OPTIMISATION OF BUSHVELD PGE SAMPLING PROGRAMMES THROUGH THE UNDERSTANDING OF SHORT AND ULTRA-SHORT RANGE VARIABILITY OF VARIOUS STYLES OF MINERALISATION IN LONMIN'S PGE DEPOSITS.

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DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Master of Science in Engineering (Mining) at the University of Witwatersrand, Johannesburg.

It has not been submitted before for any degree or examination to any other University.

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13th day of November 2013

ABSTRACT

The short and ultra-short range Platinum Group Element (PGE) grade and thickness variability of the UG2 at the Marikana operations in the Western Bushveld were examined using statistical, geostatistical and geological observations. Methods are presented that can be used to optimise underground channel sample spacing and multiple short deflection drilling. The high relative nugget effect for PGE grade results in smoothed estimates close to the local area average and opportunities for selective mining are minimal in the UG2. Robust grade estimates can still be achieved by a very significant reduction in the amount of channel sampling over that currently being conducted. The information gained from multiple deflection drilling was found to be invaluable both from creating enhanced geological interpretation of the borehole as well as a much improved level of confidence than what would be achieved from a single borehole intersection.

DEDICATION

To my wife, Dorcas, and my children, Lebogang, Dumisane and Thandiwe, who have been ever patient.

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LIST OF SYMBOLS

a	The range distance of a semi-variogram
abs	Absolute
C	Sill value of a semi-variogram
$C0$	Nugget effect structure of a semi-variogram
$C1, C2, C3$	Sill values for individual semi-variogram structures
d	A distance
D	Density
g	Value of an attribute of interest
g^*	Estimated value of an attribute of interest
h	Lag vector representing separation between two spatial locations
l or b	Length along or between a line/s
n	Number of samples
$N(h)$	Number of pairs at h used for the calculation of semi-variance.
R^2	Square of the correlation co-efficient
$R1, R2, R3$	Ranges of semi-variogram structures
s	A sample
t	Students T statistic
tt	True Thickness
u	the vector of spatial coordinates (direction and distance)
w	Weight applied
x	variable
γ	Semi-variance
$\gamma^*(h)$	experimental semi-variance at h
λ	the Lagrange Multiplier
$\bar{\gamma}(A,A)$	The average semi-variogram between the discretised points of the block
$\bar{\gamma}(S,A)$	The average semi-variogram between the discretised points of the block and the samples.
$\bar{\gamma}(S,S)$	The average semi-variogram between the samples
σ^2 / S^2	Variance
σ / S	Standard Deviation

LIST OF ABREVIATIONS

PGE	Platinum Group Element. In this study in a grade context is analogous to 3PGE+Au
UG2 -	Upper Group 2 Chromitite Layer
RLS -	Rustenburg Layered Suite
LGS -	Lebowa Granite Suite
RGS -	Rashoop Granophyre Suite
IRUP -	Iron Rich Ultramafic Pegmatite
3PGE+Au	Platinum, Palladium, Rhodium and Gold combined.
PSlope	Slope of Regression
KE%	Kriging Efficiency
HW	Hangingwall (above the reef)
FW	Footwall (below the reef)
QAQC	Quality Assurance and Quality Control
4E	4 Elements (Pt, Pd, Rh and Au). Also an assaying procedure used to determine the combined grade of the 4E.
4T	An assaying procedure used to separately determine the individual 4E elements
6E	6 Elements (Pt, Pd, Rh, Au, Rh and Ir). Also an assaying procedure used to determine the individual grade of the 6E's.
7E	7 Elements (Pt, Pd, Rh, Au, Rh, Ir and Os)
ICP	Inductively Coupled Plasma
LTC	Low Temperature Cupellation
HTC	High Temperature Cupellation
OES	Optical Emission Spectrograph
MS	Mass Spectrograph
FACF	Fire Assay Correction Factor
EPL	Eastern Platinum Limited
WPL	Western Platinum Limited
g/t	Grams per metric tonne (a unit of grade)
SPD	Stope Preparation Drive
RSE	Raise
MRM	Mineral Resource Management
ARD	Absolute Relative Difference

1. INTRODUCTION

In South Africa, Platinum Group Elements (PGE's; Platinum, Palladium, Rhodium, Ruthenium, Iridium and Osmium) are largely sourced from two narrow tabular reefs, namely the Upper Group 2 Chromitite Layer (UG2 or UG2 Reef) and the Merensky Reef. These occur in both the Western and Eastern Limbs of the Bushveld Complex. Since the 1990's an additional source has been from the more massive Platreef unit in the Northern Limb. Both UG2 and Merensky Reef are known for their geological continuity, they being traced almost uninterrupted for tens of kilometres. However, what is often less well emphasised is the high variability in grade and thickness (and therefore PGE content) and internal reef stratigraphy over very short distances (several metres or even tens of centimetres).

This study examines the short range variability of PGE grade and thickness in detail for the UG2 Reef at Lonmin Platinum's Marikana Operations (Marikana), using both multiple borehole drilled intersections (ultra-short range variability) and closely spaced channel samples (short range variability) in combination with geological observations of the reef exposures. By better understanding the short range variability of the reef, both geostatistically and geologically, the sampling programmes can be optimised thus resulting in significant cost and time savings for a given mine or project.

Using large quantities of underground sampling data and surface borehole intersections this project aims to determine:

1. What are the geological, statistical and geostatistical characteristics of the PGE mineralisation over short distances?
2. What value do multiple drilled intersections add in terms of confidence in resource estimations and our understanding of the geological framework of the mineralisation?
3. Are there more optimal sampling patterns than those currently used?

The introduction commences with a high level description of the geology at Marikana, followed by a brief introduction to the field of Geostatistics in order to provide context to the study. Given that the nugget effect is an important contributor to the confidence in estimations over a short distance, a discussion on the nugget effect, sourcing a number of opinions from scientific literature, is included. This is followed by a section on Kriging that describes the commonly used Kriging equations, as well as parameters that are derived from this system of equations that were used in this study to judge the effect of different sampling patterns.

A section on data collection is included that describes both the channel sampling and diamond drilling data. This is followed by statistical and geostatistical analysis that aims to characterise the data sets and describe the spatial behaviour of the UG2 with particular emphasis on the nugget effect.

The first experimental study is an investigation into the use of multiple intersections obtained from short non-directional deflection drilling. This is linked to some of the outcomes of the underground channel sample spacing study in terms of how a change in the nugget effect contributes to the error in a stope estimate obtained by using channel sample data.

The second experimental study is an investigation into the optimisation of underground channel sample spacing. This commences with a description of the data, a summary of pertinent work completed by other workers, detailed statistical analysis and then findings.

The key findings of the research report are then summarised in the concluding section.

1.1 The Geology of Lonmin's PGE Mineral Resources

PGE Mineral Resources in South Africa are almost exclusively contained with rocks of the Bushveld Complex. The Bushveld Complex is the largest known layered intrusion on Earth with an outcrop area of 29,450 km^2 and further sub-outcrops of 36,550 km^2 (von Gruenewaldt, 1977). The Bushveld Complex contains by far the majority of the world's PGE Mineral Resources as well as other commodities such as Chromium and Vanadium. The PGE's are intimately associated with Gold, Nickel and Copper, which form important bi-products of PGE mining in South Africa.

The Bushveld Complex consists of three suites of plutonic rocks; the Rustenburg Layered Suite (RLS), the Rashoop Granophyre Suite (RGS) and the Lebowa Granite Suite (LGS); however it is the Rustenburg Layered Suite, an eight kilometre thick succession of layered mafic and ultramafic rocks, that contains the PGE, Cr and V deposits.

The RLS is exposed in three major lobes or limbs; the Western Limb, the Eastern Limb, and the Northern Limb, as well as the Bethal Limb that is hidden beneath younger sedimentary cover and a number of smaller satellite intrusions.

PGE mining takes place in all three major limbs of the Bushveld Complex, Lonmin's Mineral Resources being present within all of the major limbs (**Figure 1-1**; Lonmin, 2012):

- the Western Limb (Marikana Operations and Pandora);
- the Eastern Limb (Limpopo Operations, Loskop Joint Venture); and
- the Northern Limb (The Akanani Project).

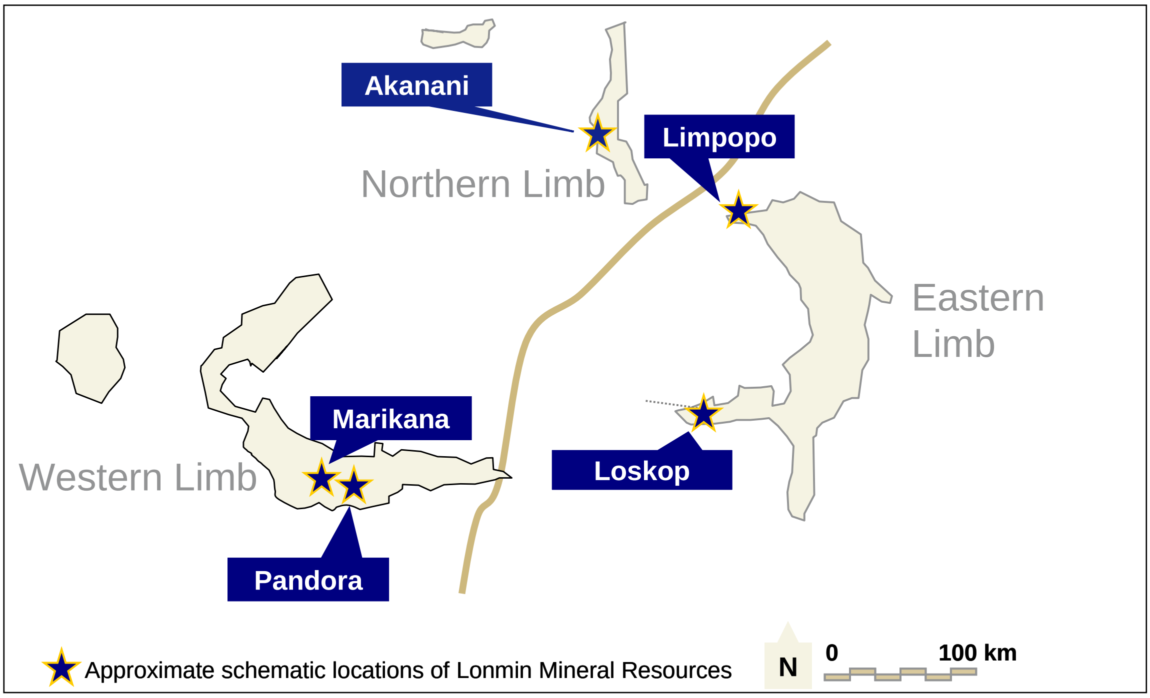


Figure 1-1 Locations of Lonmin's South African Mineral Resources (Lonmin, 2012)

Lonmin's Mineral Resources in the Eastern and Western Limbs, with the exception of Loskop, are hosted within the well-known Merensky and UG2 reefs.

Both of these units occur in the Upper Critical Zone of the RLS, which is a series of mafic and ultramafic cyclic units within which all of the known economic PGE mineralised layers in these areas occur. The Merensky Reef is a layer of mineralisation, typically in the order of one metre thick that occurs within, or in close association with, the Merensky Pyroxenite Unit. The UG2 Reef consists of the UG2 Chromitite Layer, containing most of the PGE mineralisation, and may include less well mineralised units or portions of these units either underlying or overlying the main UG2 Chromitite Layer (Lonmin, 2012).

Bushveld PGE Mineral Resources are frequently disturbed by geological conditions which may result in losses to the Mineral Resource area. The areas affected are classified as geological losses, which are commonly caused by potholes, faults, intrusive dykes and Iron Rich Ultramafic Pegmatite (IRUP).

Marikana UG2 Reef

The UG2 Reef accounts for approximately 75% of the PGE production at Lonmin's Marikana Operations (Marikana), the remaining 25% being sourced from the Merensky Reef. As at 30 September 2012, 445 Mt of UG2 Mineral Resource at a grade of 4.95 g/t 3PGE+Au over an average true thickness of 1.21 m was estimated to be remaining (Lonmin, 2012).

At Marikana, the UG2 Reef normally comprises a massive chromitite layer, which varies in thickness over the property but is generally between 1.0 m and 1.4 m thick. The reef dips to the north generally at between 8° and 14°, although it can vary more than this, particularly in the vicinity of geological disturbances. The hangingwall to the UG2 Reef is pyroxenite and the top contact is sharp and planar. In contrast, the lower contact with the underlying pegmatoidal pyroxenite, norite or anorthosite is irregular, with the chromite forming cuspate and even carrot like protrusions into the underlying lithologies. Localised areas of internal waste can occur and the internal waste is necessarily included in the Mineral Resource. Subtle changes to the thickness and grade of the UG2 Reef lead to separation into a number of domains, or styles, of UG2 mineralisation on the property that are important considerations in Mineral Resource management. For example, in the west of the property the Leader

Chromitite separates from the main UG2 Chromitite creating a zone of "split reef". This affects the far western areas of the mine lease.

A number of thin (from 0.01 cm to approximately 10 cm) chromitite layers occur a variable distance above the UG2. These are termed the UG2a Markers at Lonmin and are analogous to the Triplets described in other parts of the Bushveld Complex. In the western area they are close to the UG2 Reef hangingwall and, in some areas, they lie directly on top of the UG2 Reef appearing to coalesce. Towards the east the separation increases to several metres.

The UG2 Reef is disturbed by a number of north-north west trending faults and dykes as well as a number of IRUP bodies and numerous potholes (**Figure 1-2**). The faults and dykes disturb the reef and cause losses of available area for mining.

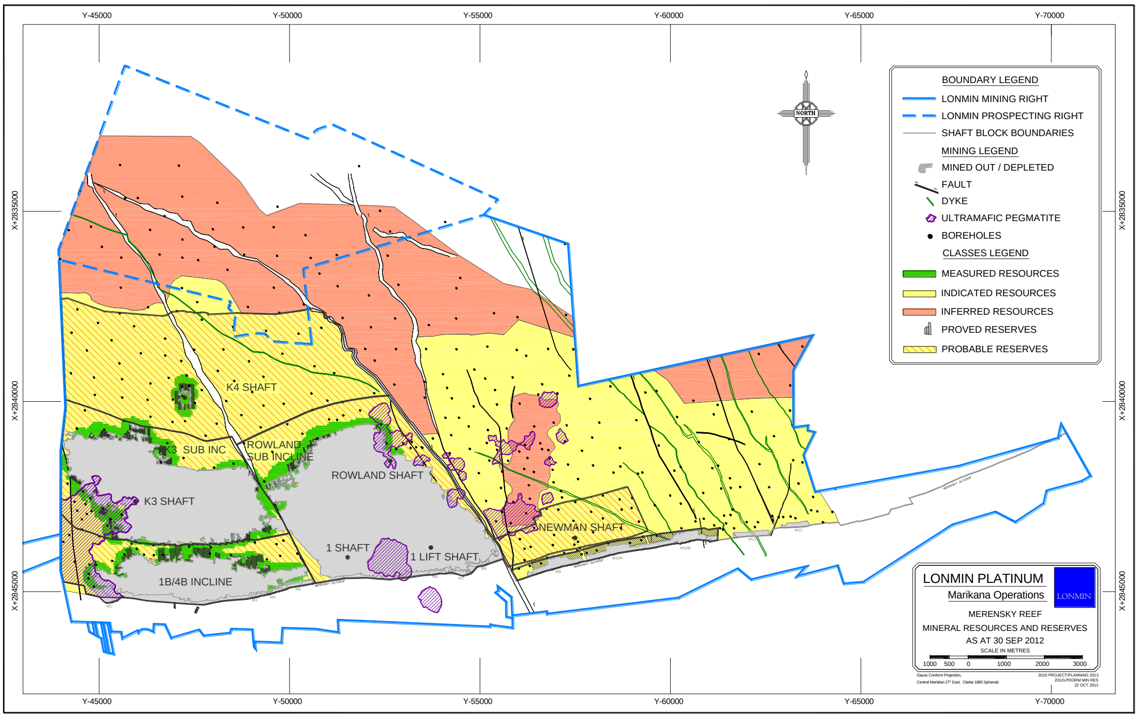


Figure 1-2 Marikana UG2 Reef Resource and Reserve Plan showing the location of major faults, dykes and IRUP (Lonmin, 2012).

Potholes are circular or elliptical areas in which portions of the footwall succession of the UG2 Reef are absent, so that the reef and its hangingwall layers transgress into this area with an inward or centripetal dip (**Figure 1-3**).

The pothole depression is normally defined by the bottom contact of the UG2, which is not at normal elevation, and clearly crosscuts stratigraphic layering in the footwall. Potholes are generally not mined due to their irregular nature which promotes high dilution. Potholed UG2 Reef can thin out to the point that only a few centimetres of the reef exists or even becomes completely absent. This extreme thinning of the reef cannot be mined without including a large amount of dilution and thus this reef has little to no economic value.

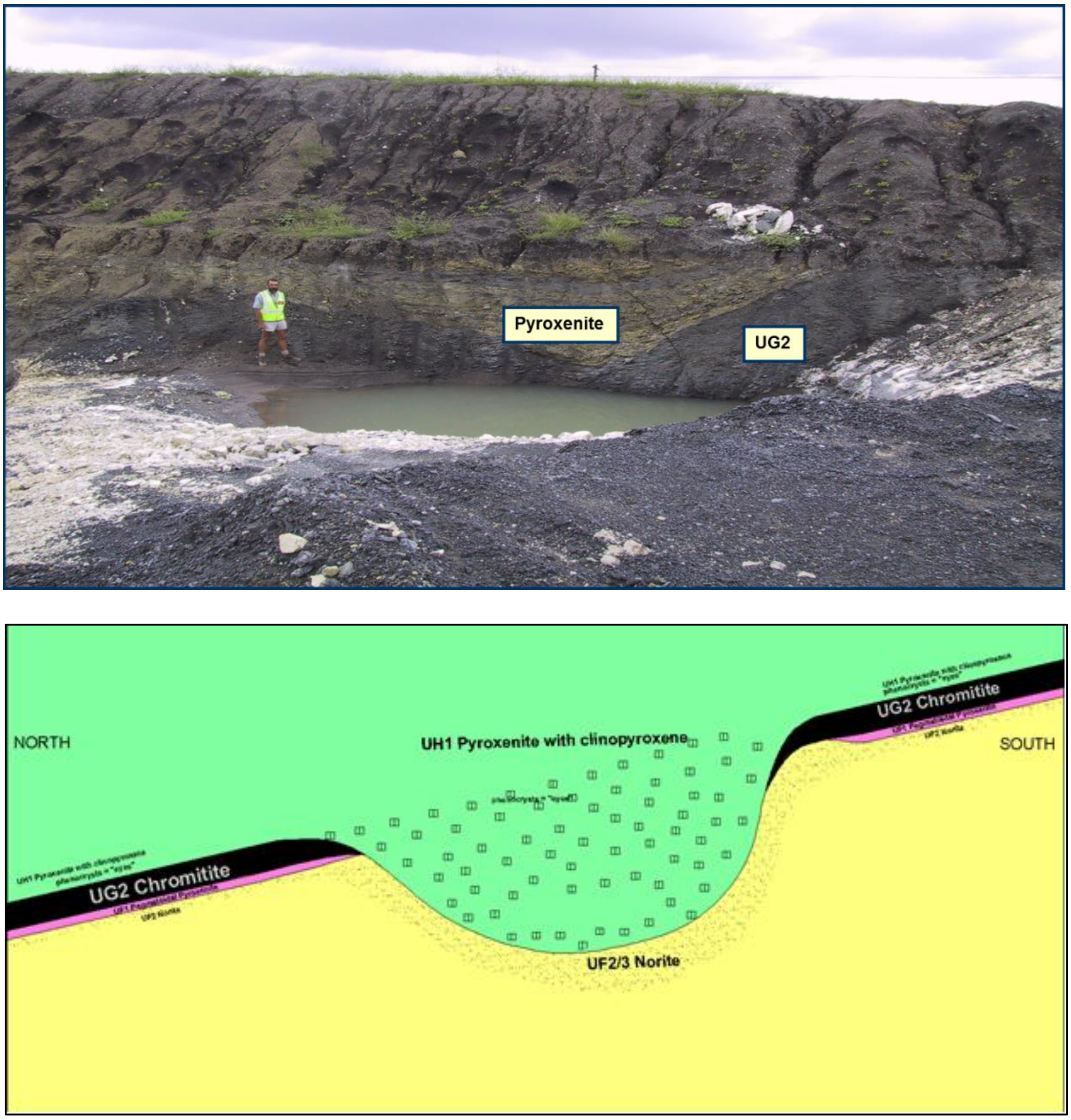


Figure 1-3 Photograph and Schematic Representation of UG2 potholes (courtesy of Lonmin).

The frequency and location of potholes are difficult to predict, although they do occur in all UG2 and Merensky reef deposits. Their location can be partially determined by geological drilling and they are well defined during mining development and stoping. Three dimensional seismic surveys are routinely carried out from surface at Lonmin, which serve to identify larger potholes (greater than 100 m in diameter) as well as other major reef disturbances. Identifying potholes in underground workings is by recognising certain characteristics typical to potholing:

- The reef layer thins out rapidly from normal reef thickness to less than 50 cm over a short distance (approximately five metres) and simultaneously dips down steeply into the footwall units.
- Where reef would be expected to occur under non-potholed conditions, hangingwall units are encountered. In general, the larger and more catastrophic the pothole is then units are encountered from further up the stratigraphy.
- Low angle jointing is associated with potholes and is more frequent on the edges of the pothole.

Potholes vary in size, from diameters of several tens of metres or less to several hundred metres. At Marikana, the proportion of the area affected by potholes varies, but on average they occupy 10.7% of the western area of the mine and 3.9% of the eastern area (Hoffmann, 2010). In individual stope areas, the area affected by potholes may be considerably higher.

The IRUP bodies replace the chromitite with magnetite, resulting in an increase in hardness, slumping and reef thinning. Modification of the PGE mineralisation is known to occur, which together with the increased hardness, creates additional complexity in extraction of PGE's from the reef.

Base metal-sulphides constitute less than 1% of the ore. In normal UG2 Reef, the major base metal sulphide minerals are pentlandite, chalcopyrite and pyrrhotite. Minor amounts of pyrite and millerite also occur. The average diameter of the base metal sulphide composite grains is approximately 30 microns. The platinum group minerals are predominantly Pt-Pd-Ni-sulphide (braggite), Pt-sulphide (cooperite), Pt-Rh-Ir-Cu-sulphide (malanite), and Ru-Os-Ir-sulphide (laurite). These minerals are generally associated with base metal sulphides and occur at sulphide grain boundaries with chromite or silicates, at sulphide-sulphide grain boundaries, or as finer grained inclusions in sulphide. Chromite rarely hosts platinum group minerals, with the exception of laurite. The average grain size of the platinum group minerals is in the region of 6 to 10 microns.

1.2 Geostatistics

In the early 1960's following substantial empirical work by authors in South Africa, Georges Matheron published a paper on the Theory of Regionalised Variables (Clark, 1979). Matheron (1971:5) describes a phenomenon that "spreads in space and exhibits a certain spatial structure" as regionalised. Furthermore he described Geostatistics as "the application of the theory of regionalised variables to the estimation of mineral deposits, with all that this implies" Matheron (1971:5). Deutsch, 2002 considers Geostatistics to encompass the study of variables that change in space or even time. Geostatistics deals with spatially auto-correlated data; that is correlation between elements of a series and others from the same series separated from them by a given interval (New Oxford American Dictionary, 2010).

The application of Matheron's work to the estimation of an unknown value at some location within an ore deposit is commonly referred to as Kriging, named after a well-known worker in this field, Dr. Danie Gerhardus Krige, a South African mining engineer who pioneered the field of geostatistics.

Geostatistics is based upon a model of the spatial variability of the data. This model is known as a semi-variogram, which describes the spatial or temporal variance of the attribute of interest.

The Semi-Variogram

The semi-variogram (γ) is a graph (and/or formula) describing the expected difference in value between pairs of samples with a given relative orientation (Clark, 1979).

Semi-variance is one half of the variance of differences between a number of points a distance h apart. It is calculated as:

$$\gamma(h)^* = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [x(u_i + h) - x(u_i)]^2$$

Where:

h = lag vector representing separation between two spatial locations

$\gamma(h)^*$ = experimental semi-variance at h

$N(h)$ = number of pairs at h

u = the vector of spatial coordinates (direction and distance)

x = variable

In most practical situations, data are not perfectly arranged in a regular grid and therefore the calculation of the semi-variance at set distances apart requires that tolerances are applied in order to be able to compute a number of paired differences. Therefore, distances are divided into a number of intervals called lags which are defined by a distance and a tolerance that is typically half the lag separation (**Figure 1-4**).

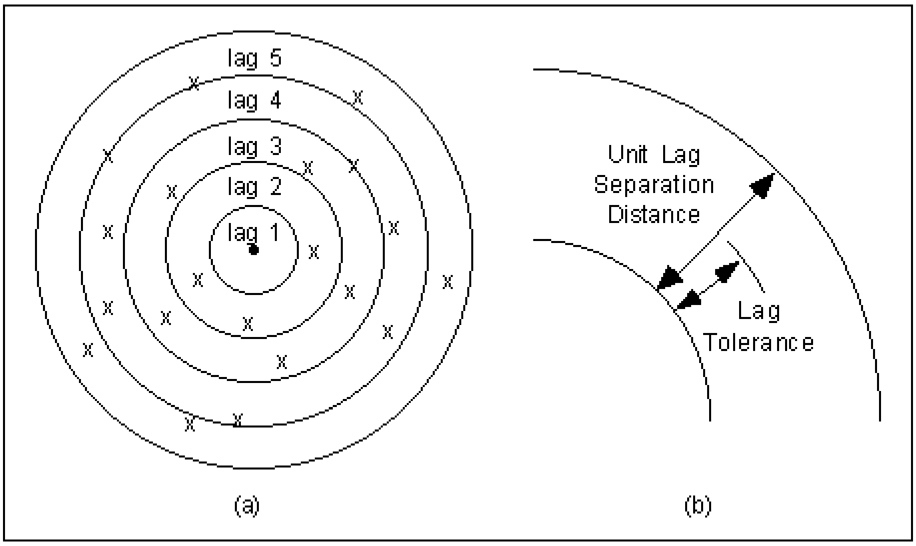


Figure 1-4 Illustration of lag tolerance (Source www.ems-i.com).

In practice, samples are not aligned along exactly straight lines and for a directional semi-variogram, angular tolerance is required. This is constrained by a maximum distance from the direction of interest, known as the bandwidth (**Figure 1-5**).

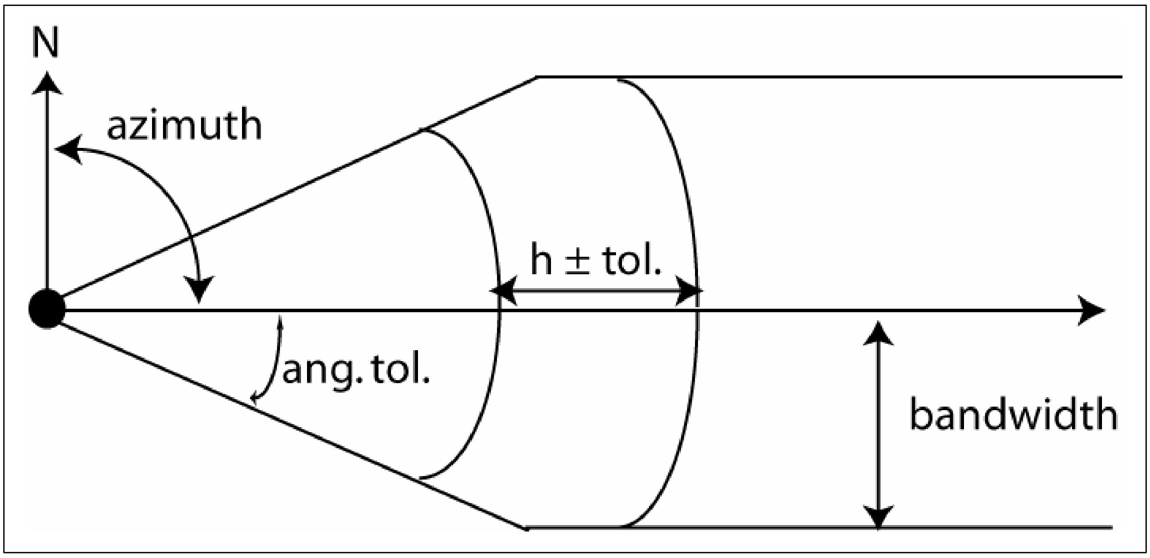


Figure 1-5 Illustration of angular tolerance and bandwidth (Bohling, 2005)

The semi-variance is calculated for a number of lags and is plotted on a graph showing lag on the x axis and the semi-variance on the y axis. The points on the graph are experimental points and thus the semi-variogram is commonly known as an experimental semi-variogram. A semi-variogram model is then fitted to the experimental data, which typically consists of between one and three structures as well as a nugget effect. This process is repeated for several different directions in order to ascertain any directional differences in the spatial continuity. Where such directional differences exist, the spatial distribution is described as anisotropic. Where no directional differences can be ascertained, the spatial distribution is described as isotropic.

It is common practice to estimate the nugget effect by extrapolating the semi-variogram model to the y axis (**Figure 1-6** and **Figure 1-7**). Duplicate sampling data can also be used when the sample spacing between the original and duplicate sample is very small in terms of the sampling scale. Typically in a mineral deposit, this may be in the order of 10’s cm.

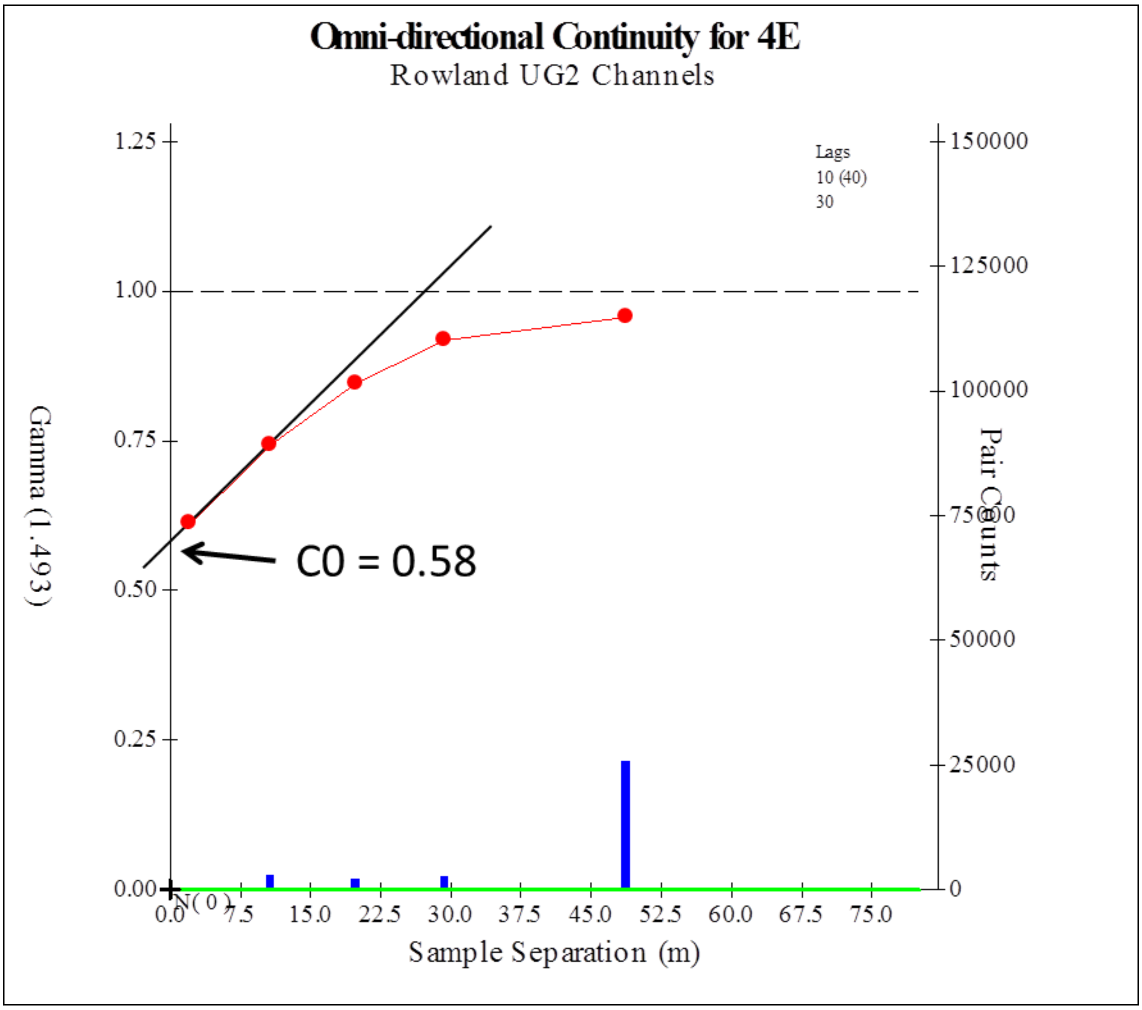


Figure 1-6 Example of determination of nugget effect by extrapolation

Most commercial Mineral Resource modelling software contains modules that calculate experimental semi-variogram data and provide for ease of fitting an appropriate model. In this study, Snowden Supervisor was used for calculating and modelling semi-variograms. For simplification purposes, the semi-variance can be normalised using Snowden Supervisor by dividing the semi-variogram value by the estimated sample variance so that the sum of the individual sill structures is equal to one.

As well as providing insight into the spatial structure of various attributes of a deposit, the semi-variogram model is used in calculations such as the Kriging process often used to estimate Mineral Resources.

A number of models exist that can be used to model the experimental data, the most commonly used for mineral resource estimation being the spherical model:

$\gamma(h) = C\left(\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3}\right)$ where $h \leq a$

and

$\gamma(h) = C$ where $h \geq a$

h	= lag distance
$\gamma(h)^*$	= experimental semi-variance at h
C	= sill value
a	= the range distance

The Nugget Effect

The nugget effect was described by Matheron (1971:77):

"At the scale of a dozen or a hundred metres, a transition phenomenon, which has, for instance, a range of the order of centimetres is no longer apparent on the experimental $\gamma(\mathrm{h})$, except as a discontinuity at the origin, or "nugget effect". In a general way, all nugget effects are reflections of a transition structure, the dimensions of which are considerable exceeded by the working scale: the details and the characteristics of this prior structure have long since ceased to be perceptible, and the larger scale has preserved a single parameter – the nugget constant – which gives a kind of overall undifferentiated measure of the "intensity" of this hidden structure."

Matheron stresses the importance of scale and considers that the nugget effect is actually a separate structure of the semi-variogram that that has a semi-variance that increases rapidly near the origin over a very small range (**Figure 1-7**). Furthermore, he mentions that at a small scale, the samples are not points they having a large volume compared to the range of the nugget effect. The variance due to the nugget effect is then in an inverse ratio to the sample volume (Matheron, 1971).

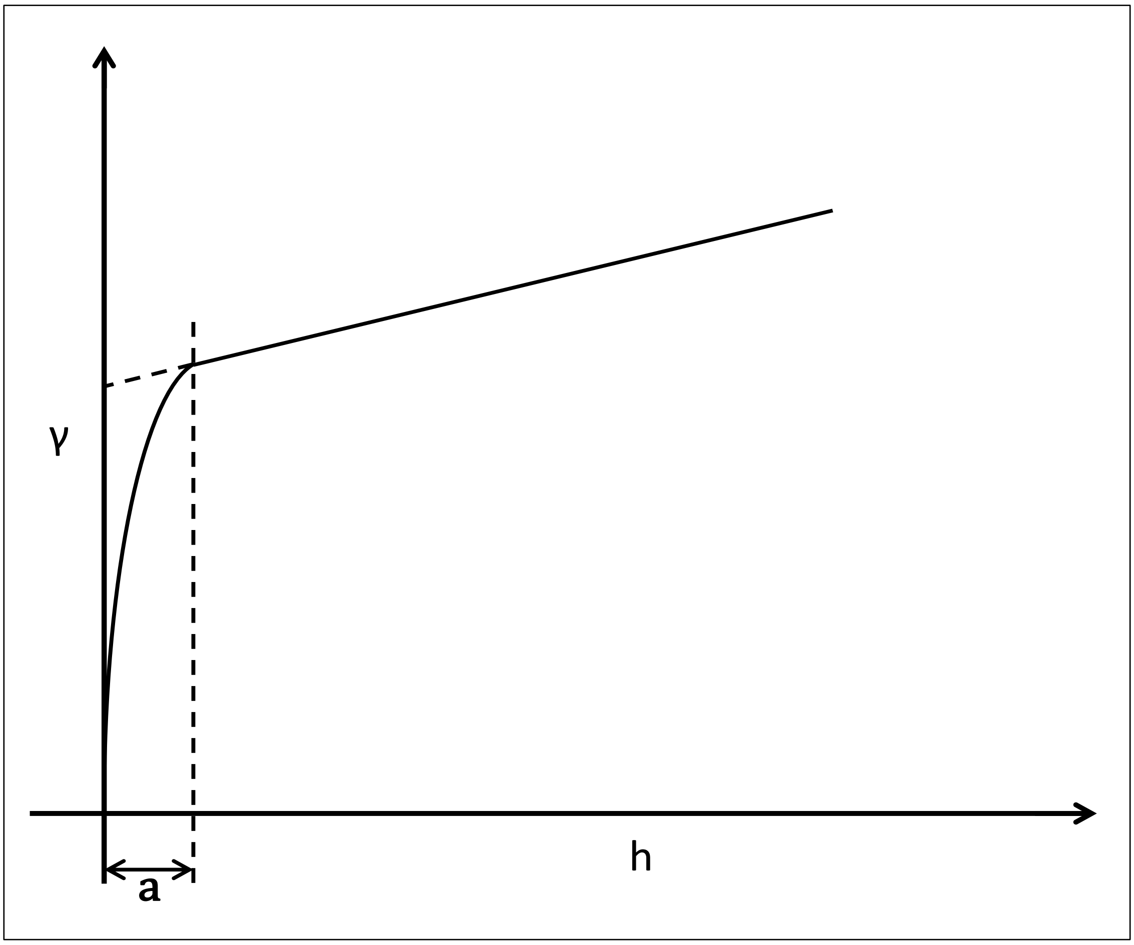


Figure 1-7 Illustration of nugget effect structure (After Matheron, 1971). In this diagram *a* is a range with a very small distance

Clark (1979:9) states that "even with completely random phenomena the semi-variogram must be zero at distance zero as "two samples measured at exactly the same position must have the same value". There has been considerable discussion on what exactly constitutes nugget effect, it being recognised that there are a number of components to it in addition to the ultra-short range structure described by Matheron.

Clark carried out work on the Geevor Tin Mine in Cornwall described in a 2010 paper entitled "Statistics or geostatistics? Sampling error or nugget effect?". Using duplicate assay data and underground channel samples spaced six inches (15.24 cm) apart, she found that the error associated with the assaying used (a vanning process) was only 6.6% of the semi-variance at a 6 inch spacing. The remaining 93.4% of the variance was attributed to spatial error at the sampling interval. In other words, the variance between two samples 6 inches apart is approximately 15 times higher than between replicate assays on the same sample (Clark, 2010).

In Carrasco's 2010 paper entitled "Nugget effect, artificial or natural?" he ascribed nugget effect to three causes:

- A microstructure or process noise, namely a component of the phenomenon with a range shorter than the sample support (true nugget effect). This is equivalent to that described by Matheron (1971).
- A structure with a range shorter than the sampling interval.
- Measurement errors. Sampling and/or assaying errors can create an artificial nugget effect which he called the "human nugget effect".

In his study Carrasco (2010) concluded the following:

- Relevant nugget effect represents short-range structures which actually belong to the process under study and irrelevant nugget effect does not belong to the process; it is induced by incorrect sampling, sample preparation, and chemical analysis.
- With good understanding of the phenomenon requiring estimation, the QAQC procedures for sampling, sample preparation and chemical analysis and an estimation of the relevant and irrelevant components, the process can be optimised.
- The estimation of both components (relevant and irrelevant) is possible if the errors are spatially independent and spatially independent of the variable under study, and a duplication of the sampling and chemical analysis procedures is available.
- The magnitude of the nugget effect is very dependent on sample support, sampling density, sampling quality, assaying procedures, and the nature of the phenomenon under study.

Carrasco (2010) also showed how high nugget effect leads to a high degree of smoothing. **Figure 1-8** illustrates that kriging weights become similar as the proportion of the nugget effect to the total semi-variance increases, regardless of the location of the sample. All the blocks belonging to the same high nugget domain will have a grade estimate very similar to the mean and therefore selective mining will result in a large proportion of blocks being wrongly assigned as ore or waste. Therefore, by understanding the cause of the nugget effect, it may be possible to reduce the irrelevant proportion and thus reduce misclassification.

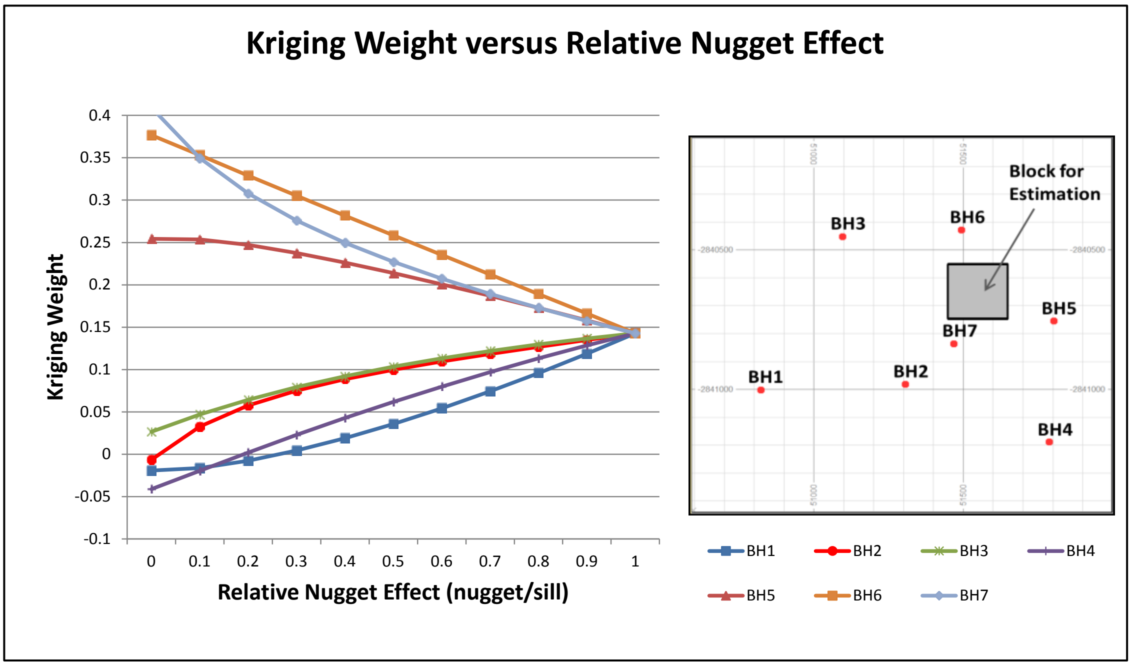


Figure 1-8 Kriging weights versus nugget effect (Adapted from Carrasco, 2010).

Golden Software's computerised modelling programs allows for the categorisation of the nugget effect into two components when modelling a semi-variogram; the error variance and the micro variance. These are summarised as follows (Barnes, undated):

- The error variance measures the reproducibility of observations. This includes both sampling and assaying (analytical) errors.
- The micro variance is the unknown semi-variogram at separation distances of less than the typical sample spacing.

In Pitard's (1993) paper "Exploration of the Nugget Effect" he also ascribes the geostatistical nugget effect to two different components:

- "true in-situ" nugget effect (i.e. small-scale intrinsic variability of the grade, or "chaotic component")

- a number of variability components due the various aspects of sampling and sample preparation and assaying procedures (“un-true” nugget effect).

Bongarçon (1994) made a number of comments on Pitard's (1993) paper and suggested that the two types of nugget components, the "true" and the "un-true", are not independent. He mentioned that microstructures actually play the main role in the value of sampling variances, especially the variance due to Fundamental Sampling Error and Grouping and Segregation Error. He mentioned that once the ore has been broken up and mixed up, the auto-correlation structures that constituted the natural segregation of the mineralisation in place are destroyed; however the microstructures at scales smaller than the rock particle sizes remain as they were when the ore was still in place.

Lyman (2011) showed that by sampling down to the microscopic level, the semi-variogram should not have a nugget effect and that the non-zero intercept of the semi-variogram is purely due to preparation and analysis of the sample. This conclusion was arrived at by demonstrating that there is no connection between the in-situ heterogeneity and the particulate heterogeneity. This is because the particulate intrinsic heterogeneity is determined by the extent to which a phase of interest will liberate at a given level of comminution, which rarely has anything to do with the in-situ properties of the rock. He recommended that semi-variograms used for geostatistical calculations and for estimation of sampling variance should be corrected for the presence of the preparation and analytical variance.

In conclusion, the nugget effect can be understood to be partly the result of ultra-short range structures, at a scale smaller than the sample support, and those due to errors in the sampling and assaying component. Errors due to the sampling and assaying process may include an element of true nugget effect due to microstructures at scales smaller than the rock particle size. However this may be of little practical relevance in estimation as it cannot be separated from the sampling and assay errors. True nugget effect could also be described as ultra-short range variability as it actually has a range, albeit very small compared to the scale of the data used for estimation. In this study, the term short range variability is used to describe the variability of the data over a range of less than the sample spacing in a close spaced sample programme, which is equivalent to the micro-variance described by Barnes.

Kriging

In Mineral Resource estimation, variations of kriging are commonly used to estimate various attributes such as grade, density and thickness. Simplistically the process is based on a semi-variogram that is used to apply weights to samples based on their location relative to the point being estimated and their location with respect to one another. The estimate is unbiased in that the sum of the weights is equal to one. Block kriging is commonly carried out, which involves representing a block by a matrix of points (discretised points). Each point is estimated and then the results are averaged to form a block estimate.

There are a number of variations of Kriging, the most widely used in Mineral Resource grade estimation being Ordinary Kriging (OK). In order to ensure that the sum of the individual weights add up to exactly one, the Lagrange Multiplier is part of the Ordinary Kriging matrix. The Lagrange Multiplier can be output from the Ordinary Kriging process and the magnitude of this value is an indication of how much extrapolation is taking place. When the Lagrange Multiplier is high, consideration may be given to other estimation techniques such as Simple Kriging.

The basic Ordinary Kriging equation is as follows:

$$g^* = \sum_{i=1}^{n} w_i g_i$$

Where the grade estimate (g*) is the summation of i number of samples each with a weight (w).

The weights are derived by solving a system of simultaneous equations where the sum of the weights is equal to one. A block is represented by a number of points (discretisation points) that are estimated individually and combined to represent the block. For example, with three samples and four discretised points the Ordinary Kriging matrix of equations is as follows:

$$w_1\gamma(s_{1,1}) + w_2\gamma(s_{1,2}) + w_3\gamma(s_{1,3}) + \lambda = \gamma(s_1 d_1) + \gamma(s_1 d_2) + \gamma(s_1 d_3) + \gamma(s_1 d_4))/4$$

$$w_1\gamma(s_{2,1}) + w_2\gamma(s_{2,2}) + w_3\gamma(s_{2,3}) + \lambda = \gamma(s_2 d_1) + \gamma(s_2 d_2) + \gamma(s_2 d_3) + \gamma(s_2 d_4))/4$$

$$w_1\gamma(s_{3,1}) + w_2\gamma(s_{3,2}) + w_3\gamma(s_{3,3}) + \lambda = \gamma(s_3 d_1) + \gamma(s_3 d_2) + \gamma(s_3 d_3) + \gamma(s_3 d_4))/4$$

Where

w_i = weights

$\gamma(s_{i,i})$ = the semi-variogram values for the distance between pairs of samples

$\gamma(s_i d_j)$ = the semi variogram values for the distance between the sample and the discretised point in the block to be estimated

λ = the Lagrange Multiplier.

Three variance components arise from Ordinary Kriging, which are useful in understanding the confidence in an estimate:

$\bar{\gamma}(A, A)$ The average semi-variogram between the discretised points of the block.

$\bar{\gamma}(S, A)$ The average semi-variogram between the discretised points of the block and the samples.

$\bar{\gamma}(S, S)$ The average semi-variogram between the samples.

If all weights are equal the Extension Variance is calculated as:

$$\sigma_e^2 = 2\bar{\gamma}(S, A) - \bar{\gamma}(S, S) - \bar{\gamma}(A, A)$$

The extension variance can be used to describe the error when extending a grade from a point into a panel and its usefulness in this study is discussed in later sections.

Kriging Variance is:

$$\sigma_k^2 = \sum_{i=1}^{n} w_i\, \bar{\gamma}(S_i, A) - \bar{\gamma}(A, A) + \lambda$$

Block Variance is:

$\sigma_b^2 = \sigma^2 - \bar{\gamma}(A,A)$

Two measures of the reliability of the Kriged estimate are often used; Kriging Efficiency (KE%) and Slope of Regression (PSlope). Kriging Efficiency is a measure of how smoothed the estimate is as reflected in the difference in variance between the 'true' block values and the kriged values. Slope of Regression is a measure of the degree of conditional bias, which is important in assessing the ability to predict the grade of the block above a cut-off grade. An estimate with a negative Kriging Efficiency is unreliable as the Kriging Variance is higher than the Block Variance. This means that the true variance between the blocks is less than that of the local kriged estimate. In these cases an alternative estimation method to Ordinary Kriging should be used such as Simple Kriging or even the domain average. An estimate with a slope of regression of less than 0.5 is too smoothed and not appropriate for selection above a desired cut-off grade. Krige (1997) considers an acceptable estimate to have a slope of regression of greater than 95%. In the authors experience slopes of regression are rarely this high and acceptable estimates may be achieved that have lower slopes of regression.

$$PSlope = \frac{\sigma_b^2 - \sigma_k^2 + \lambda}{\sigma_b^2 - \sigma_k^2 + 2\lambda}$$

$$KE\% = \frac{\sigma_b^2 - \sigma_k^2}{\sigma_b^2}$$

2. DATA COLLECTION AND PREPARATION

The data that informs this study were collected using two different sampling methods; channel sampling of the reef exposed in underground development and diamond drilling of boreholes from surface through the reef. The data were extracted from the Lonmin database in March 2012. In general, the channel samples inform estimates of small areas used for relatively short term predictions, including metal accounting, where high confidence in the estimate is required. The surface boreholes inform estimates of large areas used for longer term planning where the confidence requirement is less.

2.1 Channel Sampling

Channel samples are collected by means of a rotary saw with a diamond set blade that cuts samples from rock faces; normally on-dip raise sidewalls and less commonly strike drives. Two parallel grooves approximately 2 cm deep and 5 cm apart are cut perpendicular to the reef dip from the hangingwall continuously through the reef down to the footwall. One groove is cut 90° into the face and the other at approximately 30° so that a wedge shaped sample is produced. The volume of sample per cm sample length is approximately 8.8 cm^3. A continuous series of samples are taken through the reef at specific intervals (**Figure 2-1**). The position of the channel is measured relative to underground survey stations and the rock type, layer code and sample length is recorded by the evaluator. Each sample is assigned a bar code and bagged underground. A 2 cm deep and 5 cm wide groove remains in the rock face that is easily audited as the need arises.

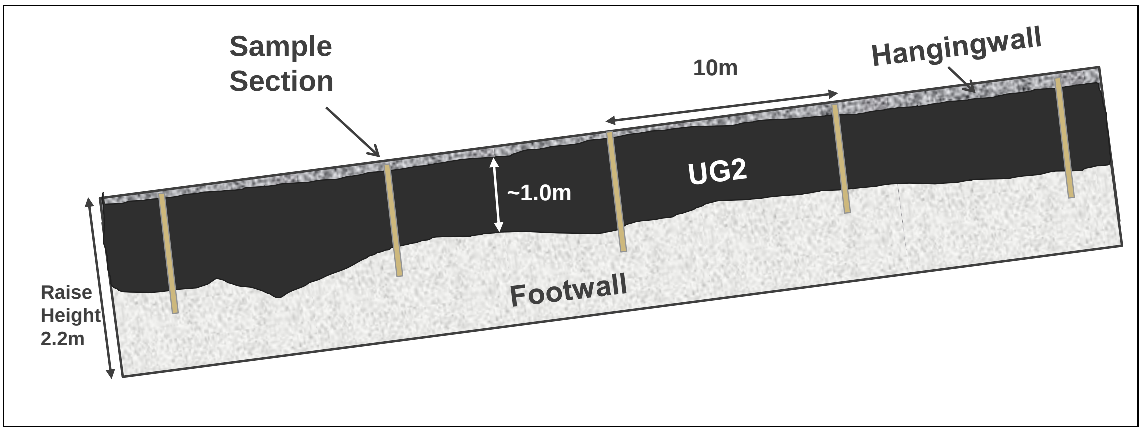


Figure 2-1 Schematic representation of a portion of a UG2 raise sidewall showing channel sample sections

After the samples are cut and bagged they are verified by a supervisor on surface, and then weighed in air and water for Specific Gravity determination using Archimedes Principle. The sample information collected underground and the density information are then captured into the mine spatial database. The samples are dispatched daily to the Lonmin Mine Laboratory and the assays values are later received directly into the mine spatial database.

2.2 Boreholes

Diamond drilling of boreholes is conducted predominantly from surface down through the Merensky and UG2 reefs until several 10's of metres into the footwall lithologies. Diamond drilling produces cylindrical cores that are logged, sampled and assayed in order to collect a variety of geological data, including the reef thickness, specific gravity and metal grades.

The reef is first drilled though by a BQ size mother hole using wire-line drilling techniques. The BQ core is of relatively small diameter (36.4 mm) and is kept as the reference core for the hole and is not normally sampled. Once the depth of the reef down the hole has been defined by the mother hole, a number of TBW size deflections are drilled through the reef using conventional drilling techniques. The TBW size core is larger diameter than BQ; 44.9 mm in diameter. Deflection drilling is completed by inserting a one degree wedge into the mother hole which deflects the drill bit to one side and provides for additional reef intersections a small distance away. At Lonmin, three deflections are drilled using wedges inserted first 5 m, secondly 10 m and then thirdly 15 m above the reef hangingwall position. Once these three deflections have been drilled, a fourth deflection is drilled out of the hole formed by the third deflection from 5 m above the reef hangingwall position. Deflection drilling continues by inserting further wedges until four complete and representative cores of the reef intersection are obtained. These short deflections provide a cluster of intersections a small distance away from each other. In a normal four deflection situation, assuming the wedges were positioned as mentioned previously, the furthest away from the mother hole that the fourth intersection can be is calculated to be approximately 0.35 m (**Figure 2-2**). **Figure 2-3** is a picture taken underground at Lonmin's K4 shaft showing the position of the deflection holes relative to each other. In this borehole (RS111), the reef is intersected by the deflection holes at between approximately 6 cm and 35 cm apart.

Calculated Maximum Distance (cm)
of deflection from mother hole on reef plane

	d1	d2	d3	d4
HW	8.7	17.5	26.2	34.9
FW	10.5	19.2	27.9	36.6

$$d1_h = d2_h = d3_h \approx d4_h$$
$$d1_f = d2_f = d3_f \approx d4_f$$

Note: Not to Scale
Horizontal distance greatly exaggerated
Reef Plane adjusted to horizontal

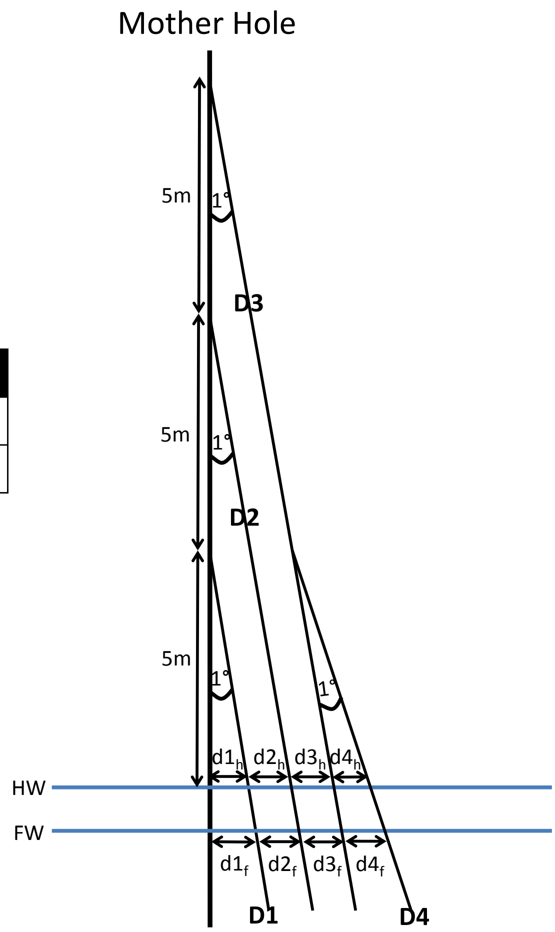


Figure 2-2 Illustration of maximum theoretical intersection spacing in the reef plane for a normal four deflection situation.

Reef intersections that are badly broken or suffer core loss through poor drilling practice are rejected. Standard practice at Lonmin is to collect four TBW size cores of the reef, three of which are sampled for resource evaluation and the fourth is set aside for destructive metallurgical or rock engineering test-work.

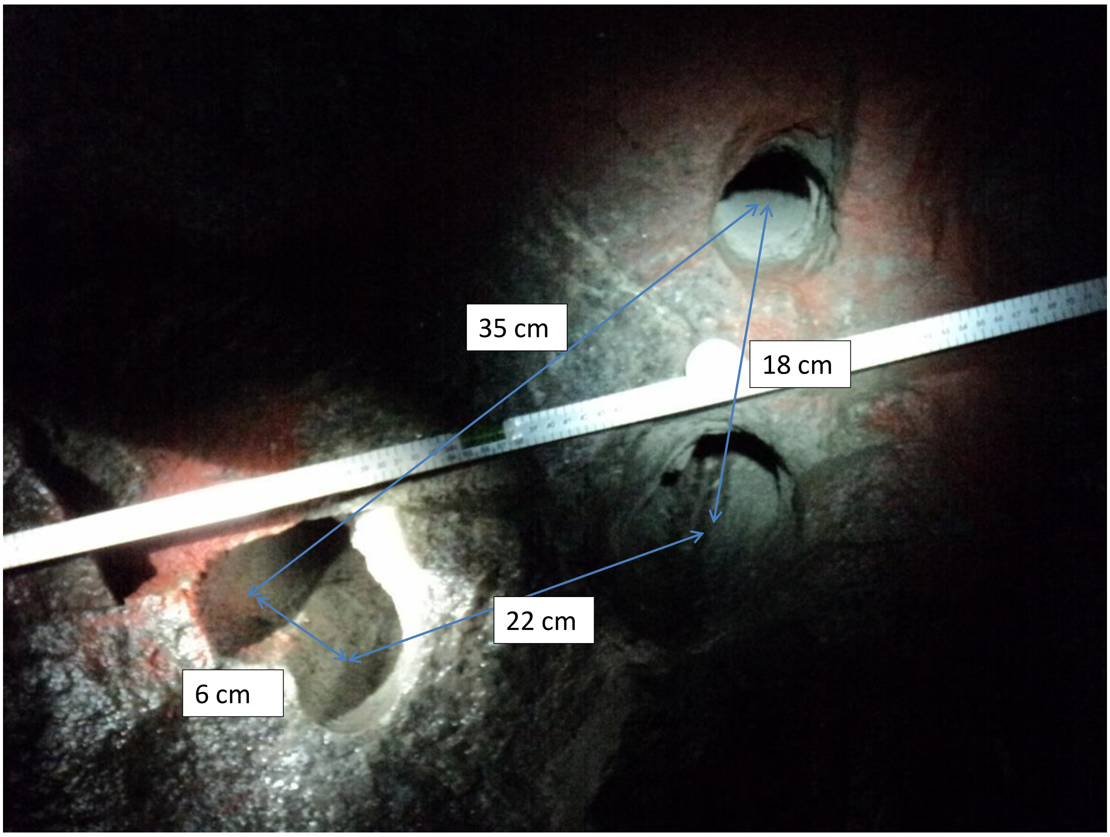


Figure 2-3 Photo of actual intersection of borehole RS111 in a Raise at K4 Shaft (view is looking approximately upwards into the hangingwall.).

The reef intersections are logged by suitably qualified and experienced geologists in ideal working conditions at the Lonmin exploration core yard outside Marikana. All sampling and logging work is checked and verified by a Senior Geologist before cutting takes place and later dispatch to the laboratory. The cores are marked up for sampling using exactly the same pattern and layer coding as used for the channel samples (**Figure 2-5**). Cores are cut longitudinally in half; one half is used for assay and the other half is kept for reference or for future check sampling. The samples are weighed in air and water for Specific Gravity determination at the Lonmin core yard. The volume of sample per cm core length is approximately 7.9 cm^3, similar to the 8.8 cm^3/cm yielded by a cut channel sample.

The samples are sent to a number of independent commercial laboratories that are suitably accredited to do the assay and that have been selected on the basis of their demonstrated ability to provide the high quality of service required. Each sample sent for assay is allocated a unique reference number so that its identity is maintained throughout the process through to final return of assay values. A comprehensive quality assurance and quality control (QAQC) protocol exists consisting of the insertion of blank samples, standard reference materials and duplicate samples. Together with the commercial laboratories own QAQC protocol this ensures that the assays of the surface borehole cores are of the highest possible quality. Assays are later received from the laboratory in digital and hard copy certified form and the digital data is captured directly into Lonmin's borehole database using an automated script.

A variation from the standard borehole sampling technique is used for evaluation of near surface mineralisation, less than 60 m depth, that is typically targeted for open pit mining. The weathered nature of the near surface material leads to poor core recovery using the small diameter BQ drilling. Instead a larger diameter HQ (63 mm diameter) core is produced. It is not practical to drill deflections at this depth and using the large hole diameter, so instead each HQ size core is cut longitudinally in quarter yielding a sample volume of approximately 7.8 cm^3/cm length of core, which is similar to that of the TBW half core. The same sampling process takes place as for the TBW size core and each of three of the quarters are sent separately for analysis with the fourth remaining for reference purposes and/or future check sampling or metallurgical test work.

2.3 Assaying of PGE's

A number of different techniques are used to assay for PGE's. The three methods that have been most commonly used for the Lonmin samples are the following:

- Assay of 4E by lead-silver fire assay followed by low and high temperature cupellation (HTC) with a gravimetric finish (the 4E procedure).
- Assay of Pt, Pd, Rh and Au by lead-silver and lead-palladium fire assay followed by low temperature cupellation (LTC) with an Inductively Coupled Plasma (ICP) finish (the 4T procedure).

- Assay of Pt, Pd, Rh, Au, Ru and Ir (and sometimes Os) by nickel sulphide fire assay followed by a hydrochloric acid leach with an ICP finish (the 6E or 7E procedure).

In all cases the samples are prepared by crushing of the sample to nominal 2 mm and then milling down to a target of better than 90% passing through 75 µm. An aliquot of normally between 25 g and 30 g is taken from the milled and homogenised sample pulp.

All of the channel sample assays were conducted at the Lonmin laboratory using a 4E procedure.

The surface borehole assays were carried out at a number of different laboratories using one of either the 4E, 4T or 6E procedures (**Figure 2-4**). In 2011, a re-assay exercise was carried out on the pulp rejects in order to gather more data on individual PGE grades in the areas that only had 4E data. This provided a source of duplicate assays that are useful in comparing the nugget effect between those assayed by 4E and 6E procedures. The surface borehole data was coded by assay method (4E, 4T and 6E) so that statistical analysis could be performed for each assay method separately.

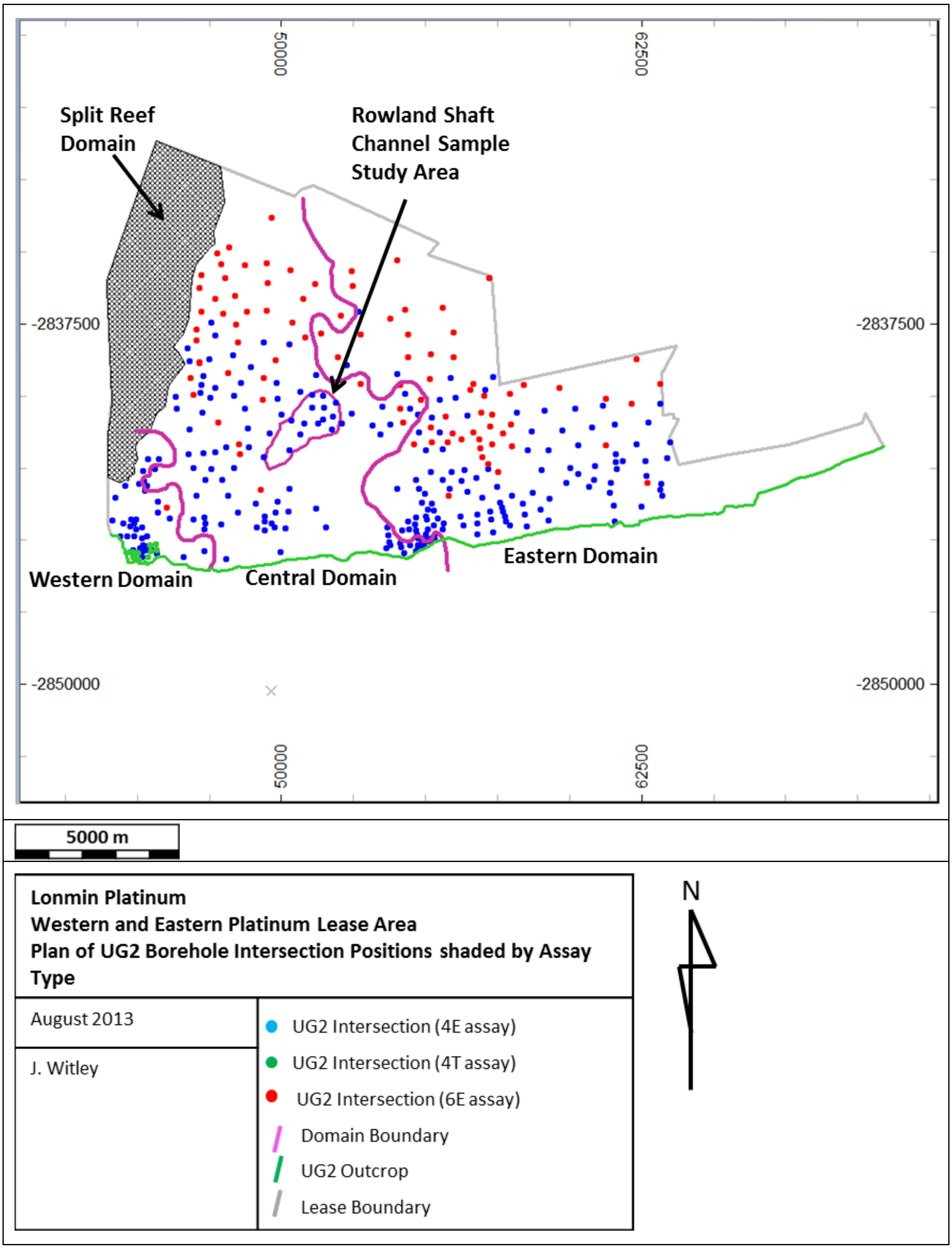


Figure 2-4 Borehole UG2 Reef intersections shaded by assay type

The 4E procedure

This method is the standard method used at Lonmin for the channel sample assays. Approximately eleven thousand channel samples are assayed at the Lonmin Laboratory at Marikana every month, thus the method needs to be cost effective with high throughput and fast turnaround times. The method is summarised as follows (compiled from Lonmin internal procedures, Randolph (undated) and Lotter et al, 2000):

- The aliquot is accurately weighed.
- The aliquot is mixed with a flux containing lead compounds and a source of carbon as a reducing agent (normally maize meal).
- Silver nitrate solution is added. Silver prevents losses of some of the PGE's during the procedure.
- The mixture is fused for 1.5 hours at around 1200°C. During this process the PGE's are collected into the molten lead, which separates from the gangue/slag on cooling.
- The lead button (containing the PGE's and silver) is removed from the slag (de-slagging).
- The lead button is subjected to low temperature cupellation for 50 minutes at around 950°C so that the lead is removed by a combination of absorption into the calcined magnesite pots and volatilisation so that a silver-PGE prill remains.
- Lead foil is added to the prill and then subjected to high temperature cupellation for 90 minutes at around 1350°C in order to remove the silver and any remaining volatile Ru, Ir and Os that has not already been burnt off in the low temperature cupellation stage. This part of the process is critical as incorrect furnace conditions can lead to excessive losses of PGE's or retention of silver in the prill.
- The remaining prill of PGE is weighed on a micro balance and, based on the sample mass, the grade of the sample is determined. The lower limit of the assaying method is dependent on the size of the PGE prill that can be visually detected by the weighing staff. Experience at Lonmin has shown that a sample grade of less than approximately 0.4 g/t 4E may report "no prill detected", which is the human visual equivalent to below detection limit of equipment.

This method provides for a single combined value for 3PGE+Au. Losses are known to occur that are highest in the low grade samples and are lowest in the high grade samples. Randolf (undated) estimated that the losses are as follows:

> 100g/t ± 3% relative loss

> 5g/t ± 5% relative loss

< 1g/t ± 15% relative loss

Losses of a similar magnitude on assays of Merensky Reef at Anglo American Platinum Limited's Rustenburg Mines have been reported by Lotter et al (2000).

The 4T procedure

This procedure involves a duplicate assay using a lead-silver collector for the first leg and a lead-palladium collector for the second leg. The advantages of this method is that the volatilisation of Rhodium in the cupellation stage is minimised and that two assays for gold and platinum are provided resulting in enhanced precision.

A quantity of silver or palladium, usually > 2mg, is added to the sample/flux mix and this is fused as in the normal lead collection method.

- The lead is removed by low temperature cupellation, leaving either a silver or palladium prill.
- The prills are dissolved in acid and individual precious metal concentrations are determined using either an Inductively Coupled Plasma- Optical Emission Spectrograph (ICP-OES) or Mass Spectrograph (ICP-MS).
- Pt, Pd and Au are determined in the silver prill and Rh, which is insoluble in silver, is determined in the Pd prill.

This method provides for quantitative analyses of Pt, Pd, Au and Rh and if these are summed gives the 4T result.

The 6E procedure

This procedure is also a fire assay, but instead of lead a nickel sulphide compound is used to collect the precious metals. The cupellation stage is not necessary and volatilisation of PGE's is avoided.

- The sample is mixed with a flux containing nickel and sulphur

- As for the lead fire assay the sample is fused at 1200°C.
- The PGE's are collected in nickel sulphide that is formed during the fusion, which separates from the slag on cooling.
- The slag is removed from the button
- The button is milled followed by dissolution and filtration
- The individual precious metals are then determined from the leached residue using ICP-OES or ICP-MS.

Summary of Assay Techniques

Each of the three commonly used procedures has advantages and disadvantages that are summarised in **Table 2-1** (summarised from Randolph (undated) and personal communications with laboratory staff). Differential losses of individual PGE's occur during the assay procedure and summation of the Pt, Pd, Rh and Au to provide a 3PGE+Au grade using different assay methods may introduce an additional source of error. Some companies use a Fire Assay Correction Factor (FACF) in order to correct for losses encountered during the 4E procedure to attempt to make it more comparable to assays derived from a 4T or 6E procedure. Lonmin does not use a FACF, mainly because the FACF varies at different grades and furnace conditions and as such may introduce an additional source of error.

Table 2-1 Advantages and Disadvantages of the three commonly used PGE assay procedures for Lonmin samples

Method	Advantages	Disadvantages	Comments
4E	High throughput. Fast turnaround times. Does not need expensive instrumentation. Can be used for process control.	Excessive loss of the more volatile PGE's (particularly Rh). Only provides combined 3PGE+Au assay. High detection limit. Very sensitive to HTC, which can result in over- or under-assay. Not recommended for metallurgical accounting.	Used by Lonmin for all channel samples. Assays conducted by the Lonmin Laboratory.
4T	Accurate. Low detection limit. Reasonable cost. Elimination of HTC - losses and retentions are minimised and power consumption reduced. Provides individual grades for Pt, Pd, Rh and Au. Better precision for Pt and Au. Rh losses are reduced. Can be used for metallurgical accounting.	Much lower throughput than 4E. Longer turnaround times. Requires expensive instrumentation. More expensive to perform than 4E. Requires 2 fusions.	Used in some exploration boreholes at Marikana (particularly pre-2005). Assays conducted using independent and accredited commercial laboratories.
6E/7E	Very accurate. Low detection limit. Elimination of LTC and HTC - reduced power consumption and PGE losses are minimised. Provides individual grades for Pt, Pd, Rh, Au, Ru and Ir, and can determine Os. Can confidently be used for metallurgical accounting.	Much lower throughput. Longer turnaround times. Requires expensive instrumentation. Most expensive to perform.	Used in all exploration borehole samples at Marikana since 2005. Assays conducted using independent and accredited commercial laboratories.

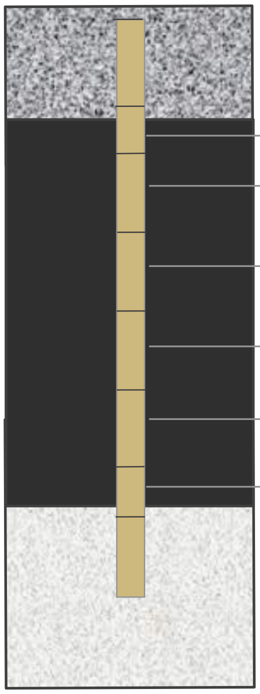
The 6E procedure is the most accurate of the three methods, but is also the most expensive. The 4E method provides for high volumes of assays at relatively quick turnaround time, but control of the assay is challenging resulting in varying degrees of accuracy and a generally lower assay for 3PGE+Au than may be achieved using the 4T or 6E procedures.

Given that the different assay methods provide for a different quality of assay it follows that the variance of the data may vary and that the measurement error portion of the nugget effect may also vary. This could have an impact on the accuracy of the Mineral Resource estimates.

2.4 Data Preparation and Validation

The channel data had previously been validated for acceptance into the mine database and thus were all complete intersections that had been checked by the Evaluation Department for inconsistencies. A further modification to the data that was made was that any data situated within a mapped pothole was rejected. This was necessary as some of the more subtle effects of potholes, in particular reef thinning at their edges, will not be recognised by the validation techniques used by the evaluation department.

Composites of the channel samples and borehole samples were calculated so that a single value for each attribute exists for each complete reef intersection (**Figure 2-5**). For mine accounting the process is slightly different, whereby the grades of the individual layers are estimated for use in under-break assessments. This is the traditional “histogram” approach used for metal accounting, an approach that is outside of the scope of this study.



Description	Layer Code	Thickness (m)	SG	4E g/t	Thickness *SG*4Eg/t	Thickness *SG
HW Waste	81	0.2	3.2	0.2	-	-
2cm HW 8 cm UG2	80	0.1	3.8	2.0	0.760	0.380
UG2	79	0.2	3.9	4.0	3.120	0.780
UG2	78	0.18	3.9	7.0	4.914	0.702
UG2	42	0.19	3.9	4.0	2.964	0.741
UG2	41	0.2	3.9	6.0	4.680	0.780
8cm UG2 2 cm FW	40	0.1	3.8	10.0	3.800	0.380
FW Waste	39	0.2	3.2	0.5	-	-
Total	**UG2**	**0.97**	**3.88**	**5.38**	20.238	3.763

Each section made up of several sample assays now represents a single point with one grade, thickness and density value

Figure 2-5 Illustration of sample pattern for UG2 Reef and the compositing calculation that is used to produce a single value for each reef intersection.

The borehole data within the study area consisted of partially validated data for 400 boreholes drilled from surface. Furthermore, some of the faulted and potholed intersections were included in the database and these had to be identified and filtered out as part of this study. This validation proceeded as follows:

- Visual inspection for positional errors.
  - Boreholes that plotted away from the lease area.
  - Boreholes that plotted away from expected plane of intersection (elevation errors)
- Lateral composite difference approach:
  - Intersections were composited into single reef intersections as normal.
  - Lateral composites for each borehole were calculated (i.e. the intersections obtained from deflection drilling were all averaged into a single composite). The absolute percentage difference

between each intersection and the borehole average was calculated for PGE (g/t) and true thickness.

- Any intersections with high difference from the mean value and boreholes with high variance were examined for faulting, potholing and other data issues.

A histogram showing percentage difference of deflection to the average of the borehole value shows a tail giving the histogram a positive skew. These tail values were examined and many of them were found to be a result of unrepresentative intersections. The histogram showing the validated data differences is much less skewed with most of the outlier data being rectified or discarded if found to be invalid (**Figure 2-6**).

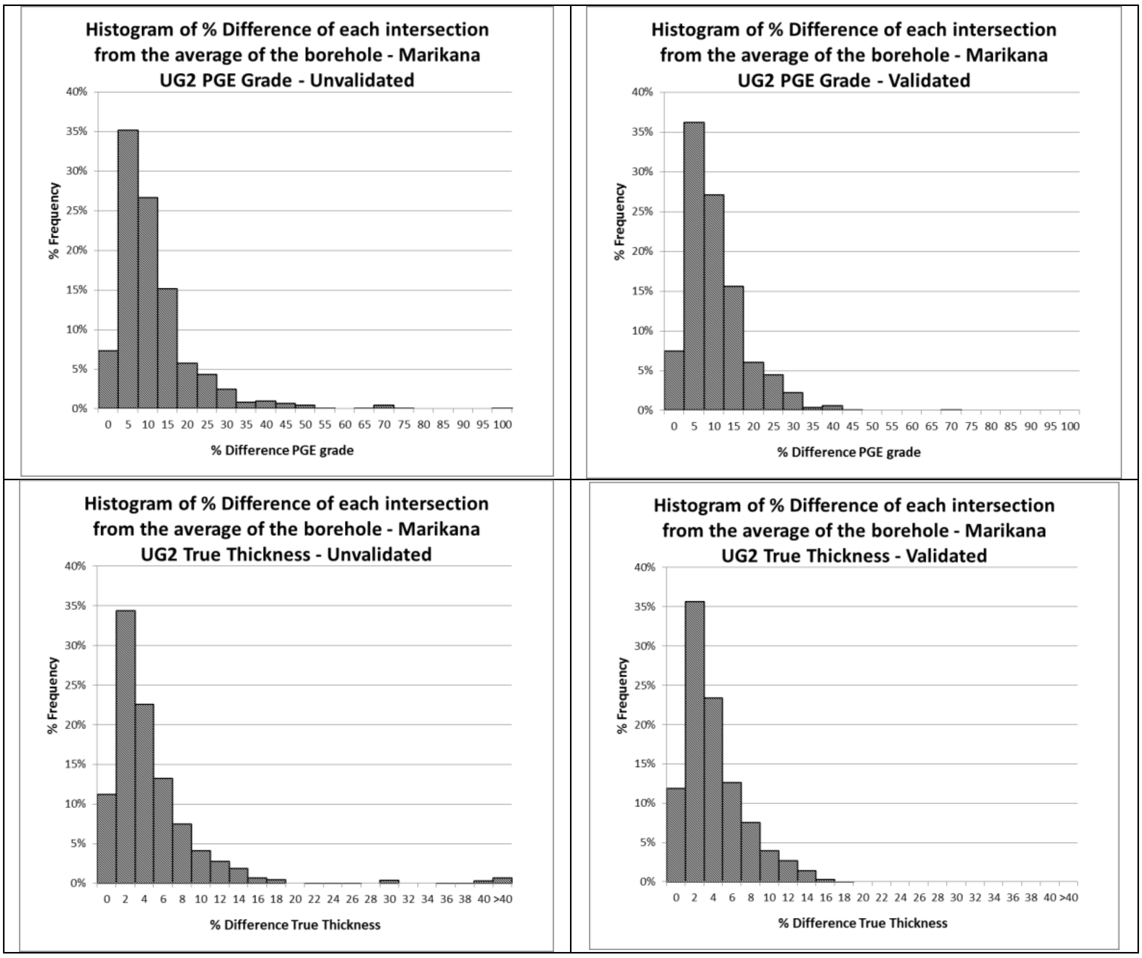


Figure 2-6 Histograms of absolute percentage difference of each deflection from its borehole mean value (un-validated data on left, validated data on right)

The validation process identified 30 boreholes that had geological or data problems (Appendix 1). These boreholes were rejected from the dataset. In addition, there were seven intersections identified that had high variability in the borehole as a result of database errors. These were fixed in the database and then included in the dataset.

The lateral composite difference approach, as described earlier in this report, is a useful tool in determining whether a borehole intersection is representative. The effects of potholes are often subtle and may not be identified in the core-yard by all geologists. The high variability over very short distances apparent in potholed reef is often identified in multiple deflections by changes in the internal stratigraphy, such as the amount of internal pyroxenite or anorthosite, large thickness variations, varying grade distribution between deflections and irregular core angles.

Deflection drilling was not carried out on many of the boreholes and as a result there are only single intersections for these holes. In some instances, deflections were not drilled as the mother hole was clearly affected by a geological condition, such as a pothole. Several of these were not sampled as it was anticipated that the grades of these intersections would not be of use in Mineral Resource estimation. Single intersections that are unusual in grade or thickness should be treated with caution in an estimation data set.

2.5 Selection of detailed study area

The Marikana Lease area together with the valid UG2 borehole and channel sample intersections, Rowland Shaft channel sample study area, the Split Reef area and the location of significant faults are shown in **Figure 2-7**.

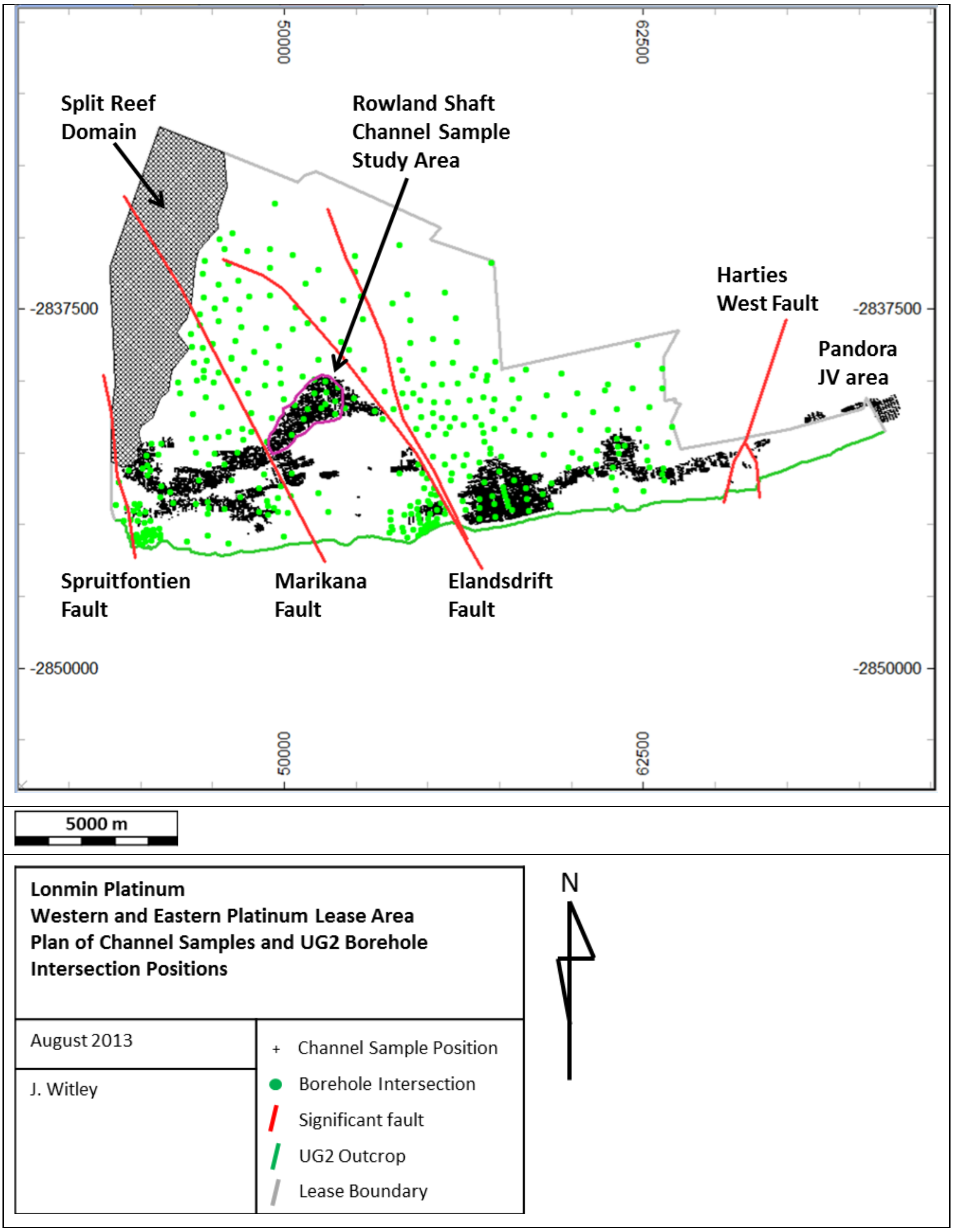


Figure 2-7 Location of Channel Sample and Borehole UG2 Reef intersections.

The UG2 Reef at Marikana falls into a number of thickness domains. For the purpose of this study the data was divided into three domains; a thick domain to the east, a narrow central domain and a thick western domain (**Figure 2-8**). The split reef domain in the west was not considered, there being too few data.

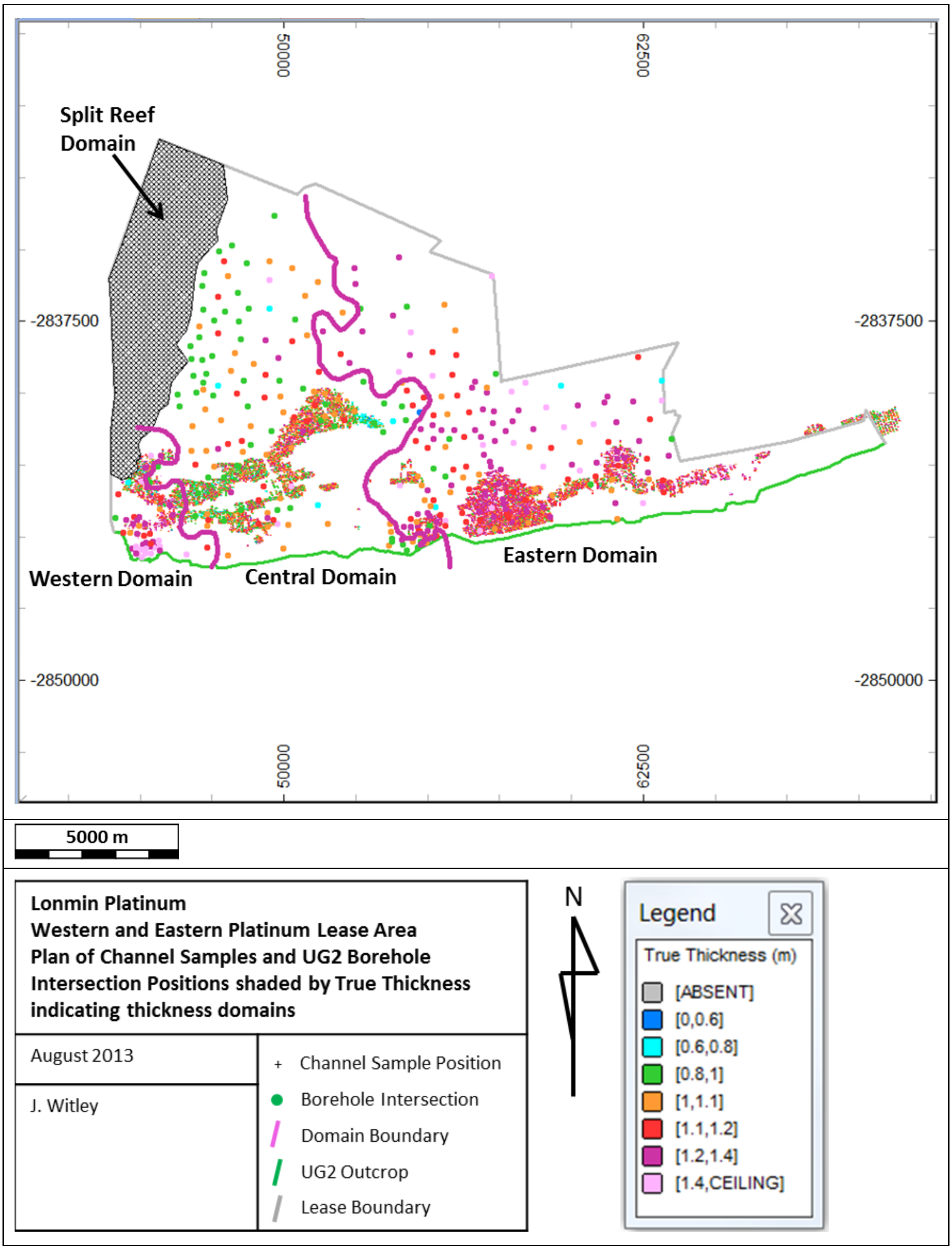


Figure 2-8 Location of thickness domains and UG2 intersection data shaded by true thickness.

Prior to 2000, underground channel sampling at Lonmin was by chip sampling using a hammer and chisel. It has been recognised that the quality of the chip sample is less than that of a cut sample (Magri and McKenna, 1986) and Lonmin phased in a change from chip sampling to diamond saw cut sampling over a number of years. By 2005, all of the mines at Marikana were using diamond saws to collect the samples. The last areas to abandon chip sampling were the eastern incline shafts comprising Eastern Platinum Limited (EPL), these being E1, E2 and E3 inclines. One of the earlier mines to convert to diamond saw cut sampling was Rowland Shaft and therefore a large amount of channel sample data collected by means of diamond saw area available at Rowland Shaft.

Rowland shaft has relatively simple UG2 Reef geology compared to areas further to the west. The abundance of potholes is lower than the western areas and the reef itself is not complicated by thin layers of chromitite in the immediate hangingwall that can be incorrectly identified and erroneously included with the UG2. A large occurrence of IRUP exists in the eastern area of Rowland, which locally has higher reef variability and forms a separate reef thickness domain. Therefore, due to the less complicated geology and high quality sampling it was considered that the best quality data should be from the western area of Rowland and thus this area was chosen for the channel optimisation study (**Figure 2-7** and **Figure 2-9**).

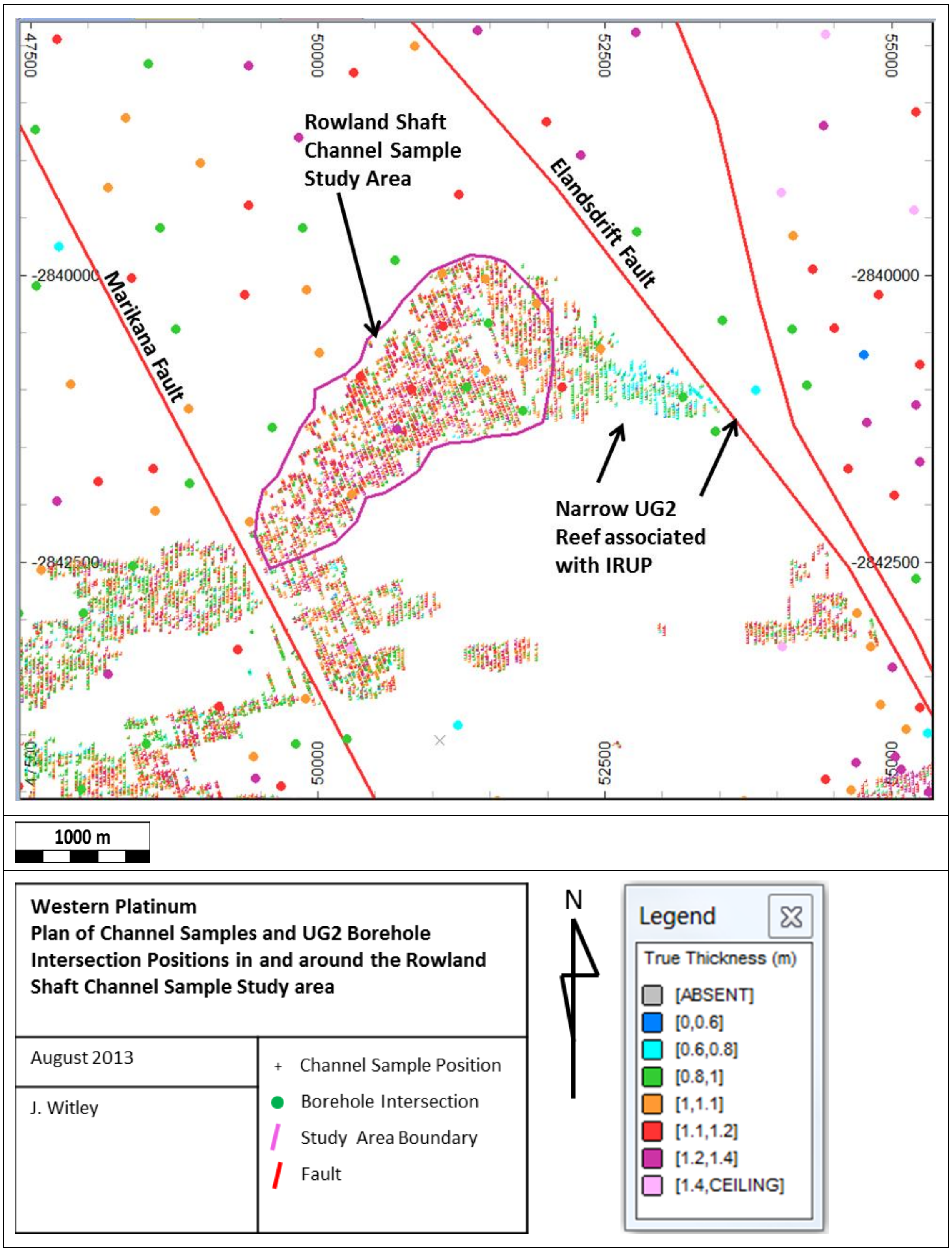


Figure 2-9 Location of thickness domains and UG2 intersection data shaded by true thickness.

The surface borehole cores have generally been diligently logged and sampled by qualified geologists and all the valid data over the Marikana property were used for the deflection study. There were only 13 surface boreholes drilled within the Rowland Shaft channel sample study area.

3. STATISTICAL AND GEOSTATISTICAL ANALYSIS

3.1 Statistical Analysis

The PGE grade and true thickness attributes were examined for each area, sample type and assay type. The data were de-clustered to 500 m by 500 m squares using a technique that averages the samples in a square by weighting according to the number of data in each square. This accounts for the borehole deflections and removes any clustering effects caused by the dense sampling in the channel sampling areas. The 500 m by 500 m cell de-clustering size was chosen as it is approximately the average borehole spacing. Statistical tests at 95% confidence were carried out in order to assess the significance of differences in PGE grade and true thickness between domains and assaying method.

Summary statistics of the borehole and channel sample data are shown in **Table 3-1** for PGE grade and **Table 3-2** for true thickness. Histograms of the channel sample data for the Central and Eastern Domains are shown in **Figure 3-1**.

Table 3-1 De-clustered summary statistics for PGE Grade

Domain	Sample Type	Assay Type	Number of Composites	Mean (g/t)	Variance (g/t^2)	CV	Skewness
Central	Borehole	6E	118	5.69	0.75	0.15	0.82
	Borehole	4E	273	5.50	1.49	0.22	0.65
	Channel	4E	13004	5.22	1.64	0.25	1.21
Rowland*	Channel	4E	3849	5.31	1.53	0.23	1.47
Eastern	Borehole	6E	141	4.98	0.84	0.19	0.43
	Borehole	4E	93	5.03	1.18	0.22	0.81
	Channel	4E	13956	4.90	1.33	0.24	1.01
Western	Borehole	6E	2	5.07	0.40	0.12	-
	Borehole	4E	76	5.24	1.36	0.22	0.08
	Channel	4E	3523	4.97	1.60	0.26	0.87

Note Rowland channel samples are a sub-set of the Central Channel sample data

Table 3-2 De-clustered summary statistics for true thickness

Domain	Sample Type	Assay Type	Number of Composites	Mean (m)	Variance (m^2)	CV	Skewness
Central	Borehole	6E	118	1.02	0.027	0.16	2.63
	Borehole	4E	273	1.01	0.021	0.15	-0.32
	Channel	4E	13004	1.05	0.026	0.15	-0.11
Rowland*	Channel	4E	3849	1.10	0.017	0.12	0.16
Eastern	Borehole	6E	141	1.24	0.037	0.16	-0.16
	Borehole	4E	93	1.23	0.041	0.17	2.02
	Channel	4E	13956	1.15	0.022	0.13	-0.21
Western	Borehole	6E	2	1.37	0.046	0.16	-
	Borehole	4E	76	1.26	0.042	0.16	-0.31
	Channel	4E	3523	1.12	0.035	0.17	-0.51

*Note Rowland channel samples are a sub-set of the Central Channel sample data.

A. PGE | B. True thickness

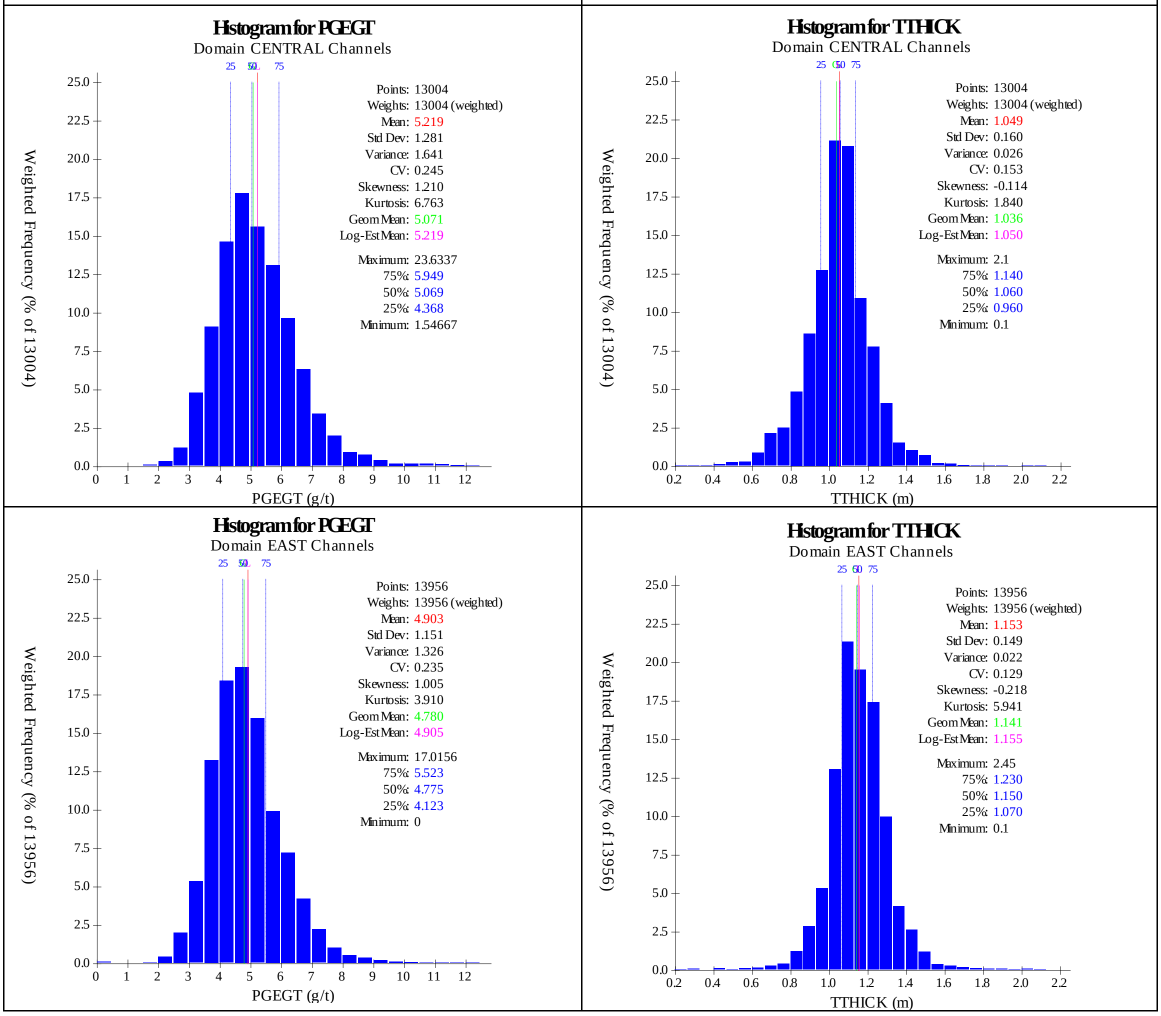


Figure 3-1 Histograms of PGE grade (A) and true thickness (B) for the Central (above) and Eastern (below) domains.

Both the PGE and true thickness histograms exhibit a near bell shape, the PGE grade histograms being slightly positively skewed and the true thickness histograms being slightly negatively skewed with the exception of the Central borehole samples assayed by the 6E method and the Eastern boreholes assayed by the 4E method.

The de-clustered average borehole grades tend to be slightly higher than the de-clustered average channel sample grades in all three domains; however at 95% confidence the means are not significantly different. The boreholes assayed by the 6E method do not consistently have higher mean grades than those assayed by the 4E method. True thickness for boreholes assayed by 4E or 6E method are within 1 cm of each other except for Western domain where only two borehole intersections were assayed by the 6E method. Mean UG2 Reef true thickness is 4 cm higher for the channels compared with the boreholes in the Central domain and 9 cm less for the Eastern domain. Within these two domains, the differences in the means are not significant between sample types. This is expected as true thickness has no dependence on the assay method. The difference between the borehole thickness and channel sample thickness in the Western Domain of 0.14 m is significant. Much of the drilling in the Western domain took place in and around a feature known as the Marikana pothole. This geological structure is a mega-pothole where the UG2 Reef thickness is locally higher, partly due to the UG2a Markers lying directly on the UG2.

The PGE grades assayed by the 6E method are consistently less variable than those assayed by the 4E method. The difference in the variance is significant at 95% confidence. Co-efficient of variation (CV) is low for both PGE grade (between 0.15 and 0.26) and true thickness (between 0.13 and 0.17).

The mean PGE grade of the Central channel intersections is significantly higher at 95% confidence than that of the Eastern intersections (5.22 g/t Central versus 4.90 g/t Eastern). The Eastern UG2 Reef tends to be thicker (1.05 m Central versus 1.15 m Eastern) and at 95% confidence the difference between the mean thicknesses is significant.

In summary, both the mean PGE grade and true thickness of intersections in the Eastern and Central Domains are different from one another at 95% confidence.

The PGE grades obtained through the 4E method are significantly more variable than those obtained using the 4E method but the mean values are comparable.

3.2 Estimation of the Semi Variogram and Nugget Effect

Semi-Variogram Model for Rowland UG2 (base case)

The UG2 changes in its grade and thickness characteristics over large distances on the scale of kilometres. Areas need to be selected that are small enough to be stationary but large enough to contain sufficient data to be able to produce a reliable variogram model without trend. Clearly distinguishable trends are observed over the 20 km strike Marikana lease these being a gradual increase in thickness from east to west and a decrease on 4E grade. Down dip trends also occur as well as abrupt changes across larger structures such as the Marikana Fault and the Elandsdrift Fault that bound the Rowland Shaft Block on its western and eastern side respectively. Within Rowland Shaft block, a large IRUP intrusion occurs in the eastern section of the mine where changes in dip and strike occur and the UG2 is narrower. The channel sample data that was selected for calculation of experimental semi-variograms was therefore restricted to the west and central areas of Rowland away from the IRUP occurrence and eastwards of the Marikana Fault. The Rowland semi-variogram was calculated using the same data as were selected for the channel sample study.

The channel samples are normally spaced approximately 10 m apart in the dip direction; however there are several instances where sampling has been closer spaced. The nugget effect can be estimated by extrapolation of the short lag experimental data (**Figure 1-6** and **Figure 1-7**). An alternative to extrapolation of the nugget effect is by calculation using deflections drilled from surface boreholes. The short reef intersection deflection drilling from surface boreholes at Marikana typically intersect the UG2 Reef several times a very short distance apart (in the order of tens of centimetres) and therefore should define the ultra-short range semi-variogram structure. As there are few surface boreholes in the Rowland shaft area, boreholes were selected from a larger area including and outside of Rowland Shaft block. The borehole data were only used to estimate the proportion of nugget effect semi-variance, as their use in semi-variogram calculation over longer ranges will introduce trend. Therefore, the nugget effect structure of the semi-variogram was calculated using surface boreholes and the rest of the semi-variogram experimental data were calculated using the channel sample data. The non-declustered data were used.

Snowden Supervisor software was used for calculating and modelling the semi-variograms. The nugget effect was calculated from the surface borehole deflection data using a Microsoft Excel spreadsheet. The data were set to zero elevation and semi-variograms were calculated in a two dimensional plane, the dip being slight and constant in this area. Traditional omni-directional semi variograms were calculated, the data showing no significant directionality at the scale of the Rowland Shaft area. Semi-variograms were calculated without any data transformation, the distribution of both the 4E grade and true thickness data being close to normal. A check was performed using a Normal Scores transformation and the experimental data were found to be almost identical to those calculated with untransformed data. Two lag separations were used; 10 m for the short range area of the semi-variogram until a sample spacing of 40 m and a 30 m lag from then on. Normalised semi-variograms were used whereby the sill is set to one, relative to the estimated sample variance.

The experimental semi-variogram data was modelled with three spherical structures and a nugget effect for true thickness, 4E grade and the accumulation of grade and thickness (PGEcmgt). The models are robust and the semi-variograms appear very reliable with a number of experimental points clearly defining a spherical structure. The negligible trend in the area is apparent by the experimental data resting very close to the sill once the sill has been reached, without continuing to increase in value at larger lags. The proportion of nugget effect variance to total variance of the borehole data is 56% for 3PGE+Au grade in the central area assayed by the 4E assay technique and this was applied to the Rowland semi-variogram. This compares with a nugget effect of 0.58 should extrapolation have been used (**Figure 1-6**). The calculated normalised nugget effect for the surface boreholes that fall within the Rowland study area is 0.61, which is similar to the overall borehole nugget effect for the Central area of 0.56.

Although the nugget effect for 4E g/t makes up a large proportion of the total variance, the actual variance, 1.493 $(g/t)^2$, is low. Continuity is demonstrated for up to 170 m, however 92% of the variance has been accounted for in the first 45 m. The range of the true thickness semi-variogram is similar to that of 4E g/t, although the nugget effect only contributes to 17% of the variance and the longer range portion of the semi-variogram accounts for a larger proportion of the variance than 4E, implying better continuity. The semi-variograms for 4E grade and 4E accumulation are very similar.

The semi-variograms for 4E grade, true thickness and grade accumulation (PGE cmgt) are shown in **Figure 3-2** and the model parameters in **Table 3-3**.

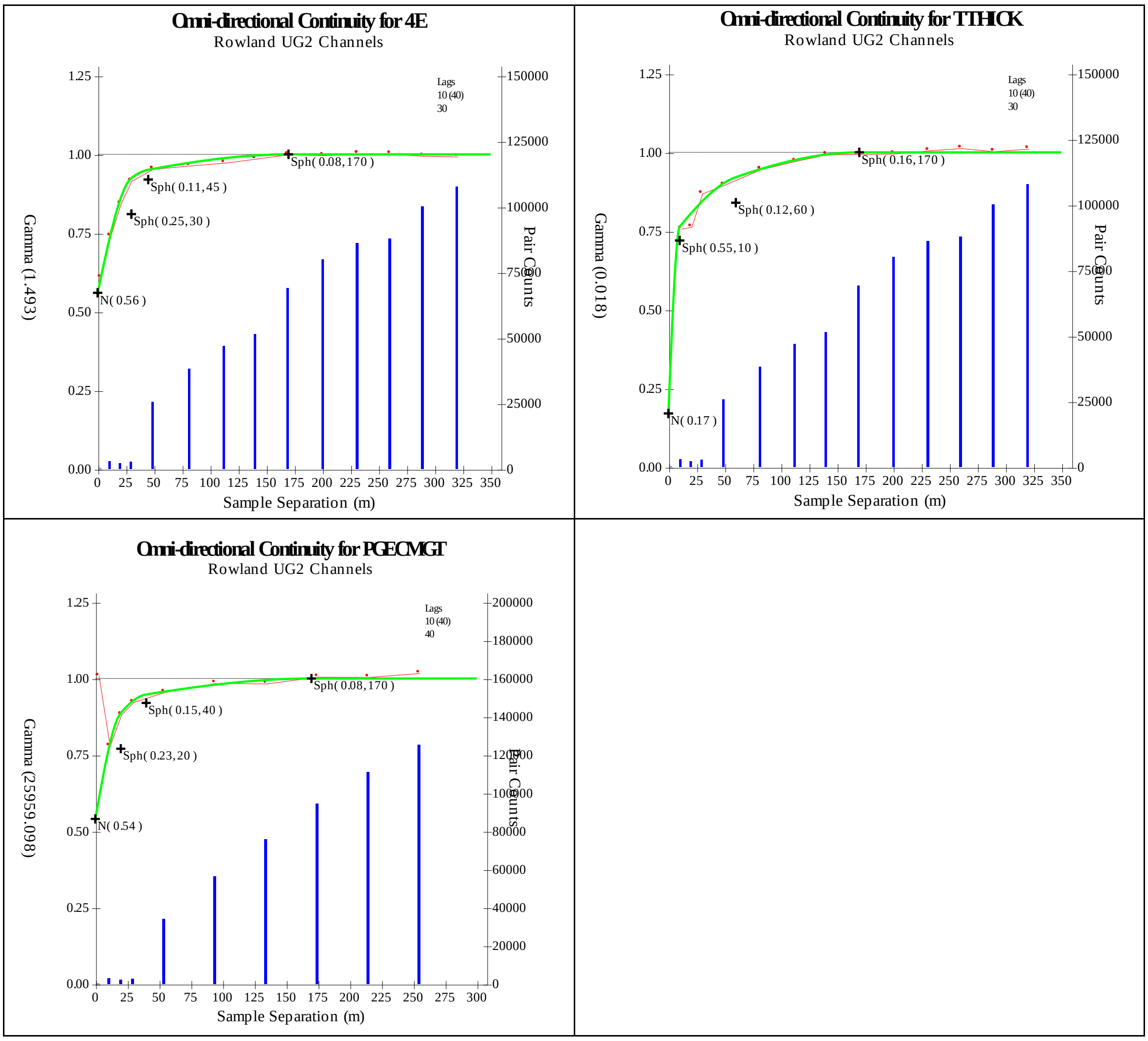


Figure 3-2 Semi-variograms for 4E grade, true thickness, and PGE cmgt – Rowland Shaft.

Table 3-3 Semi-Variogram Parameters for Rowland UG2.

Attribute	Nugget Effect (C0)	Range of First Structure (R1)	Sill 1 (C1)	Range of Second Structure (R2)	Sill 2 (C2)	Range of Third Structure (R3)	Sill 3 (C3)
4E g/t	0.56	30	0.25	45	0.11	170	0.08
True Thickness	0.18	10	0.55	60	0.12	170	0.15
4E cmg/t	0.54	20	0.23	40	0.15	170	0.08

The Rowland UG2 semi-variogram model was used as the base case semi-variogram for the geostatistical test-work later on in this report. The nugget effect is examined in greater detail in the following section, the results of this analysis being used to evaluate any optimisation of the estimation that may arise from a different semi-variogram to the base case.

Estimation of the nugget effect for Marikana UG2 Reef

Due to the high proportion of the nugget effect that is indicated by the deflections, smoothing will occur resulting in low confidence grade estimates that are close to the domain average, thus limiting any opportunity for selectivity. Any reduction in nugget effect will result in a better estimate and this warrants more detailed investigation.

Using the multiple intersections derived from the deflections, the nugget effect was calculated by the sum of the squared differences between each of the possible pairs in each hole divided by twice the total number of pairs. The nugget effect was calculated in this way by means of a spreadsheet for the eastern area and the western area of Marikana (excluding the split reef domain and the narrow reef area in the south west of the property). Separate nugget effects were calculated for the following data types:

- The deflection grade data that was obtained through the 4E assay technique. This is older data collected prior to about 2002.

- The deflection grade data that was obtained through the 6E assay technique. This is the newer data that was collected from around 2002 until present day.
- Ten HQ size diamond drillholes were drilled in the U2 pit in the south western area of the Marikana lease in 2011. These relatively large diameter cores were cut in quarter and between two and four of the quarters were assayed separately using the 4T process.
- In 2011, a re-assaying programme took place with the purpose of providing more individual PGE and Au (Pt, Pd, Rh, Ru, Ir and Au) assays in the areas where the drilling was older and only single 4E assay data were available. Fifteen UG2 deflections were re-assayed. This provided a comparison of the 4E method with the 6E method.

The deflection data gives information on the grade and width variability a few 10's of cm apart and provides information on the combined nugget effect and ultra-short range variability. The HQ quarter core produces data the same distance apart as the sample support and therefore provides a measure of nugget effect without any range but including the sample and assay error component.

The nugget effect and summary statistics for each assay method, area and attribute of interest are shown in **Table 3-4**,**Table 3-5** and **Table 3-6**. The nugget effect calculated by using the core duplicates is shown in **Table 3-7** and those derived from the pulp duplicates in **Table 3-8**. The normalised C0 is relative to the variance of the deflection data.

Table 3-4 Statistics of Deflections and Nugget Effect – PGE Grade

	n	Mean (g/t)	Variance (g/t^2)	SD (g/t)	CV	n pair	C0 (g/t^2)	C0 normalised
Central 4E	236	5.61	1.53	1.24	0.22	229	0.85	0.56
Eastern 4E	279	5.06	1.08	1.04	0.21	312	0.57	0.53
Western 4E	67	5.02	1.18	1.09	0.22	60	0.42	0.36
Total Area 4E	**582**	**5.28**	**1.35**	**1.16**	**0.22**	**601**	**0.66**	**0.49**
Central 6E	115	5.76	0.75	0.87	0.15	113	0.40	0.52
Eastern 6E	140	5.06	0.76	0.87	0.17	136	0.40	0.53
Western 6E	2	-	-	-	-	-	-	-
Total Area 6E	**257**	**5.37**	**0.88**	**0.94**	**0.17**	**250**	**0.40**	**0.46**
All Central	351	5.66	1.28	1.13	0.20	342	0.70	0.55
All Eastern	419	5.06	0.97	0.98	0.19	448	0.52	0.53
All Western	69	5.02	1.16	1.07	0.21	61	0.42	0.37
Total Area	**839**	**5.31**	**1.20**	**1.10**	**0.21**	**851**	**0.58**	**0.48**

Table 3-5 Statistics of Deflections and Nugget Effect – true thickness

	n	Mean (m)	Variance (m)	SD (m)	CV	n pair	C0 (m^2)	C0 normalised
Central 4E	236	1.03	0.02	0.14	0.13	229	0.003	0.18
Eastern 4E	279	1.23	0.02	0.15	0.12	312	0.005	0.21
Western 4E	67	1.25	0.04	0.19	0.15	60	0.009	0.25
Total Area 4E	**582**	**1.15**	**0.03**	**0.18**	**0.16**	**601**	**0.005**	**0.14**
Central 6E	115	1.01	0.02	0.14	0.14	113	0.003	0.15
Eastern 6E	140	1.25	0.04	0.20	0.16	136	0.004	0.11
Western 6E	2	-	-	-	-	-	-	-
Total Area 6E	**257**	**1.14**	**0.04**	**0.21**	**0.19**	**250**	**0.004**	**0.08**
All Central	351	1.02	0.02	0.14	0.13	342	0.003	0.17
All Eastern	419	1.24	0.03	0.17	0.14	448	0.005	0.17
All Western	69	1.25	0.04	0.19	0.15	61	0.010	0.27
Total Area	**839**	**1.15**	**0.04**	**0.19**	**0.17**	**851**	**0.004**	**0.12**

Table 3-6 Statistics of Deflections and Nugget Effect – PGE cmgt

	n	Mean (cmgt)	Variance (cmg/t^2)	SD (cmg/t)	CV	N pair	C0 (cmg/t^2)	C0 - normalised
Central 4E	236	575	21116	145	0.25	229	11356	0.54
Eastern 4E	279	623	25034	158	0.25	312	12114	0.48
Western 4E	67	625	23600	154	0.25	60	9167	0.39
Total Area 4E	**582**	**604**	**23766**	**154**	**0.26**	**601**	**11531**	**0.49**
Central 6E	115	579	12003	110	0.19	113	4995	0.42
Eastern 6E	140	634	21917	148	0.23	136	9432	0.43
Western 6E	2	-	-	-	-	-	-	-
Total Area 6E	**257**	**610**	**18034**	**134**	**0.22**	**250**	**7390**	**0.41**
All Central	351	576	18092	135	0.23	342	9254	0.51
All Eastern	419	626	23966	155	0.25	448	11299	0.47
All Western	69	627	23026	152	0.24	61	9025	0.39
Total Area	**839**	**605**	**21995**	**148**	**0.24**	**851**	**10315**	**0.47**

Table 3-7 Statistics of U2 Pit Quarter HQ core duplicates and Nugget Effect

	n	Mean	Variance	SD	CV	N Pair	C0	C0 (normalised)
PGE grade (4T)	31	4.16	0.48	0.69	0.17	34	0.033	0.07
True Thickness	31	1.64	0.095	0.31	0.19	-	-	-
PGE cmgt	31	683	27870	167	0.24	-	-	-

Table 3-8 Statistics and nugget effect of pulps, original 4E re-assay by 6E

	n	Mean	Variance	SD	CV	N Pair	C0	C0 (normalised)
PGE grade (by 4E)	14	4.35	0.34	0.59	0.13	14	0.12	0.32
PGE grade (by 6E)	14	4.32	0.46	0.68	0.16			

The nugget effect calculated for deflections assayed using the 4E procedure is significantly higher than that of the 6E data (**Figure 3-3**). For the Central Area, the nugget effect for the 4E data is more than twice as high as the 6E data

(0.85 g/t^2 versus 0.40 g/t^2) and for the Eastern Area the difference is 42% (0.57 g/t^2 versus 0.40 g/t^2). Despite the differences in the nugget effect between the two assay methods, the normalised PGE nugget effect is almost the same and so the same normalised nugget effect can be used for both data sets.

The true thickness nugget effect is low (between 0.003 m^2 and 0.005 m^2) and similar between the Eastern and Central Areas, although the Western Area exhibits more variability. The normalised true thickness nugget effect for the Western and Central Areas are the same (0.17).

The PGE cmgt nugget effect exhibits similar trends to that of grade, but is generally lower, due to the influence of the low variance true thickness component.

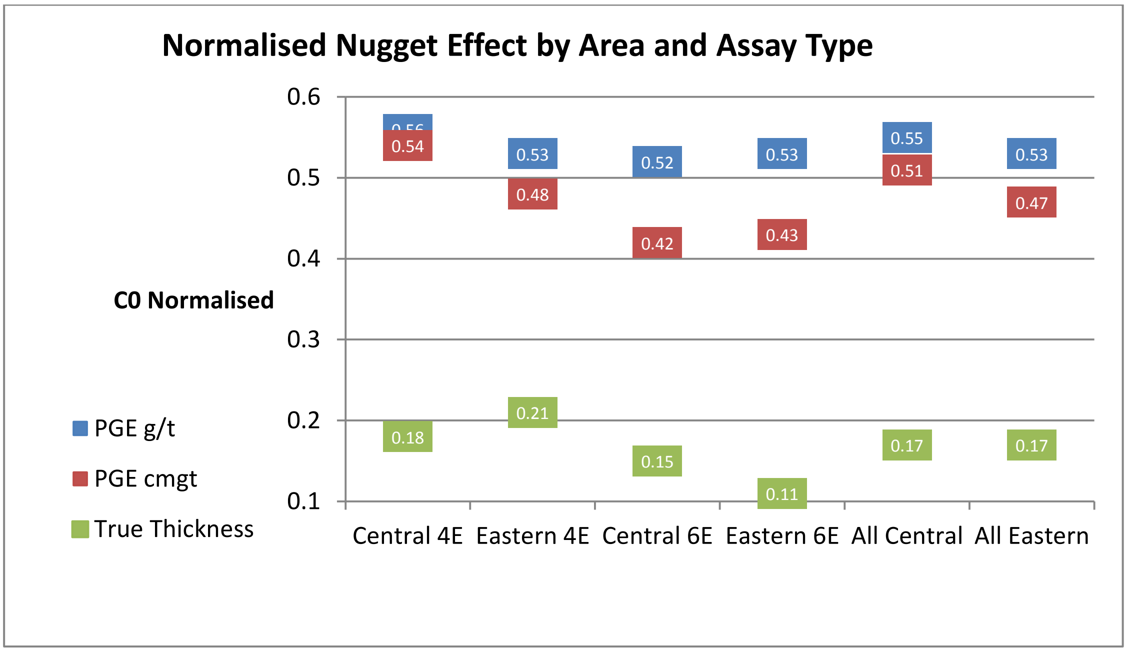


Figure 3-3 Normalised Nugget Effect for 4E grade by area and assay type

The 4T data indicate a very low nugget effect of 0.03 g/t^2, which is less than a 10th of the nugget effect for any of the PGE nugget effects determined from the drilled deflections. The 4T data is restricted to different quarters of split large diameter core in a small area of the Marikana UG2 property and there are no comparable data for samples assayed using either the 4E or 6E procedure. The very low nugget effect indicates that the assaying is precise. It may also indicate that the "true" nugget effect is very low and the high nugget effect proportions are due to small scale structures with ranges less than the deflection spacing.

Explanation of True thickness Nugget Effect

As stated by Clark (1979) "*even with completely random phenomena the semi-variogram must be zero at distance zero as two samples measured at exactly the same position must have the same value*". This holds true for reef thickness, where there should be no sampling error as thickness is a direct measurement, every measurement taken at the same place should be exactly the same. Using extrapolation of the channel sample data to the y axis, a high nugget effect proportion is estimated, somewhere in the region of 65% of the sample variance. Using the deflection data, the nugget effect is 17% of the sample variance. The apparent nugget effect that results from extrapolation appears to be high and indicates that most of the variance is found within a short distance (less than the sample spacing of 10 m) rather than over the entire area. This is consistent with geological observations, where high thickness variability over short distances is created by the combination of an undulating footwall and a planar hangingwall (**Figure 3-4**). The 0.17 nugget effect derived from the deflections explains that portion of thickness variance over a very short distance (within 30 cm). Examination of **Figure 3-4** shows that it is very likely that four intersections spaced 10's cm apart will result in quite different reef widths. This also demonstrates the importance of drilling the deflections in the first place in order to ensure that this variability is ironed out and a locally abnormally thin or thick single intersection is identified so that it does not influence a wide area.

In the absence of the measurement portion of the nugget effect for true thickness and the impossibility of range shorter than the sample support, the semi-variogram model should actually be modelled with zero nugget effect. A very short range structure with a range in the order of 0.3 m would then complete the semi-variogram while honouring the spatial observations.

Figure 3-4 Examples of footwall variability in Lonmin's UG2 operations, explaining high relative short range variability component observed in UG2 Reef thickness. (Photos courtesy of Lonmin)

4. MULTIPLE INTERSECTIONS FROM DEFLECTIONS IN ESTIMATION

Multiple intersections drilled from the same borehole are used in a number of ways. They provide an accurate measure of nugget effect, they are used directly in grade estimation and they provide important information on the geological variability over short distances, which may identify non-representative boreholes.

4.1 Sensitivity of Nugget Effect to Number of deflections

In order to understand how many boreholes are required to obtain a robust estimate of the nugget effect the following test was carried out:

- The nugget effect for both true thickness and PGE grade for deflections assayed by the 4E method was calculated only for the boreholes that had three deflections. This was carried out for both the Central and Eastern areas.
- The nugget effect was calculated for the total data set, and again by successively reducing the number of deflections by 10. In this way the differences in the nugget effect by using fewer and fewer deflections could be assessed.
- For each reduced data set, 10 iterations of the nugget effect were calculated by selecting different combinations of drillholes using a random number generator.
- The nugget effect was again calculated for each iteration and number of boreholes using only two of the three deflections.
- Four sets of data were created for each for the Eastern and Central areas:
    - Using three deflections
    - Using the 1st and 2nd deflections
    - Using the 1st and 3rd deflections
    - Using the 2nd and 3rd deflections.

The 6E data could not be assessed in this way, there being too few boreholes with three deflections with which to make a meaningful comparison.

Plots showing the nugget effect versus the number of boreholes used for each of the iterations are shown in **Figure 4-1**, **Figure 4-2**, **Figure 4-3** and **Figure 4-4**. The nugget effect plus or minus 10% and 20% of that calculated using the total data set for three deflections is shown on the plots. These demonstrate that the nugget effect is quite sensitive to the number of boreholes used. Only when 40 boreholes are used, each with three deflections, does the nugget effect stabilise close to that of the total data average but not always within ±20% of the total data estimate.

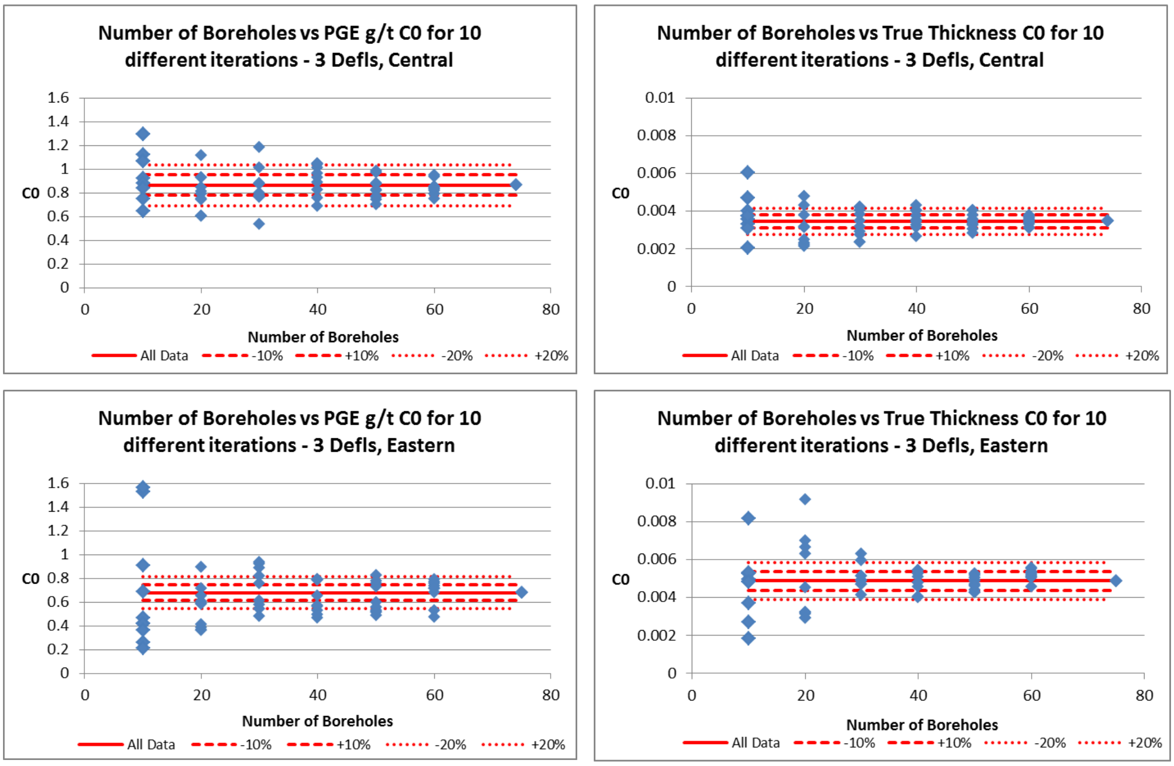


Figure 4-1 Sensitivity of nugget effect to number of boreholes – three deflections. PGE on left, true thickness on right, Central area top, Eastern area bottom.

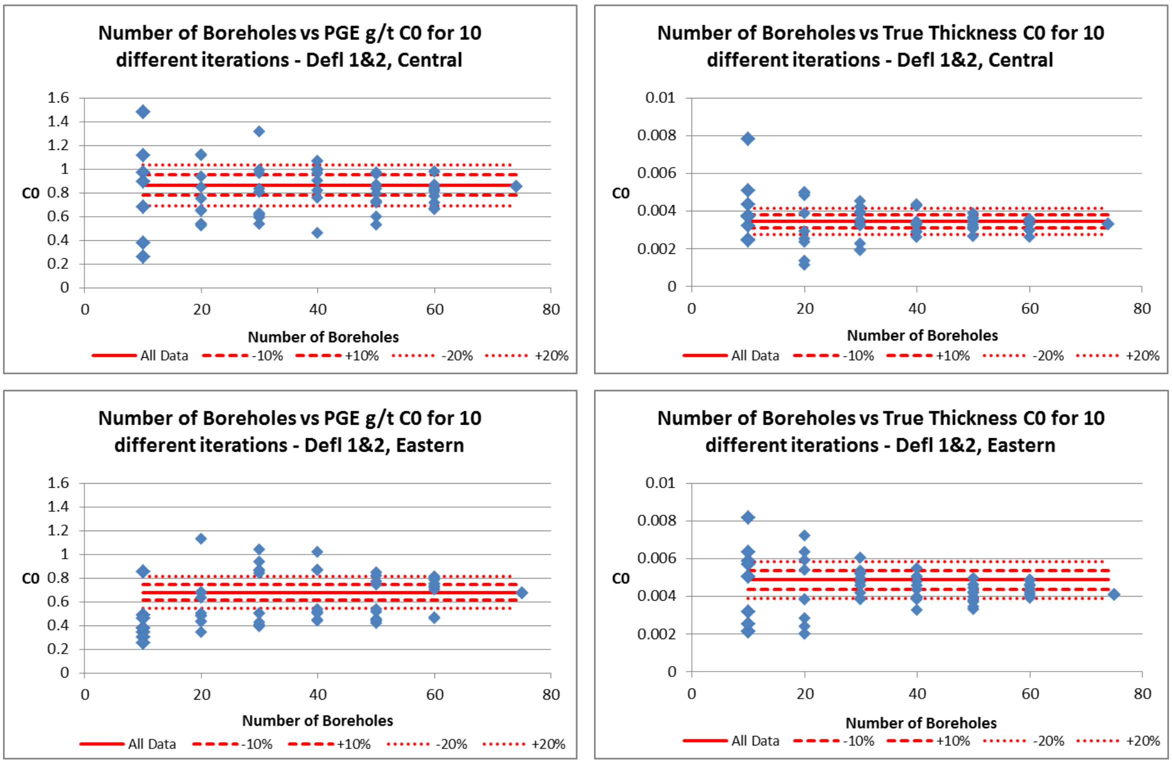


Figure 4-2 Sensitivity of nugget effect to number of boreholes – two deflections (1,2). PGE on left, true thickness on right, Central area top, Eastern area bottom.

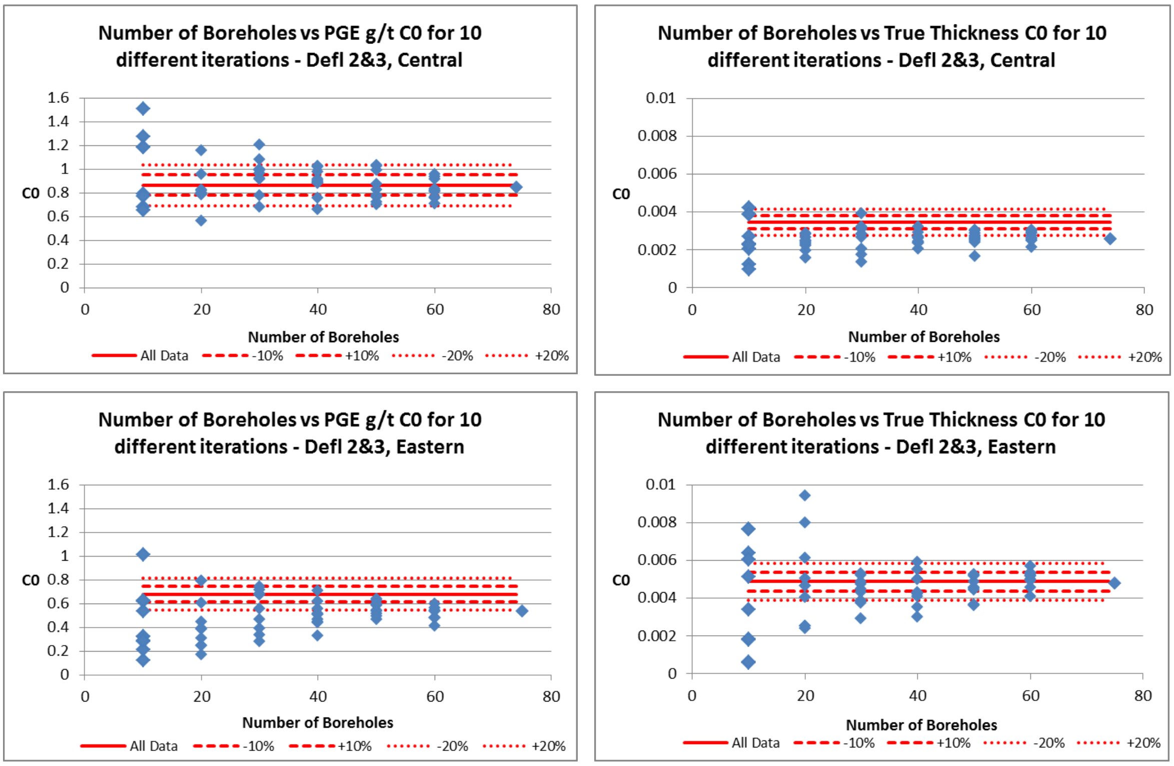


Figure 4-3 Sensitivity of nugget effect to number of boreholes – two deflections (2,3). PGE on left, true thickness on right, Central area top, Eastern area bottom.

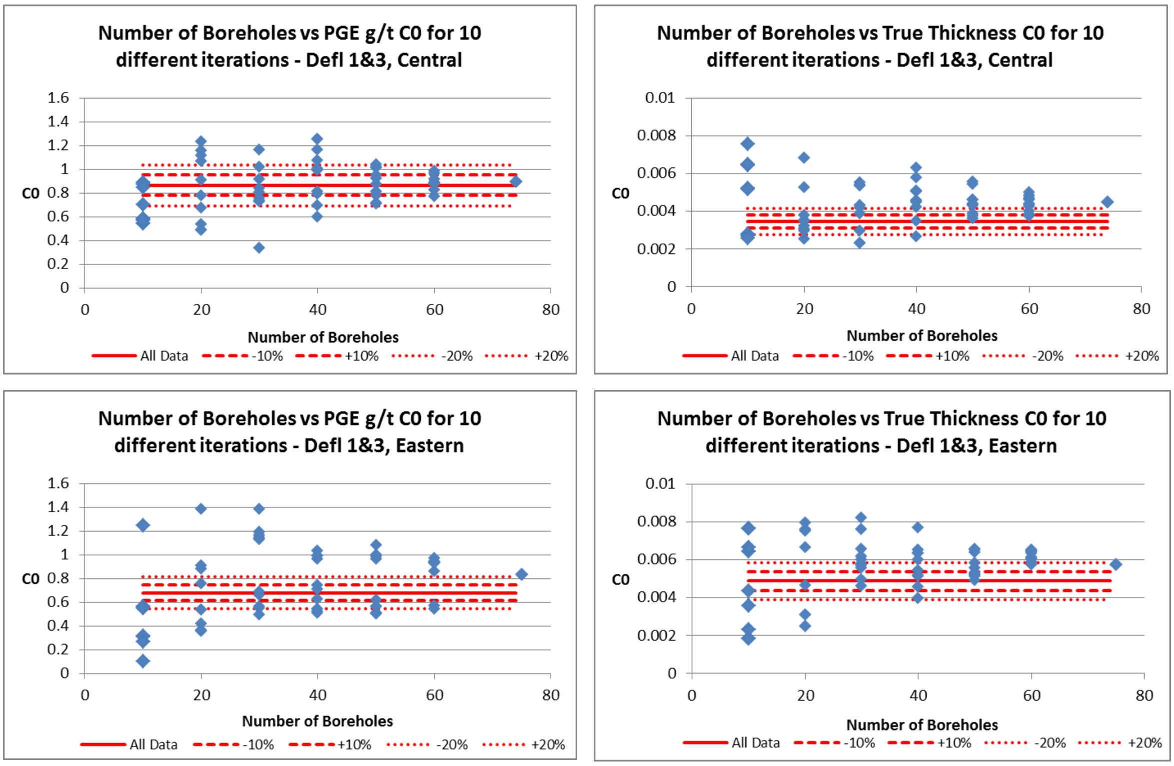


Figure 4-4 Sensitivity of nugget effect to number of boreholes – two deflections (1,3). PGE on left, true thickness on right, Central area top, Eastern area bottom.

When using two deflections, the nugget effect is more sensitive to the number of boreholes used. More boreholes will therefore be required in order to obtain a stable estimate of the nugget effect.

4.2 Estimate of error in a borehole

The nugget effect of the semi-variogram can be used to assess the error in the grade of the borehole calculated from its deflections. The error is calculated using the t statistic as follows:

$$t_{n-1;0.05} \times {S_{C0}}/{\sqrt{n}}$$

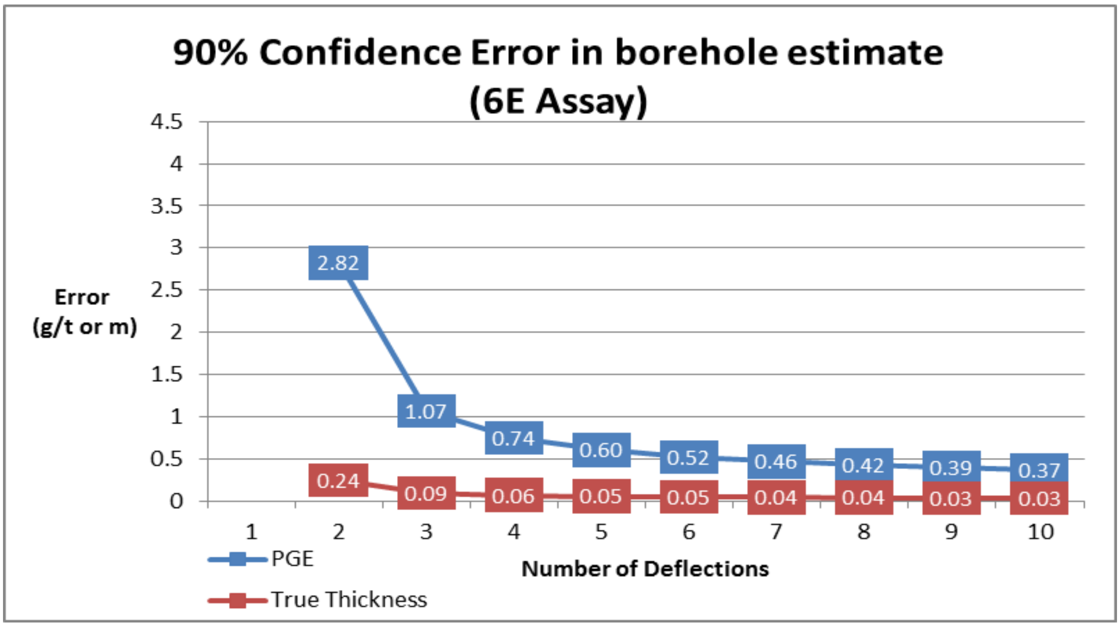
Where:

t_{n-1} = the percentage points of the *t* distribution at various degrees of freedom.

S_{C0} = the nugget effect standard deviation

n = the number of intersections.

The error was calculated for different numbers of intersections using the nugget effect for samples assayed by 6E and by 4E procedures as well as for true thickness (**Figure 4-5**).



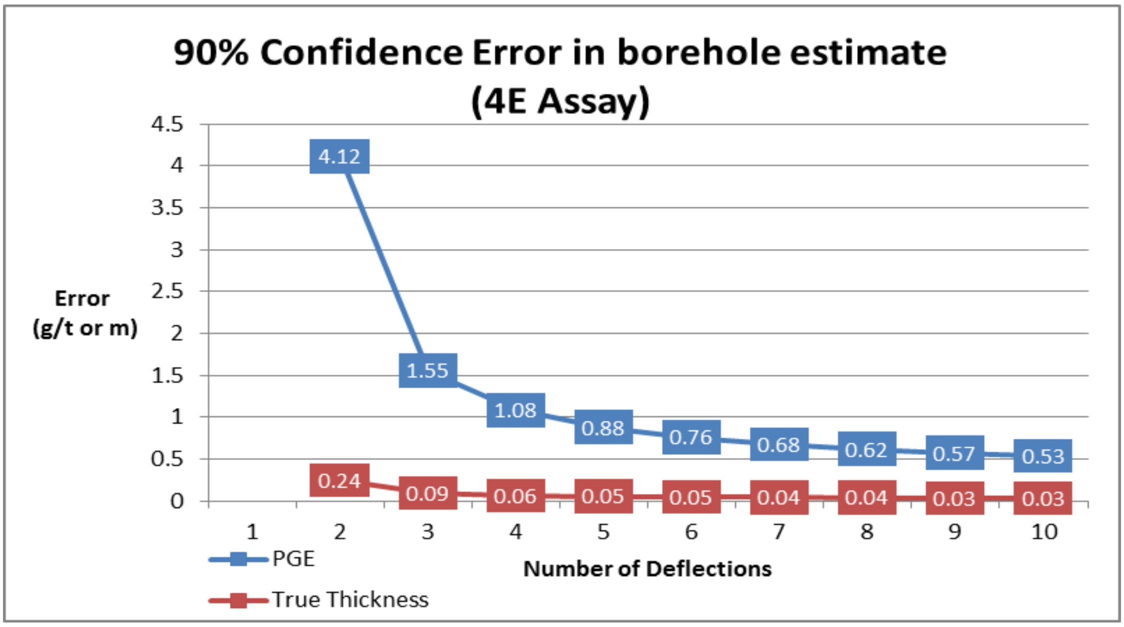


Figure 4-5 90% Confidence error in estimate of the borehole grade and thickness from different numbers of deflections.

A significant decrease in the error is noticeable with increasing number of deflections from two to five. An increase in the number of deflections from two to three results in the error being more than halved, and a reduction in error of approximately 30% is experienced when the number of deflections is increased from three to four. The true thickness error is less than 10 cm (around 10% relative error) when three deflections are available but the PGE grade error is well over one gram per tonne (around 20% relative error). The PGE grade error using the 4E procedure to assay the samples is approximately 50% higher than that if the 6E procedure is used. Should four deflections be drilled and assayed instead of three, the PGE grade error would reduce from 1.07 g/t to 0.74 g/t, resulting in a relative error of approximately 15% should the PGE grade be 5 g/t.

In addition to the error calculation, the benefit of each additional deflection can be simplistically judged by the weight assigned to each additional deflection. As the number of deflections increases the relative weight assigned to each will necessarily decrease as illustrated in **Figure 4-6**.

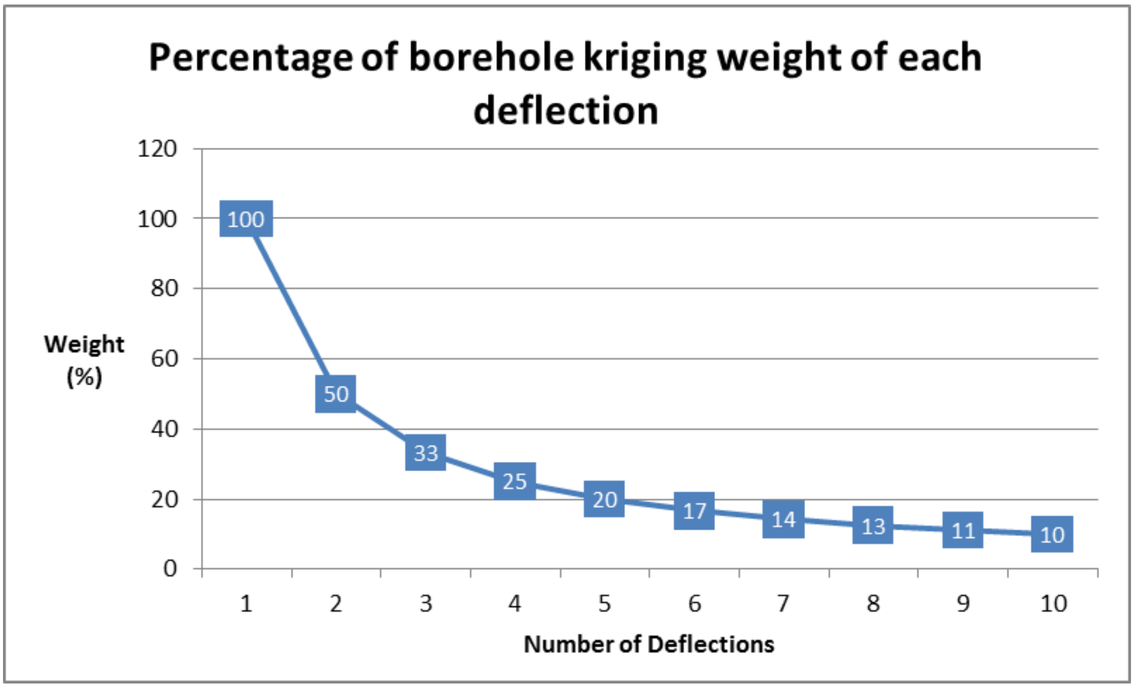


Figure 4-6 Borehole kriging weight of each deflection as a percentage of the total borehole kriging weight.

The relative kriging weight of each deflection reduces rapidly as more deflections are used. Once more than five deflections are available there is little additional weight applied and therefore little benefit to the estimate.

4.3 Use of deflections in estimation

Deflections drilled from mother holes at Marikana result in a number of reef intersections a short distance away from each other - from a few centimetres to several 10's of centimetres. The deflection holes are not surveyed down the hole and the wedges themselves are not inserted in any particular direction (non-directional wedges). When estimating Mineral Resources using the intersections obtained through deflection drilling, assumptions need to be made about their distance apart and direction from one another. Alternatives also exist such as estimating using lateral composites rather than individual intersections.

Current practice at Marikana is to add a nominal distance of 20 cm to each deflected intersection in a clockwise direction so that the first deflection is 20 cm north of the mother hole, the second 20 cm east, the third 20 cm south and the fourth 20 cm west, forming a cross arrangement. Should there be more than four deflections then 40 cm is added north of the mother hole and so on.

The kriging weights for each deflection in the helical arrangement were examined in order to understand how the kriging manages the clustered deflection data. The exercise was repeated using number of different deflection arrangements. The grade and thickness estimates were compared so that the sensitivity of the estimate to different arrangements could be assessed:

1. The cross arrangement at 20 cm (the base case).
2. The cross arrangement at 500 cm separation
3. The cross arrangement at 1 cm.
4. Using no separation
5. Using a North-South line separated by 20 cm N, 40 cm N, and 20 cm S of the mother hole.
6. Using a North-South line separated by 1 cm N, 2 cm N, and 1 cm of the mother hole.
7. Using an East-West line separated by 20 cm E, 20 cm W and 40 cm W of the mother hole.
8. Using an East-West line separated by 1 cm E, 1 cm W and 2 cm W of the mother hole.

A 500 mN by 500 mE block (the standard size block used in areas outside of the channel sampling) was selected within a typically drilled area in Rowland Shaft Block (**Figure 4-7**). The block chosen for test estimation has two surface boreholes located within it, WP016 and WP024, which are approximately 220 m apart. These two holes are relatively closely spaced, the remaining boreholes being more typically spaced between 400 m and 500 m away from each other and spread around the test block. All of the boreholes within the test area intersected the reef with three valid assayed deflections.

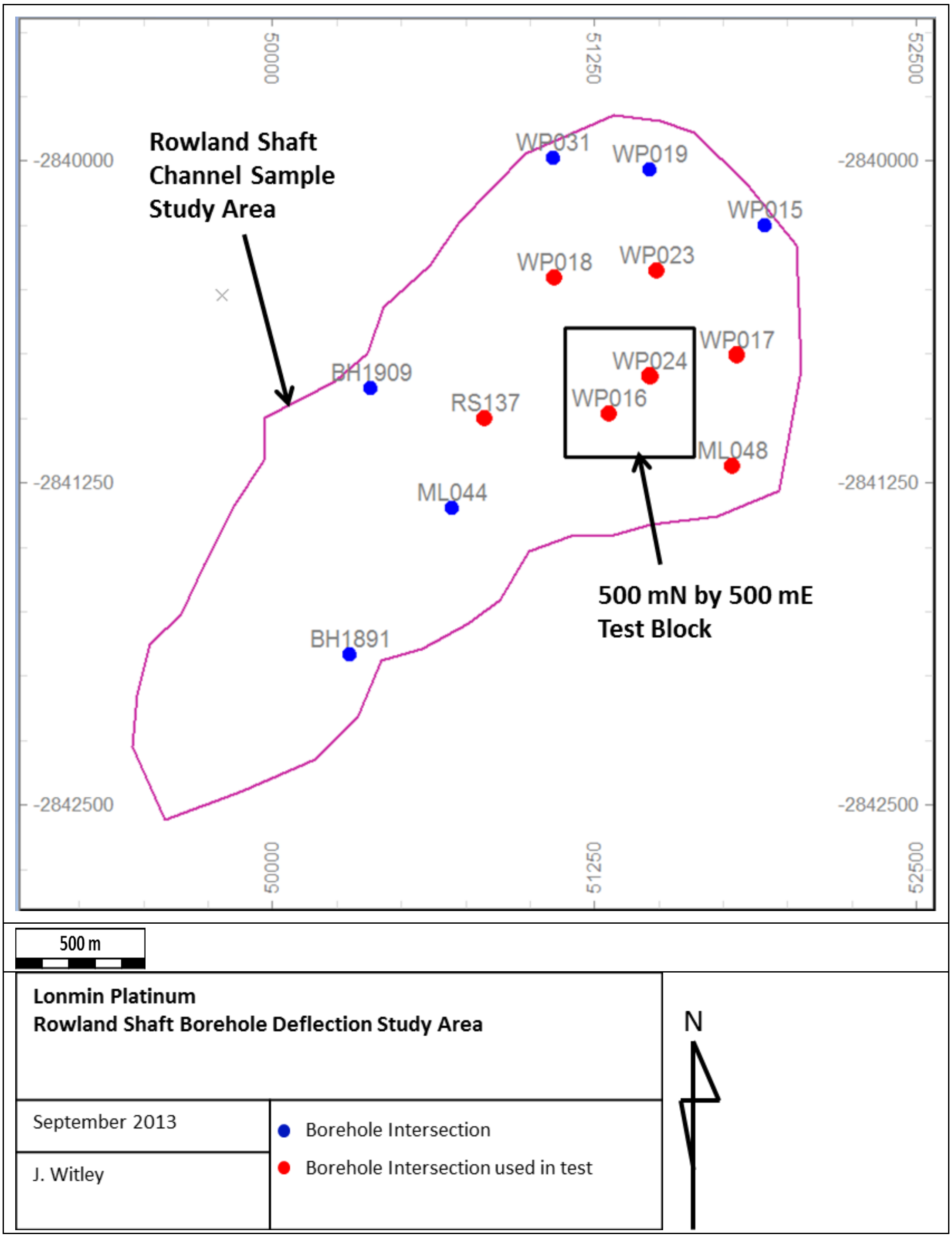


Figure 4-7 Location of Borehole Deflection Study Block in Relation to Boreholes Intersections and Rowland Channel Sample Study Area

The tests on the different spatial arrangements were conducted using the standard Rowland UG2 semi-variogram. In addition, this semi-variogram was adjusted so that the nugget effect was re-assigned to an ultra-short range ("R0.5" of 0.2 m) in order to assess whether any difference in the estimate would arise (**Table 4-1**). Further examination was made of the kriging weights by varying the number of deflections for each borehole.

Table 4-1 Normalised semi-variogram scenarios for PGE Grade

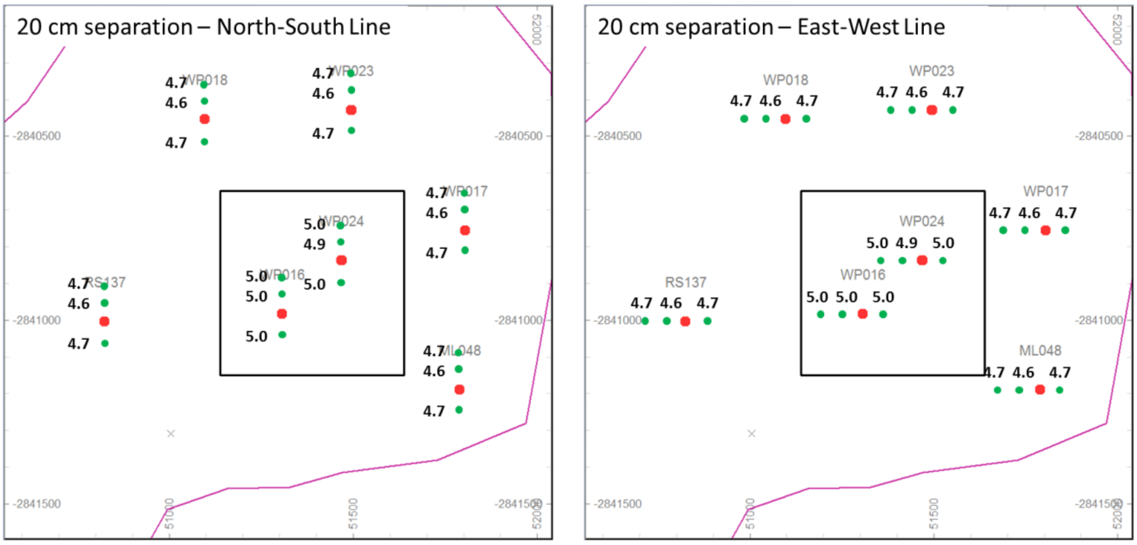
Scenario	C0	R0.5	C0.5	R1	C1	R2	C2	R3	C3
Base Case	0.56	-	-	30	0.25	45	0.11	170	0.08
No nugget (all short range)	0.00	0.2	0.56	30	0.25	45	0.11	170	0.08

Figure 4-8 shows the kriging weights applied in a block estimate using the standard cross arrangement at a variety of deflection separations.

Note that the distance between the mother hole (red dot) and the deflections (green dots) is greatly exaggerated. North is directly upwards. The block to be estimated is 500 mN by 500mE.

Figure 4-8 Kriging weights for each deflection using a variety of deflection separation distances and a standard cross arrangement (Rowland UG2 Semi-variogram).

Figure 4-9 shows the kriging weights using 20 cm deflection separation and either a North-South or East West arrangement.



Note that the distance between the mother hole (red dot) and the deflections (green dots) is greatly exaggerated. North is directly upwards. The block to be estimated is 500 mN by 500mE.

Figure 4-9 Kriging weights for each deflection using a 20 cm deflection separation distances and a line arrangement North-South and East-West (Rowland UG2 Semi-variogram).

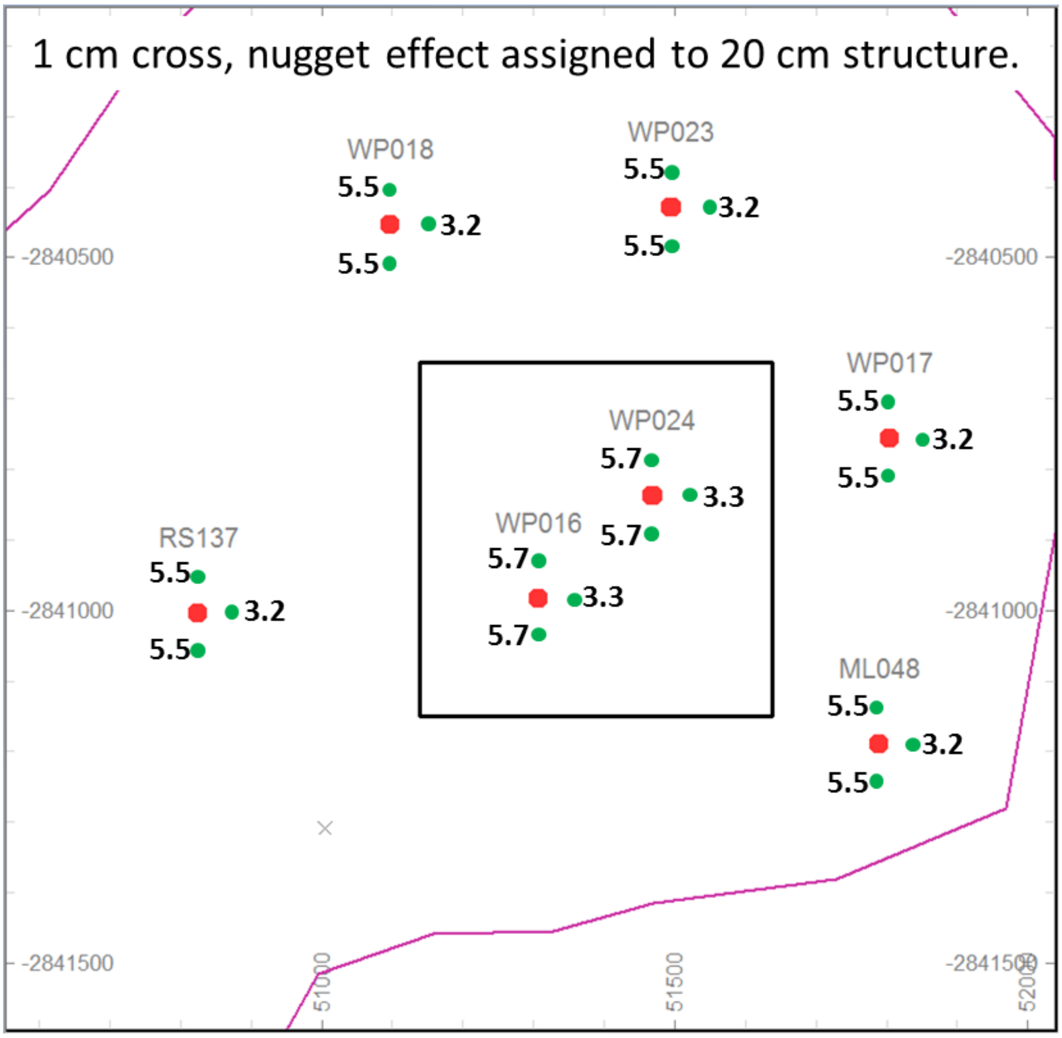
The following observations were made from the tests using three deflections per hole and the standard Rowland UG2 semi-variogram:

- Using the standard 20 cm cross arrangement, the kriging weight applied to each deflection is similar but not exactly the same. The holes within the block get slightly more weight than those outside of the block.
- Using the 500 cm cross arrangement the second deflection in each case attracts approximately 8% lower kriging weight than the first and the third deflections for holes outside of the block. Within the block the same pattern is observed but the difference between the kriging weights is less (approximately 6%). The difference in weights is caused by a principal known as ”shielding” whereby a sample in the same line of sight to the block to be estimated is given a lower weight than the other sample close by in the same direction from the block.
- Using the 1 cm cross arrangement each deflection within a hole attracts the same weight as is the case if no separation is used at all.
- The average variogram value for each deflection is the same within the same borehole (1.000 for those outside the block and 0.994 and 0.995 for WP016 and WP024 respectively). Insignificant changes to the average

variogram value are seen whether deflections are spaced 1 cm, 20 cm or 500 cm from the mother hole at five decimal places.

- Using a North-South line arrangement introduces slight screening effect at 20 cm deflection spacing. However at 1 cm spacing the screening effect is almost non-existent. The East-West arrangement gives the same results.
- The estimated PGE grade for the block is 5.73 g/t in all cases except where the 5 m spacing is used and then the estimated grade is 5.72 g/t.

The effect of ascribing the nugget effect to an ultra-short range structure (as shown in **Table 4-1**) was assessed using the 20 cm separation in the cross arrangement. For the PGE estimate, using a semi-variogram with a nugget effect of zero and applying the deflection nugget to a 20 cm range gives the same results as if a nugget structure is used. However, should the separation between the deflections be less than the range of the ultra-short range structure then the kriging weights become imbalanced (**Figure 4-10**). The imbalance in the weights is severe enough that a different estimate will result. In the test case of using a deflection separation distance of 1 cm, one of the deflections in each borehole is assigned only approximately 60% of the weight of the other two. The PGE estimate is 5.70 g/t compared with the estimate of 5.73 g/t that is obtained should the weights be correct. It should be noted that if a semi-variogram with zero nugget is used and there is no applied distance between deflections, then an error in the kriging matrix occurs. This error is logical as it is not possible to have two different values in exactly the same position without explaining the difference with a nugget effect.

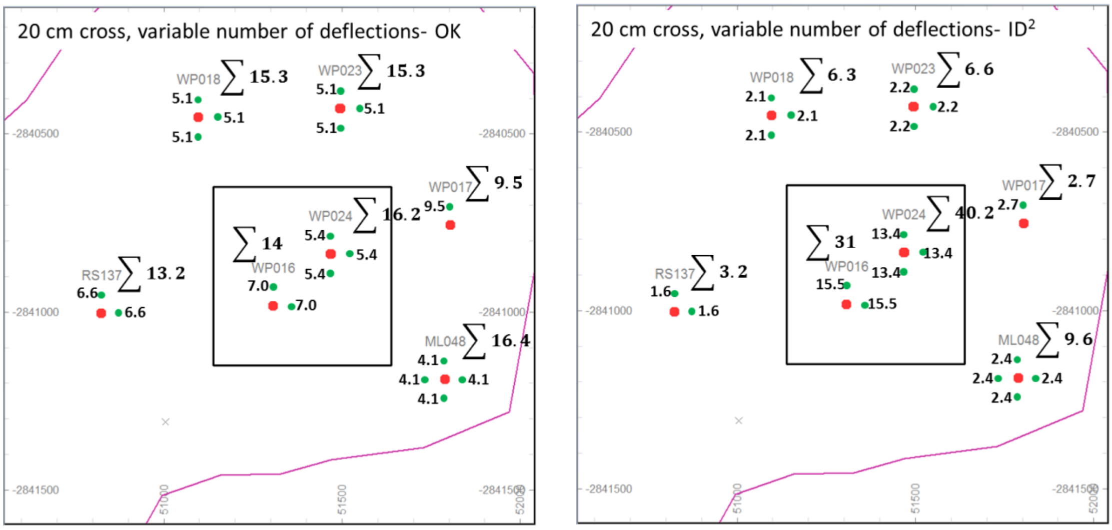


Note that the distance between the mother hole (red dot) and the deflections (green dots) is greatly exaggerated. North is directly upwards. The block to be estimated is 500 mN by 500mE.

Figure 4-10 Kriging weights for each deflection using a 1 cm deflection separation distance and a cross arrangement - Rowland UG2 Semi-variogram with zero nugget effect and 20 cm ultra-short range structure.

The kriging weights assigned to deflections when different numbers of deflections exist for each hole were examined using the standard Rowland UG2 PGE grade semi-variogram and the cross arrangement (**Figure 4-11**). The total weights for each borehole are not the same. Boreholes with fewer deflections have a lower total weight, but the individual weights assigned to each deflection are higher. In practice the ordinary kriging system recognises that there are more deflections and gives a higher confidence (weight) to those boreholes with more deflections. At the same time there is a degree of de-clustering that takes place so that the individual weights assigned to a deflection out of a borehole with four is less than the individual weight assigned to a deflection out of a borehole with three. This is an extremely important property of Ordinary Kriging applied to multiple deflection borehole grids. Although the amount of weighting is quite small, the advantage over a simple averaging or Inverse Distance techniques is that the borehole deflection samples are de-clustered and at the same time the total borehole is assigned a weight appropriate to the confidence in that hole by virtue of the number of deflections. Even with boreholes spaced further apart than the semi-variogram range Ordinary Kriging plays a part in the estimation when variable numbers of intersections occur for each borehole.

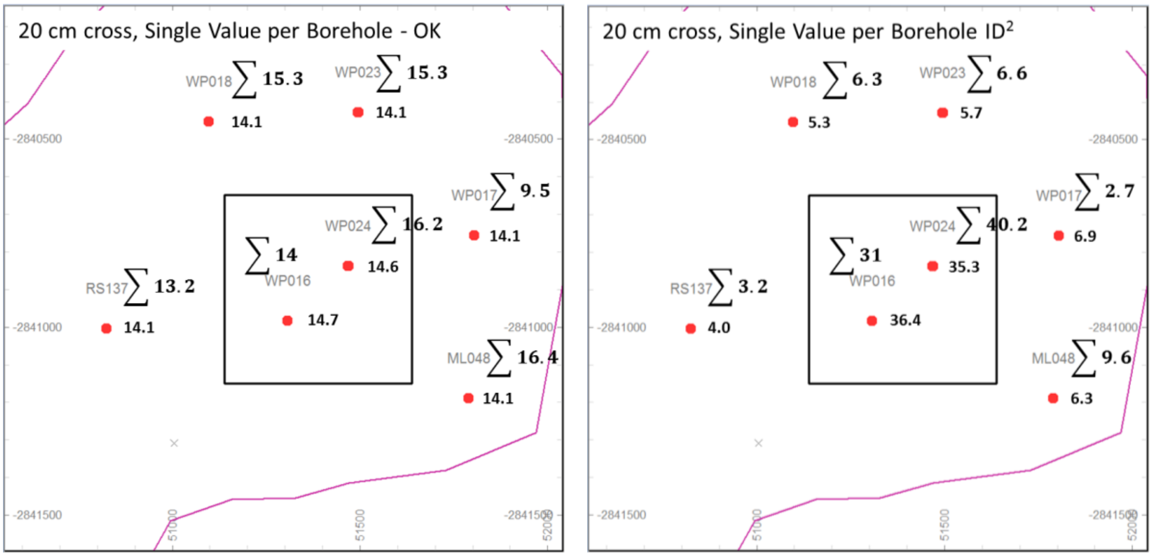
Should Inverse distance squared be used, the weights assigned to the individual deflections do not take into account the deflections a few cm's away and no de-clustering takes place, each intersection being treated as a separate borehole taking no cognisance of a close neighbour (**Figure 4-11**). Intuitively the ability of Ordinary Kriging to both de-cluster the data and assign confidence to a borehole on its number of deflections seems the most appropriate.



Note that the distance between the mother hole (red dot) and the deflections (green dots) is greatly exaggerated. North is directly upwards. The block to be estimated is 500 mN by 500mE.Σx.x is the sum of the weights.

Figure 4-11 Weights for each deflection and sum for each borehole using a 20 cm deflection separation distance and a cross arrangement for both Ordinary Kriging and Inverse Distance Squared - Rowland UG2 Semi-variogram.

Multiple intersections can be averaged into lateral composites prior to estimation. Should exactly the same number of intersections exist for each borehole the total weights will be the same whether or not lateral composites are used, but the weights assigned to lateral composites cannot take into account the differing numbers of intersections. This is illustrated in **Figure 4-12** for both Ordinary Kriging and Inverse Distance Squared. The sum of the weights is shown on both diagrams as derived from the variable number of intersection estimate. For the Ordinary Kriging example the weights are almost the same for each lateral composite borehole, very different than the sum of the weights from individual intersections. In the case of Inverse Distance Squared the weights differ between boreholes, although the excessive weight applied to holes with more individual intersections is removed.



Sum of weights for variable number of intersections shown.
North is directly upwards. The block to be estimated is 500 mN by 500mE. Σx.x is the sum of the weights.

Figure 4-12 Weights for each lateral composite borehole using a 20 cm deflection separation distance and a cross arrangement for both Ordinary Kriging and Inverse Distance Squared - Rowland UG2 Semi-variogram.

4.4 Deflection drilling for geological and data understanding

In normal UG2 Reef, actual differences between intersections on the scale of the separation caused by short deflection drilling are expected to be small. Large differences are due to relatively small scale geological features, such as faults and potholes. UG2 intersections affected by these features cannot be considered representative of the area that it is intended to estimate, which at Marikana is an area of 500 m by 500 m (the standard block size used). Inclusion of intersections that are not representative of the area that is to be estimated will result in estimates of poor accuracy and therefore it is common practice to reject boreholes affected by small scale geological disturbances.

The use of techniques such as percentage difference of an intersection from the mean of the borehole (refer to the data validation section of this report), greatly assists in identifying unrepresentative intersections. Out of the 400 boreholes in the study area, 30 boreholes were rejected on the basis of differing values obtained from intersections a few cm apart, 21 were rejected on the basis of geological disturbances and a further 9 were rejected due to unresolvable data issues. 7 other holes were identified that had data issues that were easily rectified. Without multiple intersections obtained from deflection drilling many of the unrepresentative intersections may be included in the estimation data base, it often being difficult to recognise some of the more subtle features in potholes on the basis of a single intersection.

4.5 Summary of Deflection Study Findings

At least 40 boreholes, each with three or more deflections, are required to obtain a stable estimate of the nugget effect.

Confidence in the grade and thickness of the UG2 reef intersected in a borehole is greatly increased by intersecting the reef several times.

- An increase in the number of intersections from two to three results in the error being more than halved, and a reduction in error of approximately 30% is experienced when the number of deflections is increased from three to four. The true thickness error is less than 10 cm (around 10% relative error) when three intersections are available but the PGE grade error is well over one gram per tonne (around 20% relative error). Should four deflections be drilled and assayed instead of three, the PGE grade error will reduce from 1.07 g/t to 0.74 g/t, which is a relative error of approximately 15% should the PGE grade be 5 g/t.

- The PGE grade error in a borehole using the 4E procedure to assay the samples is approximately 50% higher than that if the 6E procedure is used.
- The relative kriging weight of each deflection reduces rapidly as more deflections are used. Once more than five deflections are available, there is little additional weight applied and therefore little benefit to the estimate.

A number of important findings arose from the way in which multiple intersections are handled in the estimation procedure:

- The kriged estimate is not sensitive to the spacing applied between the intersections at very small distances. The application of larger distances should be avoided as the shielding effect causes the weights to become different for each intersection, when in practice they should be exactly the same for each intersection drilled from the same hole. Lonmin should consider using a smaller separation than the current 20 cm to ensure that the Kriging weights for deflection within the same hole are exactly the same. A 1 cm separation will suffice.
- The cross arrangement currently used is correct as it avoids the shielding effect experienced if the intersections are arranged in line. However the practice of moving one intersection so it is directly behind another (as would be the case with the fifth intersection 20 cm behind the first) must be avoided as it will cause an imbalance in the kriging weights applied.
- Should an ultra-short range structure be used to explain some of the calculated nugget effect from multiple intersections (as should be the case with true thickness) then the range of this structure must be less than the assigned intersection spacing, otherwise imbalances in the kriging weights occur.
- For all practical purposes there will be no difference in the estimate should an ultra-short range structure be used as opposed to all the deflection variance being attributed to the nugget effect.
- The use of lateral composites does not take into account the higher confidence that one would wish to assign to a boreholes with more

intersections. However, if an Inverse Distance type approach is used that does not automatically de-cluster the data then lateral composites will avoid over-weighting of boreholes with many intersections.

Deflection drilling greatly assists in the identification of boreholes affected by geological disturbances such as faults and potholes. Using techniques that examine the variability between intersections drilled from the same borehole identify data validation issues that might otherwise be overlooked.

5. OPTIMISING CLOSE SPACED (UNDERGROUND CHANNEL) SAMPLE GRIDS

The aim of this aspect of the study is to understand the variability and spatial distribution of the data so that sample grids for stope scale estimation at Lonmin's Marikana UG2 mines can be optimised. The stope scale estimates are intended to be Measured Resources that are normally converted to Proved Mineral Reserves. The estimates are used for monthly evaluation of ore produced and should be the highest confidence in-situ grade estimates at the mine. The Measured Resources are also used for prediction of metal production in the year ahead through their use in the annual technical budgeting process. Estimation of the Measured Resource proceeds through two separate processes:

1. <u>The stope scale evaluations that are used to account for monthly ore production and that guide instructions to mine:</u> At Marikana, the majority of production is from up- and down-dip stopes that are approximately 35 m in strike length by approximately 200 m on dip, depending on the dip and local adjustments to the stope layout. Once stoping has commenced, stope panels are not sampled. The initial central raise from which stoping takes place is carried by the lead and lag panels on either side of the raise throughout the stope's life and the sampling carried out during raise development is used for monthly evaluation. Evaluation of reef mined from a panel during any one month consists of assessing mined stope and reef width, measured several times during the month, against the initial estimation of width and grade derived from the average of the raise channel sampling.

   UG2 stopes, or portions of, are below cut-off grade as a result of observed geological disturbances and excessive internal or external dilution, rather than low grade areas estimated from the channel sampling. Therefore the purpose of the stope grade estimations is largely for metal accounting, rather than a decision to mine or not.

2. <u>The kriged block models that are used for forecasting metal production via the annual technical budget process:</u> In and adjacent to the areas

that have been sampled underground, 50 m by 50 m block model cells are populated with true thickness, 4E accumulation (cmgt), 4E grade (g/t) and density using Ordinary Kriging estimation. The block models are input into mine scheduling software, which output forecasts of monthly production for the following two years.

Metal accounting uses a classical statistical technique and the preparation of estimates for production forecasting uses a geostatistical technique. In considering the sample spacing optimisation, both statistical and geostatistical methods are used in order to fully assess the impact on both types of estimate.

5.1 Previous Sampling Optimisation Studies Carried out at Marikana

In 1991, Isobel Clark of Geostokos Limited carried out an investigation on optimum sampling intervals for the UG2 and Merensky Reef at Western Platinum Ltd (Clark, 1991).

The area of Clark's investigation occurred around 11 and 12 levels at the 1 Shaft area that occurs up dip from Rowland Shaft. Clark was advised that subtle differences occur within the UG2 Reef between the Eastern and Western side of the mine, although no statistical significance to the differences could be found. It should be noted that these areas are not synonymous with the Western and Eastern Domains identified in this study, the areas investigated by Clark falling within the Central Domain.

Clark (1991) found that the channel width (true thickness) distribution was not symmetrical and that the values could be explained by a two normal distributions, the main one explaining 80% of the values with a mean of 1.085 m and a standard deviation of 0.2m (variance of 0.04 m^2) and a second with a mean of 1.005 m and a standard deviation of 0.055 m (variance of 0.003 m^2). The first distribution is similar to that of the Rowland study area (mean of 1.10 m and variance of 0.017 m^2). The low variance and tendency around 1 m of the second distribution suggest that the data set used may contain a number of default 1 m values. The grade data was represented as the accumulation of PGE grade and true thickness (cmgt). Clark found that these values were highly skewed and were best modelled using a three parameter log normal distribution.

Further examination of the data by Clark revealed that no relationship existed between the channel width and grade data and therefore she considered estimation of accumulation values to be unwise since cmgt is the product of two disparate and uncorrelated variables.

It should be noted that no evidence of log normal distribution for PGE grade, true thickness or PGE accumulation was found in the data examined in this study. In 1991, chip sampling was used to sample the underground reef exposures. The data used in this study is derived from accurate diamond saw cut channel sampling and has been rigorously validated. Magri and McKenna (1986) did a study on diamond saw sampling versus chip sampling at the Randfontein Estates and Western Areas Gold Mines and showed that the cut sample data is an improvement over chip sampling and that the sampling spacing for cut samples can be larger than for the chipped ones. This may help to explain the difference in the statistical distributions of the two data sets.

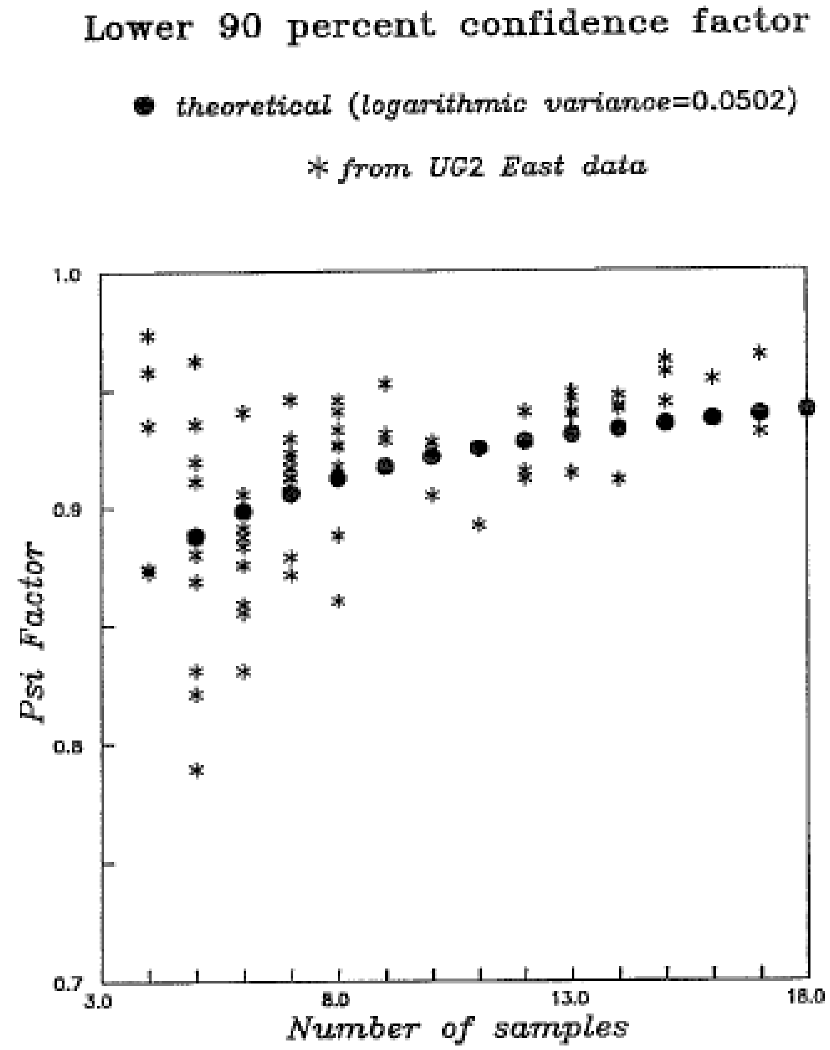
Clark modelled the thickness data using an omnidirectional spherical model with a nugget effect of 225 cm^2 and a range of 90 m with total semi-variance of 365 cm^2, in other words the nugget effect accounted for approximately 62% of the total variance. It should be noted that this nugget effect would be similar to that modelled at Rowland in this study should the model be extrapolated to the semi-variogram axis rather than using the borehole deflections. Clark mentioned that as a rule-of-thumb one would want to sample a quarter to a third of the range of influence, suggesting that for channel width a 25 to 30 m sample spacing will be adequate, however it is important to bear in mind that the large nugget effect will widen confidence limits.

Clark used a fictional data set and sampling intervals between 1 m and 250 m along a 250 m by 30 m stope panel to calculate the standard error of each sampling interval as well as the lower 90% confidence limit. This gives an indication of what sample spacing may be required. Assuming a reef width of 1.10 m an estimate can be achieved within ±10% using only 6 samples, which equates to a spacing of about 40 to 50 m (**Table 5-1**).

Table 5-1 Standard error and lower 90% confidence limit for stope true thickness estimates using varying sample intervals (Clark, 1991).

Number of samples	Sample Interval (m)	Standard Error	Lower 90% Confidence Limit (cm)
251	1	1.2300	1.58
126	2	1.5525	1.99
84	3	1.8231	2.34
63	4	2.0586	2.64
51	5	2.2696	2.91
42	6	2.4664	3.16
36	7	2.6484	3.40
32	8	2.8150	3.61
28	9	2.9813	3.82
26	10	3.1320	4.02
23	11	3.2810	4.21
21	12	3.4264	4.39
20	13	3.5539	4.56
18	14	3.6966	4.74
12	21	4.5341	5.81
9	28	5.2627	6.75
8	35	5.8359	7.48
6	42	6.5362	8.38
6	50	6.9768	8.94
4	75	8.5776	11.00
3	100	10.2488	13.14
3	125	10.8159	13.87
2	250	13.9905	17.94

For the PGE grade data, Clark modelled the logarithmic transformed PGE grades after adding a constant of 1.517 g/t. The nugget effect accounted for 76% of the total semi-variance suggesting that there is weak spatial structure and that a more random type estimator (such as Sichel's t) may be as, or more, valid than a geostatistical approach. Sichel's theory for different numbers of samples was applied to between 4 and 50 samples using the logarithmic variance of the data in order to calculate the theoretical confidence factor. The actual confidence factor was then calculated using the logarithmic variance and the average grade of the stope. The two values are compared in **Figure 5-1**, which shows that the actual confidence limits vary considerably as opposed to the regular theoretical curve. The theoretical curve shows that with six or more samples grade estimates within ±10% can be achieved at 90% confidence. However when using the actual data the variability in confidence is high and only when 14 samples or more are taken do the empirical data converge to the theoretical data indicating that a sample interval of closer than 18 m is required in a 250 m long raise.



The black dots represent the theoretical confidence factor (Sichel t) and the asterixes the emprical results from the actual data.
Source: Clark (1991).

Figure 5-1 Lower 90% Confidence factor for theoretical calculation and empirical calculation on actual data (Clark, 1991).

A short study was conducted by Johan Roos (Chief Evaluator at Lonmin Marikana Karee Mine) in 2008 in order to determine if the sample spacing on the UG2 could be widened from 10 m to 20 m. The study area was 18 and 19 levels at Karee Mine, which fall within the Central domain and partly in the Western domain as defined in this study. Every second channel was removed and the estimates between the 10 m and 20 m spaced sampling were compared. Roos noticed that although the average PGE grades of the study area were the same, the scatter between estimates was high and a bias occurred at the higher and lower grade intervals. The bias was documented as 4% at grades of greater than approximately 5.5 g/t and -3% at grades of less than approximately 4.80 g/t. Because of this bias and large differences between the individual estimates based on the 10 m and 20 m spacing, he recommended that the sample spacing remain at 10 m. The author of this report considers that the overall bias is not material. The number of samples used for each estimate was not documented, so it is not possible to fully assess the cause of the large discrepancies observed between the two sets of estimates.

5.2 The existing sampling layout and estimation methodology

The current standard sampling grid for UG2 at Marikana, in a typical up-dip mining layout, comprises channel samples spaced 10 m apart on dip and 35 m apart on strike, this being the raise spacing. Ideally this results in approximately 20 sampled channels per stope, should the sampling be complete (**Figure 5-2**). The stope preparation drives (SPD's) are not normally sampled.

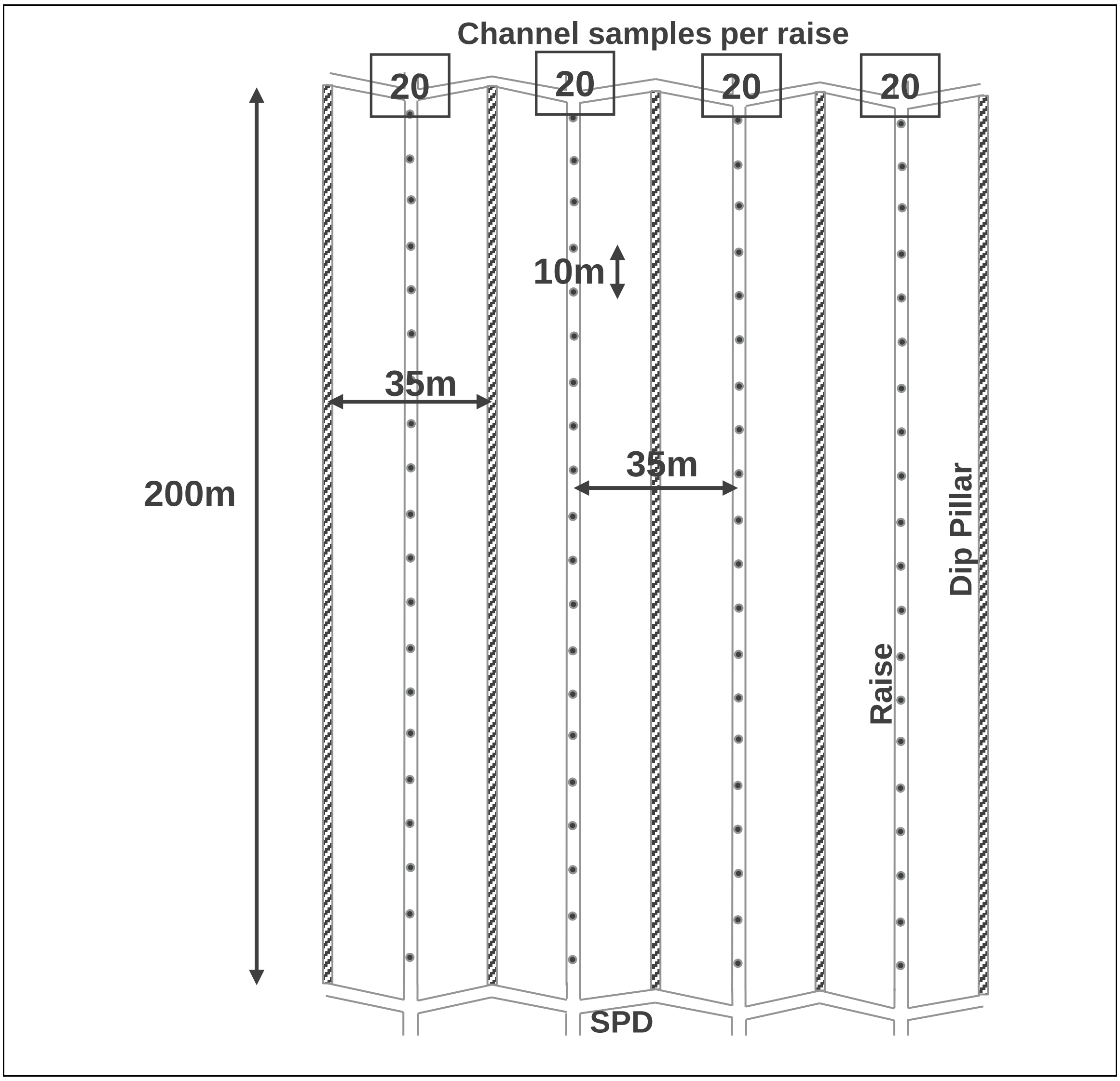


Figure 5-2 Simplified dip stope layout showing current "ideal" sampling pattern. (Black dots within the raises indicate sample positions)

In reality it is rare that there are 20 valid sample sections per raise due to a variety of reasons (**Figure 5-3**):

- UG2 reef is disturbed by potholes. In the Rowland area, potholes affect approximately 10% of the stope area (Hoffmann, 2010). Potholed reef is not mined so the potholed raise exposures are not sampled, they being unrepresentative of the mining area. On average 10% of the sample section positions will not be sampled due to potholes.
- A number of sample sections are rejected should they fail due to quality issues that are found during validation. For example, an entire sample section will fail if a sample is missing or if a laboratory error occurs that

has not been resolved. The proportion of sample sections rejected by the evaluation department is approximately 5% (Roos, pers. comm.).

- Not all sample sections are sampled at their planned positions:
  - Sampling will not be conducted if all or a portion of the reef is in the hangingwall or the footwall. This situation is common on the edge of potholes, next to faults and where the reef undulates severely.
  - Some samples are not taken due to logistical reasons, such as stope preparation work taking place before final sampling could take place and premature removal of water and compressed air services.

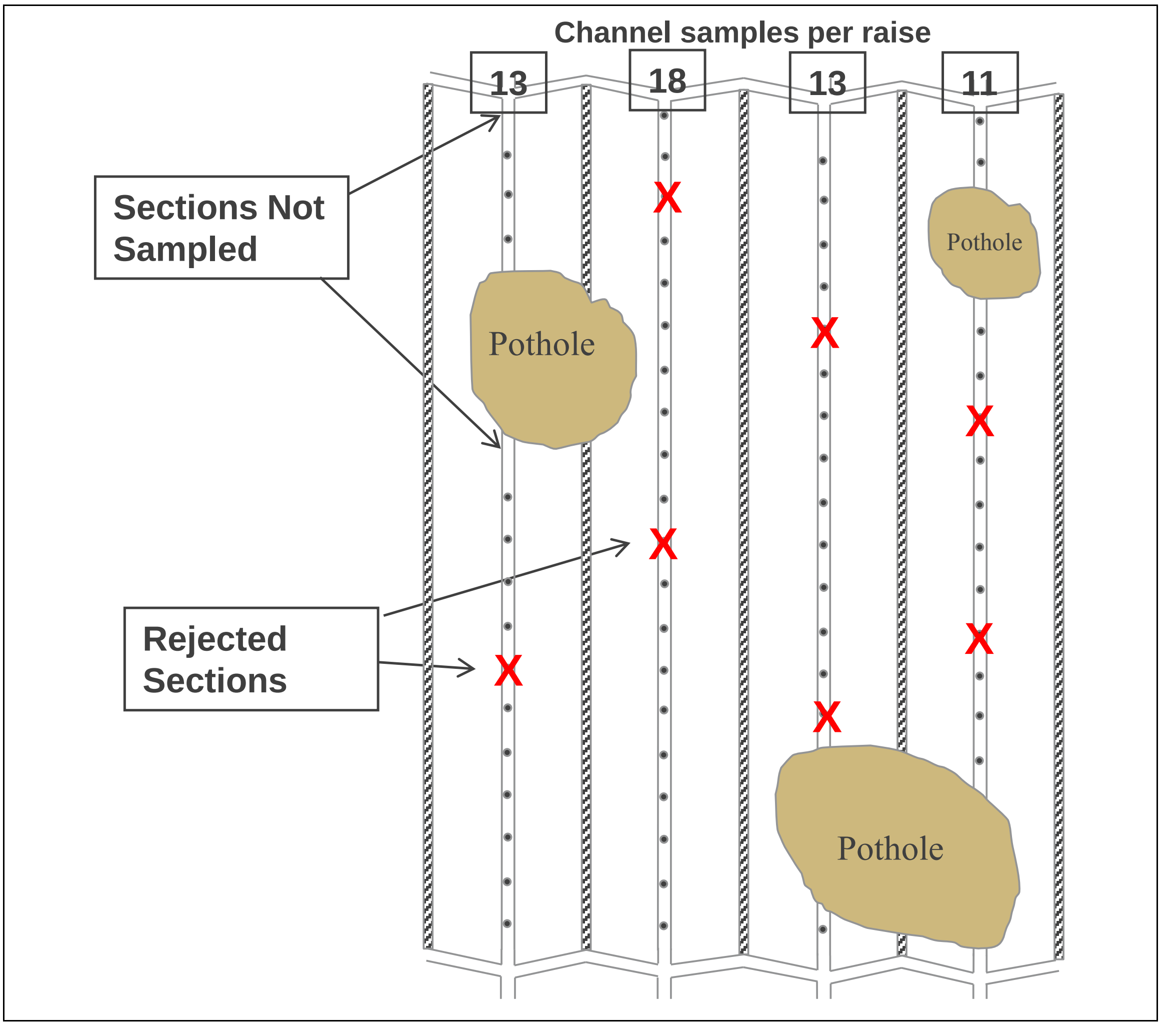


Figure 5-3 Simplified dip stope layout showing schematic final sampling pattern after geological disturbances, quality and logistical issue have been taken into account. (Black dots within the raises indicate sample positions).

Until October 2009, the Mineral Resource Management (MRM) system used at Lonmin only allowed for those channels sample sections falling within the stope to be evaluated to be selected for the stope scale estimation. Since then, the system has been changed and the evaluator has the option to source samples from adjacent raises to use for the evaluation of the stope. The standard at Lonmin is that a minimum of 15 channel samples must be used for a stope scale evaluation. Should the stope to be evaluated happen to contain less than 15 valid channels in its raise, then all the channels from the nearest adjacent raise are added and so on until the required minimum number of channels is obtained (**Figure 5-4**). The minimum number of samples required is similar to that of Clark's recommendation of 14.

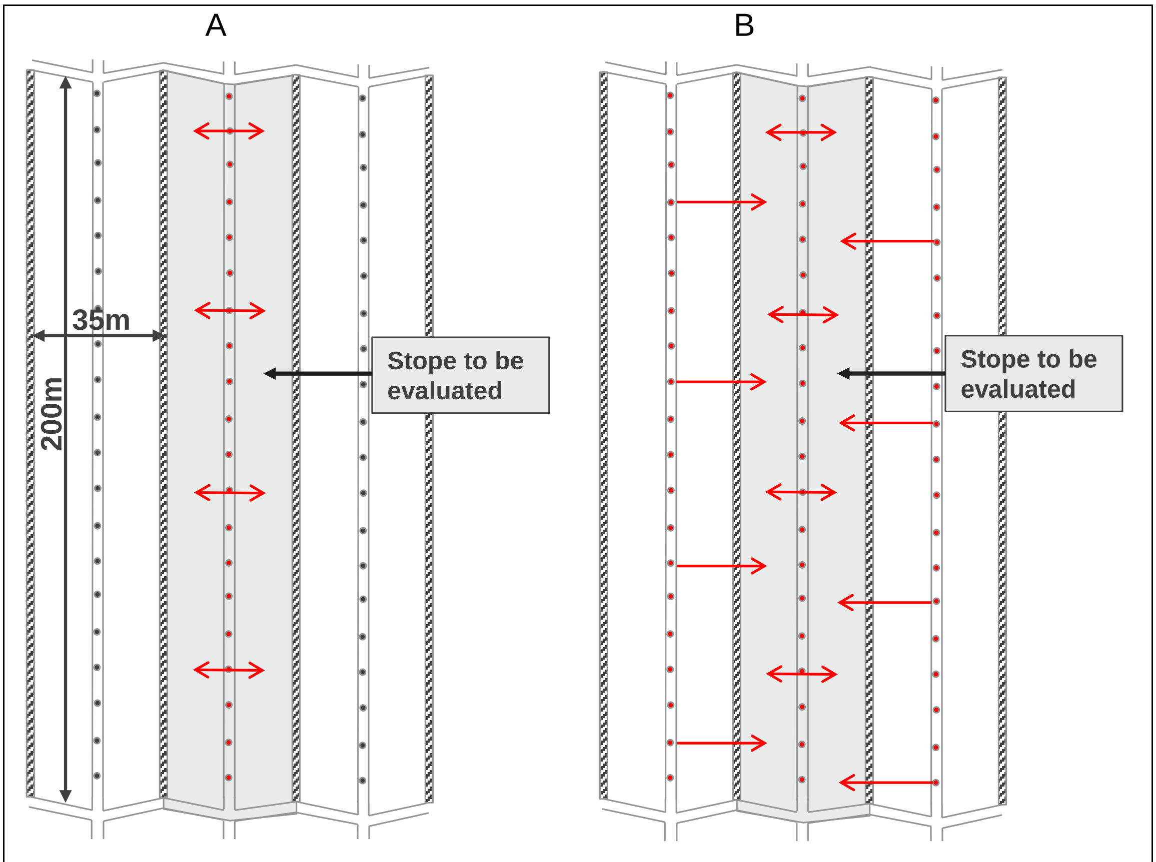


Figure 5-4 Schematic representation of channel samples selected for evaluation (red circles) using single raise selection (A) and multiple raise selection (B).

Once the samples have been selected, the stope estimation is conducted using averaging techniques with no spatial weighting, the estimation simply being:

$$g^* = \frac{\sum_{i=1}^{n}[g_i . tt_i . D_i]}{\sum_{i=1}^{n}[tt_i . D_i]}$$

$$D^* = \frac{\sum_{i=1}^{n}[tt_i . D_i]}{\sum_{i=1}^{n} tt_i}$$

$$tt^* = \frac{1}{n}\sum_{i=1}^{n} tt_i$$

Where:

g = grade in g/t

D =density in g/cm^3

tt =true thickness in metres

The actual process used to evaluate stoping at Lonmin is somewhat more complicated in that averages of each 10 cm slice through the reef are calculated as well as a number of slices into the hangingwall and footwall. This approach is commonly known as “histogram” estimation in the South African PGE industry. The histograms are used to estimate grades of over-break (dilution) and under-break (portions of the reef inadvertently left in the stope floor or roof); a subject that is outside of this study.

The following sections of the report aim to investigate what sample interval is required in order to predict the grade and thickness of a stope. Given that less variable and more normal grade distributions were found for the UG2 Reef cut channel data compared with Clark’s (1991) findings on the chipped channel data, this study was conducted with a view to widening the channel sample spacing further from the current spacing and re-assesses the findings of Clark in 1991.

5.3 Statistical analysis of the estimates resulting from the current channel sample data set

Analysis of the actual valid channel sample data set proceeded by investigating the difference in stope estimates obtained by selecting data in three ways:

1. Single raise selection: The pre-2009 evaluation method, using only the samples cut from the raise within the stope.

2. <u>Two raise selection:</u> Whereby samples cut from the raise within the stope to be evaluated and an adjacent raise along strike are used in the evaluation.
3. <u>Three raise selection:</u> Whereby samples cut from the raise within the stope to be evaluated and two adjacent raises along strike are used in the evaluation.

As the data are near normally distributed, the tests were carried out using untransformed data.

It is important to note that this is a simple statistical technique that does not take cognisance of the spatial distribution of the data. Introducing more distant raises increases the sample variability and therefore levels of confidence will likely be over-optimistic.

Number of channels samples for estimation

There are 410 raises within the study area and between one and 21 channel sample sections have been taken from each raise. Only 19% of the 410 stope raises in the sample set contain 15 or more sections (**Figure 5-5**). This is significantly lower than what should be expected, indicating that the reject rate may be considerably higher than that estimated by Evaluation Department. It should be noted that the validation process is two stage, firstly by the evaluation department and secondly by the Mineral Resources Department using an automated script (Datamine macro) with strict criteria for acceptance in the final estimation database.

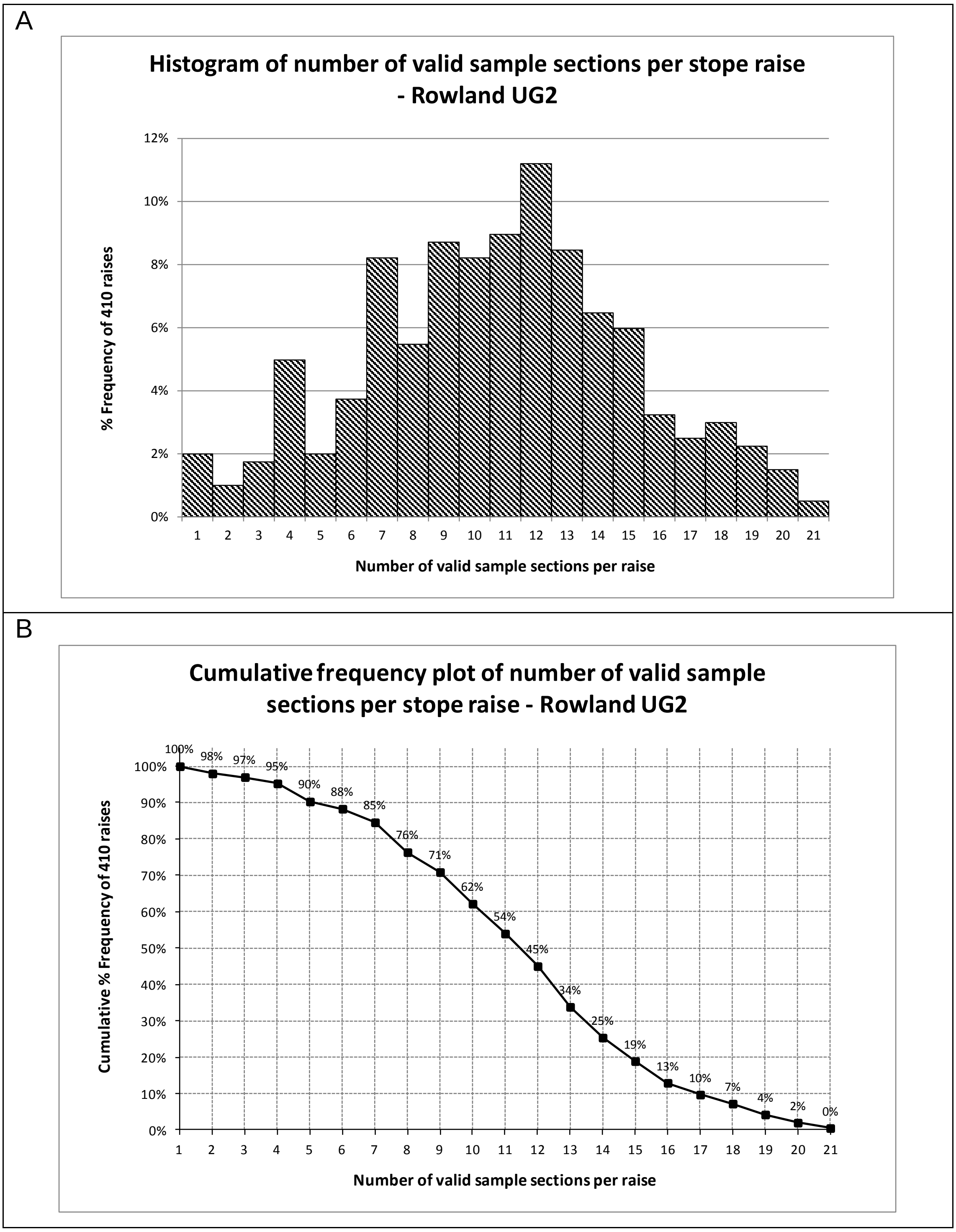


Figure 5-5 Histogram (A) and Cumulative Frequency Plot (B) showing number of valid sample sections per raise.

If the samples from two raises are selected for the estimate, then the number of samples available to estimate the stope increases so that 84% of the stopes can be evaluated using more than 15 sample sections (**Figure 5-6**).

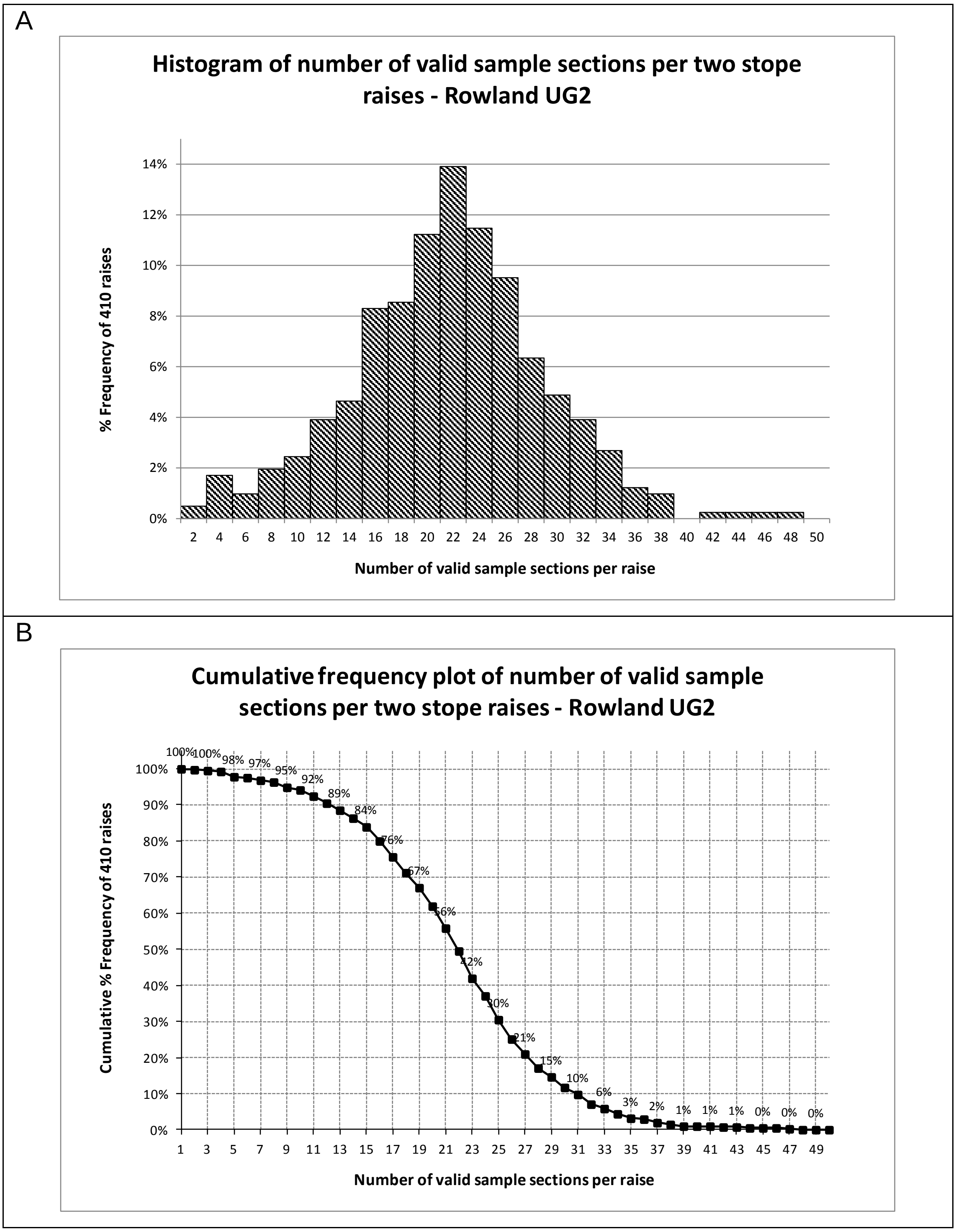
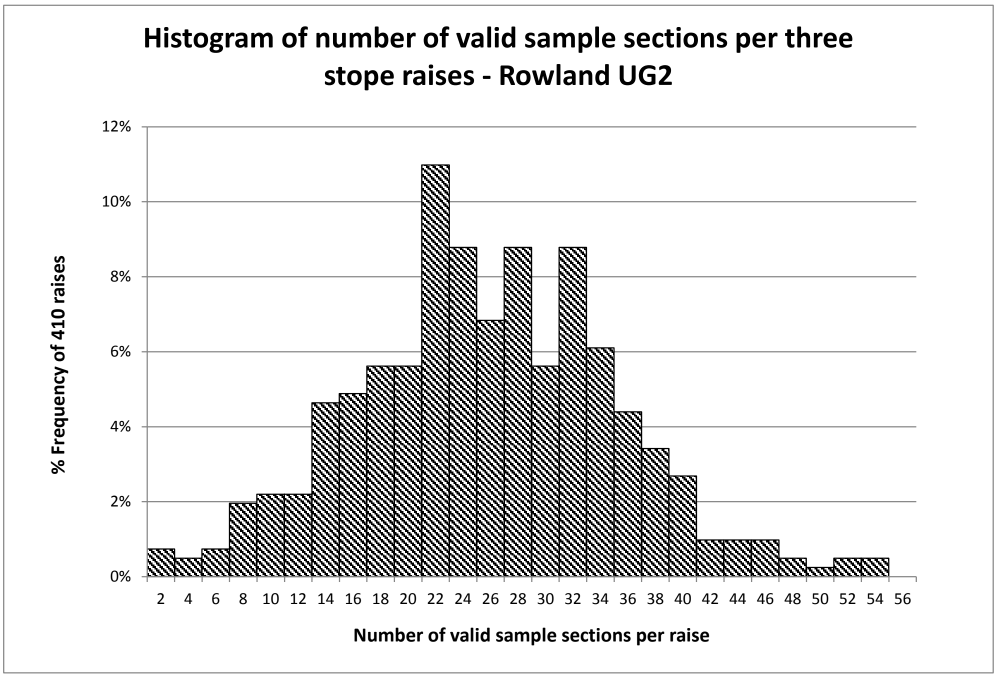


Figure 5-6 Histogram (A) and Cumulative Frequency Plot (B) showing number of valid sample sections available on a two raise selection basis

Should the samples from three raises be used for the estimate then the number of sections available to estimate the stope increases further so that 87% of the stopes can be evaluated with more than 15 sections (**Figure 5-7**).

A



B

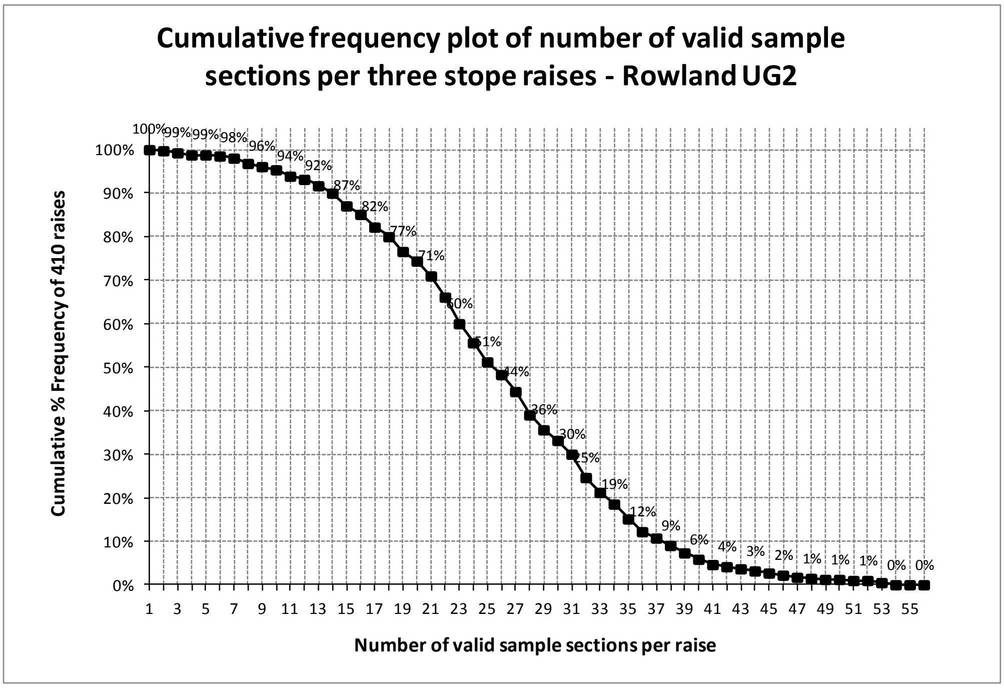


Figure 5-7 Histogram (A) and Cumulative Frequency Plot (B) showing number of valid sample sections available on a three raise selection basis

Using the present method of evaluation, 85% of the stopes can be evaluated by at least 15 samples by selecting from the stope raise and two adjacent raises compared with the 19% of raises that fulfilled the minimum number of sample criteria when only single raise selection was allowed.

Comparison of the estimates produced using the three different data selection methods.

Estimates were compared in the following ways:

- Scattergrams that graphically plot one estimate against another. Included in these graphs are 10% and 20% error lines.
- Comparison of mean and variance.
- Absolute relative difference (ARD) between the new estimates and the original estimates was calculated by:

$$ARD = abs\left[\frac{(g_o^* - g_n^*)}{g_o^*}\right].100$$

Where:

g_o^* = the original estimate of the attribute

g_n^* = the new estimate of the attribute,

and then plotted graphically.

Table 5-2 shows the mean number of samples, mean estimate of 4E grade and thickness, and variance of the estimates for samples selected from either one, two or three raises within the Rowland study area.

Table 5-2 Mean and variance of 4E grade and true thickness estimates using one, two and three raise selection.

	One Raise Selection	**Two raise selection**	**Three Raise selection**
Average number of samples per estimate	10.8	21.2	25.2
Mean of estimates 4E g/t	5.40	5.40	5.39
Variance of estimates 4E g/t^2	0.33	0.19	0.18
Mean of estimates True thickness (m)	1.10	1.10	1.10
Variance of estimates True thickness (m^2)	0.005	0.003	0.002

Two raise selection provides twice the number of samples on average for each estimate compared to single raise selection and the variance of the 4E grade estimates is reduced by 42%. The increase in the average number of samples for three raise selection is less than what should be expected mainly due to there being areas where three raises cannot be selected by the search such as at the strike limits of the mine and areas where the stope layouts are not regular. As should be expected, the average of the estimates does not change significantly between the three selection methods but the variance decreases with increasing numbers of samples.

The scattergrams compare the estimated value obtained for a panel using one estimation criteria against another; each dot on a scattergram represents a single panel. They reveal that that the correlation between single raise and two raise selection estimates is reasonable although there is significant scatter (R^2 of 0.58 for 4E grade and 0.68 for true thickness; **Figure 5-8**). Comparing estimates using a single raise with that of three raises, results in a slightly poorer correlation (R^2 0.53 and 0.56 for 4E grade and true thickness respectively; **Figure 5-9**). Estimates derived from the multiple raise selection are more smoothed compared to those from the single raise as evidenced by the flatter linear regression line. The correlation between estimates using two and three raises is better (R^2 of 0.69 for 4E grade and 0.73 for true thickness; **Figure 5-10**) and the linear regression is closer to singularity indicating that the additional smoothing from the additional samples sourced from three raises compared to two raises is minimal. All except one of the estimates from three raises is within 20% of the estimate from two raises for 4E grade and within 10% for true thickness.

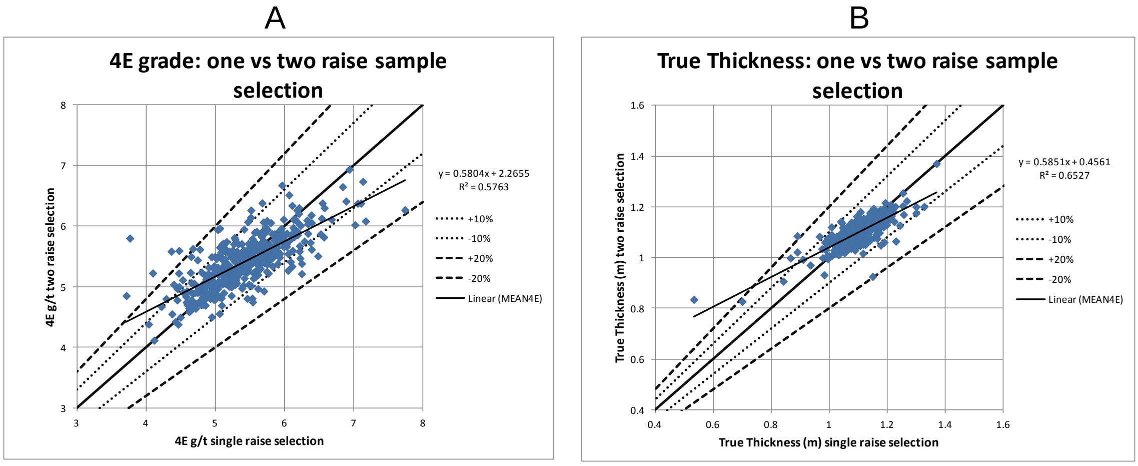


Figure 5-8 Scattergrams for 4E grade (A) and true thickness (B) – Single Raise Estimate versus Two Raise Estimate

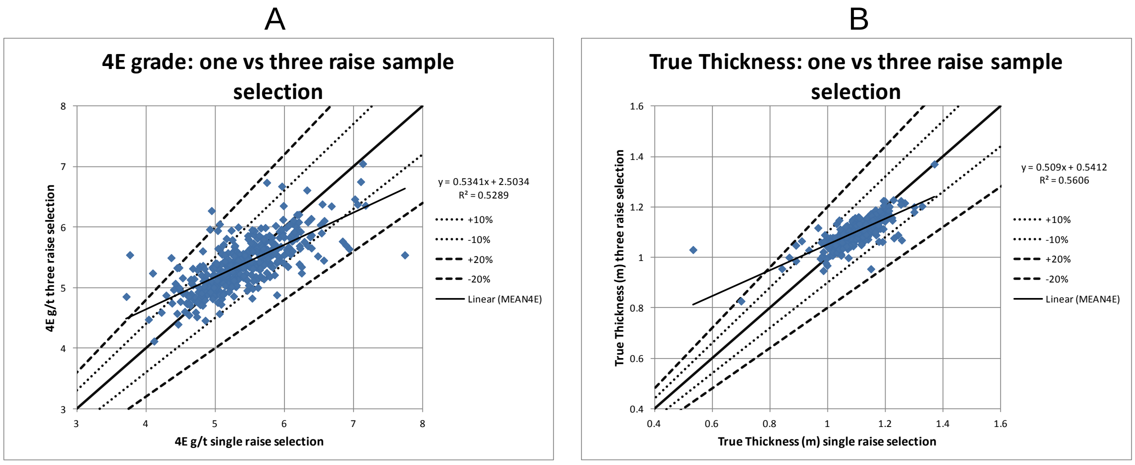


Figure 5-9 Scattergrams for 4E grade (A) and true thickness (B) – Single Raise Estimate versus Three Raise Estimate

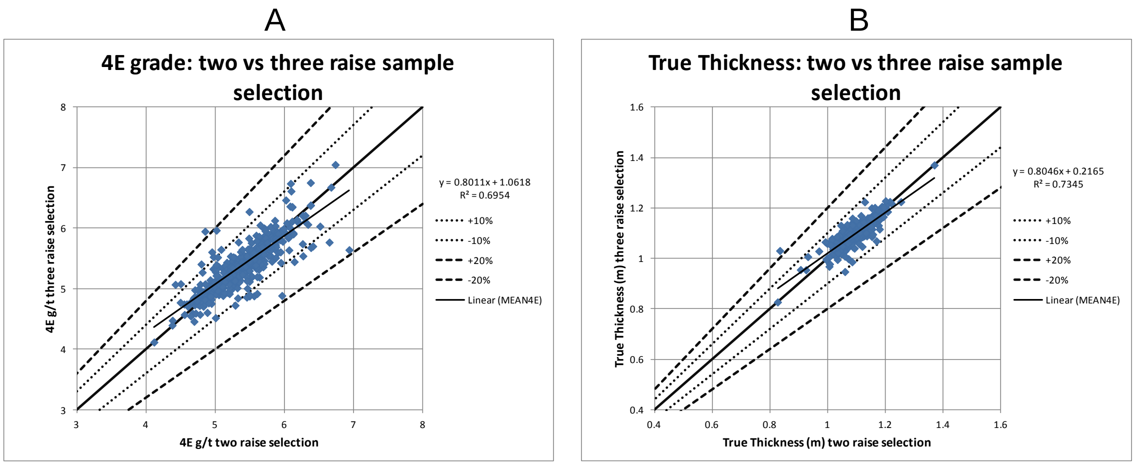
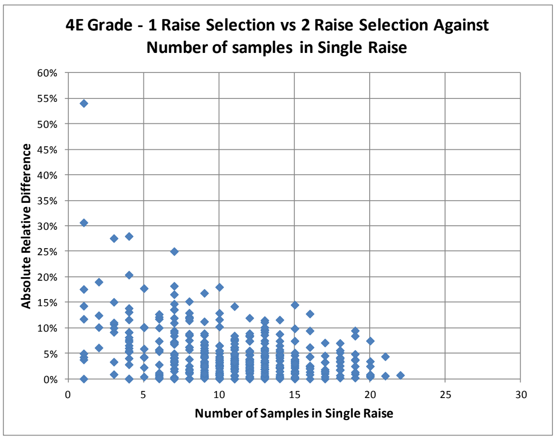


Figure 5-10 Scattergrams for 4E grade (A) and true thickness (B) – Two Raise Estimate versus Three Raise Estimate

When plotting the Absolute Relative Difference (%) between the estimates using two or three raise selection and one raise selection against the number of samples sourced from the single raise, it is evident that the larger differences are where there were few samples available for estimation in the single raise. Once there are 15 or more samples available in the single raise, the 4E grade estimates using two or three raise are mostly within 10% of those of the single raise. For true thickness, more than 12 samples are required for the corresponding two or three raise estimates to always be within 10% of the original estimate (**Figure 5-11** and **Figure 5-12**).

A



B

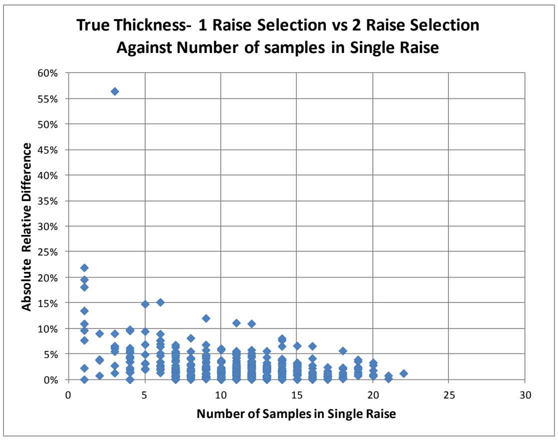
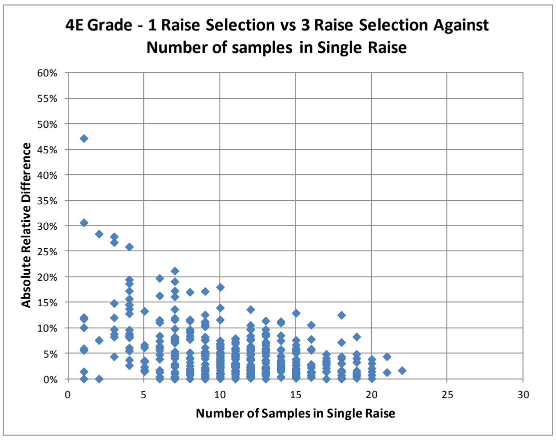


Figure 5-11 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in single raise – Single Raise Estimate versus Two Raise Estimate

A



B

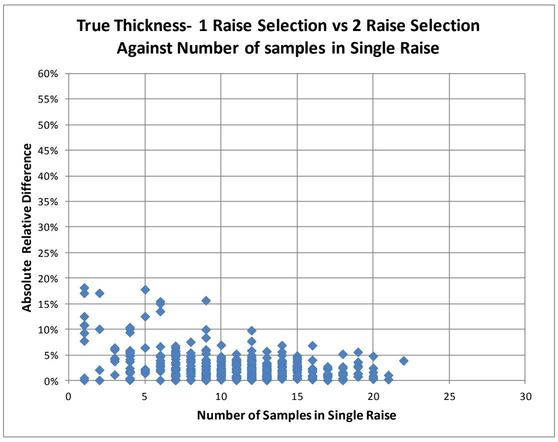


Figure 5-12 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in single raise – Single Raise Estimate versus Three Raise Estimate

Confidence in the estimates on the current data spacing using the three different data selection methods.

To give a quantitative error in the estimate, when using the mean as the estimator, the Students *t* distribution was used. The *t* distribution was used in order to determine the percentage error relative to the mean at 90% confidence for all of the raises that contained more than one sample section. An automated script was created that calculates the mean and variance for 4E g/t and true thickness for each of the raises in turn and then outputs the statistics as a single file that can be imported into Microsoft Excel for further analysis. This work was conducted using CAE Studio 3 software (Datamine) and the output of the STATS process calculates population variance and therefore it was necessary to recalculate the sample variance. The error was then derived using the following equation:

$$t_{n-1;0.05} \times {}^{s}/_{\sqrt{n}}$$

Where t_{n-1} = the percentage points of the *t* distribution at various degrees of freedom.

The error obtained was divided by the mean value of the n samples to give a percentage error of the estimate for each raise. The exercise was repeated for the two and three raise selection methods.

The percentage error for each estimate, for all of the individual raises using single raise selection, was plotted against the number of samples used. The average error was also plotted as well as the percentage of the estimates that had an error of less than 10% at 90% confidence (**Figure 5-13**).

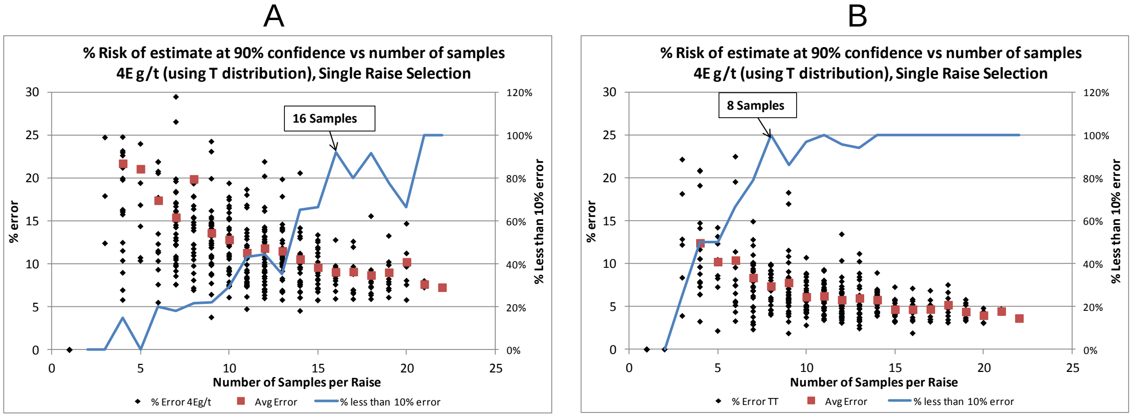


Figure 5-13 Percentage error versus number of samples for the single raise selection for 4E grade (A) and true thickness (B)

The percentage error graphs revealed the following:

- The percentage error for true thickness is considerably less than for 4E grade.
- The spread of error is high but decreases as the number of samples increase and stabilises when more than eight samples are used, the error tending to decrease slowly with increasing numbers of samples above eight.
- The average error reduces sharply with increasing number of samples. The average error does not reduce significantly once 15 or more samples are used.
- For 4E grade, the average error is less than 10% when 15 or more samples are used and for true thickness, when seven or more samples are used.
- When 16 or more samples are used for the estimate, generally greater than 80% of the 4E grade estimates have an error of less than 10%. For true thickness 8 samples are required.

The same analysis was conducted using two and three raise selection (**Figure 5-14** and **Figure 5-15**).

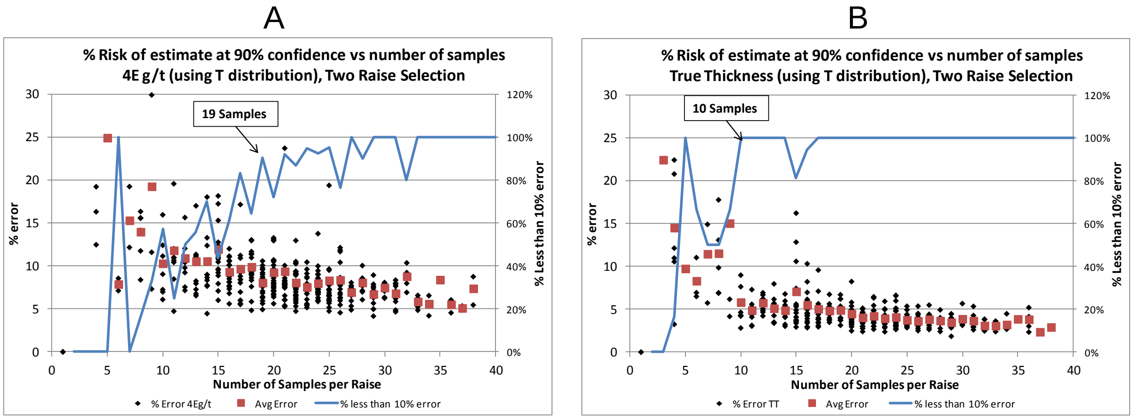


Figure 5-14 Percentage error versus number of samples for the two raise selection for 4E grade (A) and true thickness (B)

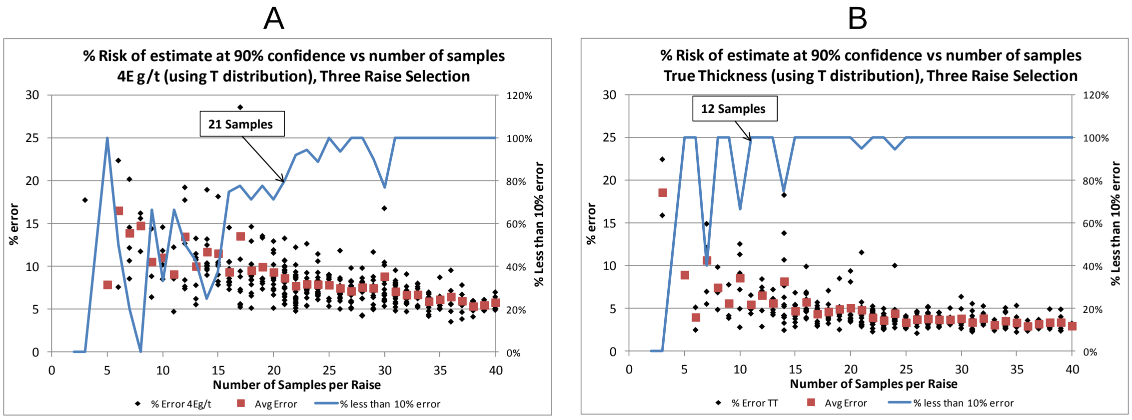


Figure 5-15 Percentage error versus number of samples for the three raise selection for 4E grade (A) and true thickness (B)

The errors are slightly higher for two raise selection when using the same number of samples as one raise selection and higher again for three raise selection. Instead of 16 samples to achieve an error of less than 10% more than 80% of the time in a 4E grade estimate in single raise selection, 19 samples are required when using two raises and 21 samples when using three raises. The same pattern occurs for true thickness. Instead of 8 samples to achieve an error of less than 10% more than 80% of the time, 10 samples are required when using two raises and 12 samples when using three raises.

It is interesting to note that as the samples are sourced from raises further away that the number of samples that are required to achieve the same estimation error, increases. This is likely a function of grade-distance relationships, which will be examined in more detail in later sections of this report.

A sensitivity test was carried out for a number of well sampled raises with more than 17 sample sections (**Figure 5-16**). For each raise, the mean and sample variance was calculated for the original sample set and again for the sample set minus one sample section and again minus two sample sections and so on. The different samples were randomly removed from the data set in many iterations and each new mean and sample variance was calculated from the new data set. A Datamine Macro was written for this purpose that used a random number generator to remove different sample sections rather than exhaustively examining the statistics for all possibilities. Using the mean and variance of each of the iterations, the percentage error for each different set of sample section data was calculated using the *t* distribution.

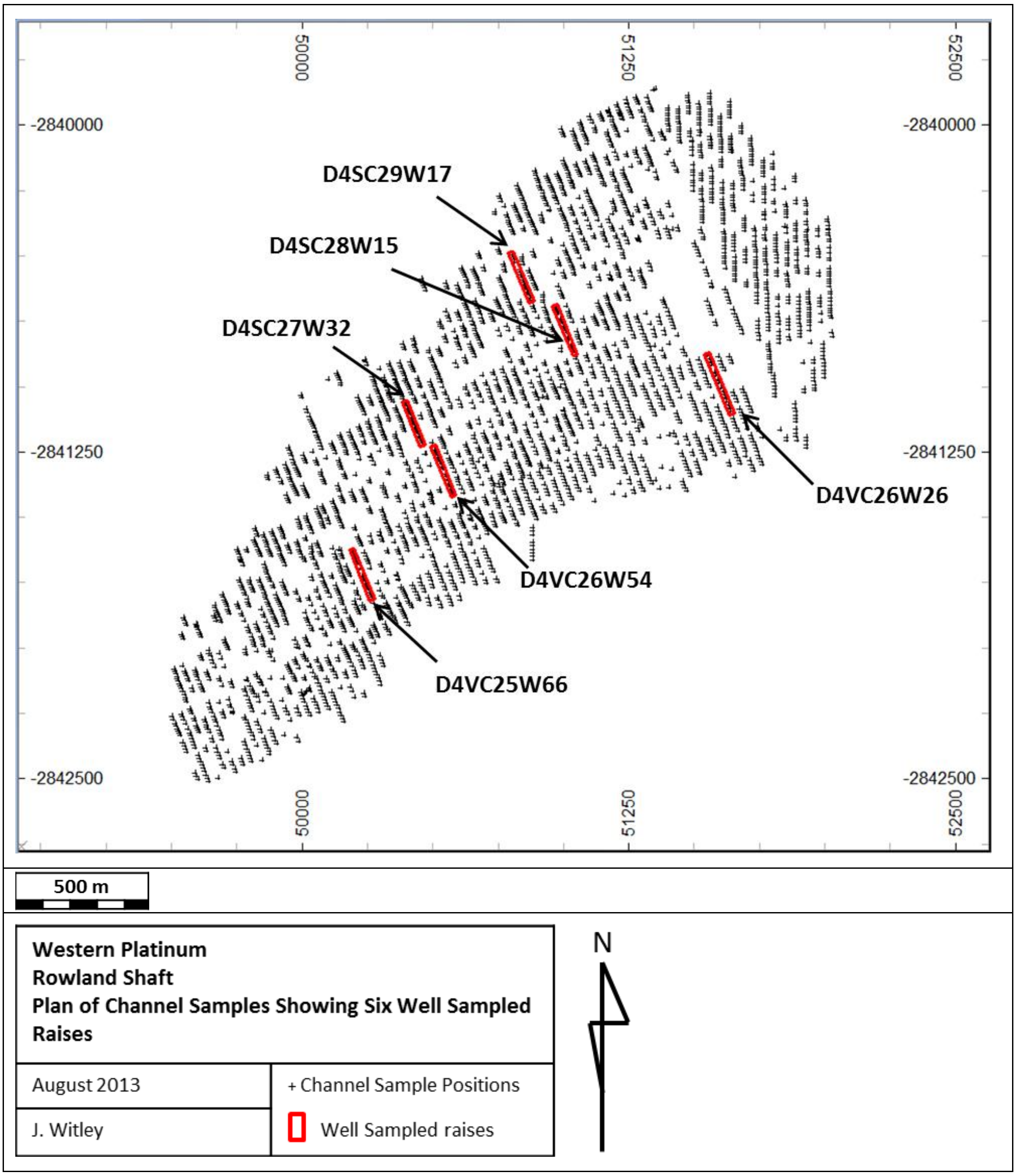


Figure 5-16 Location of the well sampled raises at Rowland shaft used for the number of samples sensitivity test.

When examining the error in the estimate using combinations of different samples from a single well sampled raise, for example D4VC36W26, similar observations can be made as with the total data set. The variability in the error is very high when sourcing few samples and the average error only reduces to below 10%, at 90% confidence, when 15 samples or more are used for the estimate for 4E grade and seven samples for true thickness. This is illustrated in **Figure 5-17** where the error in each estimate using different samples selected at random is represented by black dot and the average of the error for the number of samples is shown as a red line.

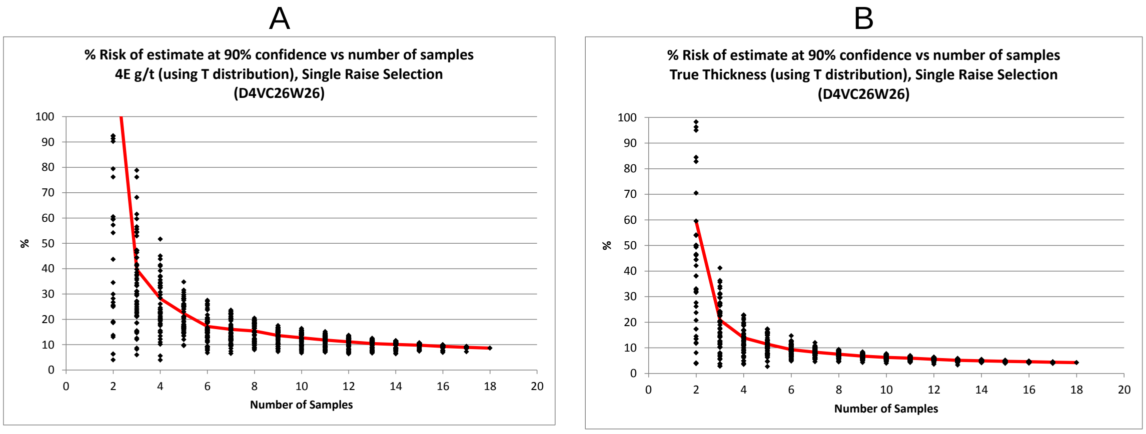


Figure 5-17 Percentage error versus number of samples for the single raise selection for 4E grade (A) and true thickness (B) – Stope D4VC26W26

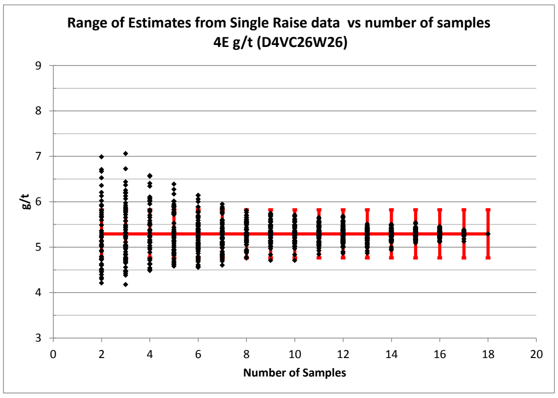
The same exercise was carried out for five more well sampled raises and queried as to how many samples were required for the average error to be less than 10% at 90% confidence. The results are shown in **Table 5-3**.

Table 5-3 Number of samples required to produce an average error of less than 10% at 90% confidence for a number of well sampled raises – single raise selection

	D4VC26 W26	D4VC25 W66	D4VC26 W54	D4SC27 W32	D4VC28 W15	D4VC29 W17
4E grade	15	11	14	16	>18	>18
Thickness	7	6	12	5	9	9

For four of the raises, between 11 and 16 samples are required for a 4E grade estimate to have an error of less than 10%. Two of the raises contained more variable data and an error of less than 10% could not be achieved with the available data.

As the number of sample sections that were taken from each raise is finite, the mean grade of the stope calculated using increasing amounts of samples will gradually approach the mean of the total number of samples in the stope, as fewer samples additional to those already used to calculate the mean are available. For the D4VC26W26 stope, once more than 11 samples are used the 4E grade estimate is within 10% of the estimate using all the data in the raise. For true thickness, more than five samples provide for an estimate within 10% of the total data mean for the 18 samples in the raise (**Figure 5-18**). This is not always the case and for the six well sampled raises, between 8 and 16 samples will produce a 4E grade estimate within 10% of the total data mean and between 5 and 9 samples for true thickness (**Table 5-4**).



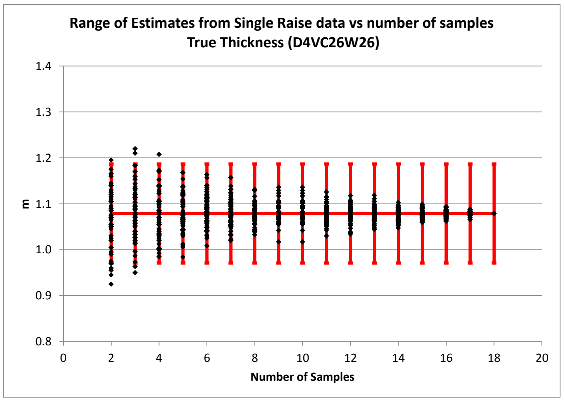


Figure 5-18 Change in estimate versus number of samples for the single raise selection for 4E grade (A) and true thickness (B) – Stope D4VC26W26 (horizontal red line is the average value of the complete sample set and vertical red lines are ±10% of the average value of the complete sample set).

Table 5-4 Number of samples required to produce an average value within 10% of the total data average for each raise - well sampled raises, single raise selection

	D4VC26 W26	D4VC25 W66	D4VC26 W54	D4SC27 W32	D4VC28 W15	D4VC29 W17
4E grade	11	8	11	10	13	16
Thickness	5	5	9	6	6	7

Although interesting to examine the variability of the samples in any one raise in this way, use of the mean of the estimate is not of great use in sample optimisation as it is self-fulfilling given the finite number of data. It does serve to illustrate how the use of limited numbers of data in an estimate can give inaccurate estimates and that estimates using few samples should be avoided.

For example, D4VC26W26 is a typical grade raise (average 5.29 g/t 4E) with about average sample variability. At an arbitrary 4E cut-off-grade of 4 g/t, using any number of samples from the raise to produce an estimate, will always result in a decision to mine based on grade alone, there being only one sample channel with a 4E grade of less than 4 g/t (**Figure 5-18**). D4SC28W15 has almost the same average 4E grade (5.23 g/t 4E) as D4VC26W26 but the channel sample grade is more variable. Using a 4E cut-off-grade of 4.0 g/t, it is possible to incorrectly categorise this stope as below cut-off grade when four or less samples are used. If only 2 samples are used, the probability of categorising this stope as waste (below 4 g/t 4E) is 14%. When using only 3 samples this probability reduces to 7% and when using 4 samples the probability of incorrect ore-waste categorisation is 4% (**Figure 5-19**). Although the chances of incorrect ore-waste classification are small, less than five is too few samples to make a mine/do-not-mine decision given the high grade variability in the stope. One would certainly not wish to take this risk after incurring high stope development costs, particularly when compared with the low cost of sampling.

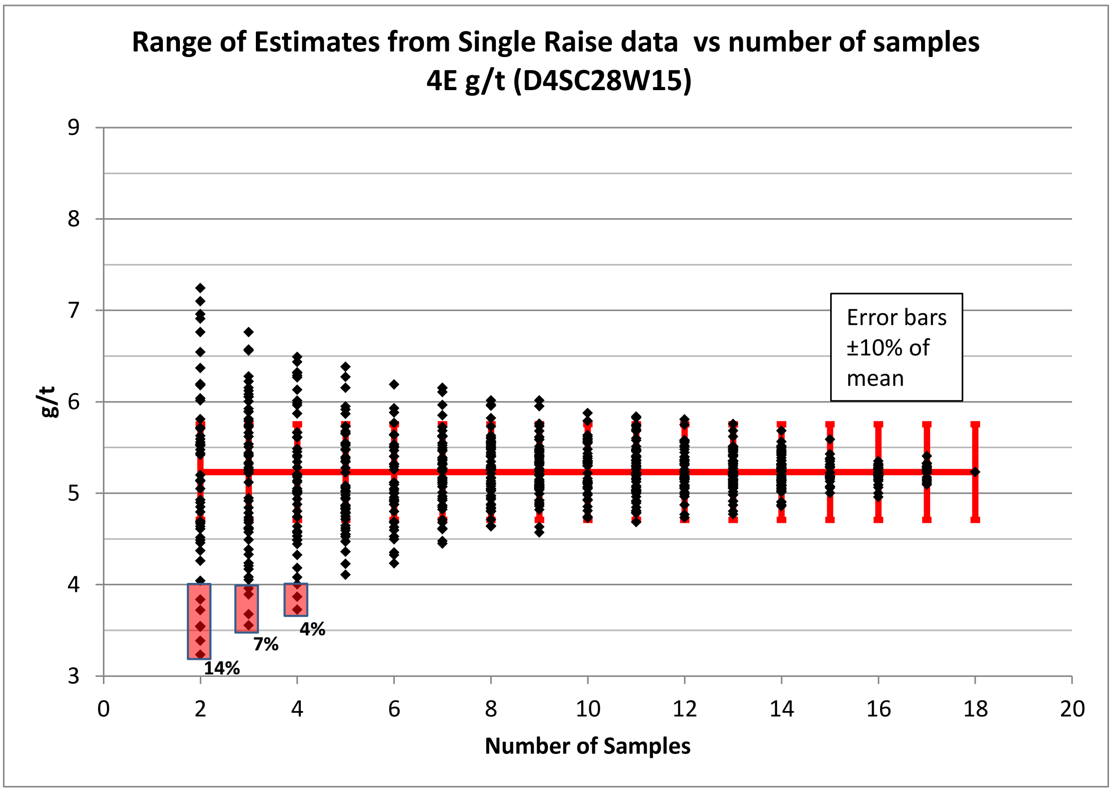


Figure 5-19 Change in estimate versus number of samples for the single raise selection for 4E grade – Stope D4SC28W15, showing percentage of estimates below 4 g/t 4E (horizontal red line is average value of the complete sample set and vertical lines are ±10% of the average value).

Fortunately 5 or more sample sections have been taken from 90% of the raises in the study area (**Figure 5-5**), thus the chances of incorrect ore-waste categorisation in the study area is likely to be very small.

Summary of the findings of the comparison between estimates using samples from one raise, two raises or three raises.

- Overall the average of the estimates is the same whether the samples from one, two or three raises are used to estimate a stope. The variance between the estimates is lowered as more samples are used from outside of the stope to be estimated.
- Estimates are smoothed more as more raises are used and more samples are sourced - estimates are a local mean.
- The difference between 4E grade estimates from a single raise and multiple raises is normally less than 10% when approximately 16 or more

samples are used for an estimate, whether two or three raises are used. For true thickness the differences are significantly less.

- Using the *t* statistic it was revealed that:
  - the percentage error in an estimate for true thickness is considerably less than for 4E grade.
  - The spread of error is high, but decreases as the number of samples increase. For an estimate sourcing samples from a single raise the spread of the error is limited when more than eight samples are used and the error tends to decrease slowly with increasing numbers of samples above eight.
  - For 4E grade, 80% of the estimates have an error of less than 10% when 16 or more samples are used sourced from a single raise. This increases to 19 when using two raises and 21 when using samples from 3 raises. This may be a function of distance-variance relationships in that the variance between samples is expected to increase as they are further away from each other.
  - When examining six well sampled raises separately, it was revealed that between 11 and 16 samples are required for a 4E grade estimate to have an error of less than 10% for four of the raises. Two of the raises contained more variable data and an error of less than 10% could not be achieved with the available data.
- Using too few samples can result in incorrect ore-waste categorisation.
- True thickness estimates are always more accurate than 4E grade estimates due to the low variance in true thickness.
- Based on achieving an error of less than 10% at 90% confidence, a minimum number of 16 sample sections for a stope estimate of 4E grade for metal accounting purposes are required, when selecting samples from a single raise. This is similar to the current Lonmin standard of 15. However, when two raises are used, 19 or more sample sections are required.
- 67% of all estimates within the study data set used 19 or more samples when selecting from two raises (**Figure 5-6**). This compares with only

13% of the estimates using 16 or more sample sections when sourcing samples from a single raise (**Figure 5-5**).

- The decision to move from selecting samples from only one raise to selecting samples from two or more raises was correct, as the chances of obtaining a reliable estimate is increased by more than five times.

5.4 Investigation into widening the sample grid

In investigating whether the sample grid can be widened and still provide an adequate number of samples for a reasonable estimate, two conditions were considered using the actual data; firstly removing every second sample section to produce 20 m sample spacing in each raise, and secondly removing every third sample section to create 30 m sample spacing in each raise. By using actual data from the 410 raises in the study area, the test grids are based on what would practically be achieved, after taking-account of the un-sampled or invalid sections, rather than the more perfect situation of a theoretical grid.

A Datamine Macro was written for this purpose that automatically created the reduced sample grid and the resulting sample mean and variance for each stope. The number of samples used for each stope estimate was also automatically recorded. For the 20 m spacing, two sets of estimates were created (every odd channel number and every even channel number) and for the 30 m spacing three sets of estimates were created (starting from every 1st, 2nd and 3rd channel).

The multiple sets of estimates were appended into a single file for each sample spacing. Using the mean and variance of the samples contributing to each estimate, the percentage error for each different set of sample section data was calculated using the *t* distribution. This was repeated for both the two and three raise selection.

A comparison was conducted to ensure that the reduced sample sets do not result in significantly different average estimates for the total study area (**Table 5-5**). As expected with a near normal distribution, selecting samples from either one, two or three raises and either 10 m, 20 m or 30 m channel sample spacing does not result in significant differences between the means of the 4E grade or true thickness stope estimates. There is an increase in variance of the estimates as the number of samples used for each stope estimate decreases and the spacing between them increases (**Table 5-5**). The variance of true thickness is so low that the reduced sample set does not affect the average thickness estimate even by 1 cm.

Table 5-5 Mean and variance of 4E grade and true thickness estimates of the reference and new estimates

	One Raise			Two Raises			Three Raises		
	10 m	**20 m**	**30 m**	**10 m**	**20 m**	**30 m**	**10 m**	**20 m**	**30 m**
Mean 4E g/t	5.40	5.41	5.41	5.40	5.40	5.40	5.39	5.39	5.39
Variance 4E g/t^2	<u>0.33</u>	0.50	0.66	0.19	<u>0.27</u>	<u>0.35</u>	0.18	0.24	<u>0.31</u>
Mean True thickness (m)	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10	1.10
Variance True thickness (m^2)	<u>0.005</u>	0.007	0.009	0.003	<u>0.004</u>	<u>0.005</u>	0.002	0.003	<u>0.004</u>

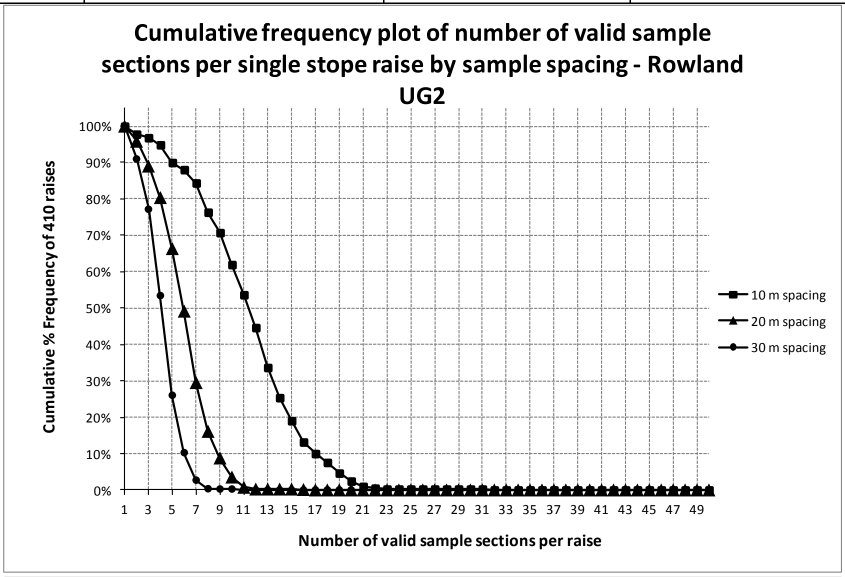
The number of sample sections available for each estimate for the 10 m, 20 m and 30 m sample spacing using one, two and three raise selection were queried and plotted on cumulative frequency plots (**Figure 5-20**). This shows that widening the sample spacing results in fewer samples per estimate than what is required to achieve a target of less than 10% error at 90% confidence; i.e. 16 for single raise, 19 for two raises and 21 for 3 raises (**Table 5-6**).

Table 5-6 Percentage of raises containing more than the required number of sample sections to ensure an optimal estimate (10% error at 90% confidence) more than 80% of the time.

	% frequency of requirement obtained		
	Original 10 m spacing	**20m spacing**	**30m spacing**
Single raise (16 samples required)	13%	0%	0%
Two raises (19 samples required)	67%	2%	0%

Three raises (21 samples required)	71%	4%	0%

A



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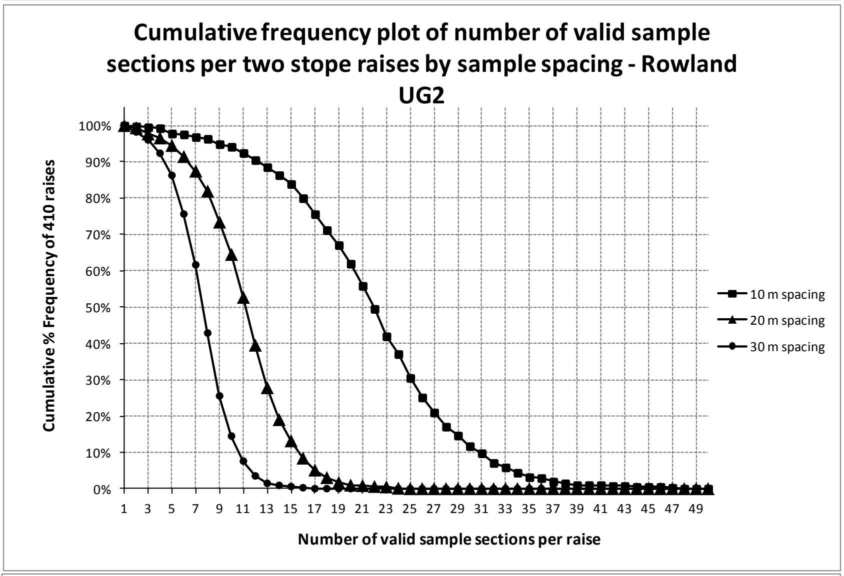


C

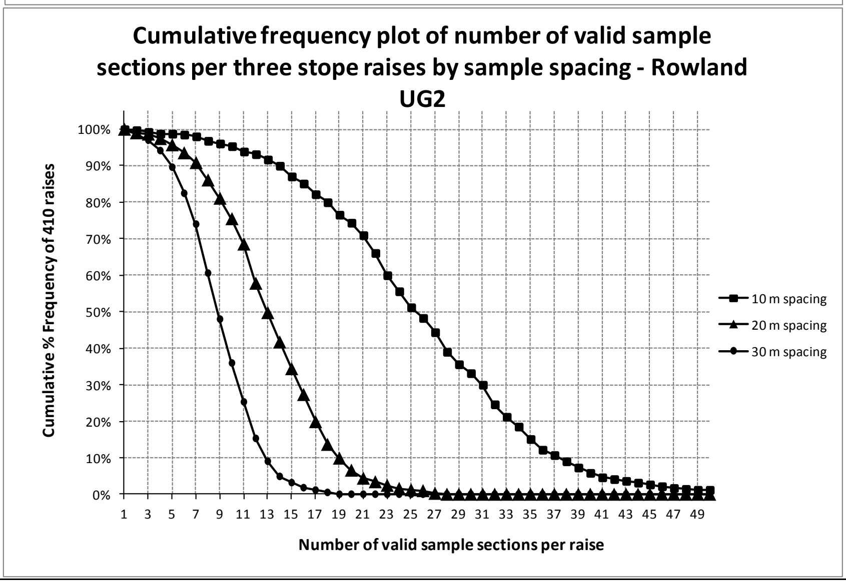


Figure 5-20 Number of samples sourced from reduced grid – (A) one raise, (B) two raises, (C) three raises

Examination of **Table 5-5** reveals that the sample variance of the subset drawn from 10 m spacing in a single raise is similar to the variance of the 20 m spaced samples drawn from 2 raises and the 30 m spaced samples drawn from 3 raises. Intuitively this would be correct as the numbers of samples from each of the aforementioned sample sub-sets should be almost the same. However spatial relationships should result in an increase in variance as the sample spacing increases, despite a similar number of sample sections.

In order to understand the overall effect of the differing sample spacing and the one or three raise sample selection, grade curves were constructed by ranking the grade estimate for each panel from highest to lowest for each sample spacing and selection type in a similar way as a grade tonnage curve (**Figure 5-21**).

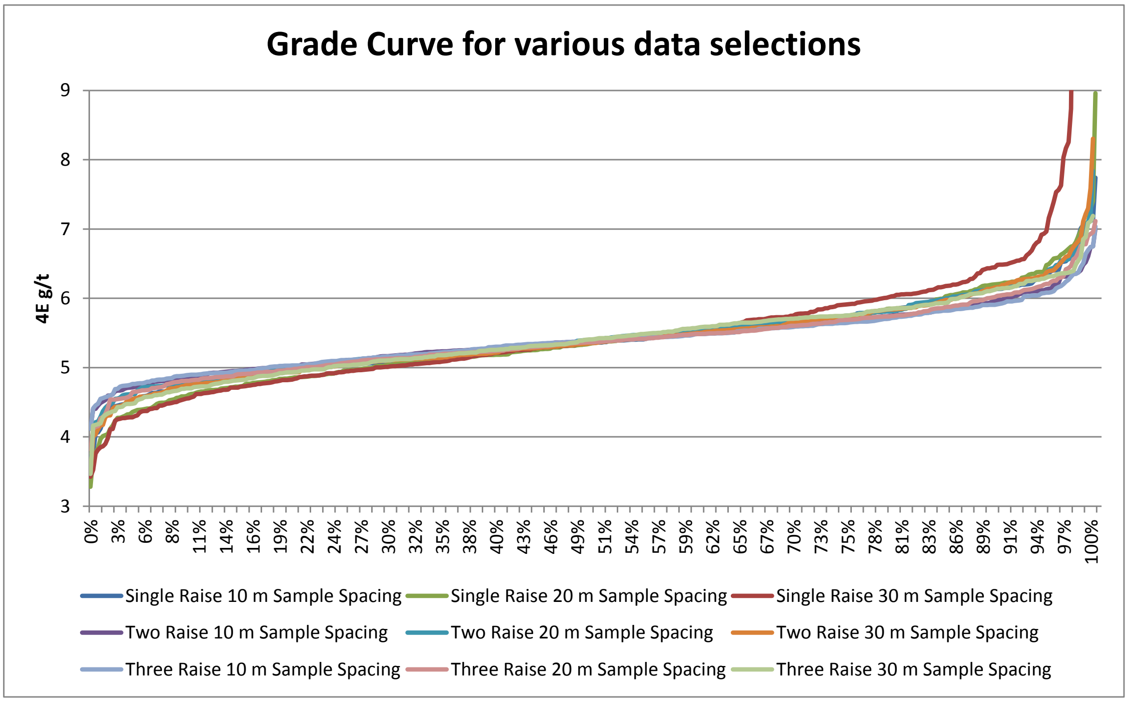


Figure 5-21 Grade curve for stope blocks based on sample spacing at 10 m, 20 m and 30 m grid for single raise, two and three raise data selection sets.

Most of the grade curve is relatively flat indicating that the UG2 at Marikana does not lend itself well to selective mining. Only 2% of the stope estimates are less than 4 g/t and these low grade estimates are generally informed by few samples. Four of the curves are very similar, the 10 m sample spacing for single raise selection, the 20 m and 30 m spacing for two raise selection and the 30 m sample spacing for three raise selection (**Figure 5-22**). This is not surprising as three of these situations will on average result in the same number of samples per grid and the different sample sets from these situations have almost the same variance (**Table 5-5**).

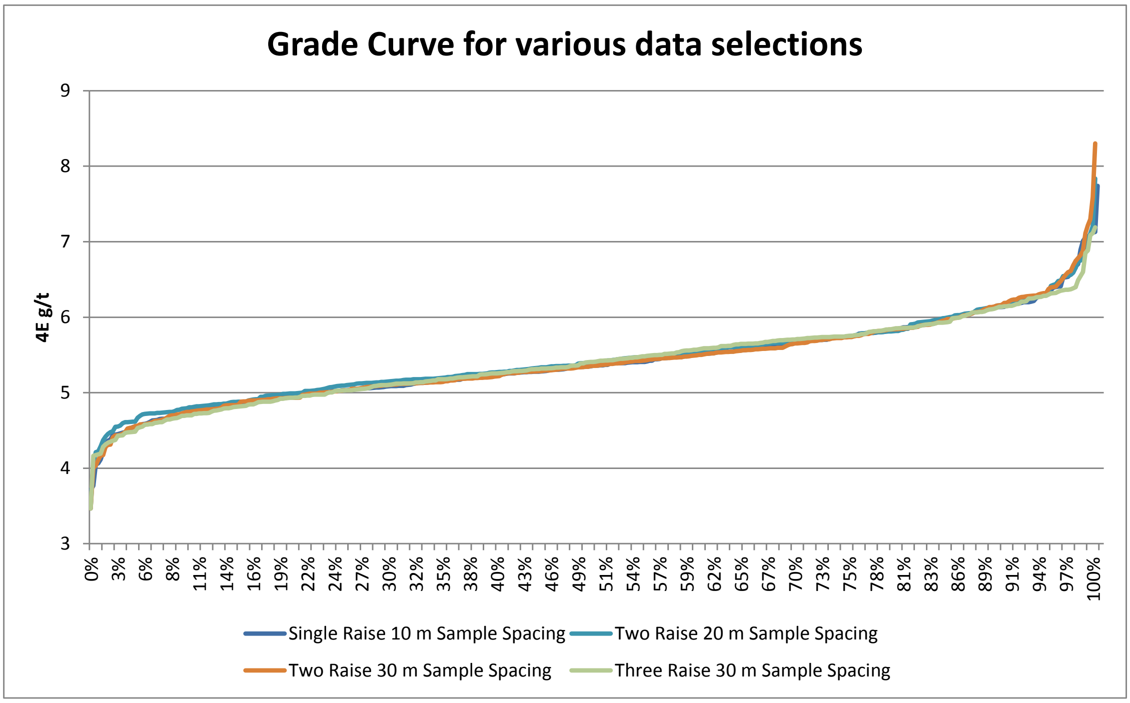


Figure 5-22 Grade curve for 10 m spacing single raise selection, 20 m spacing two raise selection, and 30 m spacing two and three raise selection.

Despite the samples being further apart, the similarity of the grade curves shown in **Figure 5-22** demonstrates that it is the number of samples that affects the estimate when using simple averaging as an estimator more than the distance between them at this scale. Using any of these four sample spacing and raise selection alternatives, overall the grade will be the same and any opportunity for selectivity will be similar.

Scattergrams comparing estimates using 10m and 20 m sample spacing in a single raise show that correlation is reasonable (**Figure 5-23**). Comparing estimates using 10 m with that of 30 m spacing, results in a slightly poorer correlation with noticeably more scatter than with 20 m spacing (**Figure 5-24**).

Estimates derived from fewer samples at the increased sample spacing are slightly less smoothed than those derived from the 10 m spaced samples. The discrepancy in slope of regression increases, when increasing the spacing relative to the 10 m spacing, as a result of the increased variance in the estimates.

A B

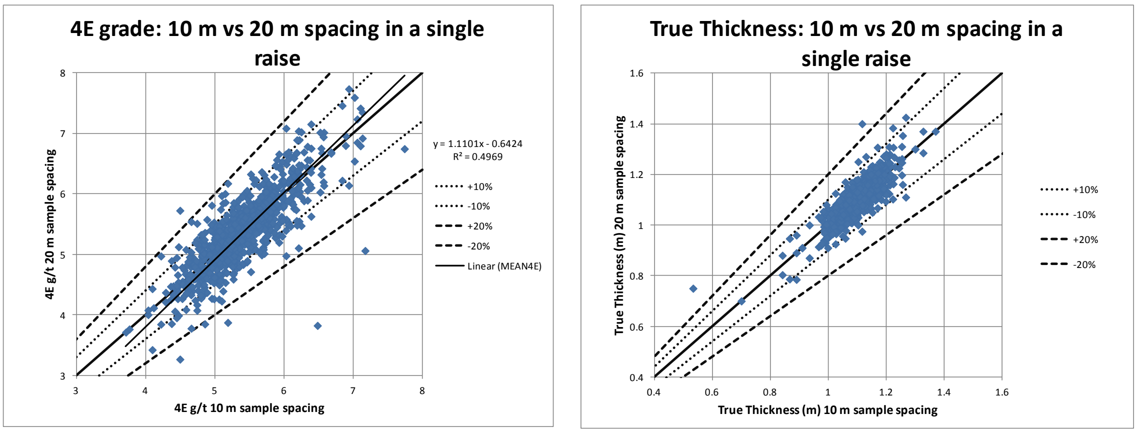


Figure 5-23 Scattergrams for 4E grade (A) and true thickness (B) – Single Raise Estimate 10 m spacing versus 20 m spacing

A B

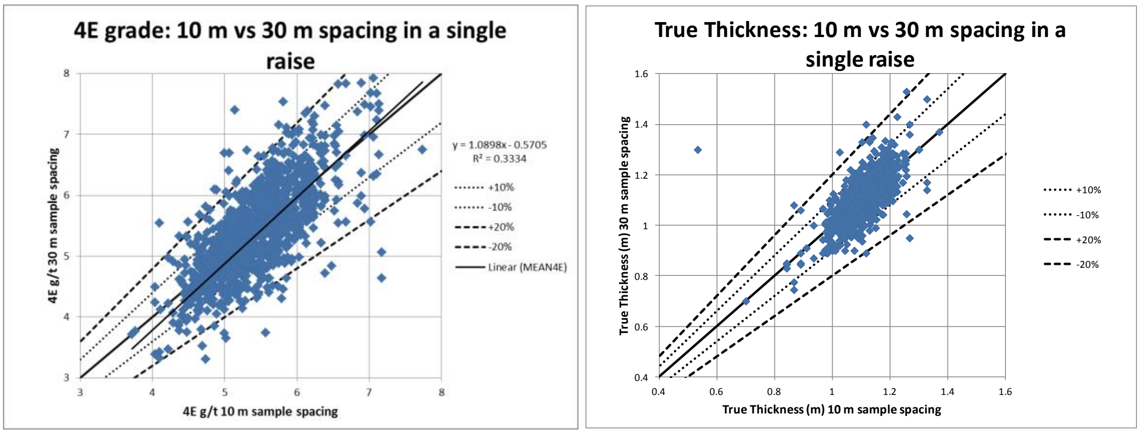


Figure 5-24 Scattergrams for 4E grade (A) and true thickness (B) – Single Raise Estimate 10 m spacing versus 30 m spacing

By plotting Absolute Relative Difference (%) against number of samples sourced from the single raise at the various sample spacing, it is evident that the larger differences are where few samples were available for estimation in the reduced data set. Where there are 8 or more samples available at 20 m spacing, the 4E grade estimates are mostly within 10% of the original estimate (**Figure 5-25**). When comparing the 10 m spacing with the 30 m spacing there is more variability and the number of available samples is fewer, consequently the number of samples required for estimates to be within 10% of the original estimate cannot be reliably determined (**Figure 5-26**). Given that only 16% of the raises have 8 or more channel samples at 20 m spacing, the chance of repeating a 4E grade estimate with the reduced spacing is poor when selecting from a single raise. To repeat an estimate within 20% with the reduced spacing is more attainable.

Only five samples are required to be able to repeat the 10 m spaced sample estimate mostly within less than 10% for true thickness.

A B

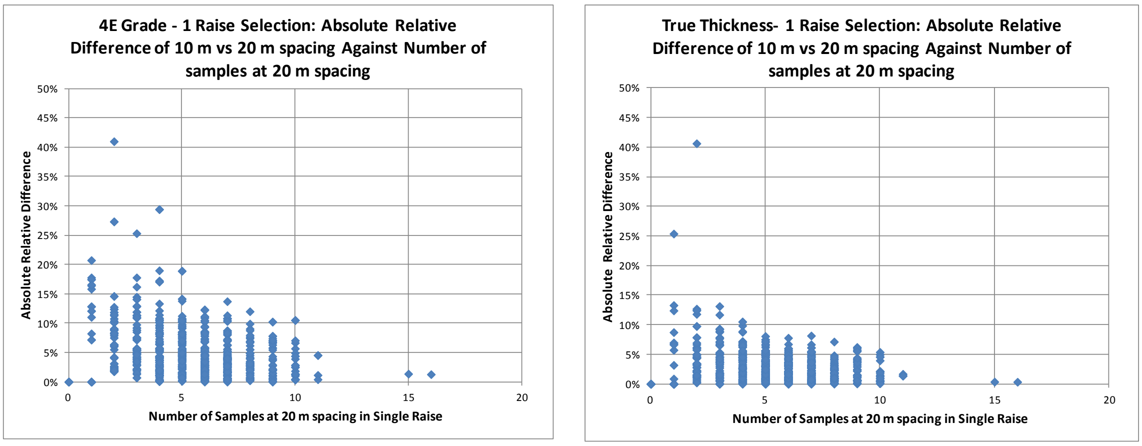


Figure 5-25 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in single raise – 10 m sample spacing estimate versus 20 m sample spacing estimate

A B

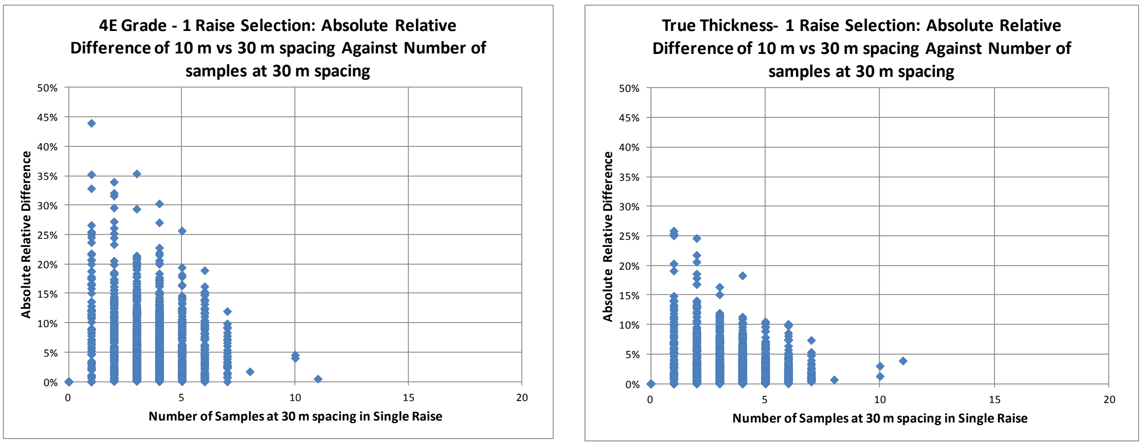


Figure 5-26 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in single raise – 10 m sample spacing estimate versus 30 m sample spacing estimate

Following the same logic for two raise selection gives better repeatability between estimates than one raise selection (**Figure 5-27** and **Figure 5-28**).

A B

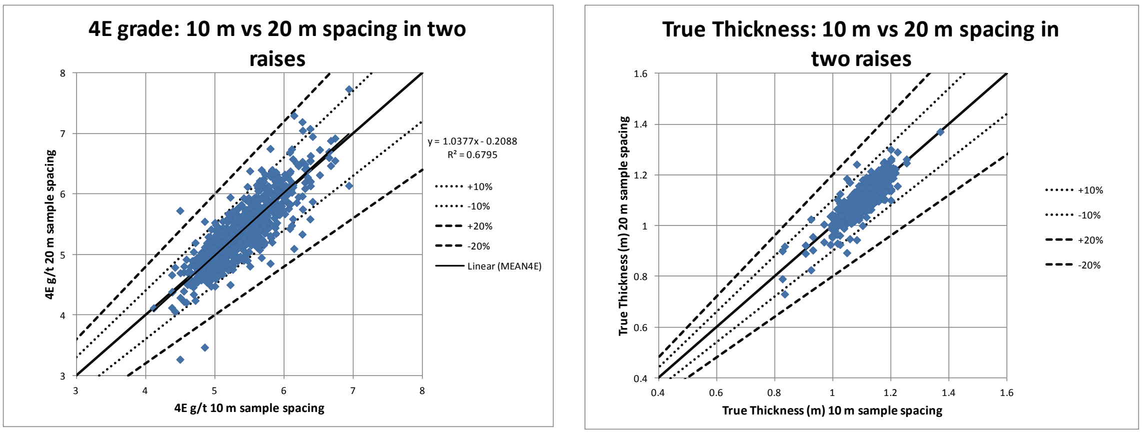


Figure 5-27 Scattergrams for 4E grade (A) and true thickness (B) – Two Raise Estimate 10 m spacing versus 20 m spacing

A B

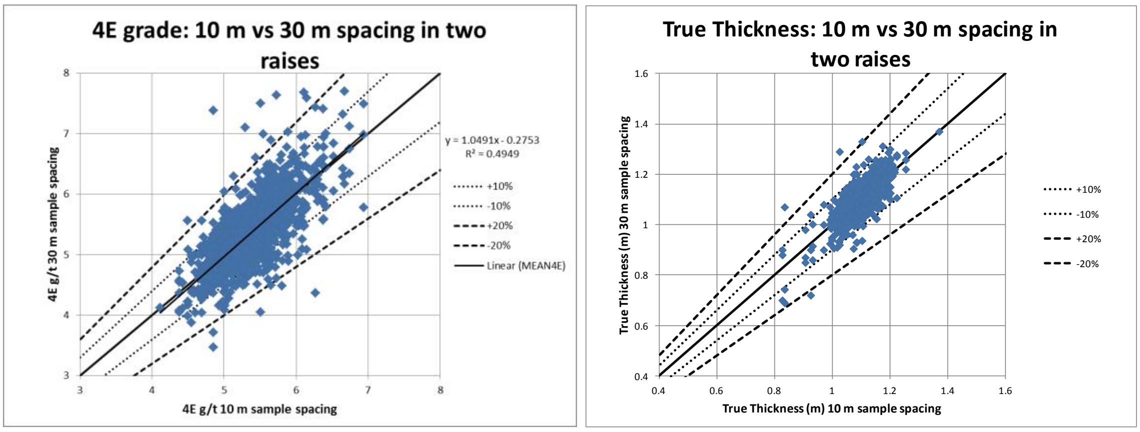


Figure 5-28 Scattergrams for 4E grade (A) and true thickness (B) – Two Raise Estimate 10 m spacing versus 30 m spacing

Once there are 8 or more samples available at 20 m spacing in a two raise selection, the 4E grade estimates are mostly within 10% of those using 10 m spaced channel samples, which is similar to the single raise selection. However, selecting 8 samples or more can be achieved 82% of the time with two raise sample selection rather than 16% for single raise sample selection (**Figure 5-20**). For true thickness, 6 or more samples are required for the corresponding 20 m spaced sample estimates to be mostly within 10% of the original estimate (**Figure 5-29**), which is easily achieved in the reduced sample set.

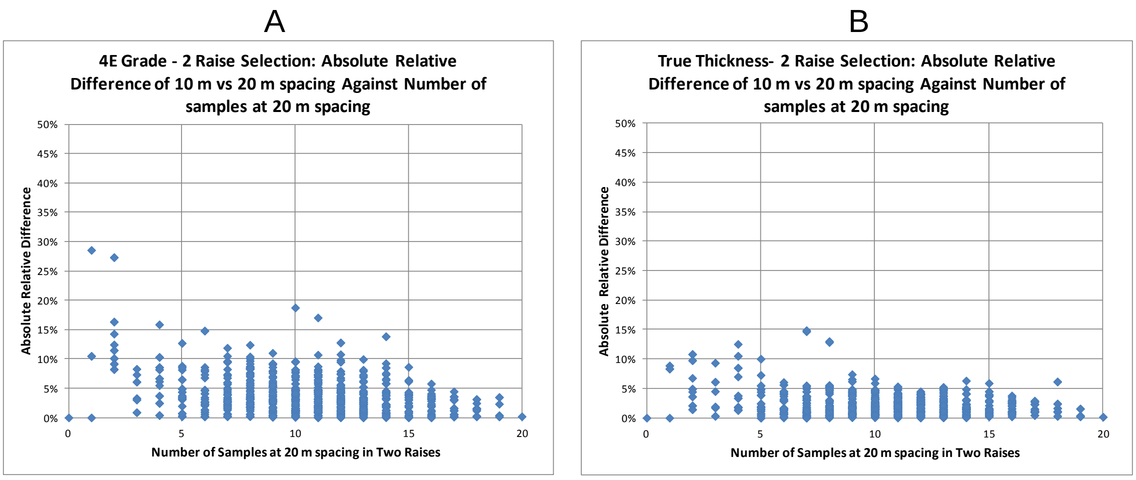


Figure 5-29 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in two raises – 10 m sample spacing estimate versus 20 m sample spacing estimate

By increasing the spacing to 30 m and selecting from two raises, 12 samples are required to attain a 4E grade estimate within 10% of that achieved by the 10 m sample spacing most of the time (**Figure 5-30**). However, there are generally too few samples in the data set; 12 or more samples being available for only 3% of the estimates (**Figure 5-20**).

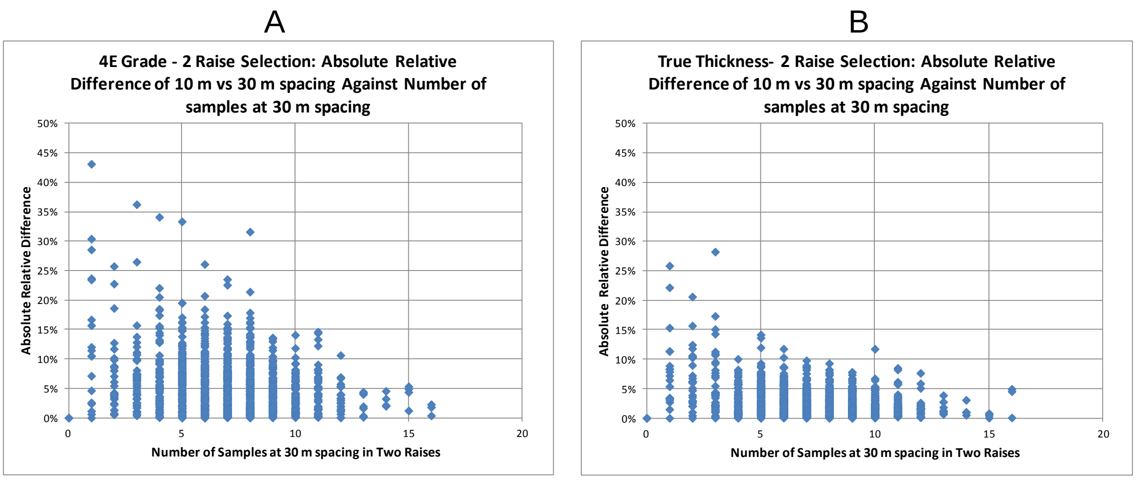


Figure 5-30 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in two raises – 10 m sample spacing estimate versus 30 m sample spacing estimate

By selecting from three raises, there is better repeatability between estimates using 10 m sample section spacing and 20 m sample spacing than with selecting from two raises (**Figure 5-31**). For true thickness, cases where the 20 m spacing compares outside of 10% with the reduced spacing are rare. With 30 m spacing many estimates are outside of the 10% limits even for true thickness (**Figure 5-32**).

A B

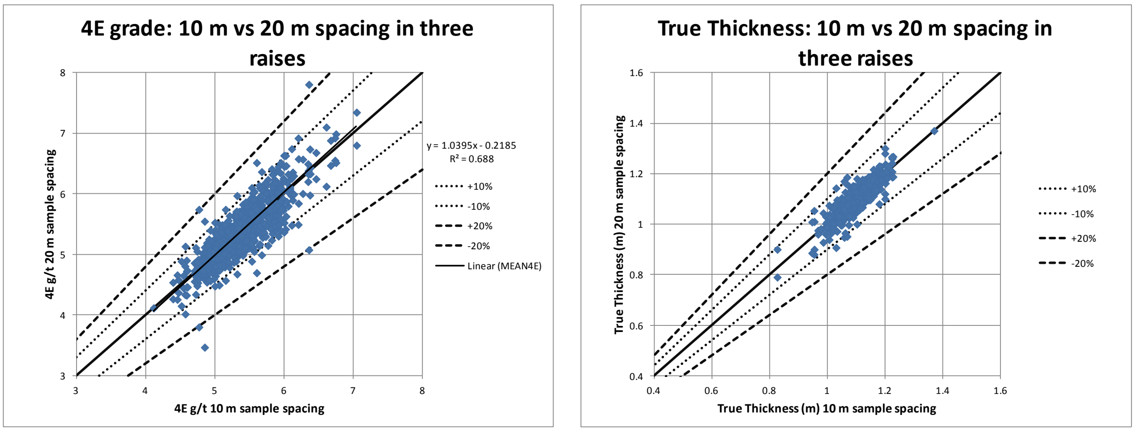


Figure 5-31 Scattergrams for 4E grade (A) and true thickness (B) – Three Raise Estimate 10 m spacing versus 20 m spacing

A B

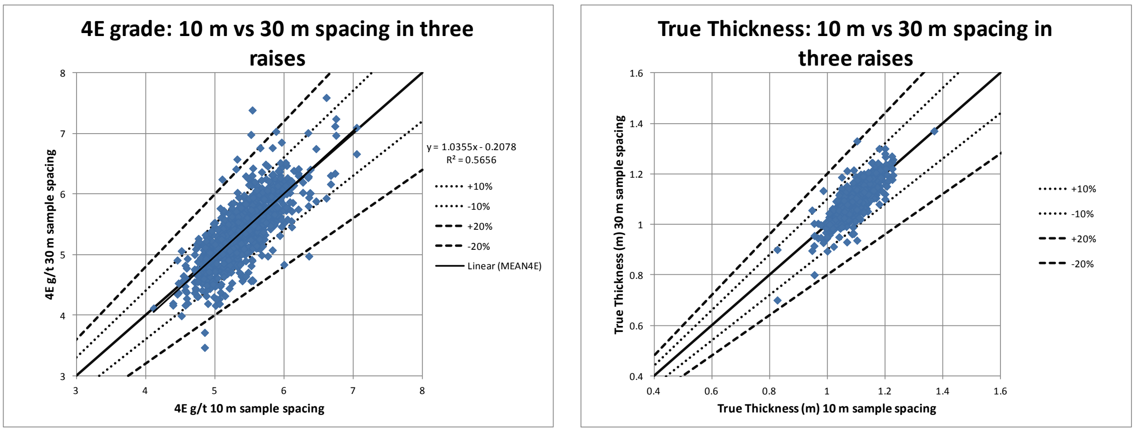


Figure 5-32 Scattergrams for 4E grade (A) and true thickness (B) – Three Raise Estimate 10 m spacing versus 30 m spacing

Once there are 10 or more samples available at 20 m spacing in a three raise selection, the 4E grade estimates are mostly within 10% of those achieved with 10 m spacing (**Figure 5-33**). In 75% of the estimates, ten or more channel sample sections are available (**Figure 5-20**).

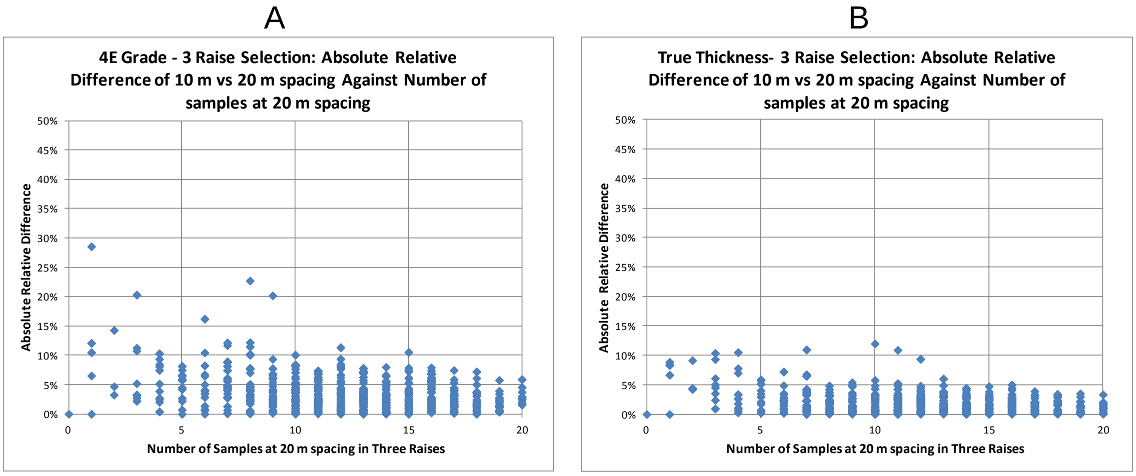


Figure 5-33 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in three raises – 10 m sample spacing estimate versus 20 m sample spacing estimate

By increasing the spacing to 30 m and selecting from three raises, 13 samples are required to attain a 4E grade estimate within 10% of the estimate by using 10 m spacing, most of the time (**Figure 5-34**). 13 or more samples are available for about 9% of the estimates.

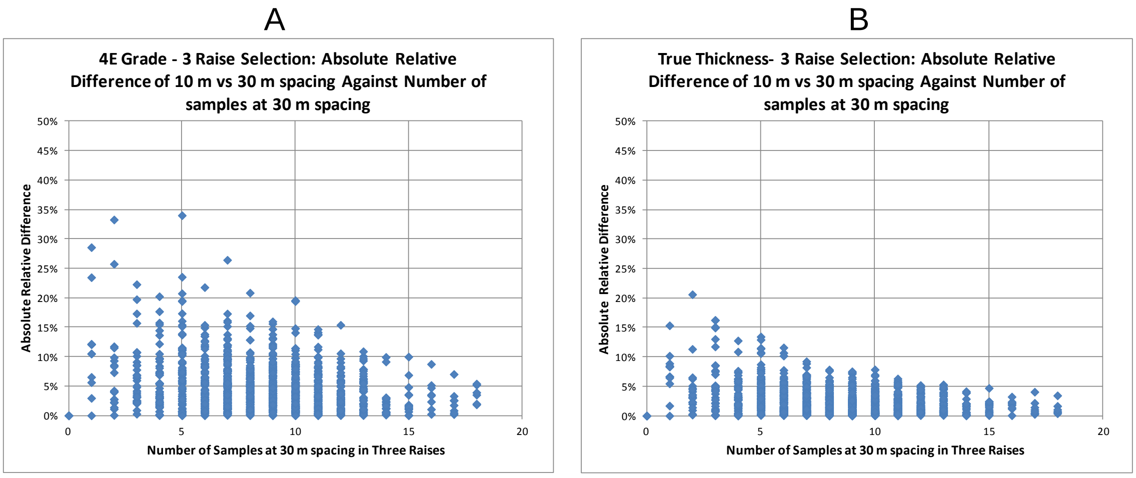


Figure 5-34 Absolute Relative Difference (%) for 4E grade (A) and true thickness (B) versus number of samples in three raises – 10 m sample spacing estimate versus 30 m sample spacing estimate

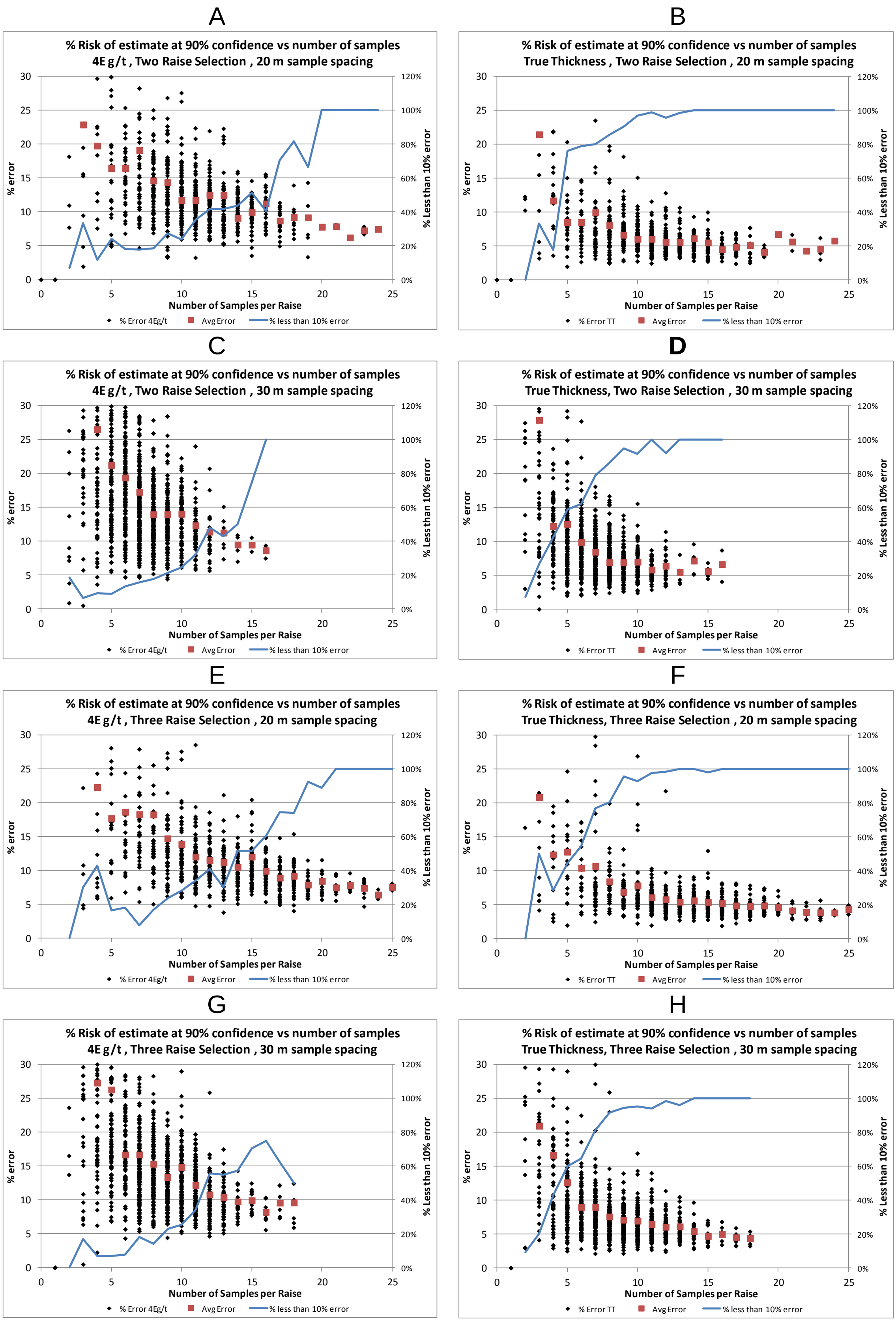


Figure 5-35 Percentage error versus number of samples for various raise selection and sample spacing for 4E grade and true thickness

Although the estimates compare well between using samples spaced either 10 m or 20 m apart when selecting from multiple raises, this does not imply the same accuracy at the increased sample spacing. The *t* distribution was used to test the error at 90% confidence limits for sample spacing of 20 m and 30 m for two and three raise selection (**Figure 5-35**).

A summary of the number of samples required for an error of less than 10% at 90% confidence for each selection method and channel sample section spacing is given in **Table 5-7** for 4E grade and **Table 5-8** for true thickness, together with the percentage of estimates that use sufficient samples

Table 5-7 Number of samples required for a 4E grade estimate with an error of <10% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	**Two Raises**			**Three Raises**		
Spacing	10m	10m	20m	30m	10m	20m	30m
Number of samples required for 80% of estimates to have an error < 10% @90% confidence (*T*)	16	19	20	15*	21	19	-*[1]
Percentage of raises in database fulfilling criteria	13	67	1	0.5	71	10	0

*Insufficient data to determine criteria within reason
*[1] Number of raises with enough samples to determine criteria

Table 5-8 Number of samples required for a true thickness estimate with an error of <10% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	**Two Raises**			**Three Raises**		
Spacing	**10m**	**10m**	**20m**	**30m**	**10m**	**20m**	**30m**
Number of samples required for 80% of estimates to have an error < 10% @90% confidence (*T*)	8	10	7	8	11	8	7
Percentage of raises in database fulfilling criteria	76	94	87	43	94	86	74

Despite that the estimates compare well with each other when reducing the sample spacing, only the 10 m spaced samples selecting from two or three raises will provide 4E grade estimates with an error of less than 10% at 90% confidence in more than two thirds of the estimates. When the spacing of the samples is increased there are insufficient samples available to provide such an accurate estimate at either 20 m or 30 m spacing even when three raises are used to select samples from. As fewer samples are required to provide a high confidence estimate for true thickness, 20 m sample spacing when selecting from two raises and 30 m sample spacing when selecting from three raises can be used with confidence.

If the target is set at 10% error there seems little scope for increasing the channel sample spacing. In order to assess what opportunities exist at slightly higher margins of error, the same tests were performed using 15% and 20% error at 90% confidence (**Table 5-9**, **Table 5-10**, **Table 5-11** and **Table 5-12**).

Table 5-9 Number of samples required for a 4E grade estimate with an error of <15% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	**Two Raises**			**Three Raises**		
Spacing	10m	10m	20m	30m	10m	20m	30m
Number of samples required for 80% of estimates to have an error < 15% @90% confidence (*t*)	11	10	10	11	9	11	11
Percentage of raises in database fulfilling criteria	54	94	65	8	96	69	25

Table 5-10 Number of samples required for a true thickness estimate with an error of <15% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	**Two Raises**			**Three Raises**		
Spacing	10m	10m	20m	30m	10m	20m	30m
Number of samples required for 80% of estimates to have an error < 15% @90% confidence (*t*)	4	5	5	4	5	6	5
Percentage of raises in database fulfilling criteria	95	98	95	92	99	94	90

Table 5-11 Number of samples required for a 4E grade estimate with an error of <20% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	Two Raises			Three Raises		
Spacing	10m	10m	20m	30m	10m	20m	30m
Number of samples required for 80% of estimates to have an error < 20% @90% confidence (*t*)	7	6	8	7	7	6	7
Percentage of raises in database fulfilling criteria	85	98	82	62	98	94	74

Table 5-12 Number of samples required for a true thickness estimate with an error of <20% at 90% confidence, and percentage of estimates that achieve this accuracy.

	One Raise	Two Raises			Three Raises		
Spacing	10m	10m	20m	30m	10m	20m	30m
Number of samples required for 80% of estimates to have an error < 20% @90% confidence (*t*)	4	5	5	4	5	4	4
Percentage of raises in database fulfilling criteria	95	98	95	92	99	97	94

By assessing the accuracy of the estimate with increasing channel sample spacing, it is revealed that should the sample section spacing be increased from 10 m to 20 m the accuracy in the estimates would decrease from approximately 10% to 15% (**Figure 5-36**). A further increase in the sample section spacing to 30 m would result in the error in the estimates increasing to approximately 20% at 90% confidence.

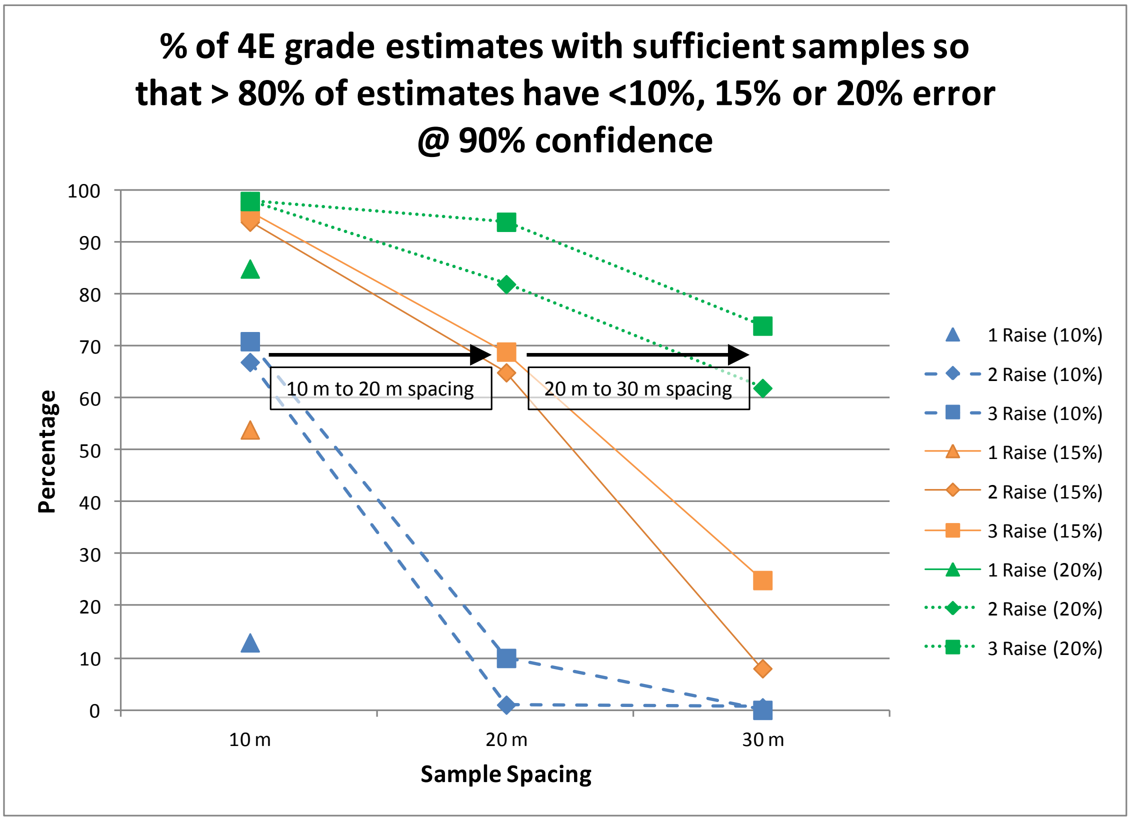


Figure 5-36 Summary of change in error with increased sample spacing

Summary of the findings of the Comparison between estimates using reduced sample spacing from one raise two raises or three raises.

- If the sample spacing is increased from 10 m to 20 m or 30 m and selection is from a single raise, the new estimates do not correlate consistently within 10% of the original estimate, there now being too few samples.
- As the sample spacing increases, more samples are required to achieve an estimate similar to that of the 10 m spacing for the same stope.

- By selecting samples spaced 20 m apart sourced from either two or three raises, slightly more than 80% of the 4E grade estimates using 10 m spacing can be repeated within ±10%.
- When the sample spacing is increased to 30 m there are insufficient samples to repeat a reasonable proportion of the 10 m spaced sample 4E grade estimates within 10%. Even when selecting from three raises only 15% of the stopes are estimated with the required number of samples.
- Although the estimates compare well between using samples spaced either 10 m or 20 m apart when selecting from multiple raises, this does not imply the same accuracy at the increased sample spacing.
- Using increased sample spacing there are insufficient data to estimate the 4E grade of a stope within less than 10% error at 90% confidence, when selecting from either one, two or three raises.
- True thickness can be estimated within less than 10% error at 90% confidence with increased sample spacing. A 20 m sample spacing when selecting from two raises and a 30 m sample spacing when selecting from three raises can be used with confidence.
- An increase in the channel sample spacing from 10 m to 20 m roughly equates to an increase in error from 10% to 15%, and to 20% for 30 m sample section spacing.
- Should the additional error incurred with the increased sample spacing be acceptable (from 10% to 15%), the sample spacing may be reduced to 20 m.
- By carefully managing the sampling programme to ensure that fewer channel sample sections are rejected due to quality issues and that sample coverage is more complete at the revised spacing the sample spacing can be increased to 20 m with little loss in accuracy in the estimate.
- Given that estimates achieved with 20 m spacing are largely within 10% of those achieved with 10 m spacing it is unlikely that there will be a noticeable difference in the stope estimates with the increased spacing and that for the mine overall no difference will be experienced.

- It should be noted that these conclusions are based on statistical assessments assuming no spatial relationship and geostatistical methods are preferred.

5.5 Geostatistical tests based on theoretical sample grids

The statistical tests described in the previous section are useful in terms of the estimation method that is used at Lonmin for stope block evaluation (the arithmetic mean of samples). However, it should be borne in mind that what is actually being estimated by the method used by Lonmin is the grade and thickness of the reef in a raise, rather that the grade and thickness of the panel. The assumption is that there is no error in extending the one dimensional raise average over the two dimensional stope area. To practically test the error in estimating the stope using the grade and thickness of the raise (an estimate that has its own error), detailed stope face sampling will be required in order to determine the "actual" grade of the stope within close limits. Reconciliation between the grade of a stope estimated from the raise sampling and the "actual" grade can then follow. Several stopes (perhaps 30) will need to be sampled in detail in order to judge the significance of the test. Due to the logistical difficulties in sawing enough face samples at regular intervals during stoping, this test has not been conducted at Lonmin and instead we need to turn to more theoretical tests in order to determine whether our sampling grid is appropriate.

Clark (1979) described five important points in estimation:

1. When estimation is performed an error is made in the prediction.
2. The magnitude of the error is a function of the structure and type of deposit and by the mineral itself.
3. The structure can probably be described by the semi-variogram in the absence of significant local trend.
4. The estimation error variance can be calculated if the semi-variogram model is known.
5. If we use an extension type of estimator, such as arithmetic mean of the samples, then the extension variance may be written:

   $\sigma_e^2 = 2\bar{\gamma}(S,A) - \bar{\gamma}(S,S) - \bar{\gamma}(A,A)$

   i.e. the 'reliability' of the estimator depends on three quantities:

- the relationship of the samples to the area being estimated $\bar{\gamma}(S, A)$,
- the relationship amongst the samples $\bar{\gamma}(S, S)$, and
- the variation of grades within the area being estimated $\bar{\gamma}(A, A)$.

Clark (1979) described a number of auxiliary functions that can be manoeuvred in order to estimate the error of estimates for a variety of scenarios and provided tables of several auxiliary functions that are required to conduct the calculation. By modifying the examples provided by Clarke (1979) to the Lonmin dip stoping layout and the multiple structured Rowland UG2 variogram, it is possible to judge the reliability of the estimation using different sampling patterns. Microsoft Excel spread sheets were created so as to automate the tests as far as possible without using specialised software.

Auxiliary Functions

This description of the auxiliary functions used for the extension variance tests has been summarised from Clark (1979). As we are using spherical models to estimate the semi variogram model, only the formulae for spherical models are presented.

The F function

This function in a one dimensional situation *F(l)* is the average semi-variogram for all possible pairs of points that can exist along the line *l*, i.e. it is the variance of grades within a line over length *l.* This variance must be removed from the system when using a point semi-variogram if we only consider the average grade over length *l* as it corresponds to the difference between a semi variogram for points and a semi variogram for a line. *F(l)* can be calculated using the following formula for a spherical model:

$F(l) = \frac{C}{20}\frac{l}{a}\left(10 - \frac{l^2}{a^2}\right)$ where $l \le a$

$F(l) = \frac{C}{20}\left(20 - 15\frac{a}{l} + 4\frac{a^2}{l^2}\right)$ where $l \ge a$

The *χ* function

This function in a one dimensional situation (*χ(l)*) is the average semi-variogram for all possible pairs of points formed between a sample at the end of a line and all possible points that can exist along the line *l*. For a spherical model it is calculated using the following formula:

$\chi(l) = \frac{C}{8}\frac{l}{a}\left(6 - \frac{l^2}{a^2}\right)$ where $l \leq a$

$\chi(l) = \frac{C}{8}\left(8 - 3\frac{a}{l}\right)$ where $l \geq a$

The semi variance between two points ($\gamma(h)$) using the spherical model is as follows:

$\gamma(h) = C\left(\frac{3}{2}\frac{h}{a} - \frac{1}{2}\frac{h^3}{a^3}\right)$ where $h \leq a$

$\gamma(h) = C$ where $h \geq a$

In order to understand the accuracy of the estimates used for metal accounting, we need to understand the error in a panel estimate rather than just a single raise. Clark (1971) described a set of auxiliary functions in two dimensions that are mostly generalisations of the one dimensional ones.

- $F(l;b)$: the average semi-variogram for all possible pairs of points that can exist in a panel with dip length l and a strike length b,
- $\chi(\mathrm{l};\mathrm{b})$ the average semi-variogram value between a line (M) of length b and an adjacent panel (M$^{/}$) of length b and breadth l,
- $\gamma(\mathrm{l};\mathrm{b})$ the average semi-variogram value between all points on one line (M) of length b and another (M$^{/}$) parallel to it, l distance away, and of the same length (this is used for evaluating panels using samples sourced from two or more raises).

An additional function is required should we wish to calculate extension variance for a panel that has both raises and drives that are sampled. $H(l;b)$ represents the average semi-variogram values between two lines (M and M$^{/}$) of lengths, b and l, at right angles to each other.

The uses of the auxiliary functions in a panel situation are illustrated schematically in **Figure 5-37**. The two dimensional formulae for the auxiliary functions are complex. (Clark (1971) has provided a table of the values of the auxiliary functions for a normalised spherical model with a range of 1 and a sill of 1 (Appendix 2a, b, c and d). By simply dividing the lengths of the panel (*l* and *b*) by the variogram range (*a*) and reading the value from the table (with some linear interpolation when required) then multiplying by the sill, the values can be found.

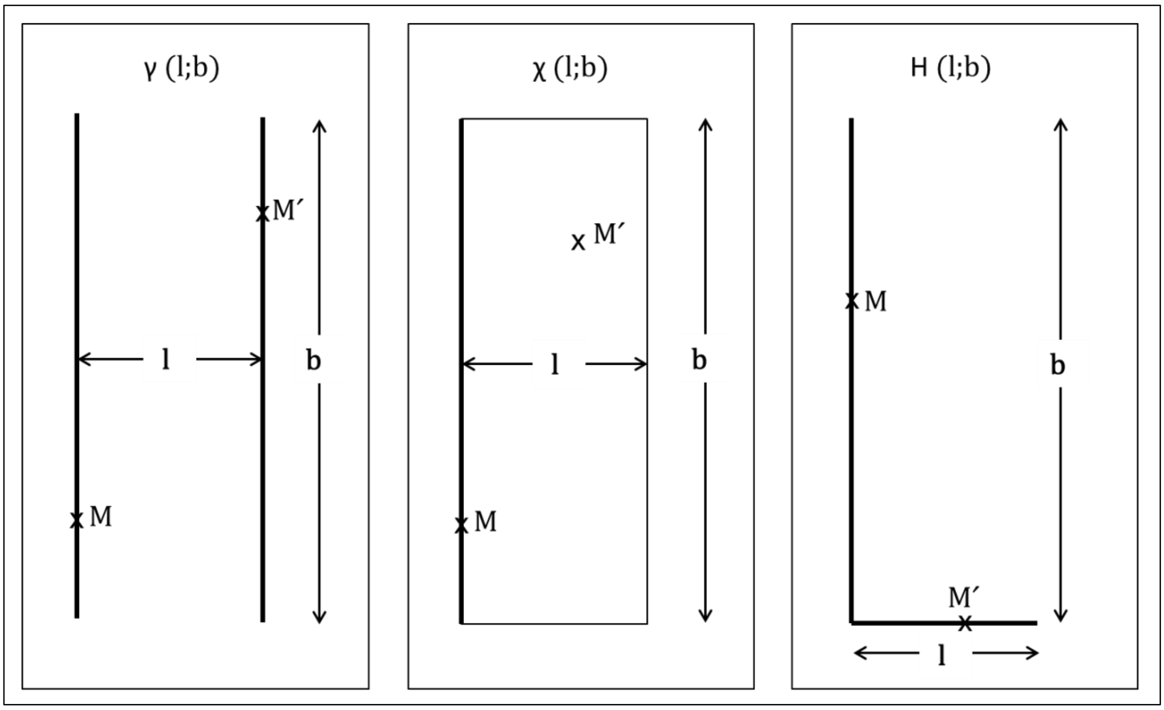


Figure 5-37 Schematic representation of the 2D auxiliary functions (reproduced from Clark, 1971)

Using Auxiliary Functions to understand the error in a raise estimate

For the purpose of simplification, a raise can be described as a one dimensional feature, its length being many times the distance of its width. Samples taken along a raise that have been composited to a single grade value can be described as points along a line. Clark (1979) described a process that can be used to understand the estimation error of samples equally spaced along a line. This concept was also used by Magri and McKenna (1986) to judge the error of channel saw sampling compared with chip sampling at Randfontein Estates and Western Areas Gold Mines.

The estimate of the grade (g^*) is simply the average grade of the samples along the line:

$$g^* = \frac{\sum_{i=1}^{n}[g_i]}{n}$$

and the extension variance is:

$\sigma_e^2 = 2\bar{\gamma}(S, A) - \bar{\gamma}(S, S) - \bar{\gamma}(A, A)$

$\bar{\gamma}(S, A)$ is the average semi-variogram value between the sample point and each point that can exists along the line.

$$\bar{\gamma}(S, A) = \frac{\sum_{i=1}^{n} [\bar{\gamma}(S_i, A)]}{n}$$

The semi-variogram values $\gamma(S_i, A)$ for points on the end of a line are each calculated using the auxiliary function *χ(l)* for a spherical model. However, points are not on the end of a line. Even the first and last sample channel points in a raise are not exactly at the beginning or end of a raise, the first sample being taken half the sample interval from the bottom. Each point along the raise forms two segments, the distance from the point to the top of the raise and the distance of the point to the bottom of the raise. The $\gamma(S, A)$ value for every sample has to be calculated twice using the length of the segment from the sample to the top of the raise and the length of the segment from the sample to the bottom of the raise. The two values obtained are then averaged according to their respective segment lengths to give a single $\bar{\gamma}(S_i, A)$ value for each sample point. For example, for the first sample along a 200 m raise where sample channels have been taken every 10 m intervals, there is a segment of 5 m from the bottom of the raise to the sample and another segment of 195 m from the sample to the top of the raise.

$\bar{\gamma}(S, S)$, is the average semi-variogram value between each of the points along the line, including themselves, which have a value of zero.

$$\bar{\gamma}(S, S) = \frac{\sum_{i=1}^{n} \sum_{j=1}^{n} [\bar{\gamma}(S_i, S_j)]}{n^2}$$

The third term $\bar{\gamma}(A, A)$ is the average semi-variogram for all possible pairs of points that can exist along the line i.e. $F(l)$.

The extension variance can now be calculated using the components calculated as described.

As the calculations are laborious and complex, a Microsoft Excel spreadsheet was compiled that uses the omnidirectional spherical variogram model for 4E as shown in **Figure 3-2**. The sample variance, raise length and the sample spacing are input to automatically calculate the extension variance of a line and thus the confidence in the estimate of the extension estimator (the arithmetic mean). This spreadsheet can now be used to test the differences in sample spacing in any raise rather like in the statistical methods outlined earlier.

The error in the estimate of the average PGE grade of a line was calculated for a number of sample spacings. The resulting values are shown graphically in **Figure 5-38**.

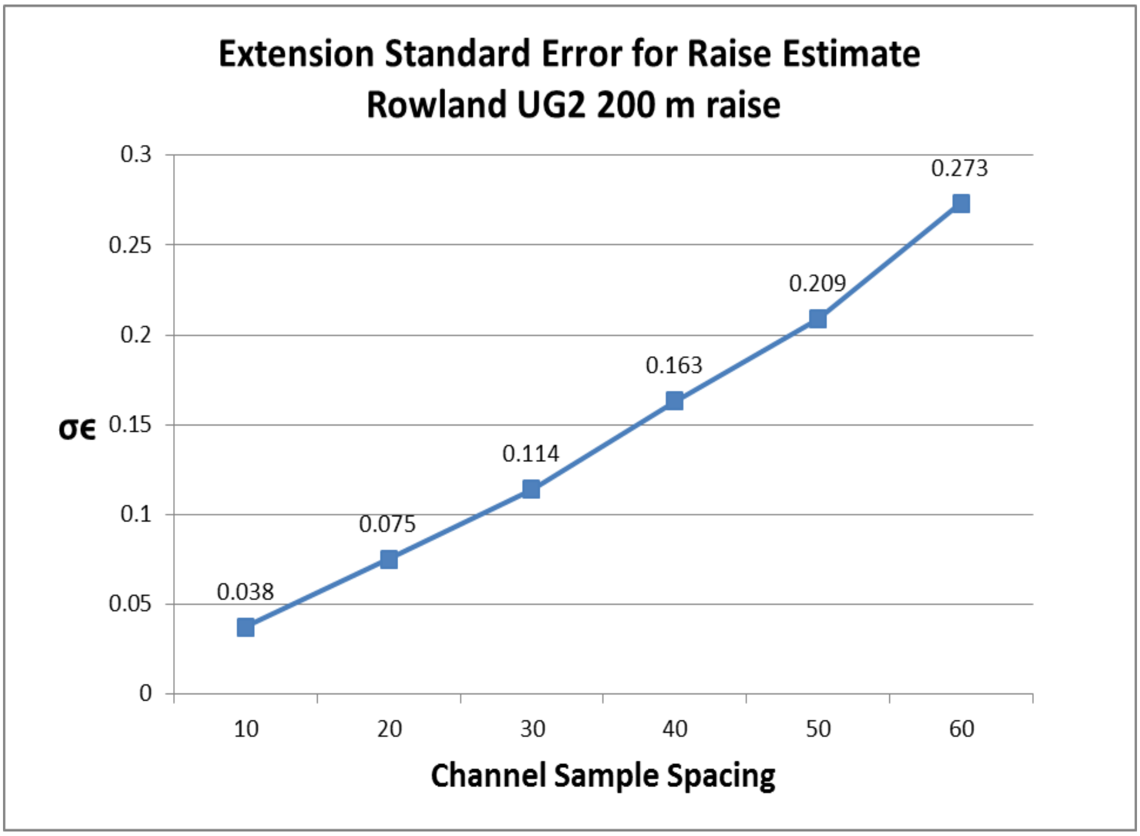


Figure 5-38 Graph showing the extension standard error of PGE grade for a number of sample spacings in a 200 m raise line (calculated using the Rowland UG2 semi-variogram).

The error increases linearly in the closer spaced intervals then starts to increase more sharply as the data become sparser. This is to be expected given the near linear shape of the semi-variogram in the shorter range intervals. As the sample spacing is doubled, the error is close to doubled also. An increase in sampling spacing from 10 m to 20 m results in the extension standard error increasing by 0.038 (from 0.037 to 0.075). Increasing the sample spacing further, to 30 m, results in an extension standard error of 0.114, an increase of 0.076 over the 10 m sampling.

Using the assumption that the samples are spaced evenly throughout the raise and that the Rowland UG2 standard semi-variogram applies then the extension standard error for each raise can be calculated. Comparing the standard deviation of the samples within each of five well sampled raises shows that the extension standard error is in the order of twenty times less than the sample standard deviation (**Table 5-13**).

Table 5-13 Extension Standard Error for each of the five well sampled raises

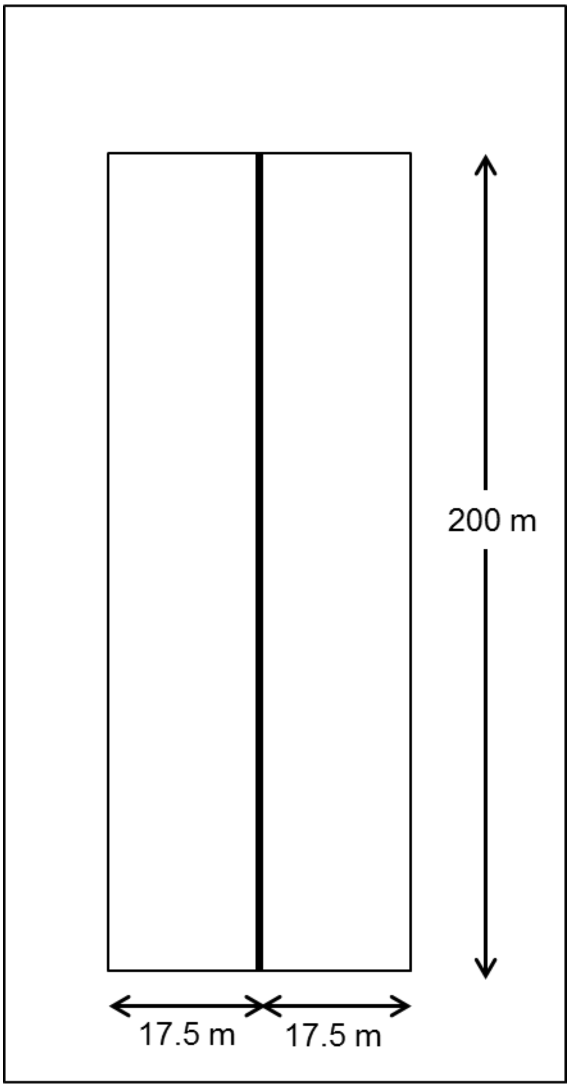
	D4VC26 W26	**D4VC25 W66**	**D4VC26 W54**	**D4SC27 W32**	**D4VC28 W15**	**D4VC29 W17**
Number of samples	18	17	18	18	18	18
Mean PGE grade	5.29	5.55	5.12	5.22	5.23	5.32
Variance	1.178	0.810	0.993	1.340	1.92	4.088
Sample Standard Deviation	1.085	0.917	0.996	1.158	1.386	2.022
Extension Standard Error	0.046	0.040	0.042	0.049	0.059	0.086

Using Auxiliary Functions to understand the error in a panel estimate

The process used for estimating the extension variance of a panel was derived from estimation of two dimensional areas outlined by Clark (1971) and adapted to the sampling configuration at Rowland.

The majority of the panels at Rowland Shaft are 35 m wide along strike and 200 m long along dip. The inter-level spacing is 35 m and the dip is approximately 10° to the north. The panel dip length and strike are determined by mining factors rather than consideration of sampling and therefore any changes to the area of the estimate or the development sampled needs to adhere to the mining layout. A number of tests on the reliability of the estimate were carried out using combinations of panels and additional sampling drives however the study was largely focussed on varying sample intervals within the 35 m spaced raises and the raise spacing.

Simplistically the estimate of the line (raise or drive) is taken to be known with 100% certainty (no error) and this known value is extended into a panel so that the panel estimate is the known value of the raise. An example of the error in applying the value in the raise to that of the panel using the Rowland panel layout (single central raise) is illustrated in **Figure 5-39**, which is followed by a worked example.



S = 200 m long raise
A = 200 m by 17.5m panel

Panel length = 17.5 m
Raise Length = 200 m

Error in Left Panel = Error in Right Panel

Normalised Semi-Variogram Model Values:
C0 = 0.56
C1 = 0.25   R1 = 30
C2 = 0.11   R2 = 45
C3 = 0.08   R3 = 170

Figure 5-39 Applying the value in the raise to that of the panel using the Rowland panel layout (single central raise)

- Calculate $\bar{\gamma}$(A,A)
- Standardise l,b

  $17.5/30, 200/30 = 0.58, 6.67$

  $17.5/45, 200/45 = 0.39, 4.44$

  $17.5/170, 200/170 = 0.10, 1.18$
- Read off and interpolate linearly normalised values from table of F values (Appendix 2b)

$F(l,b)_{r1} = 0.929$

$F(l,b)_{r2} = 0.857$

$F(l,b)_{r3} = 0.514$

- Multiply by normalised sill value

$\bar{\gamma}(A,A)1 = 0.929 * 0.25 = 0.232$

$\bar{\gamma}(A,A)2 = 0.857 * 0.11 = 0.094$

$\bar{\gamma}(A,A)3 = 0.514 * 0.08 = 0.041$

- Sum and add C0

$\bar{\gamma}(A,A) = 0.232 + 0.094 + 0.041 + 0.56 = 0.928$

- Calculate γ(S,S) for each semi-variogram structure using the following formula:

$F(l) = \frac{C}{20}\frac{l}{a}\left(10 - \frac{l^2}{a^2}\right)$ where $l \leq a$

$F(l) = \frac{C}{20}\left(20 - 15\frac{a}{l} + 4\frac{a^2}{l^2}\right)$ where $l \geq a$

$F(l)_1 = \frac{0.25}{20}\left(20 - 15\frac{30}{200} + 4\frac{30^2}{200^2}\right) = 0.223$

$F(l)_2 = \frac{0.11}{20}\left(20 - 15\frac{45}{200} + 4\frac{45^2}{200^2}\right) = 0.093$

$F(l)_3 = \frac{0.08}{20}\left(20 - 15\frac{170}{200} + 4\frac{170^2}{200^2}\right) = 0.041$

- Sum and add C0

$\gamma(S,S) = 0.223 + 0.093 + 0.041 + 0.56 = 0.916$

- Calculate γ(S,A)
- Read off and interpolate linearly standardised values from table of χ values (Appendix 2c)

$\chi\ (l,b)_{r1} = 0.941$

$\chi\ (l,b)_{r2} = 0.871$

$\chi\ (l,b)_{r3} = 0.520$

- Multiply by normalised sill value

$\gamma(A,A)_1 = 0.941 * 0.25 = 0.235$

$\gamma(A,A)_2 = 0.871 * 0.11 = 0.096$

$\gamma(A,A)_3 = 0.520 * 0.08 = 0.042$

- Sum and add C0

$\gamma(A,A) = 0.235 + 0.096 + 0.042 + 0.56 = 0.933$

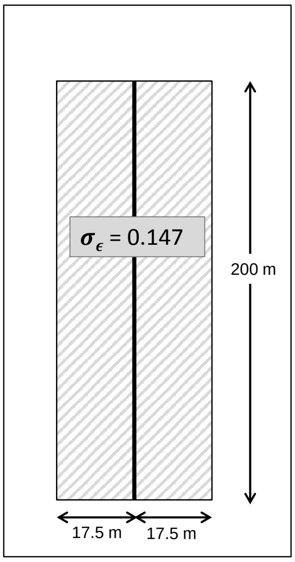
- Calculate Extension Standard Error

  $\sigma_e^2 = 2\bar{\gamma}(S, A) - \bar{\gamma}(S, S) - \bar{\gamma}(A, A)$

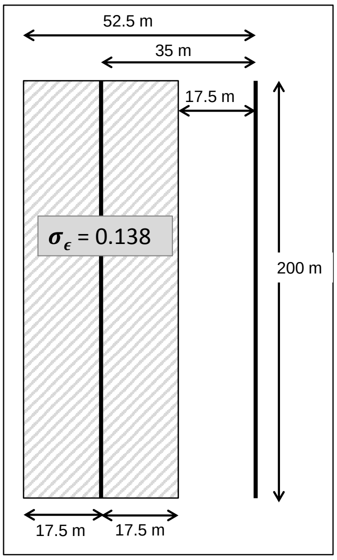
  $\sigma_e^2 = 2 * 0.933 - 0.916 - 0.928 = 0.022$

  $\sigma_e = 0.147$

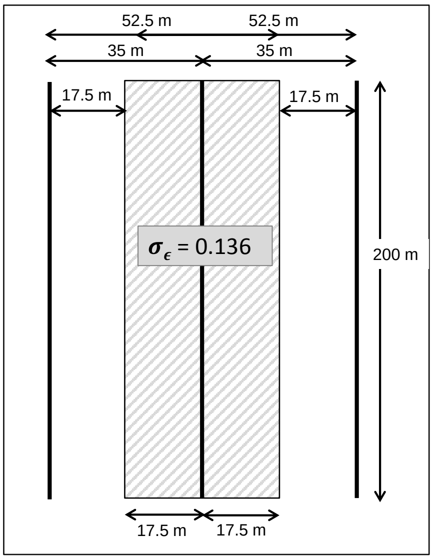
A number of manipulations to the aforementioned calculations were made in order to examine the change in extension variance for different patterns of raises sampled. The exercise was modified for a number of arrangements as shown in **Figure 5-40** to together with the corresponding σ_e values. **Figure 5-41** shows the extension standard error for a set of stopes on a level where every second raise is sampled.



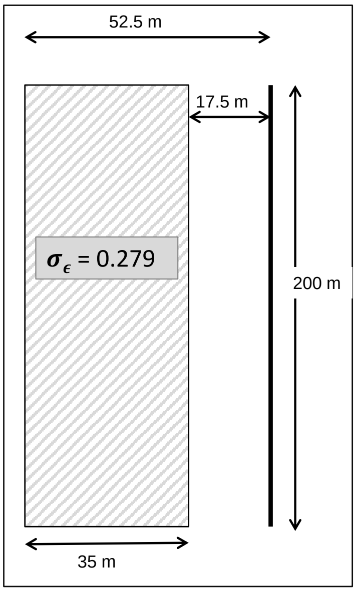
A) Estimation of Stope with a Single Central Raise



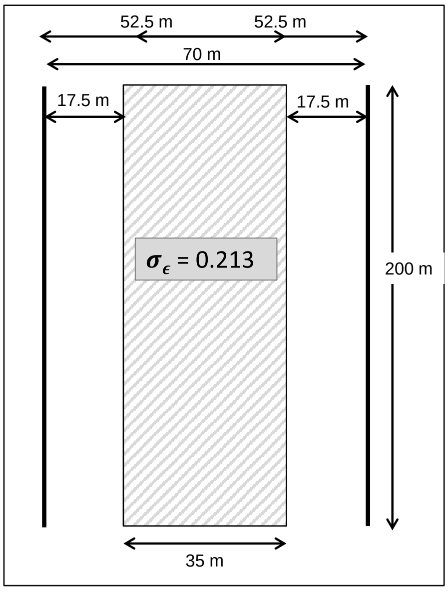
B) Estimation of Stope with a Central Raise and an adjacent raise outside



C) Estimation of Stope with a Central Raise and adjacent raises outside either side



D) Estimation of Stope with only a single adjacent raise outside



E) Estimation of Stope with adjacent raises outside either side

Figure 5-40 Changes in extension standard error by change in raise sampling configurations. (Thick vertical line indicates the raise position).

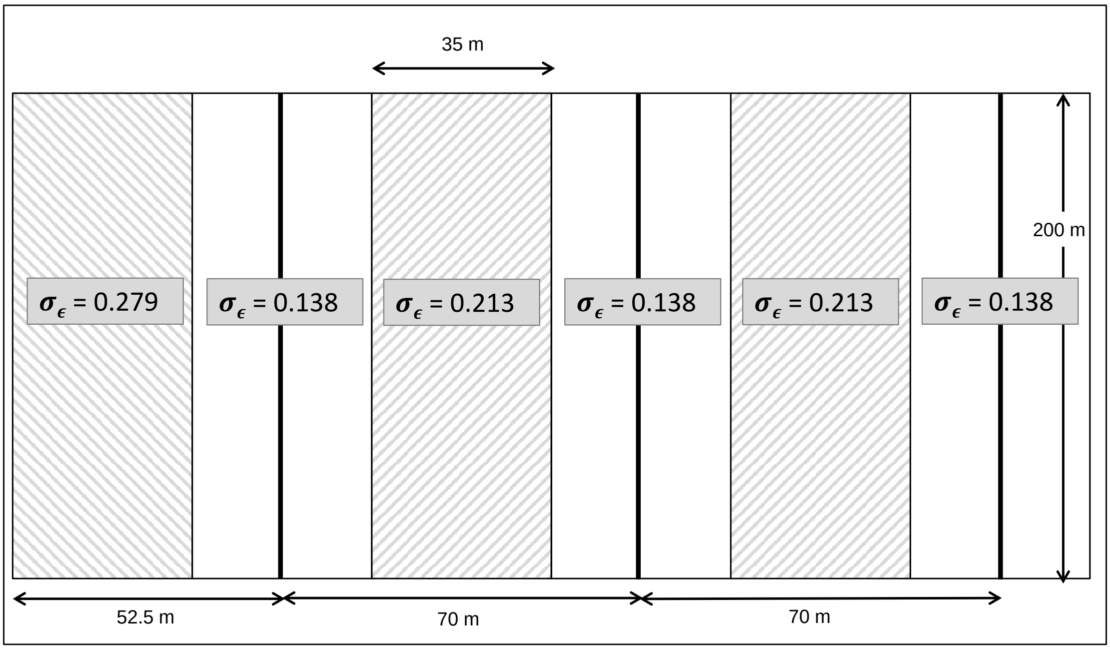


Figure 5-41 Extension standard error for stope estimates using every second raise (Thick vertical line indicates the raise position).

The extension σ_e value is markedly higher when there is no sampling inside the stope. The error is 54% higher when using raises either side of the stope compared with having a raise in the stope. Small reductions in the σ_e value occur when using peripheral sampling together with the sampling in the raise.

Skipping the sampling of every second raise increases the extension standard error from 0.138 to 0.213, which is an increase of 0.075.

Adding the raise extension variance using 10 m spaced samples (0.001408) with the extension variance created by extending the raise through the stope (0.02167) gives a standard error of:

$$\sqrt{0.001408 + 0.02167} = 0.152$$

For 20 m spaced samples the standard error is

$$\sqrt{0.005655 + 0.02167} = 0.165$$

Applying the same logic to the estimate using only raises either side and sampling at 10 m intervals gives a standard error of:

$$\sqrt{0.001408 + 0.04549} = 0.217$$

The same number of samples is taken whether increasing the sample spacing from 10 m to 20 m or by sampling every second raise at 10 m intervals. However the standard error increases by only 0.013 for 20 m spacing for all raises compared with an increase of 0.065 in every second raise (0.0325) on average. Therefore the increase in error overall is three times greater by reducing samples by sampling every second raise against the same sample reduction achieved by increasing the sample spacing.

Using the variance of the sample population used to calculate the semi-variogram 1.493 g/t^2, the error at 90% confidence in an up-dip stope at Rowland using 10 m spaced samples can be calculated as follows:

$$(0.001408+0.02167)*1.493\ \mathrm{g/t^2} = 0.0345\ \mathrm{g/t^2}$$

$$\sqrt{0.0345\ \mathrm{g/t^2}} = 0.1856\ \mathrm{g/t}$$

At a confidence limit of 90%

$$0.1856\ \mathrm{g/t} * 1.6449 = 0.3053\ \mathrm{g/t}$$

At our average grade of 5.36 g/t, the 90% confidence interval is (5.05;5.67)g/t and the percentage error is ±5.7%. Following the same logic for 20 m and 30 m spaced samples gives a 90% confidence interval of (5.03;5.69)g/t and the percentage error is ±6.2% for 20 m spacing and (4.99;5.73)g/t and the percentage error is ±7.0% for 30 m sampling. For estimation of a stope using only the raises either side, confidence intervals of (4.92;5.80)g/t, (4.91;5.81)g/t and (4.87;5.85)g/t are achieved for the 10 m, 20 m and 30 m sample spacing resulting in percentage errors of ±8.1%, ±8.5% and ±9.1% respectively.

If the sample spacing is increased to 40 m then the 90% confidence interval is (4.92;5.80)g/t and the percentage error is ±8.24% for sampling all raises and (4.82;5.80)g/t, equal to a percentage error of ±10.07%, for sampling every second raise.

Summary of the findings of using extension variance to understand the error in stope grade estimates.

- Using a semi-variogram and the principals of extension variance it is possible to compare the errors between different sampling layouts.
- The error in a raise estimate is small compared with the error in extending the raise into a stope i.e. using the average value of a raise to estimate a stope.

- The extension standard error is much smaller than the standard deviation of the samples. This will result in a lower estimate of error using geostatistical techniques than classical non-spatial statistics.
- Should a reduction in sampling be considered, the error is lower when reducing the sample spacing along a raise rather than reducing the number of raises sampled and keeping the sample spacing along the raise the same, despite that the same number of samples are taken for both alternatives.
- Only once the sample sampling is reduced to 40 m and every second raise is sampled does the percentage error become greater than ±10%.

Geostatistically, the confidence intervals are lower than attaining confidence intervals statistically. However it should be borne in mind that the geostatistical results are based on average grades and variances across the Rowland UG2 study area and there will be many stope estimates where the errors will be considerable higher due to higher local variance and few samples. The statistical analysis using actual stope grade variance data indicates that should the sample spacing be increased to 20 m then the error would be greater than ±10% at 90% confidence, however the geostatistical study using an average mean, variance and a semi-variogram indicates that increasing the sample spacing can be done while maintaining the error to acceptable limits.

5.6 Optimising Sampling Grids for Block Model Estimation for use Mine Planning

Kriged block models are used for forecasting metal production via the annual technical budget process. Lonmin's current practice is that within and adjacent to the areas that have been sampled underground, 50 m by 50 m block model cells are populated with true thickness, 4E accumulation (cmgt), 4E grade (g/t) and density using Ordinary Kriging estimation. These estimates are largely based on channel sample data, the same as those used for monthly metal accounting.

Lonmin uses CAE Studio 3 software for its block model estimates. This software is commonly known as Datamine and is widely used for Geological

Modelling, Mineral Resource Estimation and Mine Planning in the Mining Industry worldwide.

Given that the mining grid is rigid, a similar approach to that used in understanding the variability in the data and subsequent data reduction for metal accounting estimates was used, the options being to sample fewer raises and/or increase the sample spacing. The same channel sample dataset was used as for the statistical study, this being reduced into sets of data at the original 10 m spacing, 20 m spacing and 30 m spacing, as well as using every second and third raise. In effect there are nine sub-sets of the data:

1. Three subsets of data resulting from a 10 m spaced sample approach along the raises spaced 35 m, 70 m or 105 m part. The full set of data resulting from a 10 m spaced approach and raises spaced 35 m apart.
2. Three subsets of data resulting from a 20 m spaced sample approach along the raises spaced 35 m, 70 m or 105 m apart.
3. Three subsets of data resulting from a 30 m spaced sample approach along the raises spaced 35 m, 70 m or 105 m apart.

The aim of the study being to assess the deterioration of the quality of the estimate with fewer samples spaced further apart using a real database. Given that the variability of density and true thickness are very low these were not looked at in any more detail. As the 4E g/t data are by far the most variable and costly to collect, most of the error and potential benefit lies here. Although the estimates are normally performed on the accumulation of 4E grade and true thickness (4E cmgt), 4E grade was examined in this study both for the purposes of simplification and consistency with other parts of the study. The variability of the grade and accumulation data is similar and the semi-variograms are almost the same so any conclusions drawn for 4E grade will be similar for the accumulation.

The estimates achieved using the different data sub-sets were assessed through comparison of the grade estimates, Kriging Efficiency (KE%) and Slope of Regression (PSlope).

The first assessment carried out was to ascertain the amount of data available to estimate each block within the semi-variogram range from each data sub-set. The 50 m block estimates are used to predict PGE production within the next two years of production and ideally should be consistent with those of the Measured Mineral Resource category. For this confidence of estimate it is expected that only samples selected within the semi-variogram range of the block are used for the estimate and that there are sufficient numbers of samples to achieve a high confidence estimate. An omnidirectional search of 170 m (equivalent to the total semi-variogram range) was carried out and the number of samples available to estimate each block with the different sub-sets was plotted in plan (**Figure 5-42**) and summarised (**Table 5-14**).

Table 5-14 Summary of number of samples available to estimate a block for nine different sampling configurations.

Raise Spacing (strike m)	Sample Spacing (dip m)	Number of Channels	Number of Blocks within semi-variogram range of a sample	Average number of samples within semi-variogram range of a block
35	10	3849	1817	77
70	10	1977	1742	41
105	10	1299	1718	27
35	20	1830	1795	37
70	20	945	1718	20
105	20	617	1684	13
35	30	1167	1776	24
70	30	596	1685	13
105	30	395	1668	9

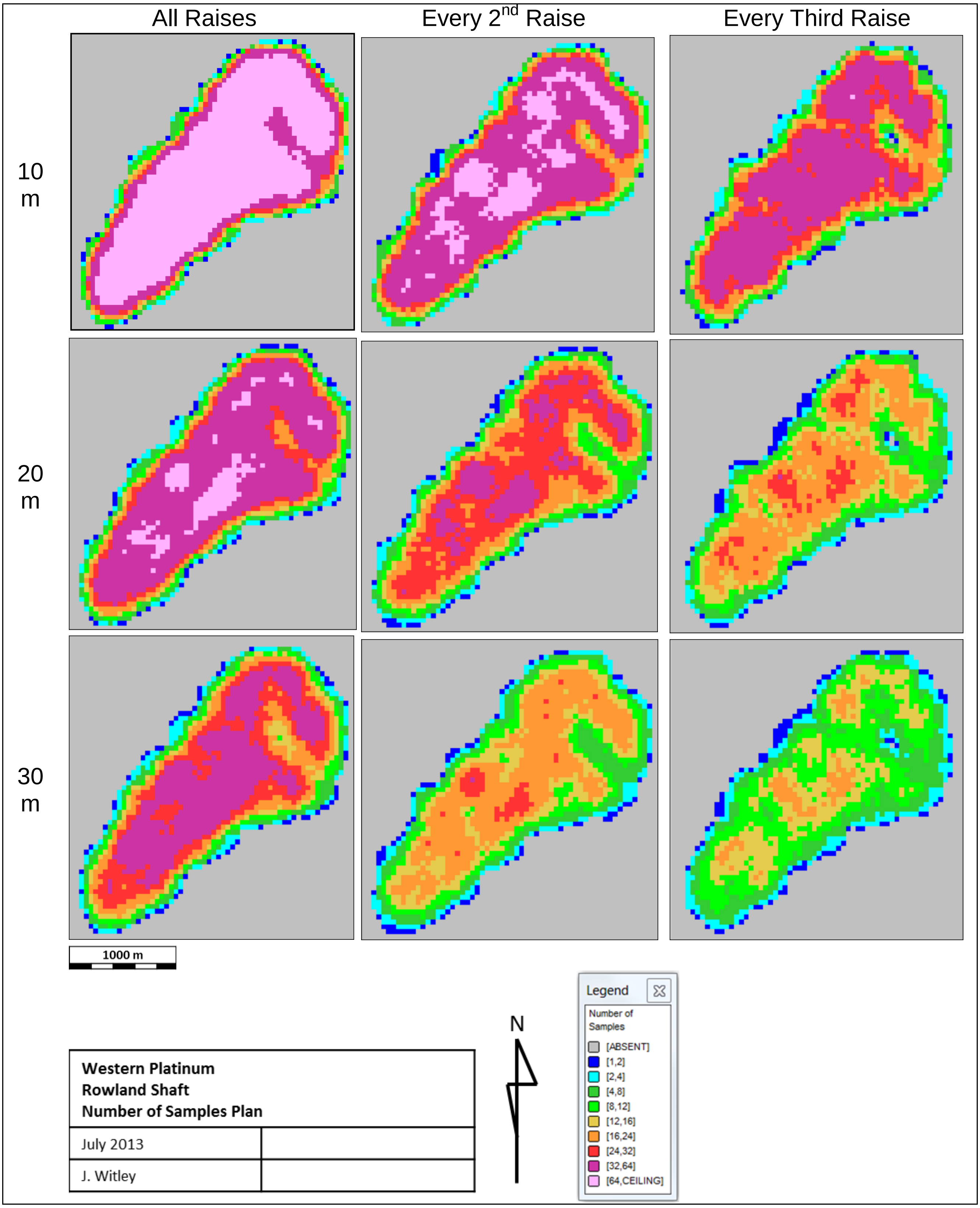


Figure 5-42 Number of Samples available to estimate a block for nine different sampling configurations.

Figure 5-42 shows that for all sample configurations except for the three sparsest grids most of the cells are estimated with at least 24 channel sample composites.

The second assessment was to look at the accuracy of each estimate in terms of KE% and PSlope. This exercise was performed using a minimum number of one composite. This is an insufficient number of samples to obtain an estimate, but the aim here of the study is to assess the accuracy in the estimates using the available data, thus low numbers of sample composites were accepted. A single omni-directional search of 170 m was used to select samples for each estimate and cells were discretised by a 5 mE by 5 mN matrix of 25 points.

Plots of Kriging Efficiency and Slope of Regression are shown in **Figure 5-43** and **Figure 5-44** for each of the nine sampling configurations.

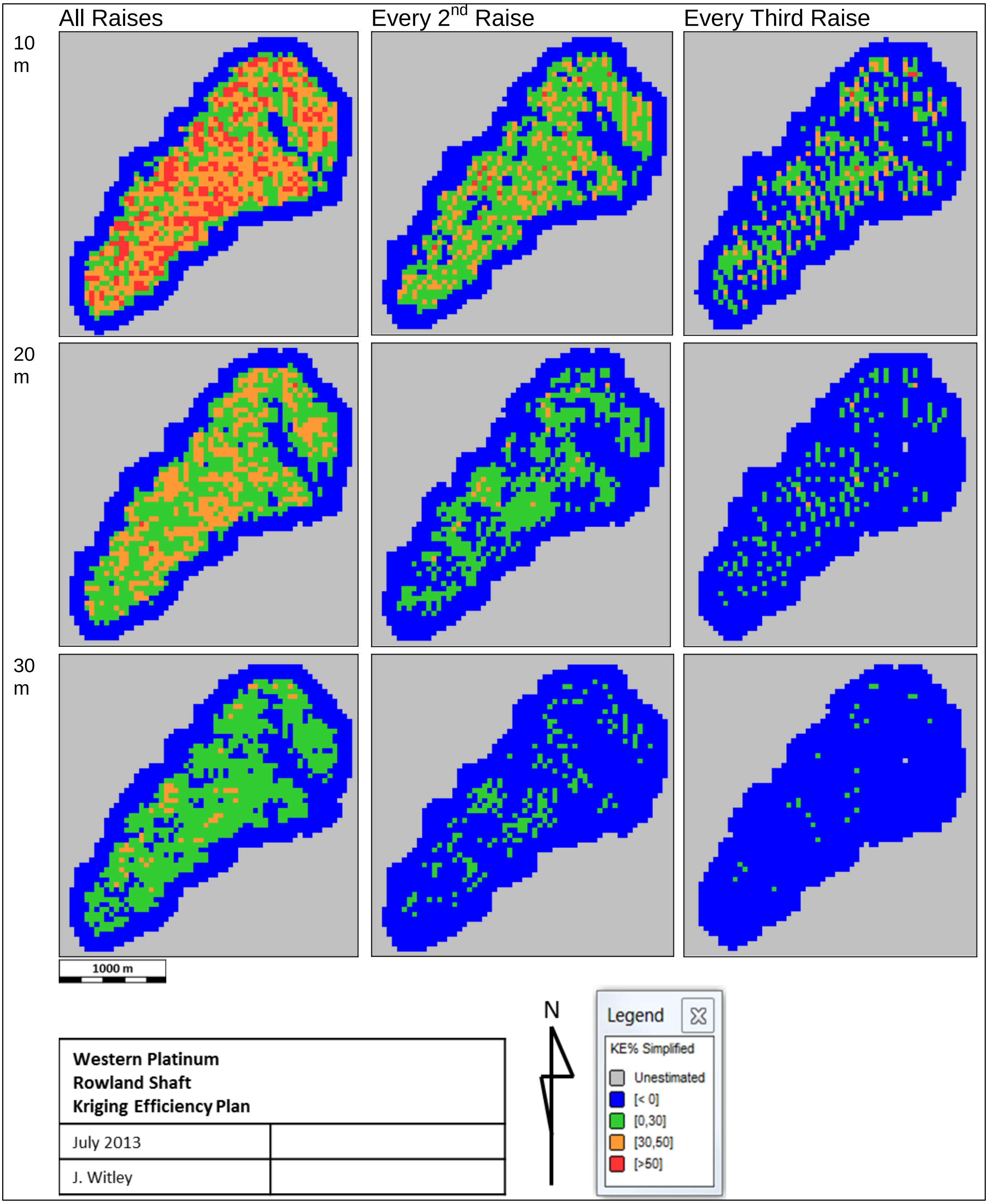


Figure 5-43 Kriging Efficiency plots for nine different sampling configurations.

The plots of KE% versus sample spacing (**Figure 5-43**) show that almost all estimates have negative Kriging Efficiency when every third raise is sampled at a 30 m or 20 m spacing and when every second raise is sampled at a 30 m spacing.

Overall, more of the blocks will be estimated at a reasonable level of confidence (KE% >30%) by sampling all of the raises at 20 m channel spacing sampling against sampling every second raise at 10 m spacing, despite there being slightly fewer samples available at the 20 m spacing. However the number of high confidence estimates (KE% > 50%) is fewer. The same relationship is observed between sampling all of the raises at 30 m channel spacing sampling against sampling every third raise at 10 m spacing.

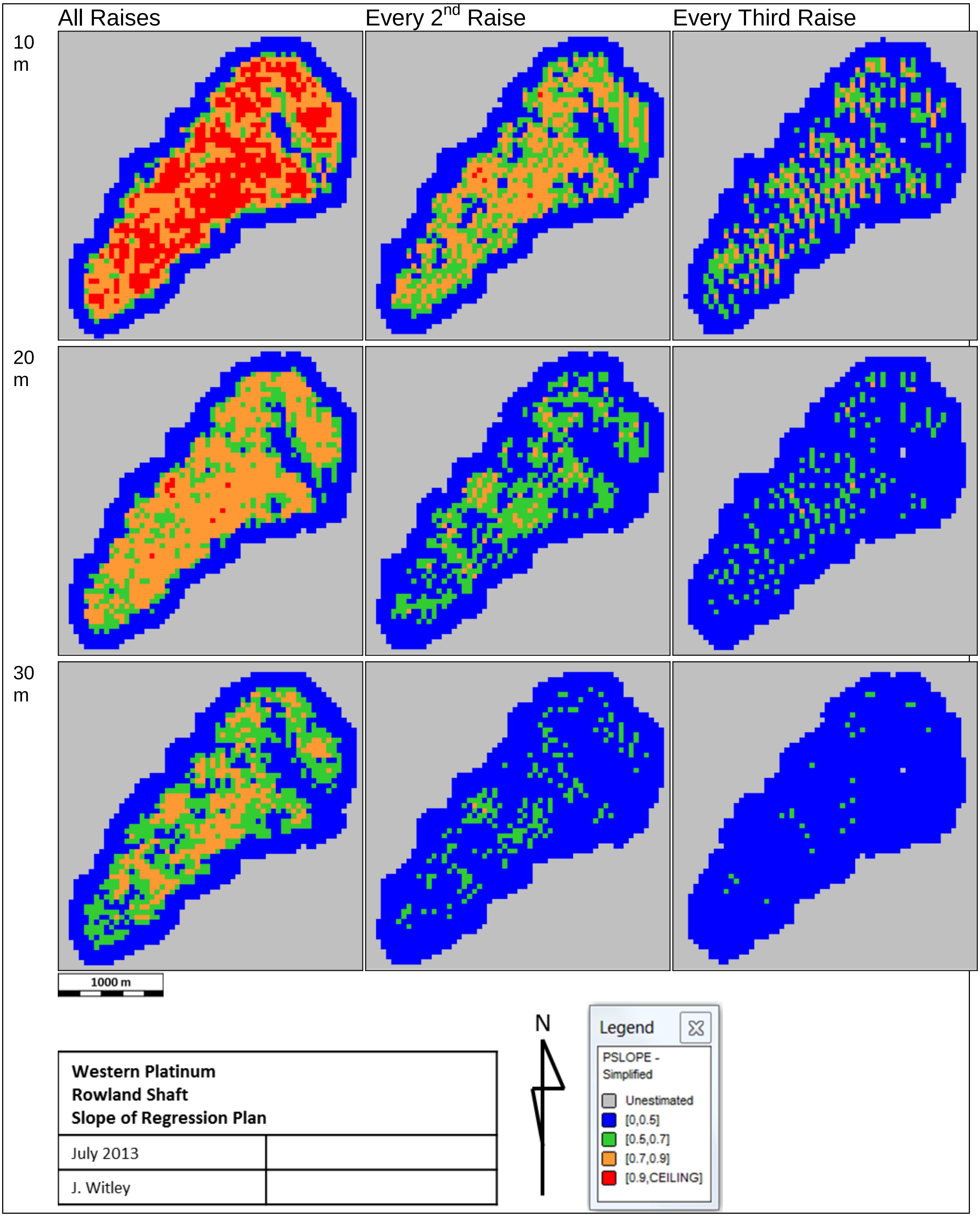
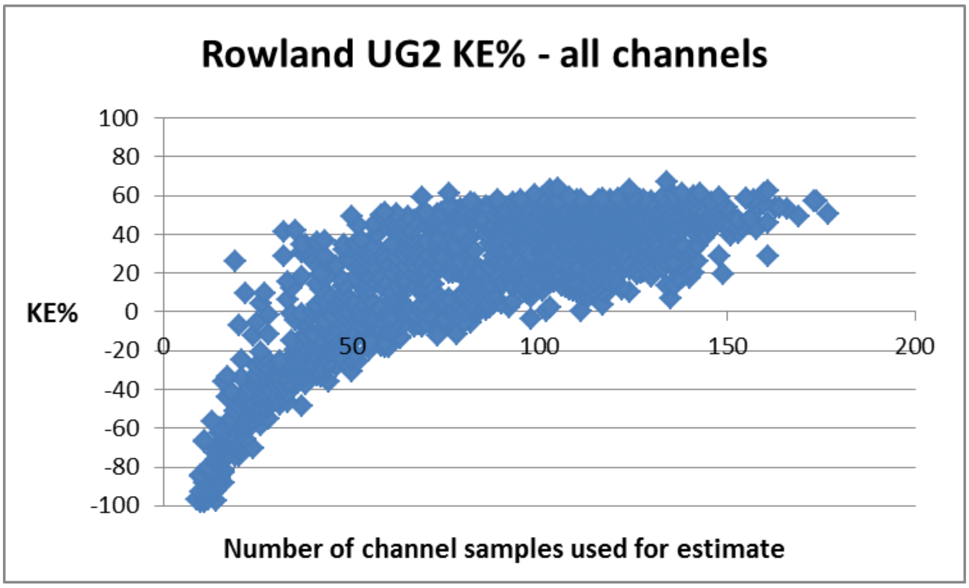


Figure 5-44 Slope of Regression plots for nine different sampling configurations.

The plots of Slope of Regression for the nine different sample patterns show a similar trend as for Kriging Efficiency. However in this case there are more higher confidence estimates (PSlope > 0.9) for reducing the spacing to 20 m along the raise as opposed to reducing the raise spacing to 70 m.

On comparing the plots of number of samples available to estimate each block within the variogram range it shows that there are more than 32 samples available to estimate many of the areas, however the KE% and PSlope are lower than required for a reasonable confidence of estimate. One example of this is the area towards the NE of the estimated area, where a northwest trending enclave of low confidence estimates (KE% < 30%, Pslope < 0.5) occurs. The relationship between number of samples and KE% or Pslope is not linear (weights being drawn from a spherical model) and the variability between the confidence in different estimates using the same number of samples is very high (**Figure 5-45**). Each block is estimated by an array of samples many different distances apart and from the block and thus the variance drawn from the semi-variogram is quite variable. In the area to the north-east estimated with many samples, but attaining low confidence, the samples are all outside of the blocks beyond the short range structure of the semi-variogram, which reaches its sill at a range of approximately 45 m. This is important to note as many Mineral Resources are assigned a high level of confidence if a certain number of samples are used to estimate within the variogram range yet estimates satisfying these criteria can still be of low confidence (negative KE% and Pslope less than 0.5).



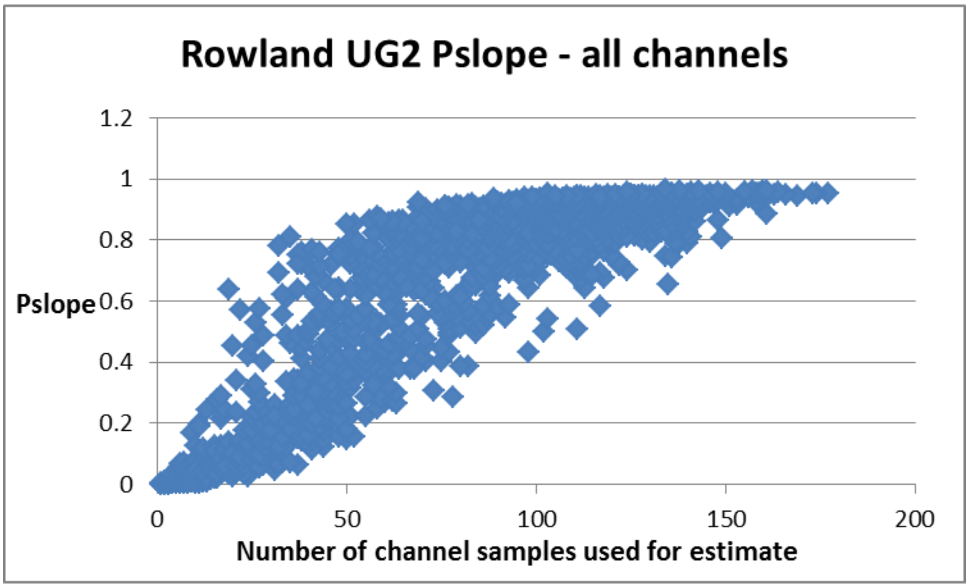


Figure 5-45 Kriging Efficiency and Slope of Regression versus number of composites

As the aim of this part of the study is to assess how well the different sample configurations predict future UG2 grade, a typical area that will be scheduled for mining in the following year was selected, this being the westernmost panels on 30 and 31 level (**Figure 5-46**).

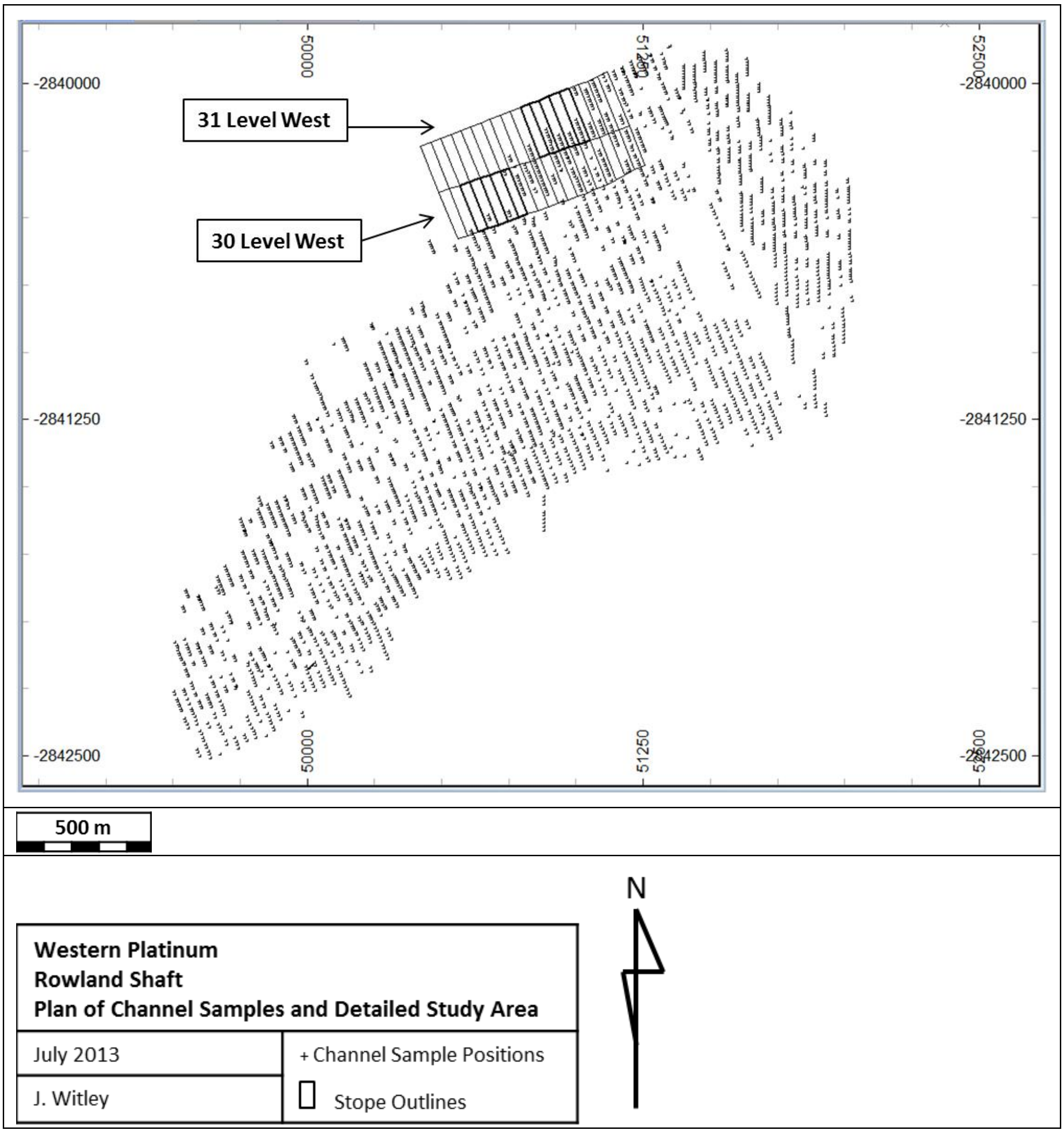


Figure 5-46 Rowland shaft channel samples and detailed study area

The specific area of 30 and 31 Level West is shown with the individual stope names in **Figure 5-47**. Note that the bold outlined stopes (30 level W17 to W21 and 31 level W09 to W13) are those areas ahead of current producing areas that would be scheduled for the following year's production. These stopes have been partially developed, either by raising from the bottom or by winzing down from the level above, and so contain some channel sample data.

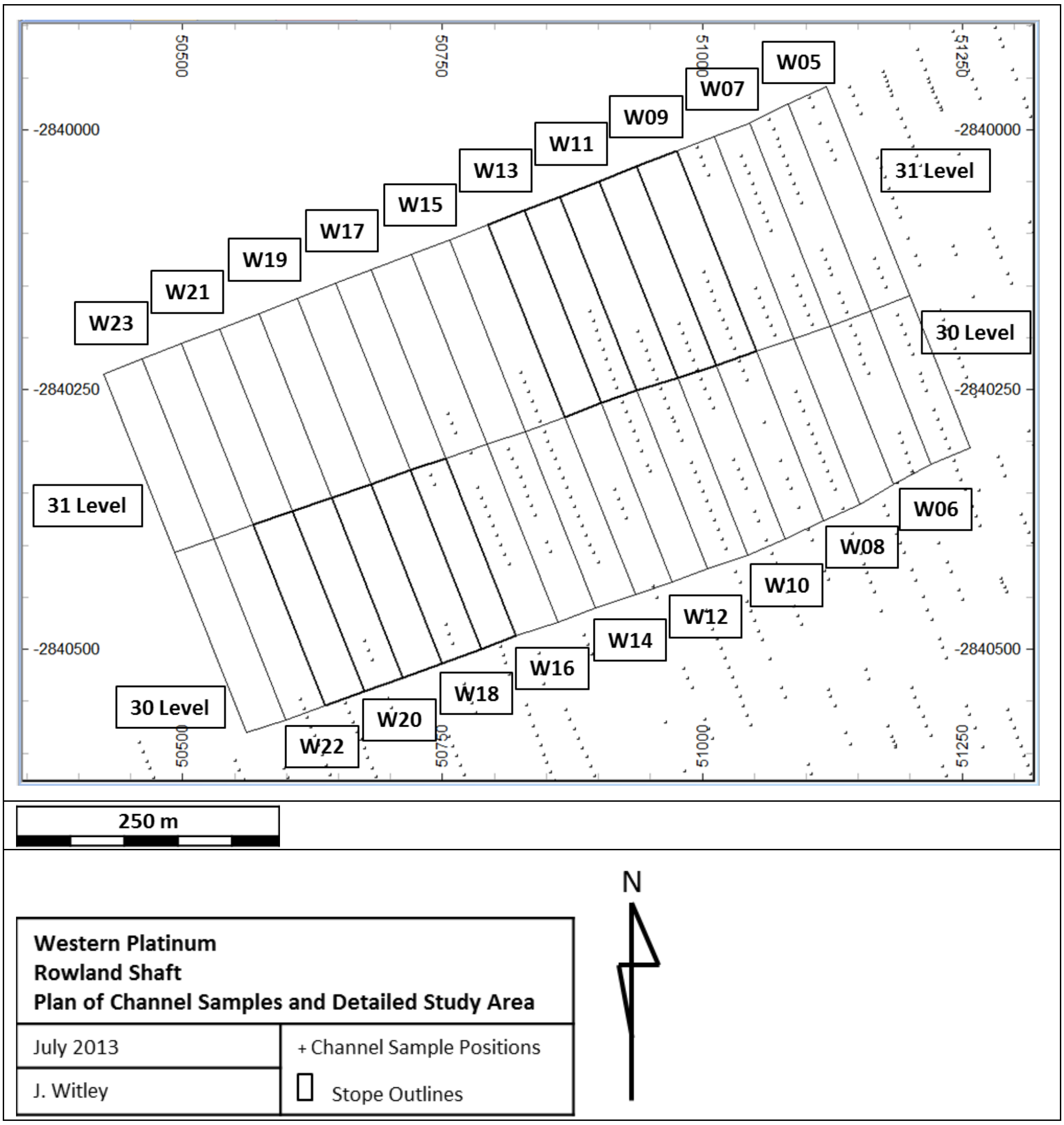


Figure 5-47 Rowland shaft detailed study area showing stope names

Kriging Efficiency maps using the nine different channel sample configurations show a rapid decrease in the accuracy of the estimates with reduction in channel samples (**Figure 5-48**). Even when all the channel samples from all raises are available for estimation, large portions of the panels scheduled to be mined are estimated with negative kriging efficiency. Several blocks in 31 level are not estimated when samples from every second raise are selected at 20 m and 30 m spacing on dip and a larger search radius would be required to estimate these blocks.

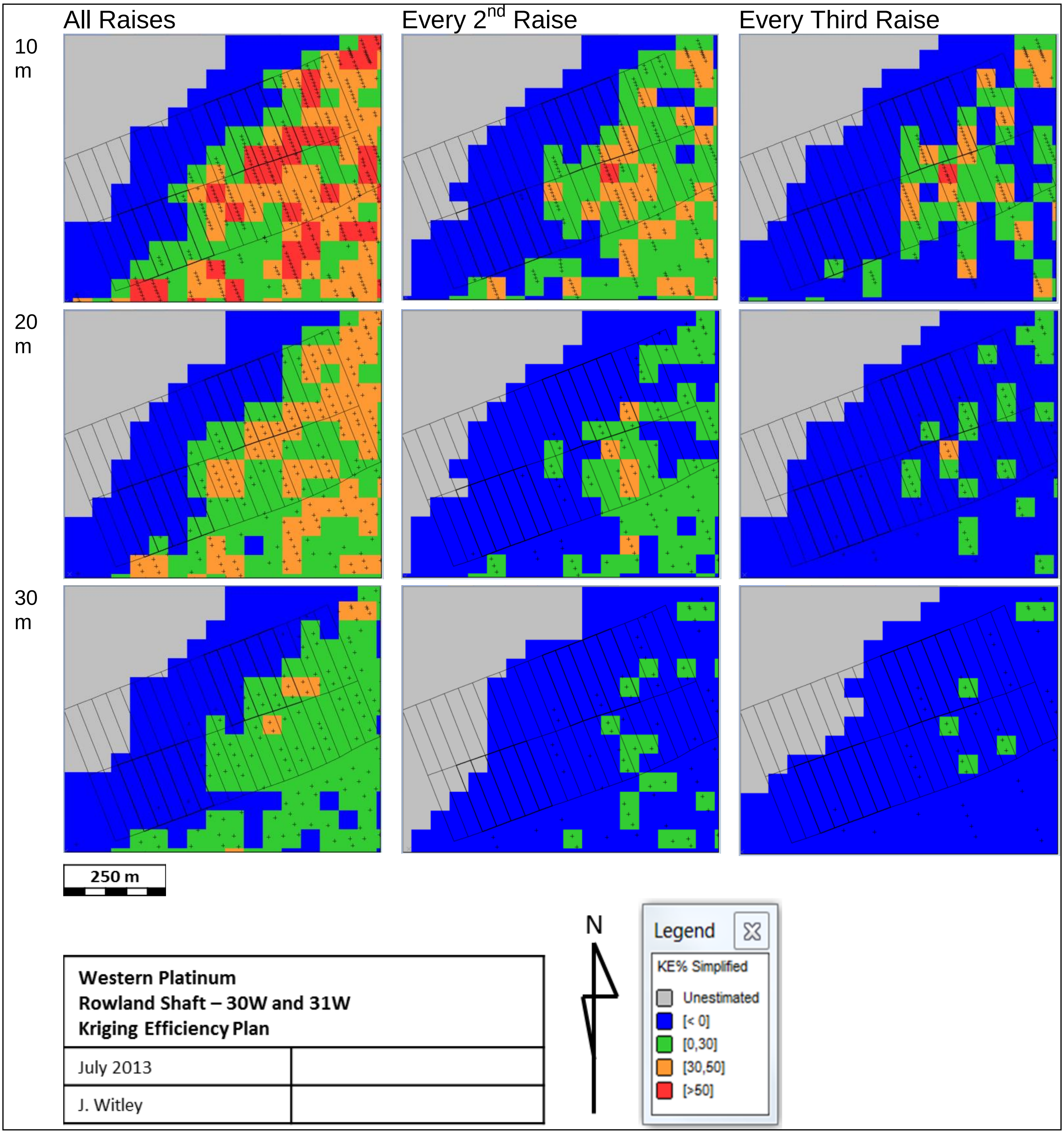


Figure 5-48 Kriging Efficiency plots for nine different sampling configurations at 30W and 31W

The block models created using the different sampling configurations were evaluated for two stopes on 30 level west and two on 31 level west. The average number of samples used for the estimation, slope of regression, kriging efficiency and Lagrange Multiplier were plotted. In addition, the 4E grade was plotted on each graph together with +10% and -10% of the estimate using all of the available data (**Figure 5-49**, **Figure 5-50**, **Figure 5-51** and **Figure 5-52**). The stopes on 30 level west are 30W17 and 30W21, the estimate for W17 is informed by a few samples within the stope and a number to the east and W21 is informed by a few samples on either side. The stopes on 31 Level west are 31W09 and 31W13, the estimate for W13 is informed by samples on either side and W09 is well informed by samples inside the stope perimeter and either side (**Figure 5-47**).

For the 30W17 Stope (**Figure 5-49**) there are between 8 and 70 channel samples available for the estimate depending on the configuration used. Kriging Efficiency is low, negative for all configurations except for that utilising all samples. Slope of regression is 0.65 when using all samples, 0.48 for the 20 m spaced sampling and somewhat lower for the rest. The Lagrange Multiplier is 0.02 when using all samples and 0.03 for 20 m sample spacing. The highest Lagrange Multiplier is 0.21, which is with the sparsest sampling grid. The grade estimates for the panel do not vary more than ±10% from the estimate using all samples for any of the sample grids. Only two estimates are outside of ±5% of that achieved with all samples used, these being for the 10 m spaced samples every second raise and 20 m spacing with every third raise being sampled.

The 30W21 Stope is the most poorly informed of the four stopes examined, there being between 3 and 32 channel samples available for the estimate depending on the configuration used. Kriging Efficiency is very low and negative, between -75% and -337% (**Figure 5-50**). Slope of regression is between 0.26 when using all samples and 0.06 for the sparsest grid used. The Lagrange Multiplier is between 0.12 when using all samples and 0.48 for the sparsest grid. The grade estimates for the panel do not vary more than ±10% for all of the sample grids except when using every second raise and 30 spaced samples along the raise. Three estimates are outside of ±5% of that achieved with all samples used, these being for 20 m spacing in all raises the 30 m spaced samples every second raise and 30 m spacing with every third raise being sampled.

The 31W09 Stope is the best informed of the four stopes examined, there being between 12 and 91 channel samples available for the estimate depending on the configuration used (**Figure 5-51**). Kriging Efficiency is low, negative for all configurations except for three, that utilising all samples (20%) 20 m spacing from every second raise (8%) and 20 m spacing from every raise (3%). For the same three sampling patterns slope of regression is greater than 50%, the rest being somewhat lower. The Lagrange Multiplier is between 0.02 when using all samples and 0.10 with the sparsest sampling grid. The grade estimates for the panel do not vary more than ±10% from the estimate using all samples for any of the sample grids. Only two estimates are outside of ±5% of that achieved with all samples used, these being for the 30 m spaced samples every second raise and 10 m spacing with every third raise being sampled.

For the 31W13 Stope there are between 7 and 58 channel samples available for the estimate depending on the configuration used (**Figure 5-52**). Kriging Efficiency is negative, between -22% and -148% for all configurations. Slope of regression is below 0.50 for all sample configurations the highest being 0.41 when using all samples. The Lagrange Multiplier is 0.05 when using all samples and the highest Lagrange Multiplier is 0.22, which is with the sparsest sampling grid. Only one grade estimate is more than ±10% or ±5% from the estimate using all samples for any of the sample grids and that is for every second raise using 30 m channel sample spacing.

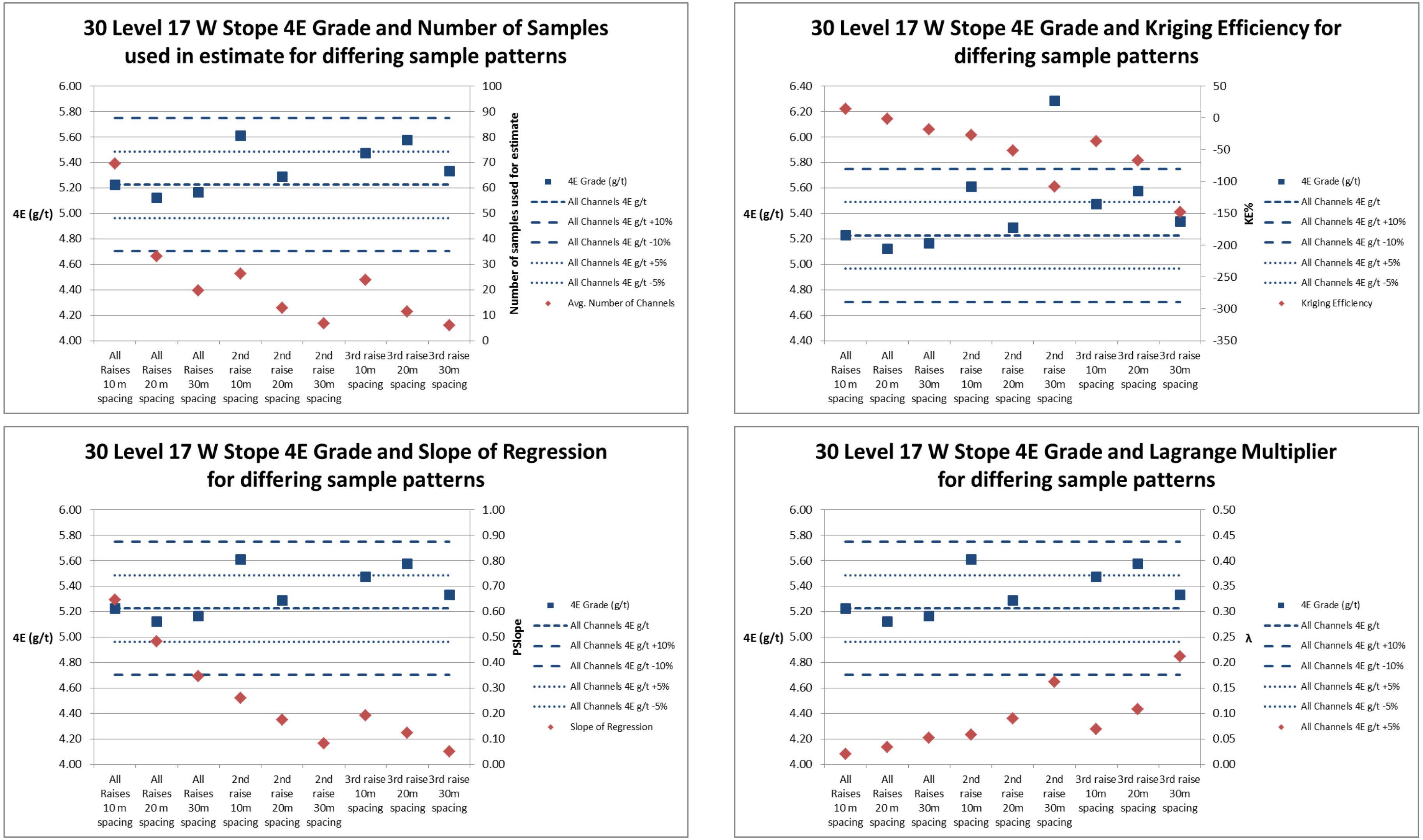


Figure 5-49 4E Grade versus Number of Channels, Kriging Efficiency, Slope of regression and Lagrange Multiplier for nine different sampling configurations at 30 Level West 17.

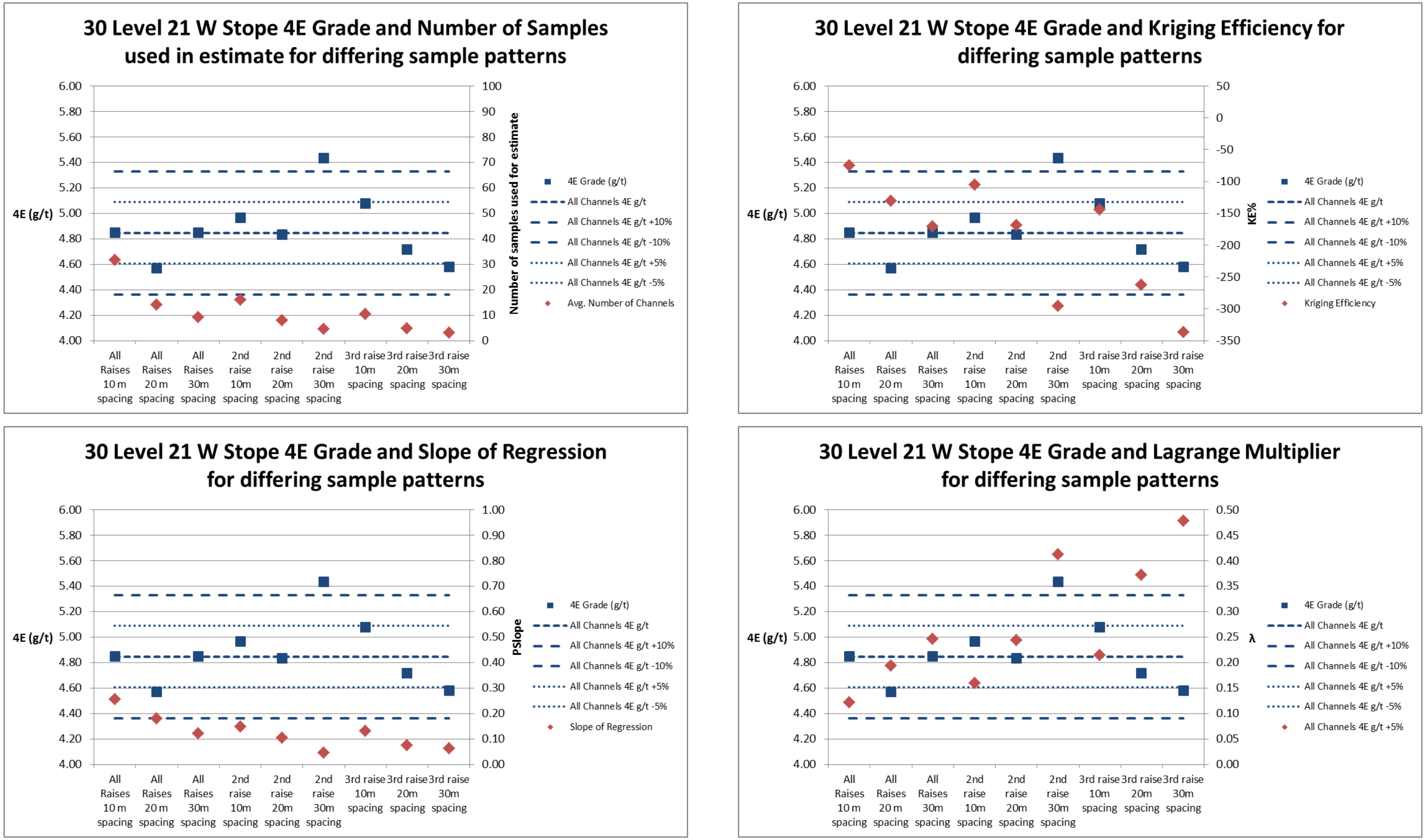


Figure 5-50 4E Grade versus Number of Channels, Kriging Efficiency, Slope of regression and Lagrange Multiplier for nine different sampling configurations at 30 Level West 21.

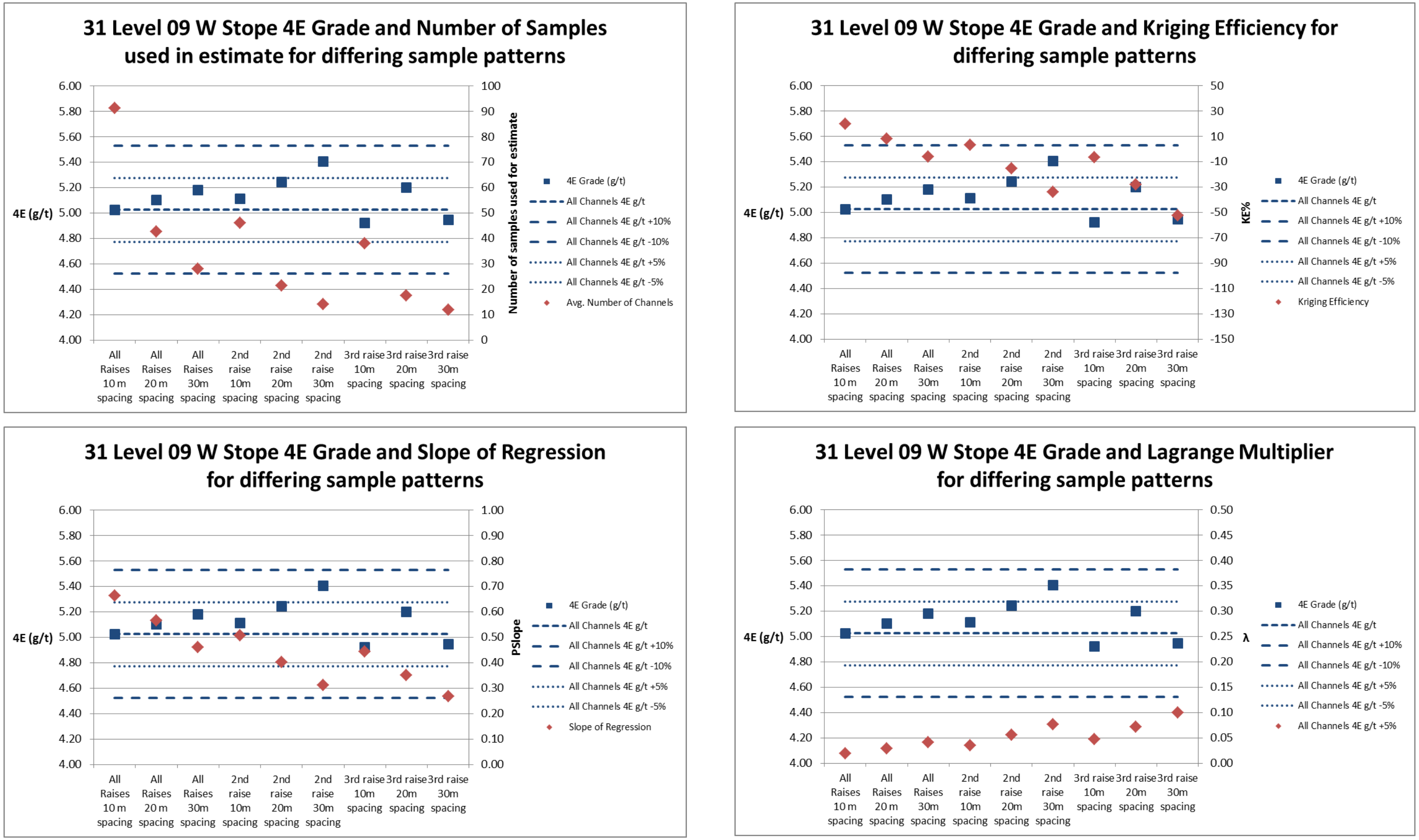


Figure 5-51 4E Grade versus Number of Channels, Kriging Efficiency, Slope of regression and Lagrange Multiplier for nine different sampling configurations at 31 Level West 09.

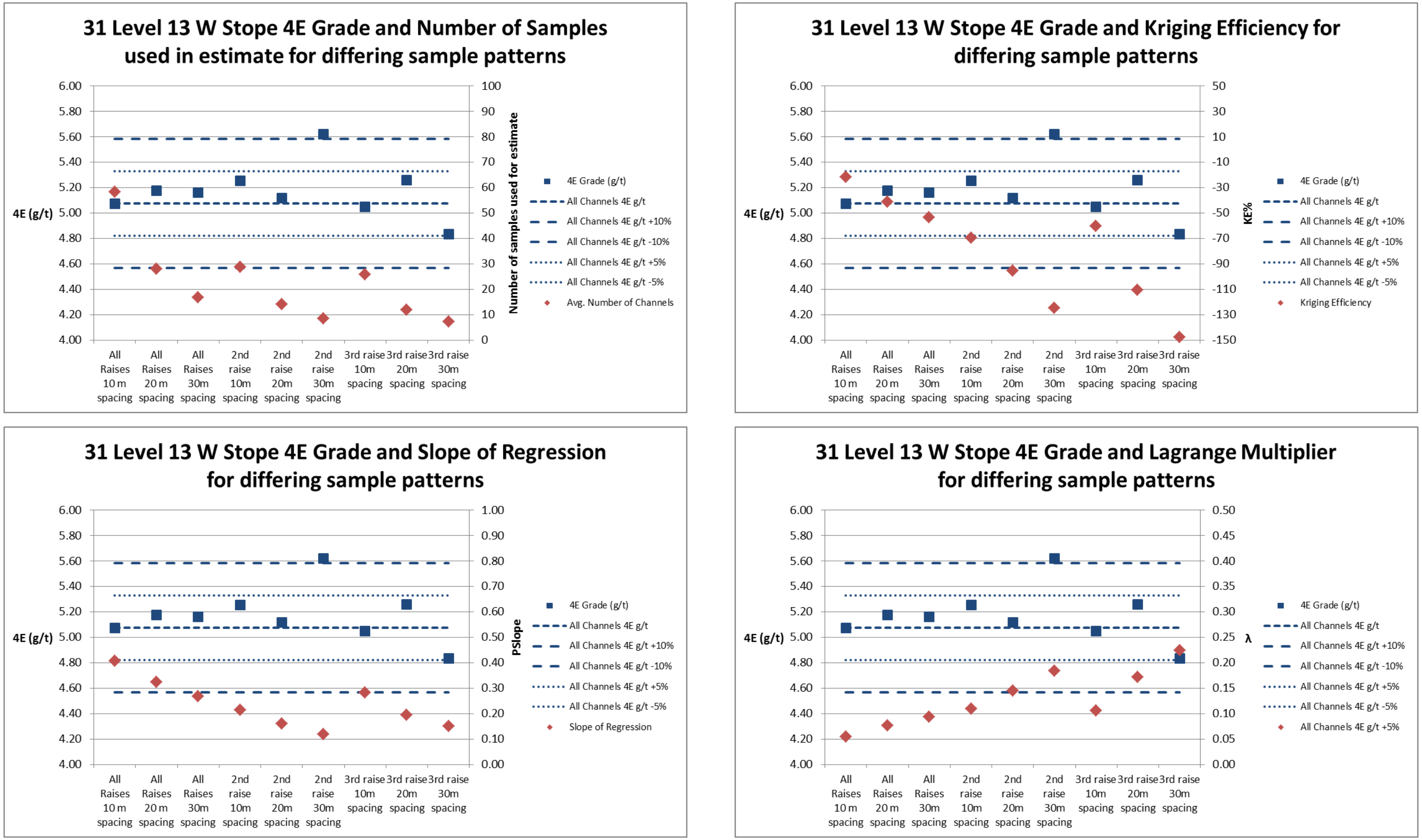


Figure 5-52 4E Grade versus Number of Channels, Kriging Efficiency, Slope of regression and Lagrange Multiplier for nine different sampling configurations at 31 Level West 13.

In summary:

- Most estimates are within ±10% of each other and in many cases within ±5% of each other whether the estimate is based on 10 m, 20 m or 30 m sampling from all, every second of every third raises. The least variability between estimates tends to be when all raises are sampled at 10 m, 20 m or 30 m spacing and every second raise at either 10 m or 20 m spacing. The small differences in the estimates reflect a high degree of smoothing over the area, likely caused by the high nugget effect and low variance as shown by the low PSlope values.
- The Lagrange Multiplier is highest using samples spaced at 20 m or 30 m along every third raise and spaced at 30 m along every third raise. This is a reflection of the higher degree of extrapolation.
- The low Kriging Efficiency and Slope of Regression is a cause for concern. Ordinary Kriged block estimates with a Kriging Efficiency of less than zero are less accurate than correctly assigned local means. The degree of smoothing is already so high (as indicated by the low Slope of Regression) that an estimate more strongly utilising a local average, such as Simple Kriging, may actually be more accurate. As the UG2 mineralisation at Lonmin is not amenable to selective mining, local accuracy is not of high priority and smoothed estimates will adequately achieve the type of estimation required for mine planning in the year ahead.
- When considering a reduction in sample spacing more regular grids are preferred over the tightly spaced sampling along wider spaced raises. This, however, is not practical for the mining method employed. The omni-directional semi-variogram shows that there is no benefit from clustered sampling in any one direction. Despite the poorer kriging efficiency and slope of regression, the small change in the estimates with fewer samples indicates that a significant reduction in the amount of sampling can be implemented without compromising the accuracy of a stope scale estimate for mine planning, particularly as the estimates only slightly away from the close spaced sample grids are highly smoothed already. Given the 35 m raise spacing and the high discard or non-

sampled rate in each raise, a 20 m spacing along the raises will achieve an approximately regular grid and source enough samples for an accurate mean at a half level scale. Should the discard rate be reduced, 30 m spaced samples along the raise may even be considered.

- Should selective mining decisions be made, they should only be done so if a stope is fully sampled and not from a block model based on extrapolation from adjacent sampling.

6. SENSITIVITY OF STOPE ESTIMATE CONFIDENCE TO NUGGET EFFECT

In section 1.2 of this report the different components to nugget effect were discussed. It was concluded that nugget effect for assayed attributes should be considered as partially a function of "true" nugget effect that has a small distance component that is a partly a function of sample support, and a component of sampling and assay error that has no distance component. Using deflections to estimate nugget effect examines the semi-variance at very short range. As the next borehole is often many times (perhaps thousands) the distance away than the intersection separation achieved through short deflection drilling, this very short range practically forms its own structure that can be modelled. For the purpose of this study, the range of this structure is denoted R0.5 and the semi-variance C0.5, which is essentially the semi-variance shorter than or equal to the distance between the deflections.

The effect on the geostatistical confidence in an estimate of changing the short-range components of the semi-variogram to reflect different components was tested using the same spreadsheet that was used for understanding the confidence in a raise (Section 5.5). The tests were based on a raise samples at a nominal interval of 10 m.

For the true thickness attribute, there cannot be either a "true" nugget effect or a measurement error. The impact on an estimate of the current (perhaps incorrect) method of assigning the true thickness short range variance to nugget effect was also determined.

The different scenarios of semi-variograms for the Rowland study area for 4E and true thickness are shown in **Table 6-1** and **Table 6-2** respectively.

Table 6-1 PGE g/t normalised semi-variogram scenarios

Scenario	**C0**	**R0.5**	**C0.5**	**R1**	**C1**	**R2**	**C2**	**R3**	**C3**
Base Case	0.56	-	-	30	0.25	45	0.11	170	0.08
No nugget (all short range)	0.00	0.2	0.56	30	0.25	45	0.11	170	0.08
50% nugget effect 50% Short range	0.28	0.2	0.28	30	0.25	45	0.11	170	0.08

Table 6-2 True thickness normalised semi-variogram scenarios

Scenario	C0	R0.5	C0.5	R1	C1	R2	C2	R3	C3
Base Case	0.18	-	-	10	0.55	60	0.12	170	0.15
No nugget (all short range)	0.00	0.2	0.18	10	0.55	60	0.12	170	0.15

The resulting normalised extension standard error for the 10 m spaced samples in a 200 m long raise are shown in **Table 6-3** for both PGE g/t and true thickness

Table 6-3 Normalised Extension Standard Error for different nugget effect scenarios (estimate of a 200 m raise with channels spaced 10 m apart)

	Base case (all nugget)	No nugget effect (all short range)	50% nugget effect 50% Short range
PGE	0.038	0.17	0.123
True Thickness	0.085	0.127	-

If we assume that the Rowland domain PGE and True thickness variance is applicable at a stope scale then the error at 90% confidence can be calculated (**Table 6-4**).

Table 6-4 Error at 90% confidence for average Rowland UG2 for different nugget effect scenarios (estimate of a 200 m raise with channels spaced 10 m apart)

	Base case (all nugget)	No nugget effect (all short range)	50% nugget effect 50% Short range
PGE (g/t)	±0.08	±0.34	±0.25
True Thickness (m)	±0.02	±0.03	-

Using the average PGE grade and true thickness of 5.36 g/t and 1.10 m respectively the percentage error for the estimate would be as shown in **Table 6-5**.

Table 6-5 Percentage Error at 90% confidence for average Rowland UG2 for different nugget effect scenarios (estimate of a 200 m raise with channels spaced 10 m apart).

	Base case (all nugget)	**No nugget effect (all short range)**	**50% nugget effect 50% Short range**
PGE	±1.4%	±6.4%	±4.6%
True Thickness	±1.7%	±2.5%	-

There are marked differences between applying all the semi-variance of the deflections as nugget effect and assuming a short range structure. There is considerably higher error should we assume all of the deflection semi-variance is nugget effect with no distance parameter. This phenomena is caused by a lower $\Upsilon(S,S)$ value when a short range structure is invoked, and therefore an increase in σ_e^2, there being less variance removed from $2\Upsilon(S,A)$ part of the equation. The lower $\Upsilon(S,S)$ value when a small distance parameter is invoked is because when there is a nugget structure, the entire semi-variance of the nugget effect is used in the calculation. When a distance parameter is added the semi-variance between samples zero distance apart (at the range of the structure) is zero and a lower average semi-variance results. This variance has been moved to further away in the semi-variogram at a range beyond the deflection spacing. This is illustrated in **Figure 6-1**.

	20	60	10	140	180
20	**0.56**	0.56	0.56	0.56	0.56
60	0.56	**0.56**	0.56	0.56	0.56
100	0.56	0.56	**0.56**	0.56	0.56
140	0.56	0.56	0.56	**0.56**	0.56
180	0.56	0.56	0.56	0.56	**0.56**

ϒ(S,S) = 0.560

	20	60	10	140	180
20	**0.00**	0.56	0.56	0.56	0.56
60	0.56	**0.00**	0.56	0.56	0.56
100	0.56	0.56	**0.00**	0.56	0.56
140	0.56	0.56	0.56	**0.00**	0.56
180	0.56	0.56	0.56	0.56	**0.00**

ϒ(S,S) = 0.448

Figure 6-1 Semi-variance between samples spaced 40 m apart with all nugget effect (left) and no nugget effect but range less than sample spacing (right).

Clark (2010) attributed a portion of nugget effect to sample precision and a larger portion to semi-variance over a short distance (less than the 6 inch sample spacing). She found that by assigning nugget effect to a short range structure gives a higher error than if it was dealt with as “true” nugget effect. She then pointed out that this is counterintuitive as there should be more confidence in the estimate if we trust our data (zero nugget effect) than if we acknowledge poor sample precision and have nugget effect at zero distance. As more of the short range variance is attributed to “true” nugget effect the less we trust our data but the more confidence we have in the results.

On examination of the impact of the variance and nugget effect of the different surface borehole assays, there are a number of assumptions that need to be used:

- The first assumption is that the data derived from 6E analysis are actually from the same stationary domain as the 4E data and that the differences in the mean and variance are a function of the assaying rather than any short range variance.
- The second assumption is that the general normalised semi-variogram calculated from the channel samples can be used with minor modification

in terms of different relative nugget effect from different sets of deflection data.

By using the mean and variance in **Table 6-6** and the modelled semi-variograms in **Table 6-7**, the error in the estimate can be calculated for each assay technique as shown in **Table 6-8**.

Table 6-6 Deflection statistics and nugget effect for PGE grade

	n	Mean (g/t)	Variance (g/t^2)	SD (g/t)	CV	N Pair	C0 (g/t^2)	C0 (normalised)
Central 4E	236	5.61	1.53	1.24	0.22	229	0.85	0.56
Central 6E	115	5.76	0.75	0.87	0.15	113	0.40	0.52
Eastern 4E	279	5.06	1.08	1.04	0.21	312	0.57	0.53
Eastern 6E	140	5.06	0.76	0.87	0.17	136	0.40	0.53

Table 6-7 PGE g/t semi-variogram for 6E and 4E assays

		C0	R1	C1	R2	C2	R3	C3
Central 4E	Normalised	0.56	30	0.25	45	0.11	170	0.08
	Actual	0.86	30	0.38	45	0.17	170	0.12
Central 6E	Normalised	0.52	30	0.29	45	0.11	170	0.08
	Actual	0.39	30	0.22	45	0.08	170	0.06
Eastern 4E	Normalised	0.53	30	0.28	45	0.11	170	0.08
	Actual	0.57	30	0.30	45	0.12	170	0.09
Eastern 6E	Normalised	0.53	30	0.28	45	0.11	170	0.08
	Actual	0.40	30	0.21	45	0.09	170	0.06

Table 6-8 PGE errors obtained by using different nugget effect for each area (estimate of a 200 m raise with channels spaced 10 m apart)

	Central 4E	Central 6E	Eastern 4E	Eastern 6E
Normalised Extension Standard Error	0.038	0.040	0.039	0.039
Error @ 90% Confidence	±0.08	±0.06	±0.07	±0.06
Percentage Error	±1.4%	±1.0%	±1.3%	±1.1%

The errors are low as is expected in a raise sampled at 10 m intervals. What is of importance is the difference in the error for an estimate between using the 4E and the 6E data. The error in a PGE estimate for the Central Area using 4E data is approximately one third higher than that for the 6E assay. For the Eastern area the error is 20% higher using the 4E data compared with that of the 6E data.

For an estimate of a stope by extension of a 200 m central raise sample along strike (as shown in **Figure 5-39**), the differences in error in the panel estimate are as significant, with the 4E error being 36% higher than the 6E estimate for the Central area (**Table 6-9**).

Table 6-9 PGE errors obtained by using different nugget effect for each area (estimate of a 35m by 200 m stope using a 200 m central raise)

	Central 4E	Central 6E	Eastern 4E	Eastern 6E
Normalised Extension Standard Error	0.147	0.155	0.153	0.153
Error @ 90% Confidence	±0.30	±0.22	±0.26	±0.22
Percentage Error	±5.3%	±3.8%	±5.2%	±4.3%

In the case of extending a raise estimate through a stope, attributing the deflection semi-variance as nugget effect versus a small range has an insignificant effect on the estimation error. This is because in this calculation $\Upsilon(S,S)$ is the semi-variance between the 200 m raise and itself rather than between samples so the value is unaffected as the ‘sample length’ of 200 m is far in excess of a 0.2 m range.

In summary:

- The way in which the short range variance is assigned has an impact on the confidence in an estimate. The more of this variance attributed to nugget effect, the higher the apparent confidence in the estimate, despite what should be considered lower confidence in the data.
- The surface borehole data obtained through 4E assay methods is less reliable than that obtained by the 6E method as evidenced by the higher nugget effect and data variance. The difference in the error is significant with the 4E error being 36% higher than the 6E estimate for the Central area.

7. SUMMARY AND RECCOMENDATIONS

7.1 The Nugget Effect and Multiple Intersections

The nugget effect consists of two main components; a portion of the nugget effect is intrinsic in-situ variability that has a range, albeit very small (at the scale of the sample support) and a portion due to errors in sampling and assaying. Separation of the two components can be made after extensive duplicate sampling and assaying programmes and the true nugget effect is often not known in a given mine or exploration project evaluated using diamond drilling techniques.

At Marikana, multiple intersections are obtained from intersections spaced a short but unknown distance apart (in the order of several 10's of cm). The semi-variance calculated from these intersections is called the nugget effect at Marikana, although it is suggested that a better description would be deflection semi-variance. The thickness variability on the scale of the deflection separation is visible in the UG2 at Marikana and it is likely that even higher differences in grade will occur. The semi-variance over this scale is described as ultra-short range variability, which can have its own structure in a semi-variogram model.

Preliminary work conducted on opposite sides of relatively large diameter core indicate that the PGE grade nugget effect may actually be very small, perhaps less than a tenth of what is implied from the deflection semi-variance. A true thickness nugget effect is applied in estimation, although it is theoretically incorrect to do so given that two measurements at the same place should be exactly the same, there being no sampling and assay error or microstructures to contend with at the measuring scale.

The shortcomings in the data to be able to apportion ultra-short range variability to its correct place in a semi-variogram affects the calculated confidence in a PGE grade estimate. The more semi-variance attributed to nugget effect, the higher the confidence in the estimate, despite what should be considered lower confidence in the data. The calculated error can be around four times higher should there be no nugget effect and the semi-variance is attributed to an ultra-short range of 20 cm instead. Test work on duplicate samples, such as opposite sides of core and juxtaposed channel samples, may be considered in order to be able to correctly calculate the nugget effect. Estimation of nugget effect by extrapolation from channel samples spaced 10 m apart will over-estimate nugget effect, particularly for true-thickness.

The older borehole assays that were conducted using 4E assaying techniques have a higher deflection semi-variance than those assayed using 6E techniques; however the normalised deflection semi-variance is almost the same. The older boreholes have higher overall PGE grade variance than the newer boreholes and therefore confidence in a grade estimate is less with the older boreholes than the newer ones.

The arrangement applied to the non-surveyed deflections around the mother hole is best by using the cross arrangement. The 20 cm separation introduces a slight and insignificant screening effect; however this increases with increasing separation. A smaller separation to the 20 cm currently applied can avoid this. Line arrangements should be avoided as screening will easily occur. Should there be a fifth intersection the current arrangement would invoke a line arrangement, which will introduce unnecessary screening. If a portion of the nugget effect is assigned to an ultra-short range structure, the spacing applied to the deflections must be equal or greater than the range of the ultra-short structure otherwise an imbalance in the kriging weights will occur.

The way Lonmin invokes a deflection arrangement is reasonable. Small improvements may be made if Lonmin adjusts the current arrangement of deflections, to a "+" for the first four deflections followed by a "x" for the following four.

The use of an ultra-short range structure in the semi-variograms may be considered should more data be available in future in order to be able to correctly apportion the deflection semi-variance. The way the nugget effect is currently applied may have an influence on the calculated confidence for true width; however classification is based largely on the PGE grade and the impact of adjusting true width semi-variograms will be minimal overall.

Ordinary Kriging is very effective in managing the data clustering created by the deflection drilling. The data are de-clustered while accounting for the higher confidence in a borehole by virtue of the number of deflections. Use of lateral composites is not recommended for estimation unless an inverse distance approach is used.

The short deflection drilling conducted at Marikana is important in assessing geological characteristics of each borehole and adds significant statistical confidence to the estimate. The use of lateral compositing to provide deflection variance statistics is a useful tool in identifying problematic boreholes or individual intersections. The grade confidence can be improved significantly with a fourth assayed intersection, however further improvements with more intersections than four will be minimal.

7.2 Underground Channel Sampling Optimisation

A number of techniques were applied in order to understand the possibility of reducing the number of channel samples taken underground. This was considered by reducing the number of raises sampled and by increasing the sample spacing in the raises, as well as a combination of the two. The techniques applied were both statistical and geostatistical.

Statistical analysis of the channel sample data revealed that as the samples are sourced from further away, more samples are needed to provide the same confidence in an estimate. For example, in order for the estimates to have an error of less than 10% at 90% confidence, 16 samples are required from within a single raise. Should insufficient samples be available in a single raise and the samples from two raises are taken, 19 samples will give the same confidence. This may be a function of distance-variance relationships in that the variance between samples is expected to increase as they are further away from each other.

In order to obtain accuracy within ±10% at 90% confidence in more than two thirds of the raises, 10 m sample spacing is required. Too few samples can be sourced within a reasonable distance of the panel to be estimated with a sample spacing of 20 m or 30 m at this confidence. Despite the poorer accuracy of estimates using the wider spacing, there is good correlation between those derived from 20 m spacing and those derived from 10 m spacing with most of the estimates being within 10% of one another. Should an increase in channel sample spacing from 10 m to 20 m be adopted, more than two thirds of the estimates will be achieved within an error ±15% at 90% confidence. The proportion of estimates within an error ±20% at 90% confidence will be more than 90%.

The geostatistical techniques revealed that if a reduction in channel samples is warranted it is best to increase the sample spacing along the raise rather than sampling fewer raises. The omni-directional semi-variogram indicates that there is no benefit from clustered sampling in any one direction. The geostatistical confidence based on the average variance of the domain was considerably lower than the confidence arrived at by using classical statistical techniques and indicated that an increase in the channel sample spacing to 20 m can be achieved while maintaining an error of less than ±10%. Geostatistical techniques on global mean and variance techniques should be used with caution as an over-estimation of confidence will result.

Block estimates of poorly sampled areas using Ordinary Kriging indicate that estimates are highly smoothed and of poor local accuracy; this is likely a function of the high proportion of semi-variance attributed to the nugget effect. Increasing the sample spacing will have little impact on these already much smoothed estimates in the forward production areas. As the UG2 mineralisation at Lonmin is not amenable to selective mining, local accuracy is not of high priority and smoothed estimates will adequately achieve the type of estimation required for mine planning in the year ahead. Should selective mining decisions be made, they should only be done so if a stope is fully sampled and not from a block model based on extrapolation from adjacent sampling.

Using the 35 m raise spacing and the high discard or non-sampled rate in each raise, 20 m planned spacing along the raises will ultimately achieve an approximately regular grid. This grid will source enough samples for an accurate predicted mean at a half level scale for mine planning. The accuracy of stope estimates by selecting every second sample in the current data base will decrease from ±10% to ±15% at 90% confidence for most of the estimates. If the decrease in accuracy at a stope scale is acceptable, the sample spacing could be reduced from 10 m to 20 m. The number of valid channel samples in each raise is less than expected. A high degree of proactive sampling management and improved quality of laboratory assays will result in a lower discard rate and better sampling coverage. The current level of accuracy could then be maintained at the 20 m spacing if 70% of the raise is sampled at 20 m spacing, as that would produce 21 samples for each set of three raises.

Approximately 11,000 channel samples are assayed by the Lonmin Mine Laboratory every month for UG2 and Merensky Reef together, about half of which are UG2 samples (Sampson -Laboratory Manager- pers. comm.). The approximate cost of assaying the UG2 samples alone is in the order of R110 per sample, which equates to an assay cost of around R7.3 million per year. An increase in the channel sampling spacing from 10 m to 20 m could result in a saving of R605,000 per month for the UG2, before considering the costs of the sampling itself. A similar study on the Merensky Reef may result in a similar outcome which could add a similar amount again to the potential saving.

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APPENDIX 1 UG2 Reef intersections identified as a result of the validation process.

Coding/Data Issues (Rejected)	Coding/Data Issues (Fixed)	Pothole	Fault
RS062 D2, D3, D4	ML44 D2	BH1825 D1, D2, D3	BH1791 D3, D4
RS147 D2	RSU2C	BH1832 D2, D3, D4	MK036D2, D3, D4
MK082 D1	RS015 D1	BH1840 D0, D1, D3	
RS042 D4	RS059 D3	MK049 D1, D3, D4	
RS092 D3	SAMTF02 D4	MK065 D1, D3, D5	
ML38 D1	WP036 D5	MK087 D2, D4, D5	
ML39 D0	MK009 D3	RS017D1	
RS0176 D1, D2, D3		RS019 D1UG2	
TN55 D0, D2 (6E only)		RS028 D1, D2, D3	
		TN024 D0	
		ML045 D1, D2, D3	
		WP25 D1, D3, D4	
		WP29 D1, D3, D4	
		RS147 D1, D2, D5	
		RS024 D1, D2, D3	
		RS066 D0	
		CR017	
		CR020	
		CR021	

Appendix 2a Auxiliary Function H(L,B) for Spherical Model with Range 1.0 and Sill 1.0 (Reproduced from Clark, 1979)

		b																			
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
l	**0.1**	0.114	0.177	0.243	0.310	0.374	0.436	0.494	0.546	0.593	0.633	0.694	0.738	0.771	0.796	0.817	0.853	0.878	0.895	0.908	0.927
	0.2	0.177	0.227	0.285	0.346	0.406	0.464	0.518	0.568	0.613	0.651	0.709	0.751	0.782	0.806	0.826	0.860	0.884	0.900	0.913	0.930
	0.3	0.243	0.285	0.336	0.390	0.445	0.499	0.550	0.597	0.639	0.674	0.729	0.767	0.797	0.819	0.837	0.870	0.891	0.907	0.919	0.935
	0.4	0.310	0.346	0.390	0.439	0.489	0.539	0.586	0.629	0.668	0.701	0.751	0.786	0.813	0.834	0.850	0.880	0.900	0.914	0.925	0.940
	0.5	0.374	0.406	0.445	0.489	0.535	0.580	0.623	0.663	0.698	0.728	0.774	0.806	0.830	0.849	0.864	0.891	0.909	0.922	0.932	0.946
	0.6	0.436	0.464	0.499	0.539	0.580	0.621	0.660	0.697	0.728	0.755	0.796	0.825	0.847	0.864	0.878	0.902	0.918	0.930	0.939	0.951
	0.7	0.494	0.518	0.550	0.586	0.623	0.660	0.696	0.729	0.757	0.781	0.818	0.844	0.863	0.879	0.891	0.913	0.927	0.938	0.945	0.956
	0.8	0.546	0.568	0.597	0.629	0.663	0.697	0.729	0.758	0.783	0.805	0.837	0.861	0.878	0.892	0.902	0.922	0.935	0.944	0.951	0.961
	0.9	0.593	0.613	0.639	0.668	0.698	0.728	0.757	0.783	0.806	0.826	0.855	0.875	0.891	0.903	0.913	0.930	0.942	0.950	0.956	0.965
	1	0.633	0.651	0.674	0.701	0.728	0.755	0.781	0.805	0.826	0.843	0.869	0.888	0.902	0.913	0.921	0.937	0.948	0.955	0.961	0.969
	1.2	0.694	0.709	0.729	0.751	0.774	0.796	0.818	0.837	0.855	0.869	0.891	0.907	0.918	0.927	0.935	0.948	0.956	0.963	0.967	0.974
	1.4	0.738	0.751	0.767	0.786	0.806	0.825	0.844	0.861	0.875	0.888	0.907	0.920	0.930	0.938	0.944	0.955	0.963	0.968	0.972	0.978
	1.6	0.771	0.782	0.797	0.813	0.830	0.847	0.863	0.878	0.891	0.902	0.918	0.930	0.939	0.945	0.951	0.961	0.967	0.972	0.975	0.980
	1.8	0.796	0.806	0.819	0.834	0.849	0.864	0.879	0.892	0.903	0.913	0.827	0.938	0.945	0.952	0.956	0.965	0.971	0.975	0.978	0.983
	2	0.817	0.826	0.837	0.850	0.864	0.878	0.891	0.902	0.913	0.921	0.935	0.944	0.951	0.956	0.961	0.969	0.974	0.978	0.980	0.984
	2.5	0.853	0.860	0.870	0.880	0.891	0.902	0.913	0.922	0.930	0.937	0.948	0.955	0.961	0.965	0.969	0.975	0.979	0.982	0.984	0.987
	3	0.878	0.884	0.891	0.900	0.909	0.918	0.927	0.935	0.942	0.948	0.956	0.963	0.967	0.971	0.974	0.979	0.983	0.985	0.987	0.990
	3.5	0.895	0.900	0.907	0.914	0.922	0.930	0.938	0.944	0.950	0.955	0.963	0.968	0.972	0.975	0.978	0.982	0.985	0.987	0.989	0.991
	4	0.908	0.913	0.919	0.925	0.932	0.939	0.945	0.951	0.956	0.961	0.967	0.972	0.975	0.978	0.980	0.984	0.987	0.989	0.990	0.992
	5	0.927	0.930	0.935	0.940	0.946	0.951	0.956	0.961	0.965	0.969	0.974	0.978	0.980	0.983	0.984	0.987	0.990	0.991	0.992	0.994

Appendix 2b Auxiliary Function F(L,B) for Spherical Model with Range 1.0 and Sill 1.0 (Reproduced from Clark, 1979)

		b																			
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
	0.1	0.780	0.120	0.165	0.211	0.256	0.300	0.342	0.383	0.422	0.457	0.520	0.572	0.614	0.650	0.679	0.735	0.775	0.804	0.827	0.860
	0.2	0.120	0.155	0.196	0.237	0.280	0.321	0.362	0.401	0.438	0.473	0.534	0.584	0.625	0.659	0.688	0.743	0.781	0.810	0.832	0.864
	0.3	0.165	0.196	0.231	0.270	0.309	0.349	0.387	0.424	0.460	0.493	0.551	0.600	0.639	0.672	0.700	0.752	0.789	0.817	0.838	0.869
	0.4	0.211	0.237	0.270	0.305	0.342	0.379	0.415	0.451	0.484	0.516	0.572	0.618	0.655	0.687	0.713	0.763	0.799	0.825	0.845	0.874
	0.5	0.256	0.280	0.309	0.342	0.376	0.411	0.445	0.479	0.511	0.541	0.593	0.637	0.673	0.703	0.728	0.775	0.809	0.834	0.853	0.881
	0.6	0.300	0.321	0.349	0.379	0.411	0.443	0.476	0.507	0.538	0.566	0.616	0.657	0.691	0.719	0.743	0.788	0.820	0.843	0.861	0.887
	0.7	0.342	0.362	0.387	0.415	0.445	0.476	0.506	0.536	0.565	0.591	0.638	0.677	0.709	0.736	0.758	0.800	0.830	0.852	0.870	0.894
	0.8	0.383	0.401	0.424	0.451	0.479	0.507	0.536	0.564	0.591	0.616	0.660	0.697	0.727	0.752	0.773	0.813	0.841	0.861	0.878	0.901
	0.9	0.422	0.438	0.460	0.484	0.511	0.538	0.565	0.591	0.616	0.640	0.682	0.716	0.744	0.767	0.787	0.824	0.851	0.870	0.885	0.907
l	**1**	0.457	0.473	0.493	0.516	0.541	0.566	0.591	0.616	0.640	0.662	0.701	0.733	0.760	0.782	0.800	0.835	0.860	0.878	0.892	0.913
	1.2	0.520	0.534	0.551	0.572	0.593	0.616	0.638	0.660	0.682	0.701	0.736	0.764	0.788	0.807	0.823	0.854	0.876	0.892	0.905	0.923
	1.4	0.572	0.584	0.600	0.618	0.637	0.657	0.677	0.697	0.716	0.733	0.764	0.790	0.811	0.828	0.842	0.870	0.890	0.904	0.915	0.931
	1.6	0.614	0.625	0.639	0.655	0.673	0.691	0.709	0.727	0.744	0.760	0.788	0.811	0.829	0.845	0.858	0.883	0.901	0.914	0.924	0.938
	1.8	0.650	0.659	0.672	0.687	0.703	0.719	0.736	0.752	0.767	0.782	0.807	0.828	0.845	0.859	0.871	0.894	0.910	0.921	0.931	0.944
	2	0.679	0.688	0.700	0.713	0.728	0.743	0.758	0.773	0.787	0.800	0.823	0.842	0.858	0.871	0.882	0.903	0.917	0.928	0.936	0.948
	2.5	0.735	0.743	0.752	0.763	0.775	0.788	0.800	0.813	0.824	0.835	0.854	0.870	0.883	0.894	0.903	0.920	0.932	0.941	0.948	0.570
	3	0.775	0.781	0.789	0.799	0.809	0.820	0.830	0.841	0.851	0.860	0.876	0.890	0.901	0.910	0.917	0.932	0.942	0.950	0.955	0.964
	3.5	0.804	0.810	0.817	0.825	0.834	0.843	0.852	0.861	0.870	0.878	0.892	0.904	0.914	0.921	0.928	0.941	0.950	0.956	0.961	0.969
	4	0.827	0.832	0.838	0.845	0.853	0.861	0.870	0.878	0.885	0.892	0.905	0.915	0.924	0.931	0.936	0.948	0.955	0.961	0.966	0.972
	5	0.860	0.864	0.869	0.874	0.881	0.887	0.894	0.901	0.907	0.913	0.923	0.931	0.938	0.944	0.948	0.957	0.964	0.969	0.972	0.977

Appendix 2c Auxiliary Function X(L,B) for Spherical Model with Range 1.0 and Sill 1.0 (Reproduced from Clark, 1979)

		b																			
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
	0.1	0.098	0.136	0.178	0.222	0.266	0.309	0.350	0.390	0.428	0.464	0.526	0.577	0.619	0.653	0.683	0.738	0.777	0.807	0.829	0.861
	0.2	0.164	0.194	0.229	0.268	0.307	0.346	0.385	0.422	0.458	0.491	0.550	0.598	0.638	0.671	0.698	0.751	0.788	0.816	0.837	0.868
	0.3	0.233	0.257	0.288	0.321	0.356	0.392	0.427	0.462	0.495	0.526	0.580	0.625	0.662	0.693	0.719	0.768	0.803	0.828	0.848	0.877
	0.4	0.302	0.322	0.348	0.378	0.409	0.441	0.474	0.505	0.535	0.564	0.614	0.655	0.689	0.718	0.741	0.787	0.819	0.842	0.861	0.887
	0.5	0.368	0.385	0.408	0.434	0.462	0.492	0.521	0.550	0.577	0.603	0.649	0.687	0.718	0.743	0.765	0.806	0.835	0.857	0.873	0.897
	0.6	0.430	0.445	0.466	0.489	0.515	0.541	0.568	0.594	0.619	0.642	0.684	0.718	0.746	0.769	0.788	0.825	0.852	0.871	0.886	0.907
	0.7	0.488	0.502	0.520	0.541	0.564	0.588	0.612	0.636	0.658	0.680	0.717	0.747	0.772	0.793	0.811	0.844	0.867	0.885	0.898	0.917
	0.8	0.542	0.554	0.570	0.589	0.610	0.631	0.653	0.674	0.695	0.714	0.747	0.774	0.797	0.815	0.831	0.861	0.881	0.897	0.909	0.926
	0.9	0.589	0.600	0.614	0.632	0.650	0.670	0.689	0.708	0.727	0.744	0.774	0.798	0.818	0.835	0.849	0.875	0.894	0.908	0.919	0.934
l	**1**	0.629	0.639	0.653	0.668	0.685	0.703	0.720	0.737	0.754	0.769	0.796	0.818	0.836	0.851	0.864	0.888	0.905	0.917	0.927	0.941
	1.2	0.691	0.699	0.711	0.723	0.737	0.752	0.767	0.781	0.795	0.808	0.830	0.848	0.964	0.876	0.886	0.906	0.920	0.931	0.939	0.950
	1.4	0.735	0.742	0.752	0.763	0.775	0.788	0.800	0.812	0.824	0.835	0.854	0.870	0.883	0.894	0.903	0.920	0.932	0.941	0.948	0.958
	1.6	0.768	0.775	0.783	0.793	0.803	0.814	0.825	0.836	0.846	0.856	0.873	0.886	0.898	0.907	0.915	0.930	0.940	0.948	0.954	0.963
	1.8	0.794	0.800	0.807	0.816	0.825	0.835	0.845	0.854	0.863	0.872	0.887	0.899	0.909	0.917	0.924	0.938	0.947	0.954	0.959	0.967
	2	0.815	0.820	0.826	0.834	0.842	0.851	0.860	0.869	0.877	0.885	0.898	0.909	0.918	0.926	0.932	0.944	0.952	0.959	0.630	0.970
	2.5	0.852	0.856	0.861	0.867	0.874	0.881	0.888	0.895	0.902	0.908	0.918	0.927	0.934	0.940	0.946	0.955	0.962	0.967	0.971	0.976
	3	0.876	0.880	0.884	0.889	0.895	0.901	0.907	0.912	0.918	0.923	0.932	0.939	0.945	0.950	0.955	0.963	0.968	0.972	0.976	0.980
	3.5	0.894	0.897	0.901	0.905	0.920	0.915	0.920	0.925	0.930	0.934	0.942	0.948	0.953	0.957	0.961	0.968	0.973	0.976	0.979	0.983
	4	0.907	0.910	0.913	0.917	0.921	0.926	0.930	0.934	0.938	0.942	0.949	0.955	0.959	0.963	0.966	0.972	0.976	0.979	0.982	0.985
	5	0.926	0.928	0.931	0.934	0.937	0.941	0.944	0.947	0.951	0.954	0.959	0.964	0.967	0.970	0.973	0.978	0.981	0.983	0.985	0.988

Appendix 2d Auxiliary Function Y(L,B) for Spherical Model with Range 1.0 and Sill 1.0 (Reproduced from Clark, 1979)

		b																			
		0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**	**1.2**	**1.4**	**1.6**	**1.8**	**2**	**2.5**	**3**	**3.5**	**4**	**5**
	0.05	0.094	0.132	0.175	0.219	0.263	0.306	0.348	0.388	0.426	0.461	0.524	0.575	0.617	0.652	0.681	0.737	0.777	0.806	0.828	0.861
	0.1	0.161	0.188	0.223	0.261	0.300	0.340	0.379	0.416	0.452	0.486	0.545	0.594	0.634	0.667	0.695	0.748	0.786	0.814	0.836	0.867
	0.15	0.231	0.252	0.280	0.312	0.347	0.383	0.419	0.453	0.486	0.518	0.573	0.619	0.656	0.687	0.714	0.764	0.799	0.825	0.846	0.875
	0.2	0.302	0.318	0.341	0.369	0.400	0.432	0.646	0.495	0.526	0.555	0.605	0.648	0.682	0.711	0.735	0.782	0.814	0.838	0.857	0.884
	0.25	0.372	0.385	0.404	0.428	0.455	0.483	0.512	0.541	0.568	0.594	0.641	0.679	0.711	0.737	0.759	0.801	0.831	0.853	0.870	0.894
	0.3	0.440	0.451	0.467	0.488	0.511	0.536	0.562	0.588	0.613	0.636	0.678	0.712	0.741	0.764	0.784	0.822	0.848	0.868	0.883	0.905
	0.35	0.507	0.516	0.529	0.547	0.568	0.590	0.612	0.635	0.657	0.678	0.715	0.746	0.771	0.792	0.809	0.843	0.866	0.884	0.897	0.917
	0.4	0.571	0.578	0.590	0.605	0.623	0.642	0.662	0.683	0.702	0.721	0.753	0.780	0.801	0.820	0.835	0.864	0.884	0.899	0.911	0.928
	0.45	0.632	0.638	0.648	0.661	0.677	0.693	0.711	0.729	0.746	0.762	0.790	0.812	0.831	0.847	0.860	0.884	0.902	0.915	0.924	0.939
l	**0.5**	0.689	0.695	0.703	0.715	0.728	0.742	0.758	0.773	0.787	0.801	0.825	0.844	0.860	0.872	0.883	0.904	0.918	0.929	0.937	0.949
	0.55	0.743	0.748	0.755	0.765	0.776	0.789	0.802	0.814	0.827	0.838	0.858	0.873	0.886	0.897	0.906	0.922	0.934	0.943	0.949	0.959
	0.6	0.793	0.797	0.803	0.811	0.821	0.831	0.842	0.853	0.863	0.872	0.888	0.901	0.911	0.919	0.926	0.939	0.948	0.955	0.960	0.968
	0.65	0.839	0.842	0.847	0.854	0.862	0.870	0.879	0.888	0.896	0.903	0.915	0.925	0.933	0.939	0.944	0.954	0.961	0.966	0.970	0.976
	0.7	0.879	0.882	0.886	0.892	0.898	0.905	0.912	0.919	0.925	0.930	0.939	0.946	0.952	0.956	0.960	0.967	0.972	0.976	0.979	0.983
	0.75	0.915	0.917	0.920	0.925	0.930	0.935	0.940	0.945	0.949	0.953	0.959	0.964	0.968	0.971	0.974	0.978	0.982	0.984	0.986	0.989
	0.8	0.945	0.946	0.949	0.952	0.956	0.960	0.963	0.966	0.969	0.971	0.975	0.978	0.981	0.983	0.984	0.987	0.989	0.991	0.992	0.993
	0.85	0.968	0.970	0.971	0.974	0.976	0.978	0.981	0.982	0.984	0.985	0.987	0.989	0.990	0.991	0.992	0.993	0.994	0.995	0.996	0.997
	0.9	0.986	0.987	0.988	0.989	0.990	0.991	0.992	0.993	0.994	0.994	0.995	0.996	0.996	0.997	0.997	0.997	0.998	0.998	0.998	0.999
	0.95	0.996	0.997	0.997	0.998	0.998	0.998	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999	0.999	1.000	1.000	1.000	1.000	1.000
	1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000