That is, when learners have fallacious perceptions about the nature of mathematical concepts, it leads to structural errors. For example an elementary arithmetic problem taken from Cramer, Post and Currier (1993) is stated below:

Sue and Julie were running equally, fast around a track. Sue started first. When she had run9 laps, Julie had run 3 laps. When Julie completed 15 laps, how many laps had Sue run?

Thirty-two out of the 33 pre-service teachers who attempted this item applied proportion to solve this problem as follows:

$$\frac{9}{3} = \frac{x}{15} \;\; so\ that\ x = 45\ .$$

The multiplicative structure is not correct for this problem as the problem has an additive structure. The error exhibited by these pre-service teachers is structural, because they failed to grasp an essential rule to the solution (Donaldson, 1963).

2.4.3.3 ***Executive errors*** refer to those errors which are due to failure to carry out procedures or manipulatives. This can happen even when the required concepts have been understood (Orton, 1983). Errors such as failure to express a fraction as a percentage are executive. For example, $\frac{1}{4} = 25\%$. The learner goes on to write $\frac{1}{4} = 0.25\%$. In this case, the learner expresses the fraction as a decimal and leave out multiplying by 100 to convert the decimal to a percentage.

Later, Movshovitz-Hadar, Zaslavsky and Inbar (1987) proposed another classification model. These authors proposed six categories of errors as discussed in the next sections.

2.4.3.3.1 Misused data errors which are due to variations between the given data in the item and how the learner relates to the data. This kind of error could also be a result of misreading the item at the beginning or during the problem solving process. This misuse of data will then be carried on to further calculations.

2.4.3.3.2 Misinterpreted language errors are mathematical errors resulting from poor interpretation of language including mathematical symbolism. Such errors may happen during modelling the problem or encoding natural language into mathematical expressions. For example, in answering the question; what is the probability that when choosing a boy at random he will be wearing contact lenses? Some learners will fail to interpret that the sample space is the

population of boys in the target population, thereby exhibiting errors if misinterpreted language.

2.4.3.3.3 Logically invalid inference errors are due to false generalisation of old knowledge into new knowledge. For example, applying the routine for multiplication of fractions over addition of fractions.

2.4.3.3.4 Unverified solutions errors (slips) occur when learners work correctly on a solution that directly does not address a given problem. The learner may correct their own error if they check their solution carefully.

2.4.3.3.5 Technical errors are those errors which occur due to failure to carry out calculations, process computational algorithms, reading data from tables, algebraic errors such as writing $3x - 2 - 2x + 1$ instead of $(3x - 2) - (2x + 1)$.

2.4.3.3.6 Random errors are non-systematic errors which do not reccur and do not make any pattern.

Although some of Movshovitz-Hadar et al. (1987) categories are subsumed by Donaldson's (1963) categories, they shed more light in to types of errors. These categories indicate the possible reasons for the errors.

2.4.4 Reasons for errors (Radatz, 1979)

Radatz (1979) also proposed a model of classification of errors in different mathematics topics and concepts based on the reasons for the errors. The categories are lack of mastery of language, difficulties in obtaining visual information, deficient mastery of prerequisite knowledge and skills, incorrect associations or inflexible thinking and application of irrelevant rules or strategies. Even with knowledge of all the types of errors and possible reasons for the errors discussed above, it is not easy to detect errors. Nesher (1987) maintains that it is not easy to detect errors if the teacher is not anticipating them.

2.4.5 Known misconceptions in probability

As mentioned above, a teacher can easily identify errors if he/she is anticipating them. Being aware of the known misconceptions equips the teacher for error identification. There are four known misconceptions in probability which include misconception of representativeness, illusion of linearity, recency effect and inappropriate distinction between simple and compound events. I describe what each misconception entails in the next sections.

2.4.5.1 Representativeness refers to estimating uncertain events based on sample reflection of population events (Hirsch & O'Donnell, 2001). For example, learners with this misconception will think that if a fair coin is tossed six times, the probability of getting an

ordered sequence HHHHHH is less likely than that of getting HTHTHT. This misconception also includes the neglect of size in comparing two probabilities, where learners ignore the relevancy of the law of large numbers (Hirsch & O'Donnell, 2001). For example, Fischbein (1999) gave the following problem to learners in grade 5 to 11:

The likelihood of getting heads at least twice when tossing three coins is smaller than/equal to/greater than the likelihood of getting heads at least 200 times out of 300 times.

The learners were asked to choose the correct phrase from the words in italics. Fischbein (1999) found that most of the learners at each grade said that the probabilities are equal. Most learners used the equality of ratios to justify their answers.

2.4.5.2 Illusion of linearity refers to improper application of linear relations where they do not apply (Van Dooren et al., 2003). For example, Fruedenthal (1973, p. 594) noted that when asking students "how many times must a die be thrown to get an equal chance of at least one six?" the students invariably answered; three. The reasoning is: probability of success in one trial is $\frac{1}{6}$. Therefore throwing 3 times, the probability becomes $\frac{1}{6} \times 3 = \frac{1}{2}$. The erroneous nature of this reasoning is revealed by the fact that, if more than 6 throws are done, the probability will be more than 1 (Van Dooren et al., 2003).

2.4.5.3 Recency effect: refers to the effect that recent events have on future events which stems from improper distinction between independent and dependent events (Fischbein & Schnarch, 1997). Consider the question, "If a couple already have two sons and are considering a third child, are they more or less likely to have a daughter the next time?" Most learners fail to realise that events such as gender of a baby are independent events and consequently the outcome of one event does not influence the outcome of the next. This misconception also includes the tendency to judge probability of the whole to be less than the probability of the parts (Kustos & Zelkowski, 2013).

2.4.5.4 Improper distinction between compound and simple events

According to Fischbein and Gazit (1984), the notion of a simple event is connected with the model of a single action while the notion of a compound event is related to a representation of more than a single operation, for many learners. Thus, for these learners, a single event has one result and a compound event has multiple results. This misconception includes using an additive procedure instead of the appropriate multiplicative procedure for calculating the number of all possible outcomes in compound events. For example, the number of possible outcomes when sums of numbers are considered in rolling a pair of dice is $6 \times 6 = 36$ not $6 + 6 = 12$.

2.4.6 Visual representations and problem solving

The importance of mental representations in problem solving has been acknowledged a long time ago by researchers such as Polya (1957). However, there is little evidence to support this idea, perhaps due to the difficulty involved in studying mental imagery (Douville & Pugalee, 2003). Corter and Zahner (2007), in their study on the process of probability problem solving, concluded that internal representation or mental model of a word problem should be constructed in order to solve the problem successfully. However, it is not always possible to get an insight into the learners' mental representations unless they create some form of external visual representations. A substantial amount of evidence has accrued to show that visual representations, diagrams in particular, can be of great assistance in problem solving (Corter & Zahner, 2007; Lesh, Landau & Hamilton, 1983; Schwartz & Martin, 2004; Suh & Moyer 2007). Many of the earlier studies concluded that external representation promotes the development of the learner's conceptual understanding of the problem. Lesh, Landau and Hamilton (1983) developed a model to explain the importance of representations in learning mathematics (see, Figure 2.3).

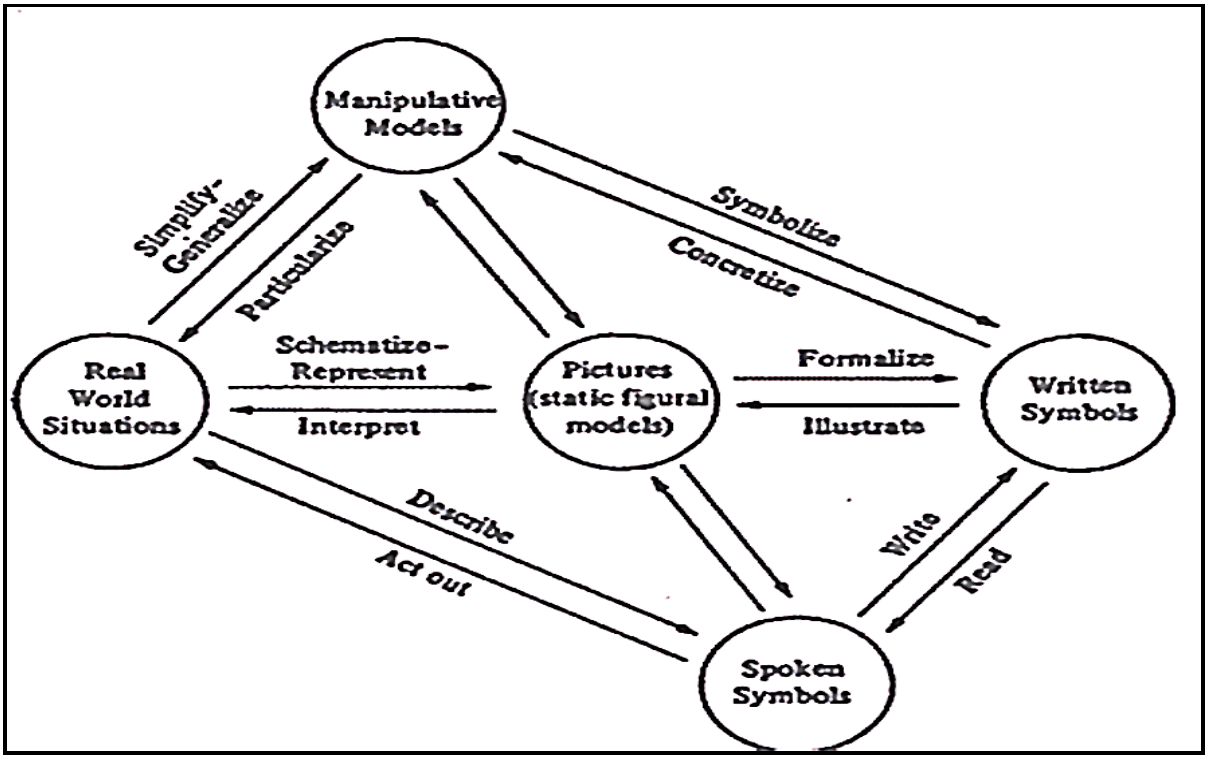


Figure 2.3: Lesh, Landau & Hamilton's (1983) model of the five types of distinct types of representation system (*Adapted from Suh and Moyer, 2007)*

Figure 2.3 illustrates that learners develop a deep understanding of mathematical ideas when they experience and make connections of the ideas in different modes. Using Lesh et al. (1983)'s work, Suh and Moyer (2007) argue that "the ability to translate among different

modes of representation indicates deeper conceptual understanding within the system" (p. 157). Suh and Moyer (2007) also concluded that representing mathematical ideas in different forms promotes relational thinking and develop algebraic thinking in learners. These external representations can be divided into learner generated diagrams and experimenter provided diagrams. Another common finding is that experimenter-provided external visual representations lead to more success in problem solving than learner constructed visual representations (Zahner & Corter, 2010).

Problem solving in probability occurs in four stages. The stages are text comprehension, mathematical representation, strategy formulation and execution of solution (Zahner & Corter, 2010). In their research on the process of probability problem solving, focusing on use of external visual representations Zahner and Corter (2010) found out that the four stages of probability problem solving do not occur in a strict linear order. External visual representations are usually created and used during representing the problem mathematically and finding a solution strategy (Zahner & Corter, 2010). These authors presented the model shown in Figure 2.4 to illustrate the possible order in which the stages can occur during the process of problem solving.

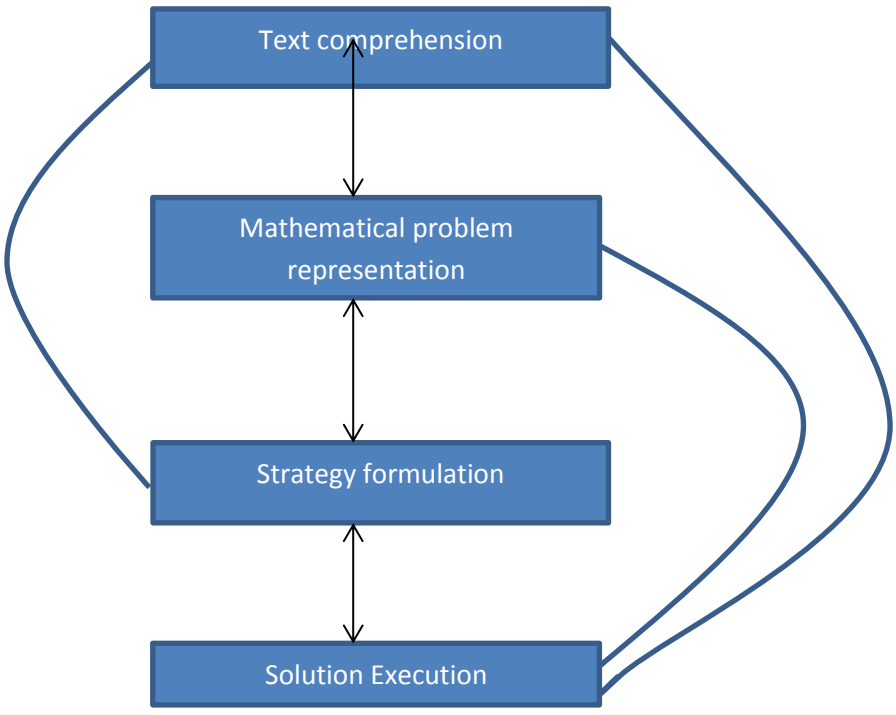


Figure 2.4: The four stages of probability problem solving (Adapted from Zahner & Corter, 2010)

Zahner and Corter (2010) also found that learners spend about 5% of their time on text comprehension, 56% on mathematics problem representation, 19% on strategy formulation and 20% of their time on solution execution. Considering the amount of time spend on

representation of the mathematics problem, representations are crucial to problem solving. It should be noted that some learners may use external representations for various reasons. It is the external representations that can be seen, assessed and analysed while the presence of an internal representation is only implied by action or response given. The representations on focus in this study range from representations that accompany text describing problems to those constructed by learners.

2.4.7 Types of external visual representations

External representations include pictures, drawings, outcome listings, tables, graphs and spatial re-organisation of given information (Zahner & Corter, 2010). Representations which depict relationships described in the problem like Venn diagrams and tree diagrams are called schematic representations while iconic or pictorial representations refer to representations depicting physical appearance of the elements like tallies, outcome listings, contingency tables and graphs (Zahner & Corter, 2010). In their earlier study, Corter and Zahner (2007) discovered that learners may use external visual representations to help in:

a) Text comprehension and summarising problem information;
b) Recording and reasoning about the situation;
c) Freeing memory storage;
d) Making abstract relationships concrete; and,
e) Coordinating results of intermediate calculations for use in later calculation steps.

It is noted that if appropriately chosen and correctly constructed, external representations can facilitate problem solving (Zahner & Corter, 2010). However, use of specific external representations is associated with specific topics. In particular, contingency tables, matrix method and venn diagrams are used for compound events while outcome listings and tree diagrams are used for sequential experiments in probability (Zahner & Corter, 2010). However it should be noted that Hegarty and Kozhevnikov (1999) found that iconic representations led to a lower success rate in solving mathematical problems than the schematic representations.

2.5 Conclusion

In this chapter, I have discussed the theoretical framework informing this study as well as the literature on the representations, errors and misconceptions in probability. In the next chapter I will discuss the research methodology and research design of the current study.

CHAPTER 3: Research Methodology

3.1 Introduction

This chapter describes the research design and methods employed to elicit data needed to answer the research questions of this study. The rationale for the choices of methods used in this study is also deliberated. The study is a case study of Grade 10 learners' responses to some probability tasks. The study aimed to investigate the errors and misconceptions that learners make in relation to solving probability problems using varied representations (see section 1.6).

3.2 Research design

In this this study I used qualitative research methods as I felt that this was the most appropriate in understanding learners' errors and misconception when using different representations to solve probability tasks. According to Brantlinger, Jimenez, Klingner, Pugach, and Richardson (2005, p. 195) qualitative research concerns a "systematic approach to understanding qualities, or the essential nature, of a phenomenon within a particular research". Also, qualitative research provides evidence on a particular phenomenon based on the exploration of specific contexts and particular individuals (Brantlinger, et al., 2005). Opie (2004) defines qualitative research as research that seeks to attain insights into how individuals interpret and modify the world about them. (Leedy, 1997) contends that qualitative researchers often collect large amounts of verbal data from a small number of participants and present their findings in descriptions which accurately reflect the situation under study. These descriptions echo with the characteristics and focus of this study. However, the credibility of a qualitative research report relies heavily on the confidence readers have in the researcher's ability to be sensitive to the data. Lincoln and Guba (1985, p. 120) argue that "If you want people to understand better than they otherwise might, provide them information in the form in which they usually experience it". I have chosen a qualitative approach mainly for the fact that a qualitative research design involves an in-depth study to understand a phenomenon which makes it a suitable choice in investigating the learners' errors and misconceptions involving representations in probability.

A qualitative approach was also chosen for this study because it has less structured protocols (Mack, Woodsong, MacQueen, Guest & Namey, 2005), which allows the researcher to use multiple data collection methods, change the data collection strategy by refining, adding or dropping techniques and in some cases the informants. Also, qualitative studies involve triangulation which increases the credibility of the results (Gill, 2008). The time frame of this project, one year, also influenced the research design and methods of data collection as more time will be needed for a quantitative study.

This study is what Borg (1987) refers to as descriptive case study. Detailed descriptions of a phenomenon are obtained from a case study and used to develop possible explanations of it or evaluate the phenomenon. This was done by examining learners' written responses to the given probability tasks. The study is also exploratory in that a selected few learners were interviewed to get an insight into the learners' thought processes as they engaged with the tasks. I was interested in both knowing the errors involved in solving probability problems using different representations and understanding the reasons behind the errors.

Since no two cases are the same, results of case studies are difficult to generalise. The results of a case study apply more specifically to the case upon which the research was based. The evidence provided by the case study can be used to seek general patterns among different studies of the same issue (Gill, 2008). Cohen and Manion (1994) argue that generalisation is possible in a case where the case study is a typical example of those we wish to generalise the results. The school under study, which is called JBC (not its true name, to protect its privacy), is a typical example of former model C[2] schools in which Afrikaans was the language of instruction before 1994[3]. Thus, JBC is a typical example of multilingual school where learners learn in English which is not their first, home or main language. However, based on the nature of the study, the aim of this study is not to generalise.

3.3 The empirical field

This study was carried out at my current school JBC. The school became English medium after 1994 as a way to address social inequalities by the South African government. Just like all other former model C schools, JBC started admitting black children and educators when South Africa became a democratic state. Therefore, children come from both the coloured and black high/low density communities around the school. The number of black learners and teachers has steadily increased, maybe because most learners and parents prefer to learn mathematics in English due to economic, ideological and political factors (Setati, 2008). As a result of the increasing multilingualism, English as a language of learning and teaching becomes a necessity because it is the common language of the school community. The school also take examinations in English with the exception of Afrikaans first additional language. All the learners take English to be their home language even though they are multilingual. Also, all the classes in the school are mixed ability.

At the time of the current study, the staff composition of the school that participated in the research was 60% black and 40% coloured educators. All the mathematics teachers are qualified to teach mathematics. Three out of the six mathematics department educators are

[2] "Model C" was a semi-private school structure established by the apartheid government of South Africa between 1990 and 1993, with decreased funding from the state, and greatly increased autonomy for schools.

foreign. One out of the three local educators is coloured and can only speak English and Afrikaans which are a second language to most black learners.

As of 1994, the school composition has gradually shifted from coloureds only to a mixture of blacks and coloureds over the years. The student body is made up of about two thirds black and one third coloured learners. The coloured learners either speak Afrikaans or English at home while there is a whole spectrum of the nine official local languages among the black learners. Therefore any given class at JBC School presents the features of a multilingual classroom. The Grade 10 classes participating in the study were chosen because I teach them. I ensured that they receive the required instruction.

3.4 Sampling

In this study, purposive sampling was used. This is where the participants are chosen for a specific purpose. The sample is chosen because it is likely to be knowledgeable and informative about the situation under study. The participants are hand-picked by the researcher on the basis of an estimation of their typicality (Opie, 2004). This is done to increase the efficacy of information obtained from small samples. Essien (2013, p. 63) contends that "the power of purposive sampling is that a few cases studied in depth yield many insights about the topic". For the purpose of this study it was important to work with learners who had received instruction on probability using different representations. The learners chosen for this study are Grade 10 learners who were all taught by the researcher.

For the purposes of this study, data were collected from my current Grade 10 mathematics classes whose parents had given informed consent for their participation in the study. The group consisted of 14 girls and 8 boys to make a total of 22 participants. The sample was a mixture of black and coloured, mixed ability learners. All the participating learners had been taught probability using different representations.

3.5 Data collection methods

Data was collected in two stages. First stage, tasks based on probability were given to the learners. The tasks took incorporated aspects of representations used to solve probability questions such as the tree diagram, contingency table, matrices and outcome listings among others. The tasks have some free response as well as multiple choice items to check consistency in learners' responses. Learners' scripts were marked, taking particular note of the errors that the students make. The errors were analysed and notes were made. Second stage, six learners were selected for semi- structured interviews based on the type of errors they made. This was done to obtain an understanding of the learners' thought processes as they were engaging with the tasks and elicit an insight into the reasons for their errors. The six learners were then interviewed and audiotaped.

3.6 Why a Task Instrument?

In order to address the research questions set out above, I opted to use tasks as the main data collection instrument. In selecting the tasks instrument, I was influenced by Nesher's (1987) work on errors and misconceptions. Nesher (1987) argues that in the realm of classroom instruction, "*being wrong* and making *errors* are negatively connotated" (p. 34). This implies that in normal classroom instruction detecting errors maybe difficult because learners may decide not to make contributions so that they do not make mistakes and keep their esteem intact. This may mean that some errors and misconceptions we need to address will go unnoticed for a long time (Nesher, 1987). Using tasks instead of lesson observation or interview provided learners with uninterrupted time to interact with the problems without fear of their peers finding out about their errors.

Nesher (1987) also noted that errors are very difficult to detect if the teacher is not anticipating them in such a way s/he have discriminating items in place. This is easier done in tasks. Every learner is presented the same tasks with discriminating items carefully selected and placed in the tasks to elicit erroneous behaviour. Each learner had equal chance of exposing their errors and misconceptions. This kind of careful planning and implementation is not easy in normal classroom instruction. Therefore, written tasks become a favourable instrument over lesson observation or oral test for my study.

According to Flanagan, Mascolo and Hardy-Braz (2009), tasks provide information regarding the areas of strength and weakness of learners. Erroneous behaviour can be regarded as a learner's weakness. However, tasks by nature, reflect behaviour or ability at a single point which can be affected by fatigue, attention or even the fact that the tasks will not count for term or report mark (Ensor, Dunne, Galant, Gumedze, Jaffer, Reeves & Tawodzera, 2002). Flanagan et al. (2009) argue that in cases where learners receive the same tasks under the same conditions, the results can be documented, empirically verified and can be used to compare grade peers. This allows for the tasks results to be interpreted and the ideas about a learner's skills may be generalised for the class or institution.

Considering that standardised tests have been criticised for having items that are often not related to the required classroom tasks and behaviours (Flanagan et al., 2009) I opted for researcher developed and curriculum based tasks. My decision was influenced by the advantages and disadvantages of locally developed tasks according to Flanagan et al. (2009) as outlined in Table 3.1

3.7 Advantages and disadvantages of locally developed and curriculum based tasks

The advantages and disadvantages of tasks that are developed locally and based on the curriculum are listed in Table 3.1

Table 3.1: The pros and cons of the implemented curriculum

Advantages	Disadvantages
Can be tailored to match program and institutional objectives.	Complex and time consuming to develop.
Specific criteria for performance can be established in relation to the curriculum.	Requires considerable leadership and coordination.
Can be used to develop locally meaningful norms	If it means that the test would be revised, it may hinder curriculum change.
Cheaper than commercial standardized test.	Vulnerable to student theft and distribution.
Maybe embedded in specific lesson or course.	Results cannot be generalised beyond the program or institution.
Easier to use in a pre and post-test approach.	

For the purposes of this study, the advantages mentioned in Table 3.1 outweigh the disadvantages. It was therefore important to use researcher developed tasks which are curriculum based with error diagnostic and misconception discriminating items especially constructed and carefully placed.

3.8 The Tasks Instrument

In designing tasks to detect errors, the teacher should be aware and anticipating the known misconceptions (Nesher, 1987). Therefore, to be comprehensive, the tasks should have items to address the four misconceptions mentioned in section 2.4.5 and discriminate one misconception from the other (Nesher, 1987). According to Brodie and Berger (2010), some errors in solving problems are a result of inappropriate visual scanning, inappropriate use of visual detail and difficulty with visual construction, use of familiar representations or routines, key word triggers and halting signals (see, section 2.3). Therefore in anticipation of these mentioned possible errors, the tasks consisted of questions where the different representations are given and some where the learners had to construct the representations.

The task items for this study were designed drawing from Hirsch and O'Donnell's (2001) test items which were designed to elicit learners' explanations while preserving the efficiency of multiple-choice formats. Section A consists of three free response questions designed to elicit the learners' conception of probability (see, Appendix A). Responses to these three questions were not included in the scoring of the test. However, responses to these items were analysed together with section B and section C. In section B, the two-part format as used by Hirsch and O'Donnell's (2001) was adopted, where the learner had to choose the correct answer for the question in the first part and choose the best suitable reason for the answer in the second part. While the first part identifies the errors, the second part provides reasons for the chosen answer. The second part was used to give me insight into the misconceptions which might be causing the errors. Section C consists of free response questions intended to elicit learners' erroneous behaviour as well as an insight into reasons for them. Below is an example of section B tasks. The task is meant to find out if the learners can read, interpret the diagram and use the available details appropriately.

A box contains 6 white cubes and 8 black cubes. The tree diagram below shows the possible outcomes of randomly choosing a cube from the box on two separate and consecutive draws (with replacement).

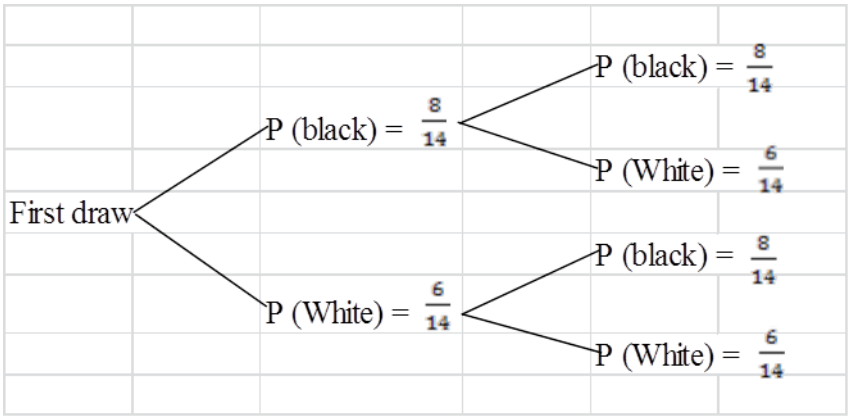


Figure 3.1: The tree diagram showing the possible outcomes of randomly choosing a cube from the box on two separate and consecutive draws

1.1.Calculate the probability that, on two successive draws, a black cube will be drawn.

a) $\frac{8}{14}$ b) $\frac{6}{14}$ c) $\frac{16}{14}$ d) $\frac{16}{49}$

1.2.Which of the following best describes the reason for your answer to Q1.1 above?

a) Probability of drawing a black cube is always $\frac{8}{14}$.

b) Probability of drawing a black cube on two successive draws is $1 - \frac{8}{14} = \frac{6}{14}$.

c) Multiplying probabilities on successive branches: $\frac{8}{14} \times \frac{8}{14} = \frac{64}{196} = \frac{16}{49}$

d) Adding probabilities on successive branches: $\frac{8}{14}+\frac{8}{14}=\frac{16}{14}$.

1.3. Calculate the probability that a black cube is drawn first and then a white cube.

a) $\frac{8}{14}$ b) 1 c) $\frac{12}{49}$ d) $\frac{6}{14}$

1.4. Which of the following best describes the reason for your answer to the Q1.3 above?

a) $\frac{8}{14}$ Because the black cube was drawn first.

b) We add probabilities of drawing each colour cube $\frac{8}{14}+\frac{6}{14}=\frac{14}{14}=1$.

c) Multiplying probabilities on successive branches with favoured outcome (Black then white) $\frac{8}{14}\times\frac{6}{14}=\frac{48}{196}=\frac{12}{49}$

d) $\frac{6}{14}$ Because the white cube is drawn last.

e) Other _____

3.9 Semi-structured interviews

In order to check consistency of the errors as well as get insight into the reasons for the errors, semi structured interviews were used. Semi-structured interviews impose a shape into the interview which prevents aimless rambling but is flexible enough to provide opportunities to probe and expand on the interviewee's responses (Opie, 2004). Semi-structured interviews also allow deviation from a prearranged text, change the wording of the question or order in which the questions are asked to suit the situation and accommodate the interviewee (Opie, 2004). As a result, the following questions are some of the questions which were used for the post-tasks interview:

a) Why did you choose to represent the problem this way?

b) Explain to me how you got this answer.

c) Why did you not respond to this item?

The primary data of qualitative interviews are verbatim accounts of what transpires in the interview. McMillan and Schumacher (2010) argue that tape recording ensures the completeness of the verbal interaction in the interview session and provides material for reliability checks. Therefore it was important to audio record the semi-structured interviews to have an accurate record of what the learners said.

3.10 Validity and Reliability

In this descriptive and exploratory case study, I sought to describe the errors that learners make when solving probability problems involving representations and explain why they made those errors. Due to the nature of the study, generalizability and evaluative validity are not my focus but descriptive, interpretive and theoretical validity are. The credibility of a

qualitative research report relies heavily on the confidence readers have in the researcher's ability to be sensitive to the data and to make appropriate decisions in the field (Brantlinger et al., 2005). In order to instil that confidence, tasks were constructed in relation to the curriculum requirements. The tasks were also piloted to check clarity of questions and were revised as was seen necessary. Written tasks were used to elicit the errors followed by interviews of selected participants to make sure the errors are consistent and elicit the misconceptions causing the errors. Interviews were audio recorded to increase reliability. Above all the considerations taken, only data on use of representations and associated with errors was considered.

3.11 Ethical considerations

Permission was sought from Gauteng Department of Education (GDE) before approaching the target school of data collection. Prior to data collection, participants were given an oral explanation and written outline information sheet of the research project's aims, nature and data collection methods. In particular, participants were informed that ethical requirements will be adhered to. Both the oral explanation and the written information sheet stressed that participation in the research project would be voluntary and that all reporting will keep participants' details anonymous.

The written information sheet contained a separate tear-off section for learners to sign giving their informed consent to participation in various sections of the research. The informed consent forms also contained a section seeking parents'/ guardian's informed consent. Two learners in the researcher's target class did not return the reply slips. In the case where parents/guardians did not consent for their custodies to participate in various parts of the research, the learners did not participate. In particular the research was done after school hours so that those learners to whom consent had not been granted will not be unduly prejudiced. Pseudonyms were used in the research report to protect the learners' identities. In addition, transcription of interviews by the researcher could have helped increase the researcher's sensitivity to data. All raw data and informed consent reply slips were kept under lock and key during the study and researcher did not discuss the errors or ignorance of one child with other learners. After this, the raw data will be destroyed.

3.13 The Pilot Study

A pilot study was undertaken in the month of May 2014 in preparation for the major study. Pilot study refers to feasibility studies otherwise known as baseline studies (Polit, 2001). These feasibility studies are trial runs done before the full scale study. A pilot study is useful in that it could give advance warning about the challenges facing the study (Polit, 2001). A pilot study will also reveal the efficiency or shortcomings of the research methods or data collection instruments and assessing the adequacy of the proposed data analysis methods.

In this study, five Grade 11 learners who had received instruction on probability using different representations in the previous year participated in the pilot study. Three of the five learners are high achievers while the remaining two learners are average performers. I needed to see if the proposed tasks were able to trigger the intended response. It was important to have this small sample as indicated because it is the learners who know what they are doing who can recognise errors in the questions than poor performers. The average performers provided me with a range of errors that I could expect in the major study. These learners were chosen for the pilot study to avoid contamination of the data. Contamination occurs when data is collected more than once from the same participants (Makonye, 2011).

The scripts were marked. An item was scored fully correct if a correct answer was given following a correct method. In case of wrong answers, the response scrutinised for errors and possible misconceptions. The pilot study made me aware of items which were not clear in terms of language. Item (5) was not quite clear as three of the participating learners asked for clarification of the question. Also, weaknesses and pitfalls of the chosen methods of data collection were revealed. Some of the errors made by the learners could not be accounted for using Brodie& Berger's (2010) discursive framework. It became necessary to search for other frameworks (for example, Donaldson, 1963; Movshovitz-Hadar, Zaslavsky & Inbar, 1987; Ratatz, 1979). As a result I was able to fine tune the tasks and method of data analysis before the major study.

3.13 Conclusion

The research design and the methodology which was used in the study were discussed in this chapter. The data collection instrument used in this study and the pilot study were also described in this chapter. The next chapter will look at the results and analysis of the probability tasks and the learner-interviews.

CHAPTER 4: Data analysis

4.1 Introduction

In the previous chapter, I discussed methods of data collection and the measures taken to ensure rigour in the research. This chapter presents the results and analysis of the data collected from the probability tasks and learner-interviews in the study. The study aimed to investigate the errors and misconceptions that learners make as they engage with probability problems involving different representations. In addition, the current study aimed to provide explanation for the observed Grade 10 learners' errors and misconceptions in the course of using different representations to solve probability problems. In order to maintain my focus, I keep the following questions in mind as I analyse the findings (section 1.7):

a) What errors do grade 10 learners make when using different representations to solve probability problems?; and,

b) How can the relationship between grade 10 learners' solution representations and, their errors and misconceptions in solving probability problems be explained?

In order to answer these research questions, it was important to collect and analyse the learners' written responses to items and also carry out interviews with some of the learners to discuss their responses with them. As a result, a qualitative study was found suitable because it accommodates interactive processes (Makonye, 2011).

Data analysis refers to a "systematic search for meaning" (Essien, 2013). According to Hatch (2003) data analysis requires organizing and interrogating data in such a way that the researcher may begin to see patterns, discover relationships, develop explanations, identify themes and make interpretations. Data can be analysed deductively or inductively. According to Miles and Huberman (2004), deductive analysis involves superimposing predetermined assumptions or theory on empirical data. The inferred assumptions are examined in the face of the empirical data to prove or disapprove the theory. Inductive analysis begins with observation and examination of processes or events leading to formation of general explanations (Opie, 2004). This implies that the research findings from inductive analysis will be used to build a theory. Hence, inductive qualitative data analysis generates theory.

This study employed the use of deductive analysis. It was more practical as Brodie and Berger's (2010) discursive framework of learner errors provided error categories involving representations. Although I am extending the use of the discursive framework to free response items, it was used to categorise errors on multiple choice items (see, Appendix 1). Hence it was important to carry out interviews to elicit clear explanations from the learners for their choice of actions and solutions.

According to Miles and Huberman (2004), deductive analysis is done by coding and memoing empirical data according to categories using the conceptual or analytical framework. Chunks of data are fitted into predetermined codes so that patterns and regularities can be revealed and noted. The memos might help the researcher to pull out important interpretations of the data which might not have been apparent at first. In this research, learners' responses were coded through comparing responses with the categories suggested in Brodie and Berger's (2010) discursive framework of learner errors. The error codes and memos provided a methodical way of assessing and capturing the learners' thinking in their written and spoken responses.

4.2 Categories used for data analysis

Learners' scripts exhibited various errors that learners made when they engaged with the tasks. In order to keep focus I employed the use of error codes adapted from Brodie and Berger's (2010) discursive framework in analysing the data. Some of the errors were difficult to classify using Brodie and Berger's (2010) discursive framework. I therefore augmented it with some categories as informed by the work done by Donaldson (1963), Movshovitz-Hadar et al. (1987) and Ratatz (1979). The following were the error codes used in this study.

Table 4.1: A summary of the error codes to analyse learners' responses

Description of error type	Error code	Related learners' response/action to the task
Inappropriate use of visual detail	VD	If some information not given is assumed or given information is ignored. It also includes misreading at the beginning or during the solution
Difficulty with visual representation construction	DC	Faulty visualisation leading to wrong answer
Inappropriate visual scan	VS	Inferring relationships between symbols without considering the underpinning mathematics
Template driven use of signifier	TD	Placing a new signifier into a familiar discursive template
Familiarity	F	Choosing a familiar routine or representation to solve a problem due to its prior meaning
Keywords trigger	KT	Words having ambiguous meaning to the learner, signalling application of a particular routine to solve the problem
Halting signal	HS	A form of answer or multiple choice destructor which causes a learner to prematurely stop

		calculations
Misinterpretation of language	ML	Failure to interpret language so as to encode it into mathematical symbolism needed to solve the problem
Logically invalid inference	LI	False generalisation of old knowledge into new knowledge
Executive errors	E	Failure in executing an algorithm
Random errors	RE	Are non-systematic errors which do not form any pattern and do not recur

The findings are presented in an integrated format for each item. The integrated format includes brief descriptions of the performance by the learners, description of learners' errors accompanied by vignettes of learners work and detailed analysis of selected errors made by the learners for each of the items. These are followed by analysis of the learners' interviews in relation to the item in the task to establish the relationship between the representations, errors and misconceptions involved. Hence the study is descriptive because it describes the errors made by the learners. It also explores the reasons for the errors and how they relate to the representations. In view of this phenomenon, the study was therefore both exploratory and descriptive (see, section 3.2 and section 3.10).

4.3 Analysis of the written probability tasks

The tasks are divided into three sections; A, B and C. Section A is made up of three items (see, Appendix A), which were meant to gauge the learners' conception of the concept of probability. Section B is made up of two step multiple choice items. In the first part, learners would respond to the problem and choose a statement which best describes the reason for their choice of answer in the second part. Section C consists of free response items. The tasks were marked paying special attention to the errors made by the learners. Responses were considered fully correct if the final answer is correct following a correct procedure. If the answer presented is wrong, the researcher scrutinised the solution for indications of partial correctness and prevalence of error. The errors were then coded in relation to the conceptual framework using the codes in Table 4.1

4.3.1 Summary of learner performance in the probability tasks

The 22 scripts collected from the participating learners were marked paying particular attention to the errors committed. The scripts were then numbered from 1 to 22 to maintain anonymity of learners. The marks obtained by the learners in the tasks were recorded and organised into a table. Scatter plots were chosen to represents the learners' marks as they clearly show the pattern of the performance without ignoring the effect of each individual point. Figure 4.1, Figure 4.2 and Figure 4.3 represent the learners' performance in the tasks.

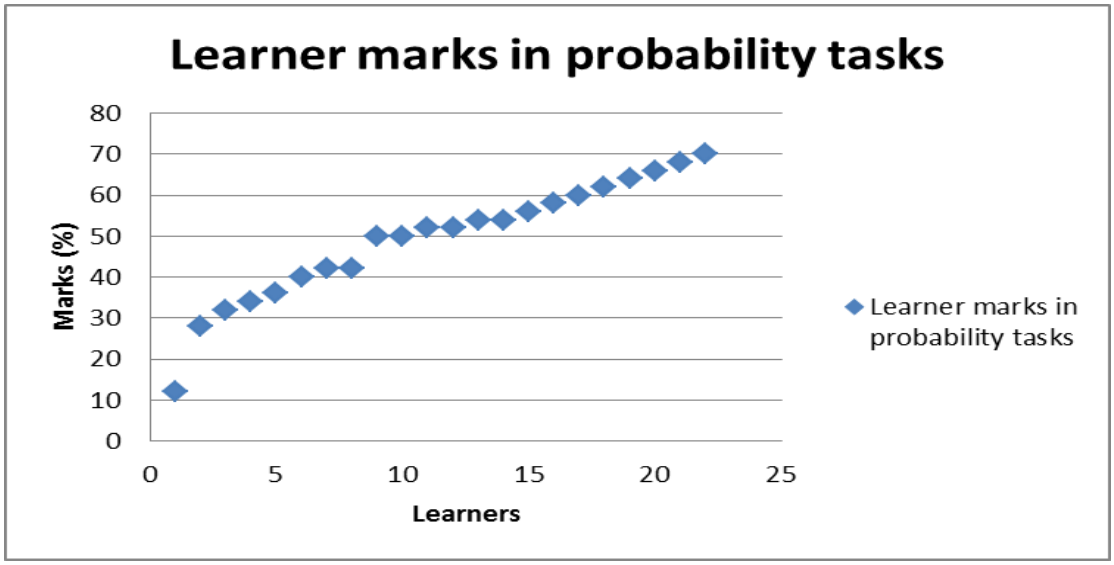


Figure 4.1: Scatter plot showing individual learners' marks in the probability tasks

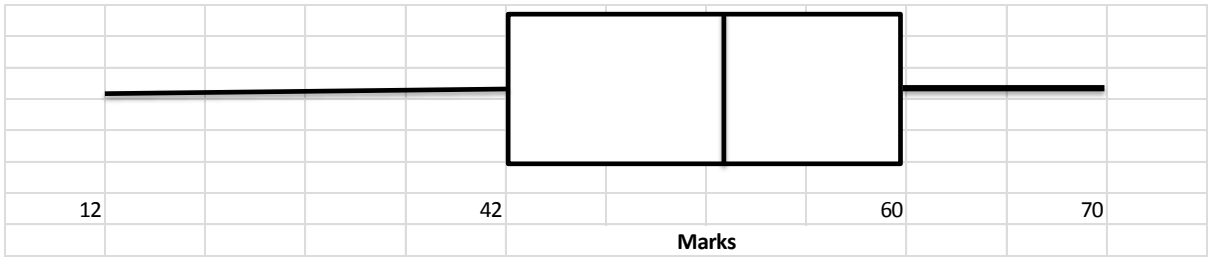


Figure 4.2: Box and whisker plot of the learners' marks in the probability tasks

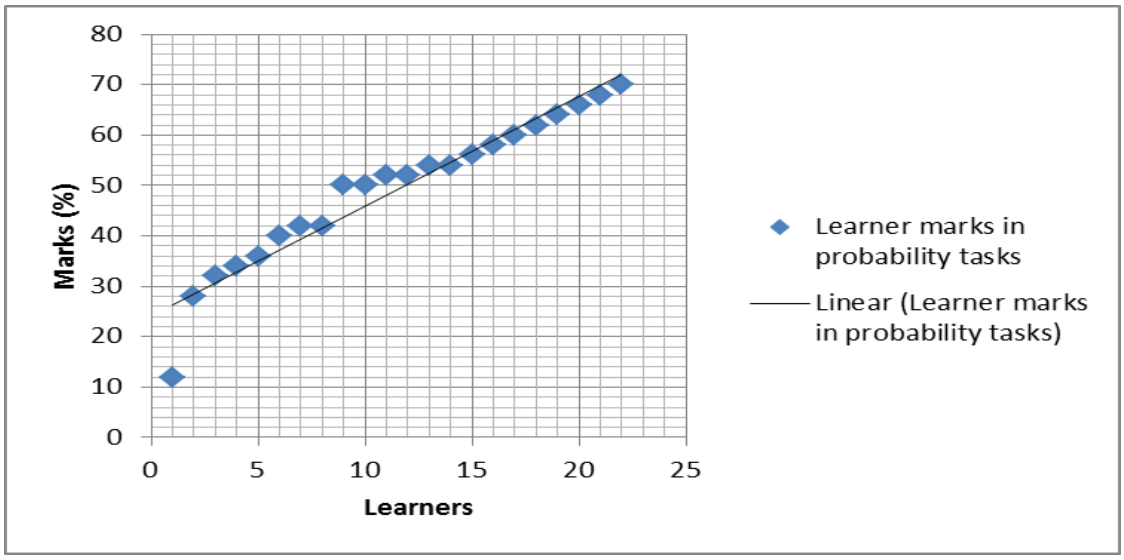


Figure 4.3: A least squares line fitted on a scatter plot of the learners' marks.

Figure 4.1, Figure 4.2 and Figure 4.3 show that the learners' marks ranged from 12% to 70%, as shown in figure 3, where half the number of learners obtained marks between 42% and 60%, one can say that their performance is mediocre. This opinion is supported by the fact that the mean mark is 24.6 which is very close to half the possible mark. In my opinion, an average performer obtains a mark equal to half the possible mark. Therefore, one can conclude that the statement is significantly correct. However, box and whisker plot shows a long whisker on the left. The data is also skewed to the right. This implies that the values are widely spaced on the left suggesting that for learners below 52%, there are larger mark differences among individuals than that of those who obtained more than 52%. The fact that 75% of the learners obtained marks greater or equal to 42%, is clearly shown on the box and whisker plot. Also, 50% of the learners obtained marks between 42% and 60%. The least squares line fitted on the scatter plot clearly shows that the lowest mark of 12% is actually an outlier. Therefore one can safely conclude that the participants' performance in the tasks was good. What follows is an item by item detailed analysis of the tasks.

4.4 Analysis of responses to tasks on Section A

4.4.1 Item 1

This item is about experimental probability. Item 1 consisted of 3 sub-items, ***a***, ***b*** and ***c***, where learners were asked to determine the probability of getting a head after the first, second and third toss of a coin. It required the learner to know that tossing a coin is an independent event where the result from one toss does not influence the result of another toss. Hence, the correct answer for part ***a***, ***b*** and ***c*** of item 1 is $\frac{1}{2}$ in each case. Table 4.2 presents the learners' responses to item 1

Table 4.2: Learners' responses to item 1 and their frequency

Item		Types of learners' responses and frequency of appearance												
		Did not answer	Describing words	Outcome symbols	0	$\frac{1}{12}$	$\frac{3}{8}$	$\frac{2}{12}$ / $\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{3}{4}$	1	> 1
1	a	1	3	4	1	1	0	0	3	0	10	0	1	0
	b	1	1	4	0	0	0	1	0	0	11	0	3	0
	c	1	1	4	0	0	1	2	0	3	6	1	0	3

For the sub-items ***a*** and ***b***, the shaded are the most occurring responses while for part ***c*** the deviation is also highlighted in darker shade. Table 4.2 shows that about 50% of the learners were able to notice that the probability of getting a head at each toss of a coin is always $\frac{1}{2}$ in the first two tosses. However, only 27% got the correct answer on part ***c***. About 14% of the learners who got correct answers on part ***a*** and ***b,*** said that the probability of getting a head in the third toss is $\frac{1}{3}$. This answer implies that the learners who gave it thought that the

probability will be slightly reduced in the third choice. It appears that these learners were estimating the probability of the third event based on sample reflection of population events (Hirsch & O'Donnell, 2001). Thus, they thought that the probability of getting an ordered sequence HHH is less likely than getting HTH or THT. Hence these learners' responses signify that they have representativeness misconception (Hirsch & O'Donnell, 2001). Four of the learners were confused by terminology in that instead of writing probabilities in number form they wrote the possible outcomes. For example, Bongani wrote:

a) H b) HH c) HHH

These answers provided by Bongani are partially correct in that they show the favoured outcomes in correct sequence. However, Bongani did not continue to determine the probability as required. Hence the favoured outcomes served as halting signals (Brodie & Berger, 2010) for Bongani. Thus, Bongani accepted that the routine is complete when he presented the favourable outcomes. This error can also be classified as an error of unverified solutions. According to Movshovitz-Hadar et al. (1987), unverified solution errors result from correct calculations but on a solution that does not directly address the given problem. In the case where the learner checks their solution, there are high chances that the learner will correct him/herself.

4.4.2 Item 2

Item 2 is also experimental probability which was intended to test learners' awareness of sample space. In changing from using a coin in item 1 to using a die, the sample space has changed from 2 to 6. Learners were asked to determine the probability of getting a six in the first, second and third roll of a fair die. The correct answer is $\frac{1}{6}$ in each case since rolling a die is a simple independent event. Therefore, the result of the previous throw does not affect the results of another throw (Jaynes, 2003). Table 4.3 shows the learners' responses and their frequencies. The shaded frequencies are the highest per section of the item. The shaded values also highlight the fact that most learners think that the probability changes with each roll. Eleven learners got the correct answer $\frac{1}{6}$, while 9 learners said the probability of getting a six in the second roll is $\frac{1}{3}$ and 10 learners said that the probability is $\frac{1}{2}$ in the third roll of the die. Thus, half the learners thought that the probability of getting a six increases with increasing number of rolls of a die. Hence, 50% of the learners failed to identify that these are consecutive independent events and were inappropriately applying the law of large numbers (Hirsch & O'Donnell, 2001) on this problem. This signifies the misconception of representativeness.

Table 4.3: Frequencies of learners' responses to item 2

Item		Types of learners' responses and frequency of appearance								
		Did not answer	$\frac{2}{7}$	$\frac{1}{6}$	$\frac{2}{6}$ / $\frac{1}{3}$	$\frac{3}{12}$	$\frac{3}{6}$ / $\frac{1}{2}$	$\frac{4}{6}$	1	>1
	a	1	0	11	1	0	6	0	1	2
2	b	2	1	2	9	0	5	1	2	1
	c	2	0	1	0	2	10	0	5	0

4.4.3 Item 3

This item was intended to test if the learners were aware of the law of large numbers. Item 3 required learners to realise that there are more chances of picking a blue marble from container B because it is a bigger sample with more of the favoured outcome. Table 4.4 shows the learners' responses to item 3 of the probability tasks:

Table 4.4: Frequency of learners' responses to item 3

ITEM 3	Did not answer	calculations	Container A	Container B
Frequency	2	1	2	17

The correct answer to item 3 is container B. Table 4.4 shows that two learners did not attempt the item and that only three learners did not get the answer correct out of the remaining twenty learners. This indicates that most learners are aware that the chances of getting the desired results increase with increasing numbers of the favoured result in the sample space. However, these maybe confirmations that about 50% of the learners were applying this frame to items 1 & 2 inappropriately as mentioned in section 2.4.5.1. The learners were therefore applying rules of one discourse to another, where it no longer holds (Sfard, 2008). Makonye and Luneta (2014) referred to this kind of error as errors of the hybridization.

Overall, only 4 out of the 22 learners in the study got every item on section A correct. This implies that only 4 learners in this study had a strong grasp of the probability concept. For the purposes of this study, the researcher was only supposed to consider sections B and C of data from scripts where learners got all parts of section A correct. However, the fact that learners were getting varying parts of Section A wrong while getting some parts of sections B and C of the probability tasks correct, signifies that the concept of probability is still slippery for most of the learners in this study. Consequently, the researcher ended up considering all the 22 scripts for data analysis to avoid working with too few data.

4.5 Analysis of responses to tasks on Section B

4.5.1 Item 4

Item 4 consisted of two sub-items, namely, item4.1 and item 4.2 (see, Appendix 1). Item 4.1 read: The first roll of a fair die results in a 3. If the die is rolled a second time, what is the chance that the second roll also results in a 3? (You can draw a diagram to help you get to the answer in space provided). The possible answers were:

a) $\frac{1}{36}$ b) $\frac{1}{5}$ c) $\frac{1}{6}$ d) Slightly less than $\frac{1}{6}$ e) Slightly more than $\frac{1}{6}$

The aim of this item was to see if learners could construct a suitable representation/s and use it/them to solve the problem. This item required learners to realise that this is a problem involving two independent consecutive events for them to choose an appropriate diagram. While 19 learners attempted the item, three learners did not attempt to solve the problem at all. This will be discussed later. Table 4.5 presents the learners' responses regarding construction of the problem representation.

Table 4.5: Frequencies of different types of representations constructed by learners

Representation	None	Calculations	Table	Matrix	Tree diagram	Venn diagram
Frequency	7	1	1	1	11	1

Table 4.5 shows that 50% of the learners chose a tree diagram. Although the tree diagram was more preferred by learners compared to other representations, none of the tree diagrams were a correct representation of the problem. It seems that learners who drew tree diagrams might have realised that the problem is a sequential experiment in which a tree diagram is a suitable representation (Corter & Zahner, 2007). However, these learners encountered difficulties in constructing the diagram since none of the tree diagrams were a correct representation of the problem. Assigning outcomes and failure to realise that the tree should have six branches were the main difficulties exhibited. It was interesting that the two learners, who got the correct answer for item 4.1, drew tree diagrams which were not correct. Prisca is one such learner. Figure 4.4 shows Prisca's tree diagram which also happens to be a typical example of the tree diagrams which were drawn by the learners. Prisca's diagram was so tiny that the researcher decided to re-draw it electronically as shown in Figure 4.4.

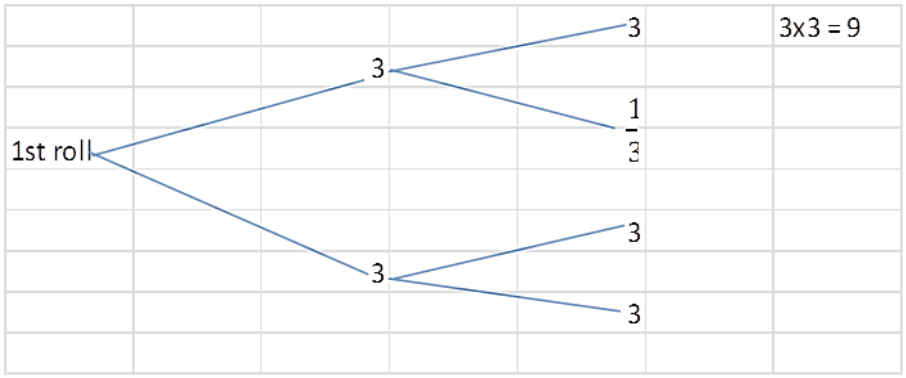


Figure 4.4: Prisca's tree diagram

There are three errors evident in Prisca's representation. First, the tree diagram has two branches at each level instead of six. This signifies a fixation to a familiar type of representation. Brodie and Berger (2010) regard this type of error as an error of familiarity. It seems Prisca did not consider that a die has six faces, which probably became a source of problem when assigning outcomes on the branches. Second, 3 is assigned as an outcome in each case except where there is $\frac{1}{3}$. Thus the outcomes did not take into account the other five faces of the die. This is a case of selective processing of information and ignoring other attributes of the problem (Donaldson, 1963). Donaldson referred to this kind of errors as arbitrary errors. Another error evident in Prisca's tree diagram is writing $\frac{1}{3}$ as an outcome of rolling a die. A die has six possible outcomes which are not fractions. Therefore the $\frac{1}{3}$ seems to be a random error as it was not repeated anywhere on the tree diagram (Movshovitz-Hadar et al., 1987). These errors indicate that Prisca was experiencing some difficulties with visual construction. I became interested to know how Prisca used this diagram or whether she used it at all, to obtain the answer. Excerpt 4.1 presents an excerpt from my interview with Prisca.

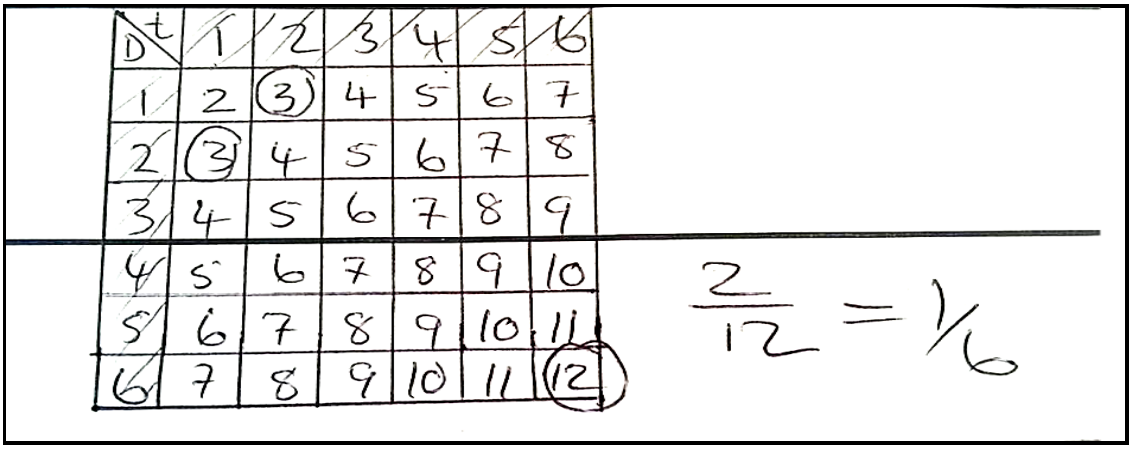
Turn	**Speaker**	**Utterance**	**Researcher's observations**
1	*Researcher*	*Please make me understand how you came up with this diagram.*	
2	*Prisca*	*Because the die landed on a 3.*	
3	*Researcher*	*Does it mean your die landed on a 3 twice in the first roll?*	
4	*Prisca*	*Ma'am[4], I wasn't really drawing.*	Difficulty with visual construction.

Excerpt 4.1: Interview with Prisca

[4] Ma'am is used in a honorific manner. It refers to a woman.

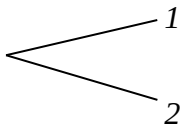
Turn 4 implies that Prisca's diagram was a way of sorting out private thoughts about the problem and she did not want to share those thoughts in the interview. Further probing for an explanation from Prisca was in vain. The researcher decided to respect the learner's privacy. It may also mean that Prisca tried to fit information from the problem into a familiar shape of the tree diagram (Brodie & Berger, 2010) but realised that it cannot fit. This is evidenced by the presence of two branches when the problem requires six branches. In this case, the learner must determine the probability of getting a 3 at the first roll as well as the second roll of the die, but a 3 is appearing on each branch in the tree diagram. Also, Prisca was applying a familiar routine in the calculation; $3 \times 3 = 9$. This routine is correct if she multiplied the probabilities of the two independent events. However, Prisca used wrong values in a correct routine for the situation at hand. Brodie and Berger (2010) classified this error as an inappropriate use of visual detail, but earlier, Movshovitz et al. (1987) referred to it as misused data errors. The fact that calculation resulted to a probability greater than 1 is a possible reason why Prisca abandoned the diagram in subsequent solving of the problem. It becomes clear that Prisca encountered difficulties in constructing the representation. Although a tree diagram is a correct choice for this problem, the diagram becomes a pitfall if not correctly constructed (Zahner & Corter, 2010).

Another example of an inappropriate use of visual detail is shown in Tongai's representation (vignette 4.1) of item 4. The matrix representation is suitable for item 4. However, Tongai correctly constructed the representation. He also correctly indicated the favourable outcomes in the diagram. His error was using 12 as the total sample space. Hence his error becomes inappropriate use of visual detail according to (Brodie & Berger, 2010). The routine used by Tongai is correct for a two-way table but incorrect for a matrix. Hence this error could also be seen as inappropriate application of a routine. Movshovitz-Hadar et al. (1987) refer to this error as a misused data error. It is misused data because there is variation between the data in the matrix and how Tongai refers to it.



Vignette 4.1: Tongai's matrix

The fact that 3 learners did not attempt item 4 at all was intriguing. Janine is one such learner. I became interested to know why she did not attempt the item. The following is an excerpt from my interview with Janine (Excerpt 4.2).

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Researcher	*Ok. I see you did not draw a diagram here or answer the question. Why is that?*	
2	Janine	*I didn't know which diagram to draw. I was so confused and I'm like ok. I didn't know which diagram to use maám. Actually I did not understand the question.*	Not understanding the question
3	Researcher	*Ok (explains the question).*	
4	Janine	*Oh, ok. I was supposed to use that tree diagram.*	
5	Researcher	*Ok. Show me. (Providing paper and pen).*	
6	Janine	*(Draws the following diagram)*  *Uuuum, you know maám, I don't understand.*	Familiarity and Difficulty with visual construction
7	Researcher	*Ok, you don't understand it. The die has how many faces?*	
8	Janine	*six maám.*	
9	Researcher	*So, if you were going to draw a tree diagram, how many branches should it have?*	
10	Janine	*six maám.*	
11	Researcher	*Six, ne? So the one you were drawing has how many branches?*	
12	Janine	*Two maám.*	

Excerpt 4.2: Interview with Janine

Turn 2 reveals that Janine did not attempt the item because she did not understand and did not know where or how to start. Being unable to produce a visual representation of the problem suggests that the learner was unable to "identify the object of their talk" (Sfard, 2007, p. 573). A problem solver should recognise similarities between the new problem and previously learnt information for them to be able to choose routines that relate to the problem (Hiebert & Lefevre, 1986; Kilpatrick et al., 2001). Janine's response in turn 2 suggests that she did not

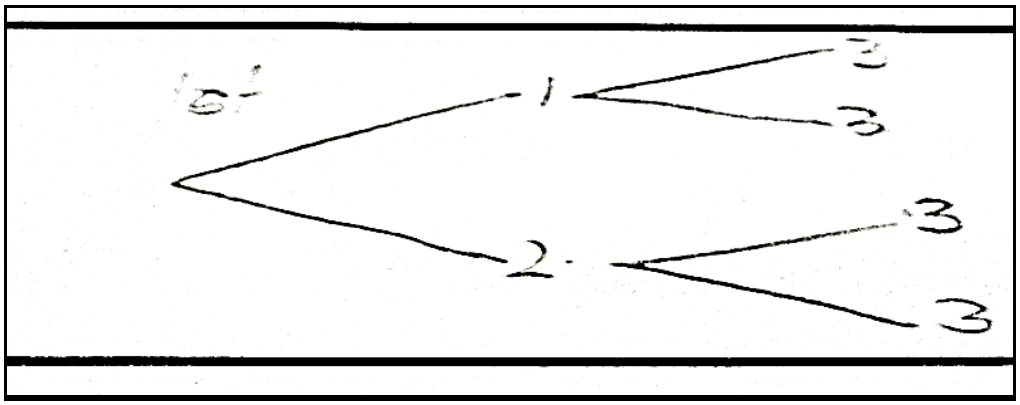
comprehend the question in such a way that she could not relate it to anything she had learnt in probability. After the researcher had explained the problem in simpler terms for her, Janine said, "*Oh, ok. I was supposed to use that tree diagram".* This signifies comprehension and successful links being made between previously learnt information and new information. However, she went on to draw a tree diagram with two branches and got stuck in assigning outcomes. This suggests that she chose a familiar representation (Brodie & Berger, 2010) but was unable to adapt it to the new problem.

Seven learners did not draw a representation but attempted items 4.1 and 4.2. The most common incorrect response to item 4.1 was $\frac{1}{6}$. The chance of getting a 3 on any one roll is always $\frac{1}{6}$ was the most occurring reason for the answer. Hence most learners did not realise that the favourable outcome is (**3;3)** where we multiply the probability of getting the first 3 with the probability of getting the second 3. Katlego did not draw a diagram but attempted the question mentioned. The following is an excerpt from my interview with Katlego.

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Researcher	*You didn't draw a diagram to assist you to answer the question. Why is that?*	
2	Katlego	*I didn't think it needed a diagram….* *The first roll does not affect the second roll, maám. The second roll does not depend on the first roll. So the probability remains* $\frac{1}{6}$.	Improper distinction between compound and simple events

Excerpt 4.3: Interview with Katlego.

While Katlego's justification is correct for simple independent events, it is not appropriate for this problem. This problem is about successive independent events. We therefore multiply the individual probabilities to get the combined probability (i.e. $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$). Katlego's response suggests an inappropriate application of probability laws due to improper distinction between simple and compound events (Fischbein & Gazit, 1984). Katlego realised that rolling a die is an independent event, but did not consider that the question is about successive independent events. This could be the reason why some learners with faulty tree diagrams chose $\frac{1}{6}$ as their answer. It implies that the faulty representation was abandoned or ignored. Mpho is one such learner who chose $\frac{1}{6}$ but had the representation shown in vignette 4.2:



Vignette 4.2: Mpho's tree diagram.

To elicit Mpho's thoughts in solving this problem, I engaged her in an interview. Excerpt 4.4 the details of my interview with Mpho:

Turn	Speaker	Utterances	Comments
1	Researcher	*You said the answer is* $\frac{1}{6}$. *How did you come up with* $\frac{1}{6}$ *from this diagram?*	
2	Mpho	*I wrote the chance for getting 3 on the first roll maám.*	Inappropriate use of routine
3	Researcher	*What about on the second roll?*	
4	Mpho	*Eish maám, I didn't think about that one.*	Improper distinction between compound and simple events

Excerpt 4.4: Interview with Mpho.

Both turns 2 and 4 indicate that Mpho did not use the diagram to determine the answer. This could be because Mpho encountered difficulties in constructing the representation resulting in loss of confidence in the diagram. Hence, one can say that learners tend to ignore their visual representations in subsequent calculation if they encounter difficulties in constructing them.

4.5.2 Item 5

The item read: *In a class of 33 learners, 6 of the 15 boys are left-handed and 5 of the 18 girls are left-handed.*

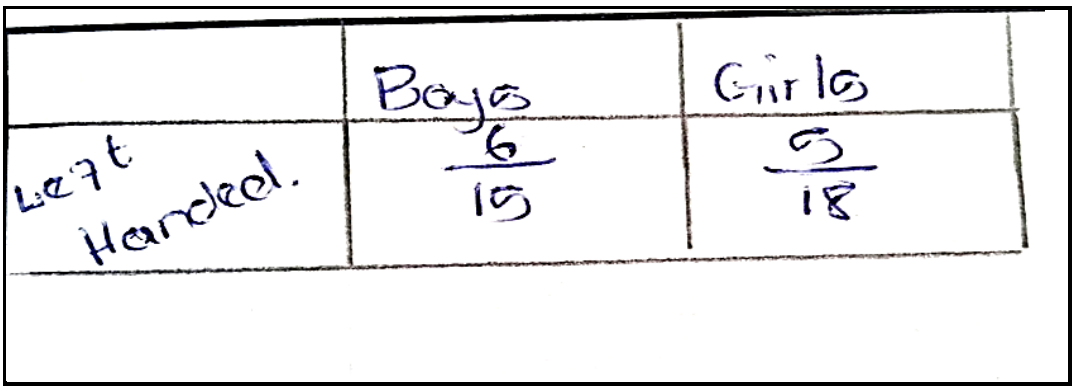
5.1. Draw a suitable diagram to represent the given information in the space provided.

Learners needed to choose and construct a representation which can easily show the combined effect of two separate events involved in the problem. Most learners found this item easy as all the participating learners attempted all parts of item 5. As shown in the Table below, 13 learners constructed 2-way (contingency) tables, six drew some sort of tables and three drew venn diagrams.

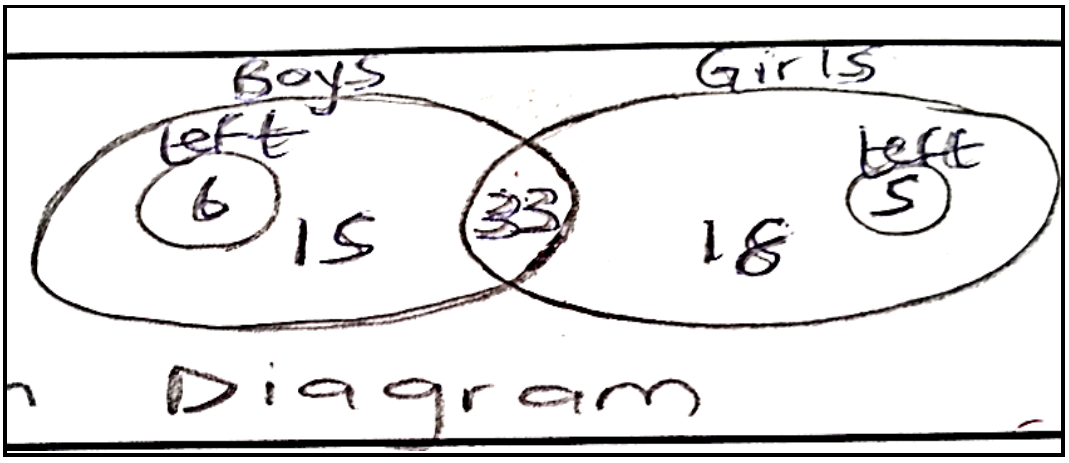
Table 4.6: Types of learners' representations and their frequency

Representation	2-way table	Venn diagram	Plain table
Frequency	13	3	6

All the 13 two-way (contingency) tables and one venn diagram were correctly constructed. However, the other 6 tables and 2 venn diagrams exhibited varying errors. The following are examples of the faulty tables and venn diagrams drawn by the learners.



Vignette 4.3: Sara's Table



Vignette 4.4: Tongai's Venn diagram

Sara's table shows two characteristics, which are gender and left-handedness. However, she clearly encountered difficulties in constructing the representation. In spite of the difficulties, Sara got correct answers for all the other parts of item 5. I engaged her in an interview to find out how or whether she used her diagram to solve the problem. Below is an excerpt of the interview with Sara:

Turn	Speaker	Utterances	Comments
1	Researcher	*Sara, tell me how you used this diagram to solve or answer the questions.*	
2	Sara	*I never used this diagram, maám. All the information that I needed is in this statement…Pointing… (In a class of 33 learners, 6 of the 15 boys are left-handed and 5 of the 18 girls are left-handed)…*	Representation ignored.
3	Researcher	*What do you mean?*	
4	Sara	*Sample space for girls is 18 and only 5 are left-handed….*	
5	Researcher	*What about here… pointing to part (5.3.3)*	
6	Sara	*Maám… total sample space is 33…and 15-6 = 9 boys are right-handed… and 18-5 = 13girls are right-handed…so 9 +13=22 learners out of 33 are right-handed.*	Use of other method to solve the problem

Excerpt 4.5: Interview with Sara

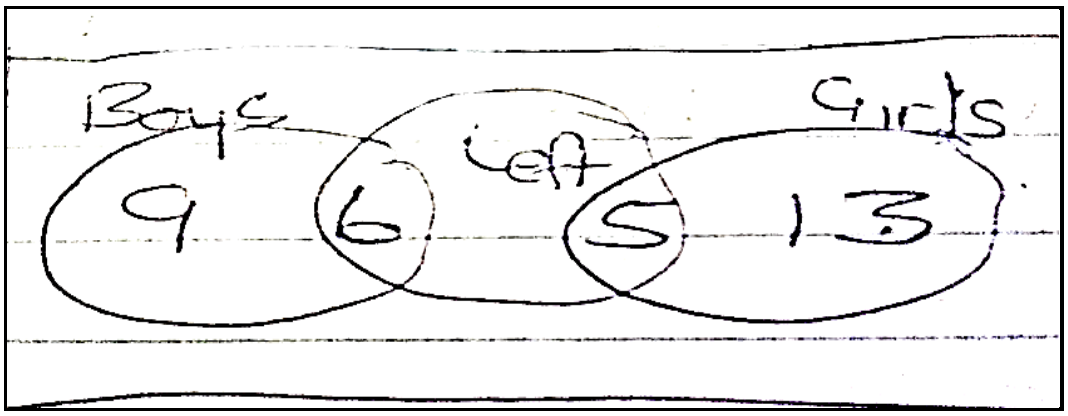
It is clear from Sara's explanation that she understood what the question required and she was able to respond without using her diagram. The reason could have been that she encountered difficulties in constructing the diagram. She could have lost confidence in it and ignored it in solving the problem. However when I talked to Tongai who drew a venn diagram, I got a different impression. The following excerpt was taken from my interview with Tongai.

Turn	Speaker	Utterances	Comments
1	Researcher	*Tell me Tongai, did you use this diagram to answer these questions (pointing to item 5)?*	
2	Tongai	*Yes maám.*	
3	Researcher	*Show me how.*	
4	Tongai	*Maám, the totals for boys and girls are shown here and those who are left-handed are also shown. I could have drawn a normal graph but I chose to draw a*	Evidence that he could have started off

		venn diagram because there is 33 learners in the class… (silence)…Eish, I don't know why I put the 33 in the middle there. Eish maám, can I start over.	using the diagram but abandoned it later.
5	Researcher	*Yes, feel free (offering pen and paper).*	

Excerpt 4.6: Interview with Tongai

Tongai went on to draw the Venn diagram shown as shown in vignette 4.5:



Vignette 4.5: Tongai's corrected Venn diagram

From Tongai's response, it is clear that he is aware that there is more than one way to solve the problem. Although Tongai claimed to have used the diagram, with the probing he realised that he could have not been able to use that diagram to solve the problem. The fact that Tongai wanted to change the diagram suggest that he could have solved the problem by other means or he corrected the visual representation mentally and then used the corrected version to solve the problem. If the assumption is correct, then Tongai's error can be classified as a random error (Davis, 1984; Movshovitz et al., 1987). This may also imply that not all learners use the representations they make for further calculations. Therefore it might be reasonable to say that, when learners encounter difficulties in visual construction or even its application in further calculations, they abandon the representation and pursue other ways to solve the problem.

About 60% of the learners cited, "because there are two characteristics involved, gender and left-handedness" as a reason for their choice of representation. Thus most learners recognised the problem as involving compound events. However, only nine learners were able to solve the problem correctly as shown in the table 4.7:

Table 4.7: Learners' responses to item 5.3.1

5.3.1	$\frac{11}{33}$	$\frac{5}{11}$	$\frac{5}{18}$	$\frac{5}{33}$	No attempt
Frequency	0	4	9	8	1

Only 9 learners got the correct answer $\frac{5}{18}$ while a common wrong answer for item (5.3.1) was $\frac{5}{33}$. Gauging from the reasons given for this answer, most of the learners either did not understand the question or were misled by the signifier in the question. Tendai is one such learner. Excerpt 4.7 shows the details of my interview with Tendai:

Turn	**Speaker**	**Utterances**	**Comments**
1	Researcher	*You gave $\frac{5}{33}$ as your answer. Please explain to me how you got it.*	
2	Tendai	*There are 5 girls who are left-handed and there is a total of 33 learners in the class.*	Inappropriate use of signifier
3	Researcher	*So this is like you are just choosing a learner from the class who happen to be a girl as well as left-handed?*	
4	Tendai	*Aaah, yes maám.*	
5	Researcher	*But from the statement "The probability that **a girl chosen from the class** at random will be left-handed" we are already aware we have a girl. This means we are particularly…*	
6	Tendai	*Focusing on the girls. Then it should have been $\frac{5}{18}$.*	realisation
7	Researcher	*Yaah*	
8	Tendai	*But maám, here (item 5) my answer was on the overall but here (pointing to item 4) was my answer of the specificness of what are the chances of getting a 3. But here (pointing to item 5) it's like the same question as the previous question. If I can say the answer is $\frac{5}{18}$, why can't I say the answer is $\frac{1}{6}$ here (pointing to item 4) with a die with 6 faces because it could be a 2. Ok, but*	Inappropriate distinction between simultaneous events and successive events.

		here I said… like over the whole field (referring to $\frac{5}{33}$*).*	

Excerpt 4.7: Interview with Tendai

It is clear from turn 8 that for Tendai, the phrase "***a girl chosen from the class***" is having an ambiguous meaning (Brodie & Berger, 2010). As a result she does not understand that there should be a routine difference when engaging with items 4 and 5. However, most learners did not experience such difficulties in answering sub-items 5.3.3 and 5.3.4 as illustrated in Table 4.8 and Table 4.9

Table 4.8: Learners' responses to item 5.3.3

5.3.3	No attempt	$\frac{119}{90}$	$\frac{22}{33}$	$\frac{13}{30}$	$\frac{13}{33}$
Frequency	1	2	16	1	2

Table 4.9: Reasons cited by learners for their answers to item (5.3.3)

5.3.4 (Reason)	No attempt	A	B	C	D	E
Frequency	1	4	14	1	2	0

Sixteen learners answered the item correctly but only 14 of those learners gave consistent reasons for their choice of answer. The other two learners gave A as their reason. I asked one of the learners why he chose that answer. He said, "$\frac{9}{15}$ *boys plus* $\frac{13}{18}$ equals $\frac{22}{33}$ " . The learner proceeded to add the fractions this way $\frac{9+13}{15+18} = \frac{22}{33}$ which is an inappropriate routine for adding fractions. Hence, one can say that the two inconsistent answers were due to inappropriate routine being applied. It is clear that the learners who chose $\frac{119}{90}$, $\frac{13}{30}$ and $\frac{13}{33}$ applied inappropriate routines because they chose reasons that are consistent with the distractors provided. The distractors were created by applying inappropriate routines.

4.5.3 Item 6

Item 6 read: A box contains 6 white cubes and 8 black cubes. The tree diagram in Figure 4.5 shows the possible outcomes of randomly choosing a cube from the box on two separate and consecutive draws (with replacement).

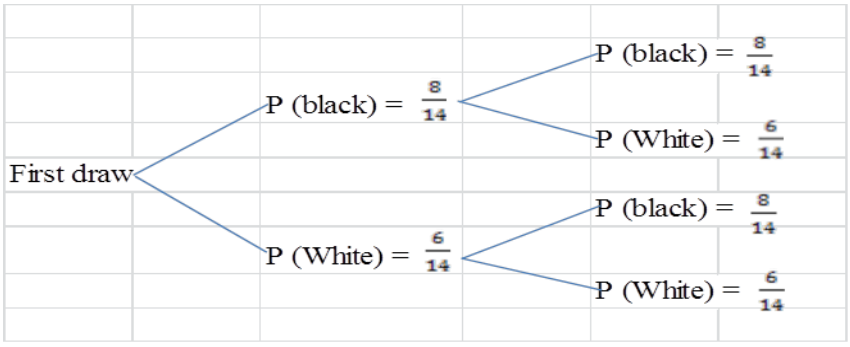


Figure 4.5: The tree diagram showing the possible outcomes of randomly choosing a cube from the box on two separate and consecutive draws

Learners were then asked to calculate the probability that, on two successive draws, a black cube will be drawn. The following were the possible answers given by learners:

a) $\frac{8}{14}$ b) $\frac{6}{14}$ c) $\frac{16}{14}$ d) $\frac{16}{49}$

All the participating learners attempted item 6. However, it appears that some of the learners were just guessing as their reasons were not consistent with the answers they chose. Table 4.10 shows the learners' responses to sub-item 6.1 and sub-item 6.2.

Table 4.10: Learners' responses to sub-items 6.1 and 6.2

6.1 Possible answers	$\frac{8}{14}$	$\frac{6}{14}$	$\frac{16}{14}$	$\frac{16}{49}$	
Frequency	8	3	4	9	
6.2 (Reason)	A	B	C	D	E
Frequency	8	2	8	4	0

On sub-item 6.1, nine learners got the answer correct while only eight learners gave a reason is consistent to their answer. One of the learners gave reason D for their answer. The distractor (D) is using an inappropriate routine for successive independent events. However, since the numerator of the correct answer and that of the distractor are the same, one can conclude that the learner's error was due to inappropriate scanning of the given details. The other wrong answers with inconsistent reasons which are not related to the chosen answer could have been a result of the learners just guessing.

P (BB) = $\frac{8}{14}$ was a common wrong answer to sub-item 6.1. All the learners who chose this answer also chose reason A for their answer which states that, "Because the black cube was drawn first". The reason offered implies that the learners did not realise that this problem

involves two successive independent events. In successive independent events, one multiplies the probabilities of the successive events to obtain the probability of the joint event.

Sub-item 6.3

The learners were asked to calculate the probability that a black cube is drawn first and then a white cube. The following possible answers were provided:

b) $\frac{8}{14}$ b) 1 c) $\frac{12}{49}$ d) $\frac{6}{14}$

Table 4.11 below shows the learners' responses to sub-item 6.3 and the reasons given for the chosen answers in sub-item 6.4

Table 1: Learners' responses to sub-items 6.3 and sub-item 6.4

6.3(possible answers)	$\frac{8}{14}$	1	$\frac{12}{49}$	$\frac{6}{14}$	
Frequency	5	7	9	1	
6.4 (Reason)	A	B	C	D	E
Frequency	5	8	9	0	0

In each case, learners chose reasons which are consistent to their answers with the exception of one learner. The one learner chose $\frac{6}{14}$ as an answer for sub-item 6.3 but gave reason B for the answer. Reason B states that: We add probabilities of drawing each colour cube $\frac{8}{14} + \frac{6}{14} = \frac{14}{14} = 1$. Since the answer and reason are not related, I took it that the learner was just guessing. Although five learners chose $\frac{8}{14}$ as their answer, the most common wrong answer offered by the learners is P (BW) = 1, with seven learners choosing it. These wrong answers indicate errors of routines and in some cases an inappropriate use of visual details as shown in the excerpt of my interview with Tongai (Excerpt 4.8).

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Tongai	*…Which means when they say on 2 successive draws they mean both draws, one after the other?*	
2	Researcher	*Yes.*	
3	Tongai	*Ok. Maám, I said* $\frac{8}{14}$ *because 1st draw, a black was chosen first and 2nd draw, a black was chosen first (indicating that the black is assigned first from top*	inappropriate use of visual details

		downwards)	
4	Researcher	*Oh, does it mean because P(B) =* $\frac{8}{14}$ *in each case?*	
5	Tongai	*Yes… The tree diagram won't help us maám.*	Confidence in the diagram is lost
6	Researcher	*Why is that?*	
7	Tongai	*But maám, we can't say* $\frac{6}{14}$ *because it must be more than or less than 14…*	Inappropriate use of routine
8	Researcher	*Ok. What about P(BW). You said the answer is* $\frac{6}{14}$.	
9	Tongai	*The answer is* $\frac{6}{14}$*because a white cube is chosen last and P(White) =* $\frac{6}{14}$.	inappropriate use of visual details

Excerpt 1: Interview with Tongai.

According to Tongai, the probability of the successive independent events is determined by the first or last event. Tongai does not seem to know any routines to apply in a tree diagram to solve this problem as he said, "*The tree diagram won't help us maám".* For him, probabilities of the successive events are tied to the last event as shown in turn 9 of Excerpt 4.8. This error can then be classified as inappropriate use of visual details.

4.6 Analysis of responses to tasks on Section C

4.6.1 Item 7

This item consisted of three free response sub-items. The item read: In a Grade 10 class of 35 learners there are 17 girls. In this class 2 girls and 1 boy wear contact lenses, 5 boys and 4 girls wear spectacles. The learners were then expected to answer the following questions:

a) Arrange the data in a two way table below.

b) What is the probability that when choosing a boy at random he will be wearing contact lenses?

c) What is the probability that when choosing a girl at random she will not be wearing contact lenses or spectacles?

Table 4.12 describes the learners' responses to item 7(a)

Table 4.12: Description of learners' responses to item 7(a)

Two-way table	Not completed	Correctly completed	Partially complete	Wrongly completed
Frequency	1	19	1	3

Only one learner did not complete the two-way table given on item 7. One learner's table was half completed, three got it wrong and 19 successfully completed the table. The large number of correctly completed tables implies that most learners easily understood how the table works and found it easy to represent information in it. Shown below is one of the 3 faulty tables which were drawn by the learners.

Gender/lenses	Boys	Girls	Total
Contact lenses	1	2	
Spectacles	4	5	
None	13	11	
Total	18	18	36

Vignette 4.6: Maria's two-way table

Maria had difficulties in representing the information in the table. It could be that she did not understand how a two-way (contingency) table works. It is unfortunate that I could not interview her, or any of the other two, because she had not given her consent for an interview but for participation in the research.

Only ten learners successfully answered the questions attached to the table while the rest showed varying degrees of errors.

Sub-item 7(b)

The question read: What is the probability that the first learner you meet in the class will wear contact lenses? Although constructing the table was easy for most learners, it was a challenge for most learners to select the correct total to use for expressing the probability. Table 4.13 represents the learners' responses to sub-item 7(b):

Table 4.13: Learners' responses to sub-item 7(b)

7(b) Answers offered	$\frac{1}{35}$	$\frac{3}{35}$	$\frac{6}{35}$	$\frac{9}{35}$	$\frac{1}{11}$	$\frac{1}{3}$	1	>1
Frequency	1	12	1	2	1	2	2	1

Twelve learners got correct answer for part (b), but the wrong answers are widely varied and there is no pattern established. For example, Nathan wrote:

$$\text{P(first learner met wears contacts)} = \text{n}\frac{5+4}{35}$$
$$= \frac{9}{35}$$

Excerpt 4.9 presents some details of my interview with Nathan.

Turn	Speaker	Utterances	Researcher's observations
1	Nathan	*…You don't know which one maám.*	
2	Researcher	*Yes. You don't know which one, but we are saying, what is the chance that the learner you meet will be wearing contacts?*	
3	Nathan	*By this table?*	
4	Researcher	*Yes*	
5	Nathan	*Eish, we don't know whether it's a boy or a girl, so I add the boys and girls together and divide with the total.*	Clear knowledge of applicable routine.
6	Researcher	*Ok. Show me which ones you are adding in the table here.*	
7	Nathan	*I add 5+4 to get 9(pointing on the table under spectacles) and divide by the total learners (pointing at 35).*	Inappropriate visual scan.
8	Researcher	*Ok. So you read off the spectacles. So to you contacts and spectacles are the same?*	
9	Nathan	*Uuuum, no maám. I just got confused there.*	

Excerpt 4.9: Interview with Nathan.

According to turn 5 in this excerpt, Nathan is clearly in the know of the applicable routine. His error is therefore due to inappropriate visual scanning since he is also aware that contacts and spectacles are not the same. As a result, he was able to rectify his mistake when I pointed it to him. However, this was not the case for part C of item 7. Only six learners gave the correct answer while $\frac{1}{35}$ was a common wrong answer as shown in Table 4.14.

Table 4.14: Learners' responses to item 7(c)

7(c) Answers offered	$\frac{1}{35}$	$\frac{1}{33}$	$\frac{3}{35}$	$\frac{6}{35}$	$\frac{1}{18}$	$\frac{1}{7}$	1	>1
Frequency	8	1	1	1	6	1	2	2

I became interested in knowing how they came up with this answer. Nathan is one of the learners who gave $\frac{1}{35}$ as an answer. Excerpt 4.10 is another excerpt of the interview with Nathan.

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Researcher	*What is the P (Boy wearing contacts)? …Ok, here we are now choosing a boy, not just a learner.*	
2	Nathan	*Ok, its* $\frac{1}{35}$ *… Yaah, it's a boy.*	Inappropriate routine.
3	Researcher	*Um. So you read off 1 from the contact lenses row and divide by 35…*	
4	Nathan	*Yaah. 35 is the total number of learners*	Inappropriate use of visual detail
5	Researcher	*Why 35, when we are told we are choosing a boy?*	
6	Nathan	*Yes maám, I chose a boy which is 1. I can't say 3 because the other 2 are girls.*	Does not see the need to focus on total number of boys.

Excerpt 2: Interview with Nathan.

In this case Nathan does not seem to be aware that this is a case to use the total of boys in expressing the probability, as we are choosing from the population of boys, not the total number of learners. This error persisted in part d of the item for him and other learners.

4.6.2 Item 8

The item read: A pet shop owner keeps all his mice in the same cage. In the cage there are:

a) 5 long-tailed white mice

b) 7 short-tailed white mice

c) 9 long-tailed black mice

d) 4 short-tailed black mice.

The learners were then asked to draw a suitable diagram or table to represent the given information in the space provided. Learners were asked to use their diagrams to answer the subsequent part items.

a) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a long tail?

b) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a black short-tailed mouse?

c) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a white, long-tailed mouse?

Table 4.15 shows how the learners responded to sub-item 8(a).

Table 4.15: Types and frequency of learners' representations of problem in item 8

Representation	None	Plain table	Two-way table	Tree diagram
Frequency	1	3	17	1

This item required learners to choose a suitable representation for the given information. A two-way table was most suitable due to the two characteristics involved in the problem. Seventeen learners correctly drew 2-way tables, one drew a faulty tree diagram, three drew faulty plain tables and one did not draw anything to represent the given information. The results show that most learners are knowledgeable and comfortable with the two-way (contingency) table. These results are also consistent with results from item 7.
Presented in Table 4.16, Table 4.17 and Table 4.18 are the learners' responses to sub-items (8b), (8c) and (8d) respectively.

Table 4.16: Learners' responses to item (8b)

8(b) Answers offered	Words describing	$\frac{9}{25}$	$\frac{12}{25}$	$\frac{14}{25}$	$\frac{1}{14}$	>1
Frequency	1	1	2	15	1	2

Table 4.17: Learners' responses to item (8c)

8(c) Answers offered	Words describing	$\frac{4}{25}$	$\frac{11}{26}$	$\frac{13}{25}$	$\frac{11}{25}$	$\frac{4}{13}$	$\frac{1}{2}$	$\frac{1}{4}$
Frequency	1	11	1	2	1	2	1	2

Table 4.18: Learners' responses to item (8d)

8(d) Answers offered	$\frac{7}{25}$	$\frac{12}{25}$	$\frac{5}{25}$ $\frac{1}{5}$	$\frac{5}{12}$	$\frac{5}{9}$	$\frac{1}{7}$	>1
Frequency	1	1	14	3	1	1	1

15 out of the 22 learners successfully answered (8b) while 11 learners got (8c) correct and 14 learners got (8d) correct. The wrong answers offered are so varied that there are no trends established in the wrong answers given for this item. The researcher concluded that these errors are random. However, an interesting error committed by one learner is shown below.

a) P(long tail) = $\frac{1}{14}$

b) P(black/short tail) = $\frac{1}{4}$

c) P(white/long tail) = $\frac{1}{5}$

This learner was not available for interview. However, it seems that the learner is taking 1 as the numerator since only one mouse is picked at a time and the number of favourable outcomes to be the denominator of the fraction. Sub-item (c) becomes correct by coincidence.

4.6.3 Item 9

A newly-wed couple decides they would like to have 4 children.

a) Complete the tree diagram below to show all possible outcomes for their children. How many possible outcomes are there?

b) List all the possible outcomes

c) How many ways are there of having 3 boys and 1 girl? List them.

d) What is the probability of having 3 boys and 1 girl? Give your answer as a fraction, a decimal and a percentage.

e) What is the probability of having 4 girls? Give your answer as a ratio.

For item 9, twenty learners correctly completed the given tree diagram following the given order while one kept changing the order at each level of the tree and one had the same sex on both branches at level 3 of the tree. Therefore, most learners found it easy to complete a partially constructed tree diagram. However, using the tree diagram to solve the problem was not easy for some learners. Table 4.19 shows the learners' responses to sub-item 9(b).

Table 4.19: Learners' responses to sub-item 9(b)

Outcomes	8	10	16	>16
Frequency	1	1	17	3

Two learners listed less than 16 outcomes, seventeen listed 16 outcomes and three learners listed more than 16 outcomes. Prisca is one of the learners who listed more than 16 outcomes. Excerpt 4.11 presents an excerpt of my interview with Prisca.

Turn	**Speaker**	**Utterances**	**Comments**
1	Researcher	*I see you completed the tree diagram. So how many outcomes are possible here?*	
2	Prisca	*Its 30 outcomes ma'am.*	
3	Researcher	*Show me how you got 30 outcomes*	
4	Prisca	*(counting outcomes assigned per branch) It's still 30 ma'am.*	Inappropriate use of visual details
5	Researcher	*When you count these as an outcome, how many children are shown at each of your outcomes?*	
6	Prisca	*One ma'am.*	
7	Researcher	*But the couple wants four children Prisca. So what counts as an outcome should have how many children?*	
8	Prisca	*4…so when we are calculating the outcomes, we must include boys and girls*	Struggling to comprehend what makes an outcome.
9	Researcher	*Yes…so an outcome should show 4 children in this case. So how many outcomes do we have?*	
10	Prisca	*Ma'am, oh, we should count it like this (pointing following branches on 4 levels)*	
11	Researcher	*Yes. Ok, this one is one, two, like that. (showing on*	

		the tree) So how many possible outcomes are there?	
12	Prisca	*16 ma'am.*	

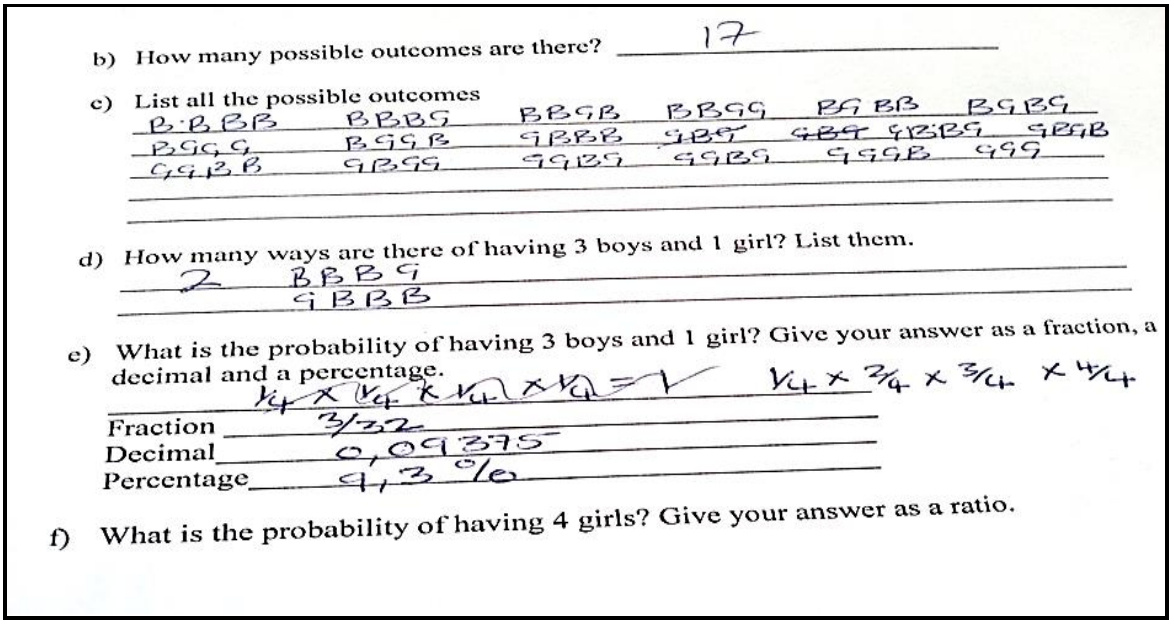
Excerpt 4.11: Interview with Prisca

Although completing the tree diagram was easy for most learners, it was not easy for some to determine what counts as an outcome in this problem as shown by Prisca in turns 4 and 8. For some learners selecting the outcomes with 3 boys and 1 girl was also a challenge as is shown in Table 4.20.

Table 4.20: Learners' responses to sub-item 9(d)

9(d)	Not attempted	<3 outcomes	3 outcomes	4 outcomes	4 outcomes	>4 outcomes
3 boys & 1girl Outcomes		listed	3 listed	not listed	Listed	listed
Frequency	1	8	5	1	6	1

Only 4 learners successfully listed the possible outcomes for 3 boys and 1 girl. Eight learners listed less than 3 outcomes and 5 learners listed 3 outcomes. This error could have been due to inappropriate visual scanning because the sample space is large. It is possible that some of the learners were just lazy to do so, as they knew that the tasks did not count for marks. In most cases, the errors on sub-item 9(e) were carried over from errors on sub-item 9(d). Tendai's work was different and I became interested in knowing how she came up with her responses. Vignette 4.7 shows Tendai's work on item 9.



Vignette 4.7: Tendai's work on item 9.

Tendai wrote that there are 17 outcomes possible on this tree diagram. When I asked her to show me how she got them, she counted and came up with 16 outcomes. So her error on this is considered a slip because she corrected herself when asked to explain. She also corrected herself on sub-item 9(d) when she counted up to 4 outcomes with 3 boys and 1 girl. Hence that error can be said to be a result of inappropriate visual scanning. However the error exhibited on sub-item 9(e) cannot be classified as a slip as revealed in the interview excerpt (see, Excerpt 4.12)

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Researcher	*Here you gave* $\frac{3}{32}$ *as your answer. Tell me how you got the answer.*	
2	Tendai	*Yooh maám, I don't know what I was writing. It's just that I was multiplying. If you do not multiply something on a tree diagram, something is wrong.*	Inappropriate routine applied
3	Researcher	*Where did you get these fractions you were multiplying?*	
4	Tendai	*Maám, I think I reduced something… oh. Ok…there are 4children maám. Probability for the first child is* $\frac{1}{4}$*, second child is* $\frac{2}{4}$*, third child is* $\frac{3}{4}$*, and the last child is* $\frac{4}{4}$*. Yaah, so I multiply them.*	

Excerpt 4.12: Interview with Tendai

From turn 2, Tendai did not use her answer from sub-item 9(d) to determine the fraction on sub-item 9(e). When I probed her further on her response, she said, "*There are 4 children, maám. Probability for the first child is* $\frac{1}{4}$*, second child is* $\frac{2}{4}$*, third child is* $\frac{3}{4}$*, and the last child is* $\frac{4}{4}$*. Yaah, so I multiply them".* For Tendai, the problem is in distinguishing that these are four successive independent events where the sample space for each event is 2. Tendai might not have realised that the probability of the gender of each child does not affect the other; therefore it will remain $\frac{1}{2}$ each time. Tendai's choice of routine was also affected by over generalisation of some probability laws, as illustrated when she said, "*If you do not multiply something on a tree diagram, something is wrong".* Thereafter, Tendai was consistently accurate with the conversion of the fraction to a decimal and percentage. However, she did not attempt sub-item 9(f). The fact that most of the incorrect responses did not seem related to sub-item 9(d) responses suggests that some learners were guessing the answer. Table 4.21 shows the learners' responses to sub-item 9(f) and their frequencies.

Table 4.21: Learners' responses to sub-item 9(f)

9(f) P(4girls)	No attempt	$\frac{1}{16}$	1:1	1:2	1:4	1:16	4:0	4:1
Frequency	8	1	3	1	4	3	1	1

Eight learners did not attempt sub-item 9(f). This could have been that they ran out of time due to poor time management during the task. However, only 3 out of the remaining 13 learners correctly responded to the item. One learner did not follow the instructions and presented the answer as a fraction. Many learners worked this item without regard to the concept of ratio. The item required the learners to identify the number of possible outcomes where all the four children are girls from the sample space and then express it as a ratio. There is evidence that some learners never considered the sample space in answering this item as illustrated in another excerpt of my interview with Prisca (see, Excerpt 4.13).

Turn	**Speaker**	**Utterances**	**Researcher's observations**
1	Researcher	*Ok. So how did you come up with 4:0?*	
2	Prisca	*Laughs….because there are 4 girls and no boys, maám.*	Disregarding the sample space.

Excerpt 4.13: Interview with Prisca

Although I did not get to interview the learners who wrote 1: 4 and 4:1 it can be inferred from Prisca's thinking that these ratios probably meant one outcome with four girls in the sample space. If this assumption is correct, then this error can be classified as inappropriate use of visual detail. Brodie and Berger (2010) propose that these kinds of errors involve variations between the data as given in the item and how the learner refers to it.

4.7 Discussions

This study focused on exploring learner errors when they solve probability problems with differing representations. The learners' written work was analysed focusing on the errors they committed. Analysis of data drew mainly from Brodie and Berger's (2010) discursive framework of learner errors. Whereas in some cases the analysis of errors was fairly straightforward, in most cases the researcher found the errors difficult to classify and analyse. This is because the discursive framework could not account for those errors. In such cases, the researcher employed other categories of errors which were a combination of Donaldson's (1963) and Movshovitz-Hadar et al. (1987) classification of learners' errors. In some cases, errors were obvious and easy to analyse, for example on item 4 where learners were required to construct a suitable representation of the problem. Sara's table (see, Vignette 4.3) is clearly

showing evidence that she is not conversant in the routines associated with construction of a contingency table (Brodie & Berger, 2010).

In some cases, there were multiple errors committed in answering one item. For example Prisca's tree diagram (see, Figure 4.4) for item 4. Prisca's tree diagram shows a random error where she assigned $\frac{1}{3}$ on one branch which is not a possible outcome on a die. Prisca also assigned 3 as an outcome on the each of the other branches which signifies arbitrary errors according to Movshovitz-Hadar et al. (1987). Prisca's tree diagram also exhibit errors of familiarity, where she drew a tree diagram with two branches at every level instead of six branches since a die has six faces. According to Brodie and Berger (2010), Prisca was probably familiar with a tree diagram with two branches and forced it to suit the current problem. In the event of multiple errors present in a learner's response, errors were classified according to the main error or main misconception causing the other errors. As a result, not all the systematic errors may have been identified in this study.

In interviews, all the six respondents showed high levels of reflection into their work. Some of the learners were even correcting their own errors after probing by the researcher. For example, Tongai who drew a faulty Venn diagram to represent the problem in item 5 where learners were requested to construct a suitable representation for the problem. Tongai said, "*Eish, I don't know why I put the 33 in the middle there. Eish maám, can I start over"* (see Excerpt 4.6 and Vignette 4.5). This kind of reflection and productive disposition towards one's work is a critical requirement for developing mathematical proficiency (Kilpatrick et al, 2001). It raises hope that in time, the learners will improve their probability problem solving skills.

4.7.1 Types and nature of common errors found in learners' work

It was noted that learners in this study made more errors in constructing and using their own representations than using given representations. For example, item 4 where learners were requested to construct a representation of their own choice was more prone to errors than item 6 where the tree diagram was provided. Only two out of twenty-two learners got item 4 correct while nine learners got item 6 correct. Hence learners experienced more success in solving problems where the representations were provided in the item. This is in line with what Zahner and Corter (2010) found in their study on the process of probability problem solving focusing on the use of external visual representations. According to Zahner and Corter (2010) experimenter-provided external visual representations lead to more success in problem solving than learner constructed visual representations. The results also show five main categories of errors and misconceptions in the learners' responses. I will discuss them in detail in the next sections.

4.7.1.1. Difficulty with visual construction of representations

The research revealed that 80% of the learners struggled with construction of visual representations especially where there are no suggested solution pathways. This was clearly depicted on items 4 and 5 of the tasks. According to Brodie and Berger (2010), an error is classified as a problem of visual construction if the learner constructs a faulty mental or external visual representation in order to solve a problem. In items 4 and 5, external visual representations were explicitly required. About 8% of the learners did not construct any representations for item 4, while 90% and 60% constructed faulty diagrams on items 4 and 5 respectively. The faulty diagrams became sources of errors in their solutions to the problems. Some learners were choosing familiar shapes or type of representations and forcing the problem to fit into them. For example, Prisca and Mpho's tree diagrams (see, Figure 4.4 and Vignette 4.2). For some learners it was simply a case of inability to make drawings. In other cases, it was an issue of over generalisation of the representation resulting in failure to modify the representation to suit the new problem. Hence these types of errors were classified as errors of difficulty with construction of visual representations.

4.7.1.2. Improper distinction between simple and compound events

The results show that improper distinction between simple and compound events was a common hindrance in construction and interpretation of representations. Learners particularly faced difficulties in distinguishing between single independent events and consecutive independent events. For example, Katlego did not draw a representation for item 4. In an interview when asked why he did not draw a diagram to represent the problem, he said, "*I didn't think it needed a diagram…. The first roll does not affect the second roll, maám. The second roll does not depend on the first roll. So the probability remains* $\frac{1}{6}$." (Excerpt 3). Katlego is partly correct in that each roll of a die is indeed an independent event. However, item 4 involves two consecutive independent events which have a multiplicative nature. This is what Donaldson (1963) called an arbitrary error due to the selective processing of information. However, the arbitrary error was stemming from an improper distinction between simple and compound events. According to Brodie and Berger (2010), if a learner inserts a new signifier into a familiar signifier such as a familiar number, shape or rule, the errors resulting from this situation are referred to as errors of signifiers. In this case, the similarity between simple and compound events posed a problem for some learners. This problem was more apparent on item 4 of the tasks. As a result for some learners, the signifiers in the problem had ambiguous meanings (Brodie & Berger, 2010) leading to wrong choice of representations or routines in the case where they had to interpret a provided representation. In this kind of error, poor language mastery also plays an important role. According to Donaldson (1963), structural errors result from lack of understanding of ideas

inherent in a mathematical problem. For example choosing an inappropriate representation type or routine for a problem is a structural error.

4.7.1.3. Application of inappropriate routines

In the case where the tree diagram is given, most learners applied inappropriate routines to determine probabilities of the favoured outcomes. Learners tended to over generalise some routines over the tree diagram. For example, Tendai knew that the probability of a single event taking place or not adds up to 1. She over-generalised this rule when she multiplied $\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}=1$ (see, Vignette 4.4) on one path on a tree diagram. The multiplication results in $\frac{1}{256}$ but Tendai wrote 1. This is what Movshovitz-Hadar et al. (1987) referred to as logically invalid inference. It is also evidence that Tendai is trying to force a certain frame on this problem. The fact that Tendai was unable to figure out why the rule did not apply in this case suggests that her understanding of the rule is procedural and superficial. Hence, the learner's conceptual grip is weak resulting in false generalisation of old knowledge into new knowledge (Movshovitz-Hadar et al., 1987).

4.7.1.4. Errors associated with familiarity

Learners tended to choose forms of representations which were familiar to them or have prior meaning to them (Brodie & Berger, 2010). For example, on item 4, 50% of the learners chose the tree diagram. The problem is a sequential experiment in which a tree diagram is a suitable representation. However, almost all but one tree diagram constructed by the learners had two branches at each level instead of six brunches to suit the problem. This is a case which resembles Einstellung effect. According to Makonye (2011) using Haylock's (1987) work, Einstellung effect refers to learners' inappropriate use of previously successful strategy to solve a new problem. This fixation to a previously successful strategy results in errors if the learner fails to modify it and culminates in confusion and unsuccessful problem solving. The findings also reveal that learners tend to ignore their representations in subsequent calculations if they realise that their modification was not successful in their construction. For example Mpho did not use her diagram to determine the probability on item 4 (see, Excerpt 4.4).

4.7.7.5. Misinterpreted language

Analysis of the research data reveal that language was a challenge for most learners as they failed to answer items due to lack of understanding what the problem requires of them. For example, four learners did not attempt item at all due to lack of understanding. According to Sfard (2007), mathematical words and narratives have a peculiar meaning in mathematics which may be different in everyday language. Learners with a weak grasp of the English language in which Mathematics is expressed would not be able to distinguish between two

closely related words or narratives. Such errors could occur during encoding mathematical language into external representations or interpretation of the representations (Movshovitz-Hadar et al. 1987). For instance, some learners ended up writing out possible outcomes instead of the probability of an event happening in items 1 and 2. For example Bongani wrote answers for item 1 as follows:

a) H b) HH c) HHH

Bongani is partially correct in that he wrote the favoured outcomes in correct sequence but did not answer the question. The outcomes became halting signals (Brodie & Berger, 2010) for the learners due to inappropriate comprehension of the problem. Hence the learners' difficulties in understanding the problem were due to failure in accessing the mathematical language in which the problem was embedded.

4.7.2 Emerging patterns

4.7.2.1 Contingency table representation more helpful than other representations

In the analysis done in this study, some patterns in learners' responses emerged. On item (5), thirteen out of twenty-two learners chose contingency tables and correctly constructed and used them successfully (see table 5). On item (7) where the learners had to complete a contingency table, only three learners out of twenty-two did not get the table correct (see table 11). On item (8), seventeen learners chose to represent the problem with a contingency table and questions were answered correctly in most cases (see table 14). Therefore results reveal that most of the learners in this study could confidently construct and skilfully use contingency tables. Consequently, most learners preferred to use contingency tables over the Venn diagram, matrix and outcome listing for problems involving compound events. In the few cases, where learners had errors in routines, it can be attributed to inappropriate use of visual details. For example on sub-item 7(b), Nathan wrote: $\mathrm{P(first\ learner\ met\ wears\ contacts)} = \mathrm{n}\frac{5+4}{35} = \frac{9}{35}$. In the interview, he corrected himself after probing. Brodie and Berger (2010) referred to this kind of error as inappropriate use of visual detail because Nathan used a wrong value in a correct routine. In some cases, it was a matter of language becoming a barrier for learners to access the appropriate routines.

4.7.2.2 Learners preferred to use the tree diagrams but faced difficulties in correctly constructing them

It is important to indicate that the learners' scripts show evidence that, for experimental probability, learners preferred the tree diagram. For example, on item 4 eleven out of the fifteen learners who drew representations chose the tree diagram (see, Table 4.5). However, three tree diagrams were partly correct suggesting that many learners faced challenges in the construction of the representation. Findings also reveal that most learners were able to assign

outcomes on a partially complete tree diagram but struggled to construct it from scratch. For example on item 9, twenty learners correctly completed the given tree diagram following the given order while one kept changing the order at each level of the tree and one had the same sex on both branches at level 3 of the tree. Not following the order indicates that the learner is not conversant with the routine of constructing a tree diagram. Therefore the error can be categorised as difficulty with construction of visual representation (Brodie & Berger, 2010). Assigning the same sex on both branches at only one level can be categorised as a random error because it was not repeated Movshovitz-Hadar et al. (1987).

4.7.2.3. Learners did not understand the questions because of probability terminology

Some learners did not attempt some of the items, particularly item 4 and item 9(f). Three learners did not attempt item 4 at all. For item 9(f), eight learners did not attempt. The research revealed that not attempting items or part thereof, may not be due to learners' not understanding the laws of probability as Hirsch and O'Donnell (2001) suggest, but lack or poor comprehension of the problem. For example, Janine did not attempt item 4. In interview, she said that she did not answer because she did not understand what the question required of her (see, Excerpt 4.2). After the researcher explained in simpler terms, Prisca was able to select a suitable form of representation for item 4. Hence a poor grasp of probability language can be said to be causing learners to submit blank responses although it may not be the only reason. The study also reveals five categories of common errors in solving probability problems using different representations. The common errors established are discussed below.

4.8 Conclusion

This chapter has drawn on the data collected to establish and describe the type of errors made by learners in solving probability problems involving representations. The next chapter concludes the study by discussing limitations of the study, possible future research and recommendations for probability instruction.

CHAPTER 5: Findings, conclusions and recommendations

5.1 Introduction

This study aimed to investigate the errors and misconceptions that grade 10 learners have when answering probability questions using different representations. This chapter concludes the study by discussing findings, conclusions, limitations, reflections and recommendations of the study.

5.2 Findings and conclusions

In the current curriculum CAPS, new topics such as probability were re-introduced into the core mathematics curriculum in South Africa. Both teachers and learners were faced with new challenges as they were ill equipped to deal with these new topics. To try and address the challenges, this exploratory case study aimed at investigating the errors and misconceptions that Grade 10 learners have when solving probability problems using varied representations. The study also intended to explain the relationship between the learners' solution representations and their errors and misconceptions in solving probability problems. The study mainly drew from Sfard's (2007) theory of commognition as it relates to representations. It was also guided by Brodie and Berger's (2010) discursive framework of classifying learner errors and misconceptions. Twenty-two learners were given probability tasks involving the use of different diagrams. The tasks were developed drawing from Nesher's theory of errors and misconceptions as it relates to error detection. In some tasks, learners had to construct their own representations as they saw fit. In other tasks, partially constructed or complete diagrams were provided. The tasks were studied paying particular attention to the errors the learners made. Six learners were interviewed to elicit the reasons for their errors in solving the probability problems.

5.2.1 The nature of learners' common errors and misconceptions in this study

Learner errors and misconceptions in solving probability via the media of different representations varied. The findings reveal five main categories of learner-errors in solving probability problems involving different representations. These are difficulty with construction of visual representations, improper distinction between simple and compound events, application of inappropriate routines, errors associated with familiarity and misinterpreted language. The findings also show that inappropriate choice of types of representations, application of inappropriate routines and some of the difficulties in visual construction of representations stemmed mainly from lack of or poor understanding of probability terms or language. Some of the difficulties with visual construction were due to learners selecting improper types of representations. Some learners' choices of representations and routines were influenced by familiarity. In some cases, the choice of representation was influenced by the fact that the routine or representation was successful in

solving previous problems. Consequently, the learners then failed to adapt the representation templets to suit the current problem. Future studies should look at ways of developing the templet adaption skills. A question arises; what strategies can be used to support the learners in learning to construct and interpret the tree diagram flexibly?

The research also revealed learners' behavioural patterns related to the different types of representations used in this study. I conclude the following:

Tree diagrams: In order to determine the probability of a sequential experiment, 50% of the participating learners preferred to use the tree diagram over other types of representations. However, none of the tree diagrams were a true reflection of the problem. Hence learners had more confidence in the efficiency of the tree diagram despite the fact that they experienced difficulties in constructing them. On the other hand, about 90% of correctly assigned outcomes resulted from partially completed tree diagrams. Therefore, one can say that learners experienced difficulties in constructing tree diagrams from scratch but they were quite competent in completing a partially constructed tree diagram. It is therefore clear that learners need more support in modelling probabilistic problems using tree diagrams so that their success rate of solving the probability problems with this representation can be boosted.

Venn diagrams: Only five Venn diagrams were drawn in solving probability problems involving compound events. Two of the five Venn diagrams were drawn by one learner. It can then be said that learners do not have confidence in the effectiveness and efficacy of Venn diagrams in solving probability problems. Otherwise, they would have soldiered on as they did with the tree diagram although they experience hardships in constructing them.

Contingency tables: In representing compound events, 90% of the learners showed preference for the contingency table over the Venn diagram, matrix and outcome listings. The learners were conversant in both the construction routines and selection of routines for further calculations associated with the contingency table. About 50% of the learners successfully solved the problems using the contingency tables. Hence the learners showed a strong grasp and are more articulate in construction, interpretation and applying routines associated with the contingency table.

Matrices: The matrix was not a popular method of representation for the participants in this study. Only one matrix was clearly and correctly drawn for item 4. The other one seemed to be a hybrid of a matrix and a contingency table. This implies that the learner was not quite conversant routines of constructing a matrix. The unpopularity also suggests that some of the learners were not even aware that there is such a type of representation.

Outcome listings: Four learners who presented outcome listings for item 1 suggesting that they were conversant with the type of representation. About 85% of the learners were able to

list possible outcomes from a tree diagram on item 9. However some lists were not exhaustive, indicating that the concept is still slippery.

5.3 Limitations of the study

The study was limited in that the size and scope of the sample was small. Data was collected from only 22 Grade 10 learners in one school. All the learners were taught by one teacher. Therefore the instruction received by the learners before the tasks can be considered to be relatively consistent. The data could have been different if it was collected from learners from different teachers and different schools. That in its own right is not a big problem because this research was qualitative, in which the purpose was to generate theory rather than prove it (Yin, 1994). Only learners taught by the researcher participated in the study for convenience purposes. As a result, the study is a case study which cannot be generalised to all schools. It therefore remains that the study was carried on just a fraction of the population although the same results could have been found over a large sample involving different teachers and schools.

Another limitation was that the tasks were too many for one sitting. As a result some of the tasks did not get enough attention from the learners as they were probably tired towards the end of the time given. The items included were to ensure that all types of representations used in the topic probability are catered for. Perhaps, taking into account two or three types of representations only would decrease the number of items in the tasks.

In adherence to ethical standards, the learners were informed that the tasks do not give bearing to their term or year mark. As a result the learners may not have given the research tasks the serious attention they deserved. Thus the data obtained may not have been a true representation of their true capabilities and deficits. There was evidence of this from the interviews when some learners confided that they just wrote answers to finish the tasks. Perhaps, the data collected would have been different had the tasks been considered as part of the year's assessments by the learners.

5.4 Reflections

As a foreign educator coming from a country where English is the only official language used for doing formal business and instruction in schools, I underestimated the impact that English proficiency has on teaching and learning of probability to English second language learners. Consequently I did not explicitly teach the different terminologies used in probability to help the learners understand. Such insights emanating from this study will help me to be more considerate of learners' backgrounds in the future.

5.5 Recommendations

Poor matric results for mathematics in South Africa, has been a cause for concern for some time. Knowing the areas where learners need support will assist teachers to develop strategies

which address the learners' needs. In view of the errors and misconceptions that learners showed in solving probability problems with different representations reported herein, I recommend that teachers should:

1. Consider explicitly teaching the meanings of the different terminology and symbols used in probability. This would promote comprehension of the problem and access to the underpinning mathematics as learners engage with probability tasks;
2. Start by providing partially complete representations of the problem for learners to complete such as partially complete Venn diagrams or tree diagrams. This will assist learners to focus on the structure of the underlying mathematics and build their confidence in using representations to solve problems. Then gradually move to tasks where the learners are required to construct their own representations;
3. Clearly distinguish single independent events from successive independent events to promote proper selection of type of representations and routines. This can be done by stating the distinguishing features of each category;
4. Make learners realise that mathematical symbols carry meaning. It is also important for learners to be able to interpret symbols and to use them carefully to represent and manipulate mathematical concepts; and,
5. Foster conceptual understanding of probability by assigning a number of different tasks to learners. Work given must be carefully graded, progressing from basic conceptual exercises to more challenging problems involving interpretation of problem, construction of representations and using the representation to solve the problem.

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Appendix 1: Probability Tasks

Section A

Write your answer in spaces provided.

Question 1



a) What is the chance that the first toss of a fair coin results in a head?
---

b) The first toss of the coin does result in a head, and the coin is tossed a second time. What is the chance that the second toss results in a head?
---

c) The coin is tossed a third time. What is the chance that the third toss results in a head?

Question 2



a) What is the chance that the first roll of a fair die results in a 6?
---

b) The first roll of the die does result in a 6, and the die is rolled a second time. What is the chance that the second roll results in a 6?
---

c) The die is rolled a third time. What is the chance that the third roll results in a 6?

Question 3

Two containers, labelled A and B, are filled with red and blue marbles in the following quantities. Each container is shaken vigorously. After choosing one of the containers, you will reach in and, without looking, draw out a marble. If the marble is blue, you win R50. Which container gives you the best chance of drawing a blue marble?

Container	Red	Blue
A	6	4
B	60	40

Section B

There are two parts to each question in this section. Both are multiple choice questions. First part you choose the correct answer for the question and in the second part you choose the letter corresponding to the reason for your answer in the first part.

Question 4

4.1). The first roll of a fair die results in a 3. The die is rolled a second time. What is the chance that the second roll also results in a 3? (You can draw a diagram to help you get to the answer in space provided). Choose the correct answer.



a) $\frac{1}{36}$ b) $\frac{1}{5}$ c) $\frac{1}{6}$ d) Slightly less than $\frac{1}{6}$ e) Slightly more than $\frac{1}{6}$

4.2) Which of the following best describes the reason for your answer to Q4.1 above? (choose the correct answer).

a) There are thirty-six possible outcomes when you roll a die twice. Getting two 3's is only one of them.
b) The second toss is less likely to be a 3 because the first toss was a 3.
c) The chance of getting a 3 on any one roll is always $\frac{1}{6}$.
d) Any of the other five numbers is more likely than a 3.
e) Other (specify):_____

Question 5.

In a class of 33 learners, 6 of the 15 boys are left-handed and 5 of the 18 girls are left-handed.

5.1) Draw a suitable diagram to represent the given information in the space provided



5.2) Choose the reason that best describes your choice of diagram.

a) Because it can be used to represent any situation.
b) Because there is a higher rate of getting a correct answer using this diagram.
c) Because there are two characteristics involved, gender and left-handedness.

d) It is useful for finding the possible outcomes of an experiment.
e) It can easily show the combined effect of two separate happenings.

5.3) Now use your diagram or otherwise to find:

5.3.1) The probability that a girl chosen at random will be left-handed.

a) $\frac{11}{33}$ b) $\frac{5}{11}$ c) $\frac{5}{18}$ d) $\frac{5}{33}$

5.3.2Which of the following best describes the reason for your answer to Q4.3.1 above?

a) There are 11 left-handed learners out of 33.
b) 5 out of the 11 left-handed learners are girls.
c) 5 out of the18 girls are left-handed.
d) 5 girls out of 33 learners are left-handed.

5.3.3) The probability that a learner chosen at random will be right-handed.

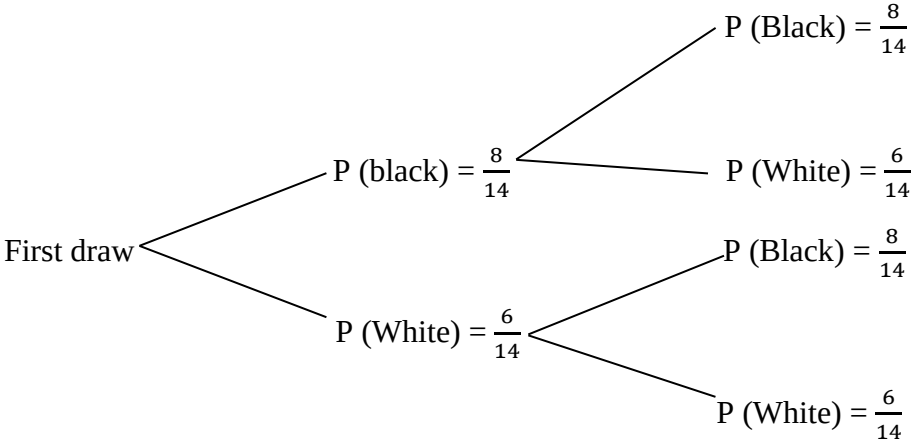
a) $\frac{119}{90}$ b) $\frac{22}{33}$ c) $\frac{13}{30}$ d) $\frac{13}{33}$

5.3.4) Which of the following best describes the reason for your answer to Q4.3.3 above?

a) We add $\frac{9}{15}$ boys plus $\frac{13}{18}$ girls.
b) $\frac{22}{33}$ Because there are 22 right-handed learners out of 33 learners.
c) We multiply $\frac{9}{15}$ boys by $\frac{13}{18}$ girls.
d) $\frac{13}{33}$ Because there are 13 right-handed girls.
e) Other (Specify)_____

Question 6

A box contains 6 white cubes and 8 black cubes. The tree diagram below shows the possible outcomes of randomly choosing a cube from the box on two separate and consecutive draws (with replacement).



6.1) Calculate the probability that, on two successive draws, a black cube will be drawn.

a) $\frac{8}{14}$ b) $\frac{6}{14}$ c) $\frac{16}{14}$ d) $\frac{16}{49}$

6.2) Which of the following best describes the reason for your answer to Q6.1 above?

a) Probability of drawing a black cube is always $\frac{8}{14}$.
b) Probability of drawing a black cube on two successive draws is $1-\frac{8}{14}=\frac{6}{14}$.
c) Multiplying probabilities on successive branches: $\frac{8}{14}\times\frac{8}{14}=\frac{64}{196}=\frac{16}{49}$
d) Adding probabilities on successive branches: $\frac{8}{14}+\frac{8}{14}=\frac{16}{14}$.
e) Other (Specify)_____

6.3) Calculate the probability that a black cube is drawn first and then a white cube.

a) $\frac{8}{14}$ b) 1 c) $\frac{12}{49}$ d) $\frac{6}{14}$

6.4) Which of the following best describes the reason for your answer to the Q6.3 above?

a) $\frac{8}{14}$ Because the black cube was drawn first.
b) We add probabilities of drawing each colour cube $\frac{8}{14}+\frac{6}{14}=\frac{14}{14}=1$.
c) Multiplying probabilities on successive branches with favoured outcome (Black then white) $\frac{8}{14}\times\frac{6}{14}=\frac{48}{196}=\frac{12}{49}$
d) $\frac{6}{14}$ Because the white cube is drawn last.
e) Other (specify)_____

SECTION C

Question 7

In a Grade 10 class of 35 learners there are 17 girls. In this class 2 girls and 1 boy wear contact lenses, 5 boys and 4 girls wear spectacles.

a) Arrange the data in a two way table below.

Gender/lenses	Boys	Girls	Total
Contact lenses			
Spectacles			
None			
Total			

b) What is the probability that the first learner you meet in the class will wear contact lenses?

_____
_____
_____

c) What is the probability that when choosing a boy at random he will be wearing contact lenses?

_____
_____
_____

d) What is the probability that when choosing a girl at random she will not be wearing contact lenses or spectacles?

_____
_____
_____

Question 8

A pet shop owner keeps all his mice in the same cage. In the cage there are:

- 5 long-tailed white mice
- 7 short-tailed white mice
- 9 long-tailed black mice
- 4 short-tailed black mice.

a) Draw a suitable diagram or table to represent the given information in the space provided.

Use your diagram to answer the following:

b) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a long tail?

_____

_____

c) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a black short-tailed mouse?

_____

_____

d) What is the probability that when the pet shop owner randomly picks a mouse from the cage, it will be a white, long-tailed mouse?

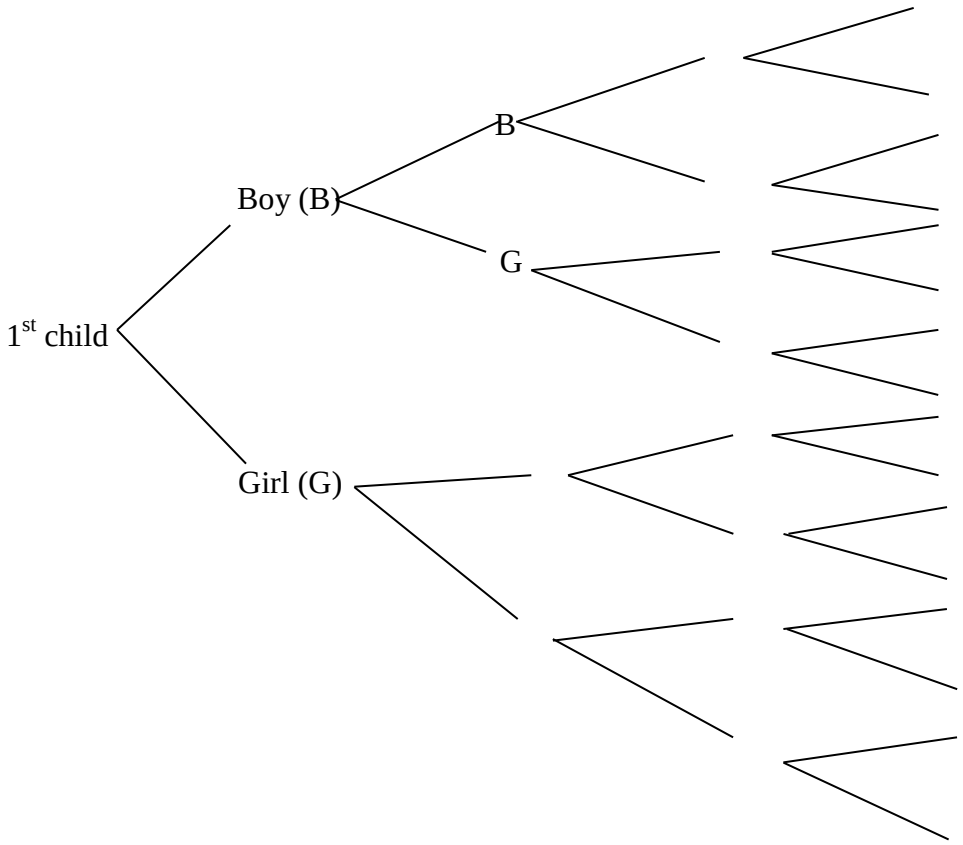
_____

_____

Question 9

A newly-wed couple decides they would like to have 4 children.

a) Complete the tree diagram below to show all possible outcomes for their children.



b) How many possible outcomes are there? _____

c) List all the possible outcomes

_____
_____
_____
_____
_____

d) How many ways are there of having 3 boys and 1 girl? List them.

_____
_____

e) What is the probability of having 3 boys and 1 girl? Give your answer as a fraction, a decimal and a percentage.

_____
Fraction _____
Decimal_____
Percentage_____

f) What is the probability of having 4 girls? Give your answer as a ratio.

_____

Appendix 2: Semi Structured interview Protocol

The researcher started the interview by explaining to the interviewee that the purpose of the interview is to get clarity on how each participant solved the probability task given. To put the participant at easy, the participant was also informed that their responses in the interview would not count for marks and does not bear on normal classroom activities. Each interview took between 10 and 20 minutes and was audio recorded.

The number of interview items and what the participant was actually asked depended on the participant's errors and their responses to the first questions. Listed below are some of the questions that were asked.

Examples of the post-task interview questions

1. Why did you choose to represent the problem this way?
2. What difficulties did you face when solving this problem?
3. I see you did not draw a diagram here or answer the question. Why is that?
4. How did you come up with (this answer) from this diagram?
5. I see you completed the tree diagram. So how many outcomes are possible here?

Appendix 3: GDE Research clearance

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

For administrative use:
Reference no: D2015 / 270

GDE RESEARCH APPROVAL LETTER

Date:	**20 August 2014**
Validity of Research Approval:	**20 August 2014 to 3 October 2014**
Name of Researcher:	**Mutara L.**
Address of Researcher:	**42 Botha Street**
	Krugersdorp
	1739
Telephone Number:	**011 665 4935; 078 194 0908**
Email address:	**579347@student.wits.ac.za; lmutara@telkomsa.net**
Research Topic:	**Exploring Grade 10 learners errors and misconceptions involved in solving probability problems with different representations**
Number and type of schools:	**ONE Secondary School**
District/s/HO	**Johannesburg North**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

Makhado
2014/08/20

1

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*
4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

.....

Dr David Makhado
Director: Education Research and Knowledge Management

DATE: 2014/08/20

Making education a societal priority

Office of the Director: Knowledge Management and Research
9th Floor, 111 Commissioner Street, Johannesburg, 2001
P.O. Box 7710, Johannesburg, 2000 Tel: (011) 355 0506
Email: David.Makhado@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix 4: Information sheet and consent forms

University of the Witwatersrand, Wits School of Education, 97 St Andrews Rd, Parktown, Johannesburg,

The Principal
JBC Secondary school
P. O. Box XXXX
Maraisburg
1700

13th August, 2014

REF**: Request for Permission to do research at Chris J Botha secondary school.**

Dear Mr. XXXX

My name is **Lydia Mutara**. I am one of your teachers and I am studying for MSc Science Education (Mathematics) in the School of Education at the University of the Witwatersrand. I write this letter to seek your permission to do research at your school (Chris J Botha).

I am carrying out a study on **Exploring the errors and misconceptions involved in solving probability problems** in schools in Johannesburg, mainly looking at **the challenges in using different representations**. I also intend to determine the reasons for the errors that students have on these concepts when they use different representations to solve probability tasks. This research is on Grade 10 mathematics learners because I noticed that since the re-introduction of probability, learners show that they do not have a good grasp on this topic. I strongly believe that diagnosing our learners' errors and misconceptions related to different representations of probability can only help us to devise strategies to teach them better.

The research will involve:

- Grade 10 mathematics learners writing on probability tasks over a period of $1\frac{1}{2}$ hours. The tasks will be written after school or break time to avoid disrupting the school times and prejudicing learners who do not participate in the research. Their scripts will be marked, taking particular note of the errors that the students make on tasks.

- Six learners will be selected for interview based on their errors on the tasks. These will be two exhibiting errors of visual mediators, two showing errors of signifiers and two exhibiting errors of routine on the tasks. The learners will be interviewed to

probe and elicit their thinking on the errors they show on the tasks.

- To accurately capture what learners say about their errors, the interviews will be audio-recorded. The interviews will take at most 10 minutes per learner per interview period during break time in my classroom, spread over three days.

Participants will not be paid for the study, advantaged or disadvantaged in any way. I would like to make it clear that participation in this study is entirely voluntary, no harm is envisaged, and all information will be treated as confidential and names not shown. Participants can choose to accept or decline to answer any questions, and can withdraw from the study at any given time. All raw data will be kept in my classroom cupboards under lock and key during and after the study. The raw data will be destroyed between 3 and 5 years after the completion of the study.

I hope to publish part or all the results of this study in academic journals. In order to maintain anonymity and confidentiality, all names I use will be pseudonyms.

Yours faithfully

Lydia Mutara Phone: 078 194 0908

Signature: _____ Date: _____

Research Supervisor: Judah Makonye Phone: 011 717 3206
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

Permission from the School Principal to carry out research

(School letter head)

Title of research: Exploring Grade 10 learners' errors and misconceptions involved in solving probability problems using different representations.

I, **XXXXX,** am the principal of **JBC Secondary School**. I have read and understood the content of the letter seeking permission for doing research on Grade 10 mathematics learners' errors and misconceptions in solving probability problems.

I permit/ do not permit **Lydia Mutara** to do the above named research at this school.

Signature: _____

Signed at _____ on this day of _____2014.

Information sheet to the Parents

University of the Witwatersrand
Wits School of Education

JBC Secondary school
P. O. Box XXXX
Maraisburg
1700

13th August, 2014

To: _____'s parent/guardian

REF: Information Sheet

My name is **Lydia Mutara**. I am your child_____'s mathematics teacher and studying MSc Science Education (Mathematics) in the School of Education at the University of the Witwatersrand.

I am carrying out a study on **Exploring Grade 10 learners' errors and misconceptions involved in solving probability problems** in schools in Johannesburg, mainly looking at **the challenges in using different representations**. I also intend to determine the reasons for the errors that students have on these concepts. This research is on Grade 10 mathematics learners because I noticed that since the re-introduction of probability, learners show that they do not have a good grasp on this topic. I strongly believe that diagnosing our learners' errors and misconceptions can only help us to devise strategies to teach them better. My research should not only benefit the institution where it is conducted, but also the South African educational system in improving the teaching and learning of Mathematics.

The research will involve:

- Grade 10 mathematics learners writing on probability tasks over a period of $1\frac{1}{2}$ hours. The tasks will be written after school to avoid disrupting the school times and prejudicing learners who do not want to participate in the research. Their scripts will be marked, taking particular note of the errors that the learners make on the tasks.

- Six learners will be selected for interview based on their errors on the tasks. These will be two exhibiting errors of visual mediators, two showing errors of signifiers

and two exhibiting errors of routine on the tasks. The learners will be interviewed to probe and elicit their thinking on the errors they show on the tasks.

- To accurately capture what learners say about their errors, the interviews will be audio-recorded. The interviews will take at most 10 minutes per learner during break time in my classroom, spread over three days.

Participants will not be paid for the study, advantaged or disadvantaged in any way.I would like to make it clear that participation in this study is entirely voluntary, no harm is envisaged, and all information will be treated as confidential and names not known. Participants can choose to accept or decline to answer any questions, and can withdraw from the study at any given time. All raw data will be kept in my classroom cupboards under lock and key during and after the study. The raw data will be destroyed between 3 and 5 years after the completion of the study.

I hope to publish part or all the results of this study in academic journals. In order to maintain anonymity and confidentiality, all names I use will be pseudonyms. I therefore ask your consent for your child's participation in the written tasks, post task interviews and for the audio recording of the interviews.

Yours faithfully

Lydia Mutara Phone: 078 194 0908

Signature: _____ **Date:** _____

Research Supervisor: Judah Makonye Phone: 011 717 3206
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

Information sheet to the Learners

University of the Witwatersrand
Wits School of Education

JBC Secondary school
P. O. Box XXXX
Maraisburg
1700

13th August, 2014

To: _____

REF: Information Sheet

I your teacher Lydia Mutara am studying for MSc Science Education (Mathematics) in the School of Education at the University of the Witwatersrand.

I am carrying out a study on **Exploring Grade 10 learners' errors and misconceptions involved in solving probability problems** in schools in Johannesburg, mainly looking at **the challenges in using different representations**. I also intend to determine the reasons for the errors that students have on these concepts. This research is on Grade 10 mathematics learners because I noticed that since the re-introduction of probability, learners show that they do not have a good grasp on this topic. I strongly believe that diagnosing our learners' errors and misconceptions can only help us to devise strategies to teach them better. My research should not only benefit the institution where it is conducted, but also the South African educational system in improving the teaching and learning of Mathematics.

The research will involve:

- Grade 10 mathematics learners writing on probability tasks over a period of $1\frac{1}{2}$ hours. The tasks will be written after school to avoid disrupting the school times and prejudicing learners who do not want to participate in the research. Their scripts will be marked, taking particular note of the errors that the learners make on the tasks.

- Six learners will be selected for interview based on their errors on the tasks. These will be two exhibiting errors of visual mediators, two showing errors of signifiers and two exhibiting errors of routine on the tasks. The learners will be interviewed to probe and elicit their thinking on the errors they show on the tasks.

- To accurately capture what learners say about their errors, the interviews will be audio-recorded. The interviews will take at most 10 minutes per learner during break time in my classroom, spread over three days.

Participants will not be paid for the study, advantaged or disadvantaged in any way.I would like to make it clear that participation in this study is entirely voluntary, no harm is envisaged, and all information will be treated as confidential and names not known. Participants can choose to accept or decline to answer any questions, and can withdraw from the study at any given time. All raw data will be kept in my classroom cupboards under lock and key during and after the study. The raw data will be destroyed between 3 and 5 years after the completion of the study.

I hope to publish part or all the results of this study in academic journals. In order to maintain anonymity and confidentiality, all names I use will be pseudonyms. I therefore ask your consent for participation in the written tasks, post-task interviews and for the audio recording of the interviews.

Yours faithfully

Lydia Mutara Phone: 078 194 0908

Signature: _____ **Date:** _____

Research Supervisor: Judah Makonye Phone: 011 717 3206
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

Parents' Informed Consent Form for written task

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Conducting Research in mathematics classrooms

Please fill in the reply slip below if you agree to let me use your child's responses to the tasks on probability. I will use my notes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Parents' Informed Consent

I _____the parent/guardian of _____

1. Hereby confirm that I have been informed by **Lydia Mutara** about the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my child's responses in the test will be processed without mentioning his/her real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. My child can at any stage, without prejudice, withdraw his/her participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) give consent for my child to write the research tasks. Yes/No

Signature of Parent: _____ **Date:** _____

Details of contact person;

Name: _____

Phone: _____

Parents' Informed Consent Form for interview

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Conducting Research in mathematics classrooms

Please fill in the reply slip below if you agree to let me use your child's responses to the tasks on probability. I will use my notes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Parents' Informed Consent

I _____the parent/guardian of _____

1. Hereby confirm that I have been informed by **Lydia Mutara** about the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my child's responses in the test will be processed without mentioning his/her real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. My child can at any stage, without prejudice, withdraw his/her participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) give consent for my child to be interviewed. Yes/No

Signature of Parent: _____ **Date:** _____

Details of contact person;

Name: _____

Phone: _____

Parent audio recording consent form

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Audio Recording in mathematics classrooms

Please fill in the reply slip if you agree to have me audio record an interview in which your child will be a participant. I will use these audiotapes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Parent's Informed Consent for Audio Recording.

I am _____the parent/guardian of _____

1. I have received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
2. I understand that **Lydia Mutara** will keep all raw data under lock and key for a period of up to 5 years. After this, the raw data will be destroyed. Yes/No
3. In view of the requirements of the research, I agree that the data collected during this audio recording can be processed in a computerized system by the researcher. (Please tick one). Yes/No
4. I accept that my child can be audio recorded should my child be picked for post-tasks interviews. (Please tick one). Yes/No

Signature of Parent: _____ Date: _____

Contact person: _____Phone: _____

Learners' Informed Consent Form for written task

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Conducting Research in mathematics classrooms

Please fill in the reply slip below if you agree to let me use your responses to the tasks on probability. I will use my notes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Learners' Informed Consent

I _____

1. Hereby confirm that I have been informed by **Lydia Mutara** about the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my responses in the test will be processed without mentioning my real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. I am aware that I can at any stage, without prejudice, withdraw my participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) give consent to write the research tasks Yes/No

Name of Learner: _____

Signature of Learner: _____ **Date:** _____

Learners' Informed Consent Form for interview

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Conducting Research in mathematics classrooms

Please fill in the reply slip below if you agree to let me use your responses to the tasks on probability. I will use my notes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Learners' Informed Consent

I _____

1. Hereby confirm that I have been informed by **Lydia Mutara** about the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my responses in the test will be processed without mentioning my real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. I am aware that I can at any stage, without prejudice, withdraw my participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) **give consent to be interviewed** Yes/No

Name of Learner: _____

Signature of Learner: _____ **Date:** _____

Learner audio recording consent form

University of the Witwatersrand
Wits School of Education

Informed Consent Form for Audio Recording in mathematics classrooms

Please fill in the reply slip if you agree to have me audio record an interview in which your child will be a participant. I will use these audiotapes for my study called:

Exploring the errors and misconceptions involved in solving probability problems using different representations.

Learner's Informed Consent for Audio Recording.

I _____(name of learner).

1. I have received, read and understood the Information and Consent sheets regarding the educational study. Yes/No

2. I understand that my teacher **Lydia Mutara** will keep all raw data under lock and key for a period of up to 5 years. After this, the raw data will be destroyed. Yes/No

3. In view of the requirements of the research, I agree that the data collected during this audio recording can be processed in a computerized system by the researcher.
(Please tick one). Yes/No

4. I accept I can be audio recorded for post-test interviews. (Please tick one). Yes/No

Signature of Learner: _____ Date: _____

Appendix 5: Ethical Clearance Letter from the University of the Witwatersrand

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

05 September 2014

Student Number: 579347

Protocol Number: 2014ECE040M

Dear Lydia Mutara

Application for Ethics Clearance: Master of Science Education

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Exploring Grade 10 learner errors and misconceptions involved in solving probability problems with different representations.

The committee recently met and I am pleased to inform you that **clearance was granted**.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Maseko

Wits School of Education

011 717-3416

cc supervisor- Dr J Makonye