Multi-Agent modelling using Intelligent Agents in Competative Games

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A dissertation submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, South Afirca, in fulfilment of the requirements of the degree of Master of Science in Engineering.

Declaration

I declare that this is my own, unaided work, except where otherwise acknowledged. It is being submitted for the degree of Master of Science in Engineering, at the University of the witwatersrand. It has not been submitted before for any other degree or examination at any other university.

Signed this ________________________ of ______________________ 2006

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Evan Hurwitz

Summary

Multi-Agent systems typically utilise simple, predictable agents. The usage of such agents in large systems allows for complexity to be achieved through the interaction of these agents. It is feasible, however, to utilise intelligent agents in smaller systems, allowing for more agent complexity and hence a higher degree of realism in the multi-agent model. By utilising the TD(λ) Algorithm to train feedforward neural networks, intelligent agents were successfully trained within the reinforcement learning paradigm. A methodology for stabilising this typically unstable neural network training was found through first looking at the relatively simple problem of Tic-Tac-Toe. Once a stable training methodology was arrived at, the more complex task of tackling a multi-player, multi-stage card-game was tackled. The results illustrated that a variety of scenarios can be realistically investigated through the multi-agent model, allowing for solving of situations and better understanding of the game itself. Yet more startling, owing to the agent's design, the agents learned on their own to bluff, giving much greater insight into the nature of bluffing in such games that lend themselves to the act.

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Foreward

This dissertation is presented to the University of the Witwarsrand, Johannesburg, South Africa, in fulfilment of the requirements of the degree of Master of Science in Engineering.

The dissertation covers the creation, utilisation and analysis of a multi-agent model utilising intelligent agents, used to model a competative card game. It comprises of three papers (all of which are written as papers for conferences or journals), namely “Optimising Reinforcement Learning for Neural Networks”; “Learning to Bluff” and “Multi-Agent Modelling Using Intelligent Agents in the Game of Lerpa”. The first paper was presented at *Game-On 2005* in Leicester in the United Kingdom, and published in the proceedings thereof. The second paper has been submitted to *Neural computation*, an MIT press journal on computational intelligence and modelling of the brain and learning, and the third submitted to *Autonomous Agents and Multi-Agent Systems,* a Springer journal dealing with multi-agent systems and multi-agent modelling. The first paper explores the stabilising of the inherently unstable TD(λ) algorithm for neural networks, in order to be used for creating intelligent agents. The second paper focuses of the creation of intelligent agents, capable of learning to predict their opponents behaviour and react accordingly. The third paper pulls the first two together, and looks into the multi-agent model arrived at once intelligent agents have been realised and integrated into a comprehensive multi-agent system.

This document complies with the univerity’s “paper-model” format. The Appendices supply some traditional game-theory, statistical analysis and describes the software used and written.

Appendix A illustrates a common example of Game-Theory, and highlights the shortcomings that this paper attempts to address.

Appendix B describes the evaluation techniques used to judge the competency of an agent playing tic-tac-toe.

Appendix C delves into the statistical analysis of the dimensionality issues discussed in the second paper, verifying the correct usage of minimising techinques when designing the card-playing agents.

Appendix D details the software tools used and created throughout this project, all of which code is available on the attached CD.

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Chapter 1: Introduction

The playing and analysis of games has traditionally been tackled through the discipline of *game-theory*, most notably advanced by John Nash and applied directly thereafter into the field of Economics. While useful up to a point, the basic assumptions within game-theory have become inviolate rules that are never questioned, and yet create serious flaws in the analysis of complex, multiplayer games. One very common such problemis known as the "trembling hand", an issue that challenges the very first precept of game-theory, namely that of the "rational player".

Multi-agent modelling, on the other hand, is being used more and more frequently to tackle problems within the field of economics in particular to understand and tackle problems whose complexity has left them beyond the scope of more traditional methods. The task of modelling a complex multiplayer game falls well within the scope of the capabilities of multi-agent modelling, provided an agent can be created with suitable learning and adaptative capabilities to represent realistic players.

Neural network technology has been widely used throughout many differring disciplines, in order to perform tasks that involve prediction and learning. The power of neural networks is their ability to learn unconstrained by the approximating function typically used in most regression-type methods. The 'universal approximator' property of neural networks allows them to learn any possible correlation, not limiting them to correlations inferred by the engineer beforehand. This makes neural networks ideal to develop game-playing strategy, since the strategy will not be limited to the strategic capabilities of the engineer.

In 1995, Gerry Tesauro utilised the TD(λ) algorithm, within the reinforcement learning paradigm, to train a neural network-based backgammon-playing agent capable of playing at international competition level. However, subsequent attemps to achieve similar results by others have encountered a large degree of failure, owing to the unstable nature of the training technique and a lack of a structured methodology in order to retain stability. Despite these misgivings, Tesauro's method still shows great promise, as evidenced by the astounding success of TD-Gammon.

This dissertation proposes a method of using intelligent agents to create a multi-agent model of a complex card game, which would allow the modelling of a game that is inadequately handled via traditional game-theory. Chapter 2 introduces the TD(λ) algorithm and the neural network background that needs to be trained using it. It then details the training and evaluation of game-playing agents in a simple game in order to arrive at a safe methodology for training neural networks with the TD(λ) algorithm. Chapter 3 illustrates the creation of an intelligent agent, applying the principles arrived at in Chapter 2. It also details the crucial issues to be considered when designing said agents, such as agent view and dimensionality. Also examined is the performance and learning capabilities of said agents from a purely computational intellignece perspective.

Chapter 4 ties the threads together, showing how intelligent agents can be brought together in a multi-agent system framework, and can then be used to model, and indeed solve, many differing scenarios within the game. This final chapter encompasses main thrust of the research.

The following papers from this dissertation have been published/submitted for publication:

1. Hurwitz E, Marwala T. *Optimising Reinforcement Learning for Neural Networks*. In Proceedings of the 6th Annual European on Intelligent Games and Simulation, Leicester, UK, 2005, pp. 13-18. ISBN: 90-77381-23-6.
2. Hurwitz E. Marwala T. *Learning to Bluff*. Submitted for publication to the journal of Neural Computation, MIT Press, U.S.A.
3. Hurwitz E. Marwala T. *Multi-Agent Modelling Using Intelligent Agents in the Game of Lerpa*. Submitted for publication to the Journal of Autonomous Agents and Multi-Agent Systems.

Chapter 2: Optimising Reinforcement learning for neural networks

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Abstract – *Reinforcement learning traditionally utilises binary encoders and/or linear function approximators to accomplish its Artificial Intelligence goals. The use of nonlinear function approximators such as neural networks is often shunned, due to excessive difficulties in implementation, usually resulting from stability issues. In this paper the implementation of reinforcement learning for training a neural network is examined, being applied to the problem of learning to play Tic Tac Toe. Methods of ensuring stability are examined, and differing training methodologies are compared in order to optimise the reinforcement learning of the system. TD(λ) methods are compared with database methods, as well as a hybridised system that combines the two, which outperforms all of the homogenous systems.*

Keywords: reinforcement, learning, temporal, difference, neural, network, tic-tac-toe.

1 Introduction

Artificial intelligence (A.I.) can only truly be considered worthy of the name when the system in question is capable of learning on its own [1], without having an expert *teacher* available to point out correct behaviour. This leads directly into the paradigm of *reinforcement learning* [1]. The advantage of using such techniques for gaming A.I. is that it would allow a gaming agent to actually learn while in-game, and adapt its own play to that of the player. Most *reinforcement learning* techniques explored utilise binary system representations, or linear function approximators, which severely hinder the scope of learning available to the artificial intelligence system. One notable exception is the work by G. Tessauro on *TD-Gammon*, in which he successfully applied the TD(λ) reinforcement learning algorithm to train a neural network, with staggeringly successful results. Following attempts to emulate his work have, however, been met with failure due to the extreme difficulties of combining backpropagation with TD(λ). These difficulties, and some solutions to them, are explored in this paper, with the A.I. system being applied to learn to play the game of tic-tac-toe by playing against itself. The reason for Tic-Tac-Toe as a choice of games is deliberately because of its simplicity, as the commonly occurring problems become easier to identify within the simpler system, and solutions are then also easier to develop.

2 Background

It is necessary to understand the workings and advantages of neural networks to appreciate the task of applying them in the reinforcement learning paradigm. It is likewise important to fully grasp the implications of reinforcement learning, and the break they represent from the more traditional supervised learning paradigm.

2.1 Neural network architecture

The fundamental building-blocks of neural networks are *neurons* [2]. These neurons are simply a multiple-input, single-output mathematical function [2]. Each neuron has a number of *weights* connecting inputs from another layer to itself, which are then added together, possibly with a *bias*, the result of which is then passed into the neuron's *activation function* [2]. The activation function is a function that represents the way in which the neural network "thinks". Different activation functions lend themselves to different problem types, ranging from yes-or-no decisions to linear and nonlinear mathematical relationships. Each *layer* of a neural network is comprised of a finite number of neurons. A network may consist of any number of layers, and each layer may contain any number of neurons [2]. When a neural network is run, each neuron in each consecutive layer sums its inputs and multiplies each input by its respective weight, and then treats the weighted sum as an input to its activation function. The output will then be passed on as an input to the next layer, and so on until the final output layer is reached. Hence the input data is passed through a network of neurons in order to arrive at an output. Figure 2-1 illustrates an interconnected network, with 2 input neurons, three hidden layer neurons, and two output neurons. The hidden layer and output layer neurons can all have any of the possible activation functions. This type of neural network is referred to as a *multi-layer perceptron* [2], and while not the only configuration of neural network, it is the most widely used configuration for regression-type problems [3].

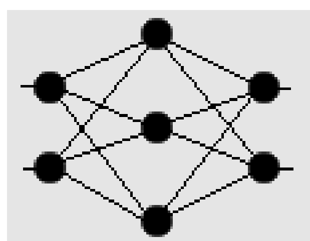

Figure 2-1. Sample Connectionist network

2.2 Neural network properties

Neural networks have various properties that can be utilised and exploited to aid in the solving of numerous problems. Some of the properties that are relevant to this particular problem are detailed below.

2.2.1 Universal approximators

Multi-layer feedforward neural networks have been proven to be universal approximators [2]. This phenomenon refers to the fact that a feedforward neural network with nonlinear activation functions of appropriate size can approximate any function of interest to an arbitrary degree of accuracy [2]. This is contingent upon sufficient training data and training being supplied.

2.2.2 Neural networks can generalise

By approximating a nonlinear function from its inputs, the neural network can learn to approximate a function [2]. In doing so, it can also infer the correct output from inputs that it has never seen, by inferring the answer from similar inputs that it has seen. This property is known as *generalisation* [2]. As long as the inputs received are within the ranges of the training inputs, this property will hold [2].

2.2.3 Neural networks recognise patterns

Neural networks are often required to match large input/output sets to each other, and these sets are often noisy or even incomplete [2]. In order to achieve this matching the network learns to recognise patterns in the data sets rather than fixate on the answers themselves [2]. This enables a network to 'see' through the data points and respond to the underlying pattern instead. This is an extended benefit of the generalisation property.

2.3 Reinforcement learning

Reinforcement learning involves the training of an artificial intelligence system by trial-and-error, reflecting the same manner of learning that living beings exhibit [4]. Reinforcement learning is very well suited to episodic tasks [4], and as such is highly appropriate in the field of game-playing, where many episodes are encountered before a final result is reached, and an A.I. system is required to evaluate the value of each possible move long before a final result is achieved. This methodology allows for online learning, and also eliminates the need for expert knowledge [4].

2.3.1 Rewards and Returns

Any artificial intelligence system requires some sort of goal or target to strive towards [2], [4]. In the case of reinforcement learning, there are two such quantities that need to be defined, namely *rewards* and *returns* [4]. A reward is defined to be the numerical value attributed to an individual state, while a return is the final cumulative rewards that are returned at the end of the sequence [4]. The return need not necessarily be simply summed, although this is the most common method [4]. An example of this process can be seen below in Figure 2-2, where an arbitrary Markov process is illustrated with rewards given at each step, and a final return at the end. This specific example is that of a *Random walk* problem.

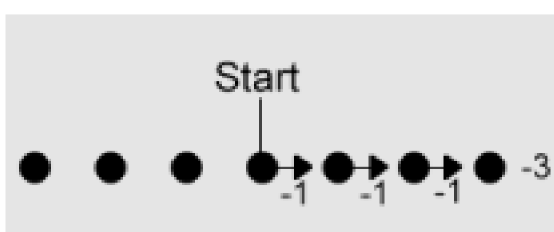

Figure 2-2. Random walk rewards and returns.

An A.I. system utilising reinforcement learning must learn to predict its expected return at each stage, hence enabling it to make a decision that has a lower initial reward than other options, but maximising its future return. This can be likened to making a sacrifice in chess, where the initial loss of material is accepted for the future gains it brings.

2.3.2 Exploitation vs Exploration

An A.I. system learning by reinforcement learning learns only through its own experiences [4]. In order to maximise its rewards, and hence its final return, the system needs to experiment with decisions not yet tried, even though it may perceive them to be inferior to tried-and-tested decisions [4]. This attempting of new approaches is termed *exploration*, while the utilising of gained knowledge to maximise returns is termed *exploitation* [4]. A constant dilemma that must be traded off in reinforcement learning is that of the choice between exploration and exploitation. One simple approach is the *ε-greedy* approach, where the system is *greedy*, i.e. attempts to exploit, in every situation with probability ε, and hence will explore with probability 1- ε [4].

2.3.3 TD (λ)

One common method of training a reinforcement learning system is to use the TD(λ) (Temporal Difference) algorithm to update one's value estimates [4] [5]. This algorithm is specifically designed for use with episodic tasks, being an adaptation of the common Widdrow-Hoff learning rule [5]. In this algorithm, the parameters or *weights* w to be altered are updated by equation (1) [5].

$$\Delta w = \alpha\left(P_{t+1} - P_t\right)\sum_{k=1}^{t} \lambda^{t-k} \nabla w P_k \tag{1}$$

The prediction P_{t+1} is used as a target for finding the error in prediction P_t, allowing the update rule to be computed incrementally, rather than waiting for the end of the sequence before any learning can take place [5]. α and λ are the learning rate and weight-decay parameters, respectively.

2.4 Neural network advantages

Neural networks have a number of advantages that can be exploited in order to optimise a reinforcement learning A.I. system. Some of these advantages are:

- Faster learning. Due to the generalisation property of neural networks, learning from one position can be generalised to learn for all similar positions.

- Positional understanding. As a result of the pattern recognition property of neural networks, the system can learn to judge positions, rather than simply remember individual state configurations.

For these reasons it is desirable to utilise a neural network as a function approximator for a reinforcement learning system.

3 Implementation

In this section we detail the implementation of the problem domain and the various methods of optimising the neural network reinforcement learning, as well as the performance measures used to evaluate the usefulness of each presented method.

3.1 Tic Tac Toe

The game of *Tic Tac Toe*, or *noughts and crosses*, is played on a 3x3 grid, with players taking alternate turns to fill an empty spot [6]. The first player places a 'O', and the second player places an 'X' whenever it is that respective player's turn [6]. If a player manages to get three of his mark in a row, he wins the game [6]. If all 9 squares are filled without a winner, the game is a draw [6]. The game was simulated in Matlab, with a simple matrix representation of the board.

3.2 Player evaluation

The A.I. system, or *player*, needs to be evaluated in order to compare different players, who have each learned using a different method of learning. In order to evaluate a player's performance, ten different positions are set up, each with well-defined correct moves. Using this test-bed, each player can be scored out of ten, giving a measure of the level of play each player has achieved. Also important is the speed of convergence – i.e. how fast does each respective player reach its own maximum level of play.

3.3 TD(λ) for backpropogation

In order to train a neural network, equation (1) needs to be adapted for use with the backpropogation algorithm [7]. The adaptation, without derivation, is as follows [7]:

$$w_{ij}^{t+1} = w_{ij}^{t} + \alpha \sum\nolimits_{K \in O} \left(P_K^{t+1} - P_K^{t} \right) e_{ijk}^{t} \qquad (2)$$

where the eligabilities are:

$$e_{ijk}^{t+1} = \lambda e_{ijk}^{t} + \delta_{kj}^{t+1} y_i^{t+1} \qquad (3)$$

and δ is calculated by recursive backpropogation

$$\delta_{ki}^{t} = \frac{\partial P_k^t}{\partial S_i^t} \qquad (4)$$

as such, the TD(λ) algorithm can be implemented to update the weights of a neural network [7].

3.4 Stability issues

The TD(λ) algorithm has proven stability for linear functions [6]. A multi-layer neural network, however, is non-linear [2], and the TD(λ) algorithm can become unstable in some instances [4] [8]. The instability can arise in both the actual weights and in the predictions [4] [8]. In order to prevent instability, a number of steps can be taken, the end result of which is in most cases to limit the degree of variation in the outputs, so as to keep the error signal small to avoid instability.

3.4.1 Input/Output representation

The inputs to, and outputs from, a typical A.I. system are usually represented as real or integer values. This is not optimal for TD(λ) learning, as the values have too much variation. Far safer is to keep the representations in binary form, accepting the dimensionality trade-off as a fair price to ensure a far higher degree of stability. Specifically in the case of the outputs, this ensures that the output error of the system can never be more than 1 for any single output, thus keeping the mean error to within marginally stable bounds. For the problem of the Tic Tac Toe game, the input to the network is an 18-bit binary string, with the first 9 bits representing a possible placed 'o' in each square, and the second 9 bits representing a '1' in each square. The output of the network is a 3-bit string, representing a 'o' win, a draw and an 'x' win respectively.

3.4.2 Activation Functions

As shown in Section 2, there are many possible activation functions that can be used for the neural network. While it is tempting to utilise activation functions that have a large scope in order to maximise the versatility of the network, it proves far safer to use an activation function that is limited to an upper bound of 1, and a lower bound of zero. A commonly used activation function of this sort is the sigmoidal activation function, having the form of:

$$f(x) = \frac{1}{1+e^{-x}} \tag{5}$$

This function is nonlinear, allowing for the freedom of approximation required of a neural network, and limiting the upper and lower bounds as recommended above. While this activation function is commonly used as a middle-layer activation function, it is unusual as an output layer activation function. In this manner, instability is further discouraged.

3.4.3 Learning rate

As the size of the error has a direct effect on the stability of the learning system, parameters that directly affect the error signal also have an effect on the said stability. For this reason, the size of the learning rate α needs to be kept low, with experimental results showing that values between 0.1 and 0.3 prove safe, while higher values tend to become unstable, and lower values simply impart too little real learning to be of any value, as a result of applying too small a change to the modified weights.

3.4.4 Hybrid stability measures

In order to compare relative stability, the percentage chance of becoming unstable has been empirically noted, based upon experimental results. Regardless of each individual technique presented, it is the combination of these techniques that allows for better stability guarantees. While no individual method presented gives greater than a 60% stability guarantee (that is, 60% chance to be stable given a 100-game training run), the combination of all of the above measures results in a much better 98% probability of being stable, with minor tweaking of the learning rate parameter solving the event of instability occurring at unusual instances.

4 The players

All of the players are trained using an ε-greedy policy, with the value of $\varepsilon = 0.1$. i.e. for each possible position the player has a 10% chance of selecting a random move, while having a 90% chance of selecting whichever move it deems to be the best move. This selection is done by determining all of the legal moves available, and then finding the positions that would result from each possible move. These positions are sequentially presented to the player, who then rates each resultant position, in order to find the best resultant position. It obviously follows that whichever move leads to the most favoured position is the apparently best move, and the choice of the player for a greedy policy. The training of each player is accomplished via *self-play*, wherein the player evaluates and chooses moves for both sides, learning from its own experiences as it discovers errors on its own. This learning is continued until no discernable improvement occurs.

As a benchmark, randomly initialised networks were able to correctly solve between 1 and 2 of the posed problems, beyond which one can say genuine learning has indeed taken place, and is not simply random chance.

4.1 Player #1 – Simple TD(λ)

Player #1 learned to play the game using a simple TD(λ) backpropogation learning algorithm. This proved to be very fast, allowing for many thousands of games to be played out in a very short period of time. The level of play achieved using this method was however not particularly inspiring, achieving play capable of solving no more than 5 of the 10 problems posed in the rating system. The problem that is encountered by player #1 is that the learning done after each final input, the input with the game result, gets undone by the learning of the intermediate steps of the next game. While in concept the learning should be swifter due to utilising the knowledge gained, the system ends up working at cross-purposes against itself, since it struggles to build its initial knowledge base, due to the generalisation of the neural network which is not present in more traditional reinforcement learning arrangements.

4.2 Player #2 – Historical database learning

In this instance, the player learns by recording each position and its corresponding target, and storing the pair in a database. Duplicate input data sets and their corresponding targets are removed, based on the principle that more recent data is more accurate, since more learning has been done when making the more recent predictions. This database is then used to train the network in the traditional supervised learning manner. A problem encountered early on with this method is that early predictions have zero knowledge base, and are therefore usually incorrect. Retaining this information in the database therefore taints the training data, and is thus undesirable. A solution to this problem is to limit the size of the database, replacing old data with new data once the size limit is reached, thus keeping the database recent. This methodology trains slower than that employed to train player #1, making long training runs less feasible than for TD(λ) learning. The play level of this method is the lowest of those examined, able to solve only four of the ten proposed problems. Nonetheless, the approach does show promise for generating an initial knowledge base from which to work with more advanced methods.

4.3 Player #3 – Fact/Opinion DB learning

Building on the promise of Player #2, a more sophisticated database approach can be taken. If one takes into account the manner of the training set generation, one notes that most of the targets in the database are no more than *opinions* – targets generated by estimates of the next step, as seen in equation (1) – while relatively few data points are in fact *facts* – targets generated by viewing the end result of the game. In order to avoid this problem, the database can be split into two sub-databases, with one holding facts, and the other holding opinions. Varying the sizes, and the relative sizes, of these two sub-databases can then allow the engineer to decide how much credence the system should give to fact versus opinion. This method proved far more successful than Player #2, successfully completing 6 of the 10 problems posed by the rating system. Its speed of convergence is comparable to that of Player #2.

4.4 Player #4 – Widdrow-Hoff based DB learning

In this instance, a very similar approach to that of Player #2 is taken, with one important distinction: Instead of estimating a target at each move, the game is played out to completion with a static player. After each game finishes, the player then adds all of the positions encountered into its database, with the final result being the target of each position. This means that no *opinions* can ever enter into the training, which trades off speed of convergence for supposedly higher accuracy. This method is not optimal, as it loses one of the primary advantages of reinforcement learning, namely that of being able to incorporate current learning into its own learning, hence speeding up the learning process. Unsurprisingly, this method trains with the same speed as the other database methods, but takes far longer to converge. It achieves a similar level of play as does Player #1, being able to solve 5 of the posed problems.

4.5 Player #5 - Hybrid Fact/Opinion DB TD(λ) learning

The logical extension to the previous players is to hybridise the most successful players in order to compensate for the failings of each. Player #5 thus utilises the Fact/Opinion database learning in order to build an initial knowledge base from which to learn, and then proceeds to learn from thence using the TD(λ) approach of Player #1. This proves more successful, since the intrinsic flaw in player #1's methodology lies in its inability to efficiently create a knowledge base, and the database method of player #3 creates that knowledge base from which to learn. Player #5 begins its learning with the expected sluggishness of database methods, but then learns much faster once it begins to learn using the TD(λ) approach. Player #5 managed to successfully solve seven of the ten problems once trained to convergence. The problem of unlearning learned information is still apparent in Player #5, but is largely mitigated by the generation of the initial knowledge base.

4.6 Player comparisons

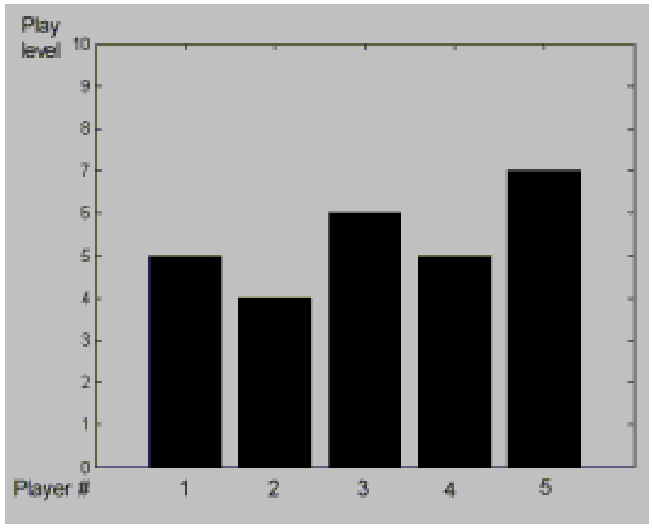

Figure 2-3. Relative player strengths

As is illustrated in Figure 2-3, the hybrid method learns to play at the strongest level of all of the methods presented. Due to the drastic differences in speed and computational power requirements, it is preferable to stay away from database-based methods, and it is thus worth noting that only the fact/opinion database method arrives at a stronger level of play than the simple TD(λ)-trained player #1, and that this methodology can easily be incorporated into a TD(λ) learning system, which produces the far more promising player #5. The fact that after a short knowledge-base generation sequence the hybrid system uses the highly efficient TD(λ) approach makes it a faster and more reliable learning system than the other methods presented. As can be seen in Figure 3, however, there is still a greater level of play strength that should be achievable in this simple game, and that has been limited by the unlearning error seen in Players #1 and #5.

For future work, it is recommended that research goes into a more efficient method of calculating eligability traces, since a better calculation of the eligibility traces would solve the problem of unlearning encountered in the TD(λ) methods, removing the play-level limit that has been encountered.

5 Conclusion

While the stability of TD(λ) methods for neural networks is in question, many precautions have been presented that greatly improve the stability of the neural network learning process. Far more important is the generalisation property of neural networks. The generalisation that is the great boon of neural networks also is its greatest weakness when applied to reinforcement learning. The ability to generalise comes at the cost of updating all position prediction with each individual prediction, which limits the development of a knowledge base. Database building techniques are useful, but also lose some of the benefits of reinforcement learning, and are hence undesirable. Lastly, the usage of database techniques to develop an initial database, followed by further learning utilising the TD(λ) approach leads to the best player in terms of player strength, illustrating the need for a teacher to 'show the ropes' to an A.I. system, before it is capable of learning on its own.

For future work, the issue on *unlearning* needs to be further explored and solved before neural network reinforcement learning is in fact a viable tool for use in gaming A.I., although once this is accomplished, the scope of the method holds great promise to allow truly adaptable A.I. agents in games, able to give players a true challenge.

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Chapter 3: Learning to bluff

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Abstract

The act of bluffing confounds game designers to this day. The very nature of bluffing is even open for debate, adding further complication to the process of creating intelligent virtual players that can bluff, and hence play, realistically. Through the use of intelligent, learning agents, and carefully designed agent outlooks, an agent can in fact learn to predict its opponents' reactions based not only on its own cards, but on the actions of those around it. With this wider scope of understanding, an agent can learn to bluff its opponents, with the action representing not an "illogical" action, as bluffing is often viewed, but rather as an act of maximising returns through an effective statistical optimisation. By using a TD(λ) learning algorithm to continuously adapt neural network agent intelligence, agents have been shown to be able to learn to bluff without outside prompting, and even to learn to call each other's bluffs in free, competative play.

1. Introduction

While many card games involve an element of bluffing, simulating and fully understanding bluffing yet remains one of the most elusive tasks presented to the game design engineer. The entire process of bluffing relies on performing a task that is unexpected, and is thus misinterpreted by one's opponents. For this reason, static rules are doomed to failure since once they become predictable, they cannot be misinterpreted. In order to create an artificially intelligent agent that can bluff, one must first create an agent that is capable of learning. The agent must be able to learn not only about the inherent nature of the game it is playing, but also must be capable of learning trends emerging from its opponent's behaviour, since bluffing is only plausible when one can anticipate the opponent's reactions to one's own actions.

Firstly the game to be modelled will be detailed, with the reasoning for its choice being explained. The paper will then detail the system and agent architecture, which is of paramount importance since this not only ensures that the correct information is available to the agent, but also has a direct impact on the efficiency of the learning algorithms utilised. Once the system is fully illustrated, the actual learning of the agents is shown, with the appropriate findings detailed.

2. Lerpa

The card game being modelled is the game of Lerpa. While not a well-known game, its rules suit the purposes of this research exceptionally well, making it an ideal testbed application for intelligent agent Multi-Agent Modelling (MAM). The rules of the game first need to be elaborated upon, in order to grasp the implications of the results obtained. Thus, the rules for Lerpa now follow.

The game of *Lerpa* is played with a standard deck of cards, with the exception that all of the 8s, 9s and 10s are removed from the deck. The cards are valued from greatest- to least-valued from ace down to 2, with the exception that the 7 is valued higher than a king, but lower than an ace, making it the second most valuable card in a suit. At the end of dealing the hand, during which each player is dealt three cards, the dealer has the choice of *dealing himself in* – which entails flipping his last card over, unseen up until this point, which then declares which suit is the *trump suit*. Should he elect not to do this, he then flips the next card in the deck to determine the trump suit. Regardless, once trumps are determined, the players then take it in turns, going clockwise from the dealer's left, to elect whether or not to play the hand (to *knock*), or to drop out of the hand, referred to as *folding* (if the Dealer has *dealt himself in*, as described above, he is then automatically required to play the hand). Once all players have chosen, the players that have elected to play then play the hand, with the player to the dealer's left playing the first card. Once this card has been played, players must then play *in suit* – in other words, if a heart is played, they must play a heart if they have one. If they have none of the required suit, they may play a trump, which will win the trick unless another player plays a higher trump. The highest card played will win the trick (with all trumps valued higher than any other card) and the winner of the trick will lead the first card in the next trick. At any point in a hand, if a player has the Ace of trumps and can legally play it, he is then required to do so. The true risk in the game comes from the betting, which occurs as follows:

At the beginning of the round, the dealer pays the table 3 of whatever the basic betting denomination is (referred to usually as 'chips'). At the end of the hand, the chips are divided up proportionately between the winners, i.e. if you win two tricks, you will receive two thirds of whatever is in the pot. However, if you stayed in, but did not win any tricks, you are said to have been *Lerpa'd*, and are then required to match whatever was in the pot for the next hand, effectively costing you the pot. It is in the evaluation of this risk that most of the true skill in *Lerpa* lies.

3. Lerpa MAM

As with any optimisation system, very careful consideration needs to be taken with regards to how the system is structured, since the implications of these decisions can often result in unintentional assumptions made by the system created. With this in mind, the Lerpa Multi-Agent System (MAS) has been designed to allow the maximum amount of freedom to the system, and the agents within, while also allowing for generalisation and swift convergence in order to allow the intelligent agents to interact unimpeded by human assumptions, intended or otherwise.

3.1 System overview

The game is, for this model, going to be played by four players. Each of these players will interact with each other indirectly, by interacting directly with the *table*, which is their shared environment, as depicted in Figure 3-1.

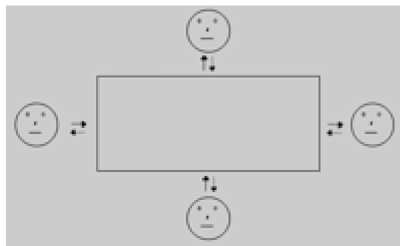

Figure 3-1. System interactions.

Over the course of a single hand, an agent will be required to make three decisions, once at each interactive stage of the game. These three decision-making stages are:

1. Whether to play the hand, or drop (*knock* or *fold*)
2. Which card to play first
3. Which card to play second

Since there is no decision to be made at the final card, the hand can be said to be effectively finished from the agent's perspective after it has played its second card (or indeed after the first decision should the agent fold). Following on the TD(λ) algorithm, each agent will update its own neural network at each stage, using its own predictions as a reward function, only receiving a true reward after its final decision has been made. This decision making process is illustrated below, in Figure 3-2.

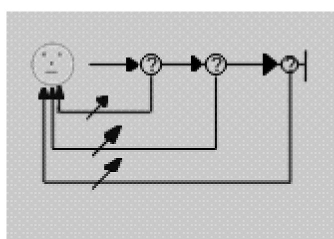

Figure 3-2. Agent learning scheme

With each agent implemented as described, the agents can now interact with each other through their shared environment, and will continuously learn upon each interaction and its consequent result.

Each hand played will be viewed as an independent, stochastic event, and as such only information about the current hand will be available to the agent, who will have to draw on its own learned knowledge base in order to draw deductions, rather than from previous hands.

3.2 Agent Aritificial Intelligence design

A number of decisions need to be made in order to implement the agent artificial intelligence (AI) effectively and efficiently. The type of learning to be implemented needs to be chosen, as well as the neural network architecture. Special attention needs to be paid to the design of the inputs to the neural network, as these determine what the agent can 'see' at any given point. This will also determine what assumptions, if any, are implicitly made by the agent, and hence cannot be taken lightly. Lastly, this will determine the dimensionality of the network, which directly affects the learning rate of the network, and hence must obviously be minimised.

3.2.1 Input Parameter Design

In order to design the input stage of the agent's neural network, one must first determine all that the network may need to know at any given decision-making stage. All inputs, in order to optimise stability, are structured as binary-encoded inputs. When making its first decision, the agent needs to know its own cards, which agents have stayed in or folded, and which agents are still to decide. It is necessary for the agent to be able to match specific agents to their specific actions, as this will allow for an agent to learn a particular opponent's characteristics, something impossible to do if it can only see a number of players in or out. Similarly, the agent's own cards must be specified fully, allowing the agent to draw its own conclusions about each card's relative value. It is also necessary to tell the agent which suit has been designated the trumps suit, but a more elegant method has been found to handle that information, as will be seen shortly. Figure 3-3 below illustrates the initial information required by the network.

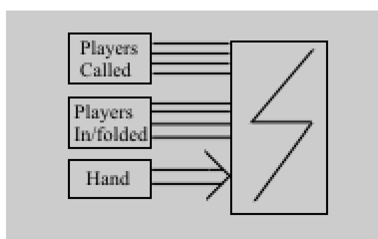

Figure 3-3. Basic input structure.

The agent's hand needs to be explicitly described, and the obvious solution is to encode the cards exactly, i.e. four suits, and ten numbers in each suit, giving forty possibilities for each card. A quick glimpse at the number of options available shows that a raw encoding style provides a sizeable problem of dimensionality, since an encoded hand can be one of 40^3 possible hands (in actuality, only ${}^{40}P_3$ hands could be selected, since cards cannot be repeated, but the raw encoding scheme would in fact allow for repeated cards, and hence 40^3 options would be available). The first thing to notice is that only a single deck of cards is being used, hence no card can ever be repeated in a hand. Acting on this principle, consistent ordering of the hand means that the base dimensionality of the hand is greatly reduced, since it is now combinations of cards that are represented, instead of permutations. The number of combinations now represented is ${}^{40}C_3$. This seemingly small change from ${}^{n}P_r$ to ${}^{n}C_r$ reduces the dimensionality of the representation by a factor of r!, which in this case is a factor of 6. Furthermore, the representation of cards as belonging to discrete suits is not optimal either, since the game places no particular value on any suit by its own virtue, but rather by virtue of which suit is the trump suit. For this reason, an alternate encoding scheme has been determined, rating the 'suits' based upon the makeup of the agent's hand, rather than four arbitrary suits. The suits are encoded as belonging to one of the following groups, or new "suits":

Trump suit

Suit agent has multiple cards in (not trumps)

Suit in agent's highest singleton

Suit in agent's second-highest singleton

Suit in agent's third-highest singleton

This allows for a much more efficient description of the agent's hand, greatly improving the dimensionality of the inputs, and hence the learning rate of the agents. These five options are encoded in a binary format, for stability purposes, and hence three binary inputs are required to represent the suits. To represent the card's number, ten discrete values must be represented, hence requiring four binary inputs to represent the card's value. Thus a card in an agent's hand is represented by seven binary inputs, as depicted in Figure 3-4.

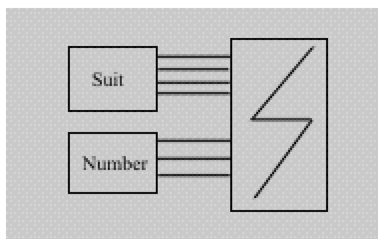

Figure 3-4. Agent card input structure

Next must be considered the information required in order to make decisions two and three. For both of these decisions, the cards that have already been played, if any, are necessary to know in order to make an intelligent decision as to the correct next card to play. For the second decision, it is also plausible that knowledge of who has won a trick would be important. The most cards that can ever be played before a decision must be made is seven, and since the table after a card is played is used to evaluate and update the network, it is necessary to represent eight played cards. Once again, however, simply utilising the obvious encoding method is not necessarily the most efficient method. The actual values of the cards played are not necessarily important, only their values relative to the cards in the agent's hand. As such, the values can be represented as one of the following, with respect to the cards of the same suit in the agent's hand:

- Higher than the card/cards in the agent's hand
- Higher than the agent's second-highest card
- Higher than the agent's third-highest card
- Lower than any of the agent's cards
- Member of a void suit (value is immaterial)

Another suit is now relevant for representation of the played cards, namely a void suit – a suit in which the agent has no cards. Lastly, a number is necessary to handle the special case of the Ace of trumps, since its unique rules mean that strategies are possible to develop based on whether it has or has not been played. The now six suits available still only require three binary inputs to represent, and the six number groupings now reduce the value representations from four binary inputs to three binary inputs, once again reducing the dimensionality of the input system.

With all of these inputs specified, the agent now has available all of the information required to draw its own conclusions and create its own strategies, without human-imposed assumptions affecting its "thought" patterns.

3.2.2 Network Architecture Design

With the inputs now specified, the hidden and output layers need to be designed. For the output neurons, these need to represent the prediction P that the network is making. A single hand has one of five possible outcomes, all of which need to be catered for. These possible outcomes are:

The agent wins all three tricks, winning 3 chips.

The agent wins two tricks, winning 2 chips.

The agent wins one trick, winning 1 chip.

The agent wins zero tricks, losing 3 chips.

The agent elects to fold, winning no tricks, but losing no chips.

This can be seen as a set of options, namely [3 2 1 0 -3]. While it may seem tempting to output the result as one continuous output, there are two compelling reasons for breaking these up into binary outputs. The first of these is in order to optimise stability, as described by Hurwitz-Marwala. The second reason is that these are discrete events, and a continuous representation would cover the range of [-3 0], which does not in fact exist. The binary inputs then specified are:

P(O = 3)

P(O = 2)

P(O = 1)

P(O = -3)

With a low probability of all four catering to folding, winning and losing no chips. Consequently, the agent's predicted return is:

$$P = 3A + 2B + C - 3D \tag{1}$$

where

$$A = P(O = 3) \tag{2}$$

$$B = P(O = 2) \tag{3}$$

$$C = P(O = 1) \tag{4}$$

$$D = P(O = -3) \tag{5}$$

The internal structure of the neural network uses a standard sigmoidal activation function, which is suitable for stability issues and still allows for the freedom expected from a neural network. The sigmoidal activations function varies between zero and one, rather than the often-used one and minus one, in order to optimise for stability. Since a high degree of freedom is required, a high number of hidden neurons is required, and thus fifty have been used. This number is iteratively achieved, trading off training speed versus performance. The output neurons are linear functions, since they represent not binary effects, but rather a continuous probability of particular binary outcomes.

3.2.3 Agent decision making

With its own predictor specified, the agent is now equipped to make decisions when playing. These decisions are made by predicting the return of the resultant situation arising from each legal choice it can make. An ε-greedy policy is then used to determine whether the agent will choose the most promising option, or whether it will explore the result of the less appealing option. In this way, the agent will be able to trade off exploration versus exploitation.

4. The intelligent model

With each agent implemented as described above, and interacting with each other as specified in Section 3, we can now perform the desired task, namely that of utilising a multi-agent model to analyse the given game, and develop strategies that may "solve" the game given differing circumstances. Only once agents know how to play a certain hand can they then begin to outplay, and potentially bluff each other.

4.1 Agent learning verification

In order for the model to have any validity, one must establish that the agents do indeed learn as they were designed to do. In order to verify the learning of the agents, a single intelligent agent was created, and placed at a table with three 'stupid' agents. These 'stupid' agents always stay in the game, and choose a random choice whenever called upon to make a decision. The results show quite conclusively that the intelligent agent soon learns to consistently outperform its opponents, as shown in Figure 3-5.

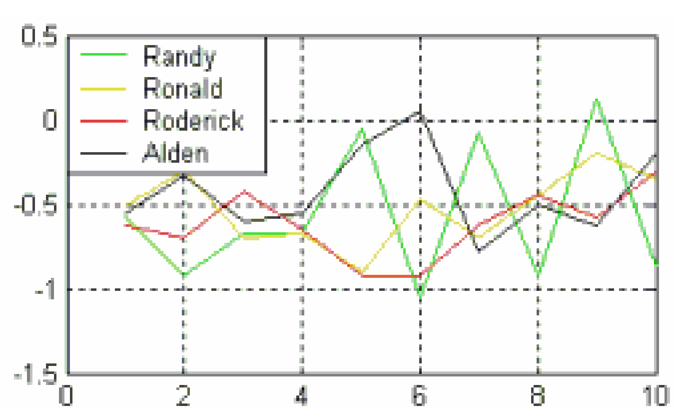

Figure 3-5. Agent performance, averaged over 40 hands

The agents named Randy, Roderick and Ronald use random decision-making, while AIden has the TD(λ) AI system implemented. The results have been averaged over 40 hands, in order to be more viewable, and to also allow for the random nature of cards being dealt. As can be seen, AIden is consistently performing better than its counterparts, and continues to learn the game as it plays.

4.1.2 Cowardice

In the learning phase of the abovementioned intelligent agent, an interesting and somewhat enlightening problem arises. When initially learning, the agent does not in fact continue to learn. Instead, the agent quickly determines that it is losing chips, and decides that it is better off not playing, and keeping its chips! This is illustrated in Figure 3-6.

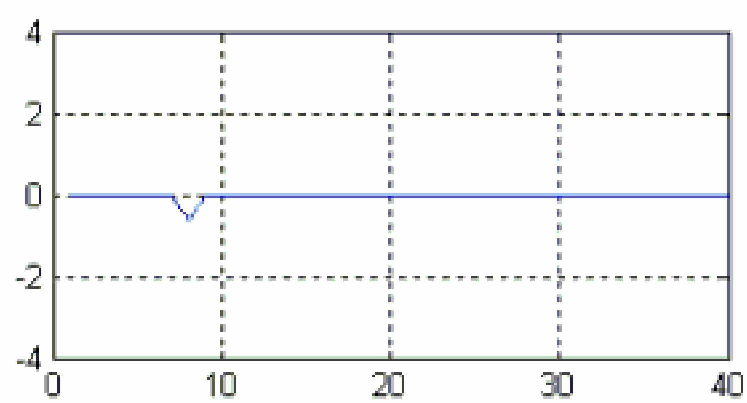

Figure 3-6. Agent cowardice. Averaged over 5 hands

As can be seen, AIden quickly decides that the risks are too great, and does not play in any hands initially. After forty hands, AIden decides to play a few hands, and when they go badly, gets scared off for good. This is a result of the penalising nature of the game, since bad play can easily mean one loses a full three chips, and since the surplus of lost chips is nor carried over in this simulation, a bad player loses chips regularly. While insightful, a cowardly agent is not of any particular use, and hence the agent must be given enough 'courage' to play, and hence learn the game. In order to do this, one option is to increase the value of ε for the ε-greedy policy, but this makes the agent far too much like a random player without any intelligence. A more successful, and sensible solution is to force the agent to play when it knows nothing, until such a stage as it seems prepared to play. This was done by forcing AIden to play the first 200 hands it had ever seen, and thereafter leave AIden to his own devices, the result of which has been shown already in Figure 3-2.

4.2 Parameter Optimisation

A number of parameters need to be optimised, in order to optimise the learning of the agents. These parameters are the learning-rate α, the memory parameter λ and the exploration parameter ε. The multi-agent system provides a perfect environment for this testing, since four different parameter combinations can be tested competitively. By setting different agents to different combinations, and allowing them to play against each other for an extended period of time (number of hands), one can iteratively find the parameter combinations that achieve the best results, and are hence the optimum learning parameters. Figure 3-7 shows the results of one such test, illustrating a definite 'winner', whose parameters were then used for the rest of the multi-agent modeling. It is also worth noting that as soon as the dominant agent begins to lose, it adapts its play to remain competitive with its less effective opponents. This is evidenced at points 10 and 30 on the graph (games number 300 and 900, since the graph is averaged over 30 hands) where one can see the dominant agent begin to lose, and then begins to perform well once again.

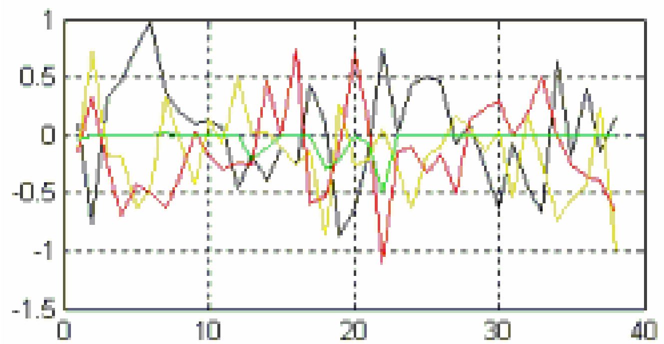

Figure 3-7. Competitive agent parameter optimisation. Averaged over 30 hands.

Surprisingly enough, the parameters that yielded the most competitive results were $\alpha = 0.1$; $\lambda = 0.1$ and $\varepsilon = 0.01$. while the ε value is not particularly surprising, the relatively low α and λ values are not exactly intuitive. What they amount to is a degree of temperance, since a higher values would mean learning a large amount from any given hand, effectively over-reacting when they may have played well, and simply have fallen afoul of bad luck.

4.3 MAS learning patterns

With all of the agents learning in the same manner, it is noteworthy that the overall rewards they obtain are far better than those obtained by the random agents, and even by the intelligent agent that was playing against the random agents. A sample of these results is depicted in Figure 3-8.

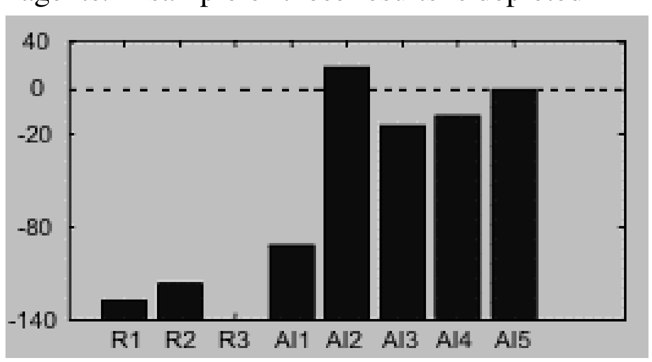

Figure 3-8. comparative returns over 200 hands.

R1 to R3 are the Random agents, while AI1 is the intelligent agent playing against the random agents. AI2 to AI 5 depict intelligent agents playing against each other. As can be seen, the agents learn far better when playing against intelligent opponents, an attribute that is in fact mirrored in human competitive learning. The agents with better experience tend to fold bad hands, and hence lose far fewer chips than the intelligent agent playing against unpredictable opponents.

4.4 Agent Adaptation

In order to ascertain whether the agents in fact adapt to each other or not, the agents were given pre-dealt hands, and required to play them against each other repeatedly. The results of one such experiment, illustrated in Figure 9, shows how an agent learns from its own mistake, and once certain of it, changes its play, adapting in order to gain a better return from the hand. The mistakes it sees are its low returns, returns of -3 to be precise. At one point, the winning player obviously decides to explore, giving some false hope to the losing agent, but then quickly continues to exploit his advantage. Eventually, at game #25, the losing agent gives up, adapting his play to suit the losing situation in which he finds himself. Figure 3-9 illustrates the progression of the agents and the adaptation described.

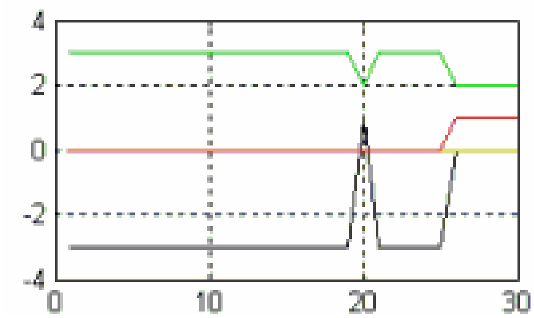

Figure 3-9. Adaptive agent behaviour

4.5 Strategy analysis

The agents have been shown to successfully learn to play the game, and to adapt to each other's play in order to maximise their own rewards. These agents form the pillars of the multi-agent model, which can now be used to analyse, and attempt to 'solve' the game. Since the game has a nontrivial degree of complexity, situations within the game are to be solved, considering each situation a sub-game of the overall game. The first, and most obvious type of analysis is a static analysis, in which all of the hands are pre-dealt. This system can be said to have stabilised when the results and the playout become constant, with all agents content to play the hand out in the same manner, each deciding that nothing better can be achieved. This is akin to Game Theory's "static equilibrium", as is evidenced in Figure 3-10.

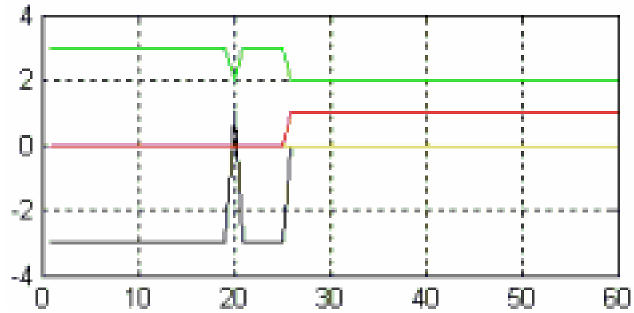

Figure 3-10. Stable, solved hand.

4.6 *Bluffing*

A bluff is an action, usually in the context of a card game, that misrepresents one's cards with the intent of causing one's opponents to drop theirs (i.e. to fold their hand). There are two opposing schools of thought regarding bluffing. One school claims that bluffing is purely psychological, while the other maintains that a bluff is a purely statistical act, and therefore no less sensible than any other strategy. Astoundingly enough, the intelligent agents do in fact learn to bluff! A classic example is illustrated in Figure 3-11, which depicts a hand in which bluffing was evidenced.

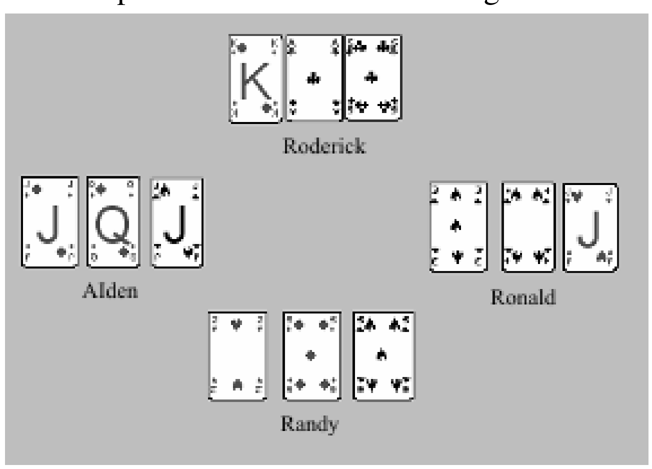

Figure 3-11. Agent bluffing

In the above hand, Randy is the first caller, and diamonds have been declared trumps. Randy's hand is not particularly impressive, having only one low trump, and two low supporting cards. Still, he has the lead, and a trump could become a trick, although his risks are high for minimal reward. Nonetheless, Randy chooses to play this hand. Ronald, having nothing to speak of, unsurprisingly folds. Roderick, on the other hand, has a very good hand. One high trump, and an outside ace. However, with one still to call, and Randy already representing a strong hand by playing, Roderick chooses to fold. AIden, whose hand is very strong with two high trumps and an outside jack, plays the hand. When the hand is played repeatedly, Randy eventually chooses not to play, since he loses all three to AIden. Instantly, Roderick chooses to play the hand, indicating that the bluff was successful, that it chased a player out of the hand! Depending on which of the schools of thought regarding bluffing one follows this astonishing result leads us to one of two possible conclusions. If, like the author, one maintains that bluffing is simply playing the odds, making the odds for one's opponent unfavourable by representing a strong hand, then this result shows that the agents learn each other's patterns well enough to factor their opponent's strategies into the game evaluation, something game theory does a very poor job of. Should one follow the theory that bluffing is purely psychological, then the only conclusion that can be reached from this result is that the agents have in fact developed their own 'psyches', their own personalities which can then be exploited. Regardless of which option the reader holds to, the fact remains that agents have been shown to learn, on their own and without external prompting, to bluff!

5. Conclusions

While the exact nature of bluffing is still unknown, it has been shown that a system involving agents capable of learning adaptively not only from the game being played, but also from their opponents, is in fact able to learn to predict its opponent's reactions. This knowledge in turn changes the statistical nature of a game being played, allowing agents to learn to bluff, based purely on rational reasoning, lending strong support to the theory that bluffing is simply *playing the odds*, and not an illogical, psychologically based action. The use of the Reainforcement learning paradigm, along with the TD(λ) algorithm for adaptively training neural networks, has been shown to meet all of the requirements to produce such agents. Lastly, the design of the agent "view", has been seen to be the most important facet of creating bluffing agents, since their view of the game as inclusive of the other players allows for the incorporation of those players into its estimation of the game's outcome. With all of these steps adhered to, artificially intelligent agents can learn to bluff!

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Chapter 4: Multi-Agent Modeling Using Intelligent Agents In The Game Of Lerpa.

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Abstract – *Game-theory has many limitations implicit in its application. By utilising multi-agent modeling, it is possible to solve a number of problems that are unsolvable using traditional game-theory. By using reinforcement learning, applied to neural networks, intelligent agents are created. Utilising intelligent agents, intelligent virtual players learn from each other and their own rewards to play the game of Lerpa. These agents not only adapt to each other, but are even able to anticipate each other's reactions, and to "bluff" accordingly should the occasion arise. By pre-dealing specific hands, either to one player or to the whole table, one may "solve" the game, finding the best play to any given situation.*

Keywords: Neural Networks, Reinforcement Learning, Game Theory, Temporal Difference, Multi Agent, Lerpa

1 Introduction

Current game analysis methods generally rely on the application of traditional *game-theory* to the system of interest [1]. While successful in simple systems/games, anything remotely complex requires the simplification of said system to an approximation that can be handled by game-theory, with its not unsubstantial limitations [2]. An alternative approach is to rather analyse the game from within the *Multi-agent Modeling* (MAM) paradigm. While this approach traditionally utilises simple agents [3], far too simplistic to handle any game of reasonable complexity, creating intelligent agents however, offers the possibility to "solve" these complex systems. The solution then becomes plausible without oversimplifying the system, as would be required in order to analyse them from a traditional game-theory perspective.

This dissertation explores the use of intelligent agents in a multi-agent system framework, in order to model and gain insight into a nontrivial, episodic card game with multiple players. The requirements for a multi-agent model will be specified, as will those for intelligent agents. Intelligent agents are then applied to the problem at hand, namely the card game, and results are observed and interpreted utilising sound engineering principles. All of this is presented with a view to evaluating the feasibility of applying intelligent agents within a multi-agent model framework, in order to solve complex games beyond the scope of traditional game-theory.

2 Game Theory

Game Theory concerns the finding of the best, or *dominant* strategy (or strategies, in the case of multiple, equally successful strategies) in order to maximise a player's winnings within the constraints of a game's rule-set [4]. Game theory makes the following assumptions when finding these dominant strategies [5]:

- Each player has two or more well-specified choices or sequences of choices

- Every possible combination of choices leads to a well-specified end-state that terminates the game.

- A numerically meaningful payoff/reward is defined for each possible end-state.

- Each player has a *perfect knowledge* of the game and its opposition. *Perfect knowledge* implies that it knows the full rules of the game, as well as the payoffs of the other players.

- All decision-makers are rational. This implies that a player will always choose the action that yields the greatest payoff.

With these assumptions in mind, the game can then be analysed from any given position by comparing the rewards of all possible strategy combinations, and then, by the last assumption, declaring that each player will choose the strategy with its own highest expected return [4].

2.1 Limitations of Game-Theory

As a result of the assumptions made when applying game theory, and of the methods themselves, the techniques developed have some implicit limitations [2]. A few of the more pressing limitations, which are not necessarily true of real-world systems, are as follows:

- Game theory methods arrive at static solutions. These methods will deduce a solution, or *equilibrium point* for a given situation in a game. In many games, however, one will find that a solution changes or evolves as players learn the particular favoured strategy of a player, and subsequently adapt to exploit the predictable behaviour that results from playing only dominant strategies, and thus ultimately defeating the player utilising the dominant strategy.

- Real players are not always *rational*, as defined above. A player may display preferences, often seemingly at odds with statistically "best-play" strategies, which can change the odds within a game. A good strategy should account for this possibility and exploit it, rather than ignore it. This problem is referred to in economic and game-theory circles as the "trembling hand".

- Game theory cannot handle more than two to three players. Due to dimensionality issues, game theory cannot be used to analyse games with a large number of players without simplifying the game to two- or three-player games, game complexity having the final word on the player limit.

- Game theory can only be applied to relatively simple games. Once again as a result of dimensionality issues, complex games have too many states to be effectively analysed using traditional game theory methods.

- In order to be analysed, many complex games are simplified by dividing the players into grouped "camps", effectively reducing multi-player games into two- or three-player games. Likewise the rules are often simplified, in order to similarly reduce the dimensionality of a game. While these simplifications may allow analysis to proceed, they also change the fundamental nature of many games, rendering the analysis flawed by virtue of examining an essentially dissimilar game.

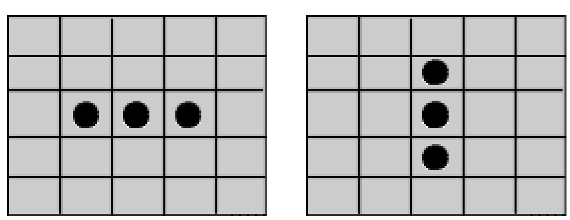
These limitations within game-theory prompt the investigation of an alternative method of analysing more complex games. This paper investigates the option of utilising *Multi-Agent Modeling*, using intelligent agents, to analyse a complex game.

3 Multi-Agent Modelling

In its simplest form, multi-agent modeling involves breaking a system up into its component features, and modeling those components in order to model the overall system [6]. Central to this methodology is the notion of *emergent behaviour*, that is, that the simple interactions between agents produce complex results [7]. These results are often far more complex than the agents that gave rise to them [7].

3.1 Emergent Behaviour

Emergent behaviour is so pivotal to the understanding and utilisation of multi-agent modeling, that a brief elaboration becomes necessary. While not, strictly speaking, an instance of MAM, John Conway's *game of artificial life* provides an excellent illustration of emergent behaviour, and the ramifications thereof [16]. In Conway's game, an MxN grid of squares (often infinite) each contains one binary value, being either *alive* or *dead*. In each iteration of the game, a *dead* square will become alive if exactly three adjacent squares are also alive, and an *alive* square will *die* if there are less than two adjacent *living* squares, or if there are more than three adjacent *living* squares, as depicted in Figure 4-1.



(a) (b)

Figure 4-1. A single iteration of *Life*

In the next iteration, the two outermost living squares in Figure 4-1(a) will die since each has only one living neighbour, and the two squares above and below the centre living square will come to life, as each has exactly three living neighbours, resulting in the situation depicted in figure 4-1(b). As one can see, the rules are incredibly simple, but the consequences of these rules, i.e. the *Emergent Behaviour*, are far from simple. Figure 4-2 shows a simple-looking shape, commonly referred to as a *glider* [16].

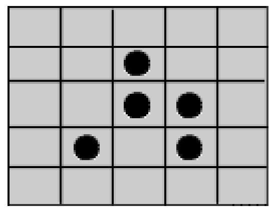


Figure 4-2. Simple Glider

This shape continues to stably propagate itself at a forty-five degree angle within the game. In contrast, the even simpler-looking shape in Figure 4-3, known as a *r-pentamino*, produces an explosion of shapes and patterns that continually change and mutate, only becoming predictable after 1103 iterations [16].

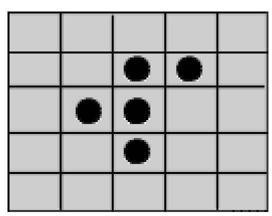


Figure 4-3. R-Pentamino

It is this same complex behaviour, emerging from interacting components following simple rules, that lies at the heart of MAM's promise, and making it such a powerful tool [7].

3.2 Advantages of Multi-Agent modeling

An agent-based model holds many advantages over standard analytical techniques, which are traditionally mathematical or statistical in nature [8]. Specifically, some of the advantages offered by an Agent-based Model are as follows [9]:

- Agents are far simpler to model than the overall system they comprise, and hence the system becomes easier to model.

- The emergent behaviour resulting from the agent interactions means that systems too complex to be traditionally modeled can now be tackled, since the complexity of the system need not be explicitly modeled.

- Large systems with heterogeneous agents can be easily handled within a multi-agent system, while this is incredibly difficult to cater for using traditional mathematics, which would make the often unrealistic demand that the components be homogenous.

3.3 Weaknesses/Limitations

While MAM has definite advantages, it is not without weaknesses. Since the emergent behaviour is arrived at empirically, and is not deterministic, it is difficult to state with any degree of certainty as to why a certain outcome has been arrived at [10]. Similarly, since emergent behaviour is often unexpected [7], it can be difficult to ascertain whether the multi-agent system (MAS) is incorrectly modeling the system in question. Thus, validation of the model becomes an important aspect of any MAS.

3.4 MAM Applications

Multi-agent modeling lends itself to a number of applications. The following are some of the more common applications of multi-agent modeling:

3.4.1 Swarm Theory

Multi-agent modeling is utilised for the development of *Swarm Theory* based systems [11]. These systems utilise many simple agents, and attempt to design simple individual rules that will allow the agents to work together to achieve a larger, common goal [11], much in the same manner that a swarm of ants will work together to collect food for the colony. These systems depend on the Engineer's ability to predict the (often unexpected) emergent behaviour of the system for given agent behaviour [11].

3.4.2 Complexity Modelling

Multi-Agent modeling is well-suited to the task of *complexity modeling* [12]. Complexity Modeling refers to modeling complex systems that are often too complex to be explicitly modeled [12]. The usage of representative agents allows for the emergent behaviour of the MAM to model the complexity within the system, rather than said complexity being explicitly modeled by the Engineer [12]. Essentially, the complexity is contained by the *interactions* between the agents, and between the agents and the system, rather than the traditional, and often insufficient, mathematical models previously used [12].

3.4.3 Economics

Fundamentally an application of Complexity Modeling, Multi-Agent modeling can be applied to economic systems [13]. This discipline, known as Applied Computational Economics (ACE), applies a bottom-up approach to modeling an economic system, rather than the traditional top-down approach, which requires full system specification and then component decomposition [13]. In order to verify ACE system veracity, the ACE model is required to reproduce known results empirically. Once this has been accomplished, the same system can then be used to predict the results of unknown situations, allowing for better forecasting and policy decision-making.

3.4.4 Social Sciences

Many attempts have been made to model social phenomena, with varying degrees of success [14]. Since social systems, by definition, involve the interaction of autonomous entities [15], multi-agent modeling offers an ideal methodology for modeling such systems [14]. The foundations of such applications have already been laid, with the groundwork being solutions to such problems as the *standing ovation* problem [14].

3.5 Making a Multi-agent Model

In order to create a multi-agent model, the smaller components that comprise the system must be specified [17]. These smaller components need to be fully modeled, so as to become the *agents* that are at the heart of the modeling technique [17]. Each agent must be capable of making decisions (often dictated by a rule-set), and these decisions may involve incomplete knowledge of its environment [17]. The agents also need to be able to receive information from their environments, and depending on the type of system being modeled, sometimes communicate with other agents [17]. The environment itself needs to be able to adjudicate the interactions between agents, but at no stage needs to be able to determine the overall ramifications of these interactions. Instead, the overall result will become apparent empirically, taking advantage of the emergent behaviour of the multi-agent system to handle the complexity modeling [17].

4 Intelligent Agents

Artificial intelligence (A.I.) can only truly be considered worthy of the name when the system in question is capable of learning on its own [18], without having an expert *teacher* available to point out correct behaviour. This leads directly into the paradigm of *reinforcement learning* [18]. Most *reinforcement learning* techniques explored utilise lookup-table system representations, or linear function approximators, which severely hinder the scope of learning available to the artificial intelligence system. One notable exception is the work by G. Tessauro on *TD-Gammon*, in which he successfully applied the TD(λ) reinforcement learning algorithm to train a neural network, with staggeringly successful results. Following attempts to emulate his work have, however, been met with failure due to the extreme difficulties of combining backpropogation with TD(λ). Some methods for overcoming these problems have been explored, allowing for the combination of these versatile techniques in order to create an intelligent agent.

4.1 What is Intelligence?

In order to create intelligent agents, it is necessary to first define intelligence, so that we may critically evaluate whether the agent created meets these criteria. In order to be considered intelligent, an entity must be capable of the following [19]:

- The entity must be able to learn.
- The entity must be able to learn from its own inferences, without being taught.
- The entity must be capable of drawing conclusions from incomplete data, based on its own knowledge.
- The entity must be able to re-evaluate its own knowledge, and adapt if necessary.

Should an agent meet these requirements, it can then be considered intelligent. In order to meet these requirements, the agents will learn within the *reinforcement learning* paradigm.

4.2 Reinforcement Learning

Reinforcement learning involves the training of an artificial intelligence system by trial-and-error, reflecting the same manner of learning that living beings exhibit [20]. Reinforcement learning is very well suited to episodic tasks [20], and as such is highly appropriate in the field of game-playing, where many episodes are encountered before a final result is reached, and an A.I. system is required to evaluate the value of each possible move long before a final result is achieved. This methodology allows for online learning, and also eliminates the need for expert knowledge [20].

4.2.1 Rewards and Returns

Any artificial intelligence system requires some sort of goal or target to strive towards [21], [20]. In the case of reinforcement learning, there are two such quantities that need to be defined, namely *rewards* and *returns* [20]. A reward is defined to be the numerical value attributed to an individual state, while a return is the final cumulative rewards that are returned at the end of the sequence [20]. The return need not necessarily be simply summed, although this is the most common method [20]. An example of this process can be seen below in Figure 4-4, where an arbitrary Markov process is illustrated with rewards given at each step, and a final return at the end. This specific example is that of a *Random walk* problem.

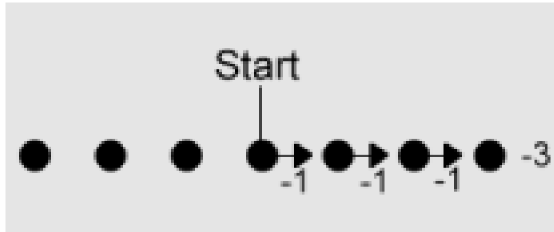


Figure 4-4. Random walk rewards and returns.

An A.I. system utilising reinforcement learning must learn to predict its expected return at each stage, hence enabling it to make a decision that has a lower initial reward than other options, but maximising its future return. This can be likened to making a sacrifice in chess, where the initial loss of material is accepted for the future gains it brings.

4.2.2 Exploitation vs Exploration

An A.I. system learning by reinforcement learning learns only through its own experiences [20]. In order to maximise its rewards, and hence its final return, the system needs to experiment with decisions not yet tried, even though it may perceive them to be inferior to tried-and-tested decisions [20]. This attempting of new approaches is termed *exploration*, while the utilisation of gained knowledge to maximise returns is termed *exploitation* [20]. A constant dilemma that must be traded off in reinforcement learning is that of the choice between exploration and exploitation. One simple approach is the *ε-greedy* approach, where the system is *greedy*, i.e. attempts to exploit, with probability ε. Hence the system will explore with probability 1- ε [20].

4.2.3 TD (λ)

One common method of training a reinforcement learning system is to use the TD (λ) (Temporal Difference) algorithm to update one's value estimates [20] [22]. This algorithm is specifically designed for use with episodic tasks, being an adaptation of the common Widdrow-Hoff learning rule [22]. In this algorithm, the parameters or *weights w* to be altered are updated by equation (1) [22].

$$\Delta w = \alpha\left(P_{t+1} - P_t\right)\sum_{k=1}^{t} \lambda^{t-k} \nabla w P_k \tag{1}$$

The prediction P_{t+1} is used as a target for finding the error in prediction P_t, allowing the update rule to be computed incrementally, rather than waiting for the end of the sequence before any learning can take place [22]. Parmaters α and λ are the learning rate and weight-decay parameters, respectively.

The prediction P can be made in a number of different methods, ranging from a simple lookup table to complex function approximators [22]. Owing to the nature of the task at hand, a function approximator with a high degree of flexibility is required, since the agents must be capable of drawing any logical links they see fit, and not be limited by our choice of function

5 Neural Networks

It is necessary to understand the workings and advantages of neural networks to appreciate the task of applying them in the reinforcement learning paradigm. It is likewise important to fully grasp the implications of reinforcement learning, and the break they represent from the more traditional supervised learning paradigm.

5.1 Neural network architecture

The fundamental building-blocks of neural networks are *neurons* [21]. These neurons are simply a multiple-input, single-output mathematical function [21]. Each neuron has a number of *weights* connecting it to inputs from a previous layer, which are then added together, possibly with a *bias*, the result of which is then passed into the neuron's *activation function* [21]. The activation function is a function that represents the way in which the neural network "thinks". Different activation functions lend themselves to different problem types, ranging from yes-or-no decisions to linear and nonlinear mathematical relationships. Each *layer* of a neural network is comprised of a finite number of neurons. A network may consist of any number of layers, and each layer may contain any number of neurons [21]. When a neural network is run, each neuron in each consecutive layer sums its inputs and multiplies each input by its respective weight, and then treats the weighted sum as an input to its activation function. The output will then be passed on as an input to the next layer, and so on until the final output layer is reached. Hence the input data is passed through a network of neurons in order to arrive at an output. Figure 4-5 illustrates an interconnected network, with 2 input neurons, three hidden layer neurons, and two output neurons. The hidden layer and output layer neurons can all have any of the possible activation functions. This type of neural network is referred to as a *multi-layer perceptron* [21], and while not the only configuration of neural network, it is the most widely used configuration for regression-type problems [23].

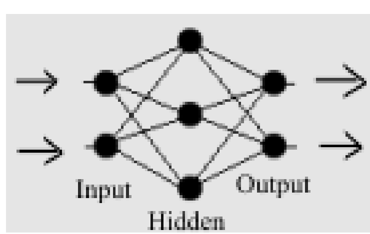


Figure 4-5. Sample connectionist network

5.2 Neural network properties

Neural networks have various properties that can be utilised and exploited to aid in the solving of numerous problems. Some of the properties that are relevant to this particular problem are detailed below.

5.2.1 Universal approximators

Multi-layer feed-forward neural networks have been proven to be universal approximators [21]. By this one refers to the fact that a feed-forward neural network with nonlinear activation functions of appropriate size can approximate any function of interest to an arbitrary degree of accuracy [21]. This is contingent upon sufficient training data and training being supplied.

5.2.2 Neural networks can generalise

By approximating a nonlinear function from its inputs, the neural network can learn to approximate a function [21]. In doing so, it can also infer the correct output from inputs that it has never seen, by inferring the answer from similar inputs that it has seen. This property is known as *generalisation* [21]. As long as the inputs received are within the ranges of the training inputs, this property will hold [21].

5.2.3 Neural networks recognise patterns

Neural networks are often required to match large input/output sets to each other, and these sets are often 'noisy' or even incomplete [21]. In order to achieve this matching, the network learns to recognise patterns in the data sets rather than fixate on the answers themselves [21]. This enables a network to 'see' through the data points and respond to the underlying pattern instead. This is an extended benefit of the generalisation property.

5.3 Training

Training of neural networks is accomplished through the use of an appropriate algorithm [24]. The two main types of training algorithms employed are backpropogation and batch updating algorithms [24]. Many algorithms exist, all with their own unique advantages and disadvantages. Commonly used backpropogation algorithms are Steepest Descent and Scaled Conjugate Gradient training methods [24], while a commonly used batch updating algorithm is the Quasi-Newton training algorithm [24]. All of these algorithms are gradient-based algorithms for multivariable optimisation, which are preferable to evolutionary methods due to their guaranteed convergence, even though global optimality cannot be guaranteed [24]. Regardless of the variation, the fundamental idea behind backpropogation methods is that weights are updated, via the optimisation method selected, from the last neuron layer back to the first [24], whereas batch updating algorithms update all weights simultaneously, making for more complex computations, although fewer required iterations [24].

5.4 Generalisation

Neural networks are capable of generalising to situations that they have not in fact been trained on, providing that they have been trained over an encompassing range of inputs [25]. In other words, a network trained on input values of 1; 4; 7; and 10 will be able to present an accurate answer to an input of 8 since the network has been trained with values both greater and smaller than 8. It will however, struggle to present a correct answer to an input of 11 since the highest input training value was only 10. The ability to generalise is at the heart of the overfitting/overtraining issue [25]. It is generally accepted that overtraining is a myth, provided that the number of hidden neurons is correct and that the training data is complete. However, since satisfying both of these conditions is nontrivial, the problem of overtraining remains a stark reality. Overtraining leads to the more specific problem of overfitting, wherein a network fits too closely to data that may be incorrect, often fitting the function to the noise rather than the underlying pattern [25]. A visual example of overfitting and in contrast good generalisation is presented in Figure 4-6.

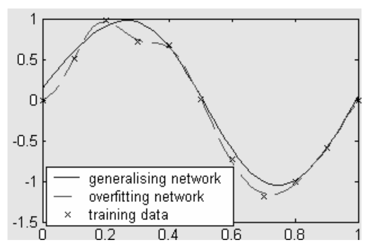


Figure 4-6. Over-fitted Vs generalising Networks

5.5 Suitability of Neural Networks for an Intelligent MAM Agent

A neural network would seem to be an exceptionally good, and obvious choice for use as a function approximator for the agent's predictor. The fact that it is a universal approximator means that the agent is capable of making any inference it deems fit to link the inputs to its observed values, thus not constraining the agent to limited human/expert knowledge, which a linear function approximator would. A comprehensive database lookup-table could provide a similar result, but it has a number of difficulties that prohibit such a measure. The first problem would be that of *dimensionality*, i.e. that there are simply too many possible states to encode, often more in total than even modern computer memory can handle. On the contrary, the compact nature of a neural network allows for all of these states to be captured using minimal memory. In addition, the training time for any sort of convergence would be extremely slow with a lookup-table method, again due to its size. Differing methods for generalising using eligibility traces [23] do exist, but all would impose an artificial linking between states that may not necessarily exist. A neural network's generalisation property, however, allows for training of similar states without imposing any such constraints on the budding Artificial Intelligence's freedom.

With these advantages in mind, the next problem is observed to be the act of training the network, since the standard algorithms are of no use in the reinforcement learning paradigm. The standard gradient-based algorithms require input/output data sets upon which to train [24], which for the following reasons our agent will not have access to:

- Datasets would require a known *correct* answer, which would prevent the agent from finding better answers than those currently accepted.

- Datasets can grow outdated swiftly, especially in a competitive gaming situation, where one's opponents can form an integral part of the game's optimum strategy.

For these reasons, the TD(λ) algorithm has been selected for use, based largely on the success of the Tessauro TD-Gammon program.

5.6 Jumping the hurdles

In order to implement the TD(λ) algorithm for training neural networks, it is better to first tackle a known, smaller problem. By first tackling such a problem, one can identify and solve the prevailing issues within the application of the algorithm, without them being clouded by issues pertaining to the more complex system. For these reasons, the problem of tic-tac-toe has been tackled, with an agent created to play against itself, learning to play the game as it plays. The simplicity of the game makes analysis of the agents easy, and the finite, solved nature of the game means that one can easily determine whether the agent has made a good or a bad decision. This last facet will not be true of larger games, where the agent could easily become better than the standard by which it is judged, and is thus very important to establish now, when evaluating the methodology.

6 Tic Tac Toe

The game of *Tic Tac Toe*, or *noughts and crosses*, is played on a 3x3 grid, with players taking alternate turns to fill an empty spot [26]. The first player places a 'O', and the second player places a 'X' whenever it is that respective player's turn [26]. If a player manages to get three of his mark in a row, he wins the game [26]. If all 9 squares are filled without a winner, the game is a draw [26]. The game was simulated in Matlab, with a simple matrix representation of the board.

6.1 Player evaluation

The A.I. system, or *player*, needs to be evaluated in order to compare different players, who have each learned using a different method of learning. In order to evaluate a player's performance, ten different positions are set up, each with well-defined correct moves. Using this test-bed, each player can be scored out of ten, giving a measure of the level of play each player has achieved. Also important is the speed of convergence – i.e. how fast does each respective player reach its own maximum level of play.

6.2 TD(λ) for backpropogation

In order to train a neural network, equation (1) needs to be adapted for use with the backpropogation algorithm [27]. The adaptation, without derivation, is as follows [27]:

$$w_{ij}^{t+1} = w_{ij}^{t} + \alpha \sum_{K \in O} \left(P_K^{t+1} - P_K^{t} \right) e_{ijk}^{t} \tag{2}$$

where the eligabilities are:

$$e_{ijk}^{t+1} = \lambda e_{ijk}^{t} + \delta_{kj}^{t+1} y_i^{t+1} \tag{3}$$

and δ is calculated by recursive backpropogation

$$\delta_{ki}^{t} = \frac{\partial P_k^t}{\partial S_i^t} \tag{4}$$

as such, the TD(λ) algorithm can be implemented to update the weights of a neural network [27].

6.3 Stability issues

The TD(λ) algorithm has proven stability for linear functions [22]. A multi-layer neural network, however, is non-linear [21], and the TD(λ) algorithm can become unstable in some instances [20] [28]. The instability can arise in both the actual weights and in the predictions [20] [28]. In order to prevent instability, a number of steps can be taken, the end result of which is in most cases to limit the degree of variation in the outputs, so as to keep the error signal small to avoid instability.

6.3.1 Input/Output representation

The inputs to, and outputs from, a typical A.I. system are usually represented as real or integer values. This is not optimal for TD(λ) learning, as the values have too much variation. Far safer is to keep the representations in binary form, accepting the dimensionality trade-off (a function of the number of inputs into the network, and hence larger using a binary representation) as a fair price to ensure a far higher degree of stability. Specifically in the case of the outputs, this ensures that the output error of the system can never be more than 1 for any single output, thus keeping the mean error to within marginally stable bounds. For the problem of the Tic Tac Toe game, the input to the network is an 18-bit binary string, with the first 9 bits representing a possible placed ‘o’ in each square, and the second 9 bits representing a ‘1’ in each square. The output of the network is a 3-bit string, representing an ‘o’ win, a draw and an ‘x’ win respectively.

6.3.2 Activation Functions

As shown in Section 2, there are many possible activation functions that can be used for the neural network. While it is tempting to utilise activation functions that have a large scope in order to maximise the versatility of the network, it proves far safer to use an activation function that is limited to an upper bound of 1, and a lower bound of zero. A commonly used activation function of this sort is the sigmoidal activation function, having the form of:

$$f(x) = \frac{1}{1+e^{-x}} \tag{5}$$

This function is nonlinear, allowing for the freedom of approximation required of a neural network, and limiting the upper and lower bounds as recommended above. While this activation function is commonly used as a middle-layer activation function, it is unusual as an output layer activation function. In this manner, instability is further discouraged.

6.3.3 Learning rate

As the size of the error has a direct effect on the stability of the learning system, parameters that directly effect the error signal also have an effect on the said stability. For this reason, the size of the learning rate α needs to be kept low, with experimental results showing that values between 0.1 and 0.3 prove safe, while higher values tend to become unstable, and lower values simply impart too little real learning to be of any value.

6.3.4 Hybrid stability measures

In order to compare relative stability, the percentage chance of becoming unstable has been empirically noted, based upon experimental results. Regardless of each individual technique presented, it is the combination of these techniques that allows for better stability guarantees. While no individual method presented gives greater than a 60% stability guarantee (that is, 60% chance to be stable given a 100-game training run), the combination of all of the above measures results in a much better 98% probability of being stable, with minor tweaking of the learning rate parameter solving the event of instability occurring at unusual instances.

6.4 The players

All of the players are trained using an ε-greedy policy, with the value of $\varepsilon = 0.1$. i.e. for each possible position the player has a 10% chance of selecting a random move, while having a 90% chance of selecting whichever move it deems to be the best move. This selection is done by determining all of the legal moves available, and then finding the positions that would result from each possible move. These positions are sequentially presented to the player, who then rates each resultant position, in order to find the best resultant position. It obviously follows that whichever move leads to the most favoured position is the apparently best move, and the choice of the player for a greedy policy. The training of each player is accomplished via *self-play*, wherein the player evaluates and chooses moves for both sides, learning from its own experiences as it discovers errors on its own. This learning is continued until no discernable improvement occurs.

As a benchmark, randomly initialised networks were able to correctly solve between 1 and 2 of the posed problems, beyond which one can say genuine learning has indeed taken place, and is not simply random chance.

6.5 Player #1 – Simple TD(λ)

Player #1 learned to play the game using a simple TD(λ) backpropogation learning algorithm. This proved to be very fast, allowing for many thousands of games to be played out in a very short period of time. The level of play achieved using this method was however not particularly inspiring, achieving play capable of solving no more than 5 of the 10 problems posed in the rating system. The problem that is encountered by player #1 is that the learning done after each final input, the input with the game result, gets undone by the learning of the intermediate steps of the next game. While in concept the learning should be swifter due to utilising the knowledge gained, the system ends up working at cross-purposes against itself, since it struggles to build its initial knowledge base, due to the generalisation of the neural network which is not present in more traditional reinforcement learning arrangements.

6.6 Player #2 – Historical database learning

In this instance, the player learns by recording each position and its corresponding target, and storing the pair in a database. Duplicate input data sets and their corresponding targets are removed, based on the principle that more recent data is more accurate, since more learning has been done when making the more recent predictions. This database is then used to train the network in the traditional supervised learning manner. A problem encountered early on with this method is that early predictions have zero knowledge base, and are therefore usually incorrect. The retaining of this information in the database therefore taints the training data, and is thus undesirable. A solution to this problem is to limit the size of the database, replacing old data with new data once the size limit is reached, thus keeping the database recent. This methodology trains slower than that employed to train player #1, making long training runs less feasible than for TD(λ) learning. The play level of this method is the lowest of those examined, able to solve only four of the ten proposed problems. Nonetheless, the approach does show promise for generating an initial knowledge base from which to work with more advanced methods.

6.7 Player #3 – Fact/Opinion DB learning

Building on the promise of Player #2, a more sophisticated database approach can be taken. If one takes into account the manner of the training set generation, one notes that most of the targets in the database are no more than *opinions* – targets generated by estimates of the next step, as seen in equation (1) – while relatively few data points are in fact *facts* – targets generated by viewing the end result of the game. In order to avoid this problem, the database can be split into two sub-databases, with one holding facts, and the other holding opinions. Varying the sizes, and the relative sizes, of these two sub-databases can then allow the engineer to decide how much credence the system should give to fact versus opinion. This method proved far more successful than Player #2, successfully completing 6 of the 10 problems posed by the rating system. It's speed of convergence is comparable to that of Player #2.

6.8 Player #4 – Widdrow-Hoff based DB learning

In this instance, a very similar approach to that of Player #2 is taken, with one important distinction: Instead of estimating a target at each move, the game is played out to completion with a static player. After each game finishes, the player then adds all of the positions encountered into its database, with the final result being the target of each position. This means that no *opinions* can ever enter into the training, which trades off speed of convergence for supposedly higher accuracy. This method is not optimal, as it loses one of the primary advantages of reinforcement learning, namely that of being able to incorporate current learning into its own learning, hence speeding up the learning process. Unsurprisingly, this method trains with the same speed as the other database methods, but takes far longer to converge. It achieves a similar level of play as does Player #1, being able to solve 5 of the posed problems.

6.9 Player #5 - Hybrid Fact/Opinion DB TD(λ) learning

The logical extension to the previous players is to hybridise the most successful players in order to compensate for the failings of each. Player #5 thus utilises the Fact/Opinion database learning in order to build an initial knowledge base from which to learn, and then proceeds to learn from thence using the TD(λ) approach of Player #1. This proves more successful, since the intrinsic flaw in player #1's methodology lies in its inability to efficiently create a knowledge base, and the database method of player #3 creates that knowledge base from which to learn. Player #5 begins its learning with the expected sluggishness of database methods, but then learns much faster once it begins to learn using the TD(λ) approach. Player #5 managed to successfully solve seven of the ten problems once trained to convergence. The problem of unlearning learned information is still apparent in Player #5, but is largely mitigated by the generation of the initial knowledge base.

6.10 Player comparisons

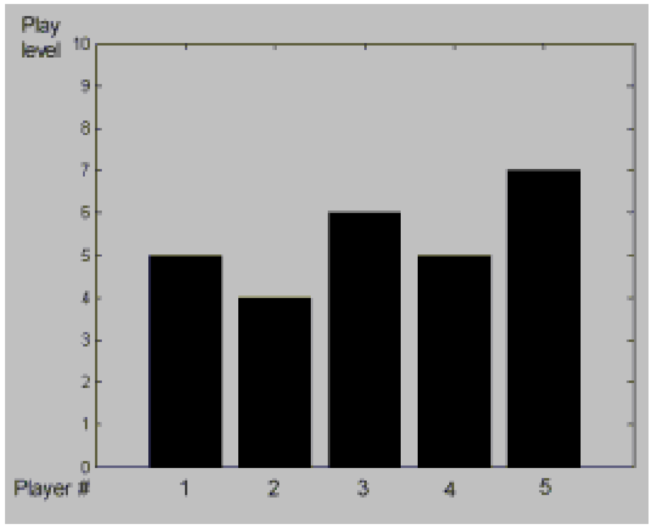


Figure 4-7. Relative player strengths

As is illustrated in figure 4-7, the hybrid method learns to play at the strongest level of all of the methods presented. Due to the drastic differences in speed and computational power requirements, it is preferable to stay away from database-based methods, and it is thus worth noting that only the fact/opinion database method arrives at a stronger level of play than the simple TD(λ)-trained player #1, and that this methodology can easily be incorporated into a TD(λ) learning system, which produces the far more promising player #5. The fact that after a short knowledge-base generation sequence the hybrid system uses the highly efficient TD(λ) approach makes it a faster and more reliable learning system than the other methods presented. As can be seen in Figure 4-7, however, there is still a greater level of play strength that should be achievable in this simple game, and that has been limited by the unlearning error seen in Players #1 and #5.

While no perfect result is achieved, the primary goal of learning and adapting is successful. The agents have been shown to learn on their own, without tutoring. They can infer from past knowledge to make estimates of unknown situations, and can adapt to changing situations. Thus the agents have satisfied all four requirements necessary to be considered “intelligent agents”.

7 Lerpa

The card game being modeled is the game of Lerpa. While not a well-known game, its rules suit the purposes of this research exceptionally well, making it an ideal testbed application for intelligent agent MAM. The rules of the game first need to be elaborated upon, in order to grasp the implications of the results obtained. Thus, the rules for Lerpa now follow.

The game of *Lerpa* is played with a standard deck of cards, with the exception that all of the 8s, 9s and 10s are removed from the deck. The cards are valued from greatest- to least-valued from ace down to

2, with the exception that the 7 is valued higher than a king, but lower than an ace, making it the second most valuable card in a suit. At the end of dealing the hand, during which each player is dealt three cards, the dealer has the choice of *dealing himself in* – which entails flipping his last card over, unseen up until this point, which then declares which suit is the *trump suit*. Should he elect not to do this, he then flips the next card in the deck to determine the trump suit. Regardless, once trumps are determined, the players then take it in turns, going clockwise from the dealer's left, to elect whether or not to play the hand (to *knock*), or to drop out of the hand, referred to as *folding* (If the Dealer has *dealt himself in*, as described above, he is then automatically required to play the hand). Once all players have chosen, the players that have elected to play then play the hand, with the player to the dealer's left playing the first card. Once this card has been played, players must then play *in suit* – in other words, if a heart is played, they must play a heart if they have one. If they have none of the required suit, they may play a trump, which will win the trick unless another player plays a higher trump. The highest card played will win the trick (with all trumps valued higher than any other card) and the winner of the trick will lead the first card in the next trick. At any point in a hand, if a player has the Ace of trumps and can legally play it, he is then required to do so. The true risk in the game comes from the betting, which occurs as follows:

At the beginning of the round, the dealer pays the table 3 of whatever the basic betting denomination is (referred to usually as 'chips'). At the end of the hand, the chips are divided up proportionately between the winners, i.e. if you win two tricks, you will receive two thirds of whatever is in the pot. However, if you stayed in, but did not win any tricks, you are said to have been *Lerpa'd*, and are then required to match whatever was in the pot for the next hand, effectively costing you the pot. It is in the evaluation of this risk that most of the true skill in *Lerpa* lies.

8 Lerpa MAM

As with any optimisation system, very careful consideration needs to be taken with regards to how the system is structured, since the implications of these decisions can often result in unintentional assumptions made by the system created. With this in mind, the Lerpa MAS has been designed to allow the maximum amount of freedom to the system, while also allowing for generalisation and swift convergence in order to allow the intelligent agents to interact unimpeded by human assumptions, intended or otherwise.

8.1 System overview

The game is, for this model, going to be played by four players. Each of these players will interact with each other indirectly, by interacting directly with the *table*, which is their shared environment, as depicted in Figure 4-8.

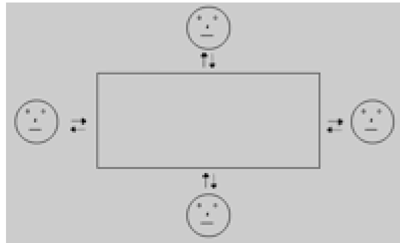


Figure 4-8. System interactions.

Over the course of a single hand, an agent will be required to make three decisions, once at each interactive stage of the game. These three decision-making stages are:

1. Whether to play the hand, or drop (*knock* or *fold*)

2. Which card to play first

3. Which card to play second

Since there is no decision to be made at the final card, the hand can be said to be effectively finished from the agent's perspective after it has played its second card (or indeed after the first decision should the agent fold). Following on the TD(λ) algorithm, each agent will update its own neural network at each stage, using its own predictions as a reward function, only receiving a true reward after its final decision has been made. This decision making process is illustrated below, in Figure 4-9.

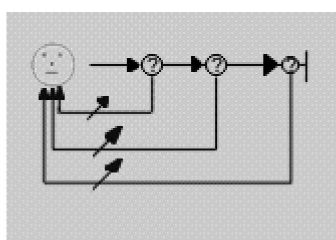


Figure 4-9. Agent learning scheme

With each agent implemented as described, they can now interact with each other through their shared environment, and will continuously learn upon each interaction and its consequent result.

Each hand played will be viewed as an independent, stochastic event, and as such only information about the current hand will be available to the agent, who will have to draw on its own learned knowledge base to draw deductions from rather than from previous hands.

8.2 Agent AI design

A number of decisions need to be made in order to implement the agent AI effectively and efficiently. The type of learning to be implemented needs to be chosen, as well as the neural network architecture. Special attention needs to be paid to the design of the inputs to the neural network, as these determine what the agent can 'see' at any given point. This will also determine what assumptions, if any, are implicitly made by the agent, and hence cannot be taken lightly. Lastly, this will determine the dimensionality of the network, which directly affects the learning rate of the network, and hence must obviously be minimised.

8.2.1 Input Parameter Design

In order to design the input stage of the agent's neural network, one must first determine all that the network may need to know at any given decision-making stage. All inputs, in order to optimise stability, are structured as binary-encoded inputs. When making its first decision, the agent needs to know its own cards, which agents have stayed in or folded, and which agents are still to decide. It is necessary for the agent to be able to match specific agents to their specific actions, as this will allow for an agent to learn a particular opponent's characteristics, something impossible to do if it can only see a number of players in or out. Similarly, the agent's own cards must be specified fully, allowing the agent to draw its own conclusions about each card's relative value. It is also necessary to tell the agent which suit has been designated the trumps suit, but a more elegant method has been found to handle that information, as will be seen shortly. Figure 4-10 below illustrates the initial information required by the network.

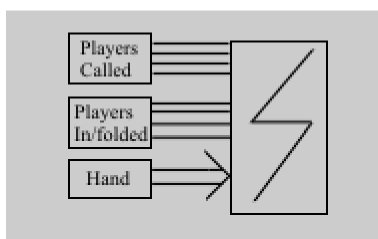


Figure 4-10. Basic input structure.

The agent's hand needs to be explicitly described, and the obvious solution is to encode the cards exactly, i.e. four suits, and ten numbers in each suit, giving forty possibilities for each card. A quick glimpse at the number of options available shows that a raw encoding style provides a sizeable problem of dimensionality, since an encoded hand can be one of 40^3 possible hands (in actuality, only ${}^{40}P_3$ hands could be selected, since cards cannot be repeated, but the raw encoding scheme would in fact allow for repeated cards, and hence 40^3 options would be available). The first thing to notice is that only a single deck of cards is being used, hence no card can ever be repeated in a hand. Acting on this principle, consistent ordering of the hand means that the base dimensionality of the hand is greatly reduced, since it is now combinations of cards that are represented, instead of permutations. The number of combinations now represented is ${}^{40}C_3$. This seemingly small change from ${}^{n}P_r$ to ${}^{n}C_r$ reduces the dimensionality of the representation by a factor of r!, which in this case is a factor of 6.

Furthermore, the representation of cards as belonging to discrete suits is not optimal either, since the game places no particular value on any suit by its own virtue, but rather by virtue of which suit is the trump suit. For this reason, an alternate encoding scheme has been determined, rating the 'suits' based upon the makeup of the agent's hand, rather than four arbitrary suits. The suits are encoded as belonging to one of the following groups, or new "suits":

- Trump suit
- Suit agent has multiple cards in (not trumps)
- Suit in agent's highest singleton
- Suit in agent's second-highest singleton
- Suit in agent's third-highest singleton

This allows for a much more efficient description of the agent's hand, greatly improving the dimensionality of the inputs, and hence the learning rate of the agents. These five options are encoded in a binary format, for stability purposes, and hence three binary inputs are required to represent the suits. To represent the card's number, ten discrete values must be represented, hence requiring four binary inputs to represent the card's value. Thus a card in an agent's hand is represented by seven binary inputs, as depicted in Figure 4-11.

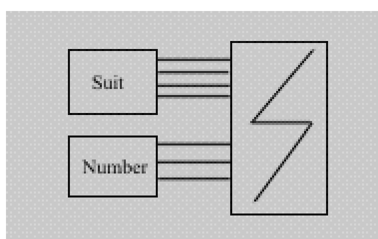


Figure 4-11. Agent card input structure

Next must be considered the information required in order to make decisions two and three. For both of these decisions, the cards that have been already played, if any, are necessary to know in order to make an intelligent decision as to the correct next card to play. For the second decision, it is also plausible that knowledge of who has won a trick would be important. The most cards that can ever be played before a decision must be made is seven, and since the table after a card is played is used to evaluate and update the network, it is necessary to represent eight played cards. Once again, however, simply utilising the obvious encoding method is not necessarily the most efficient method. The actual values of the cards played are not necessarily important, only their values relative to the cards in the agent's hand. As such, the values can be represented as one of the following, with respect to the cards of the same suit in the agent's hand:

- Higher than the card/cards in the agent's hand
- Higher than the agent's second-highest card
- Higher than the agent's third-highest card
- Lower than any of the agent's cards
- Member of a void suit (value is immaterial)

Another suit is now relevant for representation of the played cards, namely a void suit – a suit in which the agent has no cards. Lastly, a number is necessary to handle the special case of the Ace of trumps, since its unique rules mean that strategies are possible to develop based on whether it has or has not been played. The now six suits available still only require three binary inputs to represent, and the six number groupings now reduce the value representations from four binary inputs to three binary inputs, once again reducing the dimensionality of the input system.

With all of these inputs specified, the agent now has available all of the information required to draw its own conclusions and create its own strategies, without human-imposed assumptions affecting its "thought" patterns.

8.2.2 Network Architecture Design

With the inputs now specified, the hidden and output layers need to be designed. For the output neurons, these need to represent the prediction P that the network is making. A single hand has one of five possible outcomes, all of which need to be catered for. These possible outcomes are:

- The agent wins all three tricks, winning 3 chips.
- The agent wins two tricks, winning 2 chips.
- The agent wins one trick, winning 1 chip.

- The agent wins zero tricks, losing 3 chips.
- The agent elects to fold, winning no tricks, but losing no chips.

This can be seen as a set of options, namely [3 2 1 0 -3]. While it may seem tempting to output the result as one continuous output, there are two compelling reasons for breaking these up into binary outputs. The first of these is in order to optimise stability, as elaborated upon in Section 5. The second reason is that these are discrete events, and a continuous representation would cover the range of [-3 0], which does not in fact exist. The binary inputs then specified are:

- P(O = 3)
- P(O = 2)
- P(O = 1)
- P(O = -3)

With a low probability of all four catering to folding, winning and losing no chips. Consequently, the agent's predicted return is:

$$P = 3A + 2B + C - 3D \tag{6}$$

where

$$A = P(O = 3) \tag{7}$$

$$B = P(O = 2) \tag{8}$$

$$C = P(O = 1) \tag{9}$$

$$D = P(O = -3) \tag{10}$$

The internal structure of the neural network is consistent with that upon which the stability optimisation in Section 5 was done, using equation (5) for the hidden layer activation function. Since a high degree of freedom is required, a high number of hidden neurons is required, and thus fifty have been used. This number is iteratively achieved, trading off training speed versus performance. The output neurons are linear functions, since they represent not binary effects, but rather a continuous probability of particular binary outcomes.

8.2.3 Agent decision-making

With its own predictor specified, the agent is now equipped to make decisions when playing. These decisions are made by predicting the return of the resultant situation arising from each legal choice it can make. An ε-greedy policy is then used to determine whether the agent will choose the most promising option, or whether it will explore the result of the less appealing option. In this way, the agent will be able to trade off exploration versus exploitation.

9 The intelligent model

With each agent implemented as described above, and interacting with each other as specified in section eight, we can now perform the desired task, namely that of utilising a multi-agent model to analyse the given game, and develop strategies that may "solve" the game given differing circumstances.

9.1 Agent learning verification

In order for the model to have any validity, one must establish that the agents do indeed learn as they were designed to do. In order to verify the learning of the agents, a single intelligent agent was created, and placed at a table with three 'stupid' agents. These 'stupid' agents always stay in the game, and choose a random choice whenever called upon to make a decision. The results show quite conclusively that the intelligent agent soon learns to consistently outperform its opponents, as shown in Figure 4-12.

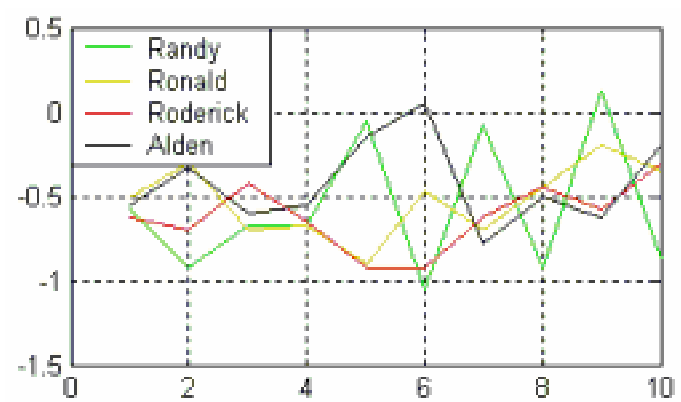


Figure 4-12. Agent performance, averaged over 40 hands

The agents named Randy, Roderick and Ronald use random decision-making, while AIden has the TD(λ) AI system implemented. The results have been averaged over 40 hands, in order to be more viewable, and to also allow for the random nature of cards being dealt. As can be seen, AIden is consistently performing better than its counterparts, and continues to learn the game as it plays.

9.1.1 Cowardice

In the learning phase of the abovementioned intelligent agent, an interesting and somewhat enlightening problem arises. When initially learning, the agent does not in fact continue to learn. Instead, the agent quickly determines that it is losing chips, and decides that it is better off not playing, and keeping its chips! This is illustrated in Figure 4-13.

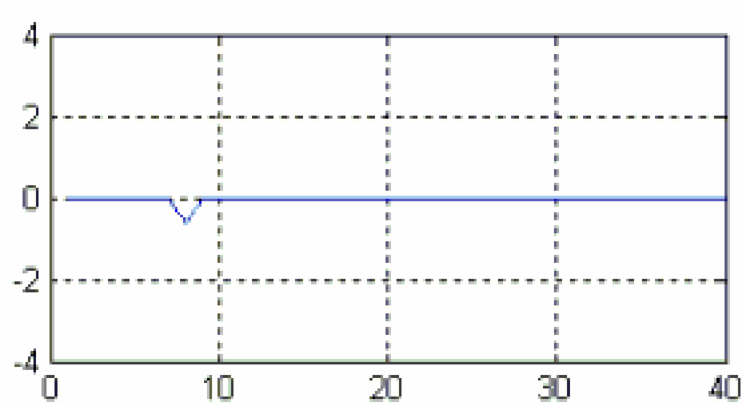


Figure 4-13. Agent cowardice. Averaged over 5 hands

As can be seen, AIden quickly decides that the risks are too great, and does not play in any hands initially. After forty hands, AIden decides to play a few hands, and when they go badly, gets scared off for good. This is a result of the penalising nature of the game, since bad play can easily mean one loses a full three chips, and since the surplus of lost chips is nor carried over in this simulation, a bad player loses chips regularly. While insightful, a cowardly agent is not of any particular use, and hence the agent must be given enough 'courage' to play, and hence learn the game. In order to do this, one option is to increase the value of ε for the ε-greedy policy, but this makes the agent far too much like a random player without any intelligence. A more successful, and sensible solution is to force the agent to play when it knows nothing, until such a stage as it seems prepared to play. This was done by forcing AIden to play the first 200 hands it had ever seen, and thereafter leave AIden to his own devices.

9.2 Parameter Optimisation

A number of parameters need to be optimised, in order to optimise the learning of the agents. These parameters are the learning-rate α, the memory parameter λ and the exploration parameter ε. The multi-agent system provides a perfect environment for this testing, since four different parameter combinations can be tested competitively . By setting different agents to different combinations, and allowing them to play against each other for an extended period of time (number of hands), one can iteratively find the parameter combinations that achieve the best results, and are hence the optimum learning parameters. Figure 4-14 shows the results of one such test, illustrating a definite 'winner', whose parameters were then used for the rest of the multi-agent modeling. It is also worth noting that as soon as the dominant agent begins to lose, it adapts its play to remain competitive with its less effective opponents. This is evidenced at points 10 and 30 on the graph (games number 300 and 900, since the graph is averaged over 30 hands) where one can see the dominant agent begin to lose, and then begins to perform well once again.

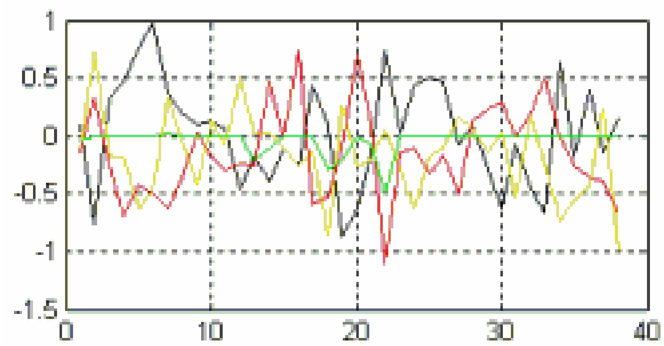


Figure 4-14. Competitive agent parameter optimisation. Averaged over 30 hands.

Surprisingly enough, the parameters that yielded the most competitive results were $\alpha = 0.1$; $\lambda = 0.1$ and $\varepsilon = 0.01$. while the ε value is not particularly surprising, the relatively low α and λ values are not exactly intuitive. What they amount to is a degree of temperance, since higher values would mean learning a large amount from any given hand, effectively over-reacting when they may have played well, and simply have fallen afoul of bad luck.

9.3 MAS learning patterns

With all of the agents learning in the same manner, it is noteworthy that the overall rewards they obtain are far better than those obtained by the random agents, and even by the intelligent agent that was playing against the random agents. A sample of these results is depicted in Figure 4-15.

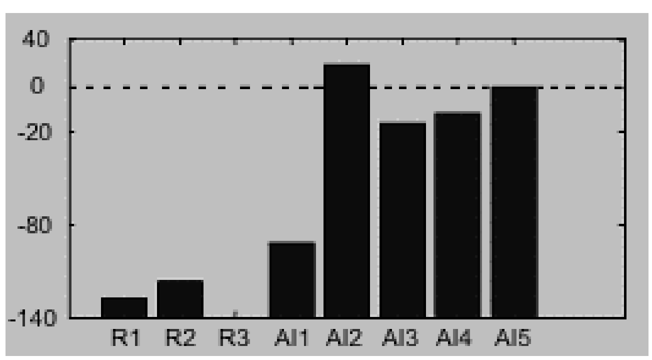


Figure 4-15. comparative returns over 200 hands.

R1 to R3 are the Random agents, while AI1 is the intelligent agent playing against the random agents. AI2 to AI5 depict intelligent agents playing against each other. As can be seen, the agents learn far better when playing against intelligent opponents, an attribute that is in fact mirrored in human competitive learning [29]. The agents with better experience tend to fold bad hands, and hence lose far fewer chips than the intelligent agent playing against unpredictable opponents.

9.4 Agent Adaptation

In order to ascertain whether the agents in fact adapt to each other or not, the agents were given pre-dealt hands, and required to play them against each other repeatedly. The results of one such experiment, illustrated in Figure 4-16, shows how an agent learns from its own mistake, and once certain of it, changes its play, adapting in order to gain a better return from the hand. The mistakes it sees are its low returns, returns of -3 to be precise. At one point, the winning player obviously decides to explore, giving some false hope to the losing agent, but then quickly continues to exploit his advantage. Eventually, at game #25, the losing agent gives up, adapting his play to suit the losing situation in which he finds himself. Figure 4-16 also illustrates the progression of the agents and the adaptation described.

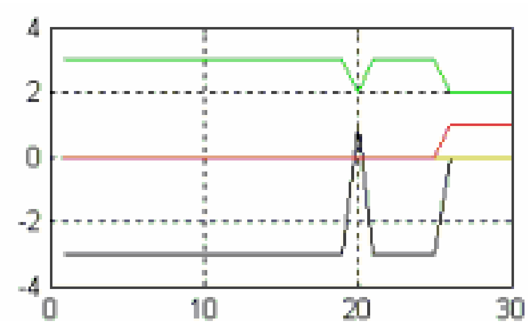


Figure 4-16. Adaptive agent behaviour

9.5 Strategy analysis

The agents have been shown to successfully learn to play the game, and to adapt to each other's play in order to maximise their own rewards. These agents form the pillars of the multi-agent model, which can now be used to analyse, and attempt to 'solve' the game. Since the game has a nontrivial degree of complexity, situations within the game are to be solved, considering each situation a sub-game of the overall game. The first, and most obvious type of analysis is a static analysis, in which all of the hands are pre-dealt. This system can be said to have stabilised when the results and the playout become constant, with all agents content to play the hand out in the same manner, each deciding that nothing better can be achieved. This is akin to Game Theory's "static equilibrium", as is evidenced in Figure 4-17.

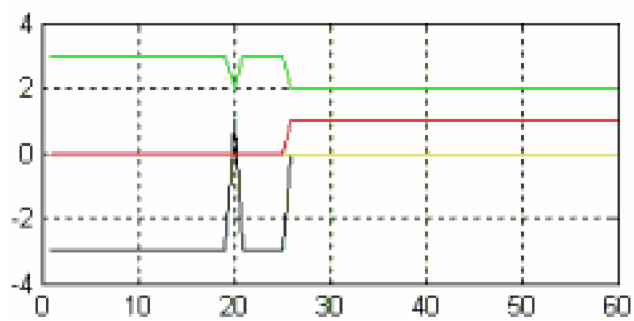


Figure 4-17. Stable, solved hand.

9.6 Bluffing

A bluff is an action, usually in the context of a card game, that misrepresents one's cards with the intent of causing one's opponents to drop theirs (i.e. to fold their hand). There are two opposing schools of thought regarding bluffing [30]. One school claims that bluffing is purely psychological, while the other maintains that a bluff is a purely statistical act, and therefore no less sensible than any other strategy [30]. Astoundingly enough, the intelligent agents do in fact learn to bluff! A classic example is illustrated in Figure 4-18, which depicts a hand in which bluffing was evidenced.

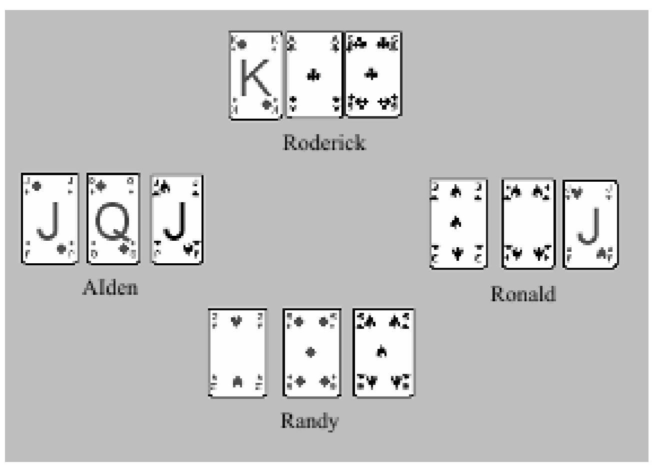


Figure 4-18. Agent bluffing

In the above hand, Randy is the first caller, and diamonds have been declared trumps. Randy's hand is not particularly impressive, having only one low trump, and two low supporting cards. Still, he has the lead, and a trump could become a trick, although his risks are high for minimal reward. Nonetheless, Randy chooses to play this hand. Ronald, having nothing to speak of, unsurprisingly folds. Roderick, on the other hand, has a very good hand. One high trump, and an outside ace. However, with one still to call, and Randy already representing a strong hand by playing, Roderick chooses to fold. AIden, whose hand is very strong with two high trumps and an outside jack, plays the hand. When the hand is played repeatedly, Randy eventually chooses not to play, since he loses all three to AIden. Instantly, Roderick chooses to play the hand, indicating that the bluff was successful, that it chased a player out of the hand! Depending on which of the schools of thought regarding bluffing one follows this astonishing result leads us to one of two possible conclusions. If, like the author, one maintains that bluffing is simply playing the odds, making the odds for one's opponent unfavourable by representing a strong hand, then this result shows that the agents learn each other's patterns well enough to factor their opponent's strategies into the game evaluation, something Game Theory does a very poor job of. Should one follow the theory that bluffing is purely psychological, then the only conclusion that can be reached from this result is that the agents have in fact developed their own 'psyches', their own personalities which can then be exploited. Regardless of which option tickles the reader, the fact remains that agents have been shown to learn, on their own and without external prompting, to bluff!

9.7 Deeper strategy analysis

While the strategy analysis already presented is useful, it is not truly practical, since for many applications one may have access to no more information than a single agent has. For such situations, one can perform a different simulation, pre-dealing only a single agent's hand. In this way, the agent will be testing its strategy against a dynamic opposition, being forced to value its hand on its own merit and not based on static results. Once the agent performs constantly with its hand, the resulting strategy will be the dominant strategy. To be said to be constantly performing, the agent should average zero or greater than zero returns when playing out according to its arrived at strategy, since negative returns would indicate playing with a weak hand that should be folded. In this way, one can determine the correct play with any given hand, effectively 'solving' the game.

9.8 Personality profiling

While Game Theory stagnates in the assumption that all players are rational, there is no such limit strangling a multi-agent model using intelligent agents. While this may not seem important on the surface, it does in fact extend the usefulness of the model far beyond that of standard Game Theory. Many poker players of reasonable skill complain bitterly about playing against beginners, bemoaning the fact that the beginners do not play as they should, as a "rational" player would, and thus throw the better players off their game. A good player, however, should not be limited to assuming rationality from his opponents, but rather should identify his opponents' characteristics and exploit their weaknesses. In order to perform this task, one needs to create "personality" types in agents, this while sounding somewhat daunting, is in fact a rather simple task. All that is required is to modify the reward function for an agent (equation #6) to reflect the personality type to be modeled. Two personality types were created, namely an *aggressive* personality and a *conservative* personality. The aggressive agent uses the reward function:

$$P = 3A + 2B + C - 2D \tag{11}$$

While the conservative agent uses the reward function:

$$P = 3A + 2B + C - 4D \tag{12}$$

Where the symbols have the following meanings:

- P denotes the expected outcome of the hand
- A denotes the probability of winning three tricks.
- B denotes the probability of winning two tricks.
- C denotes the probability of winning one trick.
- D denotes the probability of winning zero tricks.

As can be seen, in this case both agents modify the coefficient of the D term in order to skew their world view. This is certainly not the only manner in which the reward function can be modified in order to reflect a personality, but is the most obvious, since the D term represents the "risk" that the agent sees within a hand. Using the above modifications, the previously detailed strategy analysis techniques can be performed on agents with distinctive personalities. The static analysis of comes to the same results as with rational players, due to the unchanging nature of the problem, while the dynamic analysis yields more interesting results. Dynamic strategy analysis finds different dominant strategies when playing against these profiled personalities. More specifically, the aggressive player is not considered as dangerous when playing in a hand, while the conservative player is treated with the utmost of respect, since he only plays the best of hands.

10 Conclusions

Mutli-agent modeling using intelligent agents allows one to analyse and solve games that traditional game theory struggles to handle. By utilising reinforcement learning, and the TD(λ) algorithm in particular, adaptive, intelligent agents can be created to interact within the multi-agent system. While these agents will learn against both intelligent and non-intelligent agents, they learn far faster against better, more intelligent agents, achieving a higher standard of play in a shorter period of time. They also continually adapt to each other's play styles, finding dynamic equilibria within the game. The system can be used to determine "best play" strategy for any given hand, in any specific scenario. Moreover, bluffing in the game was shown to be in all likelihood a natural strategic development, an act of "playing the odds", rather than the traditional view of being a psychological play. Through modifying the reward signals received by agents, it was also shown that personality types can be modeled, allowing for hands to be "solved" taking into account non-rational opponents, a feature sorely lacking in traditional game-theory. Finally, input sensitivity analysis was found to be unsuccessful in analysing the decision-making process of the agents, by reason of the encoding scheme.

For future work, the author recommends the following avenues of research: alternate learning algorithms, possibly with better learning characteristics for nonlinear function approximators; and finding an alternate method of examining the decision-making process of the agents.

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Chapter 5: Conclusions and Further Work

In conclusion, It was found that the TD(λ) algorithm can be successfully used to train a neural network, as long as steps are taken to minimise the size of the error function, in turn minimising the probability of instability marring the training cycle. The Methodology for stabilising this training method hinges on the following critical steps :

1. Keep the inputs and outputs in a binary representation, so as to reduce the error values that can occur, preferably keeping the values between 1 and -1.
2. Utilise activation functions in both the hidden and output layers that will typically keep the error function low, such as sigmoidal activation functions. Avoid the common linear output activation function, as this can easily lead to large errors and hence instability.
3. Keep the learning rate α low, so as to avoid large changes in values that can once again lead to instability. This also falls in line with common practice to avoid overfitting, although the trade-off of this is a relatively slow learning rate. Individual problems will allow differing leeway in this regard, meaning one should experiment in order to find an optimal learning rate. (In the tic-tac-toe example, for instance, a learning rate of 0.1 was initially required to train a new network. Once the network had built up a knowledge base, however, this could be increased to 0.3 for faster learning.)

The use of reinforcement learning paradigm allowed for agents with absolutely no prior knowledge to learn meaningfully about the game, and to achieve a reasonable level of skill.

The above methodology was successfully applied to create card-playing agents, which were shown to be capable of learning from their experiences and their opponents. Careful consideration of the input structures allowed the agents to have complete information (insomuch as the game allows any one player to know) while keeping the dimensionality of the system to a minimum, allowing for swifter training cycles. So successful was the implementation, that agents in fact learned to bluff, based purely on the observations they made of their opponents playing styles. It was also noted that agents tend to learn far quicker, and achieve a much better level of play, if they play against better opponents.

Lastly, the agents then were used to model the actual game, ‘solving’ individual situations. It was shown that for varying levels of knowledge, the system can be used to determine the correct play in any given situation. This was tested for varying levels of knowledge, these being :

1. Hand known, play-styles known, and a static situation (cards never change in any hands)
2. Hand known, play-styles known, dynamic situation (opponents cards continually re-dealt, so what is the correct play based on your hand ?)

3. Hand known, play-styles known, illogical opponents. (opponents have *personalities*, rating certain game artifacts higher than their true mathematical worth. Example being a cautious player, who would rate the loss effect as -4 rather than -3, minimising the play-style of a person that becomes scared off when he loses.)

With these successfully implemented, the intelligent multi-agent model has been shown to be an effective modelling tool, useful in situations too complex to be meaningfully analysed using traditional game-theory.

For future work, research into a more robust training algorithm would be useful, as it would allow the engineer to push the limits of the agent's learning capabilities further. Research can also be done to create a unified strategy based upon the finding of the model in its various different situations. Lastly, research along similar lines into a more well-established game, such as no-limit hold 'em Poker, could allow the model to be evaluated against traditional theory, and tested upon a pool of more well-established players.

Appendix A – Game Theory example

A classic game-theory problem is that of the “prisoner’s dilemma”. In this problem, two criminals have been caught, and are presented with individual choices: They can confess, or remain silent. Should one confess and the other remain silent, the confessor will go free, and the other get ten years imprisonment. Should both confess, they will each get 5 years. If neither confesses, they will get off with a light sentence of 1 year each. Obviously, each makes his decision independently of the other, without knowledge of the other’s decision.

Using standard game-theory methodology, we construct a payoff-table to represent the outcomes of this game.

	Confess	Refuse
Confess	5,5	10,0
Refuse	0,10	1,1

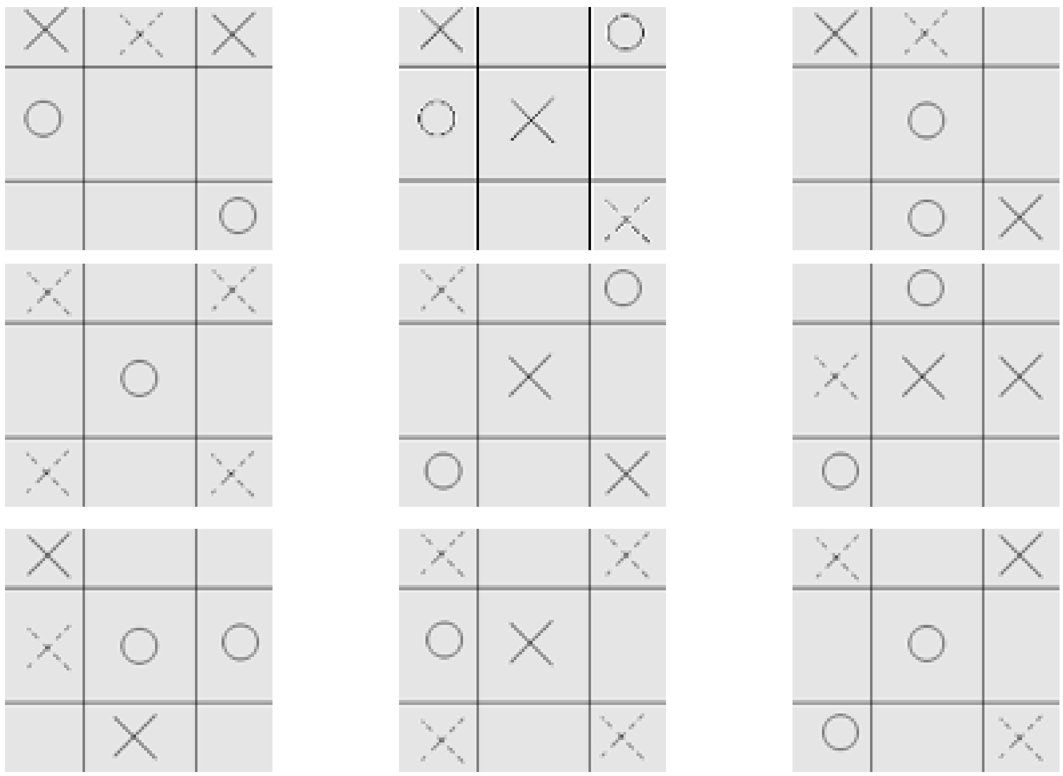
To solve the game, one evaluates the options for any given strategy. Should one player choose to confess, he will get 5 years if his friend confesses, and zero if he does not. Similarly, should he choose to Refuse, he will get 10 years if his buddy confesses, and one year if he refuses. Clearly, Confessing gives a better result regardless of his buddy’s decision, and he confesses. Using similar logic, so does his partner, and they both end up with five years, when they could have got away with one each if they had both refused.

This example illustrates the concept of a *dominant strategy*, and also illustrates one of the intrinsic weaknesses of traditional game-theory. The fact that prisoner A knows full well that the other prisoner is likely to choose to confess means that he can in fact throw more weight behind the option of him confessing, and build that into his model.

Game theory’s exclusion of player’s is the primary weakness that is addressed by the use of multi-agent modeling for solving games.

Appendix B – Tic-Tac-Toe Player Evaluation

The following positions are used to evaluate the performance of the Tic-Tac-Toe players, since they involve varying levels of difficulty, although all have definite "correct" answers. Where appropriate, multiple solutions are offered, since there is more than one correct answer to that specific problem. The correct move is represented by the Dashed cross(es).



By comparing the suggested move with the correct move, one arrives a score to objectively evaluate the playing strength of an agent.

Appendix C – Dimensionality stats

The first action taken to reduce the dimensionality of the system was to order the cards consistently, effectively reducing the number of possibilities from *permutations* to *combinations*.

A *Combination* (${}^{n}C_{r}$) can be evaluated as $\frac{n!}{(n-r)!\,r!}$, while *Permutations* can be represented as $\frac{n!}{(n-r)!}$ for integer values of n and r.

Now let P = ${}^{n}P_{r}$ and C = ${}^{n}C_{r}$

To change between C and P, let P = Cx, where x is the factor by which C is smaller than P. Therefore:

$$x = P / C$$

$$x = \frac{n!}{(n-r)!} \bullet \frac{(n-r)! \bullet r!}{n!}$$

$$x = r!$$

Therefore, by changing between Combinations and Permutations, one reduces the dimensionality of the system by a factor of r!. Since three cards are being recorded in hand, this means a reduction of 3!, which is a reduction by a factor of 6.

The next reduction in the player's hand comes in the number of suits, which is at first glance counter-intuitive. Normally, 4 suits are represented, but for this encoding scheme 5 suits are used. (Trumps, Repeated suit, highest Singleton, second-highest Singleton, Third-highest singleton). This, however, actually reduces the dimensionality of suit representation quite significantly. The reduction is best viewed in case-by-case scenarios, represented in the following table, which assumes 10 possible cards within each suit. (Key: T = Trumps, R = Repeated, S = Singleton)

Scenario	3T	2T + 1S	1T+2S	1T + 2R
Combinations	720	900	1000	900
Scenario	3S	2R + 1S	3R	
Combinations	1000	900	1000	

While the traditional, sorted representation would utilise following formula, of:
(trumpsuit)*(suit)*(number)*(suit)*(number)*(suit)*(number)
(4)(4)(10)(4)(10)(4)(10) = 256 000.

The total number of options available using the tabulated method, however, is only 6420.

This value is approximately one fortieth the size if the original card dimensions, representing a significant reduction in dimensionality as a result of the encoding system, as a result of removing hand reproduction.

Appendix D – Software Code and toolboxes

1. Introduction

This document covers the software environments and toolboxes used throughout the project described within the dissertation. The software design for the projects is detailed, covering the software necessary for both the tic-tac-toe analysis, the lerpa playing and the game viewing necessary for analysis purposes. The toolboxes utilised for the neural network training will also be discussed.

2. Basic software environment

All of the software was developed for use within the MATLAB software environment. The advantage of utilising MATLAB is its strong mathematical functionality, which proves immensely valuable when implementing and evaluating neural network systems, and in utilising the TD(λ) algorithm, used for adaptively training a neural network. In order to evaluate the played Lerpa hands, the Matlab GUIDE (GUI Development Environment) was utilised, chosen for it's optimised compatibility with Matlab m-files.

3. Toolboxes and modifications

In order to implement neural networks, the Netlab toolbox by Ian Nabney was utilised. This toolbox allows for read-to-go implementation of neural networks using standard gradient-based training algorithms, such as conjugate gradient and quasi-Newton optimisation. The modularity of the toolbox makes it simple to replace these algorithms with others, making it highly suitable for research purposes.

In order to utilise the TD(λ) algorithm to train the neural networks, the TD(λ) algorithm was manually programmed into its own m-file. This file was then used to update the neural network weights, rather than the standard training algorithms provided by the toolbox.

4. Tic-Tac-Toe implementation

The game of Tic-Tac-Toe is very simple to adjudicate, and as such the complexity of the program lies not in the evaluation of the game itself, but rather in the flow of information within the program, since training is implemented online within the reinforcement learning framework. The program action phases are as follows:

- Initiate board
- Request move from player
- Player evaluates moves
- Update board (with new move)

- Evaluate board position (win/loss/draw/unfinished)
- Repeat until finished

Diagrammatically, this is illustrated below in Figure D-1.

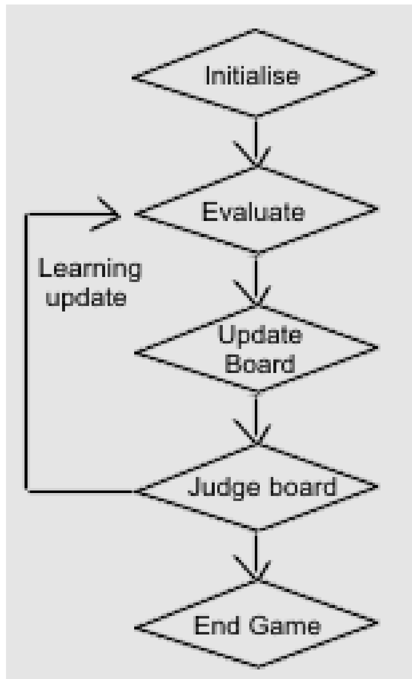


Figure D-1. Tic-Tac-Toe software design overview

External to this system is the judging function, which evaluates the strength of a playing agent after the games have been played. The use of modular functions allowed for easier evaluation of individual functions, as well as code re-use. They also allow for successful porting of the neural network updating function (using TD(λ)) from the Tic-Tac-Toe problem to the Lerpa system.

5. Lerpa implementation

A similar software system was designed for the Lerpa playout/learning as to the Tic-Tac-Toe system. The only added complication is that there are not four players, rather than only one. By keeping each function modular, the Lerpa system was made research-friendly, since a scenario can be created, and then inserted into the system at any given point in the game's cycle. Also necessary is to store the games played, in order to be able to view the plays for analysis purposes, since the game is too complex for a simple, objective evaluation such as the one utilised in the Tic-Tac-Toe problem.

Lerpa playback GUI

In order to evaluate the Lerpa-playing agents, software needed to be written to view the saved games. Since many games are played during a training run, the playback GUI needed to be able to read the many games available in a given save-file, and play back specific games either step-by-step, or continuously for easier viewing. The GUI was created using the Matlab GUIDE, which makes for easy porting of Matlab m-files into the functions of GUI. The GUI screen is illustrated in Figure D-2.

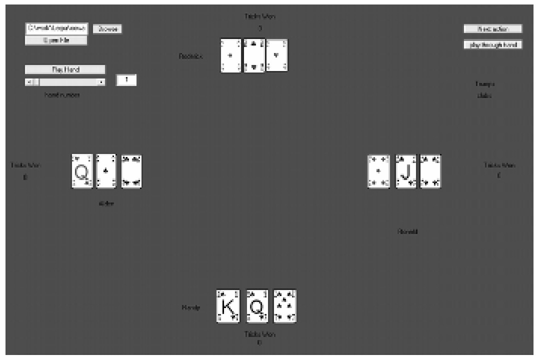


Figure D-2. Playback GUI.

The GUI allows for opening saved game files, and allows the user to choose a game to view, either by entering a number or by using the slider bar to select a game number. Once selected, the game can either be played step-by-step, using the “next action” button in the top right, or played back continuously, which plays each action with a one-second delay before the next in order to give the illusion of flowing gameplay. When played, cards are moved from the relevant player’s hand into the middle, clearly showing the playout.

The relevant player details, as well as the trump suit flipped, are shown in various text boxes around the table, indicating such details as the number of tricks won, the player’s name, and whether a player knocks or folds. In this manner, the Lerpa multi-agent system was displayed, and hence analysed.

6. Conclusions

The use of the Matlab environment allowed for easy implementation of the mathematical formulae necessary in order to implement the TD(λ) algorithm to train the neural networks. By utilising the Netlab toolbox to create neural networks, the TD(λ) algorithm could be inserted easily into the neural network training system due to the modular nature of the toolbox. By continuing to make all of the functions modular, the software was optimised for the research environment, allowing the engineer to insert scenarios into any stage of both the Tic-Tac-Toe and the Lerpa simulations. Finally, the use of the Matlab GUIDE allowed for subjective viewing of the Lerpa games played, easily incorporating the matlab m-files into the GUI.