- While the cable is winding onto the drum surface, there will be no pulse in the y direction, as the cable remains at constant radius. When a layer change occurs at position 3-4, a strong y pulse occurs. Thereafter a smaller y pulse occurs at each successive coil crossover as the cable mounts over the underlying coil layer.
- A pulse in the z direction occurs at each coil crossover, as the coiling rate is momentarily increased as the cable rises over and across the Lebus surface or lower coil layer. This pulse is accentuated at a layer change.

The various pulses occurring in the rope are illustrated in relation to the rope position in figure 1.4. The frequency at which the pulses occur is directly related to the drum winding speed. Since there is a coil cross-over every half revolution of the drum, the excitation frequency is twice that of the drum rotational frequency, for a 180° Lebus sleeve.

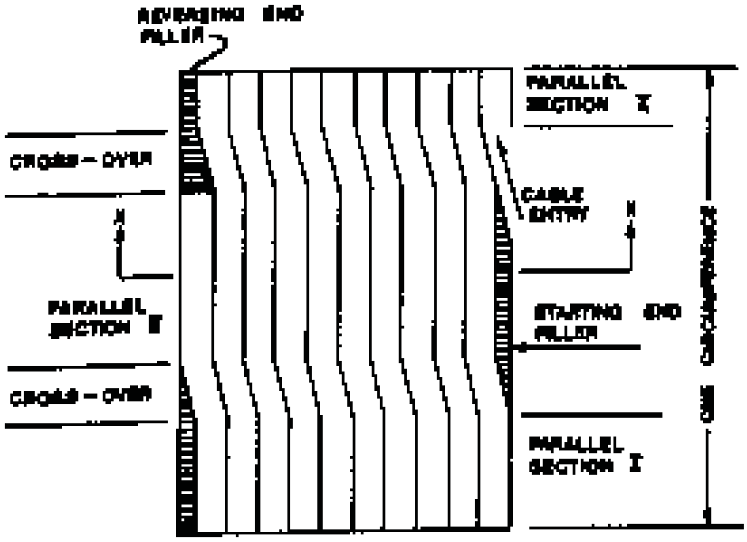

Figure 1.2: Mankowski[1982], Figure 2.4(a): Winder drum fitted with a Lebus liner

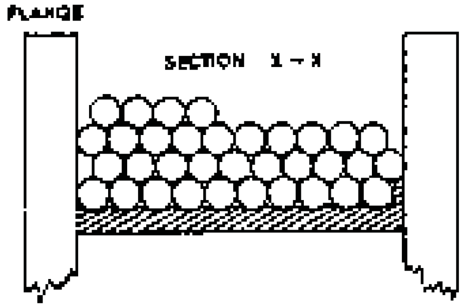

Figure 1.3: Mankowski[1982], Figure 2.4(b): Rope coiling pattern

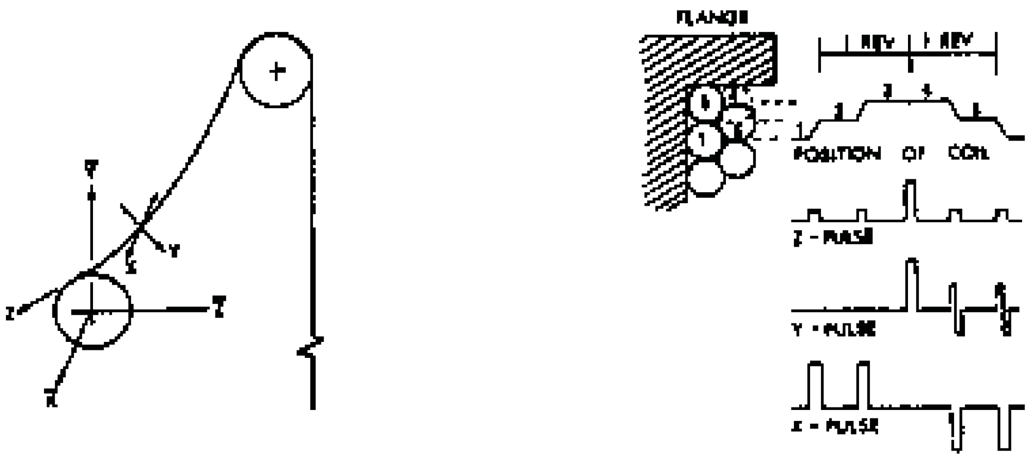

Figure 1.4: Dimitriou & Whillier[1973], Figure 8: Pulse initiation by coil cross-overs at the drum

1.2 The Study of Dynamics in the Industry

1.2.1 Longitudinal Dynamic Studies

Various aspects of hoisting technology have been examined in the past. Initially the industry was concerned with adequate rope factors of safety. This stimulated research into the longitudinal behaviour of a cable supporting an end mass under emergency braking. This aspect was first investigated by Vaughan[1903], where a lumped parameter model of the rope was employed to simulate a descending cable with a suspended mass at its end, suddenly stopped at the top support. Vaughan's [1903] analysis demonstrated that a critical depth exists, where, due to kinetic shocks, the cable is more severely stressed than that at greater depth. Perry[1906] solved a similar problem utilising the solution to the wave equation, however his analysis concentrated on the waveform and frequency of subsequent stress oscillations. These analyses assumed idealised instantaneous deceleration and thus simulated extreme loading conditions. Perry and Smith[1932] assessed the influence of mechanical breaking on winding equipment. Part of their study focused on the calculation of kinetic tension in the rope due to acceleration/deceleration or emergency breaking. They illustrated that if a ramp acceleration/deceleration profile was applied with a period equal to that of the fundamental longitudinal mode, minimum dynamic response would occur. Pollock and Alexander[1951], extended this work, refining the analysis by including higher order terms in the solution, and examining the residual response of the rope after the acceleration/deceleration had ended. Harvey and Laubscher[1965] examined the longitudinal behaviour of the hoist system, including the inertial effects of winder motor/drum and sheave, for the purpose of developing a control system capable of reducing residual response amplitudes during emergency braking. The control system developed imposed an emergency deceleration profile consisting of a ramp change until a predetermined maximum value had been obtained, holding this value constant until the end of the braking cycle. Active winder control is currently being assessed, with respect to the initial acceleration and deceleration profiles imposed during normal winding (Backeberg[1990]). This will result in lower residual response during normal operation, thus improving the fatigue life of ropes currently in use.

More recently, with the discovery of significant ore reserves between 3000-5000m levels, the aspect of rope factor of safety has been re-examined. The rationale being that a reduction in this factor may extend current hoisting technology to accommodate mining to greater depth without the use of multiple hoist systems. Naturally this would also benefit existing installations in terms of the allowable rope life, or increased payload and production rate.

Greenway[1989] re-addressed the problem as studied by Vaughan[1903], Perry [1906], Perry and Smith[1932], Pollock and Alexander[1951], deriving a model of the longitudinal response of the rope with a suspended end mass under various acceleration/deceleration profiles. This study was primarily intended to be a parametric study to assess the influence of physical parameters on the dynamic response. By utilising a non-dimensional approach, it was demonstrated that the peak response of the upper end dynamic force, and hence the ratio of dynamic to static factor of safety, is a weak function of the ratio of rope mass to attached mass. Also, the dynamic factor of safety is a weak function of shaft depth and of rope selection strategy. The aspect of reduced dynamic response with an acceleration profile of period equal to the fundamental longitudinal mode was confirmed. This study established that scope exists for reducing the rope factor of safety, through controlled ramp acceleration and deceleration profiles. In a similar vein, Greenway[1990a] evaluated the limits of hoisting from great depth. As noted, the cost of developing main and subshaft hoist systems provides strong motivation for deep single lift winding systems. Greenway[1990a] concludes that the factor of safety of the rope, and to a lesser extent, winding speed are primary parameters influencing depth and production constraints.

The static and dynamic characteristics of wire rope has received concerted attention by Costello et al[1983]. The models developed are accordingly complex and non-linear, and are summarised by Costello[1983]. An important aspect of the rope construction is its torsional response due to axial load. Butson[1981] examined this problem in the context of deep level mining. A result of his thesis confirmed that in the case of *long* wire cables rotationally restrained at the ends, the rotational coupling can be discarded, reducing the problem to an *uncoupled* wave equation. Greenway[1990b], re-examined this problem, since triangular strand rope construction, which exhibits torsional coupling is currently utilised in the South African mining industry. Greenway's analysis included an investigation of the lay length changes which occur as a consequence of the torsional response. It was concluded that the lay length change is more strongly influenced by the total rope mass, rather than the end load. Consequently, the application of such cables to deep level mining may be problematic.

1.2.2 Coupled Dynamic Studies

In the early 70's, it was realised by the mining industry that further research concerning the dynamic behaviour of the catenary system, including both lateral as well as longitudinal behaviour was required. Dimitriou and Whillier [1973] initiated research in this direction. Their analysis concentrated on the quasi-static description of the linear transverse and longitudinal natural frequencies of the catenary and vertical cable, as a function of shaft depth. It was considered that the main source of excitation was due to periodic impulses applied to the drum end of the rope as a result of coil cross-overs. Consequently an analysis of the rope movement during coiling on to the drum was conducted, resulting in a description of pulse frequency and displacement magnitudes. They demonstrated that a 180° Lebus liner would impart even harmonics of the drum rotational frequency, whilst a single groove Lebus liner would impart all harmonics of the drum rotation frequency. Furthermore, the magnitudes of the first three harmonics of the excitation would be similar. Subsequently graphical plots consisting of the natural frequencies and excitation frequency versus shaft depth were prepared[2], as illustrated in figure 1.5. It was hoped that this information would explain the phenomenon of rope whip.

Kloof Mine in Carltonville South Africa, was experiencing rope whip, and was utilised as a case study. Figure 1.5 is based on the Kloof Mine system parameters. Catenaries A,D which were of similar length exhibited the most severe vibrations. It was observed that the rope whip was more severe with a full skip ascending than an empty skip descending. With reference to figure 1.5, severe vibration began at phase 1 during ascent, continuing up until the end of the cycle. It was observed from the frequency plots that this condition came about as a result of the coil cross-over frequency coinciding with the second lateral mode of the cable. Furthermore, the fourth longitudinal mode of the system crosses the second harmonic of the coil cross-over excitation frequency line at the start of phase 1. Hence, the frequency of the longitudinal oscillation matches the frequency of the tension fluctuations induced by the amplitude of the second lateral mode of the catenary. Thus it may be expected that longitudinal and lateral oscillations would interact, perhaps mutually exciting one another.

[2]Dimitriou and Whillier, and Mankowski employed the notation $FTCn$ for the n^{th} lateral mode of the catenary, and $FLVn$ for the n^{th} longitudinal mode of the vertical cable, where the vertical cable was treated as fixed at the sheave end. Four catenaries are represented this figure namely A-D; catenaries A,B represent the overlay winder, whilst B,D represent the underlay winder.

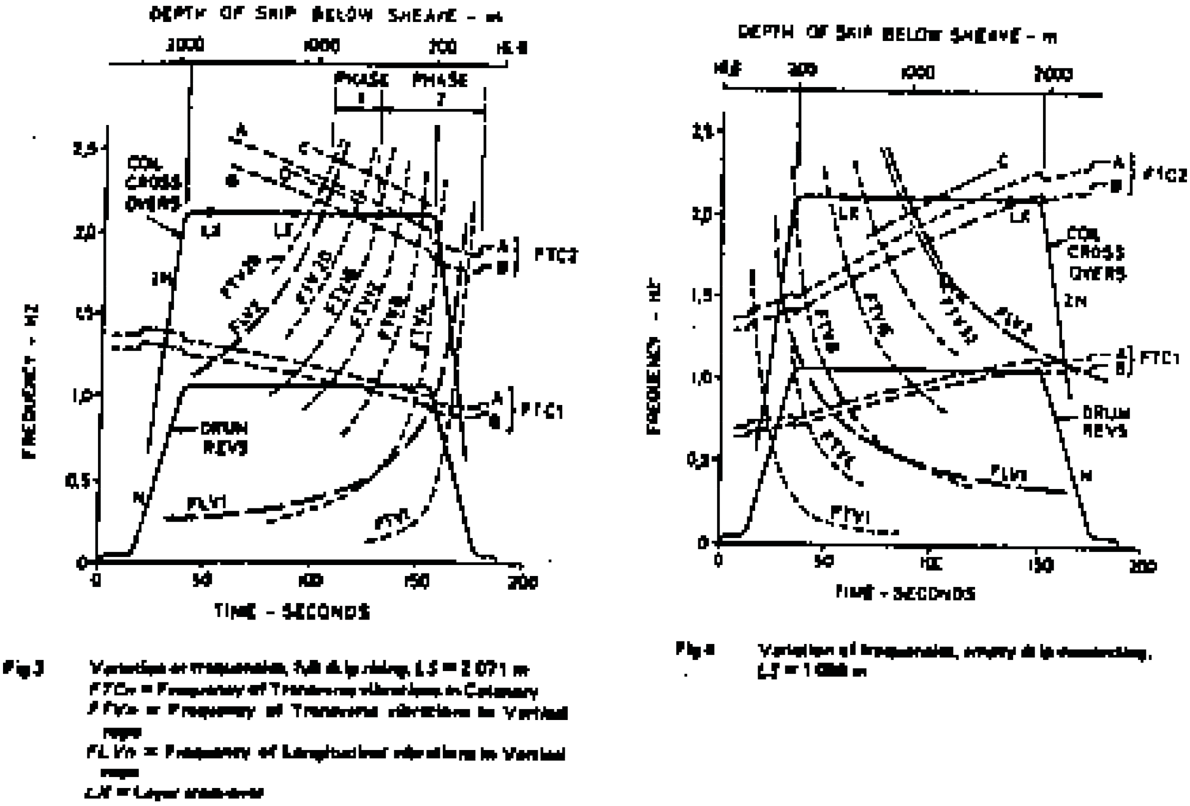

Figure 1.5: Dimitriou & Whillier[1973]: Linear frequency map

The catenary lengths for the Kloof underlay and overlay winders were:
$A = 74.4m$, $B = 79.6m$, $C = 69.8m$, $D = 75.3m$.

The passage to phase 2 coincides with a layer cross-over, and a significant axial pulse would be introduced to the system, promoting higher amplitudes. Observations of the lateral response of the vertical cable confirmed that amplitudes rose at the start of phase 1 and were approximately of equal wavelength to that of the catenary.

Dimitriou and Whillier [1973] were aware of the limitations of a linear analysis. This prompted a discussion on aspects of nonlinear system dynamics, providing a plausible description of the observed behaviour. In this discussion, the concept of both the jump phenomenon and of subharmonic resonance of the vertical rope was described. The latter referred to the occurrence of a lateral response of the vertical rope, at half the frequency of longitudinal harmonic tension fluctuations. It was stated that this is a natural consequence of string vibration, as lateral response is a subharmonic of tension fluctuation. This discussion led to a brief description of primary stability intervals associated with parametrically excited systems, and hence the importance of considering the possible influence of longitudinal tension fluctuations with respect to the lateral response of the catenary and vertical rope. The observed lateral response of the vertical rope was attributed to this effect, with regard to tension fluctuations induced by the lateral motion of the catenary. The concept of mutual excitation of the catenary by the lateral response of the vertical rope, and visa versa was briefly discussed. The possibility of an amplitude dependent natural frequency relationship pertaining to the catenary was presented as one explanation for the different behaviour between the ascending and descending cycles. Although these concepts were discussed, no formal mathematical treatment was pursued, and thus the discussion led to an intuitively described relationship between the lateral natural frequency of the catenary and the longitudinal natural frequency of the vertical rope system. On reflection, these relationships in essence described the possibility of autoparametric coupling between the vertical rope and the catenary, as well as the condition of internal resonance.

Dimitriou and Whillier [1973] concluded that a more detailed study including the non-linear aspects of the system would be required to interpret the forced nonlinear response observed at Kloof Gold Mine. However, provided care was exercised during the design stage, so as to avoid the coincidence of an excitation frequency with any linear lateral natural frequency, then to some degree, later problems could possibly be reduced or avoided. This unfortunately discards the observations concerning the parametric nature of the system, and reverts to a purely linear classical approach. This approach has been adopted by the industry in simplified form. Boshoff[1977] prepared a document for The Anglo American Corporation, describing a hoist system design methodology for the avoidance of rope whip, which still stands today. This document considers only the first mode of the catenary and the first harmonic of the coil cross-over frequency, and neglects the longitudinal modes of the coupled system. Cognisance was taken of the possible effect of transient excitations which occur at the layer change, in that these may cause miscoiling problems. For this reason, the layer change location was specifically chosen not to occur simultaneously with a catenary resonance. However, since the phase of the transverse excitation changes by 180° after a layer change, the transverse excitation may be used to precipitate the build up of a resonant condition. Usually it is not possible to wind to a great depth without inducing resonance in the catenary on either the ascending or descending cycle. It was sugested that it is preferable to accept a resonant condition on the down wind as opposed to the upwind as a design strategy.

Mankowski[1982] extended the study of the the Kloof Mine system by developing a digital computer program, capable of simulating the forced response of the system. This followed the suggestion by Dimitriou and Whillier [1973] that a comprehensive analysis was required which included nonlinear effects, as well as accounting for the coupling between the catenary and vertical rope. In this study, Mankowski[1982] investigated three different programming strategies.

Firstly, a lumped mass model of the system was employed. The model retained three translational degrees of freedom for each mass element in the catenary, whilst only considering the longitudinal motion of the vertical rope. The effect of gravity on the catenary was accounted for, as well as nonlinear geometric deformation of the cable elements. Thus the analysis was not limited to small deflection theory. The displacement function induced by the coil cross-over was simulated accurately, in the three orthogonal directions. The analysis did not account for the velocity of the cable, as this was considered a secondary effect. A simulation was performed on a system where the conveyance was close to the sheave, and the performance of the programme assessed. It was found that the formulation required substantial computational effort in order to attain accuracy and numerical stability and thus an alternative formulation based on the method of characteristics was implemented. Once again computational effort was a limiting factor, and finally a method based on Bergeron's [1961] impedance technique was employed. Expertise with the impedance method was gleaned by applying it firstly to simulate the longitudinal response of the system under harmonic excitation. Subsequently, the nonlinear lateral response of a rope with a clamped/pinned boundary condition, excited by a harmonic displacement at the clamped end, was successfully simulated. Both simulations performed satisfactorily and consequently, a final model attempting to accommodate both longitudinal and lateral behaviour was developed. In this process, the catenary was simplified to assume a parabolic shape symmetrical with respect to the span, where the axial variation in tension due to gravity was discarded. This was justified on the basis that the catenary was taut and had a sag to span ratio of less than 1:20. Only lateral response of the catenary was modelled as stated *"..each lumped mass is constrained to move in an x-y plane perpendicular to the span .."*, whilst the vertical rope was constrained to exhibit only longitudinal motion. The catenary tension was calculated on the basis of the rope stretch, and coupled to the vertical rope through an inertial balance across the sheave. The excitation at the drum was modelled as a displacement function of time, accounting for the lateral and longitudinal pulses applied to the system. The longitudinal pulse was accommodated by increasing or reducing the unstretched length of the catenary, hence directly influencing the calculation of the average tension. The axial travelling velocity was not modelled as it was considered to be unimportant, however the system parameters were varied according to the drum velocity and hence the vertical rope length, as a function of time. This model exhibited numerical difficulties in that high frequency modulations in the output occurred. As a result, a numerical damping function was employed to smooth the data.

Even though care was taken to construct the digital simulation algorithm, and detailed simulations of the lateral and longitudinal behaviour tested the robustness of each algorithm independently, the coupled system simulation became unstable at approximately 800 m during the raising cycle. "*The reason for the termination of the graphical output at 45 s, is primarily, the breakdown of the relation governing the speed of transverse disturbances in the inclined cable.... and the simulation becomes unstable*". Thus the simulated amplitude of the inclined cable was so severe that the system approached a slack rope condition. Due to this, further discussion concerning the simulated behaviour of the coupled system was restricted to the region between shaft bottom and 800 m. Mankowski[1982] attributed the instability of the simulation to the neglect of rope slip at the sheave, "*..slip at the headsheave would occur as soon as the tension in the inclined cable drops sufficiently and that slip would prevent further reduction in tension*".

With regard to the stability of the system, Mankowski[1982] states "*As a resonant condition is approached and the amplitudes increase, a form of auto-parametric excitation sets in with the result that growing amplitudes reinforce higher tension fluctuation and visa-versa. Taking into account the almost exponential rise in the velocity and tension amplitudes exhibited in the simulation results beyond 1000m, it is possible that auto-parametric excitation is one cause for the breakdown of the simulation model during resonance*".

A simulation of the Kloof winding cycle was thus performed between the depths of 1300-800 m only. Mankowski[1982] discussed the results of the simulation at length. Concerning the longitudinal motion of the skip he states: "*One puzzling aspect ... is the appearance of the fourth longitudinal mode. The fourth mode appears in all the computer output ... and is seen to be present before the completion of the acceleration profile and persists until a depth of approximately 900m where it locks in on* $2 \times FTC2$ *(the second transverse mode of the catenary)*". Mankowski[1982] detailed the dominant frequency content of the skip response versus shaft depth in figure 11.28 of his thesis[3]. This is reproduced in figure 1.6. The unshaded circles indicate the variation in the skip velocity during the simulation. The solid lines in this figure reflect the natural frequencies of the longitudinal system. These lines were calculated from a theoretical model of the longitudinal system, where the sheave inertia and skip mass were accounted for as lumped masses. The longitudinal modes are indicated in braces, where the sign indicates the relative phase between the sheave and skip motion. The dotted line indicates the relative amplitude

[3] Figures of this form were constructed by counting the cycles of the output records manually, and not by employing a Fourier analysis. Hence no indication is presented concerning the amplitudes associated with the frequency content, and in essence only the dominant modes of the wave form were extracted.

ratio between the sheave and skip, where the numerical ratios are presented on the upper ordinate of the figure. This amplitude ratio is defined as the ratio of the skip motion to the sheave motion, and hence the relative minimum at 3.75 Hz indicates a larger sheave motion than skip motion. The figure clearly indicates the presence of the fourth longitudinal mode in the simulation. Mankowski[1982] could find no plausible explanation for the persistence of the fourth longitudinal mode and states : *"... the propensity of the model to exhibit a fourth mode appears to be an inherent feature of the computer simulation model ; until a precise mathematical closed-form solution to the non-linear problem can be formulated, the phenomenon remains intractable to explanation"*.

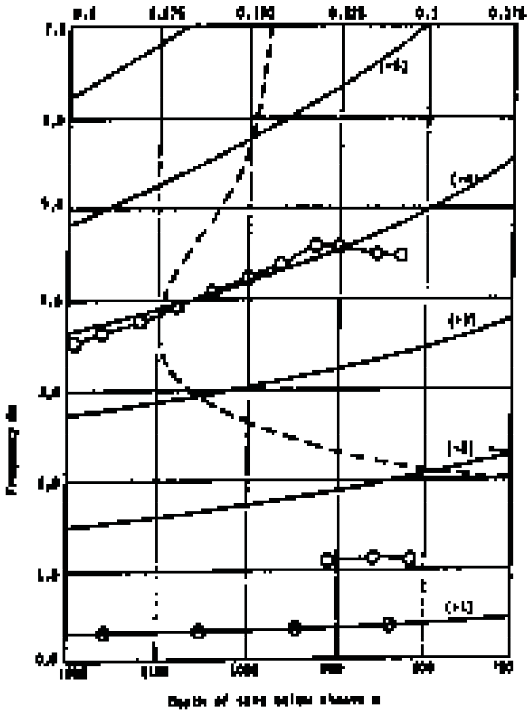

Figure 1.6: Mankowski[1982], Figure 11.28: Longitudinal frequency content of skip velocity - simulation model and theoretical modes of double mass model.

The frequency content of the response depicted in figure 1.6 was described by Mankowski[1982] as follows:

- The first longitudinal mode is excited by the acceleration profile. In practice, the first longitudinal mode may be excited by either the acceleration profile, or by the large axial pulse introduced by a layer change.

- Longitudinal response at $2 \times FTC2$ is due to the lateral response of the catenary at $FTC2$.

- The response of the skip at $FTC1$ is due to tension fluctuations in the inclined cable at frequency $FTC1$. This is associated with lateral subharmonic response of the inclined cable at frequency $\frac{1}{2}FTC1$ due to the cable rising above its equilibrium configuration.

Figure 11.29 and 11.30 of Mankowski's thesis are reproduced in figure 1.7. The first figure (figure 11.29) contains the dominant frequencies present in both the inclined cable tension and the skip velocity records from the numerical simulation. In Figure 1.7, the shaded circles serve two purposes, they indicate both the frequency of the tension variation occurring in the inclined cable and the variation in the skips velocity. The second figure (figure 11.30) presents possible excitation frequencies of the longitudinal rope, namely that at the drum revolution frequency, coil cross-over and $2\times$ coil cross-over, and the autoparametric excitation frequencies due to the transverse motion of the catenary, namely at $FTC1$, $FTC2$, $2 \times FTC2$. Mankowski[1982] attempted to describe the presence of the fourth longitudinal mode, by examining the steady state longitudinal response as a function of shaft depth, due to an excitation at these frequencies ie at $FTC1$, $FTC2$, $2 \times FTC2$. This did not prove fruitful.

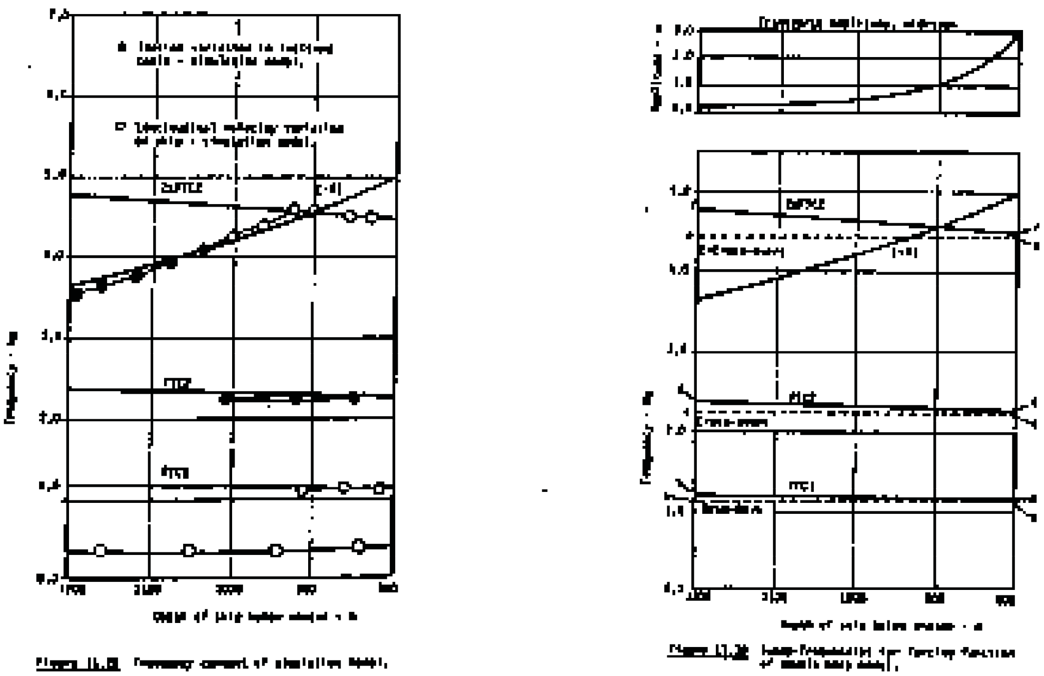

Figure 1.7: Mankowski[1982]:Frequency content of the simulation and the double mass model.

The autoparametric coupling between the catenary and vertical rope is evident in the simulation, especially in the region where the second transverse mode clearly causes longitudinal response in the vertical rope. This occurs at approximately 925m, after which the longitudinal response is dominated by the lateral response of the catenary at $2 \times FTC2$. It is interesting to note that with respect to the coupling between the longitudinal and lateral catenary motion, two conditions of internal resonance are approached whereby the longitudinal natural frequency tunes to twice the lateral frequency of the catenary. Namely at approximately 925m $FLV4 = 2 \times FTC2$, where frequency locking occurs, and towards the end of the simulation $FLV2 = 2 \times FTC1$ at 700m.

The concluding chapter of Mankowski's thesis draws attention to the correlation achieved between the simulation and observed results. Dimitriou and Whillier[1973] observed that the amplitude of lateral motion began to grow at approximately 900m. The motion settled into a clearly defined second mode, where the out of plane amplitude was largest and of the order of 1m. On occasions these vibrations continued to the end of the wind with a gradual change in mode but no perceptible change in amplitude. On other occasions severe rope whip occurred at the beginning of phase 2 (indicated in figure 1.5, at approximately 550m), following a layer change, where the in-plane amplitude reached amplitudes in excess of 2m. Mankowski's simulation predicts large amplitude catenary motion (greater than 1m amplitude) at approximately 900m. On this basis, it was judged that a fair correlation between the simulation and observed behaviour was achieved. Unfortunately a simulation of the descending skip was not presented. This would have been a useful validation of the simulation, as rope whip was not observed for the descending cycle.

Although the aspect of autoparametric excitation was identified as a mechanism affecting the stability of the simulation, the question as to whether this was representative of the system was left for future experimental and theoretical validation. As stated ".. *until further improvements in the simulation model are made to accommodate slack tensile conditions and cable slip at the headsheave, the question of parametric excitation occurring in practice cannot be answered definitely*".

Mankowski[1982] suggested that further experimental work was necessary to fully appreciate the complexities of the system, and to validate the digital simulation. In this regard rotational acceleration measurements of the headsheave would be essential. Experimental activity on operating shafts is limited due to production restraints. In light of this, Backeberg[1984] constructed a laboratory model of the system for further experimental assessment. The model consisted of an hydraulic actuator which provided axial excitation to a wire rope passing over a sheave to a suspended end mass. Figure 1.8 illustrates the laboratory model. The purpose of this work clearly stemmed from Mankowski's thesis in that it was intended to complement the simulation through correlation with experimental observation. The configuration of the model focussed the experimental results on the parametric behaviour of the system. Unfortunately a dimensional analysis tuning the laboratory model to simulate even approximately the parameters of an actual installation was not performed. In particular, the tuning of the first longitudinal mode was substantially higher than the lateral catenary modes excited during the test. Thus any possible interaction between the longitudinal and lateral catenary modes was not marked. Nevertheless, the results indicated that a relatively small axial parametric excitation was sufficient to excite the lateral modes of the catenary. The aspect of nonlinear response, namely the jump phenomenon was observed to a degree. The headsheave was observed to rotate when large amplitude catenary motion occurred, accentuating the coupling between the the catenary and vertical rope. This illustrated that slip at the sheave could arise. The scope of the investigation did not extend to the determination of the regions of parametric stability of the system, and thus these were not identified.

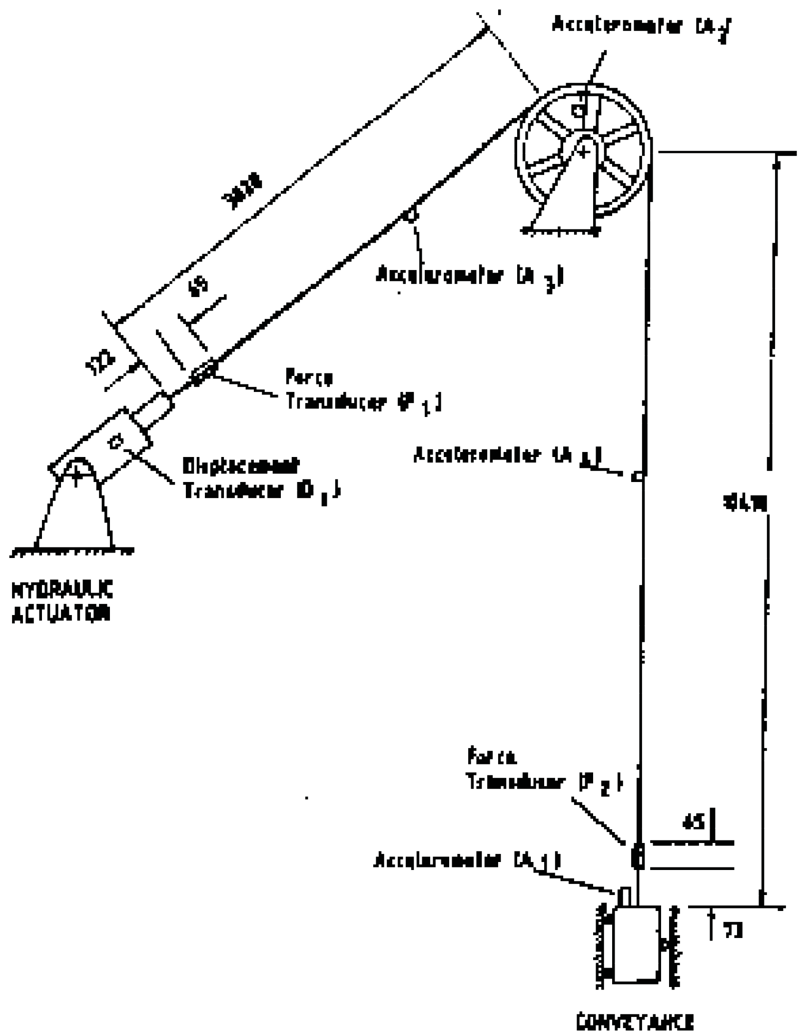

Figure 1.8: Backeberg[1984], Figure 4.1: Mine rope model

1.3 Scope for Analytical Research

As is evident from previous research, adverse catenary motion is a phenomenon which is cause for concern in the mining industry. Such motion is commonly referred to as *rope whip*. The fundamental parameters governing rope whip have not been clearly defined, and thus the dynamic integrity of hoisting systems has depended on the experience and intuition developed in the mining industry.

Mankowski[1982] identified a number of features of the hoist system which contribute to the complexity of the system behaviour. It is evident that these aspects motivated his research, and that emphasis was placed on modelling the nonlinear aspects of the system and its response numerically. Features of the system identified in the modelling process were:

- The geometrically complex nature of the cable construction.
- The inclination and sag of the catenary.
- Geometric stiffening and the associated tension variations in the catenary during large amplitude transverse motion.
- The complex boundary conditions applied to the rope at the conveyance end and at the winding drum.
- Intermediate boundary conditions at the headsheave, coupling the catenary to the vertical rope.
- Friction and slip of the cable at the headsheave.

The scope for analytical research within the context of those items identified by Mankowski[1982] is substantial. Mankowski[1982] chose to develop a digital programme modelling the effect of gravity as well as geometric stiffening, and thereby investigated the dynamic behaviour of the system.

In contrast, it is proposed here that an approach which develops the nonlinear partial differential equations of motion for the mine hoist system prior to numerical implementation, enhances the scope for describing the dynamic nature of the system. This is inferred from studies presented in the literature concerning the dynamic analyses of nonlinear taut strings and cables with pinned end conditions. These studies provide excellent paradigms for advanced studies in nonlinear dynamics. Although the techniques applied in this study are not novel, the unique characteristics of the mine hoist system, which arise due to the boundary condition at the sheave, and hence the coupling between the lateral catenary and longitudinal system response, presents a novel extension to current knowledge regarding mine hoist catenary dynamics, and perhaps a practical vehicle for further analytical development. This aspect of the coupling between longitudinal and lateral motion in the mine hoist system necessitates the retention of the longitudinal system inertia, which is commonly neglected in analyses of taut strings and cables with pinned end conditions. Furthermore the system is excited periodically, and has a nonstationary nature due to the transport velocity necessary to complete a winding cycle. The inclination of the catenary and its curvature introduce a further dimension for study.

It is likely that a comprehensive treatment including all the former aspects will lead to a situation intractable to analysis. In this context, it is necessary to capture the fundamental nonlinear aspects of the system, which promote rope whip. The autoparametric nature of the system was identified in previous studies as a potential mechanism promoting rope whip and thus warranted further research. Although literature is available on the parametric, autoparametric and internal resonance of dynamic systems, the mine hoist system provides a vehicle for a practical investigation novel to the literature. In this regard, the system has peculiar features in that the lateral natural frequencies of the catenary and vertical rope are related to higher modes by integer multiples. Specific tuning between the lateral modes and the longitudinal system modes may result in commensurate frequency relationships. Thus it is possible for the system to exhibit regions where not only parametric response, but autoparametric as well as internal resonance occurs simultaneously in the presence of external excitation. Furthermore, since the excitation is periodic, it is possible that more than one internal resonance can be stimulated simultaneously by different harmonics of the excitation frequency. If one considers that in addition to this, the dynamic characteristics of the system are non-stationary, a recipe exists for sustained research beyond the scope of this thesis. It is proposed that these features alone justify sufficient scope for an analytical study.

1.4 Scope of the Study

The autoparametric behaviour of the mine hoist system has been addressed superficially in previous research, thus this aspect of the system behaviour forms the initial focus of the study. The linear stability of the stationary system is governed by the relationship between parametric and autoparametrically induced excitation frequencies, and the tuning of the natural frequencies of the system. It is the definition of such relationships which is considered central to developing an understanding of the potential effect of adverse tuning conditions on the system response. In order to account for transient excitations and the nonstationary nature of the dynamic characteristics of the system, a nonlinear numerical simulation of the system is developed. The specific aims of the thesis are thus:

- The development of the non-linear equations of motion of the mine hoist system.
- The investigation of the autoparametric nature of the system, leading to the definition of the steady state stability of the first order response of the system.
- The development of a numerical simulation of the system which accounts for transient excitation and the non-stationary nature of the system.
- The development of a design methodology for selecting the mine hoist parameters so as to avoid rope whip.

In conclusion, the goal of the research involves proposing a strategy for selecting the system parameters so as to avoid rope whip. Conversely, this is viewed as ascertaining the hoist system dynamic characteristics which contribute to the onset of rope whip. Although the description of the large amplitude response occurring during rope whip is of secondary importance when compared to the definition of the system characteristics required to avoid rope whip, this is a necessary aspect of the study and is addressed through the development of a numerical simulation in the later part of the thesis. This gives further insight into the behaviour of existing systems exhibiting rope whip, leading to design strategies concerning the influence of transient excitation on the overall system behaviour.

Chapter 2

The Dynamics of Strings and Cables

2.1 Introduction

Cables are utilised as structural elements in many situations, a few being ship mooring cables, guy towers, suspension bridges, overhead power lines, and hoisting equipment. The specific configuration of the cable may permit a linear approximation of its behaviour, however in general, a cable exhibits nonlinear behaviour; the degree of nonlinearity being dependent on the configuration and loading to which the cable is subjected. The analysis of dynamic systems can be similarly divided into studies where a linear or a nonlinear approach is adopted. The latter may be imperative for an understanding of the phenomena associated with the observed behaviour. Prior to pursuing a particular analysis strategy with respect to the mine hoist system, it is necessary to examine the broader aspects of the possible behaviour of strings and cables.

Although this thesis is concerned with the dynamic aspects of cables, it is pertinent to briefly consider the static response of a suspended cable. This will provide an insight into the possible nonlinear nature of the problem, as well as facilitating a description of the terminology employed. Thereafter, a brief development of the linear dynamic theory, leading to nonlinear dynamic studies of cables and strings is presented.

2.2 Static Response of Cables

Irvine and Sinclair[1976], present a static analysis of an elastic cable for an arbitrary sag to span ratio, subjected to vertical point loads. Pugsley[1983] presented a review of an analysis by Pipard and Chitty, which considers an approximate method for a shallow sag cable subjected to vertical point loads. The method reduces to that presented by Irvine[1981] for shallow sag cables, which is briefly described below.

As stated by Irvine[1981], the word *catenary* derives from the Latin word for chain, meaning the profile of a chain hanging between two points, under its self weight. The mathematical description of this profile is obtained by solving the differential equations of equilibrium associated with a differential element of the chain. Figure 2.1 illustrates a cable supported at equal height, with initial sag due to its self weight. A free body diagram of a differential element of the cable is presented in the lower part of the figure . The span of the cable is defined as the chord length between the supports, whilst the sag represents the displacement of the profile from the chord. A Lagrangian co-ordinate s is employed, and is aligned along the arc length or equilibrium profile of the cable. In the theoretical development which follows, it is assumed that the element supports tensile loads only, and that the flexural rigidity is zero.

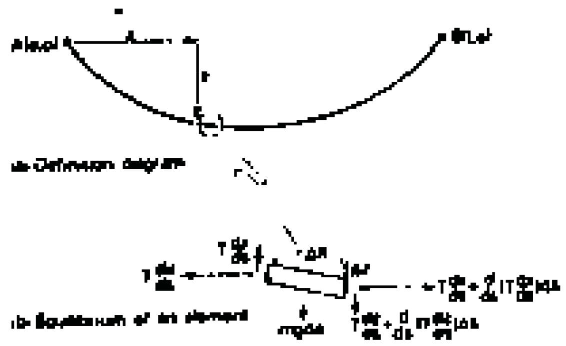

Figure 2.1: Irvine (1981): Equilibrium of an element of cable

On application of a force balance to the differential element in figure 2.1, the equations of equilibrium are obtained as :

$$\frac{d}{ds}(T\frac{dz}{ds}) = -mg$$

$$\frac{d}{ds}(T\frac{dx}{ds}) = 0$$

Where T, m refer to the tension and mass per unit length of the cable respectively. Integration of the second equation, and solving for the integration coefficient yields:

$$T\frac{dx}{ds} = H$$

Where H represents the horizontal component of the tension at the supports. Since no horizontal loads are applied to the cable, H is constant everywhere. The above equations may be manipulated into the form:

$$H(\frac{d^2z}{dx^2}) = -mg\frac{ds}{dx}$$

Under the condition of inextensiblity, $(\frac{dx}{ds})^2 + (\frac{dz}{ds})^2 = 1$, the above equation yields:

$$H\frac{d^2z}{dx^2} = -mg[1 + (\frac{dz}{dx})^2]^{\frac{1}{2}} \tag{2.1}$$

Solving this equation, subject to the boundary conditions, results in the mathematical description of the profile as a function of the span x:

$$z(x) = \frac{H}{mg}[cosh(\frac{mgl}{2H}) - cosh\frac{mg}{2H}(\frac{l}{2} - x)]$$

Although this represents the correct solution for the profile of an inextensible cable, supported at equal elevation under its self weight, significant simplifications may be made when the cable is taut, and its sag to span ratio is small.

Although Irvine [1981] advises that such an analysis is appropriate for sag to span ratios less than 1:8, Pugsley [1983] states that the results should be applicable with considerable accuracy to cables with larger sag to span ratios.

In the case where the sag to span ratio is small, $\frac{ds}{dx} \approx 1$, the cable equation (2.1) reduces to:

$$H\frac{d^2z}{dx^2} = -mg$$

$$\mathbf{z} = \frac{\mathbf{x}}{2}(1 - \mathbf{x})$$

Where $\mathbf{z} = \frac{z}{mgl^2/H}$ and $\mathbf{x} = \frac{x}{l}$. It is evident that the profile is symmetric[1] with respect to the mid-plane of the span line. The sag to span ratio δ is defined as the ratio of the cable sag at mid span, to the span length:

$$\delta = \frac{mgl}{8H}$$

Thus far, the equilibrium profile of a cable due to its self weight has been considered. The analysis can be generalised to include the case of concentrated point loads, however, in preparation for the discussion of the dynamic behaviour of a sagged cable, consideration of the mechanism by which the cable supports additional load through deformation from its equilibrium profile is pertinent. Simply stated, additional tension may be generated in the cable to first order through geometric adjustment of the profile, and to second order through stretch. Both mechanisms contribute to the nonlinear response of a sagged cable. The first mechanism dominates the response of a deep sag cable, where the axial stiffness of the cable is much greater than the geometric catenary stiffness, and thus inextensible cable behaviour dominates the response. The latter mechanism dominates the load deflection response when the cable is taut, and additional tension is generated through axial cable strain. These mechanisms may be investigated by formulating the additional tension generated in the cable when its profile changes from its equilibrium condition, under the action of external load. This load may be a consequence of direct static forces, or inertia forces. The problem is considered below.

[1] Irvine[1981] extends this analysis to include the case of an inclined cable under its self weight, and demonstrates that an approximate profile may be obtained. This profile is no longer symmetric with respect to the span or the mid point of the chord joining the supports.

Consider a cable in equilibrium, where its profile due to self weight is z. If the cable is loaded further, additional deflection w occurs. The deflection w, induces additional tension τ in the cable, and the profile changes to $z+w$. Employing a Lagrangian strain measure, it can be shown that the additional tension generated is related to the displacement of the element through Hooke's Law, and is given as:

$$\frac{\tau}{EA} = \epsilon = \frac{du}{ds}\left(\frac{dx}{ds}\right) + \frac{dz}{ds}\left(\frac{dw}{ds}\right) + \frac{1}{2}\left(\frac{dw}{ds}\right)^2$$

Where u, z, w represent the longitudinal displacement, the equilibrium profile, and the additional vertical displacement induced by the load respectively.

Since the horizontal component of the additional tension generated during the deformation is obtained from $h = \tau\frac{dx}{ds}$, the previous equation may be manipulated to the form:

$$\frac{h\left(\frac{ds}{dx}\right)^3}{EA} = \frac{du}{dx} + \frac{dz}{dx}\frac{dw}{dx} + \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

In the absence of longitudinal loads, h is constant, and this equation may be integrated with respect to x to obtain[2]:

$$\frac{hL_e}{EA} = u(l) - u(0) + \frac{mg}{H}\int_0^l w\,dx + \int_0^l \frac{1}{2}\left(\frac{dw}{dx}\right)^2 dx \tag{2.2}$$

Important observations may be made regarding the form of equation (2.2). Firstly, the cable may be influenced by the support movement, in which case $u(0), u(l)$ would not necessarily be zero. Secondly, it is influenced by the final cable profile via the term $\int_0^l w\,dx$. It is important to note that this term will be zero if an antisymmetric change w in the profile occurs. The third term $\int_0^l \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 dx$ is a measure of the change in arc length of the cable, and is thus a second order effect due to cable stretch. The dominance of these two terms is related to the cable parameter λ^2. The derivation and discussion of the cable parameter will be considered further in the section regarding the dynamics of sagged cables.

[2] Where $L_e = \int_0^l \left(\frac{ds}{dx}\right)^3 dx \approx l(1+8\delta^2)$, and w, z vanish at the limits of the integration, and the relationship $\frac{d^2z}{dx^2} = -\frac{mg}{H}$ has been employed in the reduction.

2.3 Dynamics of Strings and Cables

The development of the wave equation for a taut flat string is associated with many of the eminent personalities in the historical development of dynamics and mathematical physics. Lindsay traced the historical development of the subject in the introduction to the second revised edition of Rayleigh's "The Theory of Sound" [1945], where it is evident that numerous people have applied themselves to the study of string dynamics, for instance Pythagoras, Galileo Galilei, Hooke, Taylor, Bernoulli, D'Alembert, Euler and Lagrange. These analyses resulted in the wave equation, which is commonly presented in the literature as representative of a taut flat string undergoing small amplitude longitudinal or lateral motion. This equation may be derived on application of Newton's Second Law of motion to a differential element of the string. If small amplitude motion is assumed such that only first order terms are retained, then the equations of motion describing the longitudinal and lateral motion of the string decouple and are given as:

$$\frac{\partial^2 w}{\partial t^2} = c_t^2 \frac{\partial^2 w}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial t^2} = c_l^2 \frac{\partial^2 u}{\partial x^2}$$

$$c_t = \sqrt{\frac{T}{\rho A}}$$

$$c_l = \sqrt{\frac{E}{\rho}}$$

$$\frac{c_l}{c_t} = \sqrt{EA/T}$$

where c_t, c_l represent the lateral and longitudinal wave propagation speeds respectively, and w and u represent the lateral and longitudinal displacements respectively.

Note that T/EA is the longitudinal strain under the initial tension T, and in typical applications with steel cables, this value is small, thus the ratio

$c_t/c_l << 1$, and the longitudinal disturbances have a significantly higher propagation speed than the transverse disturbances. For a fixed length of string, the propagation speed of the longitudinal disturbance is thus sufficiently high compared to that of the lateral propagation speed to view the tension as being spatially uniform along the length of the string (Oplinger[1960]). This leads to the concept of spatially uniform but temporally variable tension. Although, when the string satisfies this condition, a state of constant tension may exist along the length of the string, additional tension change may occur during the motion due to changes in the arc length of the string. Accounting for the variation of tension due to the stretch of the string leads to the nonlinear description of the taut flat string. Thus this approach accentuates the nonlinear behaviour of a string where the catenary approaches that of a flat profile. On the other hand, the nonlinearity may be introduced via initial curvature, in which case the string is commonly referred to as a cable. In this case, additional tension may be generated during the motion due to geometric changes in the unstretched profile of the cable. In combination, these effects define the nonlinear nature of cable dynamics, and in the special case where the curvature approaches zero, and the amplitude of vibration is small, the wave equations evolve. A brief discussion outlining the salient features of string and cable dynamics follows. In this discussion, a string is defined as the limiting configuration of a cable, where the curvature is by definition zero.

2.4 Taut String Analyses

2.4.1 Nonlinear Taut String Analyses

Carrier[1945] investigated the nonlinear frequency response of a stretched string. In this analysis, the equations of motion accounted for both longitudinal and lateral inertia. A solution for the in-plane motion of the string due to an initial in-plane sinusoidal deformation was obtained by applying a perturbation method. This analysis indicated that to first order in the perturbation, the nonlinearity induces a stiffening response with increasing lateral amplitude. The nonlinear period is thus amplitude dependent, and decreases with respect to the linear period, as the lateral amplitude of motion increases. Carrier [1945] also considered the coupled out-of-plane problem, and the condition whereby the string describes a quasi-elliptic orbit. Oplinger[1960] investigated the in-plane frequency response of a nonlinear taut string, and derived equations of motion on the basis of spatially uniform but temporarily variable tension. Thus longitudinal inertia was not regarded as significant in his solution. The nonlinear nature of the system was introduced by relating the change in arc length due to the amplitude of the motion, to the additional tension generated via Hooke's Law. Application of the method of variable separation resulted in trigonometric functions for the spatial domain, and periodic elliptic functions for the temporal domain. This analysis confirmed the presence of the jump phenomenon, as well as a frequency-amplitude relationship common to nonlinear systems. Experimental correlation was achieved by constraining the string to planar motion, confirming the validity of the analysis and of the hypothesis of temporally variable but spatially uniform tension. The analysis however did not include the case of nonplanar or whirling motion. Murthy and Ramakrishna[1965], and Miles[1965] examined the nonplanar motion of the nonlinear stretched string. Murthy and Ramakrishna[1965] derived their equations by formulating the potential and kinetic energies of the system and applying Hamilton's principle. Their analysis neglected the longitudinal motion and hence longitudinal inertia, resulting in equations defining the nonlinear motion of a string where all particles of the string were constrained to move in planes perpendicular to the equilibrium chord. Although the equations provided nonlinear terms coupling the in- and out-of-plane motion, which provided a plausible mechanism describing the transition from planar to nonplanar motion, the equations developed were inconsistent with regard to the longitudinal response of the string. Anand[1966] adopted the equations defined by Murthy and Ramakrishna[1965], including a viscous damping term, and extending their analysis to examine the nonlinear forced response of the string in the presence of damping. Anand's[1966] analysis examined both the in- and out-of-plane response of the string resulting from an in-plane distributed driving force, and it was demonstrated that the stability of the planar motion, and the occurrence of the jump phenomenon was a function of the magnitude of the driving force and the degree of damping. The analysis also demonstrated that under constant damping action and excitation, the degree of *non-linearity* was described by a dimensionless parameter $\eta = (nc_l/c_t)^2$, where c_l, c_t, n refer to the longitudinal wave speed, the lateral wave speed and the mode number respectively. Since the longitudinal wave speed is fixed by the material properties, whilst the lateral wave speed is dependent on the initial tension, η is directly proportional to the square of the mode number, and inversely proportional to the initial tension. Thus a decrease in the initial tension accentuates the nonlinear behaviour. This parameter is related to the ratio of the peak additional tension generated during an oscillation, τ_p, to the equilibrium tension in the string, which directly effects the nonlinear natural period, as demonstrated by Carrier [1945]. Although this parameter may be reduced by increasing the initial tension, the neglect of the possibility of modal interaction between the longitudinal and lateral modes limits the degree to which such an approach would apply without invalidating the premise of the analysis. Anand[1969a] rederived the equations of motion, whilst examining the free response of a damped nonlinear string, due to sinusoidal initial conditions, showing that in general coupling exists between the longitudinal and transverse modes. This coupling was not accounted for in the previous analyses of Oplinger [1960], Murthy and Ramakrishna [1965] and Miles[1965]. Anand [1969a] showed that by neglecting the inertia term in the longitudinal equation of motion, a static compatibility relationship between the longitudinal and lateral response could be determined. Inclusion of this relationship in the transverse equations of motion, resulted in transverse equations of motion identical to those employed by Oplinger [1960]. Thus although the equations of motion developed by Murthy and Ramakrishna[1965] qualitatively described experimentally observed behaviour, Anand[1969a] showed these to be incorrect, and thereby confirmed the consistency of the equations formulated by Oplinger[1960] which were based on the concept of spatially uniform but temporally variable tension. Although Anand [1969a] did not employ this concept in deriving his equations of motion, it was effectively introduced by neglecting the longitudinal inertia term. Specifically, Anand's [1969a] approximation requires that the longitudinal natural frequency of the string be much higher than the lateral natural frequency and the frequency of excitation. Thus modal interaction between the longitudinal and lateral modes could be neglected. Anand[1969a] showed that with regard to the decay of the free response of the nonlinear damped string with nonplanar initial conditions, the nonlinearity and coupling between the equations of motion induces an oscillatory energy transfer between the two planes of vibration, which results in the string describing an elliptic orbit, where the axes precess during the free decay. Anand[1969b] pursued this analysis further by investigating the stability of damped forced and undamped free vibrations of the nonlinear stretched string. This was accomplished by examining the stability of the variational form of the in-plane and out-of-plane equations of motion. In the variational form, these equations reduce to coupled Hill-type equations, and the stability map was constructed by considering the roots of the characteristic equation. This analysis confirmed experimental observations, in that the in-plane oscillation becomes unstable in particular regions of the parameter space, resulting in non-planar motion. Thus the analysis demonstrated the potential complexity of the response of the non-linear taut string. Anand[1969b] also proved that in the case of free undamped vibration, planar motion is unstable and circular motion results, whilst planar motion is stable in the presence of damping. Eller[1972] provided experimental validation to Anand's [1969b] theoretical predictions. Gough[1984] related the perturbation in the orbital angular frequency and the frequency of precession of the orbital motion during free vibration decay of a nonlinear damped string to the mean square radius and area of the orbital motion respectively. The planar and nonplanar motion of a taut string due to in-plane excitation is well summarised by Nafeh and Mook[1983]. Their discussion details the locus of the in-plane and out-of-plane response amplitudes due to a constant excitation level and variable frequency, and with respect to a variable excitation amplitude at a frequency close to the in-plane natural frequency. Legge and Fletcher[1984] compare the free response of a taut string between rigid supports with that observed when one support has a mechanical impedance associated with it. They show that due to the mechanical impedance, and the nonlinear coupling between the modes, response occurs in modes not normally excited by the initial disturbance. Watzky[1992] derives the equations of motion for the large amplitude response of a stiff elastic stretched string, where both bending stiffness and torsional coupling are included. Although this study appears to be motivated primarily for assessing the vibration of musical instruments, the concept of torsional coupling is certainly relevant to mine hoist ropes. Whereas in Watzky's derivation, torsional coupling is introduced as a consequence of material torsion, in the context of mine hoist ropes, torsional response is a consequence of rope construction, where coupling exists between longitudinal and torsional motion. Such coupling has been examined by Butson[1981], Greenway[1990b] in the context of linear longitudinal models only, and provides an interesting incentive for examining the nonlinear lateral motion of such a rope.

Tagata[1977, 1983] examined the lateral nonlinear response of a taut string subjected to a longitudinal parametric excitation. This study was preceded by Lubkin and Stoker[1943] who considered the conditions governing the linear stability of a taut string to a longitudinal excitation. Tagata's[1977, 1983] analysis examined the nonlinear large amplitude response due to an axial excitation, for the first, second and third parametric instability regions of the first lateral mode.

In summary, the previous studies considered the nonlinear response of a taut string of fixed length, where longitudinal inertia is not dominant, and the lateral motion of the string is consequently governed by an equation of motion having the general form:

$$w_{tt} + 2Rw_t - (c_t^2 - \frac{c_l^2}{2l}\int_0^l (v_x^2 + w_x^2)dx)w_{xx} = f \tag{2.3}$$

where v, w represent the motion of the string orthogonal to its axis, and R represents the damping action. This equation has become synonymous with the nonlinear dynamic response of a taut string. The integral term represents the change in arc length of the string, and accounts for the nonlinear stiffening/amplitude dependence of the dynamic characteristics.

2.4.2 Travelling Taut Strings

The analysis of travelling strings initially addressed processes in the textile and related industries. According to Sack[1954], Skutsch[1897] first investigated the linear transverse vibration of axially travelling strings. Sack [1954] presented a linear analysis of the lateral response of a travelling string subjected to a lateral excitation at one support. This analysis demonstrates that the natural frequency of a travelling string decreases with increasing transport velocity, and that the related mode shapes are complex, representative of travelling waves as opposed to the real normal modes associated with stationary strings. This behaviour is attributed to the gyroscopic or Coriolis term associated with a travelling medium. The equation of motion of a travelling string undergoing small amplitude lateral motion is commonly referred to as the *threadline* equation and is given as:

$$\frac{\partial^2 w}{\partial t^2} + 2V_o\frac{\partial^2 w}{\partial t \partial x} + (V_o^2 - c_t^2)\frac{\partial^2 w}{\partial x^2} = 0 \tag{2.4}$$

The ratio of the first linear natural frequency of a string travelling axially at velocity V_o to that of a stationary string is shown to be $1 - (V_o/c_t)^2$. In hoisting applications on South African mines, $(V_o/c_t) \approx 0.14 - 0.075$, thus the error induced by neglecting the axial velocity would be of the order of 2%.[3] Mote[1966b] presented a nonlinear analysis of a flat axially moving string. This analysis indicated that in the presence of axial velocity, the nonlinear stretch behaviour of the string can contribute significantly to changes in the fundamental period. Mote [1966b] varied the parameters of a string undergoing periodic motion with an amplitude equivalent to $\frac{1}{2}$% of it's span. The results generated by Mote[1966b] are illustrated in figure 2.2, where $\beta = V_a/c_t$, $w_{max} = v/l$ and v represents the midspan deflection, P represents the initial tension, and $\tau = 2/(1-\beta^2)$ represents the nondimensional period of the fundamental. The shaded region represents cable parameters employed on South African mines. Although this result may be valid in the case of a travelling string, axially restrained to maintain a constant unstretched length of string between the supports, it is not directly applicable to the mine hoist system as the sheave end does not fulfill such a restraint. However the result is of interest as it emphasises the differing mechanisms governing the system response.

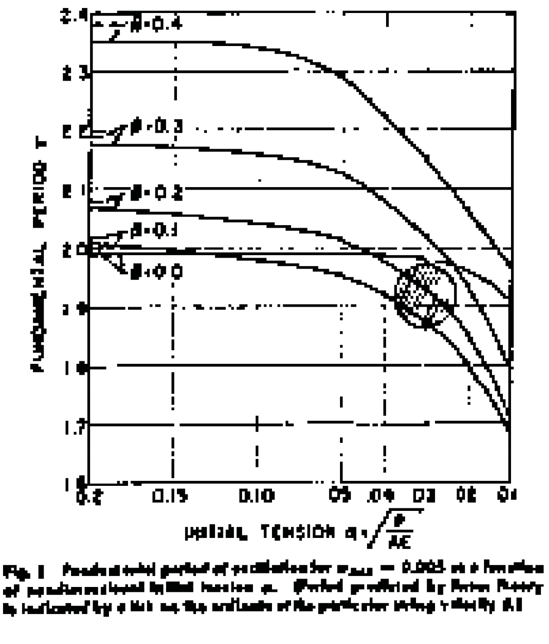

Figure 2.2: Mote (1966): Nonlinear fundamental period of an axially translating string

[3]This prompted Mankowski [1982] to discard axial velocity in his simulation of the mine hoist system.

Kim and Taborrok[1972] presented a derivation of the nonlinear equations of motion of a travelling string by considering the momentum and continuity equations of the string, as well as a mass tension constitutive law. The equations were solved by employing the method of characteristics, and applied to simulate the response of a taut flat travelling string, plucked at its mid span. Shih[1971, 1975] examined the three dimensional nature of the equations developed by Ames et al [1968], examining the phenomenon of elliptic ballooning, which is associated with a moving string under boundary excitation, where the major axis of the elliptic envelope is aligned with the direction of the excitation. It was proved that circular ballooning was unstable, a result observed experimentally by Ames et al[1968], where the motion became unstable once a circular envelope was approached, followed by a jump to second mode planar motion.

In a similar vein, Mote and Naguleswaran[1966a] investigated the linear vibrations of travelling bandsaw blades due to tension changes induced in the band by the axial velocity. Their analysis considered the flexural stiffness of the band and presented an approximate formula for determining the band natural frequency as a function of the axial velocity. The theoretical results were validated experimentally. Naguleswaran and Williams[1968] examined the parametric nature of a pulley belt or band saw blade, due to tension changes caused by eccentricities in the pulley wheels, or flaws in the blade. It was confirmed experimentally that primary parametric resonance occurred when the frequency of the tension fluctuations approached twice that of the lateral natural frequency of the travelling band. This application has been pursued further recently, Mote and Wu[1985], Wu and Mote[1986], Wang and Mote[1986, 1987].