PRIMARY MATHEMATICS IN-SERVICE TEACHING DEVELOPMENT: ELABORATING 'IN-THE-MOMENT'

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy

By
Lawan Abdulhamid
Student number: 675644

Supervisor:
Prof Hamsa Venkatakrishnan

May, 2016

ABSTRACT

This study investigates how primary school mathematics in-service teachers respond to learners' offers, over time, during classroom interactions. The study was a follow-up to a one-year long in-service 'maths for teaching' professional development course in which 33 teachers participated in 2012. Four teachers from that course were tracked in this follow-up study. Data sources within this study consisted of two cycles of observations of lessons taught by the four teachers in 2013 and 2014, and an interim video-stimulated recall (VSR) interview with each teacher, with reflections guided by the structure of Rowland et al.'s 'knowledge quartet'. A total of 18 lessons from the four teachers were video-recorded across the 2013 and 2014 observations. The notion of 'elaboration' was used in this study as an interpretive lens to examine and characterise responsive teaching actions in the South African context, with the focus narrowing over the course of the PhD to contingency situations within the knowledge quartet framework, focused on responses to learner offers. In the South African literature, the terrain of elaboration is characterised by extensive gaps in teachers' mathematical knowledge, incoherent talk, and frequent lack of evaluation of learners' offers in the classroom.

Using a grounded theory approach, I propose an 'elaboration' framework with three situations of responsive teaching (breakdown, sophistication and individuation/ collectivisation), which can be used as a tool to support the development of more responsive teaching in the South African context (and perhaps in other contexts where similar problems prevail). In this way, the study has contributed in terms of identifying some important 'stages of implementation' (Schweisfurth, 2011) that might be required to move towards the ideals of more responsive teaching that are described in the international literature, and yet remain distant from the realities of South African schooling.

Using the three markers of shifts (*extent, breadth* and *quality*) in elaboration recruited in this study, drawn from the ways in which the dimensions of responsive teaching were conceptualised, I report on the different patterns of shifts in elaboration by the four teachers. The results of this analysis indicated that all four teachers made shifts in their responses to learners' offers from 2013 to 2014 lessons in at least one or more dimensions of responsive teaching, in relation to *extent*, *breadth* and *quality* of elaborations. Findings from VSR interviews indicated associations between shifts in teachers' reflective awareness, and shifts in responsive teaching actions. Theoretically, the study contributes through characterising responsive teaching actions in contexts of evidence of limited evaluation within the elaboration framework, with a language of description for identifying and developing more responsive teaching actions in a resource constrained context.

Keywords: Responsive teaching, contingent moments, classroom interactions, primary mathematics, in-service mathematics teachers, stages of implementation, elaboration, breakdown, sophistication, individuation, collectivisation.

DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Lawan Abdulhamid

30th day of May in the year 2016

DEDICATION

To the Almighty Allah who grants knowledge and wisdom to mankind

PUBLICATIONS EMANATING FROM THIS RESEARCH

Book chapter

Abdulhamid, L. (in press). Characterizing responsive primary mathematics teaching in South African contexts. In M. Graven and H. Venkat (Eds). Improving primary mathematics education, teaching and learning: Research for development in resource constrained contexts. Basingstoke, UK: Palgrave

Journal Article

Abdulhamid, L. & Venkat, H. (2014). Research-led development of primary school teachers' mathematical knowledge for teaching: A case study. *Education as Change*, 18(1), 137-150, DOI: 10.1080/16823206.2013.877355

Conference papers

Abdulhamid, L. (forthcoming). Supporting 'elaboration' in primary school teachers' handling of incorrect answers in mathematics classrooms. Oral presentation: 13th International Congress on Mathematical Education 2016 (ICME-13), University of Hamburg, Germany.

Abdulhamid, L., & Venkat, H. (2013). Using the 'knowledge quartet' to analyse primary mathematics teaching in South Africa: the case of Sibongile. In Davis, Z. & Jaffer, S. (Eds.), *Proceedings of the 19th Annual Congress of the Association for Mathematics Education of South Africa* (Vol. 1, pp. 47 – 58). Cape Town: AMESA

ACKNOWLEDGEMENTS

I would like to thank Allah for giving me the opportunity to complete this PhD thesis. It is by His grace that this PhD has been completed. I wish to thank most deeply my supervisor, Professor Hamsa Venkatakrishnan, who has provided, without hesitation, and with unflagging support, constructive comments, suggestions and criticism, all of which provided a strong backing for this research work. I would like to thank her for her support and the intellectual energy that she gave to this work and for her ability to listen with care and critique with rigour. I thank you for providing me with an enabling environment to learn from you, and also for believing that I could successfully complete this research.

I want to thank my colleagues at the Wits Maths Connect - Primary project for giving me space to work and also for their invaluable comments and feedback during our monthly buddy PhD students' meetings. My gratitude is also extended to Dr Femi Otulaja, Dr Tony Essein, Prof. Mike Askew and Prof. Ulla Runesson for their varied inputs through seminars, feedback and comments on this work.

To my family, I would like to say thank you for your unwavering patience and support through this humbling journey. To my wife Maryam, I want to say I can never repay you for your perseverance and encouragement when it seemed to me that this research was never coming to a conclusion. To my daughters, Amina, Maryam, Fatima, Hauwa, and the PhD baby (Aminat). I believe I have inspired you to learn and to appreciate that learning has a beginning but no end. Finally, my profound gratitude goes to the principals, learners and the four teachers who consented to participate in this research, with some of them now progressing into their own postgraduate studies.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ENL, Empty Number Line

EP, Elaboration Provided

ENP, Elaboration Not Provided

KQ, Knowledge Quartet

MKfT, Mathematical Knowledge for Teaching

MKiT, Mathematical Knowledge in Teaching

PCK, Pedagogical Content Knowledge

SMK, Subject Matter Knowledge

VSR, Video-stimulated recall

WMC-P, Wits Maths Connect – Primary

CHAPTER 1

INTRODUCTION

1.1 Background to the Study

The focus of this study is on the in-depth exploration of changes in teachers' responses to contingent situations in mathematics teaching as observed over time. The participants were four South African primary teachers of mathematics. All four teachers participated in an in-service primary mathematics teacher development course focused on supporting and developing primary mathematics knowledge for teaching. However, the course itself and learning, in knowledge *per se* terms, from the course is not the primary focus of this study. Rather, the interest of the study is in tracing development in these teachers' 'ways of *being* with mathematics knowledge' in classrooms, borrowing a term used by Davis and Renert (2014). These authors elaborate their practice-based orientation to mathematical knowledge in the following terms:

> M4T [Teachers' disciplinary knowledge of mathematics] is a way of being with mathematics knowledge that enables a teacher to structure learning situations, interpret student actions mindfully, and respond flexibly, in ways that enable learners to extend understandings and expand the range of their interpretive possibilities through access to powerful connections and appropriate practice (P. 12)

Davis and Renert's empirical base is located in teachers' 'in-the-moment' decisions, with particular attention in the quote above to teachers' ways of interpreting and responding to learners' inputs. Building powerful connections, expanding and extending current understandings in classroom action have all been described in the teacher knowledge literature base as markers of strong disciplinary knowledge (see Ball, Thames, & Phelps, 2008). The problems that arise during mathematics teaching have commonly been described as relating to issues of dealing with events that were unanticipated in the teacher's thinking about lessons. Addressing these issues adequately is challenging because solutions often need to be constructed immediately, in the classroom in front of the learners (Lampert, 2001). Successful teachers have the capacity to both apply and develop their knowledge base in the context of teaching. This development of the knowledge base is particularly important as Chick (2011) has noted that teacher preparation is inevitably incomplete and in-service professional training can never cover all of the issues a teacher may encounter in their actual teaching.

My empirical focus in this study is therefore within primary mathematics lessons, exploring teachers' interpretations and responses to learners' offers over time, with interim professional development of teachers involving video-stimulated recall (VSR) interviews. VSR interviews incorporate opportunities for teachers to view and review video recordings of their own teaching in order to reflect on their practice. This approach was found to be effective in getting insight into teachers' thoughts and reflections on their own practice (Muir & Beswick, 2007), and has been used as an effective medium for promoting teacher professional learning in mathematics classrooms (Geiger, Muir, & Lamb, 2015).

Situations involving teachers' interpretations and responses to learners' offers carry strong links to the focus on 'contingency' within Rowland, Huckstep and Thwaites' (2005) 'Knowledge Quartet' (KQ). The KQ is an empirically-based conceptual framework for classifying situations in which teachers' subject matter knowledge (SMK) and pedagogical content knowledge (PCK) come into play in teaching practice in mathematics classrooms. The framework describes the interactions of four different categories of knowledge as observed in teaching practice. These are foundation, transformation, connection and contingency knowledge. While the first three of these categories are seen in situations that can reflect prior thinking about the mathematical content of lessons, contingency knowledge - according to Rowland and Zazkis (2013) - is "witnessed in teachers' responses to classroom events that were not anticipated or planned, usually triggered by an answer or a remark contributed by a student" (p. 3).

As noted already, all four teachers had attended an in-service primary 'maths for teaching' course, which focused primarily on the three 'planning-oriented' dimensions of the knowledge quartet: foundation, transformation and connection knowledge. The course's focus on these three dimensions of the KQ was driven by evidence of mathematics content knowledge gaps in the in-service course pre-test; which used items from a range of previous national and international studies including Hart et al.'s (1981) CSMS studies and Ryan and McCrae (2006) TEMT studies. These gaps aligned with the broader evidence in the South African literature on primary mathematics teachers' content knowledge (Carnoy, Chisholm, & Chilisa, 2012; National Education Evaluation & Development Unit [NEEDU], 2013;

Taylor, 2011; Venkat & Spaull, 2015). There was also evidence in the South African literature of gaps relating to primary teachers' pedagogy in the mathematics classroom. This included findings of disconnections in teaching sequences (Venkat & Naidoo, 2012), and lack of progression in teaching from more concrete to more abstract ways of working with numbers and operations (Ensor et al., 2009). Connection and progression of mathematical ideas feature prominently within the foundation, transformation and connection categories of the KQ (which has been used as a development tool), and therefore, there were strong empirical rationales for attending to these aspects in the course.

Contingency situations are more sophisticated as they require teachers to draw upon these three dimensions when needed to respond to unplanned or unexpected learners offers in the classroom. The analysis in this study narrowed during its process from the KQ's full breadth of categories to focus specifically on the nature of teacher's responses in handling these contingent moments. The aim of this study is to understand the dynamics of teacher-learners' interactions in the primary mathematics classroom, and to explore the possibilities for learning extensions and expansions, over time, in teachers' ways of handling these 'in-the-moment' interactions, through work that followed up an in-service teacher development course that focused more on the foundation, transformation and connection dimensions of the KQ.

The motivation for focusing specifically on moments of contingency emanates, on one hand, from evidence of the frequent absence of evaluation criteria in South African primary mathematics teaching in working-class schools (Hoadley, 2005). This evidence indicates, simply, a lack of any evaluative response to learners' offers, leaving children with no idea of the correctness, accuracy, efficiency, or validity of their offers. On the other hand, there are also motivations drawn from the international literature base on primary teachers' mathematical knowledge that point to responsive teaching as a key marker of pedagogic practices that are supportive of learning (Coles & Scott, 2015; Mason, 2015; Mason & Davis, 2013; Rowland, Thwaites, & Jared, 2015; Rowland & Zazkis, 2013) .

In this literature base there are assumptions about the nature of teaching as fundamentally improvisational if it is to be responsive to emergent learning, thus, pointing towards the importance of contingency knowledge and elaboration. I introduce these assumptions in the next section, before detailing the context of the study – a follow-up exploration of four teachers who participated in the first pilot of a primary 'mathematics knowledge for teaching' course in 2012. The structure of the study is detailed in the concluding sections of this chapter.

1.2 Rationales for the study

1.2.1 Looking at contingency: the South African rationale

In the South African context, concerns about low learner performance in mathematics at all levels have led to increasing attention to the nature of teachers' mathematics knowledge and pedagogy in mathematics classrooms. Gaps in the mathematical knowledge base of primary school teachers in South Africa are frequently reported (National Education Evaluation & Development Unit [NEEDU], 2013; Taylor & Taylor, 2013; Venkat & Spaull, 2015), with small-scale studies revealing incidences of limited opportunities for learners to understand mathematics in coherent ways (Venkat & Naidoo, 2012). Limited understanding of progression has also been pointed to in studies noting the ongoing use, and sometimes a 'pulling back', into concrete counting approaches to working with number instead of moving forward into more efficient, abstract strategies (Ensor et al., 2009).

Classroom evaluation practices in primary schools in South Africa provide particularly fertile ground for examining the nature of mathematical knowledge for teaching. Hoadley's (2006) study, driven by sociological concerns about differential access to knowledge for poorer and wealthier children, noted the prevalence, in working class schools, of teaching characterised by an absence of evaluative criteria (Hoadley, 2006). Hoadley described this practice in the following terms:

> The teacher engages in other work in her space and is not seen to look at what the learners are doing. She makes no comment on the work as it proceeds. No action is taken to ascertain what the learners are doing (p. 23)

The consequence of this practice is a situation in which learners may well remain unaware of the extent to which their offers and narratives are 'endorsable' from a mathematical perspective. Importantly, Hoadley noted that this absence of evaluative criteria represents a feature that has not been described as common in the developed country contexts in which the theoretical notions of evaluative criteria were initially developed. In these developed contexts, attention has been given to weaker and stronger framing of evaluative criteria, rather than an absence of evaluation (Bernstein, 1990). This particularity leads to a motivation for studying primary mathematics teaching development in relation to the kinds of 'in-the-moment' responses provided by teachers.

Broader issues and policies in the South African terrain also feed into the ways in which teacher responses are configured. Highly procedural orientations (Ally & Christiansen, 2013) coupled with selections of low cognitive demand tasks have been noted (Carnoy et al., 2012). Chorusing practices, involving collective chanting of answers have been raised as concerns in relation to the lack of openings for individuation of learning and evaluation thereof (Hoadley, 2012). Conversely, Venkat & Naidoo (2012) also point to a lack of move of individual offers into the collective classroom space (collectivising) in primary mathematics teaching.

Concerns about curriculum coverage and pacing in primary mathematics pedagogy (Reeves & Muller, 2005) also led to calls for, and subsequently, moves towards, much more tightly prescribed national curriculum specifications. Thus, currently, national mathematics curricula specify content coverage, sequencing and pacing at weekly levels (DoBE, 2011); with provincial-level interventions providing teachers with scripted lessons at the daily level (GDE, 2011). The press for coverage and standardized pacing further tend to work against openings for more responsive teaching.

1.2.2 Looking at contingency: What international literature suggests

In the mathematics education literature, mathematics knowledge in teaching – whether subject matter knowledge, curriculum knowledge or pedagogical content knowledge (Shulman, 1987) are often classified into two broad categories: (i) knowledge possessed, focusing on an abstract theoretical body of knowledge or as tacit craft knowledge held by teachers; and (ii) knowledge of process (the know-how), focusing on enactment of this theoretical information in the actual teaching practice. Within the second category, there is a body of recent international research writing (see Clark-Wilson & Noss, 2015; Coles & Scott, 2015; Mason, 2015; Rowland et al., 2015; Rowland & Zazkis, 2013) that testifies to specific and on-going interest in the ways in which teachers' mathematical knowledge is brought into play in the context of response to in-the-moment classroom events that were not anticipated or planned.

Empirically important in the distinction between the two categories is that it is one thing for a teacher of mathematics to 'have' the underlying knowledge of mathematics and its pedagogy, but quite another thing to draw upon this knowledge in teaching when needed. Mason and Davis (2013) noted this distinction in their work in a project that required teachers to teach a modelled lesson that focused on listening to and engaging with learners' mathematical ideas. They illustrate how one of the teacher-participants became frustrated when her students did not generate the same insights that had arisen in the model lesson, and in this contingent situation, then felt compelled to resort to a traditional teaching by telling and explaining. Mason and Davis noted that the issue here was not about an insufficient mathematical knowledge base to follow the modelled lesson plan, but rather, a lack of awareness of how to respond flexibly to 'in-the-moment' situations:

> …it became clear that all of the teacher-participants had more than sufficient disciplinary knowledge [knowledge of mathematics] to follow the trajectory of the lesson, so the issue was not any lack of understanding of the mathematics. They also understood the pedagogy: the intentions behind building up the example space, drawing out thoughts, bouncing back ideas, challenging interpretations, etc. They simply seemed to lack the vital connective tissue between *mathematical awareness* and *in-the-moment pedagogy*. It is one thing to notice an absence of something from a learner but quite another thing to have a sensible pedagogical action come to mind when needed (p.183).

The consequences of a lack of this connection between *mathematical awareness* and *in-the-moment pedagogy* were more limited opportunities for learners to understand the depth of the mathematical concept and important teaching points can be missed. In another similar example, Chick and Stacey (2013) narrate an account of a young primary mathematics teacher that they observed working with fractions in a Grade 5 class. The teacher had earlier built understanding of the meaning of fractions using discrete sets of objects and area models, and her preference for modelling addition of fractions was through using fraction strips. She asked learners to use fraction strips to work out ¼ + ¼. One boy claimed that $^2/_8$ was the answer. This boy avoided the fraction strips and had drawn two sets of four circles and shaded one circle in each set, and provided an explanation to support his claim as presented in Figure 1.

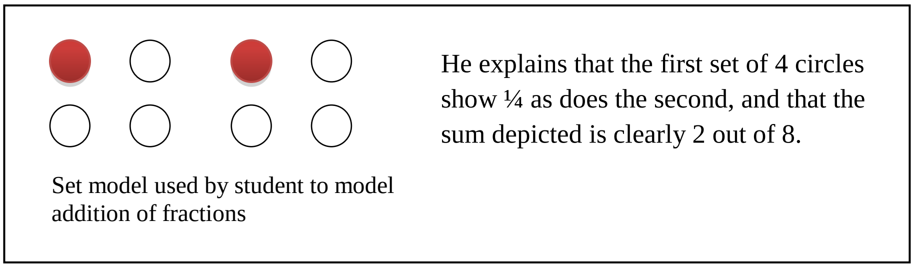


Figure 1: Chick and Stacey's (2013 P.124) example of contingent situation

They reported that the young teacher took a deep breath as she had not anticipated this learner offer. Chick and Stacey also explain that as a young teacher, her professional learning experiences had not to this point prepared her with an immediately applicable solution. More experienced teachers may have encountered similar arguments from students in the past, but for this teacher, the offer was new and she has no standard response to apply. Both of these instances occurred in-the-moment of enacting coherent and well-sequence planned lessons. Chick and Stacey noted that this young teacher knew how to explain fraction addition using the fraction strips, and her lesson was well sequenced until the moment of the unexpected. Despite a well-planned lesson then, the teacher was confused by this unexpected development.

Chick and Stacey's analysis of this incident was based on Rowland et al.'s KQ framework. They noted gaps in terms of the teacher's contingent knowledge and speculated what she might possible do mathematically and pedagogically to address this unexpected situation. Mathematically, they proposed that the teacher needed to recognise 'the role of the whole in fractions and appreciating that, for addition, both fractions are measured in relation to the *same* whole' (p. 125), which is contrary to the student's conception. Pedagogically, to meet her goal of assisting this boy's understanding, the authors suggested that the teacher should work within the context of the boy's own knowledge, behaviour, and attitudes, i.e. that she: 'must draw on her capacity to explain or give additional examples or counterexamples that might move the student beyond his current conception' (p. 125). The analysis of both of these examples of triggers of contingent situations works from a 'base' in which some evaluation of learner working is a given, with this feature forming a key contrast to the evidence seen in the South African literature.

Rowland et al. (2015) propose a classification of the origin of triggers of contingent classroom episodes with three categories. These are triggers: (i) arising from the students' ideas during the teaching/learning situation (ii) emanating from teacher insight through reflection on his/her own planned actions (i.e. teacher's in-the-moment evaluation of the lesson planning and development); and (iii) emanating from the pedagogical tools and resources that are brought to bear on the instruction, when the teacher is responding to the availability (or the unavailability) of resources. Due to the specificity of the South African context as discussed above, the present study is focused on the first trigger (arising from students' offers during classroom interactions).

In response to these triggers of contingency, Rowland et al. (2015) further propose three types of teacher responses – (i) to ignore; (ii) to acknowledge and put aside; and (iii) to acknowledge and incorporate. The present study uses the notion of elaboration to refine this typology of responses. This study therefore explores instances in which the teacher 'elaborates' in response to triggers of contingent situations in mathematics classroom as a metaphoric lens to examine responsive teaching. While 'elaboration' in the literature and in everyday usage can refer to providing a more detailed explanation without any reference to student responses, in this study, I worked with a more restricted notion of elaboration referring to teachers' responses to learner offers in contingent situations. These elaborations form the vehicles for developing responsive teaching actions, and also provide a means to bring into dialogue some of the ways of thinking about supporting the development of responsive teaching noted in the international literature as important with the specificities of the South African context.

1.3 Research questions

The main question that guided my study was:

What is the nature and extent of elaboration that teachers provide in responding to contingent moments triggered by learners' offers/contributions in primary mathematics classrooms, and how does this change over time in the context of follow-up support (involving video-stimulated recall interviews) to their participation in an in-service teacher professional development course?

In answering this research question, the following sub-questions – driven by key issues identified in primary mathematics teaching in South Africa that I have outlined in this chapter, linked to international literature - were interrogated:

1. What kinds of elaboration do teachers provide in handling incidents of incorrect mathematical offers from learners in the course of teaching?
2. What kinds of elaboration do teachers provide in response to what they view as learners' inefficient mathematical strategies or representations in the course of teaching?
3. To what extent do teachers pursue opportunities for collectivising individual offers and individuating collective offers? Collectivising translates correct mathematical offers from individual learners' into the broader classroom space; individuating translates whole class chant offers to particular learners. What modes of elaboration do teachers provide in such incidents?
4. What shifts, if any, are there over time in the kinds of elaboration identified above in the context of interim video-stimulated recall interviews?

1.4 Theoretical underpinning of the study

The focus of this study is on teachers' elaborations in response to 'in-the-moment' situations in primary mathematics classrooms. The position I take on this phenomenon is that these elaborations draw from two key bases: a psychological constructivist view of the individual cognizing teacher, drawing from an underpinning knowledge base; and an interactionist view on collective classroom practice (Bauersfeld, 1995) in which the teacher participates in and contributes to the development of collective processes through renegotiation of meaning. Cobb (1989) refers to the intersectionality of these two perspectives as an 'emergent approach', set within an interpretivist framework.

There have been calls for pedagogy to fit with learners' ways of working and learning of mathematics within an emergent approach (Bauersfeld, 1995; Cobb, Yackel, & Wood, 1993). For example, Cobb, Wood, and Yackel (1993) emphasize the need for research to unpack what it means to develop a teaching practice based on learners' ways of learning mathematics. This call suggests a teaching practice that is responsive to learning.

In response to this call, Simon (1995) advocates a theoretical model for reconstructing mathematics pedagogy within both sociological and cognitive constructivists' perspectives. Central to Simon's model is the "creative tension between the teacher's goals with regard to student learning and his responsibility to be sensitive and responsive to the mathematical thinking of the learners" (p.114). This viewpoint provides a useful insight into how quality teaching is viewed as improvisational with constant renegotiation of meaning in a social context in order to be responsive to emergent learning.

Following this line, Sawyer (2004) argues that effective teaching is fundamentally improvisational, because if the classroom is completely directed by the teacher, students cannot co-construct their own knowledge. Erickson (1982) has noted that "talk among teachers and students in … [the classroom] can be seen as collaborative improvisation of meaning and social organization from moment to moment" (p. 152). Given the multiplicity of prior experiences that characterise classroom contexts, such talk inevitably includes unpredictable events that unfold during the interaction. In this view, teaching as improvisational is conceived as creative, with teachers needing to be responsive to contingent situations in the classroom as teaching unfolds.

The emergence of unpredictable events during classroom interactions requires the teacher to quickly and improvisationally translate his or her own mathematical knowledge into a form that is responsive to the learner's level of knowledge. This kind of response is described by Mason and Spence (1999) as 'knowing-to-act in the moment' or the ability to think on one's feet as a reflective practitioner (Schön, 1987). Lampert and Ball (1999, p. 39) recommend that 'teachers be prepared for the unpredictable' because they will have to 'figure out what is right practice in the situation' and cannot entirely depend on the script or experts for what to do.

These arguments suggest that responding to learners' inputs by knowing how to act appropriately in-the-moment is a difficult task of teaching. It requires insightful and flexible responses from the teacher, and while planning and anticipating are important, it is not always possible to plan classroom responses in the context of unanticipated learners' offers. In this study, I view teacher responses to learners' offers as occurring through teacher elaboration of mathematical ideas in fundamentally creative ways.

1.5 Research design

This study was undertaken within a qualitative case study research design. The study took place as a follow-up to the 2012 year long pilot run of the in-service Wits Maths Connect-Primary (WMC-P) project's 'maths for teaching' course, in which 33 teachers, drawn from the project's ten partner primary schools in one district in South Africa, participated. The course assessments included: pre- and repeat post-tests on conceptual understanding of primary mathematics content and interim assessments on pedagogic content knowledge-related issues. The latter tasks were often framed in terms of hypothetical classroom scenarios and were designed to assess teachers' mathematical knowledge relating to connection between mathematical ideas, building mathematical progression responsively in teaching, and their access to a range of examples, explanations and representations for teaching.

Four teachers were purposively selected whose post-test performance indicated relatively strong foundation knowledge (60% and above), relatively strong in terms of their transformation and connection knowledge based on their performance during interim course assessments, and willingness to participate in this study. Data sources within this study consisted of two cycles of observations of lessons taught by the four selected teachers in 2013 and 2014, and an interim individual VSR interview with each teacher, with reflections guided by the structure of Rowland et al.'s 'knowledge quartet'. A total of 18 lessons from the four teachers were video-recorded across the 2013 and 2014 observations.

The central empirical base across the 18 lessons is focused on aspects related to additive relations. The choice of additive relation as the central mathematical content was driven by a number of key features: firstly, additive relations is an important and foundational aspect of primary mathematics curricula in South Africa and internationally; secondly, there is extensive writing in the field of mathematics education related to additive relations with particular emphasis on teaching and learning; and thirdly, a significant body of evidence in South Africa points to difficulties across learning and teaching related to this area.

In the international research base, as pointed out earlier, teachers' 'in-the-moment' responses to learners' offers tend to be analysed in relation to *what* opportunities for learning they open up, rather than initially, for *whether* learners' offers are acknowledged or evaluated at all. Thus, Ball, Hill, and Bass (2005) provide examples relating to sizing up the extent of generality of an offered procedure, and responding with appropriate follow-up questions or tasks, as instances of what it means to teach responsively. This contrast led to the need for a more grounded approach to characterizing the situations in which responses to learner offers were given in the South African context, and then analysing the nature of these responses.

In identifying and then categorizing these situations, I took a grounded theory approach (Glaser & Strauss, 1967) to data analysis of the lesson enactments. The use of this approach was considered for two reasons: firstly, the context of 'no evaluation' outlined in the opening sections meant that existing theories developed in the global North provided limited purchase; and secondly, Rowland et al's (2005) development of codes constituting their initial knowledge quartet categories had been productively developed through a similar grounded analysis approach.

In using this approach, I first identified incidents where learners offered mathematically incorrect answers to a problem, and then began to analyse the nature of teachers' responses in these situations. However, incorrect offer situations were not the only incidents in which teachers offered input that was mathematically useful. This led to further identification of incidents where teachers' provided mathematically orientated responses to learner offerings in the lessons.

Through inductive processes of constant comparison across the 18 lessons and clustering for similarities, an 'elaboration framework' emerged with three broad in-the-moment situations in which responsive teaching was commonly seen. These situations were labelled as follows: (i) *breakdown* situations - where incorrect learner offers are given; (ii) *sophistication* situations - where a correct offer is given, but is viewed by the teacher as inefficient in relation to either the representation or the strategy used by learners in producing the answer; and (iii) *individuation/collectivisation* situations - of pedagogic moves of correct and efficient learner(s) offer from either chorused offer to assessing individuals (individuation) or individual insights developed and projected to the collective classroom space (collectivization).

In each of these situations, two basic categories emerged at an early stage: elaboration not provided (ENP), or elaboration provided (EP). Elaboration not provided involved either ignoring the learner offer, or acknowledging the offer, but then moving on with the lesson without follow-up relating to this offer. Instances of elaboration provided involved incorporating and mathematically and pedagogically developing the learner offer into the lesson in responsive ways.

The ‘elaboration framework’ developed in this study is later used to provide a language of description to talk about responsive mathematics teaching. I propose in this thesis that the framework can be used as a tool to support the development of responsive teaching in a South African context (and perhaps in other developing contexts where similar problems prevail) which is marked by evidence of a frequent absence of evaluation in schools serving poorer children. In this way, the study proposes to fill a gap in terms of identifying some important ‘stages of implementation’ (Schweisfurth, 2011) that might be required to move towards the ideals of more responsive teaching that are described in the international literature, and yet remain distant from the realities of South African schooling.

1.6 Operational definitions

The following terms are used throughout this research study and are defined specifically for this study in the ways detailed below:

Contingent moment: This relates to all situations where a learner offer is given (correct or incorrect answers and insights) during instruction. Some of these situations would be viewed as ‘predictable’ and amenable to planning in the international literature, and thus, may not be considered as contingent moments in Rowland et al.’s terms. I read contingent moments in this way because of the evidence of an absence of evaluation that has been highlighted in the South African context.

Elaboration: A form of teacher response to learners’ offer by incorporating and/or developing the offer mathematically and pedagogically into the flow of the lesson.

Breakdown: A situation where an incorrect mathematical offer is given by learner(s) during the course of mathematics teaching.

Sophistication: A situation where a correct mathematical offer is given by learner(s), and is followed by a teacher’s response that suggests a view of the offer as inefficient in terms of the learner’s representation or calculation strategy.

Individuation - A situation where the whole class chant of a correct mathematical offer is pursued by the teacher to assess individual understandings.

Collectivisation: A situation where an individual learner correct mathematical offer is pursued by the teacher and shared in the collective classroom space.

1.7 How the study is structured?

In Chapter 1 I have introduced the global picture of what the present study sets out to investigate. In doing so, I have dealt with the rationale for my focus on contingent moments, emanating from the particularities related to primary mathematics teaching in South Africa, and other motivations drawn from the international literature. The research questions and gaps driving the study and the context of the study are also detailed in this chapter.

In **Chapter 2,** I discuss two distinct bodies of writings: responsive teaching and additive relations, which are central to the present study. Due to the specificity of this study, I discussed each of these areas in two parts: the international literature and the South African literature. The chapter identifies gaps in evidence about moves toward responsive teaching in South African context, which are addressed in this study.

Chapter 3 locates the study within an emergent theoretical approach. I specifically deal with the philosophical and epistemological stance of the theoretical underpinning of the study and the rationale for my choice of this theoretical lens.

Chapter 4 outlines the methodology employed in this study. It deals with research design, context of the study, selection of participants, research instruments, data sources and procedures, data analysis, trustworthiness of the research and ethical considerations.

Chapter 5 reports on the key contribution of this study – the development of a language of description to identify and develop important 'stages of implementation' towards more responsive teaching in the South African context. In developing this language, I ended up with three situations of elaborations (*breakdown*, *sophistication* and *individuation*/ *collectivisation*), with each consisting of categories and codes, which I later pulled together into what I termed the 'elaboration framework'. I illustrate, with examples of selected excerpts, how the categories of the elaboration framework were conceptualised in a context where very limited responsive teaching has been highlighted. In doing so, I offer analysis of selected incidents as ‘telling cases’ drawn from the extensive range of data of classroom practices across the four teachers. A further crucial level of analysis linked with the framework was driven by literature on the quality of mathematics teaching, related to exploring hierarchies and relationships between the emergent categories within situations. These hierarchies and relationships are discussed in this chapter.

Chapter 6 provides findings and discussions of different patterns of shifts in the kinds of elaboration provided by the four teachers between 2013 and 2014. Shifts are juxtaposed not to suggest any direct causality from either the ‘maths for teaching’ course or the VSR interview, but rather to explore the interplays over time of these professional development mechanisms with the four cases of teachers’ increasing focus on elaboration. In order to illustrate these shifts, I recruited three markers of shifts relating to ‘extent’, ‘breadth’, and ‘quality’ of elaborations – interpreted through the lenses provided by the descriptors and hierarchies within each situation of the elaboration framework presented in Chapter 5. The patterns of shifts for each individual teacher are presented, with possible associations with findings from analysis of VSR interviews. The chapter concludes with synthesis of the cross-case findings across all the four teachers.

Chapter 7 provides a summary of the key findings emanating from this study, attending to contributions to the knowledge base, and implications for South African primary mathematics teacher development and directions for future research.

CHAPTER 2

SITUATING THE STUDY IN THE LITERATURE

2.1 Introduction

The present study uses the notion of 'elaboration' as an interpretive lens to examine and characterize responsive teaching in primary school mathematics classrooms in South Africa. In doing so, the study examines situations and the nature of in-service mathematics teachers' responses that illuminate 'in-the-moment' decisions in the classroom. This chapter locates the present study in relation to literature and theory on responsive teaching. The chapter is organized into two distinct bodies of writings. The first section focuses on how responsive teaching is conceptualized and why it is seen as important both to explore quality of teaching and teaching development. The review incorporates attention to the relationship between responsive teaching and teachers' mathematical knowledge. The review also highlights some of the crucial issues that might constrain openings for responsive teaching in the South African primary mathematics teaching landscape.

The second section deals with a review of additive relations as the mathematical content area in which responsive teaching is explored. This choice was driven by a number of key features: firstly, additive relations is an important and foundational aspect of primary school mathematics curricula in South Africa and internationally; secondly, there is extensive writing in the field of mathematics education related to additive relations with particular emphasis on progression in teaching and learning; and thirdly, a significant body of evidence in South Africa points to difficulties across learning and teaching related to this area. Therefore, this literature base on additive relations acted as a vantage point for understanding and commenting on the nature of teacher contributions and elaborating on efficiency in relation to the content domain seen in this study.

2.2 Conceptualizing responsive teaching

The characterization of responsive teaching as centrally creative activity in teaching that I introduced in the opening chapter provides a key viewpoint in this study for examining the quality of mathematics teaching and its development. In the following section, I develop further this notion of responsive teaching by describing what responsive teaching entails and why attention to responsive teaching is important both in relation to quality mathematics teaching and teaching development. Teachers' mathematical knowledge is identified in this review as an important and necessary condition, though not sufficient for responsive teaching action. I provide an outline of some crucial issues that tend to constrain openings for responsive teaching in the South African context. The goal of this review is to provide me with an overarching conception that can guide the analysis, assessment and development of responsive teaching actions.

2.2.1 What is responsive teaching?

Responsive teaching is considered in the context of classroom interaction with a view to increasing teachers' awareness of the need to provide appropriate follow-ups to learners' offers (answers or contributions) in ways that extend or expand possibilities for mathematics learning. Classroom interaction has been the focus of a variety of studies over the last forty years. Initiation-response-evaluation/feedback (IRE/F) interactions (Mehan, 1979; Sinclair & Coulthard, 1975) have been studied to analyze how teachers react to and evaluate students' responses or give feedback to students (Brodie, 2007; Edwards & Mercer, 1987; Wells, 1999 and others). The teacher makes an initiation move, a learner responds, the teacher provides feedback or evaluates the learner offer and then moves on to a 'new' initiation.

Feedback or evaluation are seen as the key aspects of the IRE/F model, and are considered crucial to the understanding of responsive teaching. Cobb, Yackel, and Wood (1992) state that feedback can be seen as an essential component of the teacher's role in facilitating mathematical discourse in the course of classroom interactions. Feedback simultaneously positions the teacher as a participant who can legitimize certain aspects of mathematical activity and sanction others. Over time, these authors argue that this practice supports learners to take up what was legitimated in shared classroom discourse as sociomathematical norms, giving learners the power to decide on the correctness or validity of a mathematical assertion. This kind of feedback is indicative of a move towards more responsive teaching. Research examining teachers' use of revoicing has interpreted the practice as essential feedback provided by the teacher during the process of teaching that can be described as responsive to learners' ideas (see for example, Krussel, Edwards & Springer, 2004, and O'Connor & Michaels, 1996). Revoicing entails teacher feedback response by repeating, rephrasing, summarizing, elaborating, or translating learners' offers (Forman & Ansell, 2002).

In studies focused on language, the practice of revoicing is essentially about repeating some or all of what has been said by someone else in a preceding turn as the basis of a flow in the interaction. This repetition is manifested in two forms: either as a linguistically 'exact' copy or as a reformulation (Planas & Morera, 2011). These authors argue that linguistically exact repetition involves modification of the language used and, therefore, they regard revoicing as conceptual reformation rather than linguistic repetition.

> …every instance of the use of language is a potential modification of that language at the same time as it acts to reproduce it. Thus we find it more adequate to associate revoicing to conceptual reformulation rather than linguistic repetition (p. 1359).

O'Connor and Michaels (1996) identify three main uses of revoicing in mathematics classroom interaction that have the effect of focusing productive group discussion and scaffolding conversation on the basis of what is said, when, how, and with whom. These uses are to:

1. position students in differing alignments and allow them to claim ownership of their position;
2. share reformulations in ways that credit students with teachers' warranted inferences; and
3. scaffold and recast problem-solution strategies of students whose first language is not the language of teaching.

These uses of revoicing indicate responsiveness of teaching actions, with feedback based on learners' ideas, and the vital role of a teacher in supporting learning in the classroom through building collective understanding. Building on the work of Forman and Ansell (2002) and O'Connor and Michaels (1996), Planas and Morera (2011) examine classroom processes of collective mathematical argumentation. They found two 'positive' uses of revoicing in peer interaction: reinforcing mutual understanding and fostering explanations. Responsive teaching is therefore conceptualised in this study as a form of feedback, which could be through revoicing or other means to extend or expand learners' mathematical understanding.

Scholars have investigated the nature and effects of evaluation or feedback within the IRE/F pattern of interactions for over two decades. Findings from this body of writings can be classified into two groups: 'deficit' and 'affordance' approaches to the use of IRE/F. In the former, the authors draw linkages between a teacher's lack of mathematical awareness and feedback or evaluation in her mathematics teaching. In the latter, the authors highlight the affordances created for classroom genuine learner participation. I review each group separately.

On the deficit conversation side, research has shown that the IRE/F model can easily be used by teachers in the forms that Bauersfeld (1980) described as 'funnelling'. Funnelling involves reducing the cognitive demand of the task in a situation where the teacher initiates a classroom discourse by asking a challenging question, but, when learners can't give the answer, the teacher asks follow-up questions which get easier and easier until the only option open to learners is the specific answer to the question. This results in a situation where learners are eventually answering questions far below the level of the initial task (Brodie, 2007; Forman & Ansell, 2002). Brodie (2007) provides empirical examples where she illustrates that merely engaging learners in question-and-answer exchanges does not necessarily allow for genuine learner participation in the lesson, nor move learners' mathematical thinking forward. This pattern of classroom interaction points to teaching that does not provide appropriate follow-up or feedback to learners' responses in ways that extend or expand possibilities for learning.

The affordances approach focuses on feedback that is contingent on learners' responses during classroom interaction (Forman & Ansell, 2002; Nystrand & Gamoran, 1990), and which supports genuine learner participation in the classroom (Brodie, 2007; Edwards & Mercer, 1987; Mercer, 1995). For instance, Nystrand and Gamoran (1990) developed the notion of 'uptake' to argue that teachers who work productively with IRE/F patterns of discourse shape their own feedback based on what immediately precedes in the learner's response. Here, the teacher incorporates learners' ideas into the subsequent discussion, and therefore, the teacher's next question or new initiation is contingent on the learner's idea rather than predetermined. The teacher picks up on learners' ideas, and these ideas can change the course of the discussion or require the teacher to deviate from the agenda of the lesson.

On the whole, in the affordance approach, as Brodie (2007) writes, teachers were engaged in doing three things: (i) maintaining high task demands; (ii) responding to genuine learner questions; and (iii) supporting meaningful conversations among learners. She argues that these kinds of teacher actions are most likely to support learner participation and move forward mathematical thinking in the classrooms. Wood (1998) distinguishes ‘funnelling’ from ‘focusing’ to differentiate between the deficit and affordances approaches that teachers use when giving feedback or evaluating learners’ responses. Focusing, involves encouraging students to do most of the mathematical thinking by focusing attention on particular aspects of students’ responses without guiding students in a specific, predetermined direction. Both the affordance and deficit approaches are important in the conceptualization of what it means to examine responsive teaching or lack of it.

The key issues across the findings of the studies discussed above rest on the form of feedback or evaluation given by the teachers in response to what learners can or cannot do. I therefore conceive the act of responsive teaching as fundamentally improvisational. While both the deficit and affordances studies, for instance, contributes several suggestions for how classroom interaction influences instruction, but focus here is on patterns of interaction rather than on mathematically focused response that attuned to a productive mathematical discourse in the classroom. Conceiving the ‘quality’ of mathematics teaching through the lens of responsive teaching provides one way to develop a framework for mathematical discourse in the classroom. In the next section, I discuss why attention to responsive teaching is important in examining quality of mathematics teaching and its development.

2.2.2 Why responsive teaching is seen as important?

It is practically impossible for classroom lessons to proceed completely according to plan. This is simply because learners come together in a classroom with different preferences and abilities, and the teacher is expected to jointly engage them to accomplish same educational goal and learning outcome. Therefore, unpredictable events must be expected in the course of the teacher interactions with the learners as they engage with new subject matter. Doyle (1986) describes this unpredictability element of classroom interaction in the following terms:

> Classroom events often take unexpected turns. Distractions and interruptions are frequent. In addition, events are jointly produced and thus it is often difficult to anticipate how an activity will go on a particular day with a particular group of students (Doyle, 1986, p. 395).

This unpredictability of classroom events makes teaching a complex activity as it requires significant demands from the part of the teacher. Given that these unexpected events are part of the integrated classroom ecology, for effective learning to take place teachers have to constantly be prepared to respond to these events, and give careful and insightful feedback to learners in support for learning. This kind of awareness may sometimes deviate the teacher from the agenda of the lesson in order to be responsive to what learners can or cannot do. This provides a rationale for why attending to responsive teaching is important as a measure for teaching quality and as well as a tool for improving mathematics teaching.

Many studies, particularly at primary school level, have alluded to gains in deeper students' learning and increasing capacity for students to solve more complex mathematical problems when they participate in classroom interaction that privileges reasoning and sense making rather than memorization of procedures (Fennema, Franke, Carpenter, & Carey, 1993; Hiebert & Carpenter, 1992) For instance, in Cognitive Guided Instruction (CGI), Carpenter, Fennema, Peterson, Chiang, and Loef (1989) found that teaching that built and developed on students' existing knowledge and that encouraged student to use multiple problem-solving strategies was linked with greater increases in students' learning than teaching that focused on number facts and that did not take into account students' prior understandings.

In their measures of mathematical quality of instruction (MQI), Hill et al. (2008) include *responding to students appropriately* – 'the degree to which teacher can correctly interpret students' mathematical utterances and address student misunderstandings" (p. 437) as one key indicator of teaching quality. O'Connor (1998) talks about exploring students' opportunities to interact with mathematics by gauging students' discourses as the key requirement for supporting mathematics learning.

In the next section, and important in a South African context where gaps in primary teachers' mathematical knowledge have been widely noted, I examine the intrinsic relationship between teachers' mathematical knowledge and responsive teaching.

2.2.3 Teachers' mathematical knowledge and responsive teaching

Difficulties with linking in straightforward ways knowledge measures with teaching quality have been noted in the literature. Complexity relates to the extent at which the teacher draws upon her mathematical knowledge when needed in-the-moment, making it hard to directly compare the efficacy of acquired or possessed knowledge translating into responsive teaching action. However, writing also notes mathematical knowledge as necessary, and acutely so in contexts of responsive teaching. Therefore, discussion about different domains of mathematical knowledge in teaching is relevant.

As mentioned in the opening chapter, the present study is a follow-up exploration of four teachers' ways of being with mathematics in teaching. These teachers participated in a one year long in-service 'maths for teaching' course. Following the point made above, I made selections of teachers on the basis of relative strengths in terms of their mathematics knowledge based on their course assessments in order to explore possibilities for developing responsive teaching action. The course was framed based on the dimensions of Rowland et al.'s KQ. Hence discussion about these dimensions and their interrelatedness in terms of how responsive teaching is figured within the KQ framework is relevant.

The Knowledge Quartet (KQ)

The work of Rowland's Subject Knowledge in Mathematics (SKIMA) research group focused on categorising situations in classrooms where mathematical knowledge surfaces in teaching in the context of pre-service teacher education. The detailed analysis of 24 lessons (2 from each of 12 teachers) they observed resulted in the emergence of a framework – the Knowledge Quartet (KQ). They committed several years to the development of this framework, which has subsequently been revised many times (see Rowland, 2005; Rowland, 2012; Rowland, Huckstep, & Thwaites, 2003; Rowland et al., 2005; Rowland & Turner, 2007; Rowland, Turner, Thwaites, & Huckstep, 2009; Turner & Rowland, 2011). A significant aspect of their framework is that it is not only aimed at defining what knowledge is needed for mathematics teaching and how such knowledge may be identified, but also provides a way of understanding how such knowledge is developed in teachers. As noted already, the KQ framework describes the interactions of four different dimensions of knowledge as observed in teaching practice. These are: foundation, transformation, connection and contingency.

The first category, ***foundation***, consists of the theoretical background related to mathematics knowledge, beliefs and understanding that teachers possess or acquire during training, in preparation for their roles in the classroom. It is about knowledge possessed regardless of whether it is being put to purposive use. The key components of this theoretical background are: knowledge and understanding of mathematics *per se* (i.e. SMK) and knowledge of mathematics specific pedagogy (i.e. PCK), together with beliefs concerning the nature of mathematical knowledge, the purposes of mathematics education, and the situations that provide conducive environments for mathematical learning (Rowland et al., 2003). Indicators of *foundation knowledge* in the context of primary mathematics include: knowledge of appropriate use of manipulatives and models – moving from more concrete (enactive) to more abstract (symbolic) notions of number, concentration on developing learners' understanding rather than excessively on procedures, correct writing of mathematical expressions and demonstrating knowledge of common errors and misconceptions in the planning of a lesson and taking steps to avoid them, and in the context of additive relations, awareness of progression in solution strategies from 'count all' to 'count on' to derived and recalled facts.

The second category, ***transformation***, lies at the heart of the knowledge quartet. It is concerned with mathematical knowledge-in-action as demonstrated both in planning to teach mathematics and in the act of teaching itself. In their discussion of this category, Turner and Rowland (2011) cited Shulman's notion that this category refers to the "… capacity of a teacher to transform the content knowledge he or she possesses into form[s] that are pedagogically powerful" (Shulman, 1987, p. 15). The teacher's choice and use of examples, representations, use of instructional materials and demonstrations in teaching mathematics and explanations of mathematical ideas are the critical components of this category. Explanations can encompass rationales for the choice and use of representations or examples. Within this category, indicators relate to examples, representations together with underlying rationales (either implicit or explicit) that can be considered as appropriate for demonstrating or eliciting mathematical ideas in the course of teaching. For example, to demonstrate the idea of ‘compensation’ examples such as 27+9 and 27 + 11 are useful, and can be represented using a number line.

Connection, the third category, is concerned with the coherence of the planning or teaching mathematics displayed across an episode, lesson or series of lessons. Coherence refers to the sequencing of materials for instruction, and an awareness of the relative cognitive demands of different mathematics topics and tasks (Rowland et al., 2003). It is concerned with decisions about sequencing and connectivity made by the teacher so that the lesson ‘hangs’ together and relates to the context of previous lessons and to learners’ knowledge. Such decisions reflect teachers’ ability to anticipate what is complex and break it down into steps that can be understood by the learners. In the planning of mathematics lessons, teachers should introduce ideas and strategies in an appropriate progressive order that connects well with learners’ understandings.

Contingency, the last category, focuses on the teacher’s responses to classroom events that were not anticipated in the planning of how activity would unfold in the mathematics classroom (Turner & Rowland, 2011). Contingency refers to teachers’ knowledge of the use of learners’ descriptions of their methods/strategies and reasoning in interaction. This category carries strong link to the notion of responsive teaching, which this study explores. The features of this category are evident when the teacher deviates from the agenda set out in the prepared mathematics lesson plan, or when a teacher responds to learners’ correct or incorrect mathematical offers during instruction. Rowland et al. (2003) argue that teachers with limited mathematical knowledge find it more difficult to cope within the contingency category, providing a rationale for my selection of teachers with relatively strong mathematics knowledge as a prerequisite for seeing any responsive teaching action and to explore possible teaching development.

The descriptions of the four dimensions of the KQ and contributory codes that constitute these dimensions emerged empirically from the observation of the 24 lessons within the SKIMA project. These are summarised in Table 1.

Table 1: The knowledge quartet: dimensions and contributory codes (Rowland, 2014)

Dimension	**Contributory codes**
Foundation: knowledge and understanding of mathematics per se and of mathematics-specific pedagogy, beliefs concerning the nature of mathematics, the purposes of mathematics education, and the conditions under which students will best learn mathematics	· awareness of purpose · adherence to textbook · concentration on procedures · identifying errors · overt display of subject knowledge · theoretical underpinning of pedagogy · use of mathematical terminology
Transformation: the presentation of ideas to learners in the form of analogies, illustrations, examples, explanations and demonstrations	· choice of examples · choice of representation · use of instructional materials · teacher demonstration (to explain a procedure)
Connection: the sequencing of material for instruction, and an awareness of the relative cognitive demands of different topics and tasks	· anticipation of complexity · decisions about sequencing · recognition of conceptual appropriateness · making connections between procedures · making connections between concepts
Contingency: the ability to make cogent, reasoned and well-informed responses to unanticipated and unplanned events	· deviation from agenda · responding to students' ideas · (use of opportunities) · teacher insight during instruction · responding to the (un)availability of tools and resources

It is important to note that Rowland et al.'s work is focused predominantly on pre-service teachers and in England and Wales. As in many parts of the world, there are strong expectations that pre-service teachers prepare detailed written lesson plans for observed (and often, for all) lessons. This allowed Rowland and colleagues to make claims about deviations in enactment from the plan. But there is also evidence that formal written lesson planning is less common among in-service teachers, and this makes it harder to demarcate contingent action from planned action in in-service lesson observations. However, the contextual evidence noted earlier of frequently very limited, and sometimes, no evaluative comments on learners' offers in mathematics classrooms in South Africa suggests instead, the viability of viewing all responsive evaluation or feedback comments within the contingency category.

Interrelatedness among dimensions of the KQ framework

In their conceptualization of the dimensions of the KQ framework, Rowland et al. (2005) suggest some interrelationships (not hierarchies) among the four dimensions of the framework. They comment on foundation knowledge as distinct from the other three dimensions, arguing that the latter dimensions draw from foundation knowledge.

> It [Foundation knowledge] differs from the other three units in the sense that it is about knowledge possessed, irrespective of whether it is being put to purposeful use. This distinction relates directly to Aristotle's account of 'potential' and 'actual' knowledge. ''A man is a scientist ... even when he is not engaged in theorising, provided that he is capable of theorising. In the case when he is, we say that he is a scientist in actuality'' (Lawson-Tancred, 1998, p. 267). Both empirical and theoretical considerations have led us to the view that the other three units flow from a foundational underpinning (p. 260).

A key consequence of this distinction between foundation knowledge and the other three dimensions is that the knowledge base associated with teaching mathematics is different from the knowledge base needed to do mathematics. This argument echoes the writing of Ball (1988), for example, who distinguishes between knowing some mathematics '*for yourself*' and knowing in order to be able to help others learn it. Venkat (2015) also noted the distinction between 'maths for yourself' and 'maths for others' within teachers' representations repertoires. For the former, representations are mathematical tools for problem-solving, while for the latter, they are pedagogical objects for supporting the learning of others. Rowland (2005, p. 259) argues that possession of foundation knowledge has the potential to inform pedagogical choices and strategies in "rational, reasoned approach to decision making that rest on something other than imitation or habit".

Figure 2 illustrates diagrammatically Rowland et al's proposed interrelations among the four dimensions of the KQ. Foundation knowledge is seen as mathematical knowledge for teaching (MkfT). By MkfT I mean the acquired or possessed knowledge that teachers bring to teaching situations. The other three dimensions of the KQ are seen as constituents of mathematical knowledge in teaching (MKiT). By MKiT I mean the knowledge that is manifested in action in the actual teaching practice. Manifestation is used to emphasise that shortcomings seen in MKiT do not necessarily indicate absences of that particular knowledge within MKfT. This is so because teachers, particularly during contingent teaching action, draw upon what comes to mind in the moment, which need not stem from a lack of knowledge of other alternatives.

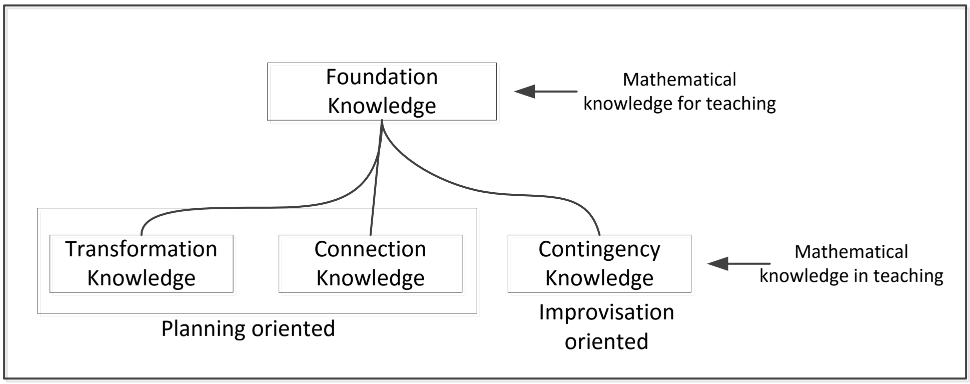


Figure 2: Interrelationship between the four dimensions of the KQ framework

A further sub-division can be made within the MKiT domain in that transformation and connection knowledge are seen as knowledge that the teacher can plan for prior to teaching enactment, while contingency knowledge is subject to improvisation in response to unplanned classroom events. Morine-Dershimer (1978, p. 84) noted "the amount of discrepancy that exists between the teacher's lesson plan and the classroom reality". Morine-Dershimer's empirical base justifies the distinction between planning and the realities of classroom events. This further suggests that awareness of the unexpected and readiness to respond to classroom realities outside planning is essential for supporting students' learning in the classroom. My focus in this study is therefore aligned to features of mathematical discourse during classroom interactions, where pedagogic practice as improvisation is key to responsive teaching actions, and viewed as distinct from the planned aspects of teaching. Explanations, in this view, can be deconstructed within aspects of the dimensions of transformation and connection knowledge of the KQ framework, and some South African literature (e.g. Adler & Venkat, 2012; Adler & Ronda, 2015) has paid specific attention to explanation within the more planned aspects of mathematical discourse in instruction (MDI), but not a part of mathematical discourse that is improvisational.

Improvisation in mathematics teaching within mathematical discourse during classrooms interaction has been the focus of many recent studies in mathematics education, and in particular, the recent publication of a Special Issue of Research in Mathematics Education (RME) (Coles & Scott, 2015; Mason, 2015; Rowland et al., 2015). This literature base attends instead to in-the-moment situations in teaching, with implications for finding ways to develop and support quality mathematics teaching that is responsive to emergent mathematics learning. In the following section, with illustrative examples drawn from empirical international studies, I discuss the relationship between teachers' mathematical knowledge base, and in-the-moment responsiveness in teaching.

2.2.4 On mathematical knowledge and in-the-moment responsiveness

Teachers' mathematical knowledge, its role in teaching, and ways to develop such knowledge among teachers have been the focus of many studies (Adler & Ball, 2009; Rowland & Ruthven, 2011). Some studies also describe various components or dimensions of such knowledge (Ball et al., 2008; Fennema & Franke, 1992; Krauss, Baumert, & Blum, 2008; Rowland et al., 2005). However, it has been noted that such categorisations of knowledge with underlying acquisition metaphors of teachers' mathematical knowledge more generally, divert attention away from teachers' ways of 'being with mathematics' as a mode of enquiry (Watson, 2008).

There is consensus among researchers that teaching actions cannot be imagined without a base in teachers' knowledge of what is to be taught. However, there is limited consensus about the extent and the nature of such knowledge (Ruthven, 2011; Zazkis & Leikin, 2010). Rowland and Zazkis (2013) suggest that understanding the mathematical knowledge needed for teaching depends fundamentally on one's perception of teaching itself.

> If teaching involved only attending to prescribed scenarios and delivering a predetermined curriculum, then it is likely that knowing that curriculum would suffice. However, teaching also involves attending to students' questions, anticipating some difficulties and dealing with unexpected ones, taking advantage of opportunities, making connections, and extending students' horizons beyond the immediate tasks. In short, teaching involves dealing with unpredictable, *contingent* events in the classroom. With this perspective on teaching, mathematical knowledge beyond the immediate curricular prescription is beneficial and demonstrably essential (p. 132).

Rowland and Zaskis' empirical base is located in teachers' 'in-the-moment' decisions while teaching, with particular attention in the quote above to teachers' ways of reflecting in action to attend to learners' questions, and broadly dealing with unexpected events. To deal effectively with such unexpected events in the classroom, teachers need the capacity to make connections between mathematical awareness and in-the-moment pedagogy. A key characteristics of this capacity are manefested through improvisation and creativity during classroom interactions.

Borko and Livingston (1989) and Yinger (1987) suggest that we can understand some aspects of in-the-moment decisions in teaching as improvisational performance. An improvisational actor enters the stage with a definition of the general situation and a set of guidelines for performing her role, rather than working from a detailed written script. Such a performer draws upon an extensive repertoire of routines and a framework of actions as the scene unfolds, incorporating them into a performance that is continually responsive to the audience and to new situations or events. This metaphor can be applied to teaching situations if these are conceived as improvisational, where the teacher begins a lesson with an outline of the instructional activity or lesson image (Morine-Dershimer, 1978), but the details and actual flow are determined by the classroom interactions as the teacher responds to what learners can and cannot yet do.

Gattegno's (2010) notion of the subordination of teaching to learning can be interpreted as located in a view of responsive teaching. Gattegno's view is in contrast to the kinds of practice where learning is subordinate to teaching (which is aligned to more rigid adherence to planned lessons), based on students memorizing and retaining facts. He proposes four tasks: invoking notions of will, sense of truth, finding how knowing become knowledge, and considering the economy of teaching as key components for enacting practices that characterise 'subordination of teaching to learning'. Using this framework, Coles & Scott (2015) analysed Scott's teaching by mapping her increasing focus on creative and co-produced mathematical processes and all the unexpected events within this on to a parallel account of one student's increasing sense of control over the subject. Their findings suggest that what changed for Scott was not centrally about new subject knowledge, but rather a new relationship to the unexpected.

Rowland and Zazkis (2013) also draw on the scenario recounted several times by Alan Bishop about the 'fraction in between fraction' problem (Bishop, 1976) to illustrate the relationship between teachers' mathematical knowledge and in-the-moment responsiveness. Alan Bishop writes:

> This happened to me many years ago, and I remember it well. You are studying fractions with a lively class of 12 year old students, and you ask them to suggest a fraction that lies between one half and three-quarters. One particularly eager student offers the answer “two thirds”. When you ask how she knows that it lies between the other two fractions, she answers: “Well you can see that on the top the numbers go 1, 2, 3 and on the bottom they go 2, 3, 4. On the top, the 2 is between the 1 and the 3, and on the bottom, the 3 lies between the 2 and the 4, so therefore two thirds must be between the other two fractions!” (In Rowland and Zazkis, 2013 p. 143)

Rowland and colleagues invited teachers from several developed countries to consider and respond to this scenario with emphases on mathematical rather than more generic features of the scenario. They categorized teachers’ responses into one of two kinds: teachers who agreed with the student focusing on the correct answer (⅔) and less about the underlying reasoning that produced the answer; and teachers who indicated unhappiness with the student’s correct answer focusing on student’s reasoning. The latter group of teachers indicated that they would respond by telling the student: ‘this is not the correct way to solve the problem’ and ‘will remind her on using arithmetic mean’ (p. 145).

Rowland and Zazkis (2013) concluded that despite the emphasis on providing mathematical responses, in most cases the teachers’ responses were mathematically limited, in the sense that their responses were evaluative (correct/incorrect) rather than unpacking of the possibilities of learning from the student’s reasoning. From their analysis of this scenario, they comment on possible ways of looking at the student’s offer mathematically as follows:

> (C1): Whenever the numerators of three fractions are *consecutive integers* and the denominators likewise, the second fraction will be between the other two.
>
> A more general version of this (though there is no evidence to suggest that she intended it) might be one of the following:
>
> (C2a): Whenever the numerators of three fractions are in *arithmetic progression* and the denominators likewise, the second fraction will be between the other two.
>
> (C2b): Whenever the second numerator is the *arithmetic mean* of the first and third and the denominators likewise, the second fraction will be between the other two. (Rowland and Zazkis, 2013 p. 148)

The consequence of this kind of analysis of the unanticipated response to contingent situations, which goes beyond mere evaluation of the student’s offer, is the creation of doors for mathematically oriented opportunities beyond the initial task. While strong conceptual knowledge of mathematics and pedagogy is necessary, Mason and Davis (2013) have also noted the distinction between teacher's mathematical knowledge and teacher's ability to make connections between mathematical awareness and in-the-moment pedagogy. This distinction suggests that the problem with the teachers' inability to respond to the student offer mathematically and pedagogically goes beyond possession of the mathematical knowledge to include mathematical awareness of the unexpected. Rowland and Zazkis (2013) argue that teachers' response to these 'triggers' of contingencies was one of three kinds: (i) to ignore (where in some cases the teacher is uncertain what the learner means, or feels that there is insufficient time to explore); (ii) to acknowledge but put aside (where the teacher accept the learner offer and deflects discussion to another time); and (iii) to acknowledge and incorporate (where the teacher takes up the learner's suggestion as a teachable moment).

More recently, Rowland et al.'s (2015) analyses of 'triggers' of contingency in a range of international empirical classroom teaching episodes, proposed an enhanced, three-part classification to triggers of contingent situations: namely; (i) students' ideas during the teaching/learning situation (like the Alan Bishop example discussed above) (ii) response emanating from teacher insight through reflection on her own planned actions (i.e. teacher's in-the-moment evaluation of the lesson planning and development); and (iii) the pedagogical tools and resources that are brought to bear on the instruction, when the teacher is responding to the availability (or the unavailability) of resources.

The first classification emanates from the student's contributions, the other two triggers (response emanating from teacher insight and response to pedagogical tools and resources) happened through teachers' monitoring and self-regulation of their actions as they perform them. They further identified three sub-types of responses within the first classification – *responding to student's ideas*. The first is the '*student's response to a question from the teacher'*; the second is a '*student's spontaneous response to an activity or discussion*'; the third is when a '*student gives an incorrect answer to a question, or as a contribution to a discussion*'. Rowland et al. (2015) state that the last two types of triggers (the teacher insight and response to pedagogical tools) were:

> … less common in our novice-teacher data, although seasoned teachers might recognize it in their own experience. The triggers in this category are the results of "reflection in action"

> (Schön, 1983), in which the teacher becomes aware, in the course of the lesson itself, that something is amiss. This awareness provokes new understanding of the content in the teacher, and a preference to modify the planned lesson agenda (p. 81).

These responses are therefore suggested as more advanced moves in responsive teaching, where the teacher is constantly engaging in reflection on her own actions as teaching unfolds beyond responding to students' ideas. This involves on-going evaluation of the lesson planning and development, and response to availability or unavailability of the cultural pedagogic tools and resources. Examples of such teacher insight during instruction are illustrated in Rowland et al. (2015, p. 82).

> The first author recalls a lesson in which his students were intended to 'see' how the number of factors of a positive integer n can be found from the powers in its prime decomposition. He introduced his exposition with the example n = 72, reasoning that this integer is relatively small, yet rich in factors. As soon as he had written $72 = 2^3 \times 3^2$ on the board he realised that this was not such a good example, since both 2 and 3 play dual roles in the decomposition, obscuring the significance of the indices as opposed to the specific primes. 72 was hastily replaced by 6125, and the reason for doing so was explained to the students later.

Recasting an example in this way through realizing the consequences of his actions as they are in the process of playing out in the classroom has been described as difficult to do. The example of n = 72 was an example selection with a foundation knowledge base: 'relatively a small integer, yet rich factors' (p. 82). However, in the course of introducing this example, the teacher noticed that the repetition of numbers in the answer could be a source of confusion or misunderstanding for students. These kinds of responses go beyond the teacher's mathematical knowledge to incorporate awareness through constant reflection in action.

Another strand of research, in-the-moment awareness is linked with teachers' beliefs. Hill et al.'s (2008) measure mathematical quality of instruction (MQI) in their coding framework by drawing extensively from literature on mathematics teaching to organise themes that characterise a measure for teaching quality. Some of these themes includes: connecting classroom practice to mathematics, richness of mathematics, responding to students appropriately, mathematical errors, and density of accurate mathematical language in instruction. With these measures within MQI framework, and the measures for mathematical knowledge for teaching, which have been linked to gains in student achievement (Hill, Rowan, & Ball, 2005), they provided case studies of teachers that support the claim for strong links between teachers' mathematical knowledge and quality of their classroom practice. However, they found in an exploratory study some factors that mediate this relationship, which were linked to teacher beliefs about teaching and learning of mathematics.

> We inspected cases for the possibility that other factors might mediate this relationship and we identified a few: teacher beliefs about how mathematics should be learned and how to make it enjoyable by students; teacher beliefs about curriculum materials and how they should be used; and the availability of curriculum materials to teachers (p. 496-497).

Teachers practices in teaching and learning mathematics depends upon various key factors. One of these key factors is teacher's beliefs concerning the teaching and learning of mathematics. Etmer (2001) argued that in conceptualising teacher beliefs, the difficulty centres on determining if, and how, they differ from teacher knowledge. According to Calderhead (1996) beliefs refer to "suppositions, commitments, and ideologies," while knowledge refers to "factual propositions and understandings" (p.715). Therefore after gaining knowledge of propositions, teachers are still free to accept or not accept them and put them into their classroom practices. For example, teachers may gain specific knowledge about how to use an empty number line in teaching addition and subtraction, and may also know that other teachers have used it successfully, yet not believe that the empty number line offers an effective tool for their classroom use.

Research on teachers' beliefs and knowledge about mathematics at elementary level reveals that many teachers describe the discipline of mathematics as a fixed body of knowledge involving numbers and their manipulation through rules and standardized procedures (Anghileri, 2006; Jackson, 1986). Griffin (2004) attributes these beliefs to teachers' own learning experiences, and its implication is an ongoing tendency to treat mathematical ideas as 'disembodied' entities by focusing instruction on ensuring that learners know various rules and standard procedures of mathematics and their application. Rowland and colleagues in their conceptualization of the knowledge quartet (KQ), like Fennema and Franke (1992), consider teacher's beliefs to be an important component that shapes knowledge-in-use. The foundation knowledge dimension of KQ encompasses SMK, PCK categories, as well as knowledge of purposes category which relates to the notion of teachers' beliefs about mathematics.

In their report of a study on effective teachers of numeracy, Askew, Brown, Rhodes, William, and Johnson (1997) point out the significance of understanding teachers' beliefs, knowledge and practice in describing their effectiveness of teaching numeracy. They suggested three aspects of beliefs that influence the teaching of numeracy, which can be linked to the aspects of beliefs mentioned in Hill et al.'s work. ;

- Beliefs about what it is to be a numerate pupil - this includes teachers' beliefs about the nature of mathematics in general, numeracy in particular, and expectations of learning outcome.
- Beliefs about pupils and how they learn to become numerate - this includes beliefs about whether or not some pupils are naturally more mathematical, the type of experiences that best bring about learning and the role of the pupils in lessons.
- Beliefs about how best to teach pupils to become numerate- these are related beliefs about teaching numeracy in terms of perception of the teacher's role in lessons and the influence of the accepted wisdom of good primary practice.

Askew et al. (1997) conclude with a model that presents the interplay and relationship between beliefs, knowledge and classroom practices. They suggested that each informs and is informed by the others and postulate that "understanding why some teachers may be more effective than others requires an examination of each of these aspects" (p.21).

The key issue emanating from this review is the fact that a mathematical knowledge base is necessary, but not sufficient for quality of mathematics instruction, and in particular for appropriate response to learners' offers in the classroom. Central to the present study is the notion of elaboration of mathematical ideas in response to unpredictable or unplanned classroom interactions. Drawing from Rowland et al. and others, I argue that these moments provide particularly fruitful contexts for examining teaching development over time in teachers' ways of being with mathematics knowledge and translating this into responsive teaching.

In the South African context, there are number of issues that might constrain openings for responsive teaching. These issues range from more broadly extensive gaps in teachers' foundation knowledge (both SMK and PCK), to problems in discursive practices during classroom interactions, such as: lack of feedback to learners, learning arranged in largely communalized rather than individuated settings, and low levels of cognitive demand. The survey of the literature relating to these issues below provides insights into ways to support the development of responsive teaching action in the South African context.

2.2.5 Problems associated with openings for responsive teaching in South Africa

In order to discuss the literature base on issues that constrain openings for responsive teaching in the South African context, it is important to be cognisant of the legacy of the apartheid era, where schools were segregated based on race. In this system, the majority of the black South African teachers were ill-trained in the then teacher education institutions. These teachers still populate many South African primary schools, particularly schools serving previously disadvantage settings (Venkat & Spaull, 2015). As Spaull (2013a) has pointed out, schooling in South Africa still operates in two education systems, one serving the minority (about 25%) of South African learners, mostly from wealthier backgrounds, which is largely functional and can favourably be compared to the standards of schooling in developed countries, and the other serving the majority drawn mostly from poorer and economically disadvantaged populations, where many of the schools are dysfunctional.

Due to this disparity of schooling in South Africa, it is important to be clear about the context in which my study is located. The schools, teachers, and learners; that I refer to in this review as well as those in my study serve populations from the second system of education. Below I discuss the two broad problems identified in the literature: teachers' foundation knowledge base and differential access to productive discourses in working-class settings and how these impact on responsive teaching.

Problems associated with foundational knowledge base

As noted already, a foundational knowledge base is necessary, though not sufficient for responsive teaching action. In South Africa, the 'necessary' element is a problem, with a significant body of evidence that points to extensive gaps in teachers' conceptual knowledge of primary mathematics (Carnoy et al., 2012; Spaull, 2013b; Taylor, 2011; Taylor & Taylor, 2013; Venkat & Spaull, 2015) and pedagogy in primary mathematics classrooms (Adler & Venkat, 2014; Askew, Venkat, & Mathews, 2012; Ensor et al., 2009; Reeves & Muller, 2005; Venkat & Adler, 2012; Venkat & Naidoo, 2012). Gaps in the foundational knowledge base indicate likely difficulties with moves towards the ideals of responsive teaching described in the international literature.

This point was emphasised again by the study of Carnoy et al. (2012) where they found that teachers with below average mathematical content knowledge taught lessons of lower quality in terms of access to mathematical content to their learners. The consequences of gaps in teachers' content knowledge base are often situations where teachers remain uncomfortable about exploring alternative learner mathematical solution strategies, contributing to a rush to completing the curriculum without emphasis on developing learners understanding. This in turn might constrain openings for responsive teaching actions.

Another strand of the problems relating to foundational knowledge base that might constrain openings for responsive teaching is associated with pedagogy in classroom. In addition to the problems of pedagogy relating specifically to additive relations teaching (Ensor et al., 2009; Venkat, 2013; Venkat & Adler, 2012), there are other studies that point to gaps in the general pedagogic mathematics knowledge base. For example, Sorto and Sapire (2011) analysed teaching quality in 38 grade six lessons in Gauteng province in South Africa. They examined closely three components of teaching quality: mathematical proficiency in the lessons; level of cognitive demand, and level of observed mathematical and pedagogical knowledge of the teachers. In terms of mathematical proficiency, their findings revealed a prevalent practice of recall of rules and definitions or performance of algorithms with no underlying concepts. Lesson planning indicated higher level of cognitive demand, but the majority of lessons were enacted with low cognitive demand.

In relation to teachers' knowledge and application of this knowledge in the classroom, they found that most teachers were unable to probe learners' conceptual understanding, and there was inappropriate use of mathematical terminology and lack of accuracy in mathematical language when explaining concepts was also evident in the lessons. These issues relate directly to gaps in the pedagogical knowledge base in classroom.

Differential access to productive discourses in working-class settings

A survey of literature into classroom-based research in primary schools points to issues relating to differential access to productive discourse within the two schooling settings in

South Africa that I highlighted earlier. Using the Bernstein's notion of framing, Hoadley (2005) examined how social class differences were reproduced through pedagogy. Framing describes the relative control teachers and learners have over selection, sequencing, pacing, evaluation and hierarchical rules in the course of classroom interactions. Framing is expressed in terms of its strength or weakness. Using Bernsteinian notation, F^{++} represents very strong framing (or teacher control) and F^{--} represents very weak framing (greater control by learners).

In the course of analysis of empirical data of classroom teaching in South Africa, Hoadley (2005) noted episodes of teaching in working-class settings where the Bernsteinian notions of strong and weak framing seemed not to apply. Hoadley devised an extension to this language of description to capture the essence of a phenomenon that she described as F^0:

> **F0** – It appears as if no attempt is made to transmit the concepts and principles in the instructional practice. What counts as a successful production in terms of instructional knowledge is therefore totally unclear. The purpose of the task/activity/discussion is unclear. Learners are unclear as to how to proceed, or they are only given criteria relating to how they should *behave* (Hoadley, 2006 p. 28)

The consequence of this practice is a situation in which learners may well remain unaware of the extent to which their offers and narratives are 'endorsable' from a mathematical perspective. As noted in her illuminating examples, Hoadley (2005) characterises the teachers' pedagogic practices in differential class settings in terms of two pedagogic modalities:

- ***Horizontal modalities*** – In the working-class settings, teachers deploy what can be described as 'restricted code', in which meanings are concrete and context-dependent, knowledge is close, local, familiar, and fragmented. Learners were learning to name the world.
- ***Vertical modalities*** **-** In the middle-class settings, teachers make available knowledge and opportunities for learning that can be described as a more 'elaborated code'; where meanings in the classroom are more context-independent, and knowledge goes beyond local space, time, context: "A potential of such meaning is disorder, incoherence, a new order, a new coherence" (Bernstein, 1986, p. 182). Learners were learning to characterise the world

Hoadley notes that in the two schooling contexts, teachers are confronted by learners who enter their classrooms with very different coding orientations. In the middle-class settings, the majority of learners construct an elaborated coding orientation from home and bring this experience to school. This orientation is consistent with what the school system privileges: i.e. the acquisition of context-independent ways of organizing experience. Therefore, classroom discourse is constructed based on elaborated codes with more focus on the content, and learners learn more mathematics. In contrast, within horizontal modalities, in the working-class context, learners largely make meaning and negotiate experience according to a restricted coding orientation from home that is brought to school settings. Hoadley's analysis shows that these teachers often did not teach the elaborated code associated with formal knowledge, and instead, 'pulled back' into a restricted code and constructed their practice along the lines of horizontal modalities. Hoadley commented that: 'the pedagogy fails to interrupt the learners' restricted orientation and does not specialise their voice with respect to the school code' (p. 265).

Broader issues and policies in the South African terrain also feed into the ways in which teacher responses are configured. Highly procedural orientations (Ally & Christiansen, 2013) coupled with selections of low cognitive demand tasks have been noted (Carnoy et al., 2012). Chorusing practices, involving collective chanting of answers have been raised as concerns in relation to the lack of openings for individuation of learning and evaluation thereof (Hoadley, 2012). Conversely, Venkat & Naidoo (2012) also point to a lack of move of individual offers into the collective classroom space in primary mathematics teaching.

Taken together, these issues point to substantial contextual evidence that a range of resource constraints encompassing histories of differential access to schooling in South Africa. Gaps in the mathematical knowledge base, and policy responses emphasising a press for coverage and standardized pacing have led to calls for, and subsequently, moves towards, much more tightly prescribed national curriculum specifications. Thus, currently, national mathematics curricula specify content coverage, sequencing and pacing at weekly levels (DoBE, 2011); with provincial-level interventions providing teachers with scripted lessons at the daily level (GDE, 2011). These issues militate against possibilities for the move towards more responsive teaching in working-class settings. This finding therefore points to the need for constructing a form of teaching practice that can support teachers in working-class settings to move towards more vertical pedagogic modalities - a missing gap in the South African writings, relating to primary school mathematics teaching that the present study aims to explore.

To address issues related to gaps in teachers' foundation knowledge base, and aspects of connections, and transformation knowledge, the WMC-P project incorporated an in-service professional development course in which the present study is located. I therefore briefly outline the work of WMC-P project.

2.3 The work of Wits Maths Connect – Primary (WMC-P) project

The WMC-P project is a 5-year longitudinal research and development project targeting primary mathematics teachers from 10 government primary schools in Johannesburg, South Africa. The WMC-P project has four interrelated objectives: (i) to improve the quality of teaching of in-service teachers at the primary school level; (ii) to improve learner performance in primary school mathematics; (iii) to research sustainable and practical solutions to the challenges of improving numeracy in primary schools; and (iv) to provide leadership in numeracy education and increase dialogue[s] around solutions for the mathematics education crisis in South Africa.

A key initiative within the broader WMC-P project was a pilot of 20-day teacher professional development – 'maths for teaching' course in 2012, with a focus on developing and deepening teachers' mathematical content knowledge from a pedagogical perspective, with framing based on the planning-oriented dimensions of Rowland et al's knowledge quartet. This frame was informed both by evidence that professional development that attends to dimensions of teachers' mathematical knowledge is more effective than professional development that focuses only on pedagogy or generic teaching skills (Garet, Porter, Desimone, Birman, & Yoon, 2001; Heck, Banilower, Weiss, & Rosenberg, 2008), and by an awareness of the mathematical knowledge related gaps described as widespread in South Africa.

The course was designed with 16 days in eight 2-day blocks across the year interspersed with 8 half days for trying out classroom tasks; 4 teachers per school were drawn from the 10 project schools. The aim of running in 2-day blocks across the school year was to ensure that participants had time to complete primary mathematics focused homework tasks, and school-based work relating to course foci across the year. This longitudinal model was informed by research evidence showing that 'once-off' or short term workshops have limited effect (Joyce & Showers, 2002), and recognition of the cumulative nature of mathematical learning.

In each 2-day block, the course was focused on a range of key areas of primary mathematics topics areas, but given the emphasis on number in primary mathematics, more emphasis was placed on number-related topics in the following sequence: Block 1 - numbers and the number system; Block 2 – additive relations; Block 3 – multiplicative reasoning; Block 4 - ratio and proportion; Block 5 - patterns, relationships and algebra; Block 6 – fractions, decimals and percentages; Block 7 – Word problems; and Block 8 – Shape, space, data handling and probability. The course included a pre- and post-test focused on conceptual understanding of primary level mathematical concepts using items drawn from a range of prior studies including Hart et al.'s (1981) CSMS studies and Ryan and McCrae (2006) TEMT studies. During the course, interim assessments based on hypothetical classroom scenarios aimed at assessing teachers' mathematics knowledge for teaching related to the topic in focus in the previous block were administered from Block 2 onwards.

The WMC-P professional development course model was premised on the fact that there is a critical need to develop primary school mathematics teachers to become more competent and confident about mathematics and its teaching, and to develop, through trialling, a programme of professional development for in-service primary mathematics teachers. The 20-day course hoped to develop teachers' planning-related mathematical knowledge in these areas with the aim of feeding into improvements in quality teaching of primary mathematics. The notion of developing primary mathematics 'content from pedagogical perspectives' entailed focus on selection of tasks and expansion of example spaces, models and representations, dealing with common learner errors, progression and connection of mathematical ideas, giving appropriate explanations of familiar traditional methods, etc. These foci feature prominently within foundation, connection and transformation dimensions of the knowledge quartet.

The broader WMC-P project had other interventions. One of these was the Lesson Starter project (LSP), focusing on improving number teaching in the Foundation Phase. The focus of the LSP was linked to the national South African Curriculum and Assessment Policy Statement (Department of Basic Education, 2011) and the provincial Gauteng Primary Language and Mathematics Strategy (http://gplms.co.za/) that together prescribed content, sequencing and teaching timeframes. Several of the WMC-P partner schools were under pressure to follow these policy drivers, so LSP focused on supporting teachers in the policy mandated 'mental mathematics' within 'whole class activity' lesson sections. The two Foundation Phase (Grades 1-3) teachers in this study participated in both the 'maths for teaching' course and the LSP project.

In the next section, I focus on the second body of writing that is relevant to this study – the literature base relating to additive relations as the mathematical content area addressed in all the lessons observed in this study. This literature base provides evidence relating to the features of this content domain. It was therefore useful in that I could use it as a vantage point for understanding and commenting on the nature of teacher's contributions seen in this study, and explore possibilities for responsive teaching within this content domain. I use the term 'additive relations' to refer to mathematical (contextualized and context-free) problems involving addition and subtraction. In this study, teachers were drawn from both Foundation (Grade 1-3) and Intermediate (Grade 4-7) phases, and therefore additive relations could include working with fractions and decimals, as well as whole numbers.

I begin this section with the South African evidence of problems associated with teaching additive relations that motivated my focus on this content area, before engaging with what international literature suggests about good teaching of additive relations. I focus specifically on: progression in problem types, strategies, models and representations; and connections and progression in the selection of examples.

2.4 Additive relations literature

2.4.1 Problems associated with the teaching of additive relations in South Africa

The South African Curriculum and Assessment Policy Statement (CAPS) specifies the importance of number related work (number operations and relationship) in primary school mathematics with the content weighting ranging from 65% in Foundation Phase to 50% at the end of intermediate phase for number learning (DBE, 2011). This weighting of content areas provides guidance on the spread of content in assessment. Early number learning (ranging from counting, number bonds to number relations) is pre-knowledge needed for additive relations understanding, and strong understanding of additive relations is a building block for understanding other mathematics content in the curriculum. Additive relations is therefore seen as a foundational content area in the primary mathematics curriculum.

Research evidence shows that many South African children perform poorly in additive relations and other number related work even in the later primary school years (Schollar, 2008). This poor performance has been linked to teachers' mathematical knowledge and pedagogy in mathematics classroom (Askew et al., 2012; Ensor et al., 2009; Venkat, 2013; Venkat & Naidoo, 2012). These issues are: disruptions in use of givens/unknowns within explanations, connections, and the move from more concrete counting to more abstract calculation strategies while teaching.

In her keynote address, Venkat (2013) shared several instances of disruptions to coherence in primary mathematics teaching. One example was presented of an episode where a teacher demonstrated lack of awareness of 'givens' and 'unknowns' within her explanations in teaching:

> A Grade 1 class are working on number bonds of 5. Each learner pair has a set of 5 bottle tops in front of them. The teacher asks for two numbers that add up to 5. Learners produce a 4/1 split. The teacher asks learners to separate the two groups. She then says: 'Put them all together and count how many you have altogether (p.7).

The mathematical purpose of the task is to draw learners' attention to bonds of 5. However, asking learners to count the 4 and 1 bottle tops when putting them back together is unnecessary given that learners were reminded that they had started with 5 bottle tops. This questioning and counting of the total, repeated several times, treats a 'given' quantity as an unknown, and thus disrupts mathematical coherence.

In another study of classroom teaching in South Africa, Venkat and Naidoo (2012) analysed connections in the language used by the teacher to communicate meaning to mathematics learning. They used systemic functional linguistics (Halliday & Hasan, 1985) and variation theory (Marton, Runesson, & Tsui, 2004) to argue that meaning is constructed through strong connections between the teacher talk within and across episodes in the lesson and activities/materials used to develop conceptual understanding. Their findings point to gaps in teachers' understanding of the nature of mathematics and the purposes underlying task sequencing and connecting in teaching number concepts. Their claim of a lack of connections of mathematical ideas within and across episodes was referred to as ‘extreme localisation’ (Venkat and Naidoo, 2012).

In another strand of evidence linked to the notion of ‘extreme localisation’ in Venkat and Nadoo’s (2012) study, Ensor et al. (2009) found that learners remained highly dependent on concrete representations for solving problems at Grade 3 level. Teachers provided limited opportunities for learners to grasp symbolic number conceptions within classroom practices that “privilege concrete modes of representation, which restrict access to more abstract ways of working with numbers” (p. 5). They posited that teachers were simply not presenting enough mathematics at a sufficiently complex level in terms of content and representations to learners.

Taking the foundational role of content of additive relation in the curriculum, and extensive evidence of problems with the teaching of this content area together provided a strong rationale for the focus on additive relations as a suitable content area in which to examine and develop responsive teaching in South African context. In the next section, I review international literature on key features of teaching additive relations. In doing so, I provide detailed discussion on progression in additive relations problem types, structure and variation of the portion of the unknown before moving into progression in models and representations. These ideas featured prominently in the empirical data of classroom teaching that I analysed in this study.

2.4.2 Progression in additive relations problem types

In researching children’s thinking in a project referred to as Cognitively Guided Instruction (CGI), Carpenter et al. (1999) provide distinctions among different types of additive relations word problems that reflected the way children thought about solving them. This problem types categorization provides a useful structure for teachers in selecting problems for instruction and thinking about how children can approach these problems to support effective classroom interactions in a progressive order. They identified four classes of word problems, and presented them as being in a hierarchy from easiest to most difficult. These problem types are useful as a starting point for thinking about this content domain. The four classes of word problems presented in hierarchical order are join, separate, part-part whole and compare. These authors also note that varying the position of the unknown within these classes can change the difficulty of the problem.

Level 1: Join problems

Join problems involve a situation in which a set is increased by a particular amount. For example "3 birds were sitting in a tree. 2 more birds flew into the tree. How many birds were in the tree then?" (Carpenter et al., 1999, p. 7). In join problems there is an action taking place with start quantity being increased by a particular quantity (the change quantity) to give another third quantity (the result of joining the two quantities). They further provide three distinct join problem types involving variation in the position of the unknown, hierarchically arranged from easiest to most difficult. Start unknown problems in the list below are much harder because the operational action underlying these change problems cannot be executed directly.

- Result unknown: Situation where the start and the change quantities are given, and a child is require to work out the result (e.g. 9+7 = __)
- Change unknown: Situation where the start and the result quantities are given, and a child is required to work out the change quantity (e.g. 9+__=16)
- Start unknown: Situation where the change and the result quantities are given, and a child is required to work out the start quantity (e.g. __+7=16)

Level 2: Separate problems

Separate problems involve a situation in which a quantity is removed from the given set. For example, "Colleen had 8 guppies. She gave 3 guppies to Rodger. How many guppies does Colleen have left?" (p. 9). In the separate problems, an action takes place where the start quantity is decreased by a particular quantity (the removed or change quantity) to leave another third quantity (the result after removing a particular quantity). Similarly, three further separate problem types with variation of the position of the unknown and hierarchically organised from easiest to harder are given as follows:

- Result unknown: Situation where the start and the removed quantities are given, and a child is require to work out the result (e.g. 8-3 = __)

- Change unknown: Situation where the start and the result quantities are given, and a child is required to work out the quantity that is being removed (e.g. 8-__=5)
- Start unknown: Situation where the removed and the result quantities are given, and a child is required to work out the start quantity (e.g. __-3=5)

Level 3: Part-part whole problems

The part-part whole problems involve a static relationship between a set and its two disjoint subsets. The difference between join/separate problems with part-part-whole problem is that while in join or separate problems, there is an action of either bringing in an additional quantity or removing a quantity over time, in part-part-whole problems, there is a static relationship between the two given quantities, which requires simultaneous, rather than sequential attention to parts and/or whole. Thus, there is no physical action of increasing or decreasing with a particular quantity. Carpenter et al argue that this lack of physical action makes start unknown problems in this category more difficult for children than the problems discussed in levels 1 and 2. For example, “10 children were playing soccer. 6 were boys and the rest were girls. How many girls were playing soccer?” (p. 9). In this problem, the 10 children are in one set; a **whole** (those playing soccer) and within this set, there are two distinct subsets; **parts** of the whole (boys and girls). Two part-part-whole problem types are described. These are:

- Whole unknown: Situation where the two disjoint subsets are given, and a child is required to work out the result (e.g. 6+4=__).
- Part unknown: Situation where one subset quantity (a part) and the result quantity are given, and a child is required to work out the other subset (a part) quantity (e.g. 6+__=10 or 4+__=10)

Level 4: Compare problems

Compare problems involve a situation that depicts comparison between two distinct disjoint sets. For example “Mark has 3 mice. Joy has 7 mice. Joy has how many more mice than Mark?” (p. 10). In compare problems, there is a referent set (Mark has 3 Mice), compared set (Joy has 7 Mice) and a difference (Joy has how many more mice than Mark). The compare problem type has to do with situations in which two sets (Mark’s mice and Joy’s) are considered simultaneously - what Carpenter and Moser (1984) describe as “static relationships”, involving ‘the comparison of two distinct, disjoint sets’ (p. 15). This contrasts with join and separate problem types, which involve an action on and transformation of a single set makes compare problems more difficult for children. Another strand of difficulty for children in interpreting compare problems is the issue of keywords. Teachers have been noted as often attributing ‘more’ to adding and ‘less’ to subtracting. This link does not always hold, and therefore emphasis on sense making of the quantitative relationships in the situation is important: for example, Mike has 14 shirts, Sam has 9 shirts. How many ***more*** shirts does Mike have than Sam? Keywords approaches frequently lead children to associate ‘***more***’ with addition calculations, which would be inappropriate in this situation. There are three compare problem types according to Carpenter et al. (1999) that are also hierarchical in terms of difficulty. These are:

- Difference unknown: Situation where the referent and the compared sets are given and a child is required to work out the difference.
- Compared set unknown – Situation where the referent set and the difference are given, and a child is required to work out the compared set.
- Referent set unknown: Situation where the compared set and the difference are given and a child is required to work out the referent set.

This map of problem types provides a useful way of thinking about possibilities for range, progression and sequencing in teachers’ selection of examples in instruction. The Carpenter et al. (199) classification is summarized in Figure 3 with arrows indicating the directions of progression in sophistication levels of classes and problem types as discussed above. Important to note that two of these classes of problems are linked to models of subtraction; separate problems are associated with ‘take away’ model, while compare problems are associated with ‘difference’ model.

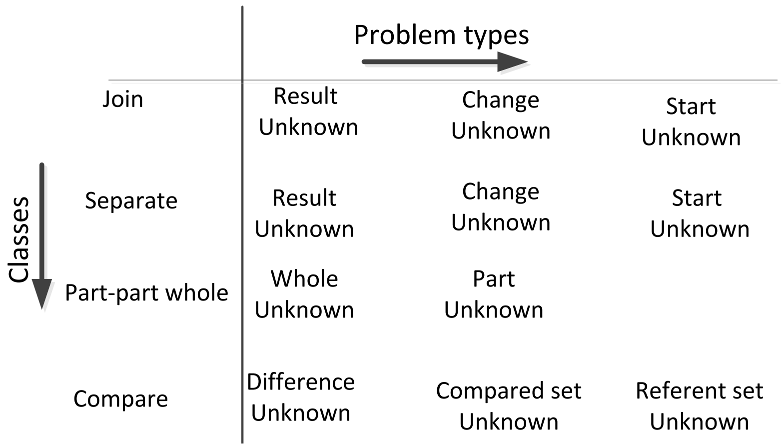


Figure 3: Summary of additive relations problem types by Carpenter et al. (1999)

Following the above categorization of problem types and classes by Carpenter et al. (1999), other researchers provide refinement of the problem types with slight differences in the terminology used. For example, Clements and Samara (2009) combined join and separate problems and called them change problems (with change plus as join and change minus as separate). Askew (2012) introduced a further refinement to Clements and Samara's categorisation by renaming 'change plus' to 'change increase', and 'change minus' to 'change decrease'. He does this in recognition of the fact that either operation (plus or minus) can be used to solve either of the change problems.

Within the CGI project, Carpenter et al. (1999) also identified three strategies commonly used for solving additive relation problems and considered these in a hierarchy of mathematical sophistication:

- Direct modelling;
- Counting; and
- Calculating.

Direct modelling for Carpenter et al. refers to the use of explicit physical representations of the quantities involved to enact the situation for a given problem. This involves the use of concrete apparatus - real objects, counters, or drawing pictures of real objects.

When a child starts counting to solve a mathematical problem either with tallies or fingers, without constructing a concrete situation to enact a problem, then for Carpenter et al. (1999), this represents progress to the second level: counting strategies. Within counting strategies, there are trajectories within addition and subtraction. While counting strategies are more efficient than direct modelling, they are inefficient when dealing with larger numbers. In these cases, more sophisticated calculating strategies need to be developed. Calculating strategies rely on understanding of number facts. A child who is calculating is able to work out 31+29 using a known fact; for example, by taking 1 from 31 and giving it to the 29, a double: 30+30 is created, that gives 60. Bridging through tens, doubling and halving, and compensation are all examples of calculating strategies.

Thompson (2010) provides a summary of the hierarchy of progression of strategies for addition and subtraction problems

- addition – counting all, counting on from first number, count on from larger number, flexible counting using 'friendly' numbers, and using recalled and derived facts.
- subtraction – counting all, counting down from (take away model), countdown to (difference model), counting up from (difference model), flexible group counting using 'friendly numbers' or 'benchmark numbers' (Mcintosh, Reys, & Reys, 1992), and using recalled and derived facts

2.4.3 Models for teaching additive relations

Two models are strongly advocated in the literature for teaching additive relations. These are: number lines, advocated in the Dutch Realistic Mathematics Education (RME) literature (Beishuizen, 1999) and part-part whole relation representations (Cobb, Boufi, McClain, & Whitenack, 1997). These models provide different conceptual views of number relations. The former is focused on an operational view, and rooted within counting strategies, with possibilities for increasing sophistication. The latter pushes towards more structural views of number relations in terms of two parts and a whole relations. I review each separately below.

The number line

There are basically two types of number line; structured (or closed) and empty number line (or open). Freudenthal (1973) suggested that a structured number line with marks for every number was a more natural model of children's informal counting strategies than arithmetic blocks. The structured number line was constituted in measurement situations and so was associated with rigid fixed distances. Gravemeijer (1994) argued though that the use of the structured number line caused counting to be conducted as a passive reading of the answer on the number line, which did not raise the level of the strategies the learners used to solve additive relations problem. This criticism of the structured number line led to the use of the empty number line in RME.

The empty number line allows learners to draw marks for themselves instead of the fixed marks in structured number lines. It facilitates the use of counting strategies, providing a means to record intermediate steps in the process. For example, in the calculation of 45 +18, the 45 is written towards the left of the line. The 18 may be seen as 5+13 and counting 5 first from 45 to get to 50 and then the remaining 13 may be counted as one straight jump from 50 to 63, or split into 10+3 and counted 60, 63 (See Figure 4). An empty number line allows children to count on in the jumps with which they are comfortable, in ones initially and subsequently, by bridging through multiples of 10 or adding 10s (Beishuizen, Van Putten, & Van Mulken, 1997). A different, and more efficient, strategy might be to count on 20, arriving at 65 on the line, and then to compensate by counting back 2 to arrive at 63 (see Figure 5).

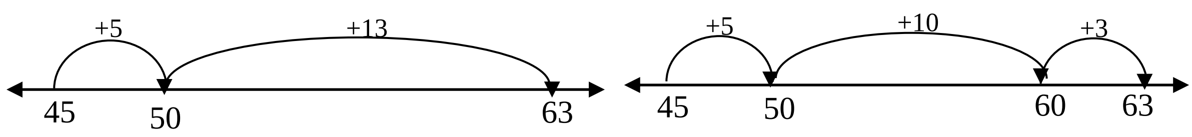


Figure 4: Empty number line representation of 45+18

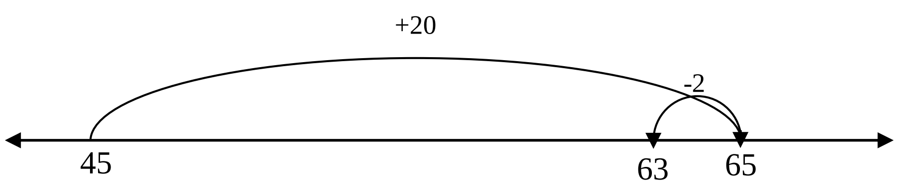


Figure 5: Empty number line representation of 45+18 using a compensation strategy

Following a comprehensive review of literature on the importance of the empty number line, Klein, Beishuizen, and Treffers (1998) articulate four reasons for using the empty number line in teaching:

- It is well-situated to link up with informal solution procedures because of the linear character of the number line.
- It provides opportunities to raise the level of learners' activity.
- It stimulates a mental representation of numbers and operations (addition and subtraction in particular).
- It can eventually help learners to develop an internalised model, where learners keep track of what they are doing without dependence on visualization, leading to a reduction of the memory load while solving a problem.

Part-part whole relations

While the empty number line pushes towards the more operational conceptions of number relations foregrounded within counting strategies, proponents of structural views advocate understanding of additive relations fundamentally as a relation between parts and wholes (Schmittau, 2003). One representation that pushes towards this structural view is the part-part whole bar diagram (see Figure 6). Understanding that numbers can be represented in many ways is central to the part-part whole relations. The structure within part-part-whole make it easier to 'see' the connections between addition and subtraction (with emphasis on number relations rather than on operations), which is vital to learners' conceptual understanding of additive relations. Activities that involves composing and decomposing numbers using a given total so that learners can focus on the parts that create the same whole (Cobb et al., 1997) help learners to develop this understanding. Numbers can be composed and decomposed in many variations for a given value. This understanding suggests teaching strategies for additive relations that encourage learners to work systematically to break up and combine numbers considering the whole, and each of the two parts, to deepen their conceptual understanding of additive relations. In particular, it also supports learners in solving additive relation problems with varied positions of the unknowns (Venkat, Ekdahl, & Runesson, 2014).

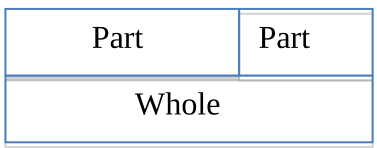


Figure 6: Part-part whole diagram

This structural view introduces the decomposition problem type which is distinct from the problem-types described above by Carpenter el al. (1999). A structural view draws attention to the idea of splitting a 'whole' into two 'parts' introduced with an activity for example, of splitting 7 monkeys between two trees (Cobb, Boufi, McClain & Whitenack, 1997). Here part-part-whole diagrams and images of number sentences are used to show the relationships between breaking down and recombining numbers (for the systematic development of number bond patterns) as the focus of attention.

The development of structural understanding of part-part whole relations is argued to allow learners to be flexible in their choice of strategy. The push in this approach is for learners to think of additive relations as combinations of parts and wholes. In any additive relations situation, problems can be represented in terms of a whole and two parts. In this classification, if the two parts are known, then the goal is to find the whole. In the other, if one part and a whole are known, then the goal is to find the other part. This kind of understanding offers a general structure to represent any additive relations problem (including all problem types described by Carpenter et al., 1999) and thus, proponents of the structural view argue that this supports learners to make generalizable number sentence statements:

Part + Part = whole

Whole – Part = part

Rather implicit, but nonetheless critical for the models and representations of additive relations discussed above, are selections of examples that motivate the need for the different concept structures and progression in strategies described. For example, learners need to be provided with example like 5 + 48 to see the necessity for using the additive commutativity property. If the teacher only presents 48 + 5, there is no motivation for this understanding. Also the need to provide examples that work across the different concept structures of addition and subtraction problem types suggested by Carpenter et al (1999) is essential. Being able to see connections and progression is therefore dependent in some ways on seeing a broader example space, and skills in selecting and sequencing examples are therefore described as significant for describing teaching quality. Haylock (2006) states:

> Pupils should be taught to: understand addition and use related vocabulary; recognise that addition can be done in any order; understand subtraction both as 'take away' and 'difference' and use the related vocabulary; recognize that subtraction is the inverse of addition; develop further their understanding of addition and subtraction; understand why the commutative law applies to addition; choose and use addition or subtraction to solve problems in 'real life', money or measures of lengths, mass, capacity or time. (P.29)

Rowland (2008) emphasises the need for the range of examples selected to reflect the range of concept structures related to an idea (additive relations in this study). For instance, in the example 5+48, a variation related to order of addends is opened. By opening this variation, a feature of the concept of addition, that the sum is independent of order of the addends is made possible to discern (Marton et al., 2004). The teachers' selection and use of representations and strategies in teaching additive relations is grounded within their selection of examples. Therefore the opportunities to learn about the breadth and depth of the concepts of additive relations, which the teacher provides through their choices of strategies and representations forms an important base for describing in-the-moment responsive teaching. While this aspect of teacher knowledge is located within transformation knowledge of the KQ framework, teacher's awareness in drawing on such ideas in responding to learners' inefficient strategies falls within the contingency category.

In addition to the above, there is also attention to two models of subtraction: as 'take away' and as 'finding difference' as key ideas within the teaching of additive relations. Flexibility in the use of strategy linked to progression is necessary for efficiency, and therefore within progression, making the distinction between the two models of subtraction important to be aware of in contingency situations.

These two models can be presented on an empty number line as strategies that offer efficient ways for solving context free subtraction problems. When the subtrahend and the minuend are close to each other (e.g. 2003 – 1998), subtraction as difference is a preference because you need fewer jumps to get to the answer. The two numbers can be written simultaneously on a number line with either 'count-down from' or 'count on' used to work out the difference (see Figure 7). If the subtrahend and the minuend are far from each other (e.g. 2005 – 7), 'take away' means fewer jumps to get to the answer (see Figure 8).

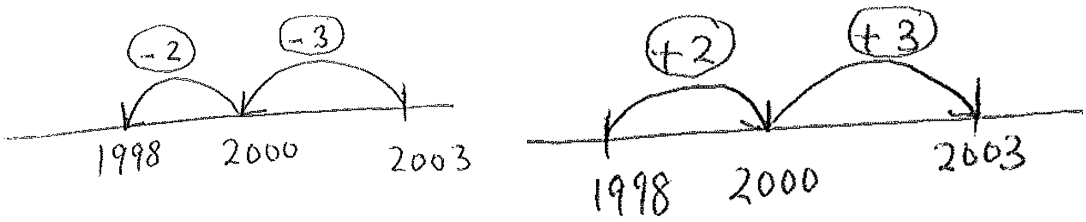


Figure 7: Solution of 2003-1998 by finding difference on ENL

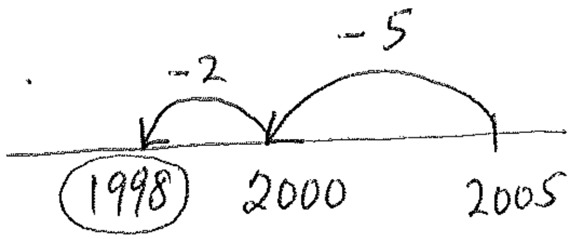


Figure 8: Solution of 2003-7 as 'take away' on ENL

An elaboration in response to learners’ solution action that points to awareness of working out an answer as ‘difference’ as well as ‘take away’ therefore draws on transformation knowledge (seen in the inclusion of appropriate examples) and connection knowledge (seen in the enactment of the examples in coherent ways) in a contingent situation.

Progression in models and representations for teaching additive relations

The discussion of the problem types and strategies for solving additive relations problems points to the importance of progression in the teaching of additive relations. Shifts in models and representations have been described as an important part of this progression. In this section, I discuss progression in the use of models and representations for additive relations. I draw on the earlier work of Bruner to argue that while there is a hierarchy of progression, this pathway is emphatically not a linear process in teaching (Roberts, 2015). Teachers need to decide when and how to use the different levels of representation to support learners mathematics learning in the classroom.

In the early years of teaching and learning of mathematics, physical and pictorial representations are widely used in order to enhance teaching and learning process. These representations act as “intermediaries between the concrete and the abstract” (Rowland et al., 2009, p. 42). Bruner (1974) suggested three hierarchical, but complementary modes of representation. The first level, which links to direct modelling, is the enactive level - where learning takes place through physical actions and includes the use of manipulatives and other concrete materials (e.g. counters, abacus, bottle tops, etc). The next level, the iconic level, is where learners make use of pictorial images in understanding the world. The highest level, the symbolic level – is where mental manipulation of numerals takes place. These levels are like steps on a ladder, where the first step is the most concrete and further up the ladder, the more abstract the form and its associated connections become. In another word, the levels are broadly progressive and relate to increasingly ‘compressed’ representations of number.

While arguing for the central role of concrete and pictorial representations in early primary mathematics teaching, Anghileri (2000) cautions against over reliance on concrete representations.

> It is important that children do not come to rely on using such materials [concrete representation] for modelling numbers, but that they develop mental imagery associated with these materials and can then work with ‘imagined’ situations (p. 112)

An over reliance on the use of concrete representations at both levels of teaching and learning has been highlighted as one of the possible reasons for learners’ poor performance in primary mathematics in South Africa (Ensor et al., 2009; Schollar, 2008). When the number range gets bigger as prescribed in the South African CAPS, working becomes increasingly long-winded and error-prone when using concrete representations.

The aim of using concrete representations at early years of primary mathematics teaching is to help learners develop mental imagery associated with number structure and therefore teaching should motivate these progressive moves in the use of representations. This discussion is important in my study, as I explore how teachers respond to what they perceive as learners’ inefficient mathematical solution actions. Delaney (2001) echoed similar concerns that the use of representations should be to allow learners to *internalise* mental images of quantity and quantitative relations, and to use these mental images when the representations are either not physically present or no longer required.

The lack of linearity in moving between concrete to abstract representation in number conceptions makes these understandings complex. Teachers need to decide when and in what ways to support the development of more concrete or more abstract notions of number. This suggests that selection and effective use of appropriate mathematical representations requires careful consideration and planning on the part of the teacher, which is largely informed by the teacher's transformation knowledge. But working with this knowledge flexibly when faced with correct or incorrect answers in classroom is part of teacher's contingency knowledge and is needed to support effective learning of mathematics.

2.5 Conclusion

Research studies of mathematics classroom interactions highlight the intricate relationship between teachers' mathematical knowledge and responsive teaching, and research in this domain points to the complexity of responsive teaching. This complexity is noted in the literature that teachers' mathematical knowledge is necessary (though not sufficient) for responsive teaching, and that responsive teaching requires increasing awareness of the triggers of contingencies in the classroom. In this chapter, I have described, with empirical examples, the conceptualization and analysis of triggers of contingency featured within the international literature. I have also noted that there are contextual issues within South African primary mathematics teaching landscape that tend to constrain openings for responsive teaching as figured in the international literature. Hence, the present study aims at categorizing and developing an in-depth and 'home-grown' analysis that can serve as stages of implementation of responsive teaching in primary mathematics classrooms in South Africa. In the next chapter I discuss the theoretical resources that I drew on to frame my research.

CHAPTER 3

THEORETICAL FRAMING OF THE STUDY

3.1 Introduction

The purpose of this chapter is to position the study theoretically. According to Bogdan and Biklen (1982) all research is guided by some theoretical orientation, whether stated or not. These authors consider good researchers to be those who are always aware of their theoretical base and use it as a guide to collect and analyse data.

My theoretical stand is located within what has been termed an emergent approach with a view to explore the nature of primary school teachers' responses to in-the-moment situations in the mathematics classrooms as discussed in the literature. I have drawn from the work of Cobb, Yackel, and Wood (1992) and Bauersfeld (1995), whose theories are grounded in both radical constructivism (Von Glasersfeld, 1991) and symbolic interactionism (Blumer, 1969). They refer to the coordination of these two perspectives as the emergent approach. I begin with an outline of the underlying philosophical and epistemological strands of the emergent theoretical approach before providing my rationale for the usefulness of this approach as a theoretical lens in this study.

3.2 An emergent approach

Attention to learners' contributions within lessons has historically formed a key part of constructivist views of learning. While constructivism as a theory of learning has been widely recruited to understand learning and learners, there have also been calls for pedagogy to fit with learners' ways of learning mathematics within constructivist perspectives (Cobb, Yackel, and Wood, 1995; Bauersfeld, 1995). Wood, Cobb, and Yackel (1995) emphasize that

> …teachers must ... construct a form of practice that fits with their students' ways of learning mathematics. This is the fundamental challenge that faces mathematics teacher educators. We have to reconstruct what it means to know and do mathematics in school and thus what it means to teach mathematics (p.127)

In response to this call, Simon (1995) advocates a theoretical model for reconstructing mathematics pedagogy within both sociological and cognitive constructivist perspectives. Central to Simon's model is the "creative tension between the teacher's goals with regard to student learning and her responsibility to be sensitive and responsive to the mathematical thinking of the learners" (p.114). This viewpoint provides the underpinning position for teaching informed by emergent approach, which is seen as improvisational and responsive to the development of students' learning as individual and as collective in the social context.

The emergent approach evolved in the context of a developmental research project, where the aim of the project was to investigate ways for supporting elementary school students' conceptual development in mathematics (Cobb and Yackel, 1995). To achieve this aim, the research team designed sequences of instructional activities for students and an approach to professional development for teachers in the context of a teaching experiment (Steffe, 1983). Their initial conceptualization of the teaching experiment was that the researcher would interact one-on-one with a single child and attempt to influence his/her constructive activities (Steffe, 1983; Steffe, Richards, & Cobb, 1983). Within this conception, learning was viewed more exclusively in psychological constructivist terms, which accounted for the development of the child's ways of reconstructing mathematical knowledge while interacting with the researcher. However, it became apparent to them that this perspective was inadequate for providing an account of how learning (and teaching) occurred in the social context of the classroom. Cobb (1989) points out that as a result of this inadequacy, the coordination of social and psychological perspectives become necessary to understand learning in the classroom context. The intersectionality of these two perspectives offered a means to develop an interpretive framework which is referred to as an 'emergent approach'.

The interpretive framework coordinates the social perspective – an interactionist perspective on collective classroom practice (Bauersfeld, 1995) and a psychological perspective – a psychological constructivist perspective on individual student (or teacher) activity as they participate in and contribute to the development of collective classroom discourse. The key features of the interpretivist framework at the classroom level are shown in Table 1. The 'emergent' approach has demonstrated the importance of individual knowledge construction and social interactions that have shown the focus on teaching as an improvised interactional activity that can reveal many insights into how learning takes place (Cobb and Bauersfeld, 1995).

Table 2: An interpretive Framework based on social and psychological perspectives

Social Perspective	**Psychological Perspective**
Classroom social norms	Beliefs about own role, others' role, and the general nature of mathematical activity in school
Sociomathematical norms	Mathematical beliefs and values
Classroom mathematical practices	Mathematical conceptions

Source: Cobb and Yackel, (1995 p. 6)

The three components that are central to the emergent approach; classroom social norms, sociomathematical norms and classroom mathematical practices are discussed.

3.2.1 Classroom social norms

In the extension of the teaching experiment where the focus was shifted from examining individual construction of knowledge to how individual knowledge construction was manifested within the social context, Cobb and Yackel (1995) reported unanticipated issues that arose during small group and whole class discussions on mathematical tasks. The students were to explain and justify their interpretations of mathematical solutions. The teacher's expectations that students could publicly explain their interpretations and solutions of tasks as planned for the teaching experiment proved difficult for children acquainted with traditional classroom practices where Cobb and Yackel noted that students: "take it for granted that they were to infer the response the teacher had in mind rather than articulate their own understanding" (p.7).

Teachers were thus confronted with the challenge to cope with the conflict between their own and students' expectations by initiating a process that was described as renegotiation of classroom social norms (Cobb, Yackel and Wood, 1989). Examples of social norms that needed to be renegotiated between the teacher and among the students included: explaining and justifying solutions, attempting to make sense of explanation given by others, indicating agreement and disagreement, and questioning alternatives in a situation where there is a conflict between different views that are presented about particular interpretations or solutions to a task. Such social norms are regarded as forming the classroom participation structure.

Social norms are clearly not conceived as psychological constructs that can be attributed to any particular individual; rather, they are a joint social construction among all participants in the classroom. While the teacher is an institutionalized authority in the classroom, multiple participants in classroom activity mean that the teacher cannot simply establish social norms; instead, these norms instead had to be initiated and renegotiated with all participants involved. One of the underlying epistemological points was that during the renegotiation process, by students making contributions, they reorganized their individual beliefs about their own role and that of others, and the overall nature of the mathematical activities (Cobb, Yackel and Wood, 1989). A social analysis conducted from the interactionist perspectives documents the evolution of the social norms, while an analysis conducted from psychological perspective documents the students' reorganization of their beliefs. The emergent approach is concerned with both analyses and treats them as complementary to each other. The social norms are seen to evolve as students' reorganize their belief systems and, conversely, the reorganization of these beliefs is seen to be enabled and constrained by the evolving social norms.

The consequence of establishing social norms at the classroom level is particularly important in my study, where it provides me with a theoretical lens to explore the nature and extent to which a social norm that views teaching as improvised and responsive to learner inputs in classrooms exists and develops over time. Conceiving teaching in this way stands in counterpoint to the prevailing contextual norms of an absence of evaluation, procedural teaching, and chorused collective learner responses. My focus was therefore on teaching practices and developments focused on the need to respond, interrogate and renegotiate learner offers as a countervailing social norm.

3.2.2 Sociomathematical norms

While analysis in relation to establishing classroom social norms is vital in exploring classroom participation structure, these norms are a generic construct that can be applied to all subject areas. In order to extend their analysis to examining social norms that were specific to students' mathematical activities in the classroom, Cobb and Yackel (1995) devised the idea of 'sociomathematical norms'. They provide examples of sociomathematical norms such as: "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation" (p. 8).

Sociomathematical norms became an explicit focus of interest in my analysis of lesson enactment within teachers' elaboration of mathematical ideas as critical in support of learning in the classroom. Within my analytical framework focused on elaborations, I have developed categories and codes examining the nature and extent to which sociomathematical norms relating to these kinds of features are in place and developed over time. Increasing focus on elaborations by eliciting alternative approaches to how learners solve the same mathematical tasks, and questioning contributions that do not appear to be mathematically different are vital markers of responsive teaching actions. In responding to teachers' requests for different solutions, Cobb and Yackel (1995) argued that "students were both learning what counts as mathematical difference and helping to interactively constitute what counts as mathematical difference in their classroom" (p. 8). On the other hand, teachers are also developing a form of practice that supports alternative and more efficient mathematical solutions in their course of interactions with learners in the classroom. This is particularly important in my study, both from the perspective of progression in mathematical concepts and the quest for moving learners to more efficient mathematical strategies and representations in solving mathematical tasks.

The development of students' intellectual autonomy was one of the goals of the project in which the emergent approach evolved. Intellectual autonomy refers to "students' awareness of and willingness to draw on their own intellectual capabilities when making mathematical decisions and judgements" (p. 9). The analysis of sociomathematical norms was found to be significantly important in understanding how teachers support learners' development of intellectual autonomy. Cobb and Yackel (1993) argue that students construct specific mathematical beliefs and values that enable them to act as increasingly autonomous members of classroom mathematical communities as they participate in the renegotiation of sociomathematical norms. Initiating and managing the process of renegotiation of sociomathematical norms by the teachers required response to unanticipated learner contributions, and teachers working 'on their feet' to handle these contingencies.

This theoretical position provides insight into a view of classroom situations and learning as collectively constructed through teacher and learner inputs, which is the underlying view in my study. While I am not explicitly focusing on the extent of establishing the social or sociomathematical norms, I am interested, in a context of evidence of very little feedback, in the nature, breadth and extent to which teacher elaborations in the context of learner offers are seen, and developed over time.

3.2.3 Classroom mathematical practices

The third aspect of the interpretive framework, classroom mathematical practices, is concerned with the development of established mathematical practices, where at some point in time interpretations of mathematical concepts or ideas are "taken-as-shared ways of reasoning, arguing, and symbolizing established while discussing particular mathematical ideas" (Cobb, Stephan, McClain, & Gravemeijer, 2010, p. 126). In contrast to social norms and socio-mathematical norms, mathematical practices are specific to particular mathematical ideas. Schoenfeld (2012) has also referred to mathematical practices, describing these as one of the two main aspects of doing mathematics, the other being mathematical content. Mathematical practices are similar to mathematical habits of mind (Schoenfeld, 2012) such as "a predilection to explore, to model, to look for structure, to make connections, to abstract, to generalize, to prove" (2012, p. 592).

With consistent use and justification of mathematical ideas in the classroom, the argument is that learners internalize such ideas to the extent that these features became an established mathematical practice of that particular collective classroom context, for example, learners who were able to devise various solution strategies for addition and subtraction problems that involved bridging through 10 or counting in 10s. As classroom sociomathematical norms, learners are expected to explain and justify their solution strategies. Later in the school year, solutions based on such interpretations of bridging through 10 or counting in 10s become an established mathematical practice by the classroom collective. Cobb and Yackel (1995) state that classroom mathematical practices were "motivated by the realization that one can talk of the mathematical development of the classroom community as well as that of individual students" (p.9).

Cobb and Yackel (1995) argue that analysis of classroom mathematical practices are of both practical and theoretical significance. Practical significance can be seen from the perspective of developmental research in that they document instructional sequences as they are realized in interaction in the classroom. This can inform on-going developmental efforts in adapting instructional sequences to realize both individual and collective learning in a social context. The theoretical significance on the other hand relates to the fact that such analysis bear directly on the issue of accounting for mathematical learning as it occurs in a social context. Against the background of the classroom social and sociomathematical norms, the mathematical practices established by the collective classroom space can be seen to constitute immediate, emergent and local situation of the students' development. Identifying sequences of such practices from the teaching perspective characterizes what counts as responsive teaching in a social classroom context, where learners participate and learn.

Martins, Towers, and Pirie (2006) used the improvisational lens to analyse collective mathematical understanding. They describe collective mathematical understanding as the kind of learning and understandings that occur when a group of students work together on a mathematical activity. Central to their analysis is the idea of co-acting which they define as

> …a processes through which mathematical ideas and actions, initially stemming from an individual learner, become taken up, built on, developed, reworked, and elaborated by others, and thus emerge as shared understandings for and across the group, rather than remaining located within any one individual. (p.156)

They make a distinction between co-actions and interactions. While in interactions there is an emphasis on reciprocity and mutuality, co-action concerns actions that are dependant and contingent upon the actions of other members of the group (Towers & Martin, 2006). Through this co-acting, an understanding emerges that is the property of the group rather than any individual. It is not that all individuals bring the same understandings to the scene but rather that individual contributions will result in something greater than the sum of the parts. Neither is it that an individual will not make his or her own personal advancements. Sawyer (2003 p.13) sees this interaction constituted by two important interrelated characteristics coined as 'collaborative emergence'. These are:

> Moment-to-moment contingency: At each moment, the possible appropriate actions are constrained to varying extents by the prior flow of the conversation. But there is always a range of possible appropriate actions, with each one offering very different future conversation paths.

> Retrospective interpretation: Each participant's contribution only acquires meaning after it is responded to by the others. In some cases, the interactional meaning of a particular statement ends up being very different from what the speaker [teacher or learners] might have intended at the time.

These two characteristics echo a view of the act of responsive teaching as engaging in a collaborative emergence as something that cannot be entirely planned by the teacher. Teachers have to be alert to dealing with the emergence of either learners' misconceptions, errors or generated informal strategies when working with mathematical concepts. Teachers need to constantly listen to learners' contributions and evaluate these in productive ways that will create opportunities for learners to co-construct new knowledge. The use of the metaphor of pedagogic practice as improvisational reveals insight on how teachers handle these moment to moment contingencies, with increasing focus of attention on appropriately dealing with unexpected events as teaching unfolds in the mathematics classroom. In this view, teaching is underlined by creative behaviors and co-produced mathematical processes that allow learners to build their own knowledge (Scardamalia & Bereiter, 2006), and through exploratory talk among teachers and learners (Mercer, 2002).

I found the 'emergent approach' - based on both psychological (cognitive) and interactionist perspectives - very useful for framing my study, and for the in-depth exploration of primary mathematics teaching development over time as teachers deal with moment-to-moment contingencies. Cognitive analysis of mathematics classroom teaching focuses on the teacher's knowledge of and about mathematics; her understanding of the mathematics of others; and how she uses this knowledge to enable learners to make their emergent learning visible through expression in the classroom. Interactionist analysis focuses on how the teacher deals with moment-to-moment events during the complex dynamic interactions between the teacher, learners and mathematics as learners encounter new knowledge. This involves how the teacher renegotiates classroom social, sociomathematical norms and established classroom mathematical practices. I explore the question of the nature and extent to which teachers construct a form of practice that fits with their students' ways of learning and teacher's '*being'* with mathematics. I am focusing on a teaching practice that is responsive to emergent learning through teacher elaborating 'in-the-moment' as a disciplined improvisation.

3.3 Conclusion

In this chapter I have discussed the broad overview of the theoretical underpinning of this study, and the theoretical assumptions upon which the study is based. I have also explained the rationale for the relevance of this theoretical lens for the present study. However, theory had to be shaped to address the problem of the study, and the particularities of the empirical data produced. In Chapter 4 and 5 I discuss the process of the interaction between theory and data in detail in a discussion of the study’s methodology and research design and the development of the language of description for the data analysis.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

4.1 Introduction

As indicated in Chapter 1, the focus of this study is an in-depth exploration of change in teachers' responses to in-the-moment situations in mathematics classroom as observed 'in-action' over time of four South African primary school teachers of mathematics. This chapter deals with the research design and methodology used in the study.

4.2 Research design

This study was undertaken within a qualitative case study approach. Case study is a research design that provides an opportunity for in-depth exploration of a bounded system (e.g. an activity, an event, a process or an individual) based on extensive data collection that results in rich description of the phenomenon under investigation (Creswell, 2007). Yin (1994) outlines case study as "an empirical inquiry that: (i) explores a contemporary phenomenon within its real context; when (ii) boundaries between phenomenon and context are not clearly evident; and in which (iii) multiple sources of evidence are used" (p. 23).

Merriam (1998) defines case study as an intensive, holistic description and analyses of bounded phenomenon. She points out that the "single most defining characteristic of case study research lies in delimiting the object of study: the case" (p. 27). The case is a unit, entity, or phenomenon with defined boundaries that the researcher can demarcate or "fence in" (p. 27), and therefore, can also determine what will not be studied. The case is "a thing, a single entity, a unit around which there are boundaries" (p. 27).

While case study approaches are commonly used in educational research, their use presents a number of challenges, because the term 'case' has not been applied uniformly and there are overlaps with other forms of research design. Merriam (1998) acknowledged that the use of case study is often misunderstood.

> Those with little or no preparation in qualitative research often designate the case study as a sort of catch-all category for research that is not a survey or an experiment and is not statistical in nature. While case studies can be very quantitative and can test theory, in education they are more likely to be qualitative. (p. 19)

For this reason, it is important to define my 'case' in this study and to outline my rationale for the choice of case study as a research design, supported by both the research paradigm and theoretical assumptions that undergird this study. I locate my study within the emergent paradigm (Guba & Lincoln, 1994; Mertens, 2005). My understanding is shaped by the view that there are multiple realities through which one can make sense of the world, and I construct my reality from my experience through interaction with the world phenomenon. This worldview is embedded in the qualitative approach, where inquiry is a process of interpretation and sense making of the phenomenon under investigation.

The theoretical assumption undergirding my study was that teaching is fundamentally improvisational if it is to be responsive to emergent mathematics learning. This led to the importance of attending to contingency situations and elaboration of mathematical ideas in the classroom. Therefore, my focus in this study is on specific aspects of teachers' contributions in response to learners' offers as observed in action over time. The aim was to categorise and develop teaching actions that are responsive to emergent mathematics learning. To better understand this phenomenon, I needed a research design based on extensive data collection that would result in rich and thick descriptions of teacher actions, and analysis of the knowledge base that these actions could be linked to in the context of intervention using VSR interviews. To do this, I employed what Yin (2003) called a multiple case study. A multiple case study enables the researcher to explore differences within and between cases. The goal here is to replicate findings across cases, in which the researcher can explore similarities and contrast between the cases (Yin, 2003). This is similar to what Stake (1995) called a collective case study.

The exploration of teachers' responses to learners' offers in primary mathematics classrooms provided the bounded system investigated over time through detailed data collection involving multiple sources of information (Creswell, 2007; Yin, 1994). As mentioned already, understanding teachers' contributions in contingent situations as observed in action, and using VSR interview as an intervention mechanism is an area of on-going interest in the mathematics education literature, and can be linked to reforms in mathematics education worldwide geared towards improving the quality of teaching and learning of school mathematics. Therefore, the kind of multiple case approach used in this study is instrumental in its purpose. Instrumental case study is used to accomplish something beyond understanding of a particular situation. It provides insight into an issue or helps to refine a theory. In this approach, the case is often looked at in depth, its contexts scrutinized, its ordinary activities detailed, and because it helps the researcher refine or build a theory, hence the case may or may not be seen as typical of other cases (Stake, 1995).

As discussed in the opening chapter of this thesis, the analyses of teachers' responses to contingent situations in international empirical studies have tended to focus on *what* opportunities for learning they open up, rather than initially, for *whether* learners' offers are acknowledged or evaluated at all. This represents a marked contrast to the realities in poorer classrooms contexts in South Africa. This contrast led to the need for a more grounded approach to characterizing the situations in which responses to learners' offers were given, and then analysing the nature of these responses. I therefore used 'contingency' in this study to describe all situations of learners' offers with the potential for some kind of elaboration. The notion of 'elaboration' is used as an interpretive lens to categorise and explain teachers' responses to such contingent moments during the teaching of additive relations.

4.3 Setting – the context of the study

This study was located in the aftermath of the first pilot of the WMC-P in-service 'maths for teaching' professional development (PD) course, in which 33 teachers drawn from the WMC-P project's ten partner schools in one district that skirts both peri-urban and sub-urban settings in South Africa, participated. The PD was designed with the aim of addressing the extensive gaps noted in primary mathematics teachers' content knowledge and pedagogy in the mathematics classrooms as outlined in Chapter 2. It was targeted at supporting the development of teachers' foundation, transformation and connection knowledge dimensions of the KQ. Thus, course tasks involved supporting teachers to build more connected mathematical understandings, in relation to representation, models and strategies and placed emphasis on building confidence (Graven, 2004) and competence in communicating and elaborating mathematical ideas. My aim within the context of the PD was to explore aspects of contingent situations – how teachers' responded to learners' offers in the classroom, and the possibilities for extension and expansion over time. This was done by tracking the development of selected teachers that participated in the PD through lesson observations and continued professional development activities involving video stimulated recall interviews.

4.4 Participants

Unlike other types of qualitative research design, there are two levels of sampling inherent in case study design (Merriam, 1998). The first is the selection of the case to be studied; the second is the sampling of the participants within the case. In this study, the case to be studied is a primary teacher developing his/her responses to learners' offers and contributions in mathematics lessons, and the contribution of VSR interview to this development. Below I describe the second level – about the selection and the characteristics of the participants for this study.

Out of the 33 teachers that participated in the WMC-P maths for teaching course, four teachers from two primary schools were purposively selected for the in-depth follow-up case study. The selection of four teachers was based on the assumption of multiple realities in the emergent framework. This meant that I proceeded with the expectation that four teachers would bring differences in terms of the ways in which they enacted instances of responsive teaching, and how they would develop this aspect over time. In the selection of these four teachers, an information-oriented selection strategy highlighted by Flynbjerg (2006) was adopted. This is a strategy for the selection of participants that maximized the utility of information from small samples.

A relatively strong mathematical concept knowledge base, considered as a key prerequisite for the possibility of responsive teaching, was considered for the selection of the participants. Teachers that demonstrated relatively strong understanding of the mathematics content knowledge (scoring 60% and above) measured by the WMC-P course post-test, with items drawn from a range of prior studies including Hart et al. (1981) CSMS studies and Ryan and McCrae (2006) TEMT studies were selected. The selection also considered teachers who are relatively strong in terms of their transformation and connection knowledge based on their performance during interim course assessments. These interim assessments were based on hypothetical classroom scenarios that required teachers to provide explanations related to connections between mathematical ideas and progression in teaching.

This ‘favourable’ selection of teachers with a relatively strong mathematical knowledge base for teaching was made based on the literature evidence noted earlier that knowledge of content and specific pedagogy are necessary (though not sufficient) for possibilities for responsive teaching. In the initial selection, I also included both foundation phase (Grade 1-3) and intermediate phase (Grade 4-7) teachers. This spread was intended to explore the hypothesis that as the content progresses across the curriculum, concepts become more abstract, and this appears to require different mathematics specific pedagogical demands (Potari, Zachariades, Christou, Kyriazis, & Pitta-Pantazi, 2007; Rowland, 2009). Also South African literature has noted specific difficulties in terms of syntactic knowledge, connections and progression among foundation phase teachers (Venkat, 2013; Venkat and Nadoo, 2012).

In addition to the above, for convenience within the data collection and the interventions, I sought a selection of two teachers, one foundation phase and the other intermediate phase, from the same school that satisfied the above criterion. Three schools were found to have satisfied these criteria, and therefore six teachers were initially selected for the study. However, two teachers from one school provided me with incomplete data sets. I therefore omitted these teachers from the final data set and ended up with four teachers that fully participated in all the data collection processes.

Based on the criteria listed above, two Foundation Phase (FP, Grades 1-3, grade-appropriate learners aged 7-9 years) teachers (Thandi and Sam[1]), and two Intermediate Phase (IP, Grades 4-6, grade-appropriate learners aged 10-12 years) teachers (Herman and Bongi) were involved in this study. Herman had taught in IP throughout his teaching experience. Bongi and Sam had experience of teaching across FP and IP grades, while Thandi had taught Fine Art and isiZulu at High School (Grade 10) for many years, before moving to teaching at FP in 2009. Thandi taught grade 3 in 2012, 2013 and 2014; Sam taught grade 2 in 2012 and 2013, and grade 4 in 2014. Herman taught grade 6 in 2012, 2013 and 2014, while Bongi taught Grade 4 in 2012 and grade 6 in 2013 and 2014. All four teachers scored 60% and above in the course post-test, with Herman having the highest score of 91% and Sam with the lowest score of 60%.

[1] All the names of the teachers and learners are pseudonyms, intended to preserve the anonymity of the participants

In the pre-course baseline questionnaire from 2012, three out of these four teachers noted mathematics specific pedagogy as the most challenging issue that they felt was needed for improving the teaching of primary mathematics and learner performance in mathematics. A summary of the background characteristics of these teachers is presented in Table 2.

Table 3: Summary of the background characteristics of the sample teachers

Biographic characteristics of the participants				Grade taught in the study period: 2012, 2013 and 2014			Primary maths content knowledge	
Participants	**Gender**	**Previous Grade(s) taught**	**Qualification(s)**	**2012**	**2013**	**2014**	**Pre-test score**	**Post test score**
Thandi	Female	10 (1992-2007) 3 (2008-date)	Diploma in Education (IsiZulu & Fine Art, 1992) Advance Cert. in Education (Foundation Phase, 2012)	3	3	3	63	84
Sam	Female	1(1995 - 1996) 5(1997 - 2003) 2(2004 - 2013) 4(2014 – date)	Diploma in Education (Maths & English, 1994)	2	2	4	44	60
Herman	Male	6,7 (1992 - 2007) 6 (2009 - date)	Diploma in Education (Maths & English, 1991) B. A (Eng & Communication studies, 2004) Honors in Maths (Wits, 2015-date)	6	6	6	78	91
Bongi	Female	5(1996 - 2000) 3(2001 - 2008) 4(2008 – 2012) 6(2013 – date)	Diploma in Education (Geography) B. Ed Planning & Policy studies	4	6	6	74	75

4.5 Data sources and procedures

The data gathering process took place over two years between 2013 and 2014, following the teachers' participation in the 2012 one year long in-service 'maths for teaching' PD course. All four teachers were tracked in the follow-up, in-depth case study. Data sources for the study included: Video recordings of classroom observations of teaching and video-stimulated recall (VSR) interviews. With the focus of this study on identifying and developing responsive teaching, the data gathering and data analysis were done concurrently, with lesson observation in 2013, initial analysis of lessons, VSR interviews and a follow-up lesson observation in 2014. In the following sections, I describe each phase of the data collection.

4.5.1 Lesson observations

Lesson sequences were observed in two cycles across all four teachers, based in two primary schools that were part of the broader WMC-P project. The first observation cycle occurred in 2013 and the second in 2014. I visited each school myself and observed and video-recorded sequences of 'normal' complete lessons lasting for about an hour each. By 'normal' lessons I mean that teachers were not asked to prepare a lesson for the purposes of my research. Rather I asked them to provide me with their timetables and work schedules and I planned my visit based on the times when they were teaching topics relating to additive relations. A total of 8 lessons (2 x 4 teachers) on additive relations were observed in 2013. This took place between February and March, 2013. A CD containing the video of recorded lesson was given to each teacher the following day, and they were encouraged to watch the lesson and identify mathematically oriented incidents of interest where they are either happy with what they had done or wanted to revisit their actions.

In 2014, I revisited the teachers' classrooms when they were teaching the same concept (additive relations) as in 2013 and within the same period (February and March, 2014). Three teachers (Thandi, Herman and Bongi) taught at the same grade level in 2014, but Sam was moved to teaching grade 4 in 2014 from Grade 2 in 2013. In 2014, three lessons each from Thandi and Herman, and two lessons each from Bongi and Sam were observed and video-recorded. Across 2013 and 2014, 18 lessons in total were observed and video-recorded – 8 lessons in 2013 and 10 lessons in 2014. A summary of the lessons observed with the four teachers is presented in Table 3.

Table 4: Summary of the 18 lessons observed in 2013 and 2014 by the 4 teachers

Teacher	Year/ Grade	Date of Observation	Duration of Lesson	Lesson Focus
Herman (5 lessons observed)	2013 Grade 6	21-02-2013	43:01	Addition and subtraction using expanded notation
		28-02-2013	38:05	Addition and subtraction of fractions
	2014 Grade 6	19-02-2014	45:33	Addition and subtraction of fractions
		27-02-2014	46:25	Revisiting addition and subtraction of whole numbers on ENL
		05-03-2014	49:22	Addition and subtraction of fractions on ENL
Bongi (4 lessons observed)	2013 Grade 6	20-02-2013	44:02	Addition and subtraction of whole numbers using ENL and vertical expanded method
		06-03-2013	54:18	Addition and subtraction involving word problems
	2014 Grade 6	18-02-2014	55:06	Column addition method
		06-03-2014	40:30	Addition of decimal numbers in context money problems
Sam (4 lessons observed)	2013 Grade 2	20-02-2013	62:15	Addition by bridging through 10 using marbles and on a number line
		06-03-2013	58:54	Part-part-whole (building addition and subtraction number sentences)
	2014 Grade 4	18-02-2014	58:13	Addition by bridging through 10 on a number line
		06-03-2014	57:03	Addition by bridging through 10 on a number line with extended number range
Thandi (5 lessons observed)	2013 Grade 3	21-02-2013	67:50	Addition and subtraction on a number line
		28-02-2013	41:16	Addition and subtraction of simple fractions using paper strips.
	2014 Grade 3	26-02-2014	53:28	Addition and subtraction of 9 and 11 by bridging through 10 on ENL
		27-02-2014	48:44	Addition and subtraction by bridging through 10 on ENL
		05-03-2014	62:03	Addition and subtraction by bridging through 10. Additional emphasis on commutativity of addition and two models of subtraction as 'take away' and as 'difference'

4.5.2 Professional development activity based on VSR interviews

Having collected the 2013 lessons observations, I initially analysed the lessons using the KQ framework as a theoretical tool. In this analysis, I focused on identifying and describing incidents that related to aspects of mathematical knowledge in teaching relevant to the four dimensions of the KQ. Using the findings from the analysis for each teacher, I designed semi-structured questions (see Appendix A) for later discussion with the teacher. Using selected incidents from the lesson in which the initial questions were drawn, I then engaged with each teacher in a VSR interview on a one-on-one basis either at the University campus or at the teacher's school (mostly at the school library) in order to minimize interruptions during the interviews.

The use of VSR interview as teacher professional learning mechanism has been found effective for identifying and examining teachers' thoughts and decisions, and the reasons for their actions in the classroom (Lyle, 2003; Muir, 2010; Pirie, 1996). There is also emerging evidence that this approach has been effective in enhancing mathematics and science teaching (Geiger et al., 2015; Santagata, 2009; van Es, 2012; Van Es & Sherin, 2008), as well as providing a powerful medium for revealing aspects of teacher's practice that were not previously considered (Geiger, et al, 2015). At the same time, Coles (2014) noted that theory related to video-based teacher learning is still underdeveloped, as little is known about the specific features that contribute to any learning successes seen. In particular, the roles of the facilitator, the number of participants or times of engagement tend to be backgrounded (Borko, Jacobs, Eiteljorg, & Pittman, 2008).

As mentioned already, teachers were given the CD of their recorded lessons, and they were told to watch the video and identify areas of interest for conversation. During the interview, the recorded lessons for each teacher were viewed again. Teachers were encouraged to stop the video if they wished to make a comment. After each lesson review, we discussed, in overview terms the strengths and the weaknesses of the lesson before specific discussion on selected incidents of the lesson that related to the following key dimensions of the KQ framework: connections between starter activities and the main lesson; decisions about sequencing and progression in the lesson; rationales for the choice and use of examples; rationales for the choice and use of representations, models and strategies; response to learner incorrect offers; and alternative explanations for big ideas in the lesson. These interviews took place between August and September, 2013. The VSR interview focused on both understanding each teacher's rationales for classroom decisions, and developing their mathematics knowledge for teaching through reflection on practice.

All the interviews were audio recorded and field notes were taken to record teachers' actions that could not be captured on the audio recorder. The interview covered discussion about the two lessons observed in 2013 and lasted a maximum of 1 hour 30 minutes with each teacher. The sequential data collection and analysis enabled me to capture rich and in-depth data about the phenomenon under investigation with a focus both on what occurred in terms of responsive teaching, and on teachers' rationales for their choices.

4.5.3 Overview of the lessons content

In this section, I provide a brief overview of the content of all the 18 lessons observed. The overview presents the general structure of the lesson, involving mental starter activities, example sequences at the whole class level, at group or individual levels. This overview is provided to assist the reader to get a sense of the lesson contents as I refer to each lesson in the analysis chapters. I present this overview according to each teacher. I refer to these lessons with a code in the order of grade, first letter of the teacher's pseudonym, lesson number and the year that the lesson was observed. For example, Herman, grade 6 teacher, second lesson that was observed in 2014 reads as 6H_2_2014.

Thandi's lessons content

Two lessons in 2013 and three lessons in 2014 were observed and video-recorded. Thandi taught all these lessons to grade 3 classes.

3T_1_2013 - Thandi began the lesson by asking groups of learners to count in tens from 10-200, starting with group 1 saying ten, group 2, twenty, group 3, thirty and returning to group 1 to say forty and so on. In the same way, she asked the class to count in 10s from 11-201 and backward from 200-0. Thandi moved on to show how numbers could be symbolically compressed from drawing ten dots strips to suggesting using 'X' to represent ten and '√' to represents a unit. This was demonstrated with the number fifty-three represented as 'XXXXX √√√'. The lesson moved on to placing numbers in the 0-100 range on a number line, and identifying the relative position of 95 and 92 on a number line marked in 10s. Thandi then informed the class that the focus of the day's lesson was addition and subtraction on a number line. Two examples were dealt with as a whole class activity: (i) 25+5 = _ and (ii) 25

= 30 - _; and one example, 40+8= _ was done as individual learner work on a worksheet. This lesson lasted 68 minutes, 30 seconds.

3T_2_2013 - Thandi stuck A-4 size papers on the board with partitions of the papers into different equal-sized parts with some parts shaded. She first asked learners to identify how many parts each whole was divided into by counting. She then asked how many parts were shaded out of the whole. Thandi asked learners to write the fraction of the shaded part to the whole in symbolic notation (e.g. ½). Learners raised their hands and the teacher chose individuals to respond. Thandi then demonstrated addition of ¼ and ¼, and ⅓ and ⅔ with the paper strips. She then moved on with subtraction of ⅖ from $^{5}/_{5}$ of a whole partitioned into 5 equal parts. This lesson lasted 41 minutes, 40 seconds.

3T_1_2014 - Thandi began the lesson with a display of the number '10' written on a flash card. She asked learners to count forward in 10s from12 – 152; 33 – 163 and then backward in 10s from 96 – 6. She then moved on to what she described as the oral mental starter. In the oral mental starter, Thandi asked a number of questions to consolidate the idea of one more and one less. The learners were asked questions in the form 'If I say a number, I want you to give me a number that is one more'; 'I want you to give me a number that is one less'. Learners raised their hands to answer and Thandi chose individuals to respond. In the main activity, Thandi introduced and used the number line for addition and subtraction. The work involved adding and subtracting 9 and 11 to numbers less than 100 using addition or subtraction of ten followed by a compensating step on the number line. Thandi worked through two strings of addition and subtraction problems (45+10, 45+11, 45+9 and 56-10, 56-11, 56-9) To add or subtract 11, 10 was first added or subtracted, and followed by addition or subtraction of 1 respectively as jumps on a number line, pointing to an awareness of using links between examples to create awareness of using 10 as a benchmark. The following tasks were given to learners to do individually: 38+10, 38+11, 38+9; and 38-10, 38-11, 38-9. The lesson lasted for 53 minutes 28 seconds.

3T_2_2014 – In the mental starter activity, Thandi asked learners to count forward and then backward in 10s within the 400 number range. She then moved on to asking a number of in the form 'what is fifteen plus ten' followed by 'what is fifteen plus ten plus two'. As before, the learners raised their hands to answer and Thandi chose individuals to respond. In the main activity, she stressed the efficiency gain in working with 10 as a benchmark for addition and subtraction. She worked through three addition problems (41+20, 49+24 and 65+29) and three subtraction problems (64-30, 73-31 and 97-38) with the whole class. The work involved, for example, working with 49+24 represented as 49+20+1+3 with counts on of 20, 1 and 3 from 49 indicated as forward jumps on the number line. For subtraction, 97-38 was represented as 97-30-7-1 respectively as backward jumps on a number line. Thandi differentiated the worksheets prepared for individual working with higher number ranges for the learners that she described as 'strong' (156+40, 249+43, 337+45, 495-49, 448 – 45) and lower number ranges for the rest of the class (35+20, 35+22, 62-40, 62-31, 62-29). The lesson lasted for 48 minutes 44 seconds.

3T_3_2014 - Thandi started the lesson with an oral mental starter activity. She asked a number of questions to rehearse the idea of comparison between two numbers and say which number was bigger or smaller (e.g. 309 and 311). She moved on with an activity that drew learners' attention to the space or gap between two numbers with questions like: 'Between 19 and 25, which number is closer to 21?' As before, learners raised their hands and she chose individuals to respond. Thandi mediated the activity by placing the 3 numbers on an empty number line and working out the gaps between them, in order to decide which number was closer to 21. In the main activity, Thandi introduced two models of subtraction - as 'difference' (when the gap or space between the two numbers being subtracted is very small) and as 'take away' (when the space or gap between the two numbers is wide). She worked through five examples on a number line (22-3, 22-19, 105-99, 105-7, 301-299). Thandi also introduced the idea of starting addition with the bigger number and adding on the smaller number using flexible group counting through multiples of 10 on a number line. She worked through another five addition examples (27+8, 6+25, 9+163, 177+6, 28+248). The following tasks were given to learners for individual work: 7+94, 172+18, 29+235, 77 – 9, 102 – 97. The lesson lasted for 62 minutes 3 seconds.

Sam's lessons content

Two lessons each in 2013 and 2014 were observed and video-recorded. Sam taught the two 2013 lessons with grade 2 learners and the two 2014 lessons with grade 4 learners. The following is a brief summary of the content of her four lessons.

2S_1_2013 – Sam commenced the lesson with an oral class chant - forward counting in 5s from 5-50 with claps for the number of groups of 5 (e.g. 1 clap for five, 2 claps for ten, 3 claps for fifteen and so on). This was followed by forward and backward counting in 10s from 0-100 and 100-0. The lesson continued with identification of the number '60' on a 100 square pasted on the board. She then asked for the number of claps for 60 if one clap stood for 10, before moving on to finding the difference between 100 and 80 through a clapping activity involving multiples of 10 and through the use of jumps of 10 on a number line. She then wrote the following sums on the board: 2+3, 3+3, 4+2, 5+4, 3+6, 4+2 and 1+6 - and asked learners to show answers on their fingers. The main focus of the lesson - addition by 'bridging through 10' – was presented through two examples: 8+5 and 7+6, using marbles as concrete apparatus and a structured number line as resource alongside her interactions and explanations. The lesson lasted for 62 minutes 15 seconds.

2S_2_2013 – Sam started the lesson by asking learners to identify odd numbers between 1 and 20 on a 100-square. This was followed by oral counts in 2s starting from 1 (counting on odd numbers). Sam then wrote the following numbers: 18 and 20, 9 and 11, 25 and 23, 19 and 21 and asked learners to say the number in between – e.g. 'What is the number between 18 and 20?' Learners raised hands and Sam pointed at individuals to respond. In the main activity, Sam had 20 marbles (10 blue and 10 yellow) hung on a string, and asked learners to count them in different ways. Learners counted in 1s, 2s, 3s, 5s and 10s. Sam then removed 5 blue and 5 yellow marbles leaving 10 marbles. Sam then split the 10 marbles into 4 and 6, and asked learners to give addition and subtraction number sentences these numbers. Learners produced; 4+6=10; 6+4=10; 10-4=6; and 10-6=4 and Sam then inserted two of these numbers onto a triad diagram, and asked learners to construct a number sentence with the missing number as the result. The second split that was dealt with was generated by a learner who offered 9 and 1. The lesson lasted for 58 minutes 54 seconds.

4S_1_2014 - Sam started the lesson by asking learners to count forward in 5s from 50 – 125. She asked learners to count forward again and then backward in 10s, 10-200 and 100 – 20 respectively. Sam then pasted 10 mental addition and subtraction problems (4+6, 4+16, 4+26, 4+36, 4+46 and 6-4, 16-4, 26-4, 36-4 and 46-4) on the board and asked learners to work out the problems in their individual mental maths exercise book for 5 minutes. A discussion followed on how individually learners worked out the problems. The first learner that Sam invited to the board drew 26 and 4 tallies, and counted them altogether. Sam asked for a more efficient strategy from the whole class and eventually got one learner who worked out 4+46 by starting with 46 and opened up 4 fingers, he then said, "47, 48, 49 and 50' as the answer. Sam commended this learner and encouraged learners to always think of more efficient strategies when working out sums. In the main activity part of the lesson, Sam demonstrated bridging through 10 on a number line. The examples 56+33 and 45+41 were dealt with as whole class teacher-guided work. She wrote down the following tasks: 53+35; 66+32 and 36+13 for pair work practice. The lesson lasted for 58 minutes 13 seconds.

4S_2_2014 - Sam started the lesson by asking learners to give an example of an odd number. The numbers 3, 5 and 7 were offered by individual learners. Sam then asked learners to count forward in 5s from 5 – 100, and then from 3-53. She asked learners to count forward again in 2s from 3-39; and in 10s from 1-91. Sam then pasted sequences of mental sums on the board (3+6=, 30+60=, 300+600= and 4+6=, 40+60=, 400+600=) and asked learners to work out answers in their mental maths exercise books. A discussion ensued about the structure of the number system with moves from units to tens and to hundreds. This involved examining the pattern of the answers obtained, for example, the case 3+6; 9 is turned to 90 in adding 30+60, and 90 is turned to 900 by adding 300+600, and so on. The unit 9 remained the same all through the connected examples. Sam then asked learners to build their own similar example sequences starting with 5+5 and 7+3. In the main lesson, Sam wrote two sums: 10+19 and 29+17 on the board. Learners were very quick at giving 19 as the answer to the first sum, but took a while before they gave the answer to 29 + 17. Sam then worked out 29+17 by breaking down 17 into 1+10+6 and indicating these counts on with jumps on a number line. She worked out two other examples (118+7 and 215+12) on a number line by bridging through 10 as a whole class activity. She then gave learners four further sums to do individually: 117+16; 223+16; 141+19 and 252+11. The lesson lasted for 57 minutes 3 seconds.

Bongi's lessons content

Two lessons each in 2013 and 2014 were observed and video-recorded. Bongi taught all the lessons with her grade 6 classes. The following is a brief summary of the content of her four lessons.

6B_1_2013 - Bongi started the lesson by telling learners a story leading to the addition of 13 and 25. She pasted 10 mental sums written on a large sheet: 88+4, 8+6, 9+10, 15+4, 22+40, 28+12, 31+10, 64+9, 9+2+8, 10+10. Two sums were worked out with the whole class, before Bongi asked learners to write down all the answers in their mental maths exercise books. In the main lesson, Bongi demonstrated a procedure for solving an addition problem on an empty number line (ENL). She started with 74+96. Writing 74 at a mark towards the start of the line, Bongi broke down 96 into 6+20+70, with each number indicated by forward jumps on the ENL. Bongi checked the answer with the traditional column addition method. She then invited a learner to work out 208+22 on an ENL on the board, before given the class pair practice work. In the pair practice, the following examples were given: 64+8, 379+21, 49+3. Bongi then moved into demonstrating a procedure involving a vertical expansion method. Starting with 485+347, she expanded 485 into 400+80+5, and 347 as 300+40+7. Bongi then added the units, tens and hundreds. She then invited one learner to work out 32672+26315 on the board using the same method. Both answers were checked with the traditional column method. The lesson lasted for 44 minutes, 2 seconds.

6B_2_2013 - Bongi started the lesson by informing learners about the focus of the lesson – 'telling stories in mathematics, demonstrating and drawing pictures'. She then pasted 10 mental sums written on a large cardboard sheet: 14+26, 79+35, 69+33, 198+5, 202+18, 128+12, 800+91, 44+28, 788+12 and 652+18. Two sums were worked out with the whole class, before Bongi asked learners to write down all the answers in their mental maths exercise book. In the main lesson, Bongi introduced the vocabulary of 'altogether' to mean addition. She then wrote down the following problem – 'In a class, there are 23 boys and 13 girls. How many more boys than girls? One learner offered 8 as the answer, and explained that he counted on from 13 to 23. Bongi accepted the offer and moved on and modelled the problem using a part-whole diagram. She then wrote down another problem: In a class, there are 22 girls and 20 boys. How many children altogether? Learners were quick in giving 42 as the answer. Bongi accepted the offer, and asked learners to model the problem using a part-whole diagram on the board. Four learners were invited but none of them were able to model the problem correctly. Bongi then modelled the problem in a part whole diagram, before giving learners pair practice to do involving examples: ---. The lesson lasted for 54 minutes, 18 seconds.

6B_1_2014 - Bongi started the lesson by telling learners a story leading to the addition of three numbers: 1000, 500 and 300. Learners quickly gave her the answer. Bongi then demonstrated a traditional column addition method with emphasis on arranging the numbers in their place value (i.e. Th H, T and U). She then pasted two sets of linked examples (e.g. 6+5+1, 60+50+10, 600+500+100, etc). In the main lesson, Bongi demonstrated the procedure for solving addition in a traditional column method with numbers in a range of 10 000). She then invited two learners to work out similar examples on the board, before she gave learners pair practice examples. The lesson lasted for 55 minutes, 6 seconds.

6B_2_2014 - Bongi started the lesson by telling learners a story about going to the school tuck-shop involving a money transaction. She displayed different common shop items (tissue paper, biscuits, cakes, etc) with marked prices involving decimals fractions. She asked learners to work out how much money would be needed to buy, for example, two tissue papers. Learners were invited to the board to work out the answer. Bongi encouraged efficient mental strategies. For example, in one of the problems: 'Tissue paper costs R13, 99. What is the cost of two tissue papers?' - Learners, in most cases, used the traditional column addition method. Bongi's response to such solution actions was to accept the answer and show learners a quicker method using rounding up the amount from R3, 99 to R4, adding twice to get R8 and then compensating by taking away 2 cents leaving R7, 98. Bongi gave learners two similar task to work with in pairs, asking them to first write down the estimated answer before working out the actual answer. The lesson lasted for 40 minutes, 30 seconds.

Herman's lessons content

Two lessons in 2013 and three lessons in 2014 were observed and video-recorded. Herman taught all these lessons to grade 6 classes.

6H_1_2013 - Herman began the lesson with mental oral starter activity where he asked a number of questions on addition of two numbers with total less than 50 and subtraction of one-digit from two digit-numbers. Learners were asked questions in the form 'seven plus three?' Learners raised their hands and Herman chose individuals to respond. In the main lesson, which was about addition by the horizontal place value expansion method, Herman prepared five different sets of number lines marked with 11- equal point intervals in 1s from 0-10, 10s from 0 – 100, 100s from 0 – 1000 and 1000s from 0 – 10 000. He stuck all these number lines on the board. Herman then moved on and defined expanded notation as ‘writing the value of each and every digit which is in that number’. He exemplified this statement with the number 367 – the value of each digit was written on the board (i.e. 300 + 60 + 7). Herman then moved on and worked out five-digit addition and subtraction problems on the board as a whole class activity by writing the value of each digit and insisted that learners count using the structured number lines while adding units, tens, hundreds, etc. Herman concluded the lesson by distributing different sets of addition and subtraction problems for learners to work in groups of four.

6H_2_2013 - Herman began the second lesson, which was about addition of mixed fractions, by asking learners, ‘What is a fraction?’ One learner offered, ‘a number’. Herman acknowledged the offer by repeating what the learner has said. He then moved to another learner. This second learner offered, ‘part of a whole’. This offer was accepted and written on the board. The teacher moved on and exemplified the meaning of fraction through paper folding. He folded an A-4 paper in halves and quarters with conversation about the number of parts the paper had been folded into and then: ‘What is the value of each part?’ Herman had already written two problems on the board: 1½ + 1½; and 1 ¾ + 1 ⅜. He provided learners with two rules for adding mixed fractions: (i) add whole numbers; and (ii) add fractions by finding the LCM of the denominators. Using these rules, Herman worked out the two problems on the board as whole class. Herman then distributed sets of two different additions and subtraction of mixed fraction problems and asked learners to work in groups of four. On completion of the work, Herman asked learners to show how they had worked out the problems to the whole class. One learner in a group stood up and explained how they worked out the problem with the teacher writing their solution on the board. Two problems were dealt with, one subtraction and the other addition problem.

4.6 Data analysis

As mentioned earlier, I gathered two sets of data: video recordings of lesson observations and the interviews based on video-stimulated recall as an interim professional development activity. I dealt with the analysis of the two data sets separately.

4.6.1 Analysis of video-recorded lessons

I took a grounded theory approach to the analysis of the lesson enactments, for the purpose of generating theory (Glaser & Strauss, 1967). Grounded theory seeks to develop theory that is grounded in data systematically gathered and analysed. Fundamental to the grounded theory approach to data analysis is the belief that knowledge may be increased by generating new theories rather than analysis of data within existing ones. Martin and Turner (1986, p. 141) define grounded theory as 'an inductive, theory discovery methodology that allows the researcher to develop a theoretical account of the general features of a topic while simultaneously grounding the account in empirical observations or data'. Emergence remains the key idea that constitutes the grounded theory approach since its initial conceptualization by Glaser and Strauss.

> It must be emphasised that integration of theory is best when it emerges, like the concepts. The theory should never just be put together (Glaser and Strauss, 1967 p. 41)

Grounded theory came to existence as a result of a reaction against 'armchair' functionalist theories in sociology (Kendall, 1999). Glaser and Strauss (1967) made a call to generate theory and refocus on qualitative data rather than verification of existing theories. This call was, and is, particularly important in two ways. Firstly, changes in social systems entail the need for caution in applying existing theories that may not fit well. Secondly, imported theories from developed country contexts in this study largely did not allow for adequate reading and interpretations of empirical data drawn from the South African developing country context. Therefore, enforcing such theories upon a dataset runs risks of what Ensor and Hoadley (2004) call 'mechanical application' of theories. The consequences of this practice may lead to missing important insights from the empirical data.

Urquhart, Lehmann, and Myers (2010, p. 359) identified four important characteristics that provide some guidelines about the uniqueness of the grounded theory approach and how researchers can use it. These are:

1. The main purpose of the grounded theory method is *theory building*.
2. As a general rule, the researcher should make sure that their prior – often expert – knowledge of the field does not lead them to pre-formulated hypotheses that their research then seeks to verify – or otherwise. Such preconceived theoretical ideas

could hinder the emergence of ideas that should be firmly rooted in the data in the first instance.

3. Analysis and conceptualization are engendered through the core process of *joint data collection and constant comparison*, where every slice of data is compared with all existing concepts and constructs to see if it enriches an existing category (i.e. by adding/enhancing its properties), forms a new one or points to a new relation.
4. '*Slices of data*' of all kinds are selected by a process of *theoretical sampling*, where the researcher decides on analytical grounds where to sample from next.

These four characteristics stipulate the scope of grounded theory, and the key point was that of conceptual sense making rooted in and emanating from the data. Another important characteristic was that of *constant comparison*, where additional data is compared with existing concepts to enrich existing categories. This process is vital in grounded theory, as the data directs relationships rather than these being theoretically determined.

Following the publication of the seminal work on grounded theory by Glaser and Strauss in 1967 several writings were done by the co-originators, which they developed further concepts and provided more clarifications on how to use grounded theory in social research. For example, Glaser (1978) introduced several key concepts that are useful in grounded theory approach, such as: 'coding families' to bring related concepts together in the data, the role of the literature and induction, theoretical sensitivity, and so on. In 1990, Strauss and Cobin jointly published systematic guidelines that detail processes and procedures for conducting grounded theory. This publication led to dispute between Glaser and Strauss about the identity of grounded theory approach. This divergence marked the beginning of two strands of grounded theory methodology.

(Glaser (1978), 1992)) is generally seen to have stayed rigid to classic grounded theory while Corbin and Strauss (1990) offer a reformulation of the grounded theory approach for novice researchers. However, the divergence is more methodological than ontological or epistemological in terms of the aspects that have been cited as the main point of divergence (see Annells, 1996; Cutcliffe, 2000 ; Urquhart et al., 2010 for more detail).

One of the key points of divergence between the two originators of grounded theory is the role of induction and verification during the process of generating theory. For Glaser (1978) prior readings drawn from the literature beyond a general idea about the problem should be avoided as any extensive focused reading might sensitise the researcher to a wide range of possibilities, which can influence the true emergence of a theory. More focused reading only occurs when emergent theory is sufficiently developed, and literature is to be used as additional data (Hickey, 1997). Indeed, Glaser (1998) discusses what he calls 'near misses' in discovering new theory; this is a process whereby as the theory begins to emerge, focused reading of close relevance might have powerful impact on the emerging theory diverting it from its true path.

For Strauss (1987) both the use of one's prior knowledge of the phenomena and the literature are early influences and, while diffuse, these understandings are seen as providing sensitivity. Indeed, in contrast to Glaser's position, Strauss recommends that a sensitizing research question should be used drawn from the researcher's experience or extensive readings and take the form of identifying the phenomenon to be studied and what is known about the subject (Corbin & Strauss, 1990). They also encourage verification of the emergent codes with literature during course of grounded theory analysis. As Kendall (1999, p. 743) noted that 'one does not need to view either approach as right or wrong; rather, the qualitative and grounded theory researcher can choose an approach, and that choice is based on the goal of the researcher's study'. Hence, it is important for the reader to note that I align more to the Straussian approach to grounded theory because it was Corbin and Strauss (1990) who focused on developing the analytic techniques and provided guidance for systematic coding of data. In particular, I rely heavily on the guidelines that detail grounded theory processes in the work of Corbin and Strauss (1990).

In the next section, I offer some reasons for why I decided to use grounded theory in my data analysis, before discussing in detail how I used the grounded theory approach in ways that led to the emergence of the 'elaboration'-focused analytical framework.

4.6.1.1 Why a 'grounded theory' approach to my data analysis?

My initial area of interest in this study was very broad, being concerned with the professional development of in-service primary mathematics teachers. The vast competing possibilities of teacher's mathematical knowledge in teaching models in the literature led to a decision to observe classroom teaching following teachers' participation in an in-service professional development, and to conduct a post observation VSR interviews.

Having gathered some initial data of classroom teaching, I created verbatim transcripts that captured all the teacher talk, teacher-learner interactions, and descriptions of the tasks and representations that were produced and used by the teachers during the course of the lesson enactments. I organised these transcripts into chunked episodes within each lesson based on shifts in the task set by the teacher, following the approach taken by Ensor et al. (2009). These authors defined task as ‘a segment of a lesson which was constituted around a single goal or theme’ (p. 14). The chunking of lessons into episodes was useful in order to have more manageable slices of data for in-depth analysis of instances of elaboration in the initial stages.

Due to the specificity of the problems noted in South African context, imported international theoretical frameworks largely did not allow for the reading and interpretations of the empirical dataset. As highlighted already, in the international literature, evaluation of learner offers is a given, in marked contrast to the realities of South African classrooms. What imported frameworks provided was a deficit conversation, which was not useful in this study from a developmental perspective (Graven, 2012). Therefore, a language of description from low level home-ground analysis was needed as a means to offer ‘stages of implementation’ (Schweisfurth, 2011) towards desired ends in relation to responsive teaching. To develop this language of description, I found the grounded analysis approach taken by Rowland et al. (2005) very useful, and reflected these authors’ focus on situations in which mathematical knowledge in teaching can be studied, which led in their work to the emergence of codes that constituted their initial KQ dimensions, and in this study to the emergence of codes that constituted dimensions of elaboration in contingency situations.

I then took each lesson transcript and examined episode by episode in grounded ways. Analysis took the form of engaging with the transcript data line by line, while identifying one or two key ideas, underlining and listing words that could represent categories, and generating sensitizing questions to focus on the data. In this phase, I kept my wording very close to the data. The following categories emerged at the initial stage:

- juxtaposing counting in 10s with plus 10;
- working with task in the vicinity of incorrect offer;
- moving between representations;

- probing learner offer with follow-up questions;
- offering more efficient strategy;
- repeating learner offer to the whole class;

These categories represent teachers' responses to classroom events that are viewed as 'low-level' evaluative or diagnostic responses in the international literature. However, given the evidence of gaps in responsive teaching actions noted already in South Africa, exemplifying this nature and range was important developmentally in relation to attempts to improve primary mathematics teaching in South Africa.

This initial data analysis helped to narrow the focus of the study from its original focus on teaching broadly using the four dimensions of the KQ, to the focus on teacher 'elaboration' of mathematical ideas in response to learners' offers during moment to moment interaction in the mathematics classroom. This focus was informed by many episodes in the lesson transcripts where some elaborations were provided, but in ways that were different to what were described in the international literature base as responsive teaching.

Grounded theory was found to be useful as its approach to data analysis helped in providing a theoretical explanation of the ways in which teachers' responded to 'in-the-moment' situations, particularly in the context of the present study where little is known about ways to support primary mathematics teachers to teach in responsive ways. My aim was to categorize, describe and explain this phenomenon with regards to specific teacher actions in response to learners' offers in South African primary mathematics classrooms.

Having narrowed my study to specific situations or moments of teacher response to learner offers in the classroom, then the issue of what constituted my '*unit of analysis'* emerged. The *unit of analysis* is the major entity that is being analysed in a study. It is the 'what' or 'who' that is being studied. In my study, it is the specific situations of contingency and how teachers responded to these situations through elaboration of mathematical ideas. These formed the empirical instances that I dealt with as 'cases' of the phenomena. The teacher's responses were analysed and interpreted from these cases through a grounded approach.

4.6.1.2 Data coding using grounded theory

Working with transcripts data from the 18 lessons, I used the detailed guidance on conducting systematic grounded theory process provided by Corbin and Strauss (1990) for the coding of the data of lesson enactments, specifically, open coding and the axial coding were used. In the following section, I detail how each of these coding methods was carried out in this study.

Open coding

Open coding is a procedure for developing categories of information. It is an interpretive process by which data are broken down analytically. Its purpose is to give new insights through thinking about the phenomena reflected in the data. I first identified the unit of analysis in lesson transcripts – the instances where a learner mathematical offer or remark triggered a teacher response of some sort. I then proceeded with the open coding – using wording that remained close to the data. An example of open coding is shown in Figure 9, using an incident where learners offered an incorrect answer. I tentatively coded the teacher's action as '*working with task in the vicinity of incorrect offer*'. This process of open coding continued until additional data stopped offering new insights about the phenomena. This is what Strauss and Cobin refer to as saturation level. At this level, I looked through the codes in the open coding, defining their unique characteristics. Eventually, I merged some codes that seemed to be describing the same thing. For example codes like: '*juxtaposing counting in 10 with addition of 10'*; '*juxtaposing plus 10 with 10 more'*; '*juxtaposing minus 5 with 5 less'* and renamed all these codes as '*verbal reframing through the use of an alternative phrase*'. I also abandoned some codes that turned out to occur less than 5 times over all the 18 lessons.

3G_1_2014_B1 – Counting in 10s from 33 - 163

9 T: (teacher claps her hands to stop the counting) right I want you to count again in tens forward again. Now I want you to count from thirty-three let's go 33

10 Class: 33, 43, 53 …103, 133 (all learners counting in chorus)

11 T: (teacher clap hands to stop the counting). I want us to start at 93 lets go ninety-three, one hundred and three, one hundred and (counting alongside with the class, teacher stops here waiting for the learners to complete)

12 Class: 93,103, 133 (all learners counting in chorus, still counted 133 after 103)

13 T: Stop (asks learners to stop counting). Let's go 103

14 Class: 103, 133 (but some learners call 113)

15 T: Remember we are counting in 10s, 103 plus 10?

16 L: 113

17 T: Yes, let's start again 93

18 Class: 93,103,113,123…163 (all learners counting in chorus, teacher counting along with the learners and stops at 113 and they carry on correctly).

Description

L9: (Now, I want you to count from thirty-three) – teacher states the **task**

L10: (After 103, learners counted 133) – **incorrect offer [Breakdown 1]**

L11: Teacher interrupts the counting sequence, ask learners to count again from 93, counts alongside with the learners and stop at 103 – **teacher restarts count at 93 - close to error, not from the start again**

L12: Learners counts 133 after 103 – **Repeat incorrect offer** [Breakdown 1 repeated]

L13: Teacher interrupts counting sequence again, ask learners to count from 103 **- teacher restarts count at 103 – zoom close to the error, not from the start again**

L14: Learners counts 103, 133 – **Repeat incorrect offer [Breakdown 1 repeated]**

L15: Teacher reminds learners that they are counting in 10 and ask (103 plus 10) **– zooms in again, juxtaposes 'counting in tens' with 'plus ten' on repeat offer of incorrect answer.**

L16: **individual learner offer 113**

L17: Yes, let's start at 93 **- She restarts count around the error once again at 93, carries through to 113 (the newly offered correct answer from an individual learner), and then asks learners to continue.**

Possible code: **Working with task in the vicinity of incorrect offer**

Figure 9: An example of first level of open coding

Axial coding

Axial coding is the intermediate stage of data analysis within grounded theory – it is a procedure of interconnecting the categories. This involves putting data together in new ways by making connections between the categories. This is achieved by exploring the:

- the *conditions* that give rise to the categories;
- the *context* in which they are embedded;
- the *strategies* that people use to manage conditions or to carry it out; and
- the *consequences* of those strategies.

At the level of axial coding, I revisited the codes that emerged from the open coding and I explored the condition, context, strategies and consequences. Going through this process, the following patterns in the data emerged:

Conditions – This was guided from the onset by the narrowed focus of the study – the specific aspects of contingency situations that triggered teacher's response to classroom events during teacher-learner interactions. The *conditions* that gave rise to the manifestations of this action were: learners' offerings, remarks or contributions in the classroom.

Context – Three contexts in which learners' offerings were embedded emerged. These were labelled as: (i) situations of incorrect learners' offers (*breakdown*); (ii) situations where correct learner offers were given, but were viewed by the teacher as working with inefficient solution methods or representations (*Sophistication*); and (iii) situations of chorused efficient offers that were moved by the teacher to assessing individual learners (*Individuation)* or where an individual learner's offer was developed and projected to the classroom space (*Collectivisation*).

Strategies – Different strategies emerged in the ways in which the teachers' responses to each of the contexts of *breakdown*, *sophistication*, and *individuation/collectivisation* were configured. Collectively, I came to view these strategies as different kinds of elaboration. For example, in the case of breakdown, two broad categories emerged: elaborations that involved 'working with task' and elaborations that involved 'working with the learner incorrect offer'. Within each of these categories further sub-categories were discernible. For example, within the category of 'working with the task', some of the sub-categories were: verbal reframing; lead-in to the task; 'switching between representations and so on. Taken together, my focus on teachers' responses and handling strategies in the context of learner offers in classroom interactions led to the development of what I later termed as the 'elaboration framework'. This has become the central analytic instrument in this study, and represents the key contribution of the study to the ways in which responsive teaching in conditions where learning is widely described as 'rote' and teaching described as 'procedural' and limited in the extent of its learner-centeredness, can be described and developed.

Consequences – At this level, I stepped back somewhat to examine the nature of the ideas that were elaborated, the implications of these elaborations for classroom learning, and how these elaborations overlap and contrast with the international literature readings of ‘in-the-moment’ teacher responses in mathematics classrooms.

An example of the four tenets of axial coding is shown in Table 4. It is important to note that the stages of open and axial coding are not linear. I constantly moved back and forth between open coding and axial coding and continually refined the categories and their interconnections.

Table 5: Summary of an example of axial coding

Evidence			Analysis and Interpretation			
Task	**Situations of elaboration**	**Excerpts from transcript**	**Condition** (*Learners' offerings*)	**Context** (*Dimensions of elaboration*)	**Strategies** *(Kinds of elaboration)*	**Consequences** *(Implications for classroom learning)*
'Number compression' – where X is used to represent ten and √ is used to represent unit. Teacher wrote down 5Xs and a √ and asked, 'What number is that?'	One learner offered 'six' as the answer	L: Six T: He says six. Ok, put down your hands. Why do you think he says six? Why do you think he looked here, looked here and here and said *six [teacher point to the board where the X's and tick were written]*. What has he done?	Incorrect offer	Breakdown	Probed learner offer with follow-up question	The teacher's *elaboration* of the incorrect learner's offer is *focused on learner incorrect offer*. In this incident the focus on leaner incorrect offer was characterized by *probed learner offer with follow-up question*. This is evident in the way the teacher initiated an investigation that interrogated the learner offer. Teacher's response potentially has consequences for the whole class by drawing attention to differences in the quantity values associated with the symbols, as in decimal place value.
Individual learner invited to the board to work out 22-19	Learner works out the problem as 'take away' by breaking 19 into 10, 2, 7 indicated by backward jumps	T: Okay, the answer is just right isn't it? C: Yes T: Twenty-two minus nineteen is? C: three T: Right it's three she got the answer correct clap hands for her. Let's all look at the board and listen. So I am going to take you from Lesley's method of getting the answer, to my new method of getting the answer. [Teacher works out the problem as a	Correct, but inefficient offer	Sophistication	Offering more efficient strategy	Teacher elaboration constitutes a progressive move to sophistication in mathematics teaching. This mode of elaboration is characterized by '*offering more efficient strategy'*. This is evident in the way the teacher accepted the offer, but she then moved on and demonstrated a more efficient model of subtraction in this particular example where the minuend and subtrahend are closer

		difference between 19 and 22. She starts at 22 and makes 3 backward jumps to 19] T: So, that is very fast. So, when you compare this one *(Teacher pointing to the number line that she did))* and that one *(Teacher pointing to the number line that Mandla did))* you can see that there is a lot of work here but it's very fast here; do you see that? C: Yes T: The reason is that I saw that twenty-two and nineteen are very close to each other. So the difference is very …				to each other.
Counting in 2s from 7	Chorused class chant 7, 9, 11, 13, … 39	T: Let's stop. What is the next number? Yes, Realogile (*teacher points to a leaner*) L: Forty-one T: Forty-one. What will be the next number? (*Points to another learner*) L: Forty-three T: Forty-three, what will be the next number? (*Points to another learner*) L: Forty-five T: Forty-five, and our finishing number? (*Points to another learner*) L: Forty-seven	Correct offer	Individuation	Confirming chorus offer with individual learner	The teacher's elaboration constitutes a pedagogic move from whole class chanted count to assessing individual's performance. This kind of elaboration is characterized by 'confirming chorus offer with individual learners'. This is evident in the way the teacher stopped the chorused count and moved on to assess individuals. A potential consequence of this response is more individuated assessment as a classroom social norm, reducing possibilities for learners to 'hide' under the collective.

Is it easy to add 20 straight away to 41?	One learner offered, 'But I know that four plus two equals six'.	L: I know that four plus two equals six T: This is not four. What is this? C: Forty T: Forty plus C: twenty T: Is what? C: Sixty T: So then forty-one plus twenty is what? The answer is sixty-one do you see that? (Pointing on the number line) C: Yes T: If forty plus twenty is sixty then forty-one plus twenty is sixty-one. Zulu siyakholwa ('Do we believe) Class: Yes	Correct offer	Collectivization	Decompressed individual learner's offer to the whole class	Teacher elaboration constitutes a pedagogic move of an individual learner offer to the whole class. This mode of elaboration is characterized by developing and unpacking the emergent individual learner's offer with the whole class. This is evident in the way the teacher made a link between the learner's offer of 4+2 to the understanding of 40+20 as a derived fact to work out 41+20. Here, learning from learner offers is projected as a classroom social norm.

4.6.2 Analysis of VSR interviews

In their research on the use of video-stimulated recall as a catalyst for teacher professional learning, Geiger et al. (2015) coordinated two frameworks to document the changes in teachers' awareness and to identify and make judgements about the forms of reflection practices teachers exhibited as a result of viewing and discussing their own teaching in order to change. These two frameworks related to 'states of awareness' and 'utilization of teacher reflective practice'. The coordination of the two perspectives became necessary in their work to sufficiently analyse the complex endeavour of effecting teacher change.

A similar approach appeared suitable in this study because my aim with the VSR interview analysis was to explore possible associations between any observed shifts (or lack of shifts) in teaching practice across 2013 and 2014 lesson enactments and the professional development using video-stimulated recall interview. Below I provide the detail of the two frameworks, and how I used them in the analysis and interpretation of the teachers' utterances from the transcripts of the interview data.

The first framework is drawn from the work of Schratz (2006) who proposed four states of awareness that teachers undergo when new innovations are introduced into the school system to document changes in practice. The four states of awareness are: unconscious incompetence, conscious incompetence, conscious competence and unconscious competence. Geiger et al. (2015) describe these states in the following terms:

> …***unconscious incompetence*** where a teacher is unaware of the limitations of their knowledge; ***conscious incompetence*** where a teacher becomes aware of their limitations in respect of a specific aspect of their teaching; ***conscious competence*** in which a teacher can address a previous weakness through deliberate planning and action; and ***unconscious competence*** where a teacher has internalised new competencies (p. 6).

Through this lens, teachers' personal states of awareness are revealed when they articulate reflectively on their practice. The highest level of change is evident when a teacher shifts from unconscious incompetence (what Schratz (2006) refers to as a teacher's comfort zone), where the teacher is not aware of limitations of her practice to unconscious competence, where the teacher has internalized new competencies. A point of worry for a teacher is evident when she is at the level of conscious incompetence awareness, but this is a move forward from unconscious incompetence because the turbulence resulting from this reflective awareness can push towards opening doors for possibilities of teacher learning, and accepting new ideas.

The second framework was drawn from the work of (Muir and Beswick (2007)). They devised a two-dimensional framework that related to the utilization of teacher reflective practice as a means of enhancing teaching. The first dimension describes three hierarchical levels of reflection; ***technical description***, where a teacher provides general accounts of classroom practice, often with a focus on technical aspects, with no consideration of the value of the experiences; ***deliberate reflections,*** where a teacher identifies 'critical incidents' and offers a rationale or explanation for the action or behaviour; and ***critical reflection***, where a teacher moves beyond identifying 'critical incidents' and providing explanations to considering others' perspectives and offering alternatives. The second dimension focuses on the *object* of reflective processes, that is, whether reflection pertains to ***self***, ***practice***, or ***students***. Geiger et al. (2015) provide a description of the association between the levels of reflection and object of reflective response as shown in Table 5.

Table 6: Levels of reflection against object of reflective response (Geiger et al., 2015, p. 7)

Levels of reflections	**Object of reflective response**		
	Self	**Practice**	**Students**
Technical	Personal role is described during a teaching event. The description is factual rather than personally insightful	Teaching activity is described in terms of technical aspects. Focus is on consequences or outcomes of their practice	Students' responses to teaching activity are described in terms of technical aspects. Focus is on consequences or outcomes of teaching practice
Deliberate	Personal role is described during a teaching event. A rationale or explanation for the personal behaviour is provided	Critical incidents are related to teaching practice, and a rationale or explanation for the practice is articulated	Students' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed
Critical	Personal role is described during a teaching event. The behaviour is critically analysed and alternative behaviours discussed	The purpose of an activity is clearly articulated, and a judgment is made about the success or otherwise of a teaching practice. When unsuccessful, an alternative practice or activity is suggested	Students' responses to teaching activity are noted, and a rationale or explanation for the response or behaviour is constructed. Potential improvements to the activity are related to an anticipated student response

Teachers' utterances were analysed based on the personal awareness competence constructs, and on the basis of level and object of reflections. In commenting about these constructs, I explore associations between each teacher's shifts in reflective awareness and shifts in responsive teaching.

4.7 Trustworthiness

The aim of trustworthiness in a qualitative study is to support the argument that the findings of the study are worth paying attention to. Lincoln and Guba (1985) posit that to ensure trustworthiness of the study as a multi-dimensional notion involves establishing; credibility, transferability, dependability and confirmability of the study.

Credibility – This notion refers to confidence in the 'truth' of the findings and is established when the results of qualitative research are believable or acceptable through credible data collection and data analysis processes. The credibility of my research was established through both cross teacher observation and the long spectrum of the data collection process-collecting data on classroom teaching over a period of two years that were then analysed rigorously as described in the data analysis chapters.

Transferability – This refers to the degree to which the findings of the study can apply or transfer to other contexts beyond the bounds of the study. This requires providing enough information so that the reader can determine whether the findings are applicable to a different setting (Lincoln & Guba, 1985). The provision of rich descriptions of teacher-learners interactions at transcript and paraphrased levels in the study makes it easier for the reader to determine if the conditions and contexts of this study are 'relatable' to other situations, and then, whether the findings might be transferrable to them.

Dependability – This refers to the extent to which the research findings are assessed to be consistent or could be repeated over time. To establish the dependability of my research, I included an 'independent audit' by frequent discussions with my supervisor and postgraduate peers within the WMC-P team on meaning and boundaries of categories. Explicit methodological detail on procedures and sharing of anonymised transcripts with emergent coding and claims, therefore built the base for dependability. My prolonged engagement in the field, as described in the procedure for data collection, also helped in identifying and describing the changes that occurred, with triangulation of observed changes via VSR interview feedback, adding to credibility and dependability of claims.

Confirmability – This refers to the extent to which the researcher can be neutral or non-judgemental when interpreting and reporting the data collected (Lincoln & Guba, 1985). They recommended four techniques for establishing confirmability of qualitative research; confirmation audit, audit trial, triangulation and reflexivity. Admittedly, objectivity in a qualitative study is problematic, since claims are highly dependent on interpretation and acknowledged as value-bound which contradicts the idea of objectivity in a quantitative sense. To establish the confirmability, I ensured that detailed backgrounds of the incidents were given, with thick description also provided for every incident before interpretations were made in addition to triangulation.

4.8 Ethical considerations

Ethical concerns are addressed as in any research in which human participants are involved in order to minimize damage or wrong doing to the participants. Access to the schools was negotiated with the principals of each primary school and the teachers gave their written informed consent to participate in the research after a discussion of the research focus and data collection plan. The teachers (and their schools) were informed by the researcher that their anonymity would be protected.

My study primarily focused on teachers, but learners and their parents/carers' were also approached with information about the study, and gave their informed consent for participation in the audio and video data capture. In the small number of cases where learners and/or parents/carers declined to participate in the study, these children were systematically excluded from the videoing during the videoing process by being asked to sit in a position outside the frame captured by the camera. All participants gave consent knowing that their participation in the study was voluntary and withdrawable at any time during the process with no fear of any consequences. Anonymised transcripts and pseudonyms are used throughout all writing in this study and the papers emanating from it.

All teachers that were approached agreed to participate in this study. My positioning as researcher and member of WMC-P project – a facilitator of the 'maths for teaching' course could well, of course, have influenced this agreement, and the ways in which these teachers participated. However, I tried to ensure that my positioning did not influence the kind of the data that was collected. In both 2013 and 2014 lesson observations, I emphasised to the teachers that I was interested in following their normal teaching practice in line with their scheme of work. I asked teachers to provide me with their time table and I planned my visit to their classes based on the times when they were teaching additive relations as part of their normal curricular routine, explaining that being the content focus of my study. The teachers were not asked to prepare special lessons to show me, nor were they asked to work with specific mathematical representations or pedagogic approaches.

4.9 Conclusion

In this chapter, I have provided a description of my research design, given reasons for the choices I made in the case study design and participants of my study. I provided reasons for the use of grounded theory for the analysis of the data collected, and how it was used in my study. Issues of ethics, validity and reliability were also dealt with. In the next chapter, I report on the key findings emanating from the grounded analysis that led to emergence of the 'elaboration' framework', consisting of codes and categories that represent stages of implementation towards more responsive teaching in resource constrained primary schools context in South Africa.

CHAPTER 5

ELABORATING 'IN-THE-MOMENT': INTRODUCING AN ELABORATION FRAMEWORK

5.1 Introduction

The constellation of limited teachers' mathematical knowledge, incoherent talk and frequent lack of evaluation criteria in teaching actions noted in South African writing on teachers and mathematics teaching makes, on the one hand, seeing what the international literature describes as possible responses to 'in-the-moment' contingencies relatively unlikely in the South African context. On the other hand though, it also makes understanding possibilities for 'in-the-moment' contingency actions from a low base important to understand within teaching development. Therefore, this chapter reports on the key contribution of this study – the development of a language of description to identify and develop important 'stages of implementation' towards more responsive teaching in the South African context. In developing this language, I ended up with three dimensions of elaborations, with each consisting of categories and codes, which I later pulled together into what I termed the 'elaboration framework'. It is important to note that while many of the emergent categories and codes may well be considered 'low-level' evaluative or diagnostic responses in the international literature, given the evidence of gaps in responsive teaching actions noted already in South Africa, exemplifying this nature and range is important developmentally in relation to attempts to improve primary mathematics teaching in South Africa.

In the following sections, I present an overview of the process that led to the three dimensions of the framework, before detailing the analysis of selected incidents of the categories and codes that constitute each dimension, noting the background to the incidents, verbatim evidence in the form of transcript excerpts, followed by interpretation of each incident and its consequences in relation to teacher's 'in-the-moment' awareness. The incidents are useful initially, in this chapter, for illustrating the kinds of elaborated responses that were seen in the South African terrain described above. The focus on elaboration was taken up as a result of contrasts between the teachers seen in the 2013 lesson observation dataset, and emerging recurring regularities of contrast between the 2013 and 2014 data sets of teaching actions as teachers responded to learner offers during the course of interactions in the classroom. My attunement to these phenomena was driven by previous findings noting their relative absence.

A further crucial level of analysis linked with the framework was driven by literature on the quality of mathematics teaching, related to exploring hierarchies and relationships between the emergent categories within dimensions. These hierarchies and relationships are discussed in the latter sections of this chapter. The framework is later used, in chapter 6, as a tool that provides me with a language of description to discuss, in-depth, the ways in which four South African primary teachers of mathematics elaborated 'in-the-moment' as they responded to learner offers in the classroom, and how the teachers' responses changed over time.

5.2 The emergence of the elaboration framework

As indicated in Chapter 4, the dataset was comprised of the video-recordings of 18 mathematics lessons that were taught by four primary school teachers of mathematics across the Foundation phase (Grades 1-3) and the Intermediate phase (Grades 4-7). I created transcripts of all the lessons that captured the teachers' talk within the lesson and the objects/representations created and/or referred to during the teachers' interactions with learners. Due to the specificity of the problems noted in the South African context, imported international theoretical frameworks largely did not allow for interpretations of some of the differences between the teachers (and seen for the teachers over time) in the empirical dataset. Therefore, a language of description was developed from a grounded analysis that created concepts that were closer to the data and which allowed for its reading. To develop this language of description, I used the grounded analysis approach taken by Rowland (2008) reflecting the focus on situations in which mathematical knowledge in teaching can be studied.

In this study, recurring empirical incidents within lessons where teachers' responded to learner offers, which linked in particular to the contingency knowledge aspect of the KQ framework led to an in-depth exploration of these incidents in this study. Three situations were identified in which these responses commonly occurred, which came to form my initial dimensions of elaboration: breakdown situations, sophistication situations, and individuation/collectivisation situations.

I use the notion of 'breakdown' to describe situations where incorrect offers are given by learners. 'Sophistication' refers to situations where a correct learner offer is given, but is viewed by the teacher as inefficient in relation to either representation or the strategy used by learner/s in producing the answer. 'Individuation and collectivisation' are situations of pedagogic moves of correct and efficient learner offers from either chorus offers to assessing individuals (individuation) or where an individual insight is developed and projected to the collective classroom space (collectivisation). In each of these situations, I inferred distinct goals of the teachers' elaboration from their responses, with these goals summarised in Table 6 below. It was this identification of key categories of teaching goals that led into my 'naming' of these three main elaboration situations with the selected labels.

Table 7: Categorization of situational nature of elaborations

Situations of elaboration	Description	Goal of elaboration
Breakdown	Incorrect learner(s) offer	Eliciting correct mathematical offer
Sophistication	Correct learner(s) offer but viewed by the teacher as inefficient	Moving to more efficient mathematical strategy or representation
Individuation/ Collectivisation	Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	Pedagogic move of chorus offer to assessing individual learners' understanding or projecting individual learner's mathematical offer to collective classroom space with some 'unpacking'

While 'breakdown' situations could be marked as such at the time of learner offers, sophistication and individuation/collectivisation could only be interpreted in these terms following the teacher's response. In terms of analytical processes, it is important to note that while empirically these categories were not mutually exclusive, for the purposes of analytical methodology, I considered all teachers' responses in the context of incorrect learner offers as breakdown situations, even when teacher's actions in resolving the breakdown situation indicated sophistication or individuation/collectivisation moves. Furthermore, the goal of 'sophistication' was inferred from a teacher's response directed at moving learners' offers to a more efficient representation or strategy in working with mathematical ideas.

The last category involved 'unpacking' the mathematical idea while moving individual offers into the collective classroom space (collectivisation), or moving a group chorus offer to assessing one or more individuals' understanding. In a situation where an individuation/collectivisation pedagogic move was present, but was in a context of sophistication, such situations were not categorised as either individuation or collectivisation. Rather, I described them in terms of sophistication because the learner offer was initially viewed by the teacher as inefficient. Hence individuation/

collectivisation situations were described only in the context where the teacher response suggested that the learner offer was not interpreted as either a breakdown or sophistication situation.

Thus, while there were overlaps between the three dimensions in the empirical space, these demarcations made for analytical purposes allowed me to separate the empirical phenomena into mutually exclusive categories. The analysis, as described in Chapter 4, followed Corbin & Strauss's (1990) systematic stages of grounded theory analysis. Teachers' responses to the three situations were initially identified and then subjected to a grounded analysis. A number of ways in which teachers responded to these situations were assigned initial descriptive codes. Eventually, I wrote down each of the initial codes on paper, and began to both group descriptors that I interpreted as having empirical similarities while also separating them into sets based on contrasts.

I looked for the similarities and contrasts in these responses in order to group codes into categories. For example, in breakdown situations, I grouped the following descriptive codes together:

- juxtaposing counting in 10s with plus ten;
- backwards jumps with subtraction;
- forward jumps with addition

I grouped these codes on the basis that all of them involved juxtaposing an initial phrase with a further phrase with an overlapping meaning or procedure. I then collected all of these codes under one assigned category – *verbal reframing*, since they were all verbal responses representing another incidence of the same object and referring to the two objects as the same thing. In relation to arguments presented in linguistic theory, these verbal reframing can be seen in terms of co-classifications that extend possibilities for ways to understand mathematical objects or processes (Halliday & Hasan, 1985) in instances where the initial verbal offering has failed to provoke the desired response.

In the following sections, I present a brief background of each dimension of elaboration drawn from 'post-analysis' further reading attuned to the aspects of the framework, before moving on and illustrating examples of categories that emerged within each dimension. A summary of the overall categories and their identifying descriptors emanating from each of the three dimensions that empirically emerged in the course of my data analysis are presented in relevant tables.

5.3 Breakdown situation of elaboration

Breakdown refers to situations where incorrect answers are offered by learners in the mathematics classroom. Incorrect answers are intrinsic parts of all mathematics learning situations. While often viewed as 'inconvenient' within teaching, there is broad agreement in the literature that errors and misconceptions are natural stage in knowledge construction and thus not only inevitable but are welcomed (Askew & Wiliam, 1995; Vosniadou & Verschaffel, 2004). However, it is not the evidence of the incorrect offers that matters from the learning perspective, but rather how the teacher uncovers and deals with the incorrect offers contingently in the classroom (Askew & Wiliam, 1995). Providing learning support which is contingent on learners' needs when incorrect answers are offered is considered effective for developing learners' understanding (Wischgoll, Pauli, & Reusser, 2015). Indeed, Koshy (2000) states that incorrect answers can be viewed as teachable moments when teachers are sensitively aware of learners' needs and can create learning opportunities.

In response to the above, the following questions could be asked: How do we characterise teachers' responses' to learners' incorrect offers in the classroom? What kinds of elaborated responses are offered by the teachers to resolve breakdown situations? What is the quality of such responses in relation to openings for learning? In all these questions, what is at stake is the nature and extent of a teacher's responses to learners' incorrect offers.

This research suggests that appropriate responses to learners' incorrect offers contribute to effective classroom evaluation and learning opportunities in the mathematics classroom. Thus, the quality of these responses depends significantly on the connection between the teacher's *mathematics awareness* and '*in-the-moment' pedagogy* (Davis & Renert, 2014). In teachers' responses to learner incorrect offers in the dataset, I initially categorised these responses into two ways: Elaboration not provided (ENP), or elaboration provided (EP). Elaboration not provided involved a range of options that have antecedents in the international and the South African literature base:

- reduced the cognitive demand of the task (Bauersfeld, 1980; Carnoy et al., 2012; Stein, Grover, & Henningsen, 1996);
- repeated learner's offer and moved on (Rowland and Zazkis, 2013; Venkat and Naidoo, 2012);
- repeated task and moved on;
- no comment and moved on (Ekdahl & Runesson, 2015)

Instances of 'elaboration provided' are occasions where the teacher incorporated and built on the learner's incorrect offer in the lesson. These instances were categorised into two types. The first of these types were *elaborations that focused on learner's incorrect offer* - where the teacher worked with the learner's incorrect offer. The second were *elaborations that were focused on the task* - where the teacher worked with the task in response to the learner's incorrect offer. Figure 10 summarises these categories, which represent the emergent 'recurring regularities' in the teachers' handling of learner offers in breakdown situations.

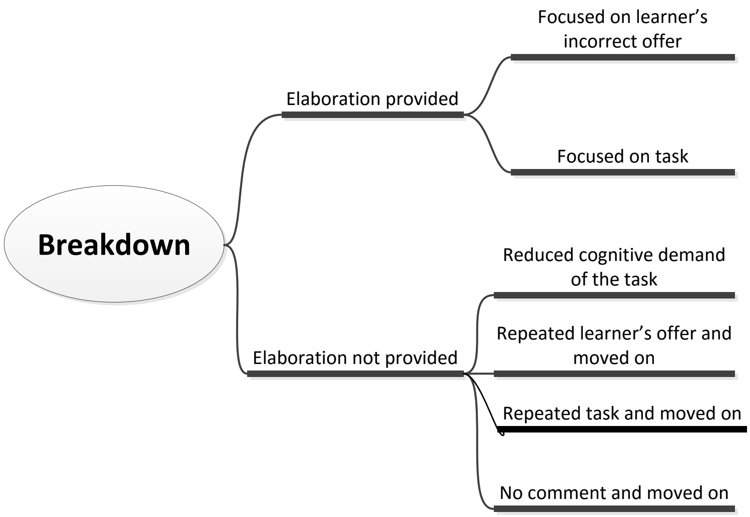


Figure 10: Teachers' handling of breakdown situations

Figure 10 presents categories that represent the ways in which teachers handled breakdown situations. I was particularly interested in describing and exemplifying, in more detail, the two broad categories where elaborations are provided. Both of these elaboration types, in the context of breakdown situations represented implementations of a move towards more responsive teaching in the South African context. I therefore proceed to describe each of these types of elaborations in turn, and provide examples drawn from the 18 lessons that I observed and analysed that illustrate the range within each category. Each of these incidents exemplifies teachers' ways of being with mathematics, or mode of enquiry, in different ways. In choosing these examples, I have selected particularly illuminating instances – 'telling cases' in Mitchell's (2002) terms - from the lessons transcripts.

5.2.1 Elaborations that focused on learner's incorrect offer

I refer here to teachers' responses where their focus of attention was on the learner's incorrect offer. The great majority of elaborations in the data set were of this kind. The following incident drawn from Sam's Grade 2 class, Lesson 2, 2013, provides one exemplification of this category. As noted already, my central thrust was on the analysis of the nature of teachers' responses rather than analysis of the incorrect offers.

Sam taught Grade 2 learners additive relations with concrete representations. After hanging a string with 10 marbles (5 blue and 5 yellow) on the board, she partitioned the marbles into four and six (Figure 11), and then drew a triad representation with the numbers: 10, 4 and 6. Sam then asked learners to give a number sentence that would go with these numbers. Learners raised their hands, and she pointed to individuals to respond. At the point where this excerpt begins, two addition number sentences had already been offered by the learners, and been written on the board by the teacher: 4+6=10 and 6+4=10. Sam then asked for a subtraction number sentence. One learner, Andiso, offered 4 – 6 =10, Sam responded in the following ways, as presented in excerpt 1: (conversational turns are numbered and brackets indicate non-verbal information; T denotes teacher, L – a learner and C- multiple learners in class chorus).

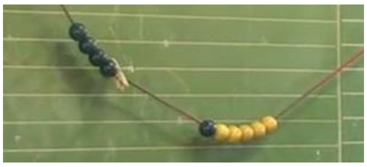


Figure 11: Partition of 10 marbles into 4 and 6

Excerpt 1

300 T: Okay, what did you want to say, Andiso? Let's give her a chance, she wants to say something

301 L: Four minus six equal to ten

302 T: (*Teacher writes '4 – 6 =10' on the board as the learner gives her number sentence).* Okay, let's go back to our marbles; can we do that?

303 C: (*Some say no, others say yes*)

304 T: No, I don't want an answer where all of you are saying yes or no. Please put up your hands if you want to say something. Okay, Thamani

305 L: (*inaudible*)

306 T: Okay, as it is. What do you say about Andiso's sum? (*Teacher points to the number sentence written on the board: 4 – 6=10*).

307 L: It's wrong

308 T: You can't say she is wrong without telling us why you think she is wrong. But you can tell us what you think about her sum. If you want, you can come here and show us (*Teacher points to the 10 marbles separated into 6 and 4*). Four take away six, so I can put six there (*Teacher moves the six marbles further away on the bead string leaving the four marbles more distinctly separate*). Is there anyone who wants to say something about her sum?

309 L1: Four is a small number

310 T: Is there anyone who wants to add to what she said?

311 L2: Six is a big number

312 T: Four is a small number; six is a big number and so? Can you take away six from four?

313 C: No

314 T: Because four is a small number. If we want to take away, do we have enough to take away?

315 C: No

316 T: Four is a small number, for now, you cannot take away six from four. But when you grow up you will see that it is possible. What must I put here? A plus and it will read? (*Teacher changes the number sentence from 4 – 6 =10 to 4 +6 =10*)

317 C: four plus six equal to ten

With the offer of '4 – 6 = 10', Sam is confronted with an incorrect answer. Her initial response was to go back to the concrete representation inviting the whole class to think about enacting the offer given by Andiso – 'Okay, let's go back to our marbles, can we do that?' (line 302).

Sam's actions here indicated an evaluative comment aimed at establishing the correctness or otherwise of the learner's offer. She engaged learners in a collaborative conversation that gave her some sense of her learners' thinking, linked to what Sawyer (2004) has termed as disciplined improvisation. The evidence of collaboration can be seen with Sam building individual learners' offers into the flow of the conversation – when one learner said, 'Four is a small number' (Line 309). Sam response was, 'Is there anyone who wants to add to what she said?'(line 310). Though Sam controlled the conversation, she incorporated learners' follow-up offers in establishing whether Andiso's offer was correct or not. She did so by building on the two learners' follow-up offers that stated the relationship between the 'four' and the 'six' as four is a smaller number than six– 'Because four is a small number. If we want to take away, do we have enough to take away?' (line 314).

Within Sam's response, it is clear that Sam knows and understands that Andiso's offer is incorrect. In itself, this response entails firstly, the presence of some evaluation (important, given Hoadley's noting of its frequent absence), but further, an evaluation that goes beyond the simple rejection of an incorrect offer that Rowland and Zaskis (2013) have noted as relatively common. Sam's response here involves initiating an investigation in which some learners were able to show that four is a smaller number than six, and therefore that one cannot take away six from four. Based on this conclusion, Andiso's offer was rejected. While more 'incidental' to the focus of my study, Sam's careful response also indicated her awareness of the possibility of the offered subtraction operation becoming possible at some point in the future, even while acknowledging that it is not possible at the current stage. This response fell within the compass of Sam's horizon mathematical knowledge (Ball et al., 2008) or connection knowledge in relation to what is anticipated as learners progress through the curriculum (Rowland et al., 2005), and is of interest in relation to South African evidence of knowledge gaps at the level of content being taught rather than at higher levels, though, this evidence has been drawn largely from Intermediate rather than Foundation Phase teachers (Venkat & Spaull, 2015).

While there is a move forward here from rejecting the learner's offer as incorrect to establishing mathematical practices of guiding the development of enquiry approaches to mathematics in the classroom (Cobb & Yackel, 1995), critical analysis of Sam's response on the basis of the international literature still points to some limitations. The rationale that established Andiso's offer of 4 – 6 = 10 as incorrect on the basis of not being able (currently) to take four away from six, is not mathematically sufficient given that it cannot generalize, for example, to providing a rationale that would apply to 6 – 4 =10 as an offer, since here, there are enough in six to take away four. Sam's response then, while exemplifying her work with the learner's incorrect offer, also suggests some limitations at the level of 'in-the-moment' pedagogy. The limitations in Sam's response indicate that the problem here is not related to her transformation or connection knowledge; rather, it would appear that she has not thought through the extent of generalizability of the rationale she has developed from learners' inputs in relation to further possible offers relating to the task. This limitation suggests a lack of awareness to draw sufficiently from these knowledge domains when needed (Davis & Renert, 2014).

This latter point notwithstanding, in the context of the above excerpt and analysis, Sam's elaboration indicated an investigation of the incorrect offer, rather than immediate rejection of the offer. I labelled Sam's response as '*probes learner's offer with follow-up question'* because she provided elaboration that interrogated the incorrect offer with no implicit or explicit rejection of the offer as incorrect from the onset. Literature suggests that engaging closely with learner thinking and reasoning provides teachers with valuable resources to use in support of emergent mathematics learning in the classroom (Franke & Kazemi, 2001; Koshy, 2000; Mason, 2015). This body of writing has identified teacher noticing and listening carefully to learners' contributions as vital in generating these valuable teaching resources, while acting contingently as a reflective practitioner (Schön, 1987). I termed this context broadly as an 'elaboration that focused on learner's incorrect offer' – as an elaboration type, in the context of breakdown situations where the teacher's response indicated a close working with the learner's incorrect offer.

A range of empirical phenomena came to be clustered within this elaboration type. In Table 7, I summarise all the codes and the code identification descriptors that emerged from analysis of the teachers' responses. The summaries that I present in the next chapter for each teacher show that phenomena that could be collected under these descriptors produced these codes as recurring regularities (Lincoln & Guba, 1984) in the dataset.

Table 8: Breakdown elaborations with focus on learner incorrect offers: coding scheme and code identification descriptors

Kinds of elaborations	Code	Code identification descriptors
Restates learner's offer and questions its correctness	FL-QC	The teacher incorporates learner's offer in a follow-up question or statement that shows implicitly or explicitly rejection of the offer. FL-QC includes questions that seek a form of yes or no response. Examples of FL-QC include phrases like: Is it? Will it be..? Can you...? Do you…? It won't? Do you think …? Examples of FL-QC: - In the context of the task 'What do we add to 27 to get 30?'The following excerpt played out: L: Add two T: No, if we add two, we are not going to get to thirty; If we add two because, we are at twenty seven. Can you think about that? Twenty-seven plus two, it won't get to thirty. - In the context of the task on 0-100 number line (NL) marked in 10s, 'Now, who can show us where the numbers 1-9 fit into the number line?' The following excerpt played out: - L: *(learner indicates the range between 0 and 90 on the NL)* - T: Okay! It is these numbers; 1-9 *(points to the space from 1- 9 on a 100-square)*, you see the space is very small. Will the space be from here to there?[pointing to the offer given by

		the learner]
Probes learner's offer with a follow-up question	FL-PQ	The teacher incorporates learner offer in a follow-up question that does not implicitly or explicitly indicate rejection of the offer. In FL-PQ teacher's purpose is to investigate the incorrect offer or seek for further clarification on why the offer is given. Teachers' responses in FL-PQ take the form of: why, how, what –type questions. Examples of FL-PQ: - In the context of working out the task '301 – 299 using the model of subtraction as difference'. The following excerpt played out: Mpho: (*Learner walks to the board and acts like a teacher*) Where must we start? Jason: Three hundred and one Mpho: (*Learner writes '301'towards the end of ENL*) Mduduzi (calls out another learner by name) Mduduzi: Minus five Mpho: (*Learner makes a backward jump of '5' from '301' and writes '-5' on top of the jump*) T: Do you understand why Mduduzi is saying minus five? C: No T: Okay let's give Mduduzi a chance to tell us, why did you say minus five Mduduzi? Do you have a reason for minus five? - In the context of task 'number compression' – where X is used to represent ten and √ is used to represent unit. Teacher wrote down 5Xs and a √ and asked, 'What number is that?' L: Six T: He says six. Ok, put down your hands. Why do you think he says six*?* Why do you think he looked here, looked here and here *(points to the X's and √* on the *board)* and said *six?* What has he done?

5.2.2 Elaborations that focused on task

Unlike the type 1 category where the teacher's response focused attention on working with the learner's incorrect offer, this category is about working with the task. Task as used here refers to a set of problems or a single complex problem that focuses learners' attention on a particular mathematical idea (Stein et al., 1996). Below, I present analysis of an incident drawn from Thandi's Grade 3 class, Lesson 1, 2014, that was chosen from the data set to exemplify the task-based elaborations of breakdown situations.

This incident, at the beginning of Thandi's lesson, was part of the oral mental starter activity that lasted for about 5 minutes. She had begun the lesson by asking learners to count in 10s from 10-200, which her class had done without difficulty. She then asked learners to count forward again in 10s from 33. Learners offered recurring incorrect counting sequences. My focus below is on Thandi's response in this context of the incorrect learners' offers.

Excerpt 2

9 T: Right! I want you to count again in tens forward, again. Now, I want you to count from thirty-three. Let's go. Thirty-three…

10 C: Thirty-three, forty-three, fifty-three, [*etc, in correct sequence*], one hundred and three, one hundred and thirty-three, one hundred and forty-three (*all learners counting in chorus*)

11 T: (*Teacher claps hands to stop the counting*). I want us to start at ninety-three, let's go: ninety-three, one hundred and three, one hundred and (*counting alongside with the class, teacher stops here waiting for the learners to continue*)

12 C: Ninety-three, one hundred and three, one hundred and thirty-three (*all learners counting in chorus, still chanted 133 after 103*)

13 T: (*Teacher claps hands to stop counting*). Let's go one hundred and three

14 C: One hundred and three, one hundred and thirty-three (*softly some learners call out 113*)

15 T: Remember we are counting in 10s, one hundred and three plus ten?

16 L: One hundred and thirteen

17 T: Yes, let's start, ninety-three…

18 C: Ninety-three, one hundred and three, one hundred and thirteen, one hundred and twenty-three, one hundred and thirty-three, one hundred and forty-three … (*all learners counting in chorus, teacher counting along with the learners and stops at 103 while learners carry on correctly*)

While the counting sequence from 33 to 103 had run smoothly, learners struggled with counting in tens at 103, evident in the recurring incorrect offers. The teacher's awareness of the specific location of this problem is inferred retrospectively (as Sawyer notes in the case of improvisational situations) from her responses. With learners saying 133 after 103, Thandi is thus confronted with a situation where she has choices to make: whether to ignore the error, or to respond to it, and if she chooses the latter, then how to respond.

Stopping the counting sequence and asking learners to count again from 93 suggests Thandi's awareness of the learners' incorrect offer, as does her counting alongside the learners and stopping at 103, where the incorrect offer was given. Learners repeated the same incorrect offer by calling out 133 after 103. Thandi stopped the counting sequence again and asked learners to count from 103. The same incorrect counting sequence was repeated. At this moment, Thandi *juxtaposes* 'counting in tens' with 'plus ten' on the repeat offer of the incorrect answer as seen in line 15. Thandi's action establishes connection between two mathematical ideas, which she draws from her connection knowledge in dealing with this trigger of contingency.

I interpreted Thandi's response as a form of elaboration constituted by a task-related response through her provision of an alternative, but equivalent, verbal representation for the task - counting in tens. In this way, the idea of 'counting in tens' is related to 'plus ten', thereby elaborating the meanings and operational processes that can be associated with counting in tens. However, it is also interesting that the 'general' idea of counting in tens is linked with the 'local' instruction to use the operation of adding 10 to 103, rather than a more general equivalence.

I labelled this task-related elaboration as '***verbal reframing***'. The verbal reframing of the task can also be understood from the systemic functional linguistics perspective of the idea of cohesive ties (Halliday & Hassan, 1991). Their notion of co-classification – presenting another instance of the same object – and referring to the two objects as the same thing, also connects and extends learner understandings. Therefore, the learners' incorrect offer was taken up here as a teachable moment, with Thandi linking the idea of 'plus 10' with counting in 10s to get to the next number. Once again, this kind of contingent response is important in the face of evidence of frequent lack of evaluation of learner offers (Hoadley, 2006), and evidence too of 'repetition' of the same instruction in the face of incorrect answers (Venkat & Naidoo, 2012), rather than the kind of elaboration seen above. This also linked to broader literature noting the importance of 'revoicing' – either with more formal mathematical language, or scaffolding with more everyday language – as a means of supporting learning (O'Connor & Michaels, 1996; Setati & Adler, 2000).

As with the first category related to responses that focused on learners' incorrect offers, a range of empirical phenomena were also clustered within the elaboration type focused on task. In Table 8, I summarise all the codes and the code identification descriptors that emerged from analysis of the teachers' responses to learners' incorrect offers. These codes were the recurring regularities (Lincoln and Guba, 1984) that emerged from the dataset.

Table 9: Breakdown elaborations with focus on task: coding scheme and code identification descriptors

Kinds of elaborations	Code	Code identification descriptors
Verbal reframing	FT-VR	The teacher uses an alternative verbal phrase in response to the learner's incorrect offer. Unlike the previous two categories here the teacher is working with the task rather than the incorrect offer. Hence, there is no explicit mention of the incorrect offer by the teacher in the follow-up response. Examples of FT-VR from the data: - [*30 was offered as an answer to 23+10*] T: No, I said twenty-three <u>plus ten, put ten more</u>. Twenty three, what is ten more? - [*Incorrect counting sequence in 10s was offered, 93,* ***103, 133****, 143*) T: Remember we are <u>counting in 10s, it's like 103 plus 10?</u>
Lead-in to the task	FT-Li	The teacher uses a different task that is analogous to the original task or another task different from the original task that can lead-in to the solution of the original task. Both FT-Li and FL-PQ involve follow-up questions. What is different between the two categories is that in FL-PQ the follow-up question incorporates the learner's incorrect offer, while in FT-Li the focus is on the task. Examples of FT-Li: - [2999 was offered as an answer to 1999 + 10) T: How many do we need to get to two thousand from 1999? <u>When we were counting we say 1998, 1999. How many do we need to get to 2000?</u> - [*9 000 was offered as an answer to 10 000 take away 10 000*] T: Unh unh! Think first. There was ten thousand, right? And we took ten thousand here. If we take ten thousand away, what is left there? Okay, fine, there are <u>five things here right, and if we take five things away, what is left there?</u>
Switching	FT-SBR	The teacher moves between different representations in response

between representations		to the learner's incorrect offer. In FT-SBR teacher restates the original task, but in a different representation (e.g. from 100-square to a number line). This differs from the FT-Li category where a different task is stated. FT-SBR may include restating the task from a context-free to a context-bound domain or vice versa. It is important to note that I didn't read this category as 'verbal reframing' because of the extensive evidence of lack of using alternative phrases in support of learning in South Africa. Therefore separating the two categories was found to be useful in reading my data. Examples of FT-MR includes: - [*Learner points at 25 on a structured number line in 10s and makes a backward jump of 5 as a solution to 25=30-__*] T: To get to twenty-five we are supposed to remove how much from thirty? <u>If you got thirty oranges, how many should you take away in order that you remain with twenty-five?</u>
Establishing generality	FT-EG	Teacher explicitly or implicitly states generic version of specific task in response to incorrect offer. The teacher's response goes beyond the specific task, with potential to be applicable to other similar situations. This category is useful in relation to Hoadley's work noting 'localizing' strategies as common in South Africa. Examples of FT-EG - [*In the context of working out the task 56 – 9 on a number line, a backward jump of 10 was made*] T: What do we do now? L: minus one T: Okay, come and show on the number line L: (*learner makes a forward jump on a number line for minus one*) T: <u>When we jump forward, is it minus or is it plus</u>? - [*Learner offered 2 15/6 as equivalent to 3 5/6*] T: Remember, we said if our denominator is six that means our one whole is divided into six equal parts. If our denominator is

		seven that means our one whole is divided into seven equal parts.
Contrasting offered and required operations	FT-CO	Teacher's response notes that the required operation is not followed, and a contrasting offer is provided. This is different from the focus on learner's offer categories despite the incorrect offer is incorporated into the follow-up, the teacher's attention is specific to the focus on task's instruction been not followed. Examples of FT-CO includes: - [*Forward counting sequence in 10s from 21; 200 was offered after 191*] T: If you say two hundred you have added nine not ten. So if you added 10 to 191, it will be… - [*60 was offered as ten less than 50*] T: Sixty is ten more, I want ten less.

Following the development of these codes, I returned to the lesson transcripts for each teacher with a more quantitative gaze. In coding the transcripts, the incidence of an incorrect offer marked the beginning of the unit of analysis. Therefore, the 'unit' of analysis in breakdown situations refers to the all interactions, verbal and written, from the moment of the incorrect offer until the breakdown is resolved. Recurring incorrect offers within the same unit of analysis and from the same source (either an individual learner, group of learners or whole class) were considered as one incident of breakdown. Where the teacher moved to another learner or to the whole class from an individual learner and another incorrect offer was given, this was then marked as a different incident of breakdown. As noted already, while possibilities for individuation/collectivisation were in play here, the breakdown situation took analytical precedence, and thus these incidents were all described in terms of breakdown situations. This is because they form within the unit of analysis of breakdown.

In each breakdown incident, the teacher's utterances/responses were coded, so it is possible to have multiple codes within one incident of breakdown. However, where a particular utterance, which had already been coded, was repeated within the same incident, the repeat utterance was not recoded or counted. This clarity relating to coding incidents is vital in interpreting the data presented for each teacher's 2013 and 2014 lessons enactment in the next chapter.

5.4 Sophistication situation of elaboration

Elementary mathematics curricula worldwide advocate the need for increasing sophistication through the move from less to more efficient strategies and representations. The South African Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011), for example, provides guidance that recommends term-by-term and grade-by-grade progression, and specifies a shift in additive relations work from the use of drawings or concrete apparatus with counting all, counting on or counting back (counting strategies) to calculation strategies including building up and breaking down numbers, doubling and halving and using resources like number lines. Therefore, responding to inefficient learners' strategies or representations is a necessary step to support understanding of the connectedness of mathematical ideas, and a step that has been noted as limited in the South African context (Ensor et al., 2009). Sophistication situations were therefore an important dimension of responsive attention by the teachers in South African, and particularly so given the widespread evidence of failure among learners well into the Intermediate Phase to move beyond counting- and repeated addition-based approaches to solving number problems (Schollar, 2008).

In the transcripts of the lessons, I identified all of the incidents where a correct offer was given by learner(s), but viewed by the teacher as inefficient. This was mostly indicated by an acceptance of the offer, followed by the word '*but*', or by asking someone to answer the question differently and in a quicker way. Here too, I looked at the teachers' follow-up responses in a grounded analysis. In doing so, all sophistication instances of elaboration across the 18 lessons were marked and then categorized through a 'constant comparison' process by examining similarities and contrasts of these responses in order to group codes into categories. A number of ways in which teachers responded to learners' offers that they viewed as inefficient were identified and coded in the dataset. As in the case of breakdown, teachers often chose to either provide no elaboration (ENP), or provide elaboration (EP). Providing no elaboration involved either acknowledging learner offers as inefficient, but then moving on with the lesson or pulling learners back to less efficient methods without any rationale for doing so. Incidents of provision of elaboration involved responses that moved the learners' offers to more efficient strategies and/or representations.

Below I provide an example that illustrates an incident of provision of elaboration in the context of sophistication situations. This example, drawn from Thandi's Grade 3, Lesson 3 from 2014, demonstrated a situation where she provided an alternative and more efficient solution method than the one offered by the learners. Thandi invited one learner, Mandla (L1), to work out a subtraction problem; '22-19' on the chalk board using an empty number line. Thandi's response to Mandla's solution action is presented in excerpt 3.

Excerpt 3

175 L1: (*learner makes a backward jump of 10 from 22 and writes '-10' on the top of the jump. He then writes '12' where the jump lands*)

176 T: Let's cross our legs and look at the board while Mandla is doing the sum.

177 L1: (*Learner makes another backward jump of 5 from 12 and writes '-5' on top of the jump and 5 where the jump lands and pauses*)

178 T: Maybe we should help Mandla

181 C: Yes

182 T: Let's help Mandla, he said, twenty-two he took away ten is he right?

185 C: Yes

186 T: He is right, now he says minus five. Will it be easy for Mandla to get the answer twelve minus five or is there other way he can subtract from there?

187 C: Yes

188 T: In order for it to be easy for him? How many can he subtract first? Khanyisile

189 L: Minus two

190 T: Minus two

191 L1: (*erases '-5' on top of the jump and writes '-2'*)

192 T: That will be better

193 L1: (*Learner writes '10' where the jump lands. He makes another backward jump of 7 from 10 and writes '-7' on top of the jump and then he writes '3' where the jump lands on the line*)

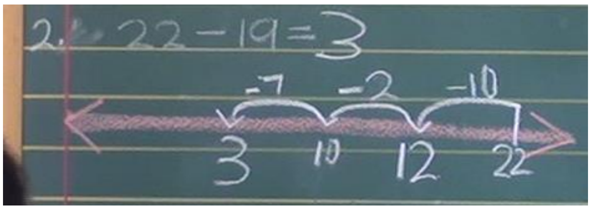


Figure 12: Mandla's representation of the solution of '22-19'

Thandi then responded to Mandla's solution in the following ways:

208 T: Okay, so we can see the process that he went through, from twenty- two subtracted ten from twelve subtracted another two from ten another seven (*Teacher points to Mandla's number line as she talk about the process*). Let's all look at the board and listen. So I am going to take you away from Mandla's method of getting the answer, to a different method of getting the answer. Twenty-two minus nineteen, I will write it up there (*Teacher writes the sum twenty-two minus nineteen on the top right of the board*)

209 C: Twenty-two minus nineteen

210 T: When we look at twenty-two and nineteen, do we see that they are very close to each other?

211 C: Yes

212 T: They are very, very close to each other. So I said to you when we did the starter that we should look at our numbers that we are adding or subtracting and then we should ask ourselves what is the difference between them? How far apart are they or how close are they? So that you can use the ***fastest method***, so I will do that sum using a different method (*Teacher draws an empty number line and marks the two numbers: 19 and 22).* Okay, now I can jump 2 backward from 22 to get to 20 and make one backward jump from 20 to get to 19 (Figure 15). How much jump do I make from 22 to 19?

223 C: Three jumps

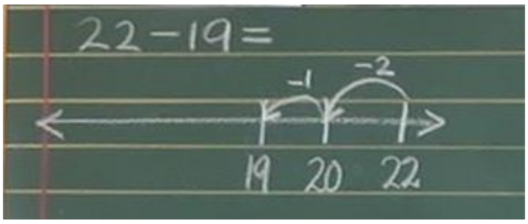


Figure 13: Teacher's representation of the solution of '22-19'

224 T: So, that is very fast. So, when you compare this one (*teacher points to her number line in Figure 15)* and that one (*Teacher points to the number line that was done by Mandla in Figure 14)* you can see that there is a lot of work here but it's very fast here. Do you see that?

225 C: Yes

226 T: The reason is that I saw that twenty-two and nineteen are very close to each other. So the difference is very …

227 C: Small

228 T: So, I get my twenty-two, I subtracted two and got to twenty and subtracted one more and got to my nineteen. So, my answer is two plus one, which is?

229 Class: Three

Excerpt 3 presents Mandla's solution action for the problem '22-19' and how Thandi responded to Mandla's solution action. In this excerpt already, we see some attunement to the strategic progressions that are widely described as important within moves towards sophistication in Thandi's invitation to the class to look for an easier option to the subtracting 5 step where Mandla pauses. Her acceptance of the bridging through ten steps as 'better' (L190) suggests awareness of the efficiency offered by strategies based on structuring number in the decimal system (Mcintosh et al., 1992).

Thandi confirmed, in agreement with the whole class, that 3 had been established as the correct answer. Thandi asked learners to clap hands for Mandla while commending him and asked him to sit down. She proceeded to remind learners about the starter activity, which was about finding which number was closer to a given number of two selected numbers. Thandi related the starter activity here as a means to decide on a faster method to use when subtracting numbers (L212). Thandi is therefore drawing from her connection knowledge in linking the work rehearsed in the starter activity to the main lesson. Thandi then moved on and demonstrated a different and more efficient method of working out subtraction problem by drawing another number line and marking the two numbers, 19 and 22, on it. She reiterated that since the numbers; 19 and 22 are very close to each other, the problem can be interpreted as the difference between the two numbers while pointing at the space between 19 and 22 on the number line. Thandi then made backwards jumps of 2 and 1 to land at 19 on the number line (see Figure 15). She then moved on with discussion about the efficiency gain in the method she demonstrated compared to the one demonstrated by Mandla. She provided a rationale for the efficiency of the method based on the two numbers 19 and 22 being very close to each other.

The literature suggests two model of subtraction as 'take away' and 'difference' (Haylock, 2008). Thandi's response to Mandla's solution of '22-19' with an offer based on seeing the problem in terms of difference indicated that her awareness of these two models of subtraction. In going on to offer her alternative strategy for working out the same problem after confirming with the class that 22-19=3 as completed by Mandla was correct, she provides a sophistication oriented elaboration.

It is important to note that Thandi's selection of examples in the broader lesson across starter and this activity were well connected. In the starter activity she included examples that contrasted the efficiency of take away and difference – as background to her working in the sophistication dimension contingently here. The links between the ideas practiced in the starter activity and then applied to this task in the main activity in the lesson suggested careful planning and sequencing of tasks and examples, with her sophistication oriented response in this instance supported by connection knowledge related to her linking between examples. It could be the case that her response here was 'planned' given the careful example sequencing and linking. The instance is important within this study's focus because her competence in drawing attention to a more efficient strategy to use in this kind of problem-solving is a move forward in the face of prevalent evidence of lack of move to more efficient methods in South Africa (Ensor et al., 2009). More broadly, teachers' awareness of the need for a growing repertoire of number facts with flexibility has been noted as important to support the development of learners' efficient working with number operations.

Thandi's response as described above constitutes an elaboration that is characterised by '*offering a more efficient strategy*'. Prior to Thandi's response, the openings provided to build efficiency through linking an example that was useful for highlighting the possibility for a more efficient strategy with an appropriate model (the empty number line) that contrasted the differences in efficiency while also displaying the 'gap' between the numbers as the key rationale for selecting the strategy to use already constitutes a vital part of the coherence and connections needed for good quality mathematical discourse in instruction (MDI) in South Africa (Venkat & Adler, 2012), and more broadly (Anghileri, 2006; Bruner, 1974; Carpenter & Moser, 1984; Drews, 2007; Ensor et al., 2009).

Thandi's follow up to Mandla's method for calculating the answer to the problem tags a sophistication move onto this coherence at the level of her tasks and representation. In the analysis above, Thandi provided an elaboration that allowed learners to discern more efficient ways of working out subtraction problems, while also using the number line representation to contrast the strategic efficiency in terms of the number of steps, while providing an image that spatially presented the 'gap' between the two numbers.

As mentioned already, teachers' responses to learners' inefficient offers were categorised into two initial groups: Elaboration provided (EP) and elaboration not provided (ENP). Elaboration not provided involved incidents where the teacher acknowledged the learner offer as inefficient and moved on, or where the teacher pulled learners back to inefficient actions. Further scrutiny of all identified incidents of provision of elaboration indicated a range of empirical phenomena which were clustered together into three kinds of elaborated responses. The first level involved *offering a more efficient strategy*; the second, *eliciting a more efficient strategy*; and the third, *interrogating learner's offer for efficiency*. Figure 14 presents these categories indicating ways in which teachers responded to what they viewed as inefficient learner offers.

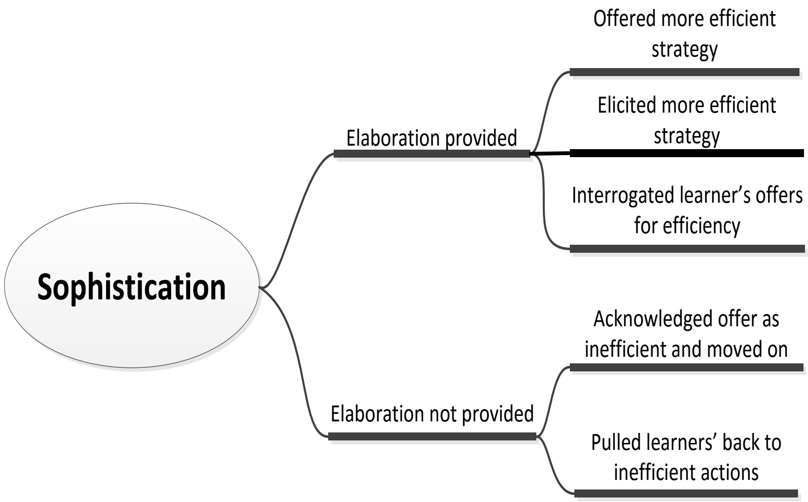


Figure 14: Teachers' responses to correct offers that were viewed as inefficient

Figure 16 presents categories that represent the ways in which teachers responded to learners' mathematically correct offers which they viewed as inefficient. In Table 9 below, I present summaries of the coding scheme and the code identifying descriptors that described the three kinds of elaborations that I labelled as incidents of provision of sophistication elaborations that empirically emerged from the dataset.

Table 10: Sophistication coding scheme and code identification descriptors

Kinds of elaboration	Code	Code identification descriptors
Offers a more efficient strategy	OES	Teacher’s response to learner’s correct offer is to accept the offer and move on to demonstrate an alternative but more efficient solution action to work out the same problem. Examples of OES indicators included: - But, you can also do much easier in this way - Teacher responds to learner’s offer of count-on with suggestion to count-on from larger number - Moving learners from a take away model of subtraction to demonstrate a difference model when the numbers are close to each other
Elicits a more efficient strategy	EES	The teacher’s response to correct learner’s offer viewed as inefficient is to elicit an alternative offer either from same learner or whole class. In EES, a teacher’s explicit or implicit statement suggests an interpretation of inefficiency of the learner’s offer before eliciting an alternative offer. Example of EES includes: -[*Learner counting backwards in 1s*] T: Will it be easy for Lesley to get the answer for 12 minus 5 or is there any other way she can subtract? -[*Learner drew 26 tallies with repeated counting and re-counting of these tallies, and wrote a ‘+’ sign followed by a symbol of 4 and thirty as the answer to 26+4*] -T: It’s correct, but is there anyone who did something differently?
Interrogates learner’s offer for efficiency	ILE	Teacher’s response to a learner correct offer viewed as inefficient is to ask learner(s) to give a rationale for that particular offer. In ILE, there is no explicit or implicit statement from the teacher that suggests inefficiency of the offer at the moment of teacher response; teacher investigates learner’s reasoning that led to the offer given. The word ‘why’ is often used within this sub-category

		Example of ILE includes: In the context of the task, 127+18 on an ENL. Jason: Plus one Solly: (*Learner writes '+1' on top of the jump*) Class: Plus five (*in chorus*) T: Okay, why do you say plus one Jason? Why? Jason: (*in silent concentration*) T: You don't know, somebody said plus five can you tell us Sthembele, why you say plus five? We are at one hundred and twenty-seven Sthembele: Because when we add one to one hundred and twenty-seven, we are very far to adding eighteen T: Very far from eighteen? Sthembele: Yes mem T: That's not how we decide which number or how many jumps. We decide how many we jump depending on this number that we are jumping from (*teacher points at 127*). So, the number that we are jumping from is one hundred and twenty-seven. So we should jump in such a way that it will be very easy for us to know the next number. Probably, a multiple of ten.

In coding the transcripts, the units of analysis were comprised of incidents where a correct offer is given by the learner(s) and the teachers' responses retrospectively suggested that they viewed the offer as inefficient. Therefore, the unit of analysis refers to the moment from the learner's (subsequently viewed as inefficient) offer, tracking all interactions, verbal and/or written, until a more efficient offer is stated or established. However, where the teacher accepted an offer and moved on, I did not code this as an incident of sophistication, even when that offer could be inferred as inefficient in relation to the literature, except where there was evidence that learners produced an offer that the teacher pulled back to a less efficient action without any overt rationale. For example, Grade 6 learners responded to the question: 'What is 3+6?' with an immediate answer of 9 as a recalled fact, but the teacher insisted that they must show this on a number line or using counters. I coded such incidents within the category of elaboration not provided.

In each incident of sophistication, the teacher's utterances/responses were coded; so it is possible to have multiple codes within one incidence of sophistication. As before, where a particular utterance, which had been coded, was repeated within the same incident the utterance was not recoded.

5.5 Individuation/Collectivisation situation of elaboration

Classroom interactions involving extensive whole class recitation work, with little or no evaluation of individual learner's understanding have been noted as prevalent in South African primary school teaching (Hoadley, 2012). Conversely, Venkat and Naidoo (2012) noted limited projection of individual learner's offers to the collective classroom space. Such 'responding' moves that attend to an individual learner while projecting and developing their offers with the whole class have been described as beneficial to broadening opportunities for learning in classrooms (Brown & Wragg, 1993; Rowland et al., 2009).

While honouring learners' different ideas, a strategy of sharing has to be done with careful insight by the teacher to ensure that the mathematical integrity of learner's ideas is established in the course of the interactions (Walshaw & Anthony, 2008). Doyle & Carter's (1984) earlier work on classroom participation reported that teachers used the strategy of 'accepting all answers' as a way of simply achieving learners' cooperation in an activity. This kind of practice has pedagogical consequences for the development of mathematical thinking, where the teacher's focus on synthesizing learner's individual contributions is largely absent.

Drawing from the above suggests that while attention to both moves - individuation and collectivisation, is necessary steps for supporting learning in the classroom, the quality of these pedagogic moves is determined by the extent to which the teacher can manage classroom interactions by focusing on synthesising individual and collective offers, while developing learners' thinking and reasoning about powerful mathematical ideas, in support of emergent mathematics learning. I identified all the incidents where correct and efficient offers were given by the learners across the 18 lessons, and subjected these responses to grounded analysis. As with the previous two dimensions, teachers often chose to either provide, or not provide, elaboration provided. Providing no elaboration involved: accepting chanted offer without assessing individual understanding, or accepting individual learner's insight without sharing and/or developing with the whole class. Intentional ignoring of a learner's suggestion without response was described as absent empirically in the international literature. Rowland et al. (2015) state:

> It is difficult to identify instances in our data where the teacher literally ignores a pupil's suggestion, except where they seem not to have heard it, in which case the teacher's lack of response cannot be regarded as intentional (p. 81)

Instances of the provision of elaborations were categorized into two types. The first is *individuating a response* - where the teacher's response to a chorused correct offer is to move to assessing individual learners' awareness of this offer; and the second is *collectivising* a *response* - where the teacher's response to an individual learner offer is to develop and project it to the whole class. The pattern of these responses is summarised in Figure 15, which constitute the different ways in which teachers provided to individuation/collectivisation situations.

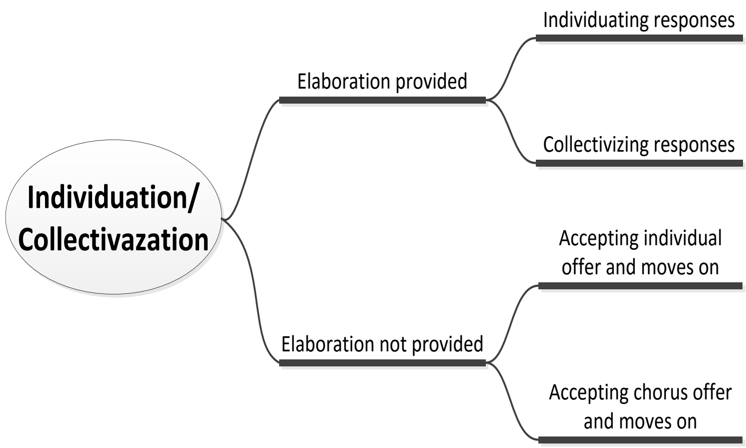


Figure 15: Teachers' responding moves to individuation/collectivisation situations

Figure 15 presents categories that represented the ways in which teachers made pedagogic moves in response to learner(s)' correct and efficient offers in the course of teaching. Below, I provide examples drawn from the data of classroom teaching relating to the two broad categories of responses where teachers provided elaborations: individuating and collectivising responses. Subsequently, a summary of all the kinds of elaborated responses that were clustered within each category is presented in a tabular form following analysis of transcript excerpts that illustrate the roots of each category.

5.5.1 Individuating responses

I refer to individuating responses to incidents where a correct and efficient offer or insight is chorused by whole class, and the teacher's follow-up response takes the form of assessing individual understanding. Though individuating responses were found in the data set, these were very limited in comparison to collectivising responses. This confirmed previous research findings in South Africa (Chick, 1996; Hoadley, 2008; Taylor & Moyana, 2005), that chorusing and rhythmic chanting are the predominant practices in working class primary classrooms, with widespread absences of individual, evaluated performance. The following incidents have been chosen from the data set to exemplify the individuating response category of elaboration, which illustrates a counter norm from prevalent practices in the South African context.

I draw on an example from Sam's Grade 4 classroom, Lesson 1, 2014, to illustrate how this response is configured and conceptualized in this study. In the lesson just prior to the excerpt below, Sam had asked learners to count in 2s starting from 7. The class counted 7, 9, 11, 13, and so on. When they got to 39, Sam asked learners to stop, and she proceeded in the following way:

Excerpt 4

81 T: Let's stop. What is the next number? Yes, Realogile (*teacher points to a leaner*)
82 L: Forty-one
83 T: Forty-one. What will be the next number? (*Points to another learner*)
84 L: Forty-three
85 T: Forty-three, what will be the next number? (*Points to another learner*)
86 L: Forty-five
87 T: Forty-five, and our finishing number? (*Points to another learner*)
88 L: Forty-seven
89 T: Forty-seven, who can tell me, did you see any pattern? Tell me
90 L: They are odd numbers
91 T: Yes, it's only odd numbers we are counting, but we are counting in twos, isn't it!
92 Class: Yes

The task enactment here moves from oral class chant to assessment of individuals' facility with the counting sequence. Sam individuates several times with different individual learners as seen in excerpt 4. There are two important features for consideration here: firstly, a pedagogy checking whether individual learners can produce the focal counting sequence rather than 'hiding' within the whole class chant; secondly, Sam, (in line 91), also elicited and elaborated more general rules for the generation of the sequence, i.e. they are all ‘odd numbers’ (offered by learners), and that odd number sequences involve ‘counting in twos’ (Sam’s elaboration involves a verbal reframing here). Thus while Sam is checking with individual learners, she draws attention to the pattern of numbers that is being generated. In this episode Sam appears to draw from a transformation knowledge base in generating an example space and renegotiation of collective meaning that counting in 2s starting from odd numbers results in the set of odd number example space. Her use of the word ‘but’ in the phrase ‘but we are counting in twos, isn’t it?’ (L91) suggests that she may be expanding a view that counting in 2s would more usually involve the even numbers.

I interpreted Sam’s response as a form of individuated elaboration that I labelled as ‘*confirming chorus offer with individual learners’*. This kind of response provided potential for evaluating individual learner understandings in ways that counter norms described as prevalent in South African primary mathematics classrooms (Hoadley, 2012). Classroom discourse literature has noted the importance of individual contributions during interactions (Mercer, 2000). In Mercer’s view, it is through engaging with individuals during interaction that collective understanding is developed.

While the incident presented above is a case of one example of an individuating response, there was another kind of elaborated responses that empirically emerged through grounded analysis. In Table 10, I summarise all the codes and the code identification descriptors that empirically emerged from analysis of the teachers’ responses that moves whole class chanted offers to assessing individuals’ understanding.

Table 11: Individuation coding scheme and code identification descriptors

Kinds of elaboration	Code	Code identification descriptors
Confirming chorus offer with individual learners	**ICf**	Teacher's response to chorus offer is to move from whole class to individuals repeating the same task. In ICf, the teacher's goal is basically checking on whether individuals can produce the same correct offer as the one previously given collectively. Examples of ICf includes: - [We have taken away more, a chorus offer] T: Put up your hands. Have we taken away more or we have taken away less than nine? Angie - [A chorus offer 73 was given as an answer to 69+4] T: Do you know the answer Nomthandazo? Do you know the answer for sixty-nine plus four? What is the answer? Nomthandazo - [Class chant – counting in 2s starting from 3, when learners get to 39, teacher stops counting sequence] T: Let's stop, what is the next number after 39? Yes Rethabile
Interrogating chorus offer with individual learners	**IIt**	Teacher's response to chorus correct offer moves from whole class to asking individuals the question of 'how' – about the procedure by which the correct offer is produced or 'why'- giving an account for the rationale to support the chorus offer. Examples of IIt includes: - [Learners offered 73 as an answer to 53+20 immediately] T: Is it easy mentally to add twenty? Can someone explain why? - T: How comes, how is it easy? Lesedi

5.5.2 Collectivising response

I refer to collectivising responses as situation where an individual's correct and efficient offer or insight is developed and/or projected to the collective classroom space. To exemplify this response type, I draw from Bongi's response to a learner's idea that could have gone unattended. Bongi had stated that it was difficult for learners to add 20 to 41 as one jump on a number line, and hence, had said that she preferred making two jumps of 10s. One learner's insight placed the teacher in a position to improvise. The following excerpt details how the incident played out:

Excerpt 5

81 T: How come, how is it easy?

82 L: Because here you add one number not two (*referring to the big jump of 20 compared to the two jumps of 10s*)

83 T: I added one number but, is it easy mentally to add twenty? Can you explain again?

84 L: Because, mem, you don't have to jump ten and jump another ten again.

85 T: Okay, I understand, but how do I know that sixty-one is forty-one plus twenty how do I know that? Khanyisile

86 L: Forty-one, we say plus ten is fifty-one

87 T: And then?

88 L: And then plus ten is sixty-one

89 T: Okay, I thought you are going to say I just know it. Forty-one plus twenty, is sixty-one, I just know it.

90 L: (*one learner raised hands*)

91 T: Do you know it?

90 L: Yes, I know that four plus two equals to six

93 T: This is not four. What is this?

94 C: Forty

95 T: Forty plus

96 C: twenty

97 T: Is what?

98 C: Sixty

99 T: So then forty-one plus twenty is what? The answer is sixty-one do you see that?

100 C: Yes

101 T: If forty plus twenty is sixty, then forty-one plus twenty is one more, which is sixty-one. Siyakholwa? (*Do we believe?*)

102 C: Yes mem

Bongi’s comment in lines 85 - 89 suggests that she had initially concluded that it would be too difficult for her learners to think about the answer to 41+20 as a recalled fact indicated by a single jump of 20 on a number line. Despite the teacher controlling the flow of the discussions in the class, she also opened up for emergent insight from the learners in her class. In the process of being open to learner inputs across Lines 84-88, Thandi firstly heard Khanyisile’s way of thinking about how to add twenty mentally as two steps quickly, but recording only the overall result from combining these two jumps, and then went on, in Line 91, to check for this understanding with another learner. In this collectivising move, she also surfaced another way of adding twenty – a place value decomposition strategy (sometimes described as a ‘1010’ strategy in the literature (Cobb, 1995; Fuson, 1992; Thompson, 1994)), while she then returned to focus on projecting the initial ‘N10’ counting on in tens strategy offer to the whole class, and adding in a link to 40+20 as a linked ‘easier’ example as she did this.

While several elaboration moves are therefore evident in this episode, this incident highlights Bongi’s flexibility in response to emergent mathematics learning, and her projecting of an individual learner’s insight to the whole class during classroom interactions. This incident highlights teacher’s awareness, readiness and thinking on her feet to draw upon both her transformation knowledge and connection knowledge in a responsive way to attend to learner’s contributions ‘in-the moment’ of teaching.

The excerpt indicates that the teacher initially had no intention to encourage a derived fact strategy to support learners’ thinking about the problem. However, one learner’s insight was taken up and unpacked by the teacher as a resource to support learners’ mental calculations. The opportunities inherent in this kind of response are captured well in a comprehensive review around what effective mathematics teachers actually do to support emergent classroom discourse (Walshaw & Anthony, 2008). These authors identified careful reflection on individual learner's insights by the teacher and sharing into the collective classroom space as an important domain for building opportunities for learning.

I labelled this kind of collectivising response as a form of elaboration characterised by '*decompressing an individual learner offer to the whole class*'. This elaboration was visible in situations where the individual offer was improvisationally developed by the teacher while projecting it to the collective classroom space. In other situations, individual offers were simply re-voiced to the whole class, and I labelled these responses as '*repeating learner offer to the whole class'*. It is important to note that a linguistically exact repetition of a learner offer may still involve modification of the language used (Planas & Morera, 2011), and therefore, *repeating* in this context is regarded as conceptual reformation of the learner offer, which can serve to extend understanding of the collective members of the class. This does not count as verbal reframing because it does not occur in a breakdown situation.

A range of empirical phenomena came to be clustered within the teachers' collectivised responses that constituted five different kinds of elaborated responses. In Table 11, I summarise all the codes and the code identification descriptors for the five kinds that emerged from analysis of the teachers' responses that moved and developed individual learners' correct and efficient mathematical offers to the collective classroom space.

Table 12: Collectivisation coding scheme and code identification descriptors

Kinds of elaboration	Code	Code identification descriptors
Confirms individual learner's offer with whole class	**CCf**	Teacher's response to an individual learner's correct offer is projected to the whole class asking the same task. In CCf, the teacher checks whether whole class can produce the same correct offer given by the individual learner. Examples of CCf includes: - T: Okay. Four (*teacher re-voices an offer given by one learner*). How many 5s are there in 20? (*addressing the whole class*) Class: 4 - [One learner offered eight 5s as 40] Excellent. Let's count on our number chart and see if we are going to get eight 5s?
Interrogates individual learner's offer with the whole class	**CIt**	Teacher's response to an individual learner's correct offer that develops and projects this offer by asking whole class the question of 'how'– relating to the procedure by which the correct offer is produced or 'why'- giving a rationale to support the individual offer. Examples of CIt includes: - [Learner offers 32 as an answer to 17+15] T: [Teacher turns to the whole class] He says 32 is the answer. Can you all tell me how he gets 32?
Repeats individual learner's offer to the whole class	**CRt**	Teacher's response to an individual learner's correct offer is projected to the whole class with the teacher reiterating the learner's solution action. In CRt, teacher re-voices the learner offer by repeating all the steps of the learner solution actions. Examples of CRt includes: T: Can you see what Refilwe has done? C: Yes T: How did she get to sixty-one? She took the bigger number, fifty is it?

		C: Yes mem T: She started with the bigger number, is it? C: Yes mem T: She put it down here, isn't it? C: Yes mem T: Then what did she say next? She said fifty plus what?
Decompresses individual learner's offer to the whole class	**CEx**	Teacher's response to an individual learner's correct offer is projected to the whole class with the teacher unpacking the learner's solution action. In CEx, teacher 'unpacks' and develops learner's offer and uses it as a teaching point beyond what was given by the learner. Examples of CRt includes: L: But I know that four plus two is equal to six T: This is not four. What is this? C: Forty T: Forty plus C: twenty T: Is what? C: Sixty T: So then forty-one plus twenty is what? The answer is sixty-one do you see that? C: Yes T: If forty plus twenty is sixty then forty-one plus twenty is sixty-one. Siyakholwa? (Do we believe?) C: Yes
Collective reasoning	**CCr**	Teacher's response to an individual learner's correct offer is projected to the whole class with the teacher facilitating a collaborative dialogue eliciting whole class reasoning on the offer given. CCr is different from CEx and CRt in the sense that in those sub-categories, it is the teacher that is revoicing (reiterating or unpacking) the individual offer to the whole class, while in CCr, the teacher facilitates and creates a collaborative form where learners contribute to a conversation about the individual offer. Examples of CRt includes: T: I am still asking you. Do you see her number line? C: Yes T: And do you see her answer? C: Yes T: What do you think?

In coding the transcripts, the unit of analysis (retrospectively interpreted) was the sequence of pedagogic moves in a teacher's response to a correct and efficient offer given either by an individual learner or the whole class. In each incident of either individuating or collectivising responses, the teacher's utterances/responses were coded, so it is possible to have multiple codes within one incident. As before, where a particular utterance which had been coded was repeated within the same incident, the utterance was not recoded.

Table 12 present the summaries of all three dimensions of elaborations and accompanying categories that constitute what I termed 'the elaboration framework'. The three elaboration situations; breakdown, sophistication, and individuation/ collectivisation are described in the first column. The second column shows categories of responses in relation to elaboration provided (EP) and elaboration not provided (ENP), with the sub-types categories illustrated and analysed in this chapter listed in the third column.

Table 13: The elaboration framework

Situations of elaboration	Categories of response	Sub-types categories
Breakdown Incorrect learner(s) offer	EP	***Learner incorrect offer-focused responses***
		Restates learners' offer and questions its correctness
		Probes the learner's offer with follow-up question
		Task-focused responses
		Verbal reframing using alternative phrase
		Lead-in to the task
		Switching between representations
		Establishing generality
		Contrasting offered and required operation
	ENP	Reduces cognitive demand of the task
		Repeats learner's offer and moved on
		Repeats task and moved on
		No comment and moved on
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	EP	Offers a more efficient strategy
		Elicits a more efficient learner's offer
		Interrogates learner's offer for efficiency
	ENP	Acknowledges correct offer as inefficient and moves on
		pulls learners' back to inefficient action
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	EP	***Individuating responses***
		Confirms chorus offer with individual learners
		Interrogates chorus offer with individual learners
		Collectivising response
		Confirms individual learner's offer with whole class
		Interrogates individual learner's offer to the whole class
		Repeats individual learner's offer with the whole class
		Decompresses individual learner's offer to the whole class
		Collective reasoning
	ENP	Accepts chorus offer and moved on
		Accepts individual offer and moved on

5.6 Hierarchies within situations of elaborated responses

The discussion so far has focussed attention on description of the range and nature of the categories that constitute the elaboration framework, and indeed, these are necessary and structurally important to the framework. However, a further crucial feature relates to whether any hierarchy exists between the sub-categories that described different elaborated responses by the teachers. My thinking about hierarchies between sub-categories was driven by classroom mathematics discourse literature and theory relating to teaching quality (Borko et al., 1992; Borko & Livingston, 1989; Cobb et al., 2010; Franke et al., 2009; Hill et al., 2008; Lampert, 2001; Nystrand & Gamoran, 1990). As Carpenter, Franke, and Levi (2003) have noted, students cannot learn mathematics with understanding without engaging in discussion and argumentation in the classroom, and that it is within the patterns of interaction and discourses created in the classroom that learners develop such understanding.

In this view, productive mathematical discourses are contingent on learner offers, and therefore exploring hierarchies in teacher's elaborated responses is particularly important in this study for two reasons. Firstly, as noted already, the categories developed in this study are grounded, in order that they might serve as 'stages of implementation' towards more responsive teaching, and this renders discussion on hierarchy important. Secondly, and related to the first, scrutiny of hierarchy within dimension provides a means for moving beyond the identification of elaborated responses to exploring possible extensions and expansions over time in teachers' responses to learner offers as a means of thinking about improvements in mathematics teaching. These rationales constitute the main thrust of what the present study aims to explore.

In the following sections, with the aid of illustrative excerpts from lesson transcripts and literature related to quality of mathematics teaching within classroom interactions, I discuss the nature of these hierarchies within each dimension of elaboration. It is important to note that quality of elaboration is viewed in relation to provision of enhanced opportunities for mathematics learning during the course of interactions.

5.6.1 Hierarchy within breakdown elaborations

Within the category of breakdown elaborations with focus on learner's incorrect offer, further scrutiny of literature led to the view that the two sub-categories of elaborations are hierarchical. The first is the teacher's response by *restating the learner's offer and questions its correctness*; and the second level is the teacher's response by *probing the learner's offer with a follow-up question*. However, no hierarchy exists within the task-related elaborations as each is simply a different kind, indicating a multiplicity of approaches to responding to learner offers in the classroom with a focus on the task.

As noted already in the description of the two categories with focus on learner incorrect offers in Table 7, the first level indicates an explicit or implicit statement of rejection of the learner offer, while the second level investigates learner thinking, and therefore explores possible rationales for why the offer is incorrect. For example, in the context of the task, 'What do we add to 27 to get 30?' - One learner offered 'two' as the answer and the teacher responded as follows:

> T: No, if we add two we are not going to get to thirty. If we add two because, we are at twenty seven? Can you think about that twenty-seven plus two it won't get to thirty. Michelle

The teacher's response here is evaluative in the sense that learners are informed that 'two' is not a correct answer, and there is some elaboration that points to the offer as incorrect because it does not produce the required outcome. This kind of response exemplifies the lower level of elaboration – response by *restating the learner offer and questions its correctness*. While this kind of response does evaluate the learner's offer, the teacher's response does not give the learner any opportunity to express her thinking. The benefit of follow-up questions that attend to students' thinking in the course of mathematics teaching is well-documented (Franke, Kazemi, & Battey, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Silver & Stein, 1996), and information obtained from student's thinking are found to serve as a vital resource that informs pedagogical decision-making in the classroom (Franke, Fennema, & Carpenter, 1997). This learner may have reasoned that there are two numbers; 28 and 29 between 27 and 30, and therefore offered 'two' as the answer, but if the possible sources of learners' thinking are not interrogated, then the teacher's options for offering elaboration linked to learner thinking are more limited.

Taking further steps to interrogate the learner thinking, without overt dismissal of the learner offer would thus move the teacher's response to the second and higher level of elaboration – *probes learner offer with follow-up question*. Fundamentally here, at the lower level, we have evaluation and acknowledgement of the incorrect offer, but no elaboration relating to how to go on to produce a correct offer, or to see why the given offer is incorrect – thus pointing towards a way of being with mathematics that is concerned primarily with the delivery of correct answers (Ekdahl & Runesson, 2015). The move, at the second level, is to probing reasons for the incorrect offer, and is thus geared towards mathematical processes as well as its outcomes. Brophy's (1999) review of generic features of quality teaching indicated that practices in classroom that engaged learners in thoughtful and sustained discourse make a difference to learning, especially when discourse is built on learners' current level of difficulties.

The distinction between the two responses thus rests on the mode of enquiry approach, with the probing of a learner's offer providing a higher level of engagement and opportunities for learners to express their own ideas. The latter sub-category is therefore interpreted as creating more productive openings for teachers to utilise a wide range of resources as teaching points (Koshy, 2000) in support of emergent mathematics learning in the classroom.

5.6.2 Hierarchy within sophistication elaborations

The literature base relating to sophistication elaborations suggests that the three sub-types are hierarchical in nature with *offering a more efficient strategy* at the least skilled level and *interrogating learner offer for efficiency* at the highest skill level. This follows Yackel and Cobb's (1996) suggestion that learners became less engaged when the teacher offered a solution to problems than when they were engaged in the reasoning and thinking that led to those solutions.

At the lower level, when teachers offer a more efficient strategy, teaching is limited to procedural rules dictated by the teacher's decision on what counts as a more efficient strategy, and this decision is in most cases amenable to planning. This contrasts with shifting learners' cognitive attention toward making sense of mathematical experiences, and being able to differentiate and be part of thinking and reasoning of what counts as more efficient mathematical working.

To exemplify the hierarchical nature of sophistication elaborated responses, I draw on an incident from Thandi's lesson. Thandi had worked through a series of examples of addition of two and three digits numbers, with focus on bridging through 10 and counting in 10s with the whole class. Across this example sequence, Thandi consistently encouraged making jumps of multiples of 10 (i.e. counting on in 10s) or making a jump that landed on a multiple of 10 (bridging through 10). Thandi wrote a sum: 127+18, and invited one learner, Solly, to work out the problem on the board. Solly drew an empty number line and wrote 127 at a mark towards the start of the ENL. He turned around and asked the class what to add first. Here Solly was imitating, acting as a teacher, in the course of the interactions.

Another learner, Jason offered 'plus one', but the class chorused, 'plus five' suggesting disagreement with Jason's offer. Thandi asked Jason to give a rationale for adding one, but Jason stayed in silent concentration. Thandi then returned to the whole class and asked Sthembele to give a rationale for adding five. Sthembele said, ‘Adding one is too far from adding 18’ – giving a rationale for the rejection of Jason’s offer of adding one, and therefore suggesting adding five. Thandi responded in the following way:

Excerpt 6

363 T: That’s not how we decide which number or how many jumps. We decide how many we jump depending on this number that we are jumping from (*teacher points at 127*). So, the number that we are jumping from is one hundred and twenty-seven. So we should jump in such a way that it will be very easy for us to know the next number. Probably, a multiple of ten. We are at one hundred and twenty-seven, so the next multiple of ten is one hundred and thirty. So how many do we need to add on one hundred and twenty-seven in order to get to one hundred and thirty. Tsanene (*Teacher points to a learner*)

364 L: Plus three

365 T: Plus three, yes, you should know that. Seven and three makes ten isn’t it?

366 C: Yes

Sthembele’s rationale provides Thandi with an opening for a teaching point in which she contrasts this rationale focused on the addend quantity (18) with her own offer based on the starting quantity (127). She states the need for an ‘easy next number’ as the rationale to efficiently decide on the intermediate jumps rather than adding any number. The idea of bridging through 10 was stressed in Thandi’s response which connects and extends learners’ understanding of efficient mental calculations (Mcintosh et al., 1992). The fluency with which learners were able to offer 3 as the required addend suggests that her sophistication related elaboration was helpful and appropriately pitched for learners in her class. This latter point is important in relation to the sophistication hierarchy given that it is possible that the teacher might offer a more efficient strategy (level 1) or elicit a more efficient strategy (level 2) in ways that remain beyond the emergent focus of learners in the class. Askew, Venkat & Mathews (2012) have noted this kind of occurrence in South African Foundation Phase classrooms.

Thus, while commenting on ‘interrogating learner offers for efficiency’ category at the highest level of response, I also acknowledge that teachers may have good reasons for providing level 1 or level 2 responses. Some of these reasons might be that in the broader classroom context, the more efficient strategy is already broadly established, and therefore the teacher might see no need to interrogate the learner inefficient offer. Remediation for the specific offer from a specific child in this situation may well focus more simply and pragmatically on eliciting or offering a more efficient strategy.

5.6.3 Hierarchy within individuation/collectivisation elaborations

As with the other two dimensions of elaboration, hierarchy also exists within individuation/collectivisation elaborations. This hierarchy is witnessed in the two categories of individuated responses - *Confirming chorus offer with individual learners* and *Interrogating chorus offer with individual learners,* with interrogation of offers viewed more highly for the reasons outlined in the previous section. A similar hierarchy operates in the first two categories of collectivisation responses - *Confirming individual learner's offer with whole class* and *Interrogating individual learner's offer with whole class*. Literature does not suggest a hierarchy across the other categories of collectivisation elaborations as each is simply of a different kind. Taken together though, they indicate multiple possibilities in the space for teachers' elaborated responses.

Within both individuated and collectivised elaborations, the key feature of the hierarchy is the distinction between *confirming* offers and *interrogating* offers with a pedagogic move from chorus offer to individual or from individual offer to the whole class. As stated already in Tables 9 and 10 – *confirming* is characterised by a teacher response that checks whether individuals or the whole class can produce the same offer, while *interrogating* is constituted by a teacher response that investigates the ‘how’ of the procedure by which the correct offer was produced and/or ‘why’- giving an account for the rationale to support the given offer.

To support the claim for the hierarchy between the two categories, I draw on an incident in the context of interpreting a word problem from Bongi's grade 6 class where a correct answer was chanted. The problem posed was: 'In a class, there are 23 boys and 15 girls, how many more boys are there than girls?' Excerpt 6 presents the interaction that unfolded.

Excerpt 7

158 T: We have twenty three boys, yes. The boys are here, one, two, three, four, five; we can get up to twenty-three. Girls, there are fifteen. But my question is how many more boys are there than girls?

159 C: Eight (*learners chorus the answer*)

160 T: Eight. What did you do? (*Teacher points to one learner*)

161 L: we counted…

162 T: Count for us, yes (*teacher invites the learner to the board*). Count for us. Count for us. Talk to them.

163 L: I count from fifteen to twenty-three.

164 T: And then can you count from there, let's go.

165 L: Sixteen, seventeen, eighteen, nineteen, twenty, twenty-one, twenty-two, twenty-three (*learner counts on from 15 to 23 and keeps track of the number of counts with his fingers*).

166 T: What is your answer?

167 L: Eight (*learner shows eight fingers*).

168 T: Eight what?

169 L: Eight boys.

Here, following a class chorus of the correct answer, Bongi moved on and asked an individual to establish how eight was produced. The learner's response was: 'We counted'. Interestingly, the learner used 'we' acting as a representative of the whole class view. Bongi then pressed the learner to demonstrate this solution action. The learner demonstrated a 'count on to' strategy – starting from 15 and counting on to 23, while keeping track of the numbers he counted with his fingers. Bongi's pedagogic move from chorus offer to assessing individual learners went beyond confirming whether the individual could state the correct answer to probing whether he could show 'how' the answer was produced. This kind of move is useful to support learners in the class who may not know how the answer was produced, and thus acts as a device that can broaden epistemological access. The caveat here is that the level of the task selected here is significantly below the number and conceptual range of the Grade 6 curriculum, and while the 'how' of answer production is probed, the strategy of counting on remains relatively low level. This emphasizes that the nature and extent of contingent responses needs to be considered in relation to task and example selections, which, in the KQ, are related to the teacher's transformation knowledge base.

The common feature of both categories is that the teacher response works from a 'base' in which some acknowledgement of learner working is a given. However, the second category in which there is some interrogation of learner offers emphasises more on learners' articulations of their mathematical thinking in contrast to confirming offers that can easily be rooted in brief question-and-answer exchanges (Brodie, 2007). As Mercer (1995) has noted, a pedagogical practice that does not attempt to synthesize learners' individual contributions tends to constrain the development of mathematical thinking.

Thus, even with the limitations related to transformation knowledge above, a pedagogical approach that is able to move learners' thinking forward involves significantly more than confirming offers, rather it involves interrogating learners' thinking into a larger mathematical world that acknowledges underlying reasoning behind mathematical procedures and rules of practice (Popkewitz, 1988). O'Connor and Michaels (1996) put it this way:

> The teacher must give each child an opportunity to work through the problem under discussion while simultaneously encouraging each of them to listen to and attend to the solution paths of others, building on each other's' thinking. Yet she must also actively take a role in making certain that the class gets to the necessary goal: perhaps a particular solution or a certain formulation that will lead to the next step… Finally, she must find a

> way to tie together the different approaches to a solution, taking everyone with her. At another level - just as important- she must get them to see themselves and each other as legitimate contributors to the problem at hand. (p. 65)

Quality mathematics teaching is inclusive and demands careful attention to learners' articulation of ideas, rather than a more basic exchange of offers or question-and-answer exchanges (Brodie, 2007). Kazemi and Stipek (2001) make the important claim that effective teaching tries to probe learners' thinking by noticing and listening carefully to what learners have to say. In another development, Yackel, Cobb, and Wood (1998) reported on empirical evidence that substantiated this claim, where teachers' focus on listening, observing, and questioning for understanding and clarification, greatly enhanced understanding of students' thinking. Contrary to the critique that such practice is impossible on the grounds that the constant probes of learner offers takes more time than the classroom could possibly offer, Jaworski (2004) provided evidence of teachers noticing and then acting knowledgeably as they interacted at critical moments in the classroom when students created a moment of opportunity in the classroom.

Critical engagement with learner thinking through probing of learner offers (Franke et al., 2009) feature prominently within all the hierarchical categories. In Table 13, I provide a re-presentation of the elaboration framework with highlighting of the hierarchies within each dimension. The highlighted areas in the framework show the nine categories in which hierarchies exist. These include: the first two categories of breakdown elaborations; the three categories of sophistication elaborations; and four categories of individuation/collectivisation elaborations.

Table 14: Hierarchies within elaboration framework

Situations of elaboration	Categories of response	Sub-types categories
Breakdown Incorrect learner(s) offer	EP	***Learner offer-focused responses***
		L1 – Restates learners' offer and questions its correctness L2 – Probes the learner's offer with follow-up question
		Task-focused responses
		Verbal reframing using alternative phrase
		Lead-in to the task
		Switching between representations
		Establishing generality
		Contrasting offered and required operation
	ENP	Reduces cognitive demand of the task
		Repeats learner's offer and moves on
		Repeats task and moves on
		No comment and moves on
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	EP	L1 – Offers a more efficient strategy L2 – Elicits a more efficient learner's offer L3 – Interrogates learner's offer for efficiency
	ENP	Acknowledges correct offer as inefficient and moves on
		Pulls learners' back to inefficient action
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	EP	***Individuating responses***
		L1 – Confirms chorus offer with individual learners
		L2 – Interrogates chorus offer with individual learners
		Collectivising responses
		L1 – Confirms individual learner's offer with whole class
		L2 – Interrogates individual learner's offer to the whole class
		Repeats individual learner's offer with the whole class
		Decompresses individual learner's offer to the whole class
		Collective reasoning
	ENP	Accepts chorus offer and moves on
		Accepts individual offer and moves on

5.6 Conclusion

This grounded categorization of teachers' responses to 'in-the-moment' contingencies in primary mathematics teaching developed in this study present some of the stage points of implementation towards more responsive teaching. It is noted that in many cases, these categories are low level evaluative responses, in contrast to how responsive teaching are discussed in the international literature. But they remain useful as a move forward in the face of the extensive teacher knowledge gaps and absence of evaluative criteria noted in the South African primary mathematics teaching landscape. Thus, it is epistemically important from the perspective of in-service teaching development.

The categories offer 'home-grown' rather than 'imported' descriptions of pedagogies with the potential for building the kinds of responsive teaching actions that are widely described as important in the mathematics education literature for supporting emergent mathematical learning. Close attention to the nature of teacher's responses, thus, represents openings for moves away from deficit characterisations based on absences, to staging point characterisations directed towards improvement. Being aware of these kinds of in-the-moment situations and possible responses are particularly important within professional development programmes for supporting moves towards responsive teaching, as they provide useful descriptions and categorizations of steps that are within the reach of current pedagogical practices in South Africa.

Another important feature of the categorisation discussed in this chapter was the dimension hierarchy. This feature of the framework is related to questions of quality of elaborated teacher responses. Together with other indicators relating to 'extent' and 'breadth' of elaborations where the focus is on looking across the range of incidents of elaboration, in the next chapter I show how the elaboration framework was also useful for analysing shifts in the nature and extent of responsive teaching using the lessons observed in 2013 and 2014.

CHAPTER 6

FINDINGS AND DISCUSSIONS OF SHIFTS IN TEACHERS' ELABORATIONS

6.1 Introduction

In Chapter 5, I exemplified in detail how I conceptualised each of the categories of responsive teaching that constitute the elaboration framework. The concern of this chapter is to present findings and discussions of the different patterns of shifts seen across the four teachers in relation to the three 'in-the-moment' situations of elaboration, namely: breakdown, sophistication, and individuation/collectivisation across the 2013 and 2014 lessons.

In the context of evidence of relatively strong mathematical knowledge for teaching and on-going development activity involving interim VSR interviews, I explore possibilities of growth in responsive teaching by the four teachers. This is done through the analysis of 2013 and 2014 data of classroom teaching, with support for the claim of shift coming through also from teacher reflections on their pedagogic actions in the VSR interviews in which shifts in awareness were possible to discern. I return to the findings from the VSR interviews following the discussion of the patterns of shifts in elaborations by each teacher. The interview data allows me to document teachers' thought and mathematically oriented decisions in the classroom, and also to explore possibilities for developing responsive teaching through teachers' reflective awareness drawn from what they acknowledge as possible limitations in their classroom decisions.

The results for each teacher were firstly categorised into the two broad categories introduced in Chapter 5: Elaboration provided (EP) and elaboration not provided (ENP), and incidents of elaboration were then coded according to the situations and categories presented in the previous chapter. In order to consider shifts in elaboration, I devised and operationalised key markers of shifts. These are presented in the next section.

6.2 Operationalisation of markers of shifts in elaboration

In order to distinguish and comment on each of the teachers' shifts in elaboration across the 2013 and 2014 lessons, I recruited three markers or indicators of shifts in elaboration relating to ***extent***, ***breadth***, and ***quality***. These markers are drawn from the ways in which the situations of responsive teaching were conceptualised as described in previous chapters. The operationalisation of these markers was based on counts of incidents where each elaboration type was seen. Both the markers and the ways in which counts of incidents proceeded are described in the following sections.

Extent of elaborations provided: This aspect is focused on a comparison of the numbers of incidents where elaborations were provided (EP) to incidents where elaborations were not provided (ENP). This is presented in percentages based on the proportion of all incidents within each situation of elaborations. While based on different aspects of additive relations and different classroom scenarios and therefore not easily comparable in any direct way, my totalling of EP and ENP incidents allowed me to compare the relative incidence of EP and ENP in each of the situations across 2013 and 2014 lessons for each teacher. A higher proportion of EP in 2014 than in 2013 provided one indication of moves forward in the *extent* of elaboration.

Breadth of elaborations provided: This aspect is focused on scrutiny of the scope of the elaboration sub-types within situations. This marker is considered in event where no hierarchy exists between the elaboration sub-types. Since all the sub-types of elaborations within the sophistication situation are hierarchical in nature, breadth of elaboration is not considered within this situation. Within the breakdown situation, for example, there were no hierarchies among the five sub-categories of elaborations that *focused on task*. So, if there was a spread of more elaboration sub-types in 2014 than in

2013, this was interpreted as a marker of shift in breadth of elaboration. This aspect derives from literature noting that multiplicity and flexibility in a teacher's response to learners' ideas is an important component of responsive teaching (Jacobs, Franke, Carpenter, Linda, & Dan, 2007; Walshaw & Anthony, 2008).

Quality of elaboration provided: This aspect is focused on scrutiny of the elaborations offered within the hierarchies of sub-types within situations. For example, within the breakdown situation, more incidents of *probing a learner's offer* rather than *restating a learner's offer and questioning its correctness* indicates a marker of 'quality' of elaboration, linked to the literature on teaching quality (Franke & Kazemi, 2001; Hill et al., 2008; Popkewitz, 1988). For sophistication elaborations, more incidents of *eliciting* and *interrogating* a learner offer for efficiency rather than *offering a more efficient strategy*, and for individuation/collectivisation situation, more incidents of pedagogic moves at the level of *interrogating* rather than *confirming* learners' offerings were also represented as markers of improving quality of elaboration.

Count of incidents of elaborations: As mentioned in the previous chapter, in each incident of elaboration the teacher's utterances/responses were coded, so it is possible to have multiple codes within one incident. However, where a particular utterance, which had already been coded, was repeated within the same incident, the repeat utterance was not counted. This forms the basis for the count of the EP and ENP incidents presented for each teacher's 2013 and 2014 lessons enactment on the elaboration framework.

6.3 Patterns in teachers' elaborations across 2013 and 2014

The analysis and findings of the patterns in teachers' elaborations across the two years are presented according to individual teachers' lessons enactments, with a synthesis of the findings across all the teachers presented after this.

I begin the analysis of each teacher's elaborations with a summary table of the results of coding their lessons across the two years. In these tables, the numbers in the table represent the frequency of each sub-type of elaboration and non-elaboration, with the sub-types set within the situations of elaboration presented in the framework in the previous chapter. I have then totalled these numbers to give the total number of incidents of 'elaboration provided' (EP) and 'elaboration not provided' (ENP) across each year's set of lessons, and represented these two totals in percentage terms using the total number of incidents across the EP and ENP in that situation as the whole value. As detailed in the previous chapter, the highlighted sub-types in the tables show the sub-types that are linked by a hierarchy.

At the preliminary level, keeping the number of incidents in the frame was useful given that the number of lessons observed in the two years was not the same for some teachers, with this data allowing me to note whether the 'rate' of incidence of EP in lessons had stayed broadly similar or had changed. In the analysis and discussion that follows, in each of the summary tables, the areas of higher incidents of EP in 2014 relative to fewer incidents of ENP provide me with pointers to the most substantial shifts for each teacher. The substantive patterns of shifts in responsive teaching for each teacher are then exemplified through excerpts and analytical commentary drawn from literature and theory. I selected these excerpts based on important qualitative contrasts, and present them in the form of narratives (Brown, 2013). I describe what *did* happen using the illustrative excerpts, and sometimes I speculate about what *could have* happened using the literature and theory based on responsive teaching in the context of additive relations as a vantage point.

Having identified and discussed pattern of shifts in responsive teaching, I then move on to consider findings from the VSR interview for each teacher focusing specifically, on the nature and range of teachers' state of awareness, and reflections on self, students and practice (Geiger et al., 2015). I conclude with a summary of key findings for each teacher relating to associations seen between awareness shifts of foci and practice shifts between 2013 and 2014 teaching. These shifts are juxtaposed not to suggest any direct causality from either the professional development course or the VSR interview, but rather to explore the interplays over time of these professional development mechanisms for the four cases of teachers' increasing focus on elaborations in this study.

6.4 Thandi's teaching

As noted in the lesson descriptions provided in Chapter 4 (see pages 87- 89), I observed two lessons on additive relation teaching by Thandi in 2013 and three lessons in 2014. Table 14 summarises the results of the coding of Thandi's 2013 and 2014 lessons using the elaboration framework.

Table 15: Coding Thandi's lessons

Situations of elaboration	Sub-types categories	2013 (2 lessons)		2014 (3 lessons)	
Breakdown Incorrect learner(s) offer	***Learner offer-focused responses***		**EP** 9 (9/13 = 69% of 2013 breakdown situations)		**EP** 26 (26/29 = 90% of 2014 breakdown situations)
	L1-Restates learners' offer and questions its correctness	5		4	
	L2-Probes the learner's offer with follow-up question	2		7	
	Task-focused responses				
	Verbal reframing	0		7	
	Lead-in to the task	0		0	
	Switching between representations	1		3	
	Establishing generality	0		2	
	Contrasting offered and required operation	1		3	
	Reduces cognitive demand of the task	1	**ENP** 4 (4/13 = 31% of 2013 breakdown situations)	2	**ENP** 3(3/29 = 10% of 2014 breakdown situations)
	Repeats learner's offer and moved on	0		0	
	Repeats task and moves on	2		1	
	No comment and moves on	1		0	
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	L1-Offers a more efficient strategy	0	**EP** 0 (0/2 = 0% of 2013 sophistication situations)	5	**EP** 12(12/14 = 86% of 2014 sophistication situations)
	L2-Elicits a more efficient learner's offer	0		3	
	L3-Interrogates learner's offer for efficiency	0		4	
	Acknowledges correct offer as inefficient and moves on	0	**ENP** 2(2/2 = 100% of 2013 sophistication situations)	2	**ENP** 2(2/14 = 14% of 2014 sophistication situations)
	Pulls learners' back to inefficient action	2		0	
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	***Individuating responses***		**EP** 4(4/11 = 36% of 2013 individuation /collectivisation situations)		**EP** 15(15/18 = 83% of 2014 individuation /collectivisation situations)
	L1 - Confirms chorus offer with individual learners	0		4	
	L2 - Interrogates chorus offer with individual learners	0		1	
	Collectivising responses				
	L1 - Confirms individual learner's offer with whole class	4		2	
	L2 - Interrogates individual learner's offer to the whole class	0		3	
	Repeats individual learner's offer with the whole class	0		2	
	Decompresses individual learner's offer to the whole class	0		6	
	Collective reasoning	0		2	
	Accepts chorus offer and moves on	4	**ENP** 7(7/11 = 64% of 2013 individuation /collectivisation situations)	2	**ENP** 3(3/18 = 17% of 2014 individuation /collectivisation situations)
	Accepts individual offer and moves on	3		1	

EP incidents occurred on 13 occasions across the two lessons seen in 2013, with 13 ENP incidents also noted. In contrast, in 2014, EP incidents occurred 53 times across three lessons, in comparison with 8 ENP incidents. This pointed to a substantially higher incidence of EP in 2014 in comparison with 2013.

On the *extent* of elaborations, Table 14 shows shifts at the level of: breakdown (up from 69% of all breakdown situations in 2013 to 90% in 2014); sophistication (up from 0% of all sophistication situations in 2013 to 86% in 2014); and individuation/ collectivization (up from 36% of all individuation/collectivisation situations in 2013 to 83% in 2014). These results collectively indicate substantial shifts in the *extent* of elaboration provided, with the highest shifts at the level of sophistication.

On the *breadth* of elaborations, there were presences in 2 out of the 5 non-hierarchical sub-types of elaborations in breakdown situations in 2013 and 4 out of 5 in 2014. No *breadth* of elaboration is considered at the level of sophistication, since all elaboration sub-types are hierarchical in nature. There was no presence of any non-hierarchical elaboration sub-type within all collectivising responses in 2013, and 3 out 3 were evident in 2014. Once again, this indicated substantial shifts at the level of *breadth* of elaborations in 2014, with more shifts at the level of collectivising responses.

On the *quality* of elaborations, in the case of breakdown situation, there were more incidents of *probing learner offers* relative to *restating learner offers and questions its correctness* in 2014 than in 2013. There were more elaborations, in the case of sophistication, at the level of *eliciting* and *interrogating efficiency* than *offering a more efficient strategy*, indicating growth in terms of the notion of '*quality*'. There were also more elaborations at the level of i*nterrogating* relative to *confirming* elaborations in the collectivising responses category in 2014 than in 2013. This finding too, indicates shifts at the level of *quality* of elaborations in Thandi's 2014 lessons.

In Table 15, I provide a summary noting where there are increases in incidence in relation to *extent*, *breadth* and *quality* within each situation of elaboration.

Table 16: Thandi's summary of shifts in relation to extent, breadth and quality

		Situations of elaboration							
Markers of shifts		***Breakdown***		***Sophistication***		***Individuation/ Collectivisation***			
		2013	2014	2013	2014	2013		2014	
Extent		69%	90%	0%	86%	36%		83%	
Breadth		2/5	4/5	NA	NA	0/3		3/3	
						Ind.	Col.	Ind.	Col.
Quality	L1	5	4	0	5	0	4	4	2
	L2	2	7	0	3	0	0	1	3
	L3	-	-	0	4	-	-	-	-

Thandi's data overall suggested marked shifts in *extent* of elaboration within sophistication and individuation/collectivisation situations, with more limited positive shifts, but shifts nevertheless, in the breakdown situation, where she started with a relatively high level of prevalence (69%). There are substantial shifts in *breadth* (within dimensions non-hierarchical sub-types) of elaborations within breakdown and individuation/collectivisation dimensions. There are also substantial shift in *quality* (within dimensions hierarchical sub-types) of elaborations within breakdown and sophistication situations, with again limited positive shifts at the level of individuating and collectivising elaborations.

To exemplify the pattern of the observed shifts by Thandi qualitatively across 2013 and 2014, I selected excerpts as 'telling cases' that indicated the areas of substantial overall quality differences seen in Table 15 across the two years. These differences were: 'breakdown-quality' difference; 'sophistication-quality' difference; and 'collectivising-quality' difference. I deal with each difference by contrasting her 'ways of being' with mathematics in her 2013 and 2014 lessons, before presenting a discussion of some possible implications of these shifts for responsive teaching.

Breakdown-quality difference

In terms of the breakdown category, Thandi's range of responses in 2013 lessons showed greater prevalence in the *learner offer-focused responses* category, and within this, greater emphasis at the lower level of the hierarchy in this category characterised by the restating of learner offers and questioning correctness. In contrast, in 2014, the majority of her responses were at the higher level of probing learner offers with follow up questions. Below, I exemplify this contrast in the quality of her breakdown elaborations across the two years.

In 2013 lesson 1, in the context of the task 25=30- _ involving using a number line to find the missing subtrahend, Thandi invited learners to work out the problem on the board. The following excerpt then played out:

Excerpt 8

285 L2: (*Learner points at 25 and indicates a backward gesture with her left hand and then pauses*)

286 T: Where do you go from 25?

287 L2: Backward

288 T: She says we start at twenty-five and go back. Does the sum say 25 minus? No, it said 25 equals. (*Teacher invites another learner*).

289 L3: (*Learner points at 25 and demonstrates a backward gesture*).

290 T: We are going backward, if we say twenty-five minus, then we move backwards. But our sum does not say that. It says twenty-five is thirty minus what? (*Thandi invites another learner*)

291 L4: (*Learner starts at 25 and demonstrates a forward jump to 30)*

292 T: What do we do next?

293 L4: We go back

294 T: Go and sit down (*teacher askes learner to go and sit down without any further comment*)

Challenges with directly modelling this problem type have been noted in the literature. In Carpenter et al.'s (1999) categorisation, missing subtrahend problems are harder to directly model as the number of jumps to make is not known. Further there is an extensive literature base on children interpreting the equals sign as a signal to operate, rather than to seek equivalence (Molina & Ambrose, 2008), making problems with the operation on the right hand side more complex through being less familiar. Thandi's response on Line 288 began with a restating of the learner offer (twenty-five and go back), and she went onto link this offer with the problem '25 minus' and questioned whether this was correct in relation to the original question. Given this analysis, this incident was coded as 'restating learner offer and questions its correctness' – the lower level of the 'learner offer-focused response' category. Further elaboration was attempted here, but it is of interest that in comparison to the 'contrasting offered and required operation' – one of the sub-categories in the 'task-focused response' group, which can be viewed as a 'single-step' contrast, Thandi's response involves a 'double-step' action – firstly linking the learner's offered action with the symbolic expression '25 minus' and then contrasting this expression with the expression in the task '25 equals'.

In the subsequent interaction after this excerpt, it is clear that most learners are not seeing the inverse relation / difference relation as the mathematical object that the teacher notes as her focus in the VSR interview. The evaluative nature of her response with explicit rejection of the learners' solution actions, without any further elaboration that potentially elicits correct solution action, appears to result in a situation where the mathematical object seemed not emerging for many learners in her class (Askew et al., 2012).

In her 2014 lesson 1, in the context of a subtraction task 38-9, Thandi had earlier introduced adding and subtracting ‘near 10’ numbers by using 10 as a benchmark. She invited one learner to work out the task on the board. The learner drew an empty number line, and marked 38 towards the end of the line. She then made a backward jump of 10 and landed at 28. The following excerpt played out:

Excerpt 9

324 L: Twenty- eight

325 T: What do we do next? Yes?

326 L: Minus one

327 T: Minus one; she says minus one, if we say minus ten and minus one how much have we subtracted?

328 C: Eleven

329 T: But, our problem says minus nine not minus eleven. (Teacher calls learner by name) Khanyisile?

330 L: Plus one

Thandi’s response to the learner offer of ‘minus 1’ having already jumped back 10, involved establishing that the learner offer was actually taking away 11, not 9, and was coded as an incident of ‘probing learner offer with follow-up question’. The literature suggests that this kind of response has the potential for extending learner understanding than overt rejection of the offer (Brodie, 2007; Nystrand & Gamoran, 1990). In contrast to Thandi’s 2013 instances of elaboration in breakdown situations where there was a prevalence of elaborations involving a restating of the learner offer and acknowledging its incorrectness, in 2014 she probed learner’s incorrect offers by establishing the possible consequence of learner solution actions without explicit rejection of the learner offer.

Sophistication-quality difference

In terms of the sophistication category, there were two incidents in Thandi's 2013 lessons that were both coded as 'pulling learner back to inefficient action'. In contrast, in 2014, there was a range of incidents where elaborations were provided, with 7 out of 12 of these responses at the higher levels in terms of the hierarchy. Below, I exemplify the contrast of sophistication difference across the two years.

In her 2013 lesson 1, upon completion of writing the 10s between numbers 0 – 100 on a number line, she asked learners to point to the position of 25. One learner pointed at the mid-point between 20 and 30. Thandi accepted this offer and wrote down 25. She then wrote down the following task: 25+5 = ___. Learners raised their hands and she invited one learner (L1) to work it out on the board using the number line. The following then excerpt played out:

Excerpt 10

259 T: Where do we start from to add twenty-five plus five? Yes (*teacher invites one learner*)

260 L1: (*Learner starts at 25, already marked on the number line, and makes a single forward jump of 5 and lands at 30.*) Thirty.

262 T: Show us where we start and how we move. Draw the jumps

264 L1: We start here and move five places (*learner uses ruler to show movement from twenty-five to thirty)*

265 T: Show us on the number line.

267 L1: One, two, three, four (*uses chalk and makes four marks between twenty-five and thirty while counting*).

268 T: Ok, move on the number line so we can see. Draw on top.

269 L1: (Learner makes the individual 5 forward jumps from 25 to 30 on the number line – see Figure 16)

270 T: Right, your answer is what? Twenty-five plus five?

270 L1: Thirty

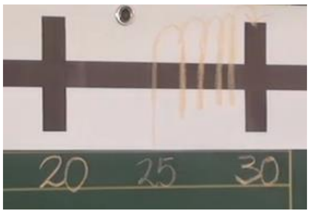


Figure 16: Solution of the task 25+5 on a number line

In the excerpt presented above, it was clear that the learner involved could work out 25+5=_ by starting at 25 and making a single jump of 5. Thandi's response was coded as pulling back because L1 demonstrated a single jump of 5, while Thandi then asked for counting on in ones. Thandi did not comment on why she insisted on the learner showing counting in ones in the VSR interview, suggesting that the pulling back was not part of her immediate frame of awareness. The move from counting in ones to flexible group counting is an important one in developing sophisticated strategies for addition and subtraction (Anghileri, 2006; Beishuizen, 1993; Mcintosh et al., 1992). The absence of such commentary in Thandi's response has been associated in the South African literature with learners ongoing operating at very low levels despite using structured resources like a number line (Venkat & Askew, 2012).

In her 2014 lesson 3, in the context of another addition task, 6+25 on a number line. Thandi invited one learner (L2) to facilitate working out the sum on the board with the whole class. He drew an empty number line and marked 25 [in the previous examples, there had been discussion about the efficiency of starting addition with the bigger number]. Thus, my focus here, as in the previous incident, is on the ways in which she dealt with the need to count on. The learner asked the class what number to add first. One learner offered 'plus 1'. He made a forward jump of 1 and wrote down 26. Another learner offered 'plus 1' again. He made another forward jump of 1 and wrote down 27 where the jump landed on the line. Another learner offered, ‘plus 4’. At this moment Thandi interrupted, and the following excerpt played out.

Excerpt 11

294 T: (*Teacher interrupts*). What number are we at twenty-seven and we are adding four? Okay, let’s put our hands down. Remember that when we said the number line is our friend; the number line has to make things easy for us, isn’t it?

295 C: Yes

296 T: It has to be easy. It just has to be easy for us. So we take numbers that are going to make it easy for us to count. This is fine (*learner erases the second jump from 26 to 27 and she left the first jump of 1 from 25 to 26*). I am not saying this is wrong, because I know that you were going to get the answer, but I just want you to get your answers quickly and easily. Now we are going to do that. We said six plus twenty-five, isn’t it?

297 C: Yes

298 T: Now let’s look at twenty-six [*teacher points to 26 with the jump of 1 already made from 25 on the number line*] and say how many do we need to add to get to the next multiple of ten? Mpho?

299 L: Plus four

300 T: Plus four, six and four is ten, so twenty-six and four is?

301 C: Thirty. (*L2 writes ‘+4’ on top of the jump and writes ‘30’ where the jump lands on the line*)

302 L2: Which number do we jump (L*2 calls out one learner by name*) Thandeka?

303 L: Plus one

304 T: Do you see that it is easier now?

305 C: Yes. (*L2 makes a forward jump of ‘1’ from ‘30’ and writes ‘+1’ on top of the jump*)

306 L2: Which number do we write? Mpunki?

307 L: Thirty-one

In the interaction presented in excerpt 11 above, Thandi encouraged learners into flexible group counting by using ten as a benchmark (Mcintosh et al., 1992). This response was coded as an incident of provision of elaboration characterised by ‘*eliciting a more efficient strategy*’. This marked a contrast to what we saw in her 2013 teaching where pulling back was the only kind of sophistication-related response seen, as exemplified in excerpt 10. In 2014 Thandi provided sophistication-related elaborations across a range of categories – with excerpt 11 coded in sub-category '*eliciting more efficient strategy*'.

Thandi's 2014 elaboration actions in the sophistication situation therefore indicated contrasts with teaching in South Africa characterised by limited progression to more flexible working with number and operations (Ensor et al., 2009).

Collectivisation-quality difference

In terms of collectivising responses, in 2013 Thandi's predominant responses were either '*accepts individual offers and moves on*' or '*confirms individual learner offer with the whole class*'. In contrast, in 2014 Thandi's predominant collectivising responses where at the sub-category of '*decompressing individual learner's offer to the whole class*' and '*interrogating learner offer with whole class*'. Below, I exemplify these pedagogic moves across the two years.

In the context of the task, 25=30-__ in Thandi's 2013 lesson, following incorrect solution actions from three learners, the next learner (L5) that was invited by Thandi started at 30 and made backward jumps of 25 broken down into 10, 10 and 5 (Figure 17).

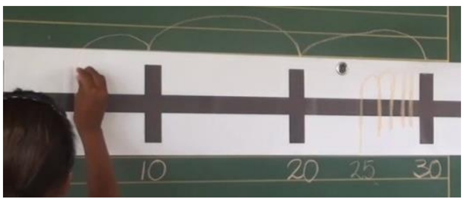


Figure 17: L5's representation of the solution of 25=30-__ on a number line

Thandi responded in the following ways:

Excerpt 12

302 T: (*Teacher interrupts, shakes her head and asks L5 to go and sit down*). It seems no learner can do this. Ok, pay attention here. What we need to do is just to take the answer from what L1 did (*points to the five unit jumps that were drawn between twenty-five and thirty by L1 in working out the preceding task; 25+5=_ using count-on strategy*). The first sum said, twenty-five plus five. L1 started at twenty-five and he made five jumps and landed at 30.

303 C: Yes

304 T: So, what is this number? (*Teacher points to the space between 25 and 30 on the number line with her two fingers*)

305 C: Five

306 T: Now, our sum is twenty-five equals thirty minus (*teacher points to the problem written as '25=30 – ' on the board*). L1 answer is 30, ne. So, we start at thirty until we reached 25, because twenty-five is the answer. What is our answer here (teacher *points to the space between 25 and 30 again on the number line*)

307 L: Five

308 T: Yes, our answer is five. So, twenty-five equals thirty minus five'

Thandi's utterance at L302 'states' rather than elicits the connection that she wants her class to focus on between the two examples. In this singular focus though, other connections between the problem situation and learner offers and between learner offers and her intended focus on the inverse connection between the two problems are dismissed. L5's solution action is ignored, even though it is a mathematically correct process for solving the problem. This incident exemplifies what Rowland and Zazkis (2013) have described as opportunities missed in contingent moments, and it is coded within the 'elaboration not provided' category. Thandi moved on to work out the problem herself without any input from the learners. This choice is of interest given that Webb and Palincsar (1996) have noted that providing solutions to learners when they struggle without actively engaging them in thinking and reasoning is either unrelated or negatively related to achievement outcomes.

Thandi states the connections between the two examples using the same number line representation, and her talk indicates a focus on using inverse operations with a 'travelling' or 'operational' metaphor that links counting up five to thirty with counting back down five to get back to twenty-five. The learner offer in this instance though points instead to a part-part-whole structural view of the problem, where 30 is seen as a whole and 5 and 25 are parts that make the whole, allowing for 'subtracting 25' as an appropriate model. Thandi's lack of follow-up of this offer suggests a lack of responsive flexibility in her preferred way of working with this problem.

In her 2014 lesson 3, in the context of the task 9+163, Thandi invited one learner (Sipho) to work out the sum on the board and requested him to work with the whole class. Sipho drew an empty number line and wrote down 163 at a mark towards the start of the line. He turned and faced the whole class and asked them what to do next. Another learner (Nthombi), offered 'plus ten' requiring Sipho to make a forward jump of 10. Sipho rejected this offer. Thandi interrupted:

> 460 T: Maybe she is right. Why do you say no? Let's see what she has to say. Let her do the rest of the sum; then we will understand her - why she says plus ten. Do what she asks you to do?

Following the teacher's interruption, Sipho made a forward jump of 10 and landed on 173. Nthombi then asked Sipho to minus one by making a backward jump of one. Sipho made the backward jump and landed on 172 as the answer (see Figure 18).

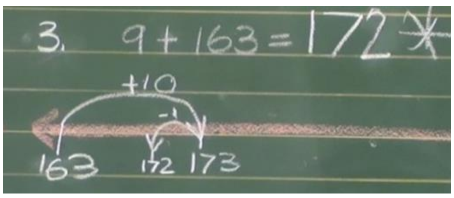


Figure 18: Nthombi's representation for the solution of the sum '9+163'

Thandi responded in this way:

Excerpt 13

465 T: But we should be able to explain. Who can try? The sum says nine plus one hundred and sixty-three. So, since we said we start with a bigger number, we start with one hundred and sixty-three and we are supposed to add nine. So, our sum stands like that *(teacher writes '163+9=' on the board). We* started at one hundred and sixty-three; it says add nine, but Nthombi says we should add ten. Do you see that?

466 C: Yes

467 T: Then we got to what number?

468 C: One hundred and seventy-three

469 T: One hundred and seventy-three, and Nthombi added ten instead of nine. So, did she add more or less than nine?

470 C: More

471 T: More by how many?

472 C: One

473 T: By one. So she went to subtract …?

478 C: One

479 T: isn't it?

480 C: Yes

Thandi allowed greater interactions among learners in her class by first requiring Sipho to solve the problem by following other learners' instructions, and imitating Thandi's prior use of the number line. While Thandi stayed outside the conversation, she interrupted to insist on following Nthombi's offer. Thandi's response here was coded as an incident of elaboration characterised by '*decompressing individual learner's offer to the whole class'*. Without Thandi's response to follow Nthombi's insight, there are indications that the idea of compensation (Klein et al., 1998) would have gone unattended. Nthombi's offer was thus used by Thandi as a teachable moment for unpacking the possibilities from the initial suggestion to add 10 offered by the learner.

In the course of this input, Thandi demonstrated interest not just in the production of the correct answer, in marked contrast to what I noted in her 2013 response, but also in the ways in which the answer was produced. She focus attention on the number relations inherent in compensation strategies that have been described as key parts of the efficiencies relating to structural number sense in the literature (Wright, Martland, & Stafford, 2006), while also sharing individual offers with the collective classroom space using the contingent learner offer as a teachable moment.

6.4.1 Discussion of Thandi's pattern of shifts across 2013 and 2014 teaching

Key differences between Thandi's 2013 and 2014 elaborations from the excerpts presented related to the *quality* of her elaborated responses. Rowland and Zazkis (2013) found that the great majority of triggers of contingent moments in the mathematics classrooms are consequences of unplanned learners' contributions. These triggers in most cases fell outside the teacher's 'lesson image' (Schoenfeld, 1998). In all the scenarios presented, there was an unexpected learner offer that placed Thandi in a position of having to improvise. My interest here is to document shifts in Thandi's responses to learner offers by contrasting her 'ways of being' with mathematics in her 2013 and 2014 lessons.

In 2013 Thandi focused more attention on getting the correct answer in a specific way without engaging with learner's offers. This was seen in the case where she encouraged unit counting in response to flexible group counting as seen in working out the task 25+5 and in the way she ignored L5 solution action for the task; 25=30-__. However, her working out of the problem presented in excerpt 12 suggests lack of error in her talk, and some attempts at explanations and connections through linking examples with the same representation, but not necessarily in ways that were responsive to what learners said or did (Nystrand & Gamoran, 1990). This evidence suggests a strong 'plan-orientation' – in which the teacher pushes for tasks to play out with focus on her intended objectives without deviation. In Chapter 2, I linked 'plan-oriented' working with the KQ's connection and transformation knowledge base that she brought into the teaching situation, with no awareness seen of the need to deviate from her planned action (Rowland et al., 2005) or to establish balance between scripted planning and improvisation (Borko & Livingston, 1989; Sawyer, 2004) in the course of her teaching.

In Thandi's 2014 teaching, there was evidence of substantial shifts in the *quality* of elaborated responses to sophistication, breakdown and collectivisation situations in her engagement with learners' thinking in responsive ways (Franke, Kazemi, & Battey, 2007; Silver & Stein, 1996). These differences suggest changes in her ways of being with mathematical knowledge (Coles & Scott, 2015) in her teaching, allowing greater interactions among her learners, and being more responsive to individual learner's contributions in the classroom, a practice that have been widely described in the literature as a marker of teaching quality (Hill et al., 2008). Overall, in 2014 Thandi demonstrated widespread moves to more responsive teaching, with greater incidence of collectivization (Venkat and Naidoo, 2012) and sophistication elaborations (Ensor, et al., 2009).

Having analysed and discussed substantial shifts in Thandi's elaborations across 2013 and 2014 lessons, I now move to findings from VSR interviews with specific focus on changes in Thandi's state of awareness (Schratz, 2006) that linked knowledge to practice, and levels and objects of reflections (Muir & Beswick, 2007) that related to the utilization of reflective practice as a means of enhancing teaching actions.

6.4.2 Findings from the VSR interviews with Thandi

In this section, I present findings from the interim VSR professional development interview conducted between 2013 and 2014 lesson observations. The VSR interview with Thandi focused primarily on her thoughts and decisions about how the task 25=30 - ___ was enacted in her lesson, and what she might do differently. Prior to the discussion about the task, I began the interview with probing the teacher to articulate her objectives for the lesson.

Researcher: Tell me what you are hoping that learners would be able to do or understand at the end of the lesson?

Thandi: Firstly, when I planned this lesson, I was having the idea that mostly our learners when they do addition and subtraction; they usually operated at a very low level. We saw some of them - they have to count in ones. They do those small sticks [referring to unit tally counts] all of them. Then, if maybe it's a subtraction of ten from eighty-five, they have to do the whole eighty-five and then they have to count back scratching one after another until they have subtracted ten. Then they have to count from one the remainder of the total sticks that were there. Those methods when you come to things like ANA [Annual National Assessment] exams you know which is timed. You find that they don't finish their work. And even if they do most of their answers are wrong. Because then it depends on whether they have counted the sticks correctly. If they make a mistake in the counting then the whole sum is wrong.

Researcher: Okay. Good, then what are you hoping that your learners would gain from this lesson?

Thandi: I was hoping that once they are able to grasp the concept of using the number line, it will minimize such mistakes - like having to count sticks one by one. Because as long as they have got their number bonds correctly they can easily get answers on the number line.

Thandi's comment indicated *deliberate* level of reflection about *students* based on mistakes and inefficiency of the calculation strategy they used. Thus, her input showed reflective awareness of the advantages and possibilities of the number line as a tool that can minimize such mistakes committed by her learners. Thandi added on advantages of the number line:

Thandi: The advantage of the number line is that you know like when you have to do a breaking down. I have given example like 800 – 457. Now there are two zeros in 800. When they do break down most of the children they have a problem in having to take whatever 100 from that 800 because the 10s is a zero, the units is nothing, they have to pull the numbers from the 100s until they get to the unit. On the way

> they get confused and they get wrong answers. But if you just give them the number line, you just say the number line I taught you, the number line, where do we start? You tell them 800 - which side of the number line? They will tell you, the right side you place the 800 there and you say, how do we jump? 400, minus 400 all at once my grade 3s do that, they get the answer so if they go to minus 7, let's say you are now subtracting the units they say minus 7 - is it easy to subtract 7 and whatever number you have landed at? They will tell you no, then what do we do? We are breaking that 7 into 5 and 2 so they say minus 5, minus 2 and they get the answer.

So far, Thandi's talk reflects what she hoped children would do with her planned tasks and representations, but does not mention that they did not do these things. Her talk indicates awareness of progression, and tasks and representations that can help with this. What is noticeably absent is a sense of needing to adapt the plan to be responsive to what her learners do with her tasks and representations. This absence suggests a view that the 'plan' is enough – and this is maybe a critique of the course's emphasis on the planned elements of the KQ framework. In relation to the contingency category, this finding indicates her state of *unconscious incompetence* with *deliberate* level of reflection about students' learning. Following this response, I asked Thandi in overview terms to talk about whether she achieved what she intended for the lesson.

> Researcher: To begin at an overview level, do you think you have achieved what you intended in this lesson?
>
> Thandi: I wouldn't say so, I wouldn't say so, because when I observed the lesson I found that there were things that I should have taught first before that lesson … I also realized that some of my learners had the idea, but I didn't give them time. For example, the girl who subtracted 25 from 30 on the number line by making a jump of ten, ten and five, so that girl I think now she was clever, because if given time she was going to get the correct answer for that sum.

Thandi added:

> Thandi: I think in that activity (*referring to the task; 25=30 - __)* I was doing in the class, the teacher herself had the idea that 25 is the answer as you can hear from the video that I stresses that 25 is our answer many times. 25 is our

answer. So, 25 is 30 minus what? I found that some learners understood it differently than how I wanted them to understand. And as a result I failed to give them that chance to explain their answers. Because I had this preconceived idea that 25 is the answer. So, 25 is 30 minus what? So I had in mind the idea that they should start from 25 and go five steps up to 30 so that they get five as the answer.

This statement marked a shift in Thandi's level of awareness through realisation of the need for flexibility in her planned oriented action. This suggests the need for bringing in her transformation and connection knowledge within contingent situation as a disciplined improvisation to respond to learner's insight (Sawyer, 2004). Her utterances in the quote above is therefore interpreted as a transition from *deliberate* (learners operated at a very low level and make mistakes) to *critical* level of reflection (things that I should have taught first before that lesson, … as a result I failed to give them that chance to explain their answers, … because I had this preconceived idea). This transition also corresponds with a move from *unconscious incompetence* to *conscious incompetence* in that Thandi recognized limitations with sequencing of her lesson, and response to learner's contribution in the classroom – a *critical* reflection about practice and students. In response to this awakening, I drew Thandi's attention to her view of the relationship between her two examples; 25+5=___ and 25= 30 – ___. Thandi responded with the following statement.

Thandi: hmmm… there is a relationship, if they know that 25 + 5 =30 then they should know like we teach them what we call family bonds. Whereby if they say, 25 + 5 = 30 then 5 + 25= 30; 30- 5 = 25 and 30 – 25 = 5. So we call this family facts when we teach them. So maybe I could have used that.

Inverse relations (and multiple relationships between a part-part-whole family of numbers more broadly) as objects are confirmed here, as well as her sense that the task sequence should have worked to 'deliver' learning, reflecting – once again – a faith in planned task and example spaces. Within this orientation, she has extended the task to family bonds, so the task has been expanded to include further possibilities and the idea of family bonds. This statement indicated a move again to a *critical* reflection about *self* ("maybe I could have used that") by considering an alternative approach to resolve breakdown situations. With probes during the interview, Thandi continued to think about better ways to communicate this idea to her learners through engaging on reflection about *self*, *practice* as well as *students*.

Thandi admitted that it was also a challenge for her as a teacher and therefore, she need to think hard to understand the concept for 'herself' before thinking of her students.

> Thandi: I can say that the trick is just the relationship between the addition and subtraction. So, then maybe if we teach them that 25 + 5 is 30, then keeping the whole as 30. I am just trying to think so hard to look for an easier way to understand this for myself. Our two parts are 25 and 5 -those two numbers are obviously smaller than the whole. So, I think if you are given a whole and one of the parts, how do they get the second part? This really is the same as 30 – 25, isn't it?

Thandi's statement indicated a move again from *deliberate* to *critical* reflection about *self* and this also pointed to a move from the state of *unconscious incompetence* to *conscious incompetence* – '*I am just trying to think so hard to look for easier way to understand this for myself*' as she is becoming more aware of the limitations of transforming 'maths for herself' to 'maths for others' (Venkat, 2015). She noticed that it is important for her to understand very well for herself before she can communicate well to others. Within this, her focus continues to be on her own explanations within teaching, and thus, the knowledge base drawn on can be linked to the KQ's transformation category.

While viewing the lesson, Thandi was able to gain greater understanding of what actually transpired in her practice rather than what she thought was happening. Like findings in other studies (Geiger et al., 2015; Rosaen, Lundeberg, Cooper, Fritzen, & Terpstra, 2008), Thandi noted that she felt that the VSR interview was a powerful medium for revealing aspects of her practice she had previously been unaware of. She specifically made comment about seeing the need to accommodate learners' contributions in the classroom, rather than imposing methods for them to memorise.

> Thandi: Yah, what I learned I can say from the interview specifically, is that as we teach from the classroom there are a lot of things that we do unaware that maybe are confusing to the learners. I saw that in the few examples in addition and subtraction that we talked about today and that we should be able to as we teach to give our learners time to demonstrate their own methods to explain themselves without having to impose our own method on them.

The teacher's reflection as a result of the VSR interview indicated ongoing learning made possible through close observation of practice, and specific reflection-oriented questions about classroom decisions. This indicated that Thandi had become more aware of the limitations of her contingent actions and decisions during classroom interactions. This shifting awareness and evidence of critical reflection about *self*, *practice* and *students*, indicated possibilities for changing classroom practice. In the following section, I link findings from Thandi's shifts in teaching practice with shifts in reflective awareness.

6.4.3 Linking Thandi's practice and reflective awareness

Thandi's teaching shown differences between 2013 and 2014 in *extent*, *breadth* and *quality* of elaborations across all the three dimensions of responsive teaching. The analysis of the VSR interview also indicated shifts from a state of *unconscious incompetence* to *conscious incompetence* and moving from *deliberate* to more *critical* reflections about *self*, *practice* and *students*. The shifts in teaching practice and that of reflective awareness were connected in some ways. Hence my focus here is to highlight the areas of association between the two observed shifts.

I have noted Thandi's shifting reflections as she become more aware of the limitations in her flexible engagement with learners' mathematically oriented contribution in her

2013 lessons. Specific reflection-oriented questions about classroom decisions produced shifts in her state of awareness from *unconscious incompetence* to *conscious incompetence* in relation to responsive teaching - where she acknowledged limitations in her own knowledge. Her engagement in thinking hard to understand and consider alternative approaches for supporting learners and resolving breakdown situations responsively linked with the evidence seen in her competence and confidence with exploring different strategies resulting from learners' contributions in her 2014 teaching practice. This finding indicates a state of *conscious competence* in her 2014 lessons in supporting emergent learning through her incorporation of and developing individual learners' contributions in her 2014 lessons.

In conclusion, Thandi's critical reflection on self, students and practice, and increasing awareness to be responsive to learner's contribution in her teaching indicates associations with the shift in extent, breadth and quality of elaborations seen from applying the elaboration framework.

6.5 Sam's teaching

As noted in the lesson descriptions provided in Chapter 4 (see pages 89- 91), I observed two lessons on additive relation teaching by Sam in 2013 and two lessons in 2014. Table 16 summarises the results of the coding of Sam's 2013 and 2014 lessons using the elaboration framework.

Table 17: Coding Sam's lessons

Situations of elaboration	Sub-types categories	2013 (2 lessons, Grade 2)		2014 (2 lessons, Grade 4)	
Breakdown Incorrect learner(s) offer	***Learner offer-focused responses***		**EP** 19 (19/29 = 66% of 2013 breakdown situations)		**EP** 14 (14/21 = 67% of 2014 breakdown situations)
	L1-Restates learners' offer and questions its correctness	9		6	
	L2-Probes the learner's offer with follow-up question	4		6	
	Task-focused responses				
	Verbal reframing	0		0	
	Lead-in to the task	0		1	
	Switching between representations	2		0	
	Establishing generality	1		0	
	Contrasting offered and required operation	3		1	
	Reduces cognitive demand of the task	0	**ENP** 10 (10/29 = 34% of 2013 breakdown situations)	0	**ENP** 7 (7/21 = 33% of 2013 breakdown situations)
	Repeats learner's offer and moved on	4		3	
	Repeats task and moves on	3		2	
	No comment and moves on	3		2	
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	L1-Offers a more efficient strategy	1	**EP** 1 (1/4 = 25% of 2013 sophistication situations)	1	**EP** 6 (6/6 = 100% of 2014 sophistication situations)
	L2-Elicits a more efficient learner's offer	0		5	
	L3-Interrogates learner's offer for efficiency	0		0	
	Acknowledges correct offer as inefficient and moves on	1	**ENP** 3(3/4 = 75% of 2013 sophistication situations)	0	**ENP** 0(0/6 = 0% of 2013 sophistication situations)
	Pulls learners' back to inefficient action	2		0	
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	***Individuating responses***		**EP** 15 (15/22 = 68% of 2013 individuation/collectivisation situations)		**EP** 24 (24/27 = 89% of 2014 individuation/collectivisation situations)
	L1 - Confirms chorus offer with individual learners	4		3	
	L2 - Interrogates chorus offer with individual learners	0		0	
	Collectivising responses				
	L1 - Confirms individual learner's offer with whole class	6		4	
	L2 - Interrogates individual learner's offer to the whole class	1		1	
	Repeats individual learner's offer with the whole class	2		8	
	Decompresses individual learner's offer to the whole class	2		8	
	Collective reasoning	0		0	
	Accepts chorus offer and moves on	1	**ENP** 7 (7/22 = 32% of 2013individuation /collectivisation situations)	0	**ENP** 3 (3/27 = 11% of 2014 individuation /collectivisation situations)
	Accepts individual offer and moves on	6		3	

EP incidents occurred on 35 occasions across the two lessons seen in 2013, with 20 ENP incidents also noted. In contrast, in 2014, EP incidents occurred 44 times across two lessons, in comparison with 10 ENP incidents. This pointed to a higher incidence of PE in 2014 in comparison with 2013.

On the *extent* of elaborations, Table 16 shows similar prevalence at the level of: breakdown (66% of all breakdown situations in 2013 and 67% in 2014); sophistication (up from 25% of all sophistication situations in 2013 to 100% in 2014); and individuation/collectivization (up from 68% of all individuation/collectivisation situations in 2013 to 89% in 2014). These results collectively indicate shifts in the *extent* of elaboration provided, with the highest shifts at the level of sophistication.

On the *breadth* of elaborations, there were presences in 3 out of the 5 non-hierarchical sub-types of elaborations in breakdown situations in 2013 and 2 out of 5 in 2014. No *breadth* of elaboration is considered at the level of sophistication, since all elaboration sub-types are hierarchical in nature. There were presences in the same 2 out the 3 non-hierarchical elaboration sub-types within all collectivising responses in both 2013 and 2014. These results collectively indicate lack of shifts in the *breadth* of elaborations provided, although the number of incidents of non-hierarchical collectivising responses has greatly increased (up from 2 incidents of two of these sub-categories in 2013 to 8 incidents in 2014).

On the *quality* of elaborations, in the case of breakdown situations, there a higher proportion of the 13 incidents of *learner offer-focused responses* were at the lower level of *restating learner offers and questions its correctness* in 2013, in contrast to equal proportions of incidents of both sub-types in 2014. There were more elaborations, in the case of sophistication at the level of *eliciting* learner offer for *efficiency* in 2014, indicating growth in terms of the notion of '*quality*'. There was a lack of shift at the level of quality of elaborations within both individuating and collectivising responses with similar proportions at the level of *confirming* offers in both 2013 and 2014 lessons.

In Table 17, I provide a summary noting differences in incidence in relation to *extent*, *breadth* and *quality* within each dimension of elaboration.

Table 18: Sam's summary of shifts in relation to extent, breadth and quality

		Situations of elaboration							
Markers of shifts		***Breakdown***		***Sophistication***		***Individuation/ Collectivisation***			
		2013	2014	2013	2014	2013		2014	
Extent		66%	67%	25%	100%	68%		89%	
Breadth		3/5	2/5	NA	NA	2/3		2/3	
						Ind.	Col.	Ind.	Col.
Quality	L1	9	6	1	1	4	6	3	4
	L2	4	6	0	5	0	1	0	1
	L3	-	-	0	0	-	-	-	-

Sam's data overall suggested marked shifts in *extent* of elaboration within sophistication and individuation/collectivisation situations, with limited change in the breakdown situation. Substantial shifts for Sam were in quality of elaboration within sophistication situations, with a higher proportion of responses at the level of *eliciting learner offers for efficiency*.

To exemplify the pattern of the observed shifts by Sam qualitatively across 2013 and 2014, I selected excerpts as 'telling cases' that indicated the area of substantial overall differences seen in Table 17 across the two years. These differences were at the level of 's*ophistication-quality*' difference. I deal with this difference by contrasting her 'ways of being' with mathematics in her 2013 and 2014 lessons, before presenting a discussion of some possible implications of these shifts for responsive teaching.

Sophistication-quality difference

In terms of the sophistication category, there were three incidents of ENP and one incident of a lower level of EP (*offering a more efficient strategy*) in Sam's 2013 lessons. In contrast, Sam range of responses in 2014 showed greater prevalence in the EP category, and within this, greater emphasis on the level of *eliciting more efficient learners' offers*. Below, I exemplify this contrast in the quality of her sophistication elaborations across the two years.

To exemplify the predominant response to sophistication in Sam's 2013 lessons, I draw on an example from her 2013 lesson 1 where she focused on addition by bridging through 10 using marbles and on a number line. Her planning and enactment of this lesson suggests a progression from using concrete apparatus (marbles) to using a number line to show procedures for bridging through 10. Sam wrote two sums on the board: 8 + 5 and 7 + 6, and said '*I know that you know how to do these sums, but today we are going to do it differently, by* ***making a ten first*** *out of those numbers*'.

Bringing out blue and yellow marbles from a container and two plates, she invited two learners to the front of the class and gave each learner a plate. One learner was asked to count eight yellow marbles onto one plate and the second was asked to count five blue marbles onto the other plate. She then stuck these marbles on the board with an addition sign in between, and a symbolic representation of the sum underneath. Sam demonstrated re-arrangement of the marbles from 8 and 5 to 10 and 3 (Figure 19). She then moved on to demonstrate the same idea with 8+5 again on a number line with 0, 10 and 20 marked. Excerpt 14 illustrates how this activity then played out

Figure 19: Making of a ten out of 8 and 5 marbles

Excerpt 14

282 T: Excellent. Okay we want to try it on a number line again. So here we want to put in our numbers (*teacher writes 1-9 between 0 and 10 and 11-19 between 10 and 20 on 0-100 number line marked in 10s*).

282 C: Yes

283 T: Okay, I have a number line which we are going to use. If it is on the number line, we say eight; can you see where eight is? Who can come and show us eight plus five on the number line? Where should we start?

284 L: (*learner points at 8 on the number line*)

285 T: Yes, that is eight. Who can come and show us eight plus five?

286 L: 1, 2, 3, 4, 5 (*learner counts five marks from 8*)

287 T: Yes we said 8 plus 5 (*teacher repeats the counting on of five*) 1, 2, 3, 4, 5. It must be here (*teacher points at 13 where she lands on the number line*). It will give us?

288 C: Thirteen

While Sam's lesson sequence suggests progression in the use of representations from more concrete artefacts (marbles) to a more abstract representation (number line), her writing in of the numbers 1-9 and 11-19 appears unnecessary if the aim is to demonstrate 'bridging through 10' as this encourages counting on in 1s – which is what the first learner does. In her response to this learner, Sam repeated the learner's counting in 1s. This indicated limited elaboration of sophistication in ongoing use, and in this case a 'pulling back', into unit counting approaches to working with number despite a move forward into the more abstract number line representation. Given this analysis, this incident was coded as '*pulling learners back to inefficient strategy'*.

In Sam's 2014 lesson 2, in the context of a mental starter activity session at the beginning of the lesson, she had pasted five addition problems on the chalkboard (with

5 further subtraction problems as well that were dealt with later) as shown in Figure 20 below.

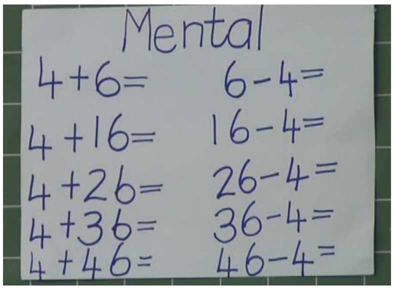


Figure 20: 10 mental addition and subtraction problems

In the setup of the task, Sam made explicit to the learners that she wanted them to quickly work out the task while observing *something* that they were going to discuss upon the completion of the task. After learners had completed all ten problems in their mental maths notebooks, Sam got all ten answers written on the board from learner inputs. She then turned back to the class and asked, '*Who can show me how you were working out the sums?*' Four learners were invited one after the other to show how they worked out the addition problems.

The first learner (L1) drew 26 tallies with repeated counting and re-counting of these tallies, and wrote down a '+' sign followed by a symbol of 4 and thirty as the answer. Sam accepted this offer and asked for '*someone who did it in a different way*'. She invited another learner (L2) to respond to the sum 4+36. L2 wrote down 4 then 36 below and worked out the sum using column addition. Once again, Sam accepted this offer, and asked for another alternative. One learner (L3) shouted, '*I am counting with hands*' and upon invitation, demonstrated starting from 4 and counting on 46 in ones orally (she said 5, 6, 7, 8, 9, 10 … 50). Sam pointed to another learner (L4), who she noted had been the first one to finish and asked L4 to show how she had worked out 4+46. L4 started with the bigger number, 46 and opening up 4 fingers one at a time, said, '47, 48, 49, 50'. Sam responded in the following ways:

Excerpt 15

155	T:	She just said forty-six. She didn’t write all these (*points to the tallies)*, this one was longer, and this one was longer (*points to column addition*). Did you hear [L4’s]?
156	L:	Yes
157	T:	So, which one is the quickest of those four girls?
158	C:	Mpunki (*referring to L4*)
159	T:	Mpunki isn’t it? Mpunki took the bigger number and added the four and she was the first one to finish. Did you notice that?
160	C:	Yes

The addition example sequence presented by Sam provides openings for using the patterned construction to build connected derived facts, a feature noted as limited in earlier South African work at primary (Venkat & Naidoo, 2012) and secondary levels (Adler & Venkat, 2014). That is, using the fact that 4+6 is equal to 10, to work out that 4+26 for example, is 10 more than 20, which is 30; and 4+46 is 10 more than 40, which is 50, and so on. However, learners’ offers showed treatments of the problems as individual sums and also, in *inefficient* ways. Sam’s response in lines 157-159 draws explicit attention to this inefficiency by pointing to ‘the quickest’ strategy as a criterion she values, and providing a partially generalized narrative in relation to the strategy offered by L4: ‘*Mpunki took the bigger number and added the four and she was the first one to finish. Did you notice that*?’ Sam’s response was coded as ‘*eliciting a more efficient strategy*’ in response to the range of learner offers, in spite of working with the individual example, rather than with the structured patterning in the example space. This marked a contrast to what we saw in her 2013 teaching where pulling back was the only kind of sophistication-related response seen in her enactment of bridging through 10 on a number line.

6.5.1 Discussion of Sam's pattern of shifts across 2013 and 2014 teaching

While Sam's VSR interview data indicated that she recognised and valued progression of mathematical ideas, her 2013 lessons showed limitations in her translating of efficiency in her moves between representations. I have showed how the idea of 'bridging through 10' was recognised by Sam, but her response to this idea on a number line indicated a disconnection between her mathematical awareness of progression and in-the-moment pedagogy (Mason & Davis, 2013; Mason & Spence, 1999). This was evident in reverting to counting in 1s with writing of the 1-9 and 11-19 initially, and in her response to the learner who counted in ones.

Sam's 2014 dataset was substantially different in terms of quality of sophistication elaborations, where she elicited and allowed different strategies to emerge from learners' perspectives and dealt well with translating her valuing of progression into classroom interaction (Knapp, 1995). Sam's introduction to the example sequence above suggested that while she had wanted learners to attend to the patterned relationship between examples, she deviated from her planned action (Rowland et al., 2015) to respond to the current level of learners' knowledge (Nystrand & Gamoran, 1990). Sam's action here can be interpreted as creating a balance between planned action and improvisation (Borko & Livingston, 1989). However, this shift in quality was only evident in her sophistication-related responses.

Having presented and discussed shifts in Sam's elaborated responses to learner offers between 2013 and 2014 lessons, I now move to findings from VSR interviews with specific focus on changes in her state of awareness and levels and objects of reflections.

6.5.2 Findings from the VSR interviews with Sam

In this section, I present findings from the interim VSR interview conducted between 2013 and 2014 lesson observations. The VSR with Sam focused primarily on her thoughts and decisions about how she enacted the idea of bridging through ten in her 2013 lesson and what she might do differently. Prior to the discussion about this idea, I began the interview with probing the teacher to articulate her objectives for the lesson.

> Researcher: Tell me what you were hoping that learners would be able to do or understand at the end of the lesson?
>
> Sam: …learners would understand that it's very simple to, like when it comes at the addition part of the lesson where I said 8+5, I wanted them to understand that when we add, make one of our number to become a 10, it is simpler to get to the answer. Since 10 is a nice and easy number to work with, to add on a smaller number it would give them the correct answer.
>
> Researcher: Do you think that your learners are well aware of 10 as a nice and easy number to work with?
>
> Sam: Yes
>
> Researcher: Okay, what do you see that allows you to say yes?
>
> Sam: Because when I say 10, he quickly thinks of 5 and 5 as you saw in the second example (referring to 7+6). They know how to make 10 quickly. They know that even if I take two from here and add to eight, I will get 10. They are getting there.

In this comment, Sam makes no distinction between the 'five-wise' decomposition strategy offered by the learner and the bridging through 10 strategy that she has advocated. There is also no reference to the evidence that several learners were unable to provide the answer by bridging through ten. Given this, Sam's focus appears to be more on what she anticipated learners to do in her lesson. Her talk indicates, as in Thandi's case, greater attention initially to the concept of bridging through 10 (perhaps in more 'result' focused ways here) drawing from her both connection and transformation knowledge. Her statement indicated that she is satisfied with her overall delivery of the lesson and the learning outcome – indicating a state of *unconscious incompetence* in relation to her pedagogical actions and *technical description* level of reflection about *self* and *students*. Through specific reflection-oriented questions during the interview, Sam realised that some of her learners didn’t understand what she meant when she asked them to make a ten.

Sam: What went well for me is that my learners were able to make the ten, but some of them, I could see that they couldn’t understand what the teacher means when she says, make a ten, what is a ten? They don’t understand the concept and why making a ten.

This statement indicated a shifting awareness with Sam beginning to see gaps in learners’ understanding of the concept of making a 10, with the broader idea of bridging through 10. This awakening is characterised by a move towards *conscious incompetence* with *deliberate* level of reflection about *students’* learning. Following this, I asked for the kind of activity she might consider to support learners become aware of the usefulness of 10 as a number that is easier to work with and used as a benchmark (Mcintosh et al., 1992). She verbalised two sequences of activities that need to be added as prerequisites for teaching addition by ‘bridging through ten’. Firstly, she noted the need to create an additional activity to support learners to make a ten, for example: ‘If I have 7 how many do I need to make a ten?’ with further similar examples. She then went on to offer the following activity:

Sam: I think I would do an activity where without knowing they will make tens. And at the end they would add on top. It will be very simple for them to add units on top of a ten and they will give you the answer mentally without counting. If you go a step further where you say fifty plus two, they don’t even need to count using counters; they will know fifty plus two more is fifty-two. So, I need to work around where they do a lot of working with ten mentally and as we go they will start to realize that ten is a useful number to work with.

The second activity was about the rationale for the usefulness of working with tens. Sam went on: ‘I would put in examples that show that ten is an easy number to add from, like putting in more on ten; 10+2, 10+3, and so on’ Once again here, drawing attention to reflecting on gaps in learner understanding is followed by a move to adapt the task and example sequence, with a focus here on attending to some of the sub-skills involved in bridging through ten. Through this, she argued that learners would see that it is easy to work with ten Sam's talking about sequencing of her lesson can be seen as development towards *conscious incompetence*, in her ability to plan tasks for the actual learning trajectory she observed with some of the learners in her lesson – the ones who were unable to deal with her original task (Simon, 1995). This transition corresponds with a move from *technical* to *deliberate* reflection about *practice* and *students*.

These additions were elicited/developed in the context of the VSR interview. Thus, an absence in the enacted practice became a presence in the post-lesson interview context. In my post-interview reflections, I noted that Sam had left a potentially useful further aspect aside – facility with partitioning the number to be added (the 5 in 8 + 5 for example), and then selecting the appropriate partition for making 10 in the problem set (Abdulhamid & Venkat, 2013). At the end of the interview Sam offered the following reflection on what was made explicit to her during our interaction in the VSR interview:

> Sam: …So activity like this will give you a reflection to know that sometimes when results are not that perfect, it is not like you should blame your learners or yourself. You need to go back - which we don't do it. Go step by step as we have done here. I could see that maybe if I have done it the way we discussed, my teaching could have been much better. Therefore for me this activity is an eye-opener where you as a teacher must always go back on whatever lesson you have given, especially when things don't go the way you wanted. Try to make reflection of yourself, your learners, and then put the two together. You would get a way forward by identifying what goes wrong and thinking of ways to deal with it.

Sam's final remark indicated potentially on-going moves to *conscious competence* level of awareness and *critical* reflection about *self*, *practice* and *students*. In the next section, I present possibilities for linking differences in Sam's teaching actions seen in 2014 with the shifting awareness and reflection discussed.

6.5.3 Linking Sam's practice and reflective awareness

Sam's teaching showed key differences in extent and quality of elaborations within sophistication situations, and slight shifts in extent and breath of elaboration were noted in individuation/collectivisation situations. Analysis of the VSR interview with Sam indicated a change in her state of awareness from *unconscious incompetence* to *conscious incompetence* levels of awareness and from *technical* description to *deliberate* reflection about *self, practice* and *students*. These changes in her state of awareness and move to critical reflection, and focus on expanding example spaces relating to key sub-skills were associated with shifts in her teaching practice in sophistication situations in contingent ways as observed on the elaboration framework.

Findings from VSR interviews indicated moves for Sam from talking about designing alternative tasks to establishing connections in sequencing of her instruction in order to make it visible for learners to appreciate what she values in the usefulness of 10 as benchmark for addition. In her 2014 teaching, I noted more incidents of eliciting learner offers for efficiency, and provision of opportunities for the emergence and evolution of learners' emergent different strategies, which were later translated into efficiency in her selection of strategies for problem solving.

Sam's concluding reflections suggests on-going learning made possible through close observation of practice, and specific reflection-oriented questions asking about classroom decisions. This evidence shows that Sam did have some of the prerequisite knowledge and skills needed to work with bridging through 10. However, the potential for improved enactment of this knowledge and skills was seen in her making explicit additional tasks for eliciting prerequisite sequences of skills through our interaction in the interview. Sam's 2014 dataset was drawn from a Grade 4, rather than a Grade 2 class, making direct comparisons more difficult. However, while the enacted task presented for discussion was focused on looking at connections between examples, her response to learner offers showed a flexible and smooth focus on establishing a single jump to the next ten as the most efficient response. Furthermore, research shows that there is significant shift in cognitive demands of teaching as the content progresses to the higher level across the curriculum. Concepts become more abstract, and Potari et al. (2007) and Rowland (2009) have noted that this appears to require a different mathematics specific pedagogical demand.

6.6 Bongi's teaching

As noted in the lesson descriptions provided in Chapter 4 (see pages 91- 93), I observed two lessons on additive relation teaching by Bongi in 2013 and two lessons in 2014. Table 18 summarises the results of the coding of Bongi's 2013 and 2014 lessons using the elaboration framework.

Table 19: Coding Bongi's lessons

Situations of elaboration	Sub-types categories	2013 (2 lessons)		2014 (3 lessons)	
Breakdown Incorrect learner(s) offer	***Learner offer-focused responses***		**EP** 7 (7/16 = 44% of 2013 breakdown situations)		**EP** 9 (9/11 = 82% of 2014 breakdown situations)
	L1-Restates learners' offer and questions its correctness	3		3	
	L2-Probes the learner's offer with follow-up question	0		2	
	Task-focused responses				
	Verbal reframing	0		0	
	Lead-in to the task	0		0	
	Switching between representations	0		2	
	Establishing generality	2		2	
	Contrasting offered and required operation	2		0	
	Reduces cognitive demand of the task	3	**ENP** 9 (9/16 = 56% of 2013 breakdown situations)	1	**ENP** 2 (2/11 = 18% of 2014 breakdown situations)
	Repeats learner's offer and moved on	4		1	
	Repeats task and moves on	2		0	
	No comment and moves on	0		0	
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	L1-Offers a more efficient strategy	1	**EP** 1 (1/4 = 25% of 2013 sophistication situations)	4	**EP** 5 (5/5 = 100% of 2014 sophistication situations)
	L2-Elicits a more efficient learner's offer	0		1	
	L3-Interrogates learner's offer for efficiency	0		0	
	Acknowledges correct offer as inefficient and moves on	0	**ENP** 3 (3/4 = 75% of 2013 sophistication situations)	0	**ENP** 0 (0/5 = 0% of 2014 sophistication situations)
	Pulls learners' back to inefficient action	3		0	
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	***Individuating responses***		**EP** 21 (21/26 = 81% of 2013 individuation/collectivisation situations)		**EP** 25 (25/29 = 86% of 2014 individuation /collectivisation situations)
	L1 - Confirms chorus offer with individual learners	1		0	
	L2 - Interrogates chorus offer with individual learners	0		0	
	Collectivising responses				
	L1 - Confirms individual learner's offer with whole class	3		6	
	L2 - Interrogates individual learner's offer to the whole class	2		2	
	Repeats individual learner's offer with the whole class	4		5	
	Decompresses individual learner's offer to the whole class	11		10	
	Collective reasoning	0		2	
	Accepts chorus offer and moves on	2	**ENP** 5 (5/26 = 19% of 2013 individuation /collectivisation situations)	2	**ENP** 4 (4/29 = 14% of 2014 individuation /collectivisation situations)
	Accepts individual offer and moves on	3		2	

EP incidents occurred on 29 occasions across the two lessons seen in 2013, with 17 ENP incidents also noted. In contrast, in 2014, EP incidents occurred 39 times across two lessons, in comparison with 6 ENP incidents. This pointed to a substantially lower incidence of ENP in 2014 in comparison with 2013.

On the *extent* of elaborations, Table 18 showed shifts at the level of: breakdown (up from 44% of all breakdown situations in 2013 to 82% in 2014); sophistication (up from 25% of all sophistication situations in 2013 to 100% in 2014); and individuation/ collectivization (up slightly from 81% of all individuation/collectivisation situations in 2013 to 86% in 2014). These results collectively indicate shifts in *extent* of elaboration provided, with high shifts at in sophistication and breakdown dimensions, and more limited positive shifts within the individuation/collectivisation dimension.

On the *breadth* of elaborations, there were presences in 2 out of the 5 non-hierarchical sub-types elaborations in breakdown situations in 2013 and in 2014 (one non-overlapping). No *breadth* of elaboration is considered at the level of sophistication, since all elaboration sub-types are hierarchical in nature. There were presences in 2 out of the 3 non-hierarchical elaboration sub-type within all collectivising responses in 2013, and 3 out 3 were evident in 2014.

On the *quality* of elaborations, in the case of breakdown situations, there was no incident of probing learner offer in 2013, but 2 incidents were present in her 2014 responses. There were increases in elaborations, in the case of sophistication in 2014, but these remained predominantly at the lower level. There were no incidents of interrogating offers in individuating elaborations both in 2013 and 2014, but there were increases in collectivising responses in 2014, but incidents of the interrogating sub-type remained the same in both 2013 and 2014. This indicated shifts at the level of *quality* in the breakdown dimension. No shifts were evident at the level of quality in sophistication and individuating/collectivising elaborations.

In Table 19, I provide a summary noting differences in incidence in relation to *extent*, *breadth* and *quality* within each dimension of elaboration.

Table 20: Bongi's summary of shifts in relation to extent, breadth and quality

		Situations of elaboration							
Markers of shifts		***Breakdown***		***Sophistication***		***Individuation/ Collectivisation***			
		2013	2014	2013	2014	2013		2014	
Extent		44%	82%	25%	100%	81%		86%	
Breadth		2/5	2/5	NA	NA	2/3		3/3	
Quality						Ind.	Col.	Ind.	Col.
	L1	3	3	1	4	1	3	0	6
	L2	0	2	0	1	0	2	0	2
	L3	-	-	0	0	-	-	-	-

Bongi's data overall suggested marked shifts in *extent* of elaboration within breakdown and sophistication situations, with more limited positive shifts, but shifts nevertheless, in the individuation/collectivization situation, where she started with a relatively high level of prevalence in 2013 (81%). The *spread* (within dimensions non-hierarchical sub-types) remains broadly the same in both 2013 and 2014, indicating no shifts in breadth of elaboration. The shift in *quality* (within dimensions hierarchical sub-types) of elaborations is only evident within breakdown elaborations, with moves from no incidents of probing learner offers in 2013 to 2 incidents in 2014.

To exemplify the pattern of the observed shifts by Bongi qualitatively across 2013 and 2014, I selected excerpts as 'telling cases' that indicated the areas of substantial quality difference shown in Table 19 across the two years. This difference was noted only at the level of 'breakdown-quality' difference. I deal with this difference through contrasting her 'ways of being' with mathematics in her 2013 and 2014 lessons, before presenting a discussion of some possible implications of this finding in relation to responsive teaching.

Breakdown-quality difference

In terms of the breakdown category, Bongi's range of responses in 2013 within learner-focused responses were at the lower level of the hierarchy in this category characterised by the *restating of learner offers and questioning correctness*. In contrast, in 2014, there were 2 out of 5 incidents within the higher level of *probing learner offers with follow up questions*. Below, I exemplify this contrast in the quality of her breakdown elaborations across the two years.

In her 2013 lesson 2, Bongi used a part/whole diagram to model word problems. Her first example as whole class work was: In a class, there are 15 girls and 23 boys. How many more boys are there than girls? Bongi enacted the problem in the following ways:

> 214 T: We must draw the story on the board. I said, in the class we have fifteen girls; here are my fifteen girls in blue they are smiling, fifteen girls. And then we have twenty-three boys. The bar for twenty-three must be long, because twenty-three is bigger than fifteen. Here are my twenty-three naughty boys. There they are good, the boys, ne? So the question is saying, how many more boys are there than girls? This means I'm looking for this gap here, which I don't know (the part/whole diagram is shown in Figure 21 below). Can you get the story now?

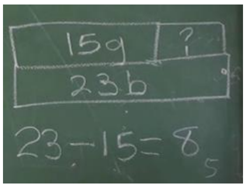


Figure 21: Part/whole diagram for modelling a compare problem

Bongi then asked learners to use the same diagram and model in the following word problem: In a class, there are 22 girls and 20 boys. How many learners are there altogether?' Learners were very quick in stating the answer as 42. Bongi accepted the answer, but also asked learners to model the problem using part/whole diagram.

Bongi invited three learners, one after the other, to draw the diagram on the board. None of these learners drew the diagram in the way that Bongi anticipated. Figure 22 captures the different diagrams drawn by the three learners. In each case Bongi's response was to restate the learner action, and comment: '*Your diagram is not telling a good story*'.

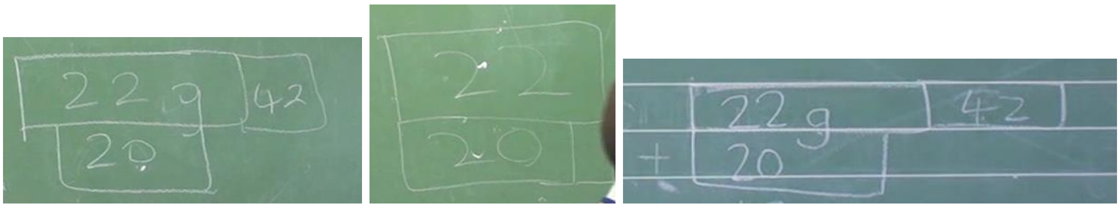
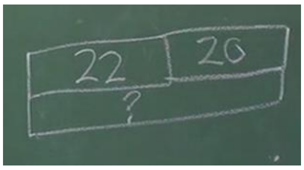


Figure 22: Incorrect part/whole diagrams drawn by three learners

Following a further incorrect offer from a learner, Bongi went on to draw the correct diagram herself, saying:

> 291 T: Okay, thank you very much. Okay, let's look at this. Let me start all over again. Okay, now I'll give you a chance, ne? We are saying in a class, here is the class, there are twenty-two girls and then there are twenty boys (she draws two bars joined together and writes 22 and 20 in each bar to represent the number of girls and boys in the class respectively). This is the class, can you see? It's a class. And then how many learners are there altogether. I will draw an empty box here, put a question mark. Can you now see the story? You are in the same class, here are twenty-two girls, and here are twenty boys.



With this offer, Bongi presents the correct model without any focus on discussing the nature of the part-part-whole relation seen in the multitude of incorrect offers from learners or in her own diagram. Furthermore, learner responses suggested that they could solve the problem without explicit need of the part-whole model. It is unclear here whether Bongi's goal in relation to this task was to introduce the model, or to use the model for problem-solving. If the former, while Bongi could make the appropriate distinctions, her response suggested no explicit talk about these relations across the compare and combine problems, and therefore, no elaboration that was responsive to learners' offers. If the latter, then the need for the part-whole model is questionable. Literature suggests the need for new representations or tools to model situation to be motivated by problems for which existing tools are insufficient or inefficient (Klein et al., 1998). Given this analysis, I coded Bongi's response as *repeat learner offer and moves on.*

The part/whole diagram as an important representation in modelling additive relations situations had been presented in the WMC-P 'maths for teaching' course. While Bongi demonstrated knowledge of the part/whole model and applying it to additive relation situations, her teaching suggested a lack of the connective tissue to use the diagram in a way that was responsive to her learners' offers. In this instance, her lack of explanations related to the relationship between quantities suggested either gaps at the level of connection knowledge in the KQ, or – if it was the case that she simply left these explanations at the level of the implicit – that she was not able to draw on her contingent knowledge in breakdown situations.

In her 2014 lesson 2, Bongi was teaching addition involving decimals in the context of money situations, and asked for the total cost of two cakes each costing R33,99. Learners offered different answers. In response, Bongi collected all these offers and initiated a conversation with the whole class to decide which of these answers is correct. The following excerpt played out:

Excerpt 16

384 T: I am having a nice cake, this one I am not going to eat it now. It’s nice and they are saying it cost thirty-three rand ninety-nine cents. I need two, how much am I going to pay?

385 C: (*Two learners raised their hands*)

386 T: Give them chance, they are still thinking. Wena you are not thinking. (*Teacher points to one learner*)

387 L: Mem, Sixty-seven rand

388 T: Sixty-seven rand? Let me write sixty-seven here (*teacher writes R67,00 on the board*) another answer?

389 L: Sixty-seven rand ninety-eight cents

390 T: Sixty-seven rand ninety-eight, (*Teacher writes “R67, 98”*) another answer?

391 L: Sixty-eight rand

392 T: Sixty-eight rand only?

393 L: Yes

394 T: Okay, *(Teacher writes “R 68,00”)* the last one?

395 L: Sixty-seven rand ninety-five cents

396 T: Sixty-seven rand ninety-five (*teacher writes “R67, 95”).* I think I am done with answers, let us now look at this one I have thirty-three rands, ninety-nine, I need two (*teacher writes ‘R33,99 + R33,99’ in vertical column form*) I need two. One way to think about calculating very quickly and to get it right is to say thirty-three rand - if I can just round it up it will give me how much?

398 C: Thirty-four

399 T: Thirty-four rand (*teacher writes 34 on the board*) I am just saying thirty-four plus thirty-four what is the answer?

340 C: Sixty-eight

341 T: Sixty-eight, very good (*teacher writes “68” on the board*) but remember I added two cents from here (*teacher points to “33, 99”*). So, what do I do? Yes

342 L: Minus two cents

343 T: Very good, I subtract from here (*teacher points to “68,00*”). So, my answer will be?

344 C: Sixty-seven rand ninety-eight

345 T: Yes, sixty-seven rand ninety-eight is the correct answer (*Teacher ticks 67,98 in the list of learner's answers that was written on the board*). So, thinking in this way, since R33,99 is very close to 34 would allow you to get the correct answer quickly and without making mistakes.

While the first learner offer was incorrect, Bongi accepted and wrote down all learners offers on the board. She then allowed other learners to participate by giving them the chance to give answers. Bongi accepted all these offers and wrote them down on the board. In contrast to her 2013 response to the incorrect offer, here Bongi launched a more interactive discussion on which of these offers was correct. While at times providing the route to follow (e.g. rounding up and noting the need for the two cent adjustment), learners interim offers, including awareness in Line 342 of the need to 'minus' the two cents, suggests that her elaboration was responsively useful. This incident was coded as probe learner offer, a higher level of learner offer-focused elaboration.

6.6.1 Discussion of Bongi's pattern of shifts across 2013 and 2014 teaching

Bongi's response to learners' offers in 2014 indicated moves forward from 2013 in relation to response to breakdown situations. In her 2013 response, Bongi ignored all learners' incorrect answers and solved the problem on her own. In her 2014, she accepted and wrote down all learners' offers (correct and incorrect answers), before working with the whole class to establish which of the answers was correct using a more efficient rounding strategy that she value to get to the answer, a practice that is described as important in extending learners understanding (Franke & Kazemi, 2001). While this is a move forward for Bongi, international literature would tend to note her guiding the solution in a specific, predetermined direction instead of encouraging learners to do most of the mathematical work.

6.6.2 Findings from the VSR interviews with Bongi

In this section, I present analysis and findings from the interim VSR interview with Bongi focused primarily on her thoughts and decisions relating to her enactment of the part/whole diagram to model word problems, and what Bongi wanted to achieve with this diagram. The following excerpt presents an early interaction:

Researcher:	We have seen in your lesson, where learners produced correct answer without the use of the diagram, but you insisted that learners must use the diagram to model the word problem. So what is it that you are saying that they are not seeing?
Bongi:	The thing which I am thinking, they missed a concept here about what we want - they are rushing the step here they are not following the stages. If I had asked something different they were going to miss. Let's suppose I was talking about the two who are absent they are not interested about those two that are absent they were only interested that if it's altogether you add quickly

Bongi seems to be saying what learners couldn't do or are missing focusing on the steps that she set out for the learners to follow, without commenting on what might have made these learners to be thinking the way they do. What is noticeably absent is a sense of needing to adapt the plan to be responsive to what her learners do with the part/whole diagram. This absence again suggest a view of a critique to the maths for teaching course that advocated the planned element of the KQ framework in terms of the use of this diagram. In relation to in-the-moment responsiveness, this finding indicates Bongi's state of awareness described by unconscious incompetence with technical level of reflection about students.

Bongi believe that lack of modelling the problem on a diagram before performing the calculation might potentially make learners to missed important information in the word problem leading to production of incorrect answers. Bongi provided example that she never asked in the class – "supposed I was talking about the two who are absent … they were only interested if it's altogether you add quickly". In relation to in-the-moment responsiveness, this indicates Bongi's state of awareness described by unconscious incompetence with technical level of reflection about students.

I further probe on why Bongi sees learners pushing towards the answer as a problem. Why she is pushing them to use the diagram, and what might she be doing differently to get learners value and use the diagram.

Researcher:	Learners job is to produce the answer, they are producing the answer and so now why teacher is burdening us with this diagram on the board. It's just like wasting our time we know what the answer is. What is it that you wanted them to see because you are pushing them to use this diagram.
Bongi:	What I wanted them to see I wanted them to see the picture because it's not always the 'altogether' question and whatever. So, I was thinking that if they can draw those pictures at least they are bringing the answer home.
Researcher:	Clearly learners are having difficulty with drawing the diagram, so one of the things we would like you to do is okay we have seen this difficulty. They are not in the page that you are on. If you have to do this lesson again, you have to do it differently. What might you be doing differently to get them?
Bongi:	I think first I need to do more dramatizing first before we could even work out this.
Researcher:	Okay, so you showed me one example of dramatizing is empty class and the principal comes and sees 22 girls lines them up and there is 20 boys, how many are they altogether? So would you be, as you are dramatizing this story you look at and you draw this altogether
Bongi:	I think I can draw this one's or else I can draw here is the class you try and draw small things for them to bring them home. And then at the end they got stuck because they did not get the concept they have the answer.
Researcher:	So here is your part/part whole for the first question (researcher draws PPW diagram for the first compared problem type with difference unknown) and then you want a part/part whole for the second question, I guess, which they were getting stuck on, I think.
Bongi:	Yes
Researcher:	Okay, so again, what you said you want to begin with the dramatization, what addition might you need to say, what question might you need to ask to get them to do that?
Bongi:	Okay, I think let's use this question that is on the board. If I am to revisit again, I think what I will do if they are failing to produce this one. What if I just

say here are the 22 girls, you draw few say they are here, they visualize the girls even if they won't get to 22. Here, I have the 20 boys you draw a few. What can we do? Which operation can we put here? How many learners altogether they would say teacher here we are going to add. Let's add because previously we were adding numbers against units, 2 digit plus 2 digits. Those who can add down here they can add. Those who can't now go back and use any other method which we used before. The number line they can go on to 22 we are adding 20 they make the jumps because then I would like to see learners applying the different methods that they learned before to do the addition calculation.

Again, the use of the part/whole diagram and the difficulty associated with learners' generalisation of 'altogether' means to add has become the central mathematical focus of the lesson for Bongi. So she is finding it hard to lose this sense. With this orientation and lack of awareness of using the part/whole model responsively beyond what she learned in the course makes the use of the model unhelpful in this context. If the use of the model is about addressing the fact that children just add using 'altogether' as a cue then her chosen example is inappropriate. Therefore, the alternative approach she mentioned through dramatizing also does not provide any means to address the problem of using altogether as a cue. As a teacher, one might want to teach the model in the context of a task where it is not needed in order to provide children with a model that will help them with more complex problems. Given this analysis, Bongi's utterances collectively do no change her state of awareness, as she remain at the level of unconscious incompetence, having not acknowledged the limitations in her use of the diagram. I probed again on other rationales for the use of the diagram, Bongi made the following statement.

Researcher: A follow up question, then what is the importance of the diagram?

Bongi: For this diagram, there are learners; we have different groups of learners. Some can interpret the word problem and get the answers straight away while others who can't, they need the diagram. As long as they see that 22 and here is 20 which means mem is asking for this full bar (referring to a whole in part-part whole diagram). What can I do? So, the bar diagram they drive the learners to choose a correct operation – whether we are adding or subtracting.

Researcher: Agreed with that if they can get the model, they can get the diagram, you have to get the diagram correct in order to get the numbers and the operation for the calculation.

Bongi: Yes

Researcher: So, the problem at the moment is that some children don't seem to get a word problem to the diagram.

Bongi: Yes, they will end up getting wrong answers if the diagram is not correct.

Researcher: So, how do we get them there? Because the problem at the moment is not about getting from the part/part whole picture to the answer, but the problem is about getting from words to the diagram.

Bongi: I think we need to go back to the word again, underline the keywords and then underline the numbers which we are using.

Bongi's statement above indicates a much clearer intention in the use of the model on the basis that some children need to be able to set up the model to calculate the answer correctly. But again her example selection is inappropriate in relation to this aim – so the issue here is with planned elements of KQ, not necessarily contingency. My probe on this matter had pushed Bongi to revert back to traditional practice of underlining keywords, which has been widely acknowledged as a problematic practice, in contrary to sense making of the mathematical word problems (Roberts, 2015). Understanding the problem with Bongi's selection of example, which seems not cohere with her aim of the using of the model might be the reason why she revert to traditional way of working by underlying keywords. I then asked her about how to solve problem like: In a class, there are 22 girls and 20 boys. How many more girls than boys? Bongi made the following statement

Bongi: Okay, let's supposed they can't grasp this one with underlining keywords. I go back again to the class, I draw this rectangle only (teacher draws a rectangular bar); I am trying to dramatize this rectangular bar here is my class. In my class, we 22 girls and then we have 20 boys. I would then ask learners whether the boys and the girls are equal in number. So, let's look at it as long as they say no; they are not equal which means some are more than the others. Let's look at the question, how many more girls are there than boys? What are you going to do? The clever ones will say mam they are 2. They don't say we are subtracting; they will just give you the answer quickly

> they are 2. They say mem, I am just looking at the number 20 and 22; the difference is 2.

With this provoking compared problem type where literature suggests that learners are struggling to solve such problem by direct modelling and underlying keyword, Bongi realizes the problem of the underlying keywords that she proposed. Bongi then reverts back to the part/whole diagram, with some attempt about acting on the problem. This again, indicated no evidence of any changes in her reflective awareness.

6.6.3 Linking Bongi's practice and reflective awareness

The areas of shifts noted in Bongi's teaching practice were predominantly at the level of extent of elaborations, with limited positive shift in quality of elaborations within breakdown situation. Analysis from the VSR interviews indicated no change in Bongi's state of reflective awareness, as her utterances in the interview do not show any move from her comfort zone. This showed that Bongi remains at the level of *unconscious incompetence*, with a move from technical to deliberate reflection. This is in line with what the literature suggests, that teachers need to realise the limitations of their knowledge before they can pay attention to the need for changing their classroom practice.

Bongi retained the plan-oriented element of her teaching in relation to the use of the part/whole diagram, a model presented in the course that she attended. She demonstrated little readiness to temper the emphasis on her planned actions to consider the need for alternative actions in contingent situations. It was evident also that there were gaps in the planned elements of KQ, particularly in her selection of examples, as if the use of the model was about addressing the fact that children just add using 'altogether' as a cue then her chosen example was inappropriate. As indicated earlier, limitations in the planned element further constrain possibilities for responsive teaching.

6.7 Herman's teaching

As noted in the lesson descriptions provided in Chapter 4 (see pages 93- 94), I observed two lessons on additive relation teaching by Herman in 2013 and three lessons in 2014. Table 20 summarises the results of the coding of Herman's 2013 and 2014 lessons using the elaboration framework.

Table 21: Coding Herman's lessons

Situations of elaboration	Sub-types categories	2013 (2 lessons)		2014 (3 lessons)	
Breakdown Incorrect learner(s) offer	***Learner offer-focused responses***		**EP** 2 (2/7 = 29% of 2013 breakdown situations)		**EP** 6 (6/19 = 32% of 2014 breakdown situations)
	L1-Restates learners' offer and questions its correctness	1		4	
	L2-Probes the learner's offer with follow-up question	0		0	
	Task-focused responses				
	Verbal reframing	0		0	
	Lead-in to the task	1		1	
	Switching between representations	0		0	
	Establishing generality	0		0	
	Contrasting offered and required operation	0		1	
	Reduces cognitive demand of the task	0	**ENP** 5 (5/7 = 71% of 2013 breakdown situations)	3	**ENP** 13(13/19 = 68% of 2014 breakdown situations)
	Repeats learner's offer and moved on	2		1	
	Repeats task and moves on	2		5	
	No comment and moves on	1		4	
Sophistication Correct learner(s) offer but viewed by the teacher as inefficient	L1-Offers a more efficient strategy	0	**EP** 0 (0/3 = 0% of 2013 sophistication situations)	3	**EP** 5(5/6 = 83% of 2014 sophistication situations)
	L2-Elicits a more efficient learner's offer	0		1	
	L3-Interrogates learner's offer for efficiency	0		1	
	Acknowledges correct offer as inefficient and moves on	0	**ENP** 3(3/3 = 100% of 2013 sophistication situations)	0	**ENP** 1(1/6 = 17% of 2014 sophistication situations)
	Pulls learners' back to inefficient action	3		1	
Individuation/ Collectivisation Correct chorus offer that is individuated or correct offer from individual learner that is collectivised by the teacher	***Individuating responses***		**EP** 5(5/13 = 38% of 2013 individuation/collectivisation situations)		**EP** 13(13/16 = 81% of 2014 individuation/collectivisation situations)
	L1 - Confirms chorus offer with individual learners	3		5	
	L2 - Interrogates chorus offer with individual learners	0		0	
	Collectivising responses				
	L1 - Confirms individual learner's offer with whole class	0		1	
	L2 - Interrogates individual learner's offer to the whole class	0		1	
	Repeats individual learner's offer with the whole class	2		2	
	Decompresses individual learner's offer to the whole class	0		4	
	Collective reasoning	0		0	
	Accepts chorus offer and moves on	1	**ENP** 8(8/13 = 62% of 2013 individuation /collectivisation situations)	0	**ENP** 3(3/16 = 19% of 2014 individuation /collectivisation situations)
	Accepts individual offer and moves on	7		3	

EP incidents occurred on 7 occasions across the two lessons seen in 2013, with 16 ENP incidents also noted. In contrast, in 2014, EP incidents occurred 25 times across three lessons, in comparison with 17 ENP incidents. This pointed to a marginally higher rate of incidence of EP in 2014 in comparison with 2013.

On the *extent* of elaborations, Table 20 showed small shifts at the level of: breakdown (up from 29% of all breakdown situations in 2013 to 32% in 2014); sophistication (up from 0% of all sophistication situations in 2013 to 83% in 2014); and individuation/collectivization (up from 38% of all individuation/collectivisation situations in 2013 to 81% in 2014). These results collectively indicate large shifts in extent of elaboration provided, with limited positive shifts at the level of breakdown.

On the *breadth* of elaborations, there was presence in 1 out of the 5 non-hierarchical sub-types elaborations in breakdown situations in 2013 and 2 out of 5 in 2014. No *breadth* of elaboration is considered at the level of sophistication, since all elaboration sub-types are hierarchical in nature. There was presence in 1 out of the 3 non-hierarchical elaboration sub-type in 2013 and 2 out of 3 in 2014. This indicates relatively limited positive shifts at the level of *breadth* of elaborations.

On the *quality* of elaborations, within the hierarchical sub-types, in the case of breakdown situations, in both 2013 and 2014 Herman's elaborated responses remain at the lower level - *restating learner offer and questions its correctness*. There were two incidents of higher level sophistication elaborations in 2014 lessons relative to none in 2013, indicating shifts at the level of sophistication-quality. The range of Herman's response to individuation and collectivisation in both 2013 and 2014 lessons showed greater prevalence at the lower level of confirming offer, indicating lack of shift in the notion of quality within individuation/ collectivisation situation.

In Table 21 I provide a summary noting where there are increases in incidence in relation to *extent*, *breadth* and *quality* within each situation of elaboration.

Table 22: Herman's summary of shifts in relation to extent, breadth and quality

		Situations of elaboration							
Markers of shifts		***Breakdown***		***Sophistication***		***Individuation/ Collectivisation***			
		2013	2014	2013	2014	2013		2014	
Extent		29%	32%	0%	83%	38%		81%	
Breadth		1/5	2/5	NA	NA	1/3		2/3	
						Ind.	Col.	Ind.	Col.
Quality	L1	1	4	0	3	3	0	5	1
	L2	0	0	0	1	0	0	0	1
	L3	-	-	0	1	-	-	-	-

Herman's data overall suggested marked shifts in *extent* of elaboration within sophistication and individuation/collectivisation situations, with no positive shifts, in the breakdown situation. There are also very limited shifts in *breadth* (within each dimension's non-hierarchical sub-types) of elaborations within breakdown and sophistication situations. Very limited shifts in *quality* (within dimensions hierarchical sub-types) within sophistication elaboration, with no shifts in quality at the level of breakdown and individuation/collectivisation situations were seen.

To exemplify the pattern of the observed shifts by Herman qualitatively across 2013 and 2014, I selected excerpts as 'telling cases' that indicated the area of substantial overall quality difference seen in Table 21 across the two years. This difference was noted at the level of 'sophistication-quality' difference. I deal with this difference through illustrating contrasts in his 2013 and 2014 lesson, before presenting a discussion of some possible implications of this finding in relation to responsive teaching.

Sophistication-quality difference

In terms of the sophistication category, Herman's responses in 2013 lessons were all at the level of pulling learner back to inefficient actions. In contrast, in 2014, the majority of his responses were at the lower level sub-category, but with at least two incidents of the higher sub-categories (given that some excerpts of these kinds of moves were coded within the breakdown category in my methodology). Below, I exemplify this contrast in the quality of his sophistication elaborations across the two years.

In 2013 lesson 1, in the context of addition by expanded notation, Herman prepared five different sets of number lines marked with 11- equal point intervals in 1s from 0-10, 10s from 0 – 100, 100s from 0 – 1000 and 1000s from 0 – 10 000. He stuck all these number lines on the board This incident is drawn to illustrate how Herman used these number lines representations in the context of working out the sum, 56 125 + 12 532 using expanded notation. Herman wrote down the value of each digit in the two numbers in expanded horizontal form as show in Figure 23:

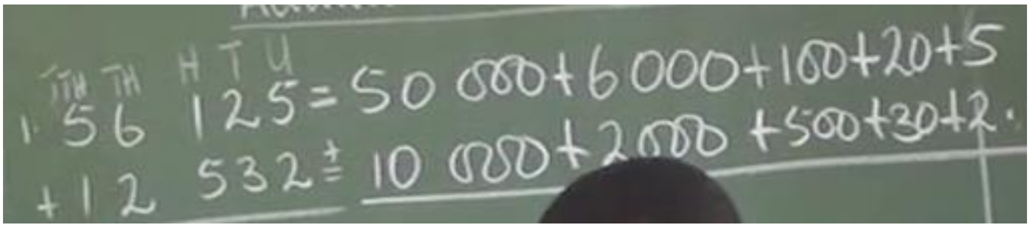


Figure 23: Addition by expanded notation

Herman's response to the use of the number lines is presented in excerpt 17 below

Excerpt 17

75 T: And we add. Using our number line, which are here, if you do not remember. We are saying now, can somebody come and show us, five plus two on this number line? Five plus two on that number line. Very quickly, five plus two thou…no, five plus two

76 L1: Seven

77 T: No, I mean, using our number line here. What do we mean?

78 L1: Seven (*points at 7 on the number line*)

79 T: How did…how did he show that? He simply went, two plus five, a seven. Okay, fine, that's correct. But can somebody show us how we make those jumps? Yes, Justice? (*Teacher invites another learner*). The answer is correct, remember? Just show it here. Five plus two is equal to? (*Teacher points to 0-10 structured number line*)

80 L2: (*Learner starts at 5 and makes two unit jumps and lands at 7 on 0-10 number line*).

81 T: Five plus two is equal to?

82 C: Seven

83 T: Seven. Right, so, if you have got any problems when you are doing these sums (*referring to individual digit sums*), use the number line, isn't it?

84 C: Yes

85 T: Right, so five plus two is equal to…

86 C: Seven

87 T: Twenty plus thirty. Somebody show us on the…ten, twenty, thirty, forty, number line. Yes, Michelle, quickly please.

88 L3: Twenty plus thirty is fifty

89 T: I said, show us on the number line

90 L3: (*Learner starts at 20 on 0-100 number line marked in 10s, makes three jumps in 10s and lands at 50*).

91 T: Twenty plus…yes, that's very correct. So, twenty plus thirty is equal to?

92 C: Fifty

93 T: Very good. Right, is equal to fifty. Now, what is one hundred plus five hundred?

94 C: Six hundred

95 T: Who can show on the hundreds number line? Yes

96 L: (Learner starts at 100, on 0-1000 number line marked in 100s, makes five jumps and lands at 600)

97 T: OK, good. Then, what is six thousand plus two thousand? Who can show on the number line?

98 C: Eight thousand

99 T: Okay, fifty thousand plus that one thousand?

100 C: Sixty thousand

101 T: I want to see answer on the number line

In the excerpt presented above, it was clear that learners could answer all the quantity value addition problems as recalled facts. Herman's response was coded as 'pulling back', because of the consistent production of the correct answers by the learners, while Herman repeatedly asking them to use the structured number lines. The way in which the number line was enacted seemed to be not responsive to the current levels of learners' knowledge in terms of thinking about progression in teaching. At the planning level, there is already a potential mismatch between knowing expanded digit values into the 10000 range, and yet needing a number line to jump in 1s/10s/100s/ etc. So the task, its representation in expanded format, and several number lines as a tool appeared to be problematic. So there are problem that can be ascribed to planning level of representation selections (KQ - transformation category), as well as working against sophistication responsively and hence coded as 'pulling back'.

In his 2014 teaching, Herman demonstrated a different pattern of response to progression in the use of the number line. Excerpt 18 illustrates this practice, in the context of Herman response to learners' solution actions in working out addition sums on a number line.

Excerpt 18

191 T: Let's go back to this sum here (*teacher points at 179+32, already done on the board by making 3 jumps of 10s and a jump of 2*), where we said one hundred and seventy-nine plus thirty-two. And it was done as ten, ten, ten plus two, which was fine isn't it?

192 C: Yes

193 T: We are now in Grade 6 isn't it?

194 C: Yes

195 T: Do we really need to add ten, ten, ten?

196 C: No

197 T: We can add like thirty very easily, isn't it?

198 C: Yes

201 T: and even thirty-two at once easily isn't it?

202 C: Yes

203 T: So I would like to see you mixing some of this to get to the answer as quickly as possible to show that you are clever right.

204 C: yes

While Herman started his lesson with a smaller number range below Grade 6 class level, the number range was extended in the subsequent examples in the lesson. Herman's response to the learner who made jumps of three 10s was coded as offering a more efficient strategy because he encouraged learners to make bigger jumps. While this was a low level sophistication response, my focus here is about how he responded to learners' use of the number line in contrast to what is seen in the 2013 incident. While there, he insists on seeing the count in units/ tens/ hundreds, etc in the context of correct offers by recall, here he encourages jumping in multiples of ten rather than accepting the offer involving jumps of ten. The number line as tool for supporting strategic efficiency is therefore used for advancing offers in responsive ways in 2014.

6.7.1 Discussion of Herman's pattern of shifts across 2013 and 2014 teaching

Key differences between Herman's 2013 and 2014 sophistication elaborations from the excerpts presented related to both the use of the representation and counting strategy. In terms of the representation, Herman moved from structured to empty number line representation. The structured number line was constituted in measurement situations and so was associated with rigid fixed distances. Gravemeijer (1994) argued that the use of the structured number line caused counting to be a passive reading of the answer on the number line, which did not raise the level of the strategies the learners used to solve a problem. This criticism of structured number line led to the origin of the empty number line in the Dutch Realistic Mathematics Education movement (RME). The empty number line allows learners to draw marks for themselves instead of the fixed marks in structured number lines.

In terms of strategy, Herman's response in 2013 is constituted by pulling learner's back to the reading of answers on the number line, while in 2014 Herman encourages more progressive moves in counting strategy by encouraging and challenging learners to make group and flexible jumps on a number line. Though Herman's response to sophistication in 2014 was at the lower level in terms of the notion of 'quality'. Despite this response being at the lower level, it is a move forward for Herman in acknowledging the need for quicker ways to get to the answer in comparison to insisting on showing answers on a structured number line, in a situation where learners can produce correct answers mentally.

6.7.2 Findings from the VSR interviews with Herman

In this section, I present findings from the interim VSR interview conducted between 2013 and 2014 lesson observations. The VSR interview with Herman focused primarily on his thoughts and decisions relating to progression in his 2013 teaching. I began the interview by probing the teacher to articulate his objectives for the lesson.

Researcher: Tell me what you were hoping that learners would be able to do or understand at the end of the lesson?

Herman: I expect my learners to be able to add and subtract mixed fractions. Right, so that was the main objective of that particular lesson. That they should be able to add and subtract mixed fractions.

Researcher: To begin at an overview level, do you think that you achieved what you intended for the lesson?

Herman: uhmm! (Takes a deep breath), partially, though, I think there were a lot of errors, sorry, I mean a lot of following rules, but not really explaining to the children what it means. As I watched the video, I think the way I tackle the first example, where I illustrate the fractions by drawing diagrams, it was a better way of progressing in that lesson. So that children are actually seeing what is happening, but then I see that I then shifted from that particular approach of using diagram to stating rules

Researcher: Tell me a little bit about these rules that you noticed in your teaching as problematic?

Herman: Well, like whereby when I said if you have got mixed fractions, first of all add the whole numbers; then the next step, find the lowest common multiple (the LCM), then the next step say, four into the LCM multiply by that one and then add the numerator and then eventually find the answer. These are just steps, which you have to follow. But during the initial stages of teaching fractions, I believe there should be a lot of diagrams that children will be seeing than using these rules containing steps. The steps will then follow when kids have concretized the fractional situations, they now know what fractions means.

Herman's response indicated a transition from technical description about students to deliberate reflection about self and practice *(... not really explaining to the children what it means; ... but then I shifted from that particular approach of using diagram to stating rules*). Herman utterances indicated privileging the use of diagrams as a "better way of progressing in the lesson" over stating rules. He noted possible ways to address this weakness through deliberate planning and action involving using diagrams, while also noting limitations in the sequencing within his lesson by moving so quickly into sets of rules without working through many examples using diagrams to model fractions situations:

Herman: Because I believe that diagrams would have enabled children to concretize what is really happening in adding and subtracting fractions. So I want to believe that the best way would have been to have more examples of adding and subtracting of fractions using diagrams, so that learners would got to understand what would follow as set of rules

Herman statement above indicates awareness of the value of diagram in concretizing fraction problems. When probed further on incidents where learners offered incorrect answers that were largely ignored, Herman made the following comment:

Researcher: Do you think it is important to deal with incorrect answers or we just need to focus attention to correct answers?

Herman: I think dealing with incorrect answers is important. As you ask the student why he gave that particular answer you can be able to trace where he got lost. But in the video, I am surprised that I just brushed him aside it was a mistake because you have to find out. I had to ask the student why he gave that particular answer so that you can actually find where the child has gone wrong or where the child… because it could be the student is losing everything, so in tackling the wrong answers as well and finding why he gave that particular answer makes it easier for the teacher to find corrective measures to make the student understand.

Herman’s utterances indicate acknowledgement of his ignoring, rather than probing, of incorrect offers. In his articulation of the consequences of ignoring incorrect learner’s offer and providing a rationale for why attending to incorrect offers is important in practice there is an expression denoting transition from deliberate reflection to critical reflection about self and practice in relation to handling learner’s incorrect offering. However, later in the interview when I asked Herman to describe a ‘good’ mathematics teacher, he stated that handling learners’ incorrect offers should not deviate the teacher from planned objectives of the lesson, suggesting a subsequent, individualised alternative instead:

Herman: … I also believe that a good mathematics teacher should be somebody who is tolerant, because you find that in teaching students will give you wrong answers that will totally frustrate you. Or that will totally lead you off from your objective of the lesson.

Researcher: Tell me what your response would be, when learner’s incorrect answer lead you off from the objective of your lesson?

Herman: Probably as a teacher you will ask the students maybe to come see you later or you go see the student later because I don't believe that you should deviate away from or leave away from the objective of the lesson because your objective is carrying almost like forty students and if you are thrown off the objective and then you tackle one incorrect answer, it means you would have lost the other 39 or 40 students following the same lesson. My believe is that objective of the lesson should be followed to the book by all, because then at the end of the lesson you are able to measure and say well the objective of my lesson was this and I have achieve it or not. If you do not achieve it, then what were my short falls? Where did I go wrong? Because I believe that a successful lesson should have a lot of the students understand what is intended by the end of the lesson.

More broadly, Herman's belief that lesson planning should be rigidly followed with no flexibility indicated lack of readiness to find a balance between planning-oriented action and improvisation oriented action. Knowledge of the importance of following up errors therefore sits at odds with beliefs about following the plan. Literature pointing to a link between knowledge and beliefs about mathematics teaching, determining teaching practice and teacher behaviour in the mathematics classroom (Fennema & Franke, 1992), suggests that interrogating learners' incorrect offering is unlikely to be implemented while teaching in Herman's case. Griffin (2004) attributes such beliefs to teachers' own learning experiences, and its implication is an ongoing tendency to treat mathematical ideas as 'disembodied' entities. Herman's belief that focusing instruction rigidly on lesson plan is important, without necessarily being responsive to in-the-moment situations, echoes this belief.

At the end of the interview, Herman made the following reflection of what had become explicit for him during our interactions:

Herman: First of all, I didn't know that when one is teaching there are mistakes which you make without even noticing and this interview involving watching your own lesson and followed by discussion, which we have done I believe it's an eye opener for me because I have seen mistakes that I didn't notice when I was teaching. It enriches the teacher as well, such discussions enriches you in a

> sense that you become a better teacher because you avoid the mistakes that you were making when you were teaching.

Herman's final remarks indicated potential for on-going moves from unconscious incompetence to conscious incompetence level of awareness through noticing of limitations in his actions while teaching and *deliberate* reflection about *self and practice*.

6.7.3 Linking Herman's practice and reflective awareness

Analysis of Herman's teaching showed differences in extent of elaboration within sophistication and extent and breadth within individuation/collectivization situations, and very limited positive shifts at the level of quality within sophistication and collectivisation situations. The VSR interviews indicated shifts in state of awareness from unconscious incompetence to conscious incompetence and transition from technical to deliberate reflections about self and practice.

The findings of shifts in Herman's teaching practice and shifts in level of awareness and reflection on teaching were connected in some ways. Firstly, the observed shifts in extent and breadth of elaborations is associated with Herman's shifts in reflective awareness of his teaching action by noting limitations of his teaching and he indicated the use of diagram as an alternative strategy that he values, with the use of diagrams seen in his 2014 teaching. Secondly, the lack of shift in quality of elaborations is interesting in relation to Herman's belief as manifested in the VSR interview that teaching should not deviate from the objective of the lesson. Hill et al. (2008) finding that association between mathematical knowledge and teaching quality is mediated by teacher beliefs about teaching of mathematics and belief about how learners learn mathematics is also of interest given Herman's high scores on the WMC-P course test, and relatively low levels of responsive teaching and limited change across 2013 and 2014.

In conclusion, Herman case suggests that developing teacher knowledge is necessary, but not sufficient for changing classroom practice without taking into cognisance the teacher beliefs about mathematics teaching and learning. His dataset emphasises that making direct links between developing teacher knowledge and classroom practice/shifts in classroom practice is complex. Calderhead (1996) emphasises the importance of making a distinction between teacher's beliefs and knowledge as a vital component of understanding changes in teaching actions. Therefore after gaining knowledge of propositions, teachers retain the option to accept or not accept them and put them into their classroom practices. Herman's case suggests that beliefs about coverage as planned largely overrode his awareness of the need to teach responsively, but in spite of this, the quality of responses focused on sophistication did substantially improve.

6.8 Synthesis across the four case study teachers

In Table 22 I provide a summary noting where there are increases in incidence in relation to *extent*, *breadth* and *quality* within each situation of elaboration by the four teachers.

Table 23: Summary of shifts in elaborations by the four teachers across 2013 and 2014

	Breakdown			**Sophistication**		**Individuation/ Collectivization**		
Teachers	**Extent**	**Breadth**	**Quality**	**Extent**	**Quality**	**Extent**	**Breadth**	**Quality**
Thandi	√	√	√	√	√	√	√	√
Sam	√	X	√	√	√	√	√	X
Bongi	√	X	√	√	X	√	√	X
Herman	X	X	X	√	√	√	√	X

Overall the results show that while all teachers did show positive shifts in terms of elaboration framework, the, patterns of elaboration and of changes in elaborated responses are very different for the four teachers. Across the four teachers collectively, there were very limited changes in the area of *breadth* (within non-hierarchical sub-types) in breakdown situations, and *quality* (within hierarchical sub-types) in individuation/collectivisation situations. Also, the fact that there were more comments across the four teachers on expanding task and example spaces in the VSR interviews, suggests greater reliance on the transformation knowledge category of the KQ. Intended changes at the transformation level of task and example spaces are linked though with evidence of the changes in responsive teaching seen in 2014 lessons (i.e. in contingency part of KQ). What this finding suggests is that the development of transformation and connection knowledge is necessary for supporting changes in responsive teaching.

Another interesting finding across the four teachers was the unique nature of Herman's data in relation to the other three teachers. Herman was the strongest among the four teachers in our measures of both mathematics conceptual knowledge and PCK; however, his teaching data suggest very limited shifts in responsive teaching. This was attributed to his beliefs about rigid adherence to plan-oriented actions in his teaching, as communicated in his VSR interview. He stated in the VSR interviews that he preferred to attend to incorrect offers with learners in individual work after teaching sessions. While limitations in Herman's patterns of elaboration across the two years provide further backing for prior evidence noting that while strong foundations in content knowledge are necessary, they do not provide any guarantees for the possibility of either quality of planned or contingent teaching, in this study, his dataset indicates that a similar claim can be made in relation to transformation and connection knowledge: that while knowledge in these dimensions is necessary, this knowledge too may not be sufficient for contingent actions in the classroom.

6.9 Conclusion

In this chapter I have presented analysis and findings of the pattern of shifts in the teachers' elaborations across 2013 and 2014 teaching in relation to *extent*, *breadth* and *quality* of elaborations. The findings from this analysis indicated that all the four teachers have shifted in their responses to learners offer in at least one or more situations of elaborations. Findings from VSR interviews indicated a change in state of awareness by the four teachers except Bongi from either unconscious incompetence to conscious incompetence, indicating awareness of limitations in their teaching practice. Shifts are also observed by the four teachers in their level and object of reflection with move towards critical reflection on self, practice and students. It was noted in this chapter that findings from VSR are associated with shifts in teaching practice.

CHAPTER 7

CONCLUSION

7.1 Introduction

Responsive teaching seen through the lens of 'elaboration' in the context of classroom interactions comprised of three in-the-moment situations (breakdown, sophistication and individuation/collectivisation) enabled me to gain an understanding of an important aspect of the quality of mathematics teaching and teaching development. The notion of elaboration was used as an interpretive lens to examine and characterise teaching actions that are responsive to learner offers during the course of classroom interactions. The position I took on this phenomenon was that these elaborations were drawn from two key bases: a psychological constructivist view of the individual cognizing teacher, drawn from an underpinning knowledge base; and an interactionist view on collective classroom practice (Bauersfeld, 1995) in which the teacher participated in and contributed to the development of collective processes through renegotiation of meaning.

To bring this research to a close, I begin this chapter with a restatement of the main foci of the research with emphasis on the analysis process. I then move on to provide a summary of the key findings emanating from this study, attending to contributions to the knowledge base, and implications for South African primary mathematics teacher development. Of course, all research has its limitations, hence I discuss limitations of this research and possible features that could have improved the research, and note possible directions for further research in the concluding section of this chapter.

7.2 What were the foci of this research?

This study had two main foci, and these were firstly, the identification of stages of implementation of responsive teaching actions in a terrain with substantial contextual evidence of limited teachers' mathematical knowledge, incoherent talk and frequent lack of evaluation criteria in teaching actions noted in the South African literature. Responsive teaching as introduced in Chapter 2 is conceptualised in the international literature in relation to creating opportunities for learning from contingent situations triggered by learner offers rather than initially talking about whether learner offers are acknowledged or not. The contextual problems outlined in this thesis, which I have noted as constraints to openings for more responsive teaching actions led to the motivation for finding interim solutions as stages of implementation towards possibilities for more responsive teaching in this context.

Research has shown that teachers' mathematical knowledge is necessary, and acutely so, in the context of responsive teaching. Therefore, in the context of work that followed the one year long WMCP in-service professional development course, I made selections of four teachers from that course on the basis of relative strengths in their foundational knowledge base. I used a grounded theory approach (Glaser & Strauss, 1967) to characterize the situations in which responses to learner offers were given, and then analysed the nature of these responses. As stated already, the use of grounded theory approach was considered on the basis that the context of 'no evaluation' of learner offers and extensive gaps in teachers' foundational knowledge base meant that existing theories developed in the international terrain provided limited purchase, particularly from the point view of development.

The second focus relates to the first, that is, in the context of evidence of relatively strong mathematical knowledge for teaching and on-going development activity involving interim VSR interviews, I explored possibilities of growth in responsive teaching by the four teachers through analysis of 2013 and 2014 data of classroom teaching. In commenting on these shifts, I recruited three markers or indicators (extent, breadth and quality) driving from the ways in which the categories of responsive teaching were conceptualized in Chapter 5. Shifts are juxtaposed to present the interplays over time of these professional development mechanisms across the four cases of teachers' increasing focus on elaborations.

The main research question that guided this investigation was:

What is the nature and extent of elaboration that teachers provide in responding to 'in-the-moment' situations in primary mathematics classrooms, and how does this change over time in the context of follow-up support (involving video-stimulated recall interviews) after participation in an in-service teacher professional development?

I used three sub-questions based on the three situations of responsive teaching, and one sub-question based on shifts in responsive teaching action to access the extent and nature of elaborated responses to learner offers and possible growth in these responses. Two cycles of video recordings of lessons' observations (from the four teachers) collected over period of two years: 2013 and 2014; and VSR interviews with individual teachers represented the data gathered to answer these four sub-questions: The key findings emanating from this research are summarised in the next section.

7.3 Summary of the key findings

There were three interrelated key findings that emanated from this research.

1. ***Identification of categories that can serve as important stages of implementation towards more responsive teaching***

Through the grounded analysis process discussed in Chapter 4 and presented in Chapter 5, three situations of elaborations in which important interim stages of implementation of responsive teaching are located were identified. In the first instance, situations of learner incorrect offers were identified as the most prevalent triggers of elaborations in mathematics teaching. However, incorrect offers were not the only triggers of elaborations, as the data indicated the presence of emergent responses to correct answers as well. In this research, I identified and described how the teacher's awareness of the need to support increasing sophistication in teaching additive relation strategies could also trigger elaboration. This kind of sophistication response was found to be particularly important in a context where limited access to more sophisticated mathematical strategies has been described as a prevalent teaching practice in South Africa (Ensor et al., 2009). Teachers often accepted or encouraged rudimentary methods, rather than responsively focusing on progression in the context of learners' offers of correct answers. This practice is characterised by acceptance of correct offers without encouraging shifts in learner thinking to more abstract forms in terms of strategy or representation used.

The need to support pedagogic move from chorus offers to assessing individuals features prominently in the literature (e.g. Walshaw & Anthony, 2008) and its frequent absent is also noted in the South African context (e.g. see Hoadley, 2012). I have proposed that thoughtful engagement with individuated responses is a useful mediational tool within moves towards more responsive teaching. Conversely, literature also suggests that developing and sharing individual learners' offers in the collective classroom space provides a useful mechanism for broadening understanding (Borko & Livingston, 1989; Sawyer, 2012) and its frequent absent was also noted in South African context (see for example, Venkat & Naidoo, 2012).

Across the three identified situations that are sources of triggers of elaborations, I further identified two broad categories of teachers' responses. These are: Elaboration provided (EP) and Elaboration not provided (ENP). Elaboration provided involves building and developing on the learner offer with possibilities of openings for learning, while elaboration not provided describe teaching practice that are characterised by ignoring learner offer or accepting the offer and then moving on without any further elaborations.

The common key feature seen from the empirical data set across the three triggers of elaboration was that appropriate response to learner offers required not just mathematical knowledge *per se*, but the teachers' *ways of being* with the mathematical knowledge (Davis & Renert, 2014). This is located in teacher's improvisational capabilities, rather than focusing on planned actions in the classroom. As noted already in the literature, mathematics teaching rarely proceeds according to plan, and in most cases learner offers brings epistemological and pedagogical digression within a lesson. Teacher's awareness and ability to draw upon their mathematical knowledge in context as a 'disciplined improvisation' (Sawyer, 2004) while responding flexibly is necessary in this situation.

As a way of example, I have shown how Thandi in her 2013 lesson rejected a correct offer from a learner whose action suggests interpretation of the task; 25=30-__ as 30-25=. The teachers' awareness and push for sophistication - wanting a 'count on' or 'count back from' based on seeing subtraction as difference, seems to work against her ability to work flexibly and responsively in bringing her mathematical knowledge into context. So while we might want to see sophistication moves, seeing them in inflexible ways may not be helpful. This speaks back to a policy context in South Africa that has pushed the need for sophistication and progression to the point of prescribing some of this, but without attention to flexible responsiveness.

The policy context also pushes for coverage in a relatively tightly prescribed sequence, and some of these pressures may have figured within Herman's insistence on the need to complete his planned task sequence in lessons without divergence to accommodate learner responses. Given these constraints, the finding of positive moves forward for all four teachers in this study is of interest, as are the combination of mechanisms associated with them (a mathematics for teaching course followed by observations and VSR interviews providing openings to study and reflect on this teaching).

The categories developed in this study provide potential staging points for moving teachers' forwards towards flexibility in response to breakdown, sophistication and individuation/collectivisation responses that might be required to move towards the ideals of more responsive teaching that are described in the international literature, and yet remain distant from the realities of South African schooling.

2. ***The categories of elaborations were found to provide a useful language of description as a tool for identifying and developing responsive teaching***

Using the categories that emerged from this research allowed me to understand the nature and extent of teachers' elaboration, with a move towards more responsive teaching actions. The elaboration framework was found to be useful in providing a language of description to talk about responsive mathematics teaching in the South African context, an area that both recent research and recent reforms at the policy level have largely ignored in the press for progression. It was found to be useful also in describing different patterns of shifts across the four cases of teachers that participated in this study as presented in Chapter 6.

Findings from this study indicated improvements in the ways in which the four teachers responded to learner offers in the classroom. It also showed differentiated patterns of responsive teaching across their teaching. Three indicators of shifts were recruited for this analysis: *extent*, *breadth* and *quality* of elaborations. My analysis showed that Thandi, whose primary mathematics conceptual knowledge score in the in-service course was moderate relative to Herman, made more substantial shifts towards responsive teaching than Herman. The setup of hierarchies within situations of the elaboration framework also showed that Herman provided lower quality elaborations than Thandi. In findings like this, I saw confirmation of earlier international research noting that while a strong foundational knowledge base in mathematics is necessary, it does not provide a 'sufficient' base for responsive teaching in classrooms. This point too, is important in the context of prevalence in South Africa of a public rhetoric that is focused solely on primary mathematics teacher knowledge. While this writing is geared towards the acknowledged necessity of a strong content knowledge base, it tends to leave aside the need for marshalling this knowledge sensitively and responsively.

My analysis shows though, that a singular focus on mathematical content knowledge is unlikely to be sufficient. Even where VSR interview data indicated awareness of the need to respond sensitively to incorrect offers, and knowledge of a variety of ways in which this could happen, there were still limitations in enacting this in whole class teaching situations. It is important to note also that the VSR data indicated more bias towards planned actions resting on connection and transformation knowledge of KQ, rather than contingency – even though responsive teaching shifts were seen across 2013 and 2014. This points to limitations associated with the course that VSR data reflects course orientations – and suggests need to pay more attention to need for flexible responsive teaching and beliefs associated with this orientation.

3. ***The Video-stimulated recall interview was found to be potentially useful in revealing unaware aspects of teachers teaching practice that they do not previously considered***

With specific reflection-oriented questions about classroom decisions, the VSR interview was found to be a useful vehicle for supporting teachers in revealing unaware aspects of their teaching practice. This in turn, may potentially enable teachers to become more critical about their actions and decisions in the classroom. By way of example, the VSR interview appeared instrumental in supporting Sam to bring statements of prerequisite skills explicitly to the fore. Her reflection as a result of the follow up post-observation VSR interview indicated ongoing learning beyond the in-service 'maths for teaching' course that she attended and her long prior teaching experience. The evidence shows that Sam did have some of the prerequisite knowledge and skills needed to work with bridging through 10. However, the potential for improved enactment of this knowledge and skills was seen in her making explicit additional tasks for eliciting prerequisite sequences of skills during the interview.

Similarly, through the VSR interview Thandi was able to focus attention on her enacted rather than her intended practice. She specifically made comments about making room to accommodate learners' contributions in the classroom, and not necessarily always imposing methods for them to memorise. This was the pattern in her practice that she noticed through viewing and reviewing the video of her teaching. Like findings in other studies (Geiger et al., 2015; Rosaen et al., 2008), the teachers in this study also found VSR interviews to be a powerful medium for revealing aspects of their teaching practice that they had not previously considered. This suggests the potential of this approach for on-going development in teachers' *ways* of *being* with mathematical knowledge during classroom interactions in improvisational ways. In Schratz's (2006) term, the teachers' state of awareness changed from unconscious incompetence to conscious incompetence, with the push evident in acknowledging limitations in their teaching practice and thinking about alternative approaches in response to these limitations.

7.4 Contributions

While deficit conversation is well-known in the South African literature relating to possibilities for responsive mathematics teaching, this study proposes a starting place for talking differently about mathematics teaching. The notion of elaboration is used as a metaphoric lens through which to reinterpret practice, and as a practical basis for teaching action, and as a means to 'bring into dialogue' some of the ways of thinking about supporting teachers to teach responsively. This research contributes to the dialogue in two ways: firstly, in describing and categorizing potential stages of implementation of responsive teaching (and providing illustrative excerpts of these stages), and secondly, in bringing into light ways to support the development of responsive teaching action in a resource constrained context. In this way, this research had made both theoretical and methodological contributions to the field, and these contributions are largely located in the ‘elaboration’ framework that emerged in the course of the analysis in this study.

7.4.1 Theoretical contribution

In their initial conceptualization of the knowledge quartet framework, Rowland et al. (2005) indicate that the planned and contingent dimensions of the KQ (i.e. transformation, connection and contingency) draw from foundation knowledge as illustrated in Figure 2 (page 40). Building on this work and conceptualizing the notion of responsive teaching within the contingency dimension of the KQ in the South African context, I found empirically that the interrelatedness among the dimensions of the KQ is complex and multifaceted. Findings from this study suggest a somewhat different conceptualization: that teachers draw from foundation knowledge (specifically including the belief aspect), transformation and connection knowledge in contingent situations. The contribution of connection and transformation knowledge in contingent situations has been signposted theoretically by Rowland et al. (2009), but a valuable empirical confirmation is offered in this study.

By way of example, I showed how Sam drew from her horizon content knowledge (Ball et al., 2008) or connection knowledge in her response to the possibilities of solving problems that lead to negative numbers to her grade 2 learners. Sam here draws from her knowledge of what is anticipated as the learners progress through the curriculum. Similarly, I have showed how Thandi juxtaposes counting in 10 with plus 10 in her response to learners’ difficulty with counting sequence. Thandi’s actions drew from a base within her transformation knowledge – using analogies to make concepts comprehensible to others. However, Herman’s ‘possession’ of knowledge relating to connection and transformation as seen in VSR interviews is held in conjunction with beliefs at the foundation knowledge level about a transmission orientation to teaching focused on the need for coverage. Thus, shortcomings in contingency actions can be interpreted as an outcome of tensions between knowledge of mathematics in all three base categories being insufficient for translating into actions in the face of beliefs and orientations to teaching that work against the need for 'in-the-moment' responsiveness relating to contingency.

Herman believes that successful mathematics teaching focuses more on achieving the objectives of the lesson, without deviating from it in responding to learners' incorrect offers. Herman's conception about how best mathematics should be taught appeared more reliant on foundation knowledge, than the other three categories overall. This empirical evidence suggests that all the three dimensions of KQ (foundation, transformation and connection knowledge) are needed in contingent situations.

7.4.2 Methodological contributions

There are two significant methodological contributions that this research has unveiled. Firstly, the study has contributed in characterising responsive teaching actions consisting of three situations or triggers of elaborations (breakdown, sophistication, and individuation/collectivisation) and their accompanying categories through a grounded theory approach that highlighted important stages of implementation of responsive teaching action in the South African context. The categories are well grounded in the context of resource constraints and were clearly defined for others to use perhaps in similar contexts, where class sizes and classroom practices reflect much of the South African terrain, for understanding the possibilities for responsive teaching.

Secondly, the elaboration framework together with VSR interviews can be used as professional development mechanisms to support the development of moves towards more responsive teaching. Findings from this research indicated that developing mathematical knowledge in in-service professional development course is critically needed in the South African context, but translating this knowledge into classroom practice is not a straight forward process. My findings indicated possibilities and limitations of a course focused on foundation, transformation and connection categories of KQ, more than contingency, and I noted that talking about teaching (espoused practice), while perhaps reflecting awareness and maybe even intentions for teaching, provides few guarantees for enacted practice. This research indicates both the need and the usefulness of following-up continuous professional development into teachers' classroom practice. In Figure 24, I propose a possible methodological approach that can be used to support teachers' development of responsive teaching action.

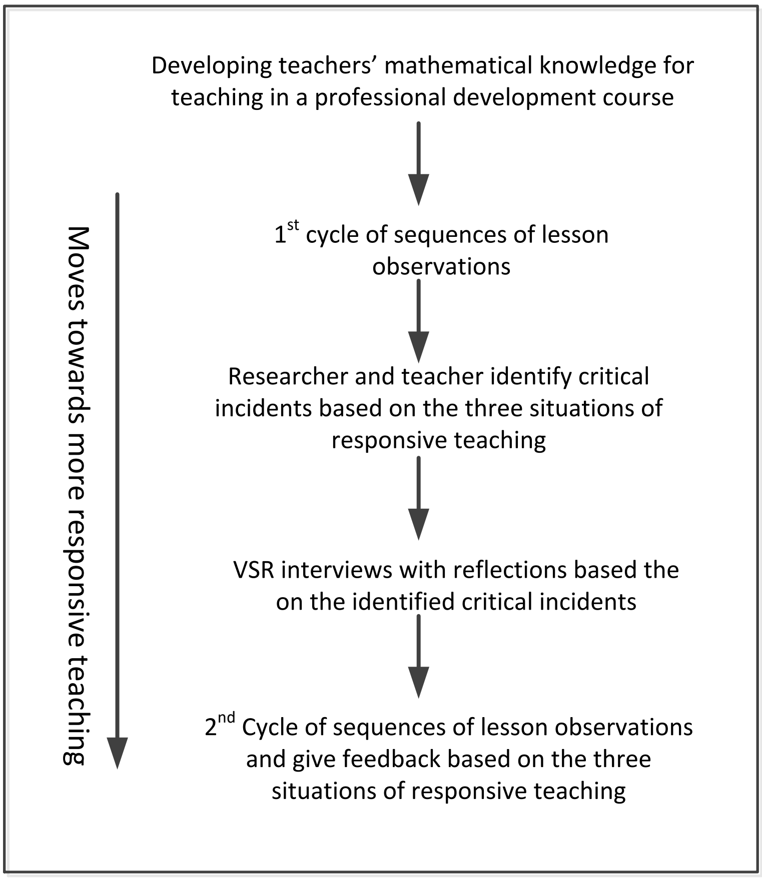


Figure 24: A possible methodology for studying and developing more responsive teaching

7.5 Implications of this research for policy on primary mathematics teacher professional development

Firstly, the findings of this research speak to South Africa's policy on professional development in two main ways. The first is the nature of teacher professional development initiatives. Evidence from this research strongly points to classroom observations showing that practice does not follow in any direct way either from knowledge or even from the practices that teachers espouse. This finding has strong support in prior evidence (H. Chick & Stacey, 2013; Coles & Scott, 2015; Davis & Renert, 2014) and provides grounds for a shift to including follow-up in classroom related support alongside or, at the end of professional development focusing on developing teacher knowledge (both SMK and PCK). The persistence of short-term in-service workshops in the South African terrain therefore, lacks a basis in evidence with uncertainty on how they feed into classroom practice.

Secondly, the findings of this study might be useful in extending professional development in ways that attend to classroom practice following participation in PD focused on developing mathematics knowledge for teaching. In doing so, the categories that emerged from these study can be put into practical use in the follow-up activities, with the categories open to use for considerations about planning for the unexpected in the classroom. For example, being ready for predictable errors and misconceptions is viewed as part of teachers' specialized content knowledge (Ball, et al., 2008). Ball and Bass (2000) add that teachers can even prepare for unpredictable uncertainties, which they can 'know in the context of the problems they have solved' (p. 90). Given that these categories were developed from the same context in which the teachers work, there are strong possibilities that they will encounter these or similar triggers of elaborations in the future and being aware of them can assist teachers to respond to learners' offers in more productive ways.

7.6 Limitations of the study

I began this study with a focus on understanding the KQ in the context of classroom teaching situations in South Africa. The study came to focus more on contingency situations and responsive teaching based on evidence of interesting changes in this aspect and key differences between the four case study teachers. But the interim data based on VSR interviews that were constructed on the basis of KQ categories provided key insights into the relationship between teachers' mathematical knowledge base seen via the KQ categories and the contingency elaborations seen in classroom practices. As such, in the VSR interviews there were conversations across all the four situations on the KQ framework. Looking back, it could have been more productive for this study to structure the VSR interviews on contingency situations, and in particular, on the three in-the-moment dimensions of responsive teaching identified in this study. In doing so, more focus would have been on how teachers responded to learner offers in the classroom, rather than the broader insights that I generated. I do not know whether structuring the VSR interview based on these categories, would have resulted in the likelihood of more substantial shifts in responsive teaching. However, at the time of the interview, the categories were not clearly conceptualised by the researcher. The broader frame though, did allow for more attention to the ways in which the planned elements of the KQ interacted with the contingency element.

Another limitation of this study relates to the features of the proposed 'elaboration' framework. Neither empirical nor theoretical considerations have pointed to a hierarchy of task-based elaboration. However, this could be a limitation of the proposed elaboration framework, in the sense that the framework does not distinguish between lower and higher level tasks. Literature suggests that 'rich tasks' are better for promoting rich task-based responses from learners and teachers. Inclusion of attention to this feature may be more likely to allow a researcher to see hierarchy in task-based responses.

7.7 Directions for further research

The three triggers of elaborations that I propose as stages of implementation towards more responsive teaching in this thesis are empirically-determined and grounded in the context. I have examined and analysed mathematics teaching on the basis of the three situations. However, it is sensible to ask whether additional data might come to light in the future that might refine, extend or inform other triggers of elaborations. As such there is need for research to explore these issues through the following questions:

1. How might sophistication elaborations be flexibly applied in the context of correct and incorrect learner offers?
2. In breakdown situations, do learner-focused responses suggest a closer engagement with learner incorrect offer than task-focused responses? If so, what is the consequence of this in relation to responsive teaching?
3. What are the consequences of hierarchies within situations of elaboration in relation to overall quality of mathematics lesson?

7.8 Self reflection

This research journey has made significant contributions to my growth as a lecturer in teacher education, and as a researcher. Professional development of primary mathematics teachers has been the focus of my research journey, as I tried to look into the persistent challenges that primary school teachers faced while teaching mathematics. With restructuring of the Nigerian basic education system in 1999, I was actively involved between 2006 and 2011 in the professional development activities of primary school mathematics teachers. This drive led me to taking up a position as a doctoral fellow at Wits University in South Africa within Wits Maths Connect – Primary (WMC-P) project. The duality of the project aims; *research* and *development* fit into my broader vision of research as useful in its own right, but powerful for supporting the development of primary mathematics teaching and learning in schools. In the WMC-P, I was privileged to engage in professional development activities with primary school mathematics teachers. I came to realise that there are very similar challenges that primary mathematics teachers faced in both countries, and perhaps in other similar developing nations. These challenges range from knowledge gaps and a frequent lack of productive discourses during classroom interactions that can open up greater opportunities for learning.

As a researcher, this journey emphasised the importance of thorough and unambiguous operationalisation when talking and describing teaching and teacher development. As a lecturer in teacher education, it invoked the need to translate experiences gained in working with in-service teachers into development of pre-service teacher education in South Africa and Nigeria. Reflecting the advice given by both Ensor and Hoadley (2004), and Schweisfurth (2011) relating to be cautious in importing lenses from developed to developing nations, I came to realise the danger of deficit conversations from the point of view of development, and rather, as suggested by Schweisfurth (2011) to think about stages of implementation.

This drive led me to a grounded theory approach to the characterization of categories of mathematics teaching that are responsive to learners' offers, which can then be organised based on literature and theory into stages of implementation that work towards more responsive teaching as described in developed nations, but attuned to the realities of a developing country context. I found it useful to reflect on the process that I engaged in for the while doing grounded theory. Glaser (2010) outlines three characteristics to be considered for researchers interested in grounded theory approach to data analysis.

> The grounded theory researcher must have three important characteristics: an ability to conceptualize data, an ability to tolerate some confusion, and an ability to tolerate confusion's attendant regression. These attributes are necessary because they enable the researcher to wait for the conceptual sense making to emerge from the data. This is just a fact (p. 4).

I strongly concur with the author's point that '*this is just a fact*' of what it means to do grounded theory. My metaphor for this was of a 'toothbrush' approach: consider a carpeted room 900 square meters in size that you are asked to clean with a toothbrush. The process is frustratingly slow, but doing so allows you to understand the characteristics and to 'see' patterns in the make-up of the ground. In the same way, grounded theory researchers work to explain emergent questions and patterns drawn from the data – what Glaser described as the 'ability to conceptualise data' and also the 'ability to tolerate some confusion'.

In this engagement I began to notice a pattern in teachers' responses that led me to a categorization of triggers of elaborations. It started with looking at learners' incorrect offer, what I described as a 'breakdown'. Along the way, I came to realise that it was not only in cases of breakdown that the teacher provided mathematically oriented responses to classroom interactions, leading to the additional categories, and subsequently, with literature, hierarchies. Further empirical data is likely to lead to refinement of the 'elaboration framework', which I would like to take up in my post-doctoral research work.

Finally, as I engaged closely with classroom data of teaching, I became more aware of the complexity of the work of teaching, particularly teaching that is responsive to learner offers. How we support learners to be effective and efficient in learning mathematics became centrally important in this context. How I, as a teacher educator, support teachers in order for them to support quality mathematics teaching has been the key motivation for this study. I believe that the process and the outcomes of this study have made a small contribution to this endeavour, while speaking back also to the ways in which contingency situations have generally been described in the international literature.

References

Adler, J., & Venkat, H. (2014). Teachers' mathematical discourse in instruction: Focus on examples and explanations. In H. Venkat, M. Rollnick, J. Loughran & M. Askew (Eds.), *Exploring mathematics and science teachers' knowledge: Windows into teacher thinking* (pp. 132-146). Abingdon, Oxon: Routledge.

Ally, N., & Christiansen, I. M. (2013). Opportunities to develop mathematical proficiency in Grade 6 mathematics classrooms in KwaZulu-Natal. *Perspectives in Education, 31*(3), 106-121.

Anghileri, J. (2000). *Teaching number sense*. London: Continuum.

Anghileri, J. (2006). *Teaching Number Sense*. London: Continuum International Publishing Group.

Annells, M. (1996). Grounded theory method: Philosophical perspectives, paradigm of inquiry, and postmodernism. *Qualitative Health Research*, *6*(3), 379-393.

Askew, M., Brown, M., Rhodes, V., William, D., & Johnson, D. (1997). Effective teaching of numeracy: *Report of a study carried out for the teacher training agency*. London: King's College, London.

Askew, M., Venkat, H., & Mathews, C. (2012). Coherence and consistency in South African primary mathematics lessons. In T. Y. Tso (Ed.), *Proceedings of the 36th Conference of the International Group for the psychology of mathematics Education* (Vol. 2, pp. 27-34). Taipei, Taiwan: PME.

Askew, M., & Wiliam, D. (1995). *Recent Research in mathematics Education*. London: HMSO.

Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, *29*(1), 14-17, 20-22, 43-46.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content Knowledge for Teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407. doi: 10.1177/0022487108324554

Bauersfeld, H. (1980). Hidden dimensions in the so-called reality of a mathematics classroom. *Educational Studies in Mathematics, 11*(1), 23-41.

Bauersfeld, H. (1995). Development and function of mathematizing as a social practice. In L. P. Steffe & J. Gale (Eds.), *Constructivism in education* (pp. 137- 158). Hillsdale, NJ: Lawrence Erlbaum.

Beishuizen, M. (1993). Mental Strategies and Materials or Models for Addition and Subtraction up to 100 in Dutch Second Grades. *Journal for Research in Mathematics Education, 24*(4), 294-323. doi: 10.2307/749464

Beishuizen, M., Van Putten, C. M., & Van Mulken, F. (1997). Mental arithmetic and strategy use with indirect number problems up to one hundred. *Learning and Instruction, 7*(1), 87-106. doi: http://dx.doi.org/10.1016/S0959-4752(96)00012-6

Bernstein, B. (1986). On pedagogic discourse. In J. G. Richardson (Ed.), *Handbook of theory and research for the sociology of education*. New York: Greenwood Press.

Bernstein, B. (1990). *The structuring of pedagogic discourse: Volume IV, Class, codes and control* (Vol. 4). London: Routledge.

Blumer, H. (1969). *Symbolic interacrionism*. Englewood Cliffs. NJ: Prentice Hall.

Bogdan, R., & Biklen, S. (1982). *Qualitative research for education: An introduction to theory and practice*. New York: Alien and Bacon, Inc.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 194-222.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and teacher education, 24*(2), 417-436.

Borko, H., & Livingston, C. (1989). Cognition and Improvisation: Differences in Mathematics Instruction by Expert and Novice Teachers. *American Educational Research Journal, 26*(4), 473-498. doi: 10.3102/00028312026004473

Brodie, K. (2007). Dialogue in mathematics classrooms: beyond question-and-answer methods. *Pythagoras*(66), 3-13.

Brophy, J. E. (1999). *Teaching*. Geneva, Switzerland: International Academy of Education and the International Bureau of Education.

Brown, G., & Wragg, E. C. (1993). *Questioning*. London: Routledge.

Bruner, J. (1974). *'Representation in childhood', in beyong the information given*. London: Gorge Allen & Unwin.

Calderhead, J. (1996). Teachers; beliefs and knowledge. In D. C. Berliner & R. C. Calfee (Eds.), *Handbook of Educational Psychology* (pp. 709-725). New York: Macmillan Library Reference.

Carnoy, M., Chisholm, L., & Chilisa, B. (Eds.). (2012). *The Low Achievement Trap: Comparing Schools in Botswana and South Africa*. Cape Town: Human Sciences Research Council.

Carpenter, T. P., Fennema, E., Franke, M. L., & Levi, L. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth: Heinemann.

Carpenter, T. P., Fennema, E., Peterson, L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.

Carpenter, T. P., Franke, M. L., & Levi, L. (2003). *Thinking mathematically: Integrating arithmetic and algebra in elementary school*. Portsmouth: ERIC.

Carpenter, T. P., & Moser, J. M. (1984). The Acquisition of Addition and Subtraction Concepts in Grades One through Three. *Journal for Research in Mathematics Education, 15*(3), 179-202. doi: 10.2307/748348

Chick, H., & Stacey, K. (2013). Teachers of mathematics as problem-solving applied mathematicians. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 121-136.

Chick, J. K. (1996). Safe-talk: Collusion in apartheid education. In H. Coleman (Ed.), *Society and the language classroom* (pp. 21-39). Cambridge: Cambridge University Press.

Clark-Wilson, A., & Noss, R. (2015). Hiccups within technology mediated lessons: a catalyst for mathematics teachers' epistemological development. *Research in Mathematics Education, 17*(2), 92-109. doi: 10.1080/14794802.2015.1046476

Cobb, P. (1989). Experiential, Cognitive, and Anthropological Perspectives in Mathematics Education. *For the Learning of Mathematics*, *9*(2), 32-42. doi: 10.2307/40247953

Cobb, P. (1995). Cultural Tools and Mathematical Learning: A Case Study. *Journal for Research in Mathematics Education*, *26*(4), 362-385. doi: 10.2307/749480

Cobb, P., Boufi, A., McClain, K., & Whitenack, J. (1997). Reflective Discourse and Collective Reflection. *Journal for Research in Mathematics Education, 28*(3), 258-277. doi: 10.2307/749781

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2010). Participating in classroom mathematical practices *A journey in mathematics education research* (pp. 117-163): Springer.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking, and classroom practice. *Contexts for learning: Sociocultural dynamics in children's development*, 91-119.

Cobb, P., Yackel, E., & Wood, T. (1992). Interaction and Learning in Mathematics Classroom Situations. *Educational Studies in Mathematics, 23*(1), 99-122. doi: 10.2307/3482604

Cobb, P., Yackel, E., & Wood, T. (1993). Learning mathematics: Multiple perspectives, theoretical orientation. In T. Wood, P. Cobb, E. Yackel & D. Dillon (Eds.), *Rethinking elementary school mathematics: Insights and issues. Journal for Research in Mathematics Education Monograph Series, Number 6* (pp. 21-32). Reston, VA: National Council of Teachers of Mathematics.

Coles, A. (2014). Mathematics teachers learning with video: the role, for the didactician, of a heightened listening. *ZDM, 46*(2), 267-278.

Coles, A., & Scott, H. (2015). Planning for the unexpected in the mathematics classroom: an account of teacher and student change. *Research in Mathematics Education, 17*(2), 128-147. doi: 10.1080/14794802.2015.1047787

Corbin, J. M., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology, 13*(1), 3-21. doi: 10.1007/bf00988593

Creswell, J. W. (2007). *Qualitative Inquiry and Research Design: Choising among five approaches* (2nd ed.). thousand Oaks,CA: Sage.

Cutcliffe, J. R. (2000). Methodological issues in grounded theory. *Journal of advanced nursing, 31*(6), 1476-1484.

Davis, B., & Renert, M. (2014). *The mathematics teachers know: Profound understanding of emergent mathematics*. New York, NY: Taylor & Francis.

Drews, D. (2007). Do resources matter in primary mathematics teaching and learning? In D. Drews & A. Hansen (Eds.), *Using Resources to Support Mathematical Thinking, Primary and Early Years*. London: Learning Matters Ltd.

Edwards, D., & Mercer, N. (1987). Common knowledge: The growth of understanding in the classroom. *London: Methuen*.

Ekdahl, A.-L., & Runesson, U. (2015). *Teachers' responses to incorrect answers on missing number problems in South Africa.* Paper presented at the ICMI

Study 23 Primary mathematics study on whole numbers, Macau, China, 431-439.

Ensor, P., & Hoadley, U. K. (2004). Developing languages of description to research pedagogy. *Journal of Education, 32,* 81-104.

Ensor, P., Hoadley, U. K., Jacklin, H., Kuhne, C., Schmitt, E., Lombard, A., & van den Heuvel-Panhuizen, M. (2009). Specialising Pedagogic text and time in Foundation Phase numeracy classrooms. *Journal of Education* (47), 5-29.

Erickson, F. (1982). Classroom discourse as improvisation: Relationship between academic task structure and social participation structure in lessons. In L. C. Wilkinson (Ed.), *Communicating in the classroom*. New York: Academic Press.

Etmer, P. A. (2001). Responsive instructional design: scaffolding the adoption and change process. *Educational Technology, 41*(6), 33-38.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York, NY: Macmillan.

Fennema, E., Franke, M. L., Carpenter, T. P., & Carey, D. A. (1993). Using children’s mathematical knowledge in instruction. *American Educational Research Journal, 30*(3), 555-583.

Flynbjerg, B. (2006). Five Misconceptions about case-study research. *Qualitative Inquiry, 12*(2), 219-245.

Forman, E., & Ansell, E. (2002). The multiple voices of a mathematics classroom community *Learning Discourse* (pp. 115-142): Springer.

Franke, M. L., & Kazemi, E. (2001). Teaching as learning within a community of practice: Characterizing generative growth. *Beyond classical pedagogy: Teaching elementary school mathematics*, 47-74.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. *Second handbook of research on mathematics teaching and learning, 1*, 225-256.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Freudenthal, H. (1973). *Mathematics as an educational task*. Dordrecht: Reidel.

Fuson, K. C. (1992). Research on whole number addition and subtraction. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 243-275). New York: Macmillan.

Garet, M. S., Porter, A. C., Desimone, L., Birman, B. F., & Yoon, K. S. (2001). What Makes Professional Development Effective? Results From a National Sample of Teachers. *American Educational Research Journal, 38*(4), 915-945. doi: 10.3102/00028312038004915

Geiger, V., Muir, T., & Lamb, J. (2015). Video-stimulated recall as a catalyst for teacher professional learning. *Journal of Mathematics Teacher Education*, 1-19.

Glaser, B. G. (1978). Theoretical sensitivity: Advances in the methodology of grounded theory (Vol. 2). Mill Valley: CA: Sociology Press.

Glaser, B. G. (1992). Basics of grounded theory: Emergence vs. forcing. *Mill Valley, CA.*

Glaser, B. G. (2010). The Future of Grounded Theory. *The Grounded Theory Review, 9*(2), 1-14.

Glaser, B. G., & Strauss, A. (1967). *The Discovery of Grounded Theory: Strategies for Qualitative research*. New York: Aldine de Gruyter.

Gravemeijer, K. (1994). Educational Development and Developmental Research in Mathematics Education. *Journal for Research in Mathematics Education, 25*(5), 443-471. doi: 10.2307/749485

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational Studies in Mathematics, 57*(2), 177-211.

Graven, M. (2012). Teacher education through re-authoring their narratives. In C. Day (Ed.), *The Routledge international handbook of teacher and school development* (pp. 127-138). Abingdon: Routledge.

Griffin, S. (2004). Teaching number sense. *Education Leadership,* 39-42.

Guba, E. G., & Lincoln, Y. S. (1994). Competing paradigms in qualitative research In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (pp. 105-117). Thousand Oaks, CA: Sage.

Halliday, M. A. K., & Hasan, R. (1985). Language, context and text: Aspects of language in a social semiotic perspective. *Victoria: Deakin University Press*.

Hart, K. M., Brown, M., Kuchemann, D., Kerslake, D., Ruddock, G., & McCartney, M. (1981). *Children's understanding of mathematics: 11-16*. London: John Murray.

Haylock, D. (2006). *Mathematics explained for primary teachers*. London: Sage.

Heck, D. J., Banilower, E. R., Weiss, I. R., & Rosenberg, S. L. (2008). Studying the Effects of Professional Development: The Case of the NSF's Local Systemic Change through Teacher Enhancement Initiative. *Journal for Research in Mathematics Education, 39*(2), 113-152.

Hickey, G. (1997). The use of literature in grounded theory. *Nursing Times Research*, *2*(5), 371-378.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D.A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 65-97). New York: Macmillan.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L., & Ball, D. L. (2008). Mathematical Knowledge for Teaching and the Mathematical Quality of Instruction: An Exploratory Study. *Cognition and Instruction, 26*(4), 430-511. doi: 10.1080/07370000802177235

Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of Teachers' Mathematical Knowledge for Teaching on Student Achievement. *American Educational Research Journal, 42*(2), 371-406. doi: 10.3102/00028312042002371

Hoadley, U. K. (2005). *Social class, pedagogy and the specialization of voice in four South African primary schools*. Unpublished PhD thesis, University of Cape Town.

Hoadley, U. K. (2006). Analysing pedagogy: the problem of framing. *Journal of Education*, *40*, 15–34.

Hoadley, U. K. (2012). What do we know about teaching and learning in South African primary schools? *Education as Change, 16*(2), 187-202. doi: 10.1080/16823206.2012.745725

Jackson, P. (1986). *The practice of teaching*. New York: Teachers College Press.

Jacobs, V. R., Franke, M. L., Carpenter, T. P., Linda, L., & Dan, B. (2007). Professional Development Focused on Children's Algebraic Reasoning in Elementary School. *Journal for Research in Mathematics Education, 38*(3), 258-288.

Jaworski, B. (2004). *Grappling with complexity: Co-learning in inquiry communities in mathematics teaching development.* Paper presented at the Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (pp. 17-36), Bergen, Norway.

Joyce, B. R., & Showers, B. (2002). *Student achievement through staff development* (3rd ed.). USA: ASCD.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, 59-80.

Kendall, J. (1999). Axial coding and the grounded theory controversy. *Western journal of nursing research, 21*(6), 743-757.

Klein, A. S., Beishuizen, M., & Treffers, A. (1998). The Empty Number Line in Dutch Second Grades: Realistic versus Gradual Program Design. *Journal* *for Research in Mathematics Education, 29*(4), 443-464. doi: 10.2307/749861

Knapp, M. S. (1995). *Teaching for Meaning in High-Poverty Classrooms*. New York NY: Teachers College Press.

Koshy, V. (2000). Children's mistakes and misconceptions. In V. Koshy (Ed.), *Mathematics for Primary Teachers*. London: Routledge.

Krauss, S., Baumert, J., & Blum, W. (2008). Secondary mathematics teachers' pedagogical content knowledge and content knowledge: validation of the COACTIV constructs. *ZDM Mathematics Education, 40*, 873-892.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Lampert, M., & Ball, D. L. (1999). Aligning teacher education with contemporary K–12 reform visions. In G. Sykes & L. Darling-Hammond (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 33-53). San Francisco, CA: Jossey Bass.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic Inquiry*. USA: SAGE Publications, Inc.

Lyle, J. (2003). Stimulated Recall: A Report on Its Use in Naturalistic Research. *British Educational Research Journal, 29*(6), 861-878.

Martin, P. Y., & Turner, B. A. (1986). Grounded theory and organizational research. *The Journal of Applied Behavioral Science, 22*(2), 141-157.

Martins, L., Towers, J., & Pirie, S. (2006). Collective mathematical understanding as improvisation. *Mathematical Thinking and Learning, 8*(2), 149-183.

Marton, F., Runesson, U., & Tsui, B. M. (2004). The space of learning. In F. Marton & B. M. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3-40). Mahwah, NJ: Lawrence Erlbaum.

Mason, J. (2015). Responding in-the-moment: learning to prepare for the unexpected. *Research in Mathematics Education*, *17*(2), 110-127. doi: 10.1080/14794802.2015.1031272

Mason, J., & Davis, B. (2013). The Importance of Teachers' Mathematical Awareness for In-the-Moment Pedagogy. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 182-197. doi: 10.1080/14926156.2013.784830

Mason, J., & Spence, M. (1999). Beyond mere knowledge of mathematics: The importance of knowing-to act in the moment. *Educational Studies in Mathematics, 38*, 135-161.

Mcintosh, A., Reys, J. B., & Reys, R. E. (1992). A proposed framework for Examining Basic Number Sense. *For the leraning of mathematics, 12*(3), 2-8, 44.

Mehan, H. (1979). *Learning lessons*. Cambridge, MA: Harvard University Press.

Mercer, N. (1995). *The Guided Construction of Knowledge: talk amongst teachers and learners*. Clevedon: Multilingual Matters.

Mercer, N. (2002). *Words and minds: How we use language to think together*. London: Routledge.

Merriam, S. B. (1998). *Qualitative research and case study applications in education*. San Fransisco, CA: Jossey-Bass, Inc.

Mertens, D. M. (2005). *Research and evaluation in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods* (2nd ed ed.). Thousand Oaks CA: Sage.

Mitchell, R. B. (2002). A quantitative approach to evaluating international environmental regimes. *Global environmental politics, 2*(4), 58-83.

Molina, M., & Ambrose, R. (2008). From an operational to a relational conception of the equal sign. Thirds graders' developing algebraic thinking. *Focus on Learning Problems in Mathematics, 30*(1), 61-80.

Morine-Dershimer, G. (1978). Planning in Classroom Reality: An In-depth Look. *Educational research quarterly, 3*(4), 83-99.

Muir, T. (2010). *Using Video-Stimulated Recall as a Tool for Reflecting on the Teaching of Mathematics.* Paper presented at the 33rd annual conference of the Mathematics Education Research Group of Australia, (pp. 438-445), Fremantle.

Muir, T., & Beswick, K. (2007). Stimulating Reflection on Practice: Using the Supportive Classroom Reflection Process. *Mathematics Teacher Education and Development, 8*, 74-93.

National Education Evaluation & Development Unit [NEEDU]. (2013). National Report 2012: The state of literacy teaching and learning in the Foundation Phase. Pretoria: National Education Evaluation & Development Unit.

Nystrand, M., & Gamoran, A. (1990). Student Engagement: When Recitation Becomes Conversation. In H. Waxman and H. Walberg (Eds.), Contemporary research on teaching. Berkeley: McCutchan.

O'Connor, M. C. (1998). Language socialization in the mathematics classroom: Discourse practices and mathematical thinking. *Talking mathematics in school: Studies of teaching and learning*, 17-55.

O'Connor, M. C., & Michaels, S. (1996). Shifting participant frameworks: Orchestrating thinking practices in group discussion. *Discourse, learning, and schooling*, 63-103.

Pirie, S. (1996). *Classroom video-recording: When, why and how does it offer a valuable data source for qualitative research?* Paper presented at the Annual Meeting of the North American Chapter of the International Group for Psychology of Mathematics Education, Panama City, Florida.

Planas, N., & Morera, L. (2011). *Revoicing in processes of collective mathematical argumentation among students.* Paper presented at the Proceedings of the seventh Congress of the European society for research in mathematics education, (pp. 1356-1365).

Popkewitz, T. S. (1988). Institutional issues in the study of school mathematics: Curriculum research. *Educational Studies in Mathematics, 19*(2), 221-249.

Potari, D., Zachariades, T., Christou, C., Kyriazis, G., & Pitta-Pantazi, D. (2007). *Teachers' mathematical knowledge and pedagogical practices in the teaching of derivative.* Paper presented at the Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education, (pp. 1955-2004).

Reeves, C., & Muller, J. (2005). Picking up the pace: variation in the structure and organization of learning school mathematics. *Journal of Education, 37*(1), 97-125.

Roberts, N. (2015). *Interpreting children's representations of whole number additive relations in the early grades.* Paper presented at the ICMI Study 23 Primary mathematics study on whole numbers, (pp. 243-250), Macau, China.

Rosaen, C. L., Lundeberg, M., Cooper, M., Fritzen, A., & Terpstra, M. (2008). Noticing Noticing How Does Investigation of Video Records Change How Teachers Reflect on Their Experiences? *Journal of Teacher Education*, *59*(4), 347-360.

Rowland, T. (2005). *The knowledge quartet: A tool for developing mathematics teaching.* Paper presented at the 4th Mediterranean conference on mathematics education, (pp. 69-81), Nicosia.

Rowland, T. (2008). Researching teachers' mathematics disciplinary knowledge. In P. Sulliyan & T. Wood (Eds.), *International handbook of mathematics teacher education: Knowledge and beliefs in mathematics teaching and teaching development* (Vol. 1, pp. 273-298). Rotterdam, the Netherlands: Sense Publishers.

Rowland, T. (2009). *Foundation knowledge for teaching: Contrasting elementary and secondary mathematics.* Paper presented at the Sixth Congress of the European Society for Research in Mathematics Education, (pp. 1841-1850), Lyon, France.

Rowland, T. (2012). Explaining explaining. In S. Nieuwoudt, D. Loubscher & H. Dreyer (Eds.), *Proceedings of Annual congress of the Association of Mathematics Education of South Africa (AMESA)* (Vol. 1, pp. 54-66). Potchefstroom, North West: AMESA.

Rowland, T. (2014). The knowledge quartet: the genesis and application of a framework for analysing mathematics teaching and deepening teachers' mathematics knowledge. *Sisyphus-Journal of Education, 1*(3), 15-43.

Rowland, T., Huckstep, P., & Thwaites, A. (2003). The knowledge Quartet. In J. Williams (Ed.), *Proceedings of the British Society for Research into Learning Mathematics* (Vol. 23, pp. 97-102). Birmingham: BSRLM.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary Teachers' Mathematics Subject Knowledge: the Knowledge Quartet and the Case of Naomi. *Journal of Mathematics Teacher Education,, 3*(8), 255-281.

Rowland, T., Thwaites, A., & Jared, L. (2015). Triggers of contingency in mathematics teaching. *Research in Mathematics Education, 17*(2), 74-91. doi: 10.1080/14794802.2015.1018931

Rowland, T., & Turner, F. (2007). Developing and using the 'knowledge quartet': A framework for the observation of mathematics teaching. *The Mathematics Educator, 10*(1), 107-123.

Rowland, T., Turner, F., Thwaites, A., & Huckstep, P. (2009). *Developing primary mathematics teaching: reflecting on practice with the knowledge quartet*. London: SAGE publications Ltd.

Rowland, T., & Zazkis, R. (2013). Contingency in the Mathematics Classroom: Opportunities Taken and Opportunities Missed. *Canadian Journal of Science, Mathematics and Technology Education, 13*(2), 137-153. doi: 10.1080/14926156.2013.784825

Ryan, J., & McCrae, B. (2006). Assessing Pre-Service Teachers' Mathematics Subject Knowledge. *Mathematics Teacher Education and Development, 7*, 72-89.

Santagata, R. (2009). Designing video-based professional development for mathematics teachers in low-performing schools. *Journal of Teacher Education, 60*(1), 38-51.

Sawyer, R. K. (2004). Creative Teaching: Collaborative Discussion as Disciplined Improvisation. *Educational Researcher, 33*(2), 12-20.

Sawyer, R. K. (2012). *Explaining creativity: The science of human innovation* (2nd ed.). New York: Oxford.

Scardamalia, M., & Bereiter, C. (2006). Knowledge Building: Theory, pedagogy, and technology. In R. K. Sawyer (Ed.), *Cambridge Handbook of the Learning Sciences* (pp. 97-118). NewYork: Cambridge University Press.

Schmittau, J. (2003). Cultural-historical theory and mathematics education. In A. Kozulin, B. Gindis, V. S. Ageyev & S. S. Miller (Eds.), *Vygotsky´s educational theory in cultural context*. Cambridge:: Cambridge University Press.

Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*(1), 1-94.

Schoenfeld, A. H. (2012). Problematizing the didactic triangle. *ZDM, 44*(5), 587-599.

Schollar, E. (2008). Final Report: The primary mathematics research project 2004-2007 - Towards evidence-based educational development in South Africa. Johannesburg: Eric Schollar & Associates.

Schön, D. A. (1987). *Educating the reflective practitioner*. San Francisco, CA: Jossey-Bass.

Schratz, M. (2006). Leading and Learning:Odd Couple'or Powerful Match? *Leading and Managing, 12*(2), 40.

Schweisfurth, M. (2011). Learner-centred education in developing country contexts: From solution to problem? *International Journal of Educational Development*, *31*, 425-432.

Setati, M., & Adler, J. (2000). Between languages and discourses: Language practices in primary multilingual mathematics classrooms in South Africa. *Educational Studies in Mathematics, 43*(3), 243-269.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Silver, E. A., & Stein, M. K. (1996). The Quasar Project The" Revolution of the Possible" in Mathematics Instructional Reform in Urban Middle Schools. *Urban Education, 30*(4), 476-521.

Simon, M. A. (1995). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145. doi: 10.2307/749205

Sinclair, J. M., & Coulthard, M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils*. United Kingdom: Oxford University Press.

Sorto, M., & Sapire, I. (2011). The teaching quality of mathematics lessons in South African schools. *Journal of Education, 51*, 1-22.

Spaull, N. (2013a). Poverty & privilege: Primary school inequality in South Africa. *International Journal of Educational Development, 33*(5), 436-447.

Spaull, N. (2013b). South Africa's Education Crisis: The Quality of Education in South Africa 1995-2011. Centre for Development and Enterprise.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage.

Steffe, L. P. (1983). The teaching experiment methodology in a constructivist research program. In M. Zweng, T. Green, J. Kilpatrick, H. Pollack & M. Suydam (Eds.), *Proceedings of the fourth international congress on mathematical education* (Vol. 1, pp. 469-471). Boston, MA: Birkhäuser.

Steffe, L. P., Richards, J., & Cobb, P. (1983). *Children's counting types: Philosophy, theory, and application*. New York: Praeger.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488.

Taylor, N. (2011). The National School Effectiveness Study (NSES): Summary for the synthesis report. Johannesburg: JET Education Services.

Taylor, N., & Taylor, S. (2013). Teacher knowledge and professional habitus. In N. Taylor, S. Van der Berg & T. Mabogoane (Eds.), *What makes schools effective? Report of the National Schools Effectiveness Study* (pp. 202-232). Cape Town: Pearson Education South Africa.

Thompson, I. (1994). Young Children's Idiosyncratic Written Algorithms for Addition. *Educational Studies in Mathematics, 26*(4), 323-345. doi: 10.2307/3482657

Thompson, I. (2010). From counting to deriving number facts. In I. Thompson (Ed.), *Teaching and learning early number* (pp. 97-109). New York: Open University press.

Turner, F., & Rowland, T. (2011). The Knowledge Quartet as an Organising Framework for Developing and Deepening Teachers' Mathematics Knowledge. In T. Rowland & K. Ruthven (Eds.), (Vol. 50, pp. 195-212). Netherlands: Springer.

Urquhart, C., Lehmann, H., & Myers, M. D. (2010). Putting the 'theory'back into grounded theory: guidelines for grounded theory studies in information systems. *Information systems journal, 20*(4), 357-381.

van Es, E. A. (2012). Examining the development of a teacher learning community: The case of a video club. *Teaching and teacher education, 28*(2), 182-192.

Van Es, E. A., & Sherin, M. G. (2008). Mathematics teachers'"learning to notice" in the context of a video club. *Teaching and teacher education, 24*(2), 244-276.

Venkat, H. (2013). Curriculum development minus teacher development≠ mathematics education. In Z. Davis & S. Jaffer (Eds.), *Proceedings of the 19th Annual National Congress of the Association for Mathematics Education of South Africa* (pp. 27-40). Cape Town: AMESA.

Venkat, H., & Adler, J. (2012). Coherence and connections in teachers' mathematical discourses in instruction: original research. *Pythagoras, 33*(3), 1-8.

Venkat, H., Ekdahl, A.-L., & Runesson, U. (2014). Connections and simultaneity: Analysing South African G3 part-part-whole teaching. In C. Nicol, P. Liljedahl, S. Oesterle & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (Vol. 2, pp. 65-72). Vancouver, Canada: PME.

Venkat, H., & Naidoo, D. (2012). Analyzing coherence for conceptual learning in a Grade 2 numeracy lesson. *Education As Change, 16*(1), 21-33.

Venkat, H., & Spaull, N. (2015). What do we know about primary teachers' mathematical content knowledge in South Africa? An analysis of SACMEQ 2007. *International Journal of Educational Development*(41), 121-130.

Von Glasersfeld, E. (1991). *Radical constructivism in mathematics education*. Dordrecht, The Netherlands: KJu we.

Vosniadou, S., & Verschaffel, L. (2004). Extending the Conceptual Change Approach to Mathematics Learning and Teaching. *Special Issue of Learning and Instruction, 14*(5), 445-451.

Walshaw, M., & Anthony, G. (2008). The Teacher's Role in Classroom Discourse: A Review of Recent Research into Mathematics Classrooms. *Review of Educational Research, 78*(3), 516-551. doi: 10.2307/40071136

Webb, N. M., & Palincsar, A. S. (1996). *Group processes in the classroom*: Prentice Hall International.

Wells, G. (1999). *Dialogic inquiry: Towards a socio-cultural practice and theory of education*. United Kingdom: Cambridge University Press.

Wischgoll, A., Pauli, C., & Reusser, K. (2015). Scaffolding—How can contingency lead to successful learning when dealing with errors? *ZDM, 47*(7), 1147-1159.

Wright, R. J., Martland, J., & Stafford, A. K. (2006). *Early numeracy: Assessment for teaching and intervention*. London: Sage.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 458-477.

Yackel, E., Cobb, P., & Wood, T. (1998). The interactive constitution of mathematical meaning in one second grade classroom: An illustrative example. *The Journal of Mathematical Behavior, 17*(4), 469-488.

Yin, R. K. (1994). *Case Study Research Design and Methods* (Vol. 5). India: SAGE Publications, Inc.

Yin, R. K. (2003). *Case study research: Design and methods (3rd ed.)*. Thousand Oaks, CA: Sage.

Appendix A: Sample of VSR interview schedule

Introduction

Thank the interviewee for allowing me to observe and video recorded his/her lesson and making time available for this interview to discuss about the lesson. Then provide an overview of the interview.

In this interview, I would like to first watch the video of your lesson together with you. We would only watch for about 10-15 minutes to allow you recall on what transpired during the lesson. Then I would like to talk with you about specific areas in the lesson in order to reflect on your practice and to understand some of the mathematics pedagogic decisions you took during planning and in the enactment of the lesson. We would also refer to the video again where necessary to help you recall on specific issues under discussion.

Semi-structured questions

1. **Opening conversation**
    a. What were you hoping that learners would be able to do, or understand, at the end of that lesson?
    b. And, if I begin at an overview level, did you feel that you achieved what you had intended for the lesson?
        i. Which learners did you think were successful? What did you see that allows you to say this?
        ii. And what were less successful learners struggling with?

2. **Discussion on lesson starter or mental activity**
    a. You began the lesson by class chant count in 5s and clapping for the multiples of 5 and then forwards and backwards counting in 10s. What were you hoping that learners would gain from this?
    b. How is the activity connecting with what you were hoping that learners would be able to do, or understand, at the end of that lesson?
    c. If no connection is seen, now how might you create the connection?
    d. If you are to teach the same lesson again, would you choose to use the same starter or mental activity? If not, which starter activity would you choose and why?

3. **Choice and use of representations**
    i. In this lesson, I saw you using marbles in two colours and a number line. However, there are many representations that could be used in teaching

addition by bridging through 10. Why did you choose these particular representations?

ii. Before your participation in our 20-day course, have you been using these representations in the same way you used in this lesson? If not what is the change?

iii. If you are to teach the same lesson again, would you choose and use the same representations? Why do you think so?

4. Choice and use of examples

i. Let's look at the examples you used in this lesson. Why do you choose these examples?

ii. What do you consider very important in the selection of these examples? What were you hoping that learners would gain from these examples?

iii. Would you select the same examples if you are to teach the same lesson? If not what kind of examples would you select and why?

5. **Dealing with incorrect answers** - For example in this lesson, you ask for the number of claps when counting backwards from 100 to 80. Learners provided incorrect answers. What do you feel in this situation? Why do you do what you did?

6. **Explanation** – Explanation is very important in teaching. What did you consider a good explanation for addition/subtraction using number line representation?

7. Describe a good mathematics teacher?

8. Describe a good mathematics learner?

9. Is there anything else that you would like to add?

Appendix B: Ethics approval

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

Student Number:
675644
Protocol Number:
2012ECE170

Date: 14-Nov-2012

Dear Lawan Abdulhamid

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Using the 'Knowledge Quartet' to explore shifts in primary school teachers mathematical knowledge for teaching in South Africa

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

Matsie Mabeta
Wits School of Education

011 717 3416

CC Supervisor: Prof. H Venkatakrishnan

Appendix C: GDE approval for WMC-P project

Reference No: D2011/69

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Enquiries: Diane Buntting (011) 843 6503

Date:	31 January 2011
Name of Researcher:	Professor H. Venkatakrishnan
Address of Researcher:	Room 2,
	WMC corridor,
	Marang Block;
	WITS School of Education
Telephone Number:	011 717 3742 / 082 099 1967
Fax Number:	0865068324
Email address:	hamsa.venkatakrishnan@wits.ac.za
Research Topic:	Improving the teaching and learning of primary school numeracy/mathematics
Number and type of schools:	TEN PRIMARY Schools
District/s/HO	Johannesburg East

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. *The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.*
2. *The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.*
3. *A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.*

Office of the Chief Director: Information and Knowledge Management
Room 501, 111 Commissioner Street, Johannesburg, 2000 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0809 Fax: (011) 355-0734

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

31 January 2011

Shadrack Phele MIRMSA
[Member of the Institute of Risk Management South Africa]
CHIEF EDUCATION SPECIALIST: RESEARCH COORDINATION

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	

Appendix D – Participants information letters and consent forms

INFORMATION SHEET: TEACHERS

Dear Teacher DATE:

My name is Lawan Abdulhamid and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring shifts in primary school teachers' mathematical knowledge for teaching in South Africa. The focus of my research is on in-service primary school teachers that are attending the Wits Math Connect professional development course.

My research involves using a questionnaire composed of questions and tasks that will be used to elicit information relating to your beliefs concerning the nature of mathematical knowledge, the purposes of mathematics education, the situation under which mathematics is best learned; mathematics content knowledge and mathematics specific pedagogy. I will also be interviewing you as an individual and audiotape our conversations; and I will videotape two cycles of your mathematics lesson/teaching on additive relation content area.

The reason why I have chosen your school is because of your active participation in the Wits Maths Connect professional development course. I am therefore, inviting you to participate in this study.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3-5 years after completion of this study.

This will be a learning experience for both of us as co-participants in this study. Your participation is voluntary, so you are free to withdraw your participation at any time without any prejudice and/or penalty. Please also note that there are no financial rewards for your participation in this study.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE

NAME: Lawan Abdulhamid

ADDRESS: LGB2, International House, East Campus, Wits University

EMAIL : lawan.abdulhamid@wits.ac.za or lawalpt@yahoo.co.uk

TEL NUMBER: 011-7173371 (office) or 0710980251

Teacher's Consent Form: Questionnaire

Please fill in and return the reply slip below indicating your willingness to fill in a questionnaire for my voluntary research project called:

Using the 'Knowledge Quartet' to explore shifts in primary school teachers' mathematical knowledge for teaching in South Africa

Permission for the use of a questionnaire

I, ________________________

Give/do not give* my consent to fill in a questionnaire

[ ]I know that I may withdraw from the study at any time without prejudice and/or penalty

[ ]I know that I can decline to answer a specific question.

[ ]I am aware that the researcher will keep all information confidential in all academic writing.

[ ]I am aware that my questionnaire will be destroyed between 3—5 years after completion of the project.

Teacher Signature: ________________________ Date: __________________

Contact person:

NAME: Lawan Abdulhamid

ADDRESS: LGB2, International House, East Campus, Wits University

TEL NUMBER: 011-7173371 (office) or 0710980251

*please delete as appropriate

Teacher's Consent Form: Interview

Please fill in and return the reply slip below indicating your willingness to be interviewed for my research project called:

Using the 'Knowledge Quartet' to explore shifts in primary school teachers' mathematical knowledge for teaching in South Africa

Permission to be interviewed

I, ________________________

Give/do not give* my consent to be interviewed.

[ ] I know that I don't have to answer all the questions and that I may withdraw from the study at any time without prejudice and/or penalty.

[ ] I am aware that the researcher will keep all information confidential in all academic writing.

[ ] I am aware that my interview transcript will be destroyed within 3—5 years after completion of the project.

Teacher's Signature: ______________________ Date: ________________

Contact person:

NAME: Lawan Abdulhamid

ADDRESS: LGB2, International House, East Campus, Wits University

TEL NUMBER: 011-7173371 (office) or 0710980251

Teacher's Consent Form: Audiotaping

Please fill and return the reply slip below and indicate your willingness to have your interview audiotaped for my research project called:

Using the 'Knowledge Quartet' to explore shifts in primary school teachers' mathematical knowledge for teaching in South Africa

Permission to be audiotaped

My name: ________________________

I give/do not give (please delete as appropriate) my consent to have the interview recorded.

[ ]I know that I may withdraw from the study at any time without prejudice and/or penalty

[ ]I know that I can stop the audiotaping of the interview at any time without repercussions.

[ ]I know that the tapes will be destroyed within 3-5 years after completion of the project and will be kept safe until then.

Teacher's Signature: ________________________ Date: ________________

Contact person:
NAME: Lawan Abdulhamid
ADDRESS: LGB2, International House, East Campus, Wits University
TEL NUMBER: 011-7173371 (office) or 0710980251

Teacher's Consent Form: Videotaping

Please fill and return the reply slip below and indicate your willingness to have your teaching videotaped for my research project called:

Using the 'Knowledge Quartet' to explore shifts in primary school teachers' mathematical knowledge for teaching in South Africa

Permission to be audiotaped

My name: ________________________

I give/do not give (please delete as appropriate) my consent to have my teaching videotaped.

[ ]I know that I may withdraw from the study at any time without prejudice and/or penalty

[ ]I know that I can stop to allow my lesson to be videotape at any time without repercussions.

[ ]I know that the videotapes will be destroyed within 3-5 years after completion of the project and will be kept safe until then.

Teacher's Signature: ________________________ Date: ________________

Contact person:
NAME: Lawan Abdulhamid
ADDRESS: LGB2, International House, East Campus, Wits University
TEL NUMBER: 011-7173371 (office) or 0710980251

INFORMATION SHEET: LEARNERS

Dear Learner DATE:

My name is Lawan Abdulhamid and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring primary school teachers' mathematical knowledge for teaching in South Africa. The focus of my research is on your teachers, because s/he has participated in our professional development course at Wits University.

I will observe and record video of your teacher as s/he teaches you mathematics. I want you to know that I will be recording using a video camera and I need you to agree that it is alright to record video of your teacher's teaching of mathematics while you are in the class. I also want you to know that you may appear in the video recording. The reason why I chose your class is because your teacher has agreed to participate in my research.

Remember, this is not a test. It is not for marks and it is not compulsory, which means that you don't have to appear in the video. Also, if you decide halfway through that you prefer not to appear in my video recording, this is completely your choice and I will position the camera in a way that will not capture you.

I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept secret in all my writing about the research. Also, all video recorded will be stored safely and destroyed after 5 years of completion of this research.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

NAME: Lawan Abdulhamid

ADDRESS: M89, Marang block, Wits Education Campus, Wits University, Johannesburg

EMAIL : lawan.abdulhamid@wits.ac.za or lawalpt@yahoo.co.uk

TEL NUMBER: 011-7173371 (office) or 0710980251

Consent Form –Learner

Learner
I am happy to appear in the mathematics lesson video
I am happy for the extract from these videos to be viewed by the researchers for research purpose.
Learner name: ____ Learner Signature:____ Date: ____

INFORMATION SHEET: PARENTS

Dear Parent/Guardian DATE:

My name is Lawan Abdulhamid and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing research on exploring shifts in primary school teachers' mathematical knowledge for teaching in South Africa. The focus of my research is on in-service primary school teachers that are attending Wits Math Connect professional development course.

My research involves observing and videotaping mathematics lesson at the school where your child is attending. I was wondering whether you would mind if I do the observation and videotaping in your child's class while he/she is present. The reason why I have chosen your child's class is because his/her teacher has agreed to participate in my study.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

NAME: Lawan Abdulhamid
ADDRESS: LGB2, International House, East Campus, Wits University
EMAIL : lawan.abdulhamid@wits.ac.za or lawalpt@yahoo.co.uk
TEL NUMBER: 011-7173371 (office) or 0710980251

Parent/Guardian Consent Form

Parent/Guardian
I consent / do not consent* for my child to appear in the mathematics lesson video
I consent / do not consent* for the extract from these videos to be viewed by the researcher and for research purpose
School name:______________________________________ Parent name:______________________________________ Child name:______________________________________ Parent Signature:__________________________________ Date:____

* Please delete as appropriate