

DEEP RADIO PROBES OF DARK MATTER

Nicola Diane Orford

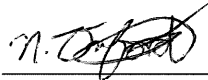
A dissertation submitted to the the Department of Physics, University of the
Witwatersrand in fulfilment of the academic requirements of the degree of Master
of Science.

Johannesburg, February 6, 2015

Preface and Declarations

The work described by this thesis was carried out at the University of the Witwatersrand, Department of Physics, from February 2012 until October 2014, under the supervision of Professor Sergio Colafrancesco.

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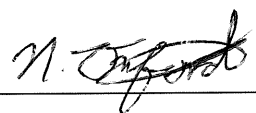
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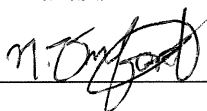
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1.

Title: Local Group dSph radio survey with ATCA (I): Observations and background sources

Status: In Submission - ArXiv pre-print available

Authors: M. Regis, L. Richter, S. Colafrancesco, M. Massardi, W de Blok, S. Profumo and N. Orford

Contributions: Data collection

Analysis of data

Source comparison with other surveys

Abstract

We explore indirect detections of Dark Matter, focusing on deep radio observations of six dwarf spheroidal galaxies (dSph), Carina, Fornax, BootesII, Hercules, Segue2, Sculptor.

We discuss the WIMP Dark Matter particle annihilation process and describe briefly the particles produced in this process. We consider the emissions, which can result from electrons and positrons produced. We describe why dSph are the best observational targets for indirect Dark Matter detection at radio frequencies.

We describe the theoretical framework for predicting Dark Matter synchrotron emissions and make some predictions for the six dSph of interest to us.

We discuss ATCA observations of these dSph and explore the background source subtraction process in detail. We obtain an upper limit on the WIMP mass and compare our results to various other experiments. We discuss prospects for this work towards attaining an indirect Dark Matter detection.

Acknowledgments

I would like to acknowledge the South African Square Kilometer array for their support, both financial and otherwise, without whom this research would not have been possible. Many thanks to Geoff Beck for writing me a LaTeX template to make writing up my thesis simpler.

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Chapter 1

Introduction

Dark Matter is one of the great enigmas of modern physics. There is evidence that baryonic matter, the matter comprising everything we see, touch and understand, forms a mere 4.87% of the universe. A mysterious form of matter, known as Dark Matter, composes a much larger 26.4%, whilst the remainder consists of an obscure form of energy – Dark Energy, *citetplanckXVI*. Dark Matter is known to interact gravitationally, yet emits no radiation as it does not interact through electromagnetic forces. See Figure 1.1.

There exist numerous indications of the presence of Dark Matter. Rotation curves of many galaxies indicate that the distribution of light in a galaxy does not correlate with the mass distribution. Many galaxies are known to lack a Keplerian drop-off in rotation velocity, whilst the stability of the galactic disks surrounding other galaxies indicates the presence of large amounts of mass away from the center of the galaxy. Gravitational lensing indicates masses of dwarf galaxies, galaxy clusters and large scale structures are greater than expected, Colafrancesco (2010).

Fluctuations of the cosmic microwave background (CMB) are smaller than required for formation of various existing structures (see Figure 1.2, indicating the presence of an additional non-baryonic component of the universe. In addition, N-body simulations of the formation of large scale structures (LSS), galaxy clusters and individual galaxies require the presence of Dark Matter, Colafrancesco (2010). These are just some of the pieces of evidence pointing to the existence of Dark Matter, yet they are only indicators of location and amount of Dark Matter, beyond which the absence of electromagnetic radiation makes Dark Matter very difficult to observe, detect or understand.

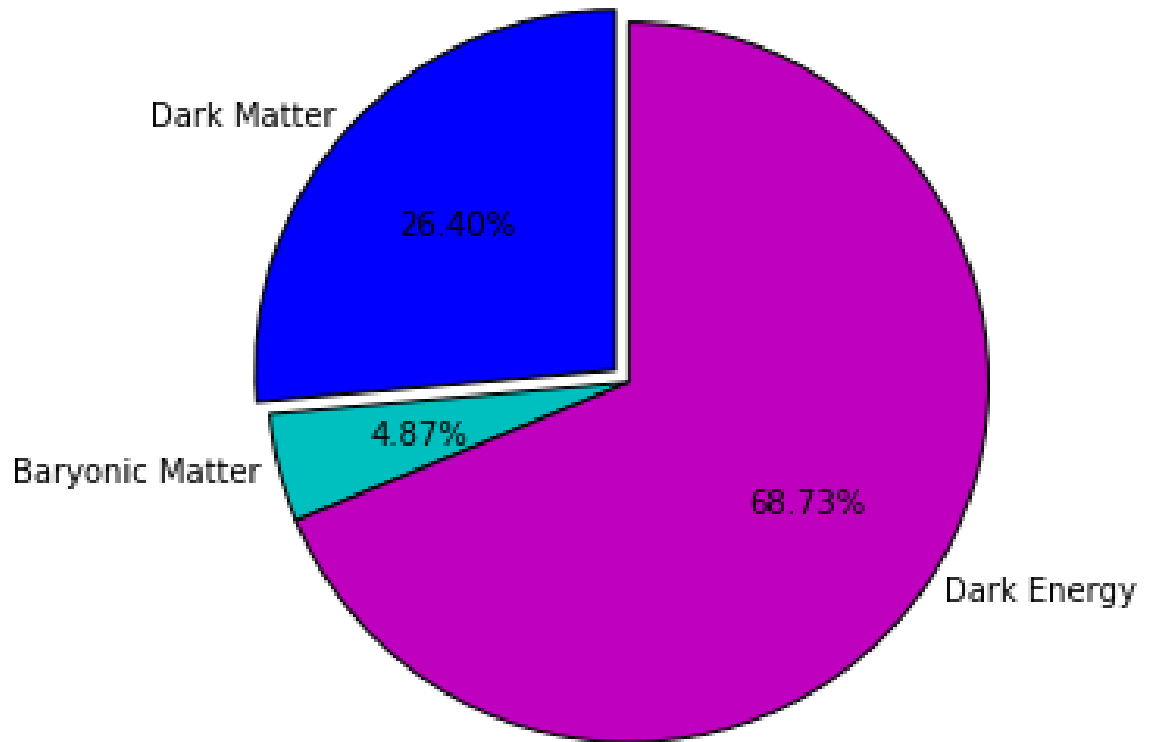


Figure 1.1: The Composition of the Universe in terms of baryonic matter, Dark Matter and Dark Energy

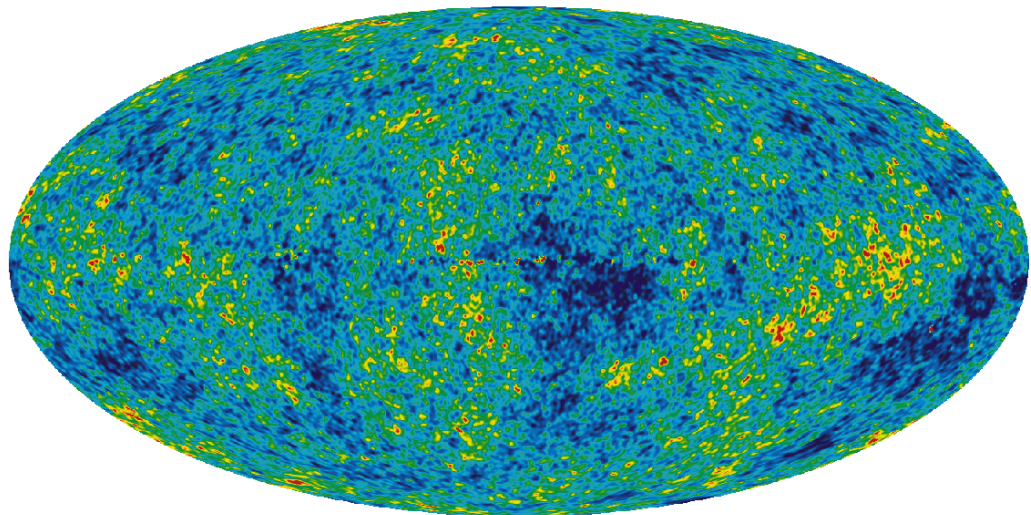


Figure 1.2: WMAP 5 Year Image of the Cosmic Microwave Background. Fluctuations are caused by the interaction of sizes indicate the presence of Dark Matter. <http://wmap.gsfc.nasa.gov/media/080997/index.html>, NASA/WMAP space science team

1.1 History of Evidence for the Existence of Dark Matter

1.1.1 Observational Evidence

The first evidence for Dark Matter was discovered by Zwicky (1933), during a study of galaxy clusters. Zwicky noted that the mass of the Coma cluster, as determined using redshifts and the velocity dispersion of the cluster, was ~ 400 times greater than that calculated from the luminous matter observed. This matter was referred to in his paper, “On the redshift of extragalactic nebulae”, as “*dunkle (kalte) materie*” or Dark (cold) Matter and thus the term Dark Matter was born.

A similar excess of non-luminous matter was observed in the Virgo cluster by Smith (1936) who postulated that the difference between the mass within the luminous boundaries of a nebula, and the total mass of the nebula “*represents a great mass of inter-nebular material within the cluster*”. Hubble (1936) referred to this missing mass discrepancy as both real and important, yet it remained largely unnoticed within the field for many years.

Babcock (1939) investigated the rotation of the Andromeda (M31) galaxy. His observations revealed anomalous behaviour of the angular velocity of galactic matter. It was expected that the velocity would decrease moving away from the center of the galaxy in correlation with the decreasing amounts of luminous mass, however an approach to constant, non-zero, angular velocity was observed. These observations led him to postulate that either a large percentage of the mass of the galaxy must be situated in the outer regions of the galaxy, as opposed to within the bright core as had been expected, or there was an unexpectedly large amount of absorption of light by dust. These observations of M31 were later confirmed and strengthened by the research performed by Rubin & Ford (1970).

Oort (1940) investigated the galaxies NGC4494 and NGC3115 and observed that “*the distribution of mass in the system appears to bear almost no relation to that of light*”. Babcock’s postulate of unexpectedly large amounts of absorption by dust in Andromeda cannot be extended to NGC3115, which is dust free. Babcock’s observations thus indicated the first evidence for significant amounts of Dark Matter present in a single galaxy, as opposed to the Dark Matter associated with galaxy clusters discussed above, van den Bergh (2001).

Further investigations of the rotations of ten high luminosity spiral galaxies by Rubin et al. (1978) indicated that the rotation curves of such galaxies are flat and this indicates the presence of massive halos extending to large radii. Rubin et al. (1980) later repeated these observations for twenty-one spiral Sc galaxies with varied mass, luminosity and radial size and found them to contain a large portion of mass

in their halo, beyond the optical galaxy.

In the 1950's, Page (1952, 1960) and Holmberg (1954), amongst others, investigated the mass of pairs of galaxies. They found these pairs to have an unexpected light to mass ratio. Similarly, Kahn & Woltjer (1959) investigated the motions of members of the Local Group. They found that Andromeda and our Milky Way appear to be orbiting one another, and thus extrapolated that the Local Group must contain a significant amount of intergalactic matter. They also noted that it seems unlikely that this intergalactic matter is predominantly composed of stars.

De Vaucouleurs (1960) and Ambartsumian (1961) argued that galaxy clusters are unstable and thus the virial theorem cannot be applied to them. This solves the missing mass problem caused by the application of the virial theorem to the velocity dispersion of galaxies within clusters. However, van den Bergh (1961, 1962) countered this argument by noting that the time scale for galaxy clusters to disperse implies there should be far fewer clusters than are presently observed. In addition, he studied the number of each type of galaxy existing within clusters and existing as isolated galaxies and noted that the greater proportion of galaxies exist within galaxy clusters. He believed this implied that galaxy clusters are stable, making the virial theorem applicable and the missing mass problem real.

Finzi (1963) tried to solve the missing mass problem by proposing that at distances greater than half a kiloparsec, the gravitational attraction between two objects decreases much more slowly than $1/r^2$. This offers some resolution to the mass of the galaxy being significantly larger when deduced from the motions of distant globular clusters as compared to the mass deduced from solar motion, as well as to the rotation curves of various galaxies. However, this theory forces the conclusion that general relativity does not apply on scales of kiloparsec order.

Penzias & Wilson (1965) observed background radiation in the radio spectrum with a temperature of $3.5K \pm 1K$, which was isotropic, unpolarised and free from seasonal variation. Penzias & Wilson (1965) also note that the spectrum of this radiation rules out known radio sources. Similar radiation was also observed by De-Grasse et al. (1959); Ohm (1961) at different radio frequencies and today is known as the CMB – radiation, which is considered to be a relic of the epoch of recombination.

The radiation forming the CMB originates from the era of recombination. This era occurred when the universe cooled sufficiently to allow for the recombination of electrons and protons to form hydrogen. This formation of neutral atoms caused the universe to become transparent as the thermal radiation could no longer be absorbed. The CMB exhibits small fluctuations, which have been studied by many experiments. Two of the most important experiments are the Wilkinson Microwave

Anisotropy Probe (WMAP), *citetwmap13*, which is a full sky survey examining the fluctuations at large scales, and the Planck CMB observatory, Planck Collaboration et al. (2013), which performs high resolution observations of these fluctuations.

CMB fluctuations represent small variations in the density of the universe. These fluctuations are influenced by the gravitational effects of baryonic matter and the interactions of the radiation with the baryonic particles. The fluctuations observed strongly suggest a greater gravitational influence than predicted from the amount of baryonic matter. Dark Matter would exert a gravitation influence without any additional interaction of the Dark Matter particles with the radiation. The CMB fluctuations thus form a strong indicator of the presence and amount of Dark Matter in the Universe.

N-body simulations of galaxies such as those done by Ostriker & Peebles (1973), show that flattened galaxies are unstable and quickly collapse to bar-like structures in the absence of large amounts of mass in the surrounding halo. In his paper on the mass of galaxies and the universe, Ostriker et al. (1974) states that the excess mass in the outer regions of galaxies can be most plausibly explained as a giant halo, composed of faint stars or other bodies. This suggestion is further discussed in section 1.1.2.

1.1.2 Red Herrings

There exists a great amount of evidence showing our understanding of the Universe is far from complete. An initial attempt to explain this enigma was the postulate that Dark Matter comprises objects known as Massive Compact Halo Objects or MACHOs. These MACHOs would consist of non-luminous or dark compact objects formed from ordinary baryonic matter, such as black holes, brown dwarves, large planets, or neutron stars. Such objects could potentially be detected through gravitational lensing effects and micro-lensing - if a MACHO passed in front of a star or other luminous object, the light from that object would be lensed around the MACHO, Bennett et al. (1996).

These objects are likely to be smaller than the resolving power of the telescope, however the effect is detectable through an increase in the luminosity of the object (micro-lensing). Micro-lensing creates a signature for the presence of the MACHO. Such events would be rare, but detectable, and attempts have been made, among others, by the MACHOs collaboration, 'Expérience pour la Recherche d'Objets Sombres' (EROS), the Optical Gravitational Lensing Survey (OGLE) as well as the successor to the MACHOs project, the SuperMACHO survey - see Alcock et al. (1998); Becker et al. (2005); Griest et al. (1991); Pratt et al. (1995); Tisserand et al. (2007).

Surveys searching for such evidence have however failed to reveal enough dark compact objects to resolve the Dark Matter problem. It is widely thought that MACHOs comprise only a small portion of the missing mass.

The absence of sufficient MACHOs to explain the excess matter in the universe leads to the possibility that our understanding of gravity is flawed, Clowe et al. (2006). Many attempts have been made to modify Newtonian dynamics in order to resolve these gravitational evidences for the presence of excess matter.

As our understanding of gravity on small and medium scales gives accurate predictions and results, theories of modified Newtonian dynamics (MOND and others) generally attempt to modify gravitational forces on large scales such as those in galaxies, clusters and other large scale structures. Any modification of our current understanding of gravity presents a difficult task - in no small part because most modifications result in the violation of the equivalence between gravitational and inertial mass. Although we know of no fundamental reason why this equivalence should exist, this relation is the cornerstone of Einstein's theory of general relativity, a concept on which vast portions of modern physics are built, Clowe et al. (2006).

In addition, evidence from colliding galaxy clusters counts strongly against a modified theory of gravity. The Bullet cluster, for example, is composed of two colliding galaxy clusters, one smaller than the other. If the extra mass seen in this galaxy results from a flawed understanding of gravity, gravitational lensing should still center on the visible masses.

However, since Dark Matter is thought to interact minimally with the luminous matter as well as itself, if the clusters contain Dark Matter, as the two clusters collide, the Dark Matter continues along its trajectory, falling towards the center of the collision, Clowe et al. (2006). The luminous matter in each cluster experiences a gravitational pull from the Dark Matter in its own cluster, as well as collisions with the matter in the other cluster, causing it to fall behind the Dark Matter. Thus if the clusters are dominated by Dark Matter with these properties, the mass will be concentrated away from the dominant visible component and this will be reflected by the gravitational lensing observed.

Observations of gravitational lensing indeed indicate that the mass within the colliding clusters is concentrated away from the luminous matter. There has been much discussion of the implications of the Bullet cluster but it deals a non-negligible blow to theories of modified gravity, which battle to explain these observations.

1.2 The Dark Universe

There exists a significant body of evidence indicating that our universe is composed of a large amount of matter, which emits no electromagnetic radiation, yet this excess matter does not seem to stem from MACHOs, nor does modified gravity appear to solve our dilemma. As described above, this evidence stems from the standard cosmology derived from observations. Information from observations of the Cosmic Microwave Background (CMB), the large scale distribution of galaxies as well as kinematics of galaxies and galaxy clusters, amongst others, tells us that the Universe is dominated by Dark Matter.

We work in a base model where the universe is considered to be a Friedmann–Lemaître–Robertson–Walker (FLRW) space-time - a space-time, which is isotropic, homogeneous and expanding. The FLRW metric is a solution to Einstein’s field equations and within this, we treat anisotropies or directional dependencies of the Cosmic Microwave Background (CMB) as small fluctuations about this metric, Bergström (2013). Within such a metric we find the universe to be flat with Dark Matter composing 26.4% of the universe, whilst the more ordinary luminous matter comprises only 4.87%. Dark Energy forms the remaining 68.73%. In other words Dark Matter composes 84.4% of the mass content of the universe.

The evidence thus far tells us that Dark Matter has existed since the formation of the universe (Alpher et al., 1953). Therefore, Dark Matter candidates must have a lifetime comparable to the age of the universe. In addition the matter is non-luminous - it emits no electromagnetic radiation and so the particles must be electrically neutral. In order to allow the particles to influence structure formation, they must be heavy, resulting in them moving more slowly. Thus, Dark Matter particles must be stable, massive, and neutral.

Possible candidates from the standard model include the Z^0 boson or the Higgs boson, H^0 . These two particles however, have a lifetime only a fraction of a second long, preventing them from influencing formation of structures. Another possibility is the neutron. The neutron is electrically neutral, yet decays rapidly when not bound to a nucleus. When neutrons are bound within nuclei, they become visible.

The proton is a stable baryon - the only known stable baryon. However, it is not electrically neutral and will thus be visible through the emission of electromagnetic radiation. A last possibility from the standard model are the neutrinos. However, the three known neutrinos are very light or massless and thus move rapidly. This means that they are unable to affect structure formation. This leads to the idea that perhaps Dark Matter lies beyond the standard model, Bergström (2013). Perhaps Dark Matter is of entirely different composition to baryonic matter - the everyday matter from which everything we have conceived and explored, thus far, is made.

Particles beyond the standard model, which have a long lifetime, are massive and are electrically neutral are collectively known as Weakly Interacting Massive Particles or WIMPs and are considered very probable candidate for explaining Dark Matter. WIMPs are thought to have been created moments after the Big Bang, undergoing continual annihilation and creation until the expansion of the universe caused the density of the Dark Matter to drop to a point where the particles are seldom close enough to annihilate one another. This annihilation is still ongoing but at a drastically reduced rate, resulting in a stable number density.

In addition, the abundance of Dark Matter observed requires that Dark Matter particles must have an annihilation cross section $\langle \sigma v \rangle_{\chi} \sim 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$ for it to have once been in thermal equilibrium. This cross section corresponds to that of particles near the weak scale, giving further support to the theory that WIMPs comprise Dark Matter, Spekkens et al. (2013).

1.3 Search for Dark Matter

The evidence thus far is convincing in it's indication that there must be some form of matter, completely different to the more standard baryonic matter. However, none of this evidence constitutes an actual detection of Dark Matter. In fact, we are in possession of very few clues as to the true nature of Dark Matter, which brings us to the question - what methods can we use to investigate Dark Matter and what information can be gleaned from such investigations? See Figure 1.3.

The best detection of Dark Matter would of course be the direct detection of a Dark Matter particle. This is however a very difficult feat to achieve as the expected interaction is that of a WIMP scattering off a nuclei elastically. This requires the detection of an event with an energy in the KeV range and is further complicated by the range of cross sections of interaction with normal matter and the range of feasible energies of Dark Matter particles. There are many direct detection experiments underway including PICASSO (Archambault et al., 2012), CDMS II (CDMS II Collaboration et al., 2010), CoGeNT (Aalseth et al., 2014), LUX (LUX Collaboration et al., 2013), XMASS (Abe et al., 2013), DAMA/LIBRA (Bernabei et al., 2010) and CRESST II (Angloher et al., 2012), amongst others. (Cushman et al., 2013)

We can also gain information on Dark Matter by taking an approach within which we detect Dark Matter indirectly. Indirect detection of Dark Matter can be further split into physical probes and inference probes. Inference probes form the main body of evidence we have used to show that Dark Matter does indeed exist. These probes infer the existence of Dark Matter. They include gravitational lensing,

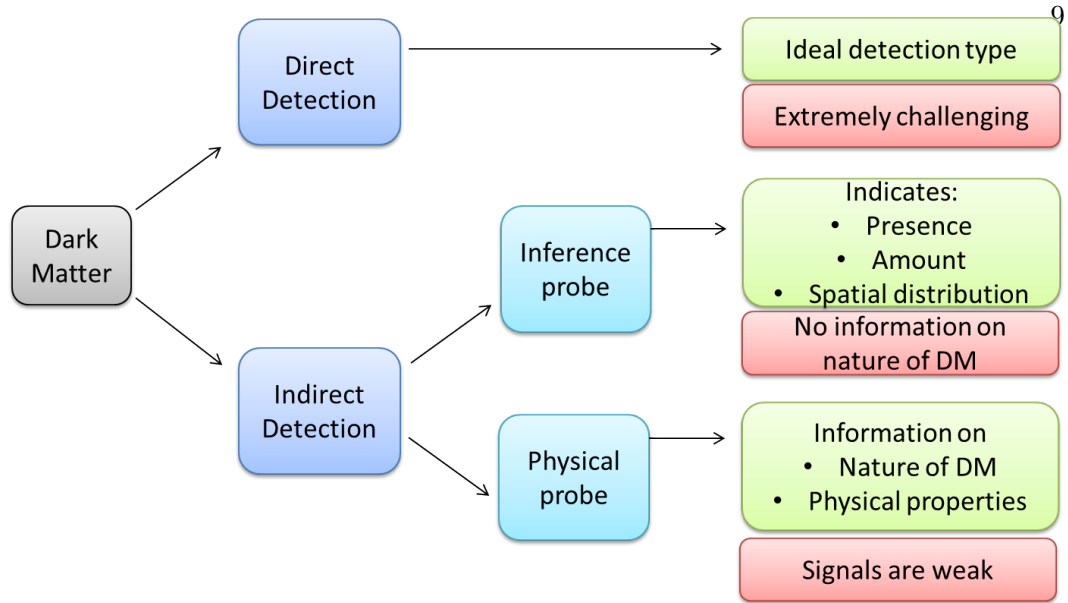


Figure 1.3: A diagram illustrating Dark Matter detection techniques and their advantages and disadvantages

rotation curves of galaxies and other probes mentioned in section 1.1. These probes provide information on the presence, amount and location of Dark Matter, however inference probes have a limitation. They are unable to provide information on the nature of the Dark Matter particles themselves.

Particle colliders can also be used as inference probes for detecting Dark Matter. During collisions of high energy particles, if Dark Matter is created, it's presence will not be seen by detectors but can be inferred from the existence of events with missing energy or momentum. These types of potential detections are best suited to low mass Dark Matter particles, Bauer et al. (2013).

Our research is focused on the remaining method of Dark Matter detection - the use of astrophysical techniques to form physical probes of Dark Matter. This is an indirect detection method, which relies on information provided by inference probes to search for information on the nature of Dark Matter particles themselves.

Indirect inference searches focus on detecting standard model particles created during WIMP annihilations or decays. These particles include electrons, positrons and neutrinos. Neutrinos resulting from neutralino annihilation can be detected directly, whilst electrons and positrons must be studied through the secondary emissions they produce.

In particular, we search for signals produced by the abovementioned leptonic particles during WIMP annihilations and decays. These particles can emit electromagnetic radiation through a number of processes and can be detected through various experiments. One such experiment is the FERMI-LAT collaboration, which has investigated the gamma-ray emissions produced by π^0 annihilation. (Bauer

et al., 2013; Colafrancesco, 2010; Spekkens et al., 2013)

To date, such γ -ray searches have yielded some of the tightest constraints on the velocity averaged annihilation rate of Dark Matter as a function of WIMP mass, Bauer et al. (2013); Spekkens et al. (2013). Our research focuses on signals produced by electrons and positrons formed in the same annihilation reaction.

1.4 Deep Search for Radio Signals from Dark Matter

The evidence for the existence of Dark Matter mentioned in section 1.1 predominantly takes the form of inference probes and whilst these tell us about the amounts and distribution of Dark Matter, it is difficult, if not impossible, to extract information about the nature of Dark Matter particles. The best evidence for Dark Matter would be a direct detection of a WIMP and many such experiments, as mentioned in section 1.3, are ongoing.

It would however be very interesting to use astronomical data to obtain more information on the nature and mass of the Dark Matter particles themselves. Our methodology involves both observational data and theoretical predictions, which we wish to combine in order to place restrictions on the mass of WIMP Dark Matter particles as a function of annihilation rate.

If we consider the annihilation of two WIMP Dark Matter particle candidates, we find that secondary leptonic products such as electrons are produced. These electrons can emit radiation through a variety of processes, revealing information about their distribution and energies and thereby allowing us to learn about the Dark Matter itself. We can use this radiation to search for signals of Dark Matter particle pair annihilations in cosmic structures.

Our first step is to consider briefly the products produced in the annihilation of neutralinos. We consider the electrons and positrons produced to be of primary interest and we briefly describe the types of emission, which can result from the interactions of WIMP annihilation products with their astrophysical environment. The three processes are synchrotron radiation, inverse Compton scattering and thermal bremsstrahlung. These processes cause electrons to emit radiation. Synchrotron radiation emits a flux in the radio frequency range. We discuss these emission mechanisms briefly in section 2.2 and we explore synchrotron radiation in greater detail in section 4.1.

Gamma ray emissions are produced directly in the neutralino annihilation. Secondary emissions result from particles produced in the annihilation. These secondary emissions resulting from WIMP Dark Matter annihilations are expected to be both

weak and diffuse. We consider various targets for observation in order to detect radio emissions and we discuss the merits of various candidates in section 2.3. We then go on to describe our chosen candidate type - dwarf spheroidal galaxies (dSph).

It is possible to obtain a theoretical electron energy density distribution of the electrons produced by the annihilation of two WIMPs, and along with properties of these dwarf spheroidal galaxies, to create a theoretical prediction of the flux predicted to be coming from the electrons as they emit radiation through the aforementioned radiative processes. We outline the theoretical framework in chapter 3. We then go on to describe specific assumptions, which can be made for dSph. In addition we calculate these expected emissions as a function of the WIMP mass for various configurations of the Dark Matter distribution and describe these predictions in chapter 4. Considering deep radio observations of dSph allows us to probe the emissions resulting from WIMP annihilations and this information can be used to place constraints on the WIMPs themselves.

In order to be able to test our theoretical predictions for the diffuse emission of WIMP Dark Matter particles, we have taken deep radio observation of six dwarf spheroidal galaxies. Our observations were designed to be able to detect a diffuse signal (large beam) whilst having a sensitivity great enough to identify point sources without reaching the confusion limit.

Our observations achieve this by obtaining a deep survey capable of mapping background sources precisely. The high resolution maps were obtained by using an Australian Compact Telescope Array (ATCA) with a core of 5 dishes and a sixth dish with a long baseline. These high resolution maps were important for obtaining astrometric data. Lower resolution maps were obtained by dropping the data from the long baseline and using the emission measured in the compact core of the array. The lower resolution maps are used for obtaining the total flux density of the sources. The full technical details of these observations of our chosen targets with the ATCA array are described in chapter 5.

The predicted emissions of interest from Dark Matter annihilation are weak. The observations have thus been taken over long time frames. Sensitivity of the instrument was maximised. Great care was taken in the data reduction process to remove and minimise unwanted radio emission. This was achieved by cleaning the obtained spectra through the removal of point sources and other background noise to produce images that predominantly consist of Dark Matter signals. The aim of this is to place as strong a limit on the annihilation process as possible. We elaborate on the details of this data reduction and the analysis processes for each dSph in chapter 5.

The main goal of our research was to search for diffuse emissions from dSph

galaxies. This diffuse emission is critical for placing bounds on the properties of Dark Matter using indirect detection techniques, and is useful for obtaining information about other cosmological and astrophysical questions not covered in this thesis. In this regard, dSph are key probes for Dark Matter, galaxy evolution and formation on small scales, as well as for use as probes for near field cosmology.

In chapter 5 we detail the analysis of point-like radio sources in the six dSph galaxies observed. We covered this issue in great detail and it formed the largest portion of the work, since the contamination of maps by contributions from background sources is a key issue in identifying diffuse radio emissions.

From this data reduction process we acquire details of the non-thermal diffuse emissions in the dSphs and elaborate on the WIMP Dark Matter constraints obtained in section 5.4. We then conclude with some perspectives for further research and the potential for placing more stringent constraints on WIMP Dark Matter using telescopes such as MeerKAT and the SKA in chapter 6.

Chapter 2

WIMP Annihilation and Dwarf Spheroidal Galaxies

2.1 WIMP Annihilation

WIMPs are the leading Dark Matter candidate being electrically neutral, heavy particles with a lifetime comparable to the age of the universe. WIMP Dark Matter particles can undergo pair annihilation and inverse pair production. These two mechanisms once controlled the thermal equilibrium of WIMPs and still occur occasionally in the current cold universe.

Particle pair annihilations of WIMPs create stable, energetic, baryonic particles, at a rate which depends on the pair annihilation rate $\langle \sigma v \rangle$, as well as the number density of the Dark Matter. The microscopic details of a WIMP model determine the final state products, Profumo & Ullio (2010). A schematic representation of such final state products is shown in Figure 2.1.

The lightest neutralino, χ of the minimal super-symmetric extension of the Standard Model (MSSM) is a leading WIMP candidate. Neutralinos annihilate to form quarks, leptons, vector bosons, and Higgs bosons, depending on their mass and physical composition.

Figure 2.1 shows that in the annihilation of two χ neutralinos, pions are formed. These can be either neutral or charged pions. Neutral pions produced in the neutralino annihilation decay further to produce gamma rays. The detection of such γ -rays is one way to obtain information about the neutralinos themselves.

The charged pions produced in the χ neutralino annihilation decay to produce both electrons and positrons. These electrons and positrons can interact in a number of ways to produce radiation which can then be studied in order to shed light on the nature of the decaying neutralinos. Specifically, the electrons and positrons can interact with charged particles to produce thermal bremsstrahlung, producing

radiation at γ and x-ray wavelengths.

The secondary electrons and positrons produced by neutralino annihilation also interact with photons to produce inverse Compton scattering or specifically interact with radiation from the CMB to produce the Sunaev-Zeldovich effect. This results in distortions of the Cosmic Microwave background.

Finally, an interaction can occur between an existing magnetic field in the area of the neutralino annihilation and the secondary electrons and positrons produced. The radiation produced in this interaction is known as synchrotron radiation and produced radio waves. We focus on the electrons and positrons produced during annihilation and focus predominantly on synchrotron radiation in order to probe the nature of Dark Matter.

We begin our search for Dark Matter by discussing the various types of radiation which can be emitted by the electrons and positrons produced in neutralino annihilation. The three dominant mechanisms are thermal bremsstrahlung, inverse Compton scattering and synchrotron radiation.

A basic understanding of the mechanisms involved will allow us to place constraints on the best astrophysical structures to study towards achieving an indirect detection of Dark Matter. We will discuss various possible structure types. We then go on to describe in detail the properties of our chosen observational target.

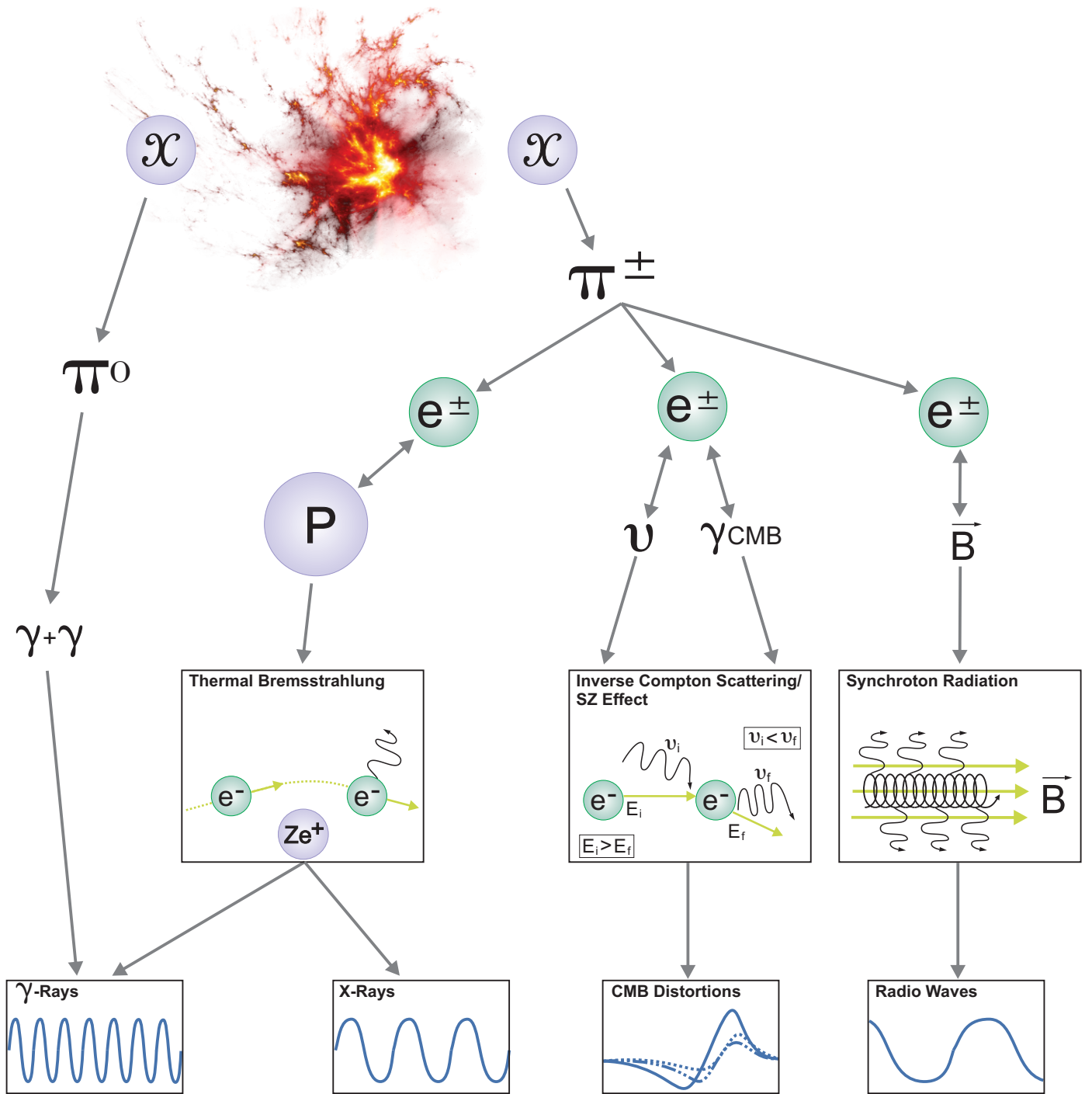


Figure 2.1: A schematic representation of Neutralino Annihilation showing the types of radiation which can result

2.2 Radiative Mechanisms

2.2.1 Thermal Bremsstrahlung

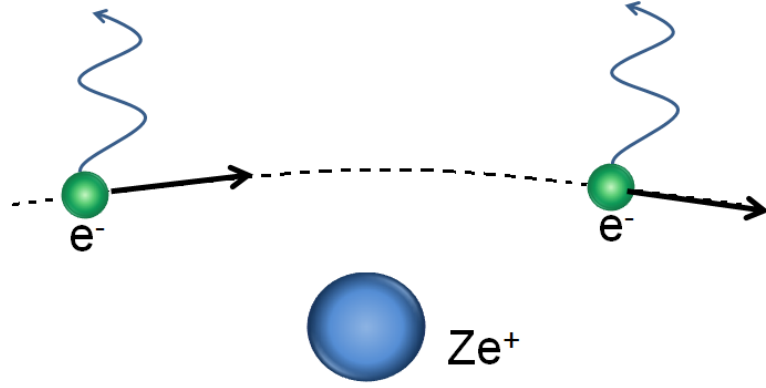


Figure 2.2: A Schematic Representation of Thermal Bremsstrahlung

Thermal bremsstrahlung occurs when a charged particle is deflected as it passes another charged particle. The deflection usually occurs when a lighter particle such as an electron or positron moves past a heavier charged particle such as an ion. The deflection causes a change in the energy of the particle, resulting in the release of a photon. See Figure 2.2.

If our neutralinos undergo annihilation within a cloud of charged particles, such radiation will result. Thermal bremsstrahlung will also result if the secondary electrons and positrons move through a cloud of ions after the annihilation occurs. The resulting radiation is typically at X-ray frequencies, thus as we are considering radio emissions, we will not discuss this process in detail.

2.2.2 Inverse Compton Scattering

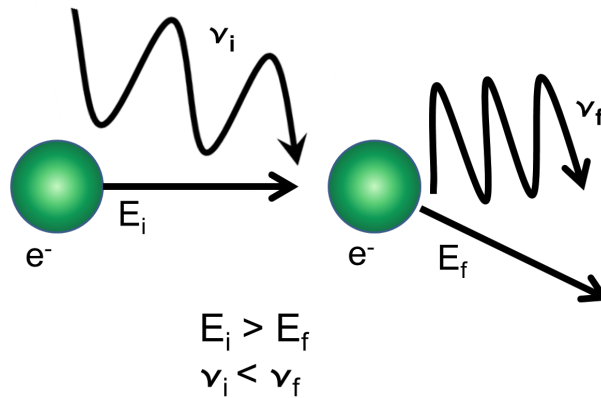


Figure 2.3: A Schematic Representation of Inverse Compton Scattering

Inverse Compton scattering occurs when an electron transfers energy to a photon. Ultra-relativistic electrons transfer their kinetic energy through inverse Compton scattering by scattering low energy photons to higher energies. This results in a shift in the frequency of the photons. The effect can also be seen when photons from the CMB pass through electron clouds and this specific phenomenon is known as the Sunyaev-Zel'dovich effect. See Figure 2.3.

The inverse Compton scattering effect can thus be used for indirect detections of Dark matter as the electrons and positrons produced can scatter photons and thus produce detectable radiation. The effect however occurs at frequencies which are of minimal interest to this work so we will not discuss inverse Compton scattering in detail.

2.2.3 Synchrotron Radiation

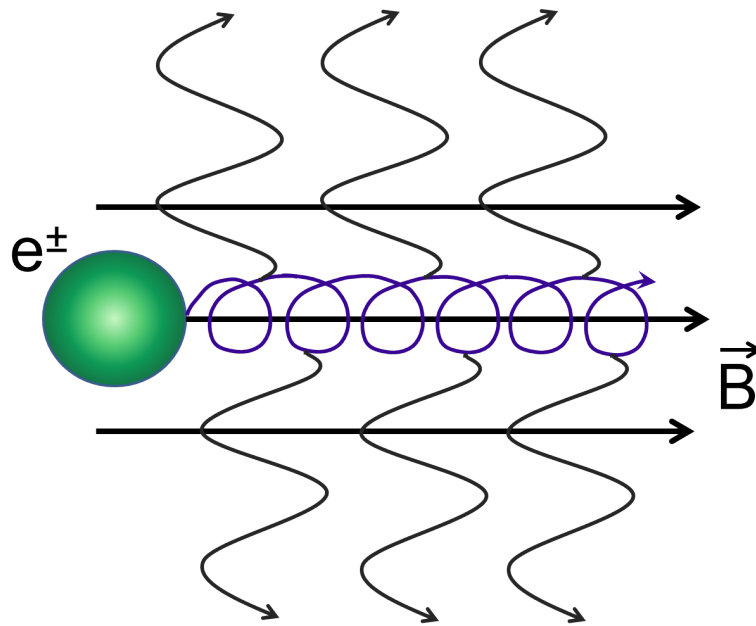


Figure 2.4: A Schematic Representation of Synchrotron Radiation

Synchrotron radiation is a process whereby charged particles are accelerated radially through a magnetic field. This acceleration results in the release of radiation. In general, the radiation released by synchrotron processes occurs at radio frequencies. Synchrotron radiation is the main process of interest in detecting emissions using radio is the main process we will be considering in the indirect detection of Dark Matter and so we discuss this effect in detail in section 4.1. See Figure 2.4.

2.3 Astronomical Targets for Detection of WIMP Annihilation

We have now discussed the general radiative mechanisms relevant to electrons and positrons produced during neutralino annihilation. In order to study WIMP annihilation signals resulting from these annihilation processes, we must consider potential astrophysical systems from which we can obtain information on the diffuse emissions produced by secondary electrons and positrons.

The emission signals expected to result from the interactions of these secondary electrons and positrons are small. We must thus study structures, which are both large and dominated by Dark Matter. We require a target system situated at a distance such that the expected flux can be detected within the limits of current instrumentation. Our final constraint results from the predicted spectrum of emissions from secondary electrons and positrons produced during neutralino annihilation. It is important to remember that the dominant emission frequencies from electrons and positrons are x-ray, γ -ray and radio frequencies. The central region of our target system must therefore be devoid of sources of diffuse radiation at radio, x-ray or γ -ray frequencies.

One may first consider the galactic center or central regions of nearby galaxies as the most likely place for us to detect Dark Matter annihilation signals. This suggestion is based on the proximity of these structures leading to high signal strengths. In addition, such regions are thought to have high densities of Dark Matter, Colafrancesco et al. (2006). See Figure 2.5.

The galactic center and the central regions of other galaxies are however very complex regions to study. These regions contain many sources of radiation, such as pulsars or supernovae remnants, which would obscure portions of the spectrum expected to result from the WIMP annihilations. In addition to this, these sources of radiation are not all fully understood. It is thus difficult to pinpoint and extract individual contributions to the spectrum. In particular, it is challenging to extract the contribution from Dark Matter.

Galaxy clusters are dominated by Dark Matter. This prevalence of Dark Matter has been revealed for many galaxy clusters through gravitational lensing, which indicates a much larger mass than the mass indicated by the light of the cluster alone. A large Dark Matter component for galaxy clusters is also revealed through studies of the radial velocities of component galaxies, as well as through comparison to the X-ray data, which tells us about the amount of baryonic matter within galaxy clusters. Galaxy clusters set interesting constraints on the properties of Dark Matter through their emission features, both thermal and non-thermal, which can



Figure 2.5: A wide angle view of the galactic center
Ivan Eder
(<http://apod.nasa.gov/apod/ap120106.html>)



Figure 2.6: The Coma cluster of galaxies
Jim Misti
(<http://apod.nasa.gov/apod/ap060321.html>)

be extended spatially and spectrally. See Figure 2.6.

The number of galaxy clusters that can be probed is limited by the sensitivity and resolution of available gamma-ray, x-ray and radio frequency instruments. In addition, galaxy clusters are characterised by the presence of intergalactic gas in the intra-cluster medium (ICM). These intergalactic gases produce emission as a result of thermal bremsstrahlung. This radiation is seen at x-ray frequencies and may obscure features of emission from secondary particles produced in neutralino annihilation.

Globular clusters are large and nearby. There are many of these structures close enough for us to study and thus to collect high quality data from. In addition, globular clusters are fairly simple structures, composed of a spherical collection of stars. There is little to no gas or dust and the stellar population is old. Globular clusters are thus relatively straightforward to analyse, however the mass to light ratio of globular clusters is low. These structures are dominated by baryonic matter and the low Dark Matter content causes the predicted signals to lie below the sensitivity threshold of current and future planned experiments. See Figure 2.7.

DSph are very similar in composition to globular clusters. DSph are approximately spherical and comprise an old stellar population with mainly cold gas, little to no dust and no cosmic ray population. In addition to this most dSph are dynamically stable. They are one of the most common types of galaxy in the universe and are the dominant galaxy type within the Local Group. Several such dwarf galaxies are found near the Milky way and the Andromeda galaxy. See Figure 2.8.

DSph and globular clusters are in fact so similar that it has been proposed that they are in fact not distinct astrophysical objects. There is however one very important distinguishing characteristic between these two types of structures, dSph are dominated by the presence of large amounts of Dark Matter. In fact, in the local Universe, they are the structures most dominated by Dark Matter. These characteristics make dSph a very important class of object for the study of Dark Matter and an ideal candidate for our research.



Figure 2.7: Omega Centauri - a globular cluster
CEDIC team
(<http://apod.nasa.gov/apod/ap140529.html>)



Figure 2.8: Leo I - a dwarf spheroidal galaxy
David Malin
(<http://apod.nasa.gov/apod/ap991003.html>)

2.4 Dwarf Spheroidal Galaxies

DSph are one of the most common types of galaxies in the universe, and are the dominant galaxy type within the Local Group as well as within the total galaxy population. DSph are among our closest neighbours.

2.4.1 Properties of Dwarf Spheroidal Galaxies

DSph are one of the simplest known galactic systems and are mainly composed of stars, (Mateo (1998)). The stellar population of dSph is usually composed of a central core with a higher concentration of stars, slowly fading into the surrounding sky with no well defined outer boundary, van den Bergh (2000).

DSph have extremely low surface brightnesses and are among the smallest, least luminous galaxies known. The dominant form of emission from dSph is in the form of optical radiation in the form of starlight. There exists very little strong radio or infrared radiation, nor do we observe strong emission lines from dSph. (Impey & Bothun, 1997; Murdin, 2001; van den Bergh, 2000)

The above observations suggest dSph contain a very small portion of interstellar gas or dust. In addition, few star forming regions or young stars are ever seen within dSph. The stellar populations of dSph comprise predominantly old- and intermediate-age stars. The presence of significant numbers of intermediate age stars suggests that whilst simple systems in their current state, dSph have undergone complex star formation histories, which are as yet poorly understood.

DSph galaxies have been well studied and are found to have very low luminosities. Many dSph have been found to be so spatially extended that based on their luminous mass alone it is hard to understand how they could be gravitationally bound. In addition, the stars comprising dSph appeared to be moving too fast relative to one another for them to remain gravitationally bound within a galaxy of such small size.

DSph are found to have a stellar population with a spread of about $100pc$. The velocity distribution of dSph is observed to be $> 5km/s$. These observations allow us to infer a dynamical mass of the order of $10^7 M_{\odot}$. We can then determine the mass to light ratio, which is found to be of the order of $(10^3 - 10^4) M_{\odot}/L_{\odot}$.

These observations indicate that only a small portion of the mass of the dwarf spheroidals comprises visible stars. In fact, dSph contain a large proportion of Dark Matter and probably contain the most Dark Matter amongst single dark galaxies. This information alone indicates that dSph may be one of the best possible obser-

vational targets for studying the nature of Dark Matter.

2.4.2 Astrophysical Significance of Dwarf Spheroidal Galaxies

DSph form a very important source for studies of small scale evolution, galaxy formation as well as other types of near-field cosmology. Our understanding of galaxy formation is challenged by the kinematics, structure and chemical composition of dSph, allowing the investigation of structure formation models at early times as well as on small scales, Mateo (1998). As low luminosity structures, dSph allow us to study the low luminosity threshold of galaxy formation.

Star formation processes within dSph require that we obtain a deeper understanding of processes such as star formation feedback, the star formation efficiency of small Dark Matter Halos as well as processes of chemical enrichment, which cause us to challenge and test the standard cold Dark Matter model.

The importance of these structures has resulted in very deep photometric data on the stellar populations of dSph. We however lack information about the population of plasma (both thermal and non-thermal), which may reside in these structures since there is minimal information about the diffuse emission of dSph at any observing frequency. The prevalence and proximity of dSph means that there are a number of dSph within the range of observable flux.

We know that dSph are extremely rich in Dark Matter. DSph have low level astrophysical backgrounds, as they lack significant sources of radio, X-ray or γ -ray emission, which would obscure diffuse emissions resulting from neutralino annihilation. In addition to these attributes, there are significant numbers of dSph found to be close enough to be accessible for indirect detection studies as their proximity allows for a diffuse flux from secondary emissions detectable within the limitations of current instrumentation. These factors show dSph form a critical probe for both inference and physical studies of Dark Matter.

Our catalog of dSph is still far from complete. Data from the Sloan Digital Sky Survey (SDSS) has allowed the discovery of many more dSph, almost doubling the number of known dSph as well as revealing the existence of ultra-faint galaxies, McConnachie (2012); McConnachie et al. (2008)

These ultra-faint dSph are significantly less luminous than any galaxy previously known, less luminous than even some globular clusters. Spectroscopic data reveals that the ultra-faint dSph are extremely low in metal content, McConnachie (2012); McConnachie et al. (2008). Analysis of the SDSS data in terms of survey completeness reveals that we may only know about a small number of dSph, with a

few hundred ultra-faint dSph still undiscovered at greater distances and in different regions of the sky not reached by the SDSS.

In order to begin developing an understanding of the star formation and evolution processes of dSph, existing photometric data about the stellar populations requires supplementation in the form of a search for point-like radio emissions within dSph. In addition, the understanding and knowledge of background sources is critical in obtaining and identifying observations of a diffuse emission from dSph.

The diffuse emissions of dSph are important to us since assessing the amount of plasma, both thermal and non-thermal, residing within dSph requires the study of their diffuse emissions. Such studies will also allow us to obtain more information on the presence and details of large scale magnetic fields. Most important to us however is that these diffuse emissions include the signals from Dark Matter annihilation so in order to explore signals from secondary annihilation products, we need to obtain a diffuse emission which is as clean as possible.

Towards filling this deficit of information about dSph as well as using these systems as an indirect probe of Dark Matter, we will present in chapter 5, observations of a sample of six dSph galaxies as well as our analysis thereof.

2.4.3 Cold Dark Matter Paradigm

DSph are important in probing the cold Dark Matter paradigm. The cold Dark Matter paradigm postulates a small velocity dispersion for Dark Matter particles in the universe. One important consequence of this postulate is the prediction of significant numbers of structures with sub-galactic sizes. Whilst this model is very successful in predicting and explaining large scale observations, there exist a number of difficulties in explaining small scale phenomena.

In particular, these difficulties include descriptions of Local Group dSph as well as the inner regions of galactic Dark matter halos. In order to resolve these issues, more information is needed regarding dSph, which are satellites of the Milky Way galaxy and are particularly important objects to study.

The density profiles of dSph are one such probe of the cold Dark Matter paradigm. The depth of the central density profile of dSph is thought to be much less than predicted by the cold Dark Matter paradigm, Walker (2013). In addition, N-body simulations of cold Dark matter predict the formation of a large number of subhalos in the formation of a halo such as the Milky Way galaxy, Regis et al. (2014b). These simulations thus predict far greater numbers of dSph galaxies than currently observed.

The discovery of a number of ultra-faint dSph discussed in section 2.4.2 begins to address this deficit. In addition, the completeness limits of the SDSS survey show that new ultra-faint dSph should be found, which will further address this shortfall of observed dSph. The thousands of dSph predicted are however, still not seen, with less than 100 dSph currently known.

Studies of dSph allow us to probe whether it may be necessary to depart from the collisionless cold Dark Matter scheme. Observations that reveal Dark Matter induced effects during structure formation or changes to the primordial power spectrum may result in smaller numbers of small-scale structures, Weinberg et al. (2013). This evidence could favour a departure from the cold Dark Matter paradigm. Baryonic physics effects such as low star formation efficiency or supernova feedbacks would be evidence in support of the cold Dark Matter scheme.

In dSph, a low gas content has been observed. This shortfall lies below what is required for star formation and could explain the inefficiency thereof. Low quantities of gas in dSph have several possible causes. Firstly, gas collection may be prevented when intergalactic gas is heated by the ionising UV background. Alternately, early feedback effects may remove gas from the shallow gravitational potential of a dSph. This removal of gas may also occur as a result of tidal streams of gas as the dSph orbits the parent halo. The gas within a dSph thus provides a large body of information and measurements thereof would be important in exploring solutions to the controversies of the cold Dark Matter paradigm. (Bullock et al., 2000; Mayer et al., 2001; Regis et al., 2014b; Weinberg et al., 2013)

Chapter 3

Theoretical Framework

We have now discussed the types of radiation, which can result from electrons and positrons produced during Dark Matter annihilations. We have also discussed which structures we can viably detect this predicted emission from. We have decided to consider dSph as our targets for observations and we discuss in detail our observations and the analysis thereof in chapter 5. We will now lay the theoretical framework required to understand the emission process in full. In addition we will discuss factors such as diffusion and their effect on the detection of Dark Matter annihilation products. We will use this framework to help us understand what we are searching for within our observations and to use the observational data to place constraints on the Dark Matter annihilation rate as a function of the neutralino mass.

3.1 Neutralino Annihilation

We will work in a Λ CDM model of structure formation. Although we have briefly mentioned that electrons and protons are produced during the annihilation of neutralinos, we must first discuss this process in depth in order to understand the signals we are trying to detect.

Neutralinos are Majorana fermions. This means that the dominant final states of neutralino annihilation comprise either heavy fermions or gauge and Higgs bosons with the light fermion states suppressed. The actual final states depend on the mass and composition of the WIMP itself.

We consider the minimal supergravity model by Ellis et al. (1984, 1983); Goldberg (1983). This model is a thoroughly defined SUSY model, which is consistent with constraints from accelerators and other phenomenological constraints. This minimal supergravity model gives a thermal relic abundance of neutralinos exactly matching the central cosmologically observed value. Use of this model enables cross comparison of our results with those of many studies such as direct Dark Matter searches and colliders. (Baer et al., 2003; Edsjö et al., 2004)

There are a handful of regions within the minimal supergravity model which have effective $\Omega_\chi h^2$ suppression mechanisms to produce sufficiently low thermal neutralino relic abundance. These mechanisms include co-annihilation of the neutralino with the next lightest SUSY particle, the occurrence of light neutralino and s fermion masses, rapid annihilations through s channel Higgs exchanges and non-negligible bino-higgsino mixing.

We consider the benchmark cases for super-symmetry suggested by Battaglia et al. (2004). These set-ups are tuned to feature a neutralino thermal relic density, which gives precisely the central WMAP estimated cold Dark Matter density. The SUSY particle spectrum is homogeneous throughout the minimal supergravity parameter space, yet the spectra resulting from the different benchmark models show $\simeq 3$ qualitatively distinct shapes based on the dominant final state resulting from the neutralino annihilations.

We are most concerned with the secondary electrons and pions produced in the neutralino annihilation. There are relatively few spectral patterns from the final state products of neutralino pair annihilations and the most relevant physical properties are the final state products and the mass of the neutralino itself. The electrons produced are subject to energy losses as well as spatial diffusion, which both contribute to the evolution of the source spectrum to an equilibrium spectrum. Protons, which are also produced in small quantities during the annihilation and can provide another source of secondary electrons. Pions produced are affected by rapid decay.

Most of the continuum spectrum at energies $E \gtrsim 1\text{GeV}$ come from the γ -rays produced by the decay of neutral pions as in Equation 3.1, Colafrancesco & Mele (2001); Colafrancesco et al. (2006). This radiation does not undergo diffusion and is directly radiated since this decay occurs on a very short time scale. There is also γ -ray emission as a result of inverse Compton scattering (ICS) and bremsstrahlung emission of the secondary electrons. This γ -ray emission is studied by the Fermi-LAT collaboration and we will make comparisons to these results in section 6.2.

$$\pi^0 \rightarrow \gamma\gamma \tag{3.1}$$

We consider three main sources of $e^\pm$ from $\chi\chi$ annihilation. Electrons are produced by prompt generation mechanisms, resulting from direct decays of vector bosons as in Equation 3.2, producing electrons with a continuum spectrum of typical energy $M_\chi/2$. A second generation of electrons is produced from leptonic decays in the final states of the initial prompt decays. These electrons have a continuum spectrum $E \lesssim M_\chi/2$ and are produced as secondary decay products as in Equation 3.3.

In addition, both π^0 and $\pi^\pm$ pions result from neutralino annihilations. Secondary electrons are produced from the decay of $\pi^\pm$, as in Equation 3.4. These secondary electrons produce radiation through bremsstrahlung with charge particles within the halo, synchrotron radiation due to interactions with a magnetised atmosphere as well as ICS of CMB and other background photons. These sources of radiation, together with the gamma radiation discussed above have a large impact on the multi-frequency spectral energy distribution (SED) of Dark Matter halos.

$$\begin{aligned} b &\rightarrow e \\ c &\rightarrow e \\ \tau^\pm &\rightarrow e^\pm \end{aligned} \tag{3.2}$$

$$\begin{aligned} W^\pm &\rightarrow \tau^\pm \rightarrow e^\pm \\ W^\pm &\rightarrow c \rightarrow e^\pm \end{aligned} \tag{3.3}$$

$$\begin{aligned} \pi^\pm &\rightarrow \mu^\pm \nu_\mu (\bar{\nu}_\mu) \\ \mu^\pm &\rightarrow e^\pm + \bar{\nu}_\mu (\nu_\mu) + \nu_e (\bar{\nu}_e) \end{aligned} \tag{3.4}$$

3.1.1 Source Function

The annihilation of two neutralinos will result in particles produced by the annihilation itself as well as particles produced during the decay and fragmentation of these primary products. For each stable particle i produced promptly in these processes we can define a source function $Q_i(r, E)$ to give the number of particles produced per unit time, per unit energy, and per unit volume.

We define $\langle \sigma \nu \rangle_{0,\chi}$ to be the annihilation rate at $T = 0$, for a particle of mass M_χ , which is set by the particle physics framework. T defines the time and $T = 0$ is the moment of annihilation. Each kinematically allowed final state, f , of the annihilation will have a branching ratio, B_f and a spectral distribution dN_i^f/dE . The branching ratios are set by the particle physics framework. The spectral distribution can be found using MonteCarlo codes such as those found in Gondolo et al. (2004); Sjostrand (1995). We define the number density of neutralino pairs at a given radius to be $\mathcal{N}_{pairs}(r)$. The number density of neutralino pairs at a given radius requires understanding of the Dark Matter halo as well as a discussion of subhalos. We will thus return to this quantity in section 3.36.

We can use the above defined quantities to determine $Q_i(r, E)$ as given in Equation 3.5. The branching ratios of the products produced in the neutralino annihila-

tion process are set by the particle physics framework.

$$Q_i(r, E) = \langle \sigma \nu \rangle_{0,\chi} \sum_f \frac{dN_i^f}{dE}(E) B_f \mathcal{N}_{pairs}(r) \quad (3.5)$$

The secondary products of the neutralino annihilation have spectral properties determined by the mass of the Dark Matter particle as well as the branching ratio. These spectral properties can then be further influenced by diffusion and energy losses, which will be discussed in section 3.1.5. The neutralino has a wide range of possible masses, ranging from GeV to TeV scales, as well as viable cross sections within the most general super-symmetric Dark Matter set-ups, Bottino et al. (2003); Profumo (2005).

An upper limit on the neutralino mass is given by Profumo (2005) in Equation 3.6, and is based on purely theoretical grounds. Co-annihilation process do not allow us to set a lower bound on the viable mass of the neutralinos. The relation given in Equation 3.7 attempts to tie the relic WIMP abundance to the pair annihilation cross section, however this relation is violated within both minimal setups and MSSM.

$$\langle \sigma \nu \rangle_0 \lesssim \left(\frac{M_\chi}{TeV} \right)^2 10^{-22} cm^3/s \quad (3.6)$$

$$\Omega_\chi h^2 \approx \frac{3 \times 10^{-27} cm^3/s}{\langle \sigma \nu \rangle_0} \quad (3.7)$$

3.1.2 Branching Ratios

The largest branching ratios of the annihilation into fermion-antifermion pairs are often into the third generation final states $b\bar{b}$, $t\bar{t}$ and $\tau^+\tau^-$, since the neutralino is a Majorana fermion, and since the Yukawa coupling of third generation quarks is much larger than the coupling of first and second generation quarks. If these fermions have super-symmetric partners similar in mass, the $\tau^+\tau^-$ branching ratio will be suppressed with respect to the $b\bar{b}$ branching ratio. These fragmentation functions are very similar and we will refer to this as the soft spectrum. Alternately we can have the formation of massive gauge bosons, W^+W^- and Z_0Z_0 , as a result of the neutralino annihilation. The fragmentation functions for these two states are almost indistinguishable and we refer to this as the hard spectrum. The case where there is a non-negligible fraction branching into $\tau^+\tau^-$ gives rise to an intermediate case between the hard and soft spectra.

3.1.3 Structure of the Dark Matter Halo

The Dark Matter Halo Profile

We consider the Λ CDM model of structure formation and study a spherically symmetric distribution of Dark Matter. This can be represented as in Equation 3.8 where a is a characteristic length scale, r is distance from the center of the structure and ρ' is a normalisation parameter. The function $g(x)$ describes the Dark Matter distribution and we can consider several possibilities for it's form.

$$\rho(r) = \rho' g\left(\frac{r}{a}\right) \quad (3.8)$$

We must either assume that the Dark Matter profile remains unaltered from the moments prior to baryon collapse or we must try to account for the effect of baryon collapse on the Dark Matter. We will consider two cases from the former as well as one extreme case from the latter in order to establish limits of the Dark Matter halo profile.

One extreme picture in which the baryon collapse has an effect on the Dark Matter profile consists of a large transfer of angular momentum within the structure, between luminous baryonic and non-luminous Dark Matter, El-Zant et al. (2001). This could result in a clumping of the baryons into dense gas clouds, followed by them sinking into the central portion of the Dark Matter halo. This modifies the profile of the Dark Matter significantly at the inner region. We label this profile BUR, see Equation 3.9, Burkert (1995).

If we assume that the baryon collapse has little or no impact on the Dark Matter profile, we can use the results found in N-body simulations of hierarchical clustering. This gives us a universal shape for the Dark Matter profile. This assumption matches the picture provided by present-day cluster morphology simulations. In this limit we consider two possibilities - the form which is non-singular, labeled as EIN, (Einasto, 1965), (see Equation 3.10) and a form containing a mild singularity towards it's center, labeled as NFW, (Navarro et al. (1996)) (see Equation 3.11). Each of the Dark Matter halo profiles discussed (NFW, EIN, BUR) are derived from N-body simulations. (Colafrancesco et al., 2006; Diemand et al., 2005a,b)

$$g_{BUR}(x) = \frac{1}{(1+x)(1+x^2)} \quad (3.9)$$

$$g_{EIN}(x) = \exp\left(\frac{-2}{\alpha(x^\alpha - 1)}\right) \quad \alpha \simeq 0.15 \quad (3.10)$$

$$g_{NFW}(x) = \frac{1}{x(1+x)^2} \quad (3.11)$$

We introduce also the virial mass, M_{vir} and concentration parameter, C_{vir} . We define the virial radius, R_{vir} to be the radius such that for a halo with mass M_{vir} ,

the mean density of the halo is equal to the mean background density, $\bar{\rho} = \Omega_m \rho_c$, multiplied by the virial over-density Δ_{vir} , as in Equation 3.12. We assume a flat cosmology, allowing us to approximate the virial over-density by the expression in Equation 3.13, where z is the redshift. (Bryan & Norman, 1998; Bullock et al., 2001; Colafrancesco et al., 1994, 1997)

$$M_{vir} = \frac{4}{3}\pi\Delta_{vir}\bar{\rho}R_{vir}^3 \quad (3.12)$$

$$\Delta_{vir} = \frac{18\pi^2 + 82(\Omega_m(z) - 1) - 39(\Omega_m(z) - 1)^2}{2 - \Omega_m(z)} \quad (3.13)$$

Further, we define r_{-2} to be the radius at which the Dark Matter profile has an effective logarithmic slope of -2. The concentration parameter is then given by Equation 3.14. The values of x_{-2} for the Dark Matter profiles we consider are listed in Table 3.1.

$$\begin{aligned} c_{vir} &= \frac{R_{vir}}{r_{-2}} \\ &= \frac{R_{vir}}{x_{-2}a} \end{aligned} \quad (3.14)$$

Profile	x_{-2}
$g_{BUR}(x)$	$\simeq 1.52$
$g_{EIN}(x)$	1
$g_{NFW}(x)$	1

Table 3.1: Calculated Values of x_{-2} for BUR, EIN and NFW Dark Matter Halo Profiles

Navarro et al. (1997) showed that larger concentrations occur in lighter halos and thus there exists a strong correlation between the virial mass and concentration parameter. This correlation is important in discussing mean density and will be referred to again when discussing substructures in the Dark Matter halo.

We consider the model from Bullock et al. (2001) describing this correlation. A halo of mass M at epoch z is assigned a collapse redshift, z_c when it has mass M_c . This is defined by the relation in Equation 3.15. This typical mass of collapse corresponds to a fixed fraction F of M where $F=0.015$. (Bullock et al., 2001; Navarro et al., 1997)

$$M_c(z_c) \equiv FM \text{ with } F = 0.015 \quad (3.15)$$

The collapsing mass is defined by Equation 3.16. $\sigma(M)$ is the present day root mean squared density fluctuation for a sphere of mass M . $\delta_{sc}(z)$ is the critical over-

density required for collapse in a spherical model. The critical overdensity, δ_{sc} is given in Eke et al. (1996).

$$\sigma(M_c(z)) = \delta_{sc}(z) \quad (3.16)$$

The fluctuation of the power spectrum can be related to the power spectrum fluctuation, $P(k)$, through the relationship defined in Equation 3.17. In this relation, $\bar{W}$ is the top-hat window function on the scale given in Equation 3.18. $\bar{\rho}$ is the mean matter density and relates to the critical density by Equation 3.19. In addition the power spectrum is parametrized through Equation 3.20 where k^n is the shape of the primordial power spectrum and $T^2(k)$ is the transfer function, which is associated with a specific Dark Matter scenario.

$$\sigma^2(M) \equiv \int d^3k \bar{W}^2(kR) P(k) \quad (3.17)$$

$$R^3 = \frac{3M}{4\pi\bar{\rho}} \quad (3.18)$$

$$\bar{\rho} = \Omega_m \rho_c \quad (3.19)$$

$$P(k) \propto k^n T^2(k) \quad (3.20)$$

As given in Bardeen et al. (1986) and modified by Peacock (1999) to include baryonic matter, we take $T^2(k)$ for an adiabatic Cold Dark Matter (CDM) model. The transfer function is further modified at large k by the introduction of a multiplicative exponential cut-off corresponding to the scale at which there is free streaming of WIMPs as in Diemand et al. (2005c). In addition to this, we fix the primordial power spectral index $n = 1$. This leads to the normalisation of the spectrum $P(k)$ as $\sigma_8 = 0.897$.

The first toy model we consider is by Bullock et al. (2001). Here we consider the characteristic density of the halo at redshift z to be equal to the density of the universe at the collapse redshift z_c of the Dark Matter halo. This allows us to rewrite the virial concentration as in Equation 3.21, Bullock et al. (2001), where the constant K is fitted through N-body simulations. This is applicable down to the free streaming mass scale for Dark Matter Halos, $10^{-6} M_\odot$. The dependence of the concentration parameter on mass becomes insignificant at smaller masses where objects tend to collapse at the same redshift.

$$\begin{aligned} c_{vir}(M, z) &= K \frac{1 + z_c}{1 + z} \\ &= \frac{c_{vir}(M, 0)}{1 + z} \end{aligned} \quad (3.21)$$

We consider also the toy model described by Eke et al. (2001). In this model they define the collapse redshift as in Equation 3.22. In this equation $D(z)$ represents the linear growth theory factor. σ_{eff} is given in Equation 3.23 and forms an effective amplitude on the mass scale for the power spectrum, modulating $\sigma(M)$. This definition of z_c also makes z_c dependent on both the shape and the amplitude of the power spectrum. C_σ is a free parameter fitted to the data and M_p is the mass of the halo within the radius where maximum circular velocity is reached. This means the toy model by Eke et al. (2001) has a similar scaling in redshift as the toy model by Bullock et al. (2001), yet the concentration parameter is less dependent on the halo mass. This allows us to express the concentration parameter as in Equation 3.24.

$$D(z_c)\sigma_{eff}(M_p) = \frac{1}{C_\sigma} \quad (3.22)$$

$$\begin{aligned} \sigma_{eff}(M) &= \sigma(M) \left(-\frac{d\ln(\sigma)}{d\ln(M)}(M) \right) \\ &= -\frac{d\sigma}{dM}M \end{aligned} \quad (3.23)$$

$$c_{vir}(M, z) = \left(\frac{\delta_{vir}(z_c)\omega_M(z)}{\delta_{vir}(z)\omega_M(z_c)} \right)^{\frac{1}{3}} \frac{1+z_c}{1+z} \quad (3.24)$$

The toy model proposed by Bullock et al. (2001) has a mismatch against observational data at low mass but this discrepancy is even larger for the Eke et al. (2001) toy model. This results from the weaker dependence of the concentration parameter on the mass of the Dark Matter halo. In addition the model by Eke et al. (2001) is very sensitive to the power spectrum specified, and the extrapolation collapses under small logarithmic derivatives of $\sigma(M)$.

We consider also the inner shape of the Dark Matter halo profile. In order to do this we consider the radial velocity dispersion of the halo population projected along the line of sight for a tracer population. This is a reliable observable quantity. Our halo is assumed to be spherically symmetric and to have no bulk rotation. We thus relate our observable to the total mass profile $M(R)$ as in Equation 3.25 where $\nu(r)$ is the density profile of our tracer population, given in Equation 3.27, Colafrancesco et al. (2006). We define β to be the constant over radius anisotropy parameter given by Equation 3.26 and $I(R)$ is the surface density at the projected radius R . $\sigma_r^2(r)$ is the radial velocity dispersion and $\sigma_\theta^2(r)$ is the tangential velocity dispersion.

$$\sigma_{los}^2(R) = \frac{2G}{I(R)} \int_R^\infty \nu(x) M(x) x^{(2\beta-2)} \int_R^x \left(1 - \beta \frac{R^2}{r^2}\right) \frac{r^{-2\beta+1}}{\sqrt{r^2 - R^2}} dr dx \quad (3.25)$$

$$\beta \equiv 1 - \frac{\sigma_\theta^2(r)}{\sigma_r^2(r)} \quad (3.26)$$

$$\nu(r) \propto \frac{1}{(r/r_s)(1 + r/r_s)^2} \quad \text{with } r_s = 7.05' \quad (3.27)$$

We obtain the constraints on the Dark matter profile by considering the contribution it makes to the total mass profile. The total mass profile also comprises a gas component, which can be given as in Equation 3.28. In galaxy clusters the total mass profile would also comprise terms due to spiral and E-S0 galaxies with their corresponding density profile normalised by their mass-to-light profile to give the observed luminosity. In Equation 3.28, $n_0 = 3.42 \times 10^{-3} \text{ cm}^{-3}$, $r_c = 10.5'$ and $b = 0.75$.

$$n(r) = n_0 \left(1 + \left(\frac{r}{r_c}\right)^2\right)^{-1.5b} \quad (3.28)$$

Substructures within the Dark Matter Halo

Local density variations play an important role since the signals produced through WIMP pair annihilation scale as the square of WIMP density, Geller et al. (1999). Substructures within the Dark Matter halo will have a parallel effect to that discussed for the parent halo, we again introduce the following variables for our subhalo.

Each subhalo is identified through its virial mass M_s and concentration parameter c_s , parallel to Equation 3.12 and Equation 3.14. This gives us a corresponding length scale a_s and density ρ'_s as in Equation 3.8. We assume a spherical distribution of subhalos with each subhalo spherically symmetric and comprising the same profile shape as the parent halo. This allows us to specify completely the probability distribution for the subhalo number density by specifying M_s , c_s and the radial coordinate r for the subhalo position, Diemand et al. (2005a)

We consider only the case where dependence on these factors can be factorized, since this simplified case is sufficient for our purposes. In order to do this we introduce the subhalo mass function (Equation 3.29) where M_{cut} is the free streaming cut-off mass. $A(M_{vir})$ is a normalization derived by setting the total mass contained in subhalos to be a fraction f_s of the total virial mass of the parent halo as in Equation 3.30. We assume $f_s = 0.5$. (Chen et al., 2001; Green et al., 2005; Hofmann et al., 2001)

$$\frac{dn_s}{dM_s} = \frac{A(M_{vir})}{M_s^{1.9}} \exp \left[- \left(\frac{M_s}{M_{cut}} \right)^{\frac{-2}{3}} \right] \quad (3.29)$$

$$f_s M_{vir} = \int_{M_{cut}}^{M_{vir}} dM_s \frac{dn_s}{dM_s} M_s \quad (3.30)$$

We also introduce $\mathcal{P}_s(c_s)$, a log-normal distribution of concentration parameters around a mean value determined by the substructure mass. The mean concentration is linked to the subhalo mass similarly to the trend outlined in Equation 3.21, the toy model by Bullock et al. (2001) and Equation 3.24, the toy model by Eke et al. (2001). Substructures however tend undergo tidal stripping and collapse in higher density environments, effects which both lead to higher concentrations of up to $\approx 1.5x$ larger for subhalos than the parent halo. We make the ansatz given in Equation 3.31 with F_s independent of M_s , Diemand et al. (2005c)

$$\langle c_s(M_s) \rangle = F_s \langle c_{vir}(M_{vir}) \rangle \text{ with } M_s = M_{vir} \quad (3.31)$$

In addition we must consider the spatial distribution of our subhalos. Traces of tidal stripping found using numerical simulations indicate that radial distribution is less concentrated than the smooth Dark Matter component giving Equation 3.32, which is normalized by the requirement given in Equation 3.33.

$$p_s(r) \propto g\left(\frac{r}{a'}\right) \text{ with } \frac{a'}{a} \simeq 7 \quad (3.32)$$

$$4\pi \int_0^{R_{vir}} r^2 p_s(r) = 1 \quad (3.33)$$

We can now write our factorized subhalo probability distribution as Equation 3.34.

$$\frac{dn_s}{dr^3 dM_s dc_s} = p_s(r) \frac{dn_s}{dM_s}(M_s) \mathcal{P}_s(c_s) \quad (3.34)$$

3.1.4 Number Density of Neutralino Pairs

The number density of neutralino pairs at a given radius is determined by combining the contribution from the smooth Dark Matter component and the contributions from each subhalo. We can write the smooth Dark Matter contribution as the difference between the cumulative Dark Matter halo profile and the term bound in

subhalos at a given radius. Contributions from subhalos are taken in the limit of unresolved substructures. We can then write $\mathcal{N}_{pairs}(r)$ as in Equation 3.36, considering only the spherically averaged variables, Colafrancesco et al. (2006)

$$\begin{aligned} \mathcal{N}_{pairs}(r) = & \left(\frac{(\rho' g(\frac{r}{a}) - f_s M_{vir} p_s(r))^2}{2M_\chi^2} \right) + \\ & \left(p_s(r) \int \frac{dn_s}{dM_s} \int \mathcal{P}_s(c'_s(M_s)) dc'_s dM_s \right) \times \int \frac{(p'_s g(\frac{r_s}{a_s}))^2}{2M_\chi^2} d^3 r_s \end{aligned} \quad (3.35)$$

We then define $\Delta_{M_s}^2$ as in Equation 3.36, giving the average enhancement due to a subhalo of mass M_s and Δ^2 (Equation 3.37) to be the sum over all such contributions weighted over the subhalo mass function times mass. $\Delta_{M_s}^2$ increases with decreasing M_s , flattening out at the mass scale at which structures generally collapse, for all three Dark Matter profiles listed in Table 3.1.

We introduce the quantity $\tilde{\rho}_s$, which is defined in Equation 3.38 such that in the limit $a' = a$, when the radial distribution of substructures traces the Dark Matter profile, $\tilde{\rho}_s = \rho'$, the halo normalization parameter. Then if we normalise the density to the mean matter density of the universe today, we can rewrite $\mathcal{N}_{pairs}(r)$ in a compact form as in Equation 3.39.

$$\Delta_{M_s}^2(M_s) \equiv \frac{\Delta_{vir}(z)}{3} \int \mathcal{P}_s(c'_s) \frac{I_2(c'_s x_{-2})}{(I_1(c'_s x_{-2}))^2} (c'_s x_{-2})^3 \quad (3.36)$$

$$\text{where } I_n(x) = \int_0^x y^2 (g(y))^n dy$$

$$f_s \Delta^2 \equiv \frac{1}{M_{vir}} \int \frac{dn_s}{dM_s} M_s \Delta_{M_s}^2(M_s) \quad (3.37)$$

$$\tilde{\rho}_s \equiv \frac{M_{vir}}{4\pi(a')^3 I_1(\frac{R_{vir}}{a'})} \quad (3.38)$$

$$N_{pairs}(r) = \frac{\tilde{\rho}^2}{2M_\chi^2} \left(\frac{(\rho' g(\frac{r}{a}) - f_s \tilde{\rho}_s g(\frac{r}{a'}))^2}{\tilde{\rho}^2} + f_s \Delta^2 \frac{\tilde{\rho}_s g(\frac{r}{a'})}{\tilde{\rho}_s} \right) \quad (3.39)$$

As shown in Colafrancesco et al. (2006), the enhancement of the Dark Matter profile due to subhalos is primarily of interest when the neutralino source is extended. The effect of subhalos in compact systems is negligible.

3.1.5 Diffusion Effects

The emission produced by the interactions of the secondary electrons produced during neutralino annihilation is affected by the diffusion and energy losses these particles undergo. Neglecting the effects of re-acceleration and convection, we can take these effects into account using Equation 3.40, where dn_e/dE is the equilibrium spectrum, $D(E, \mathbf{x})$ is the diffusion coefficient, $b(E, \mathbf{x})$ is the energy loss term and

$Q_e(E, \mathbf{x})$ is the source function.

$$\frac{\partial}{\partial t} \frac{dn_e}{dE} = \nabla \left(D(E, \mathbf{x}) \nabla \frac{dn_e}{dE} \right) + \frac{\partial}{\partial E} \left(b(E, \mathbf{x}) \frac{dn_e}{dE} \right) + Q_e(E, \mathbf{x}) \quad (3.40)$$

We assume our energy loss term and diffusion coefficient are positionally independent. This gives us Equation 3.41.

$$\frac{\partial}{\partial t} \frac{dn_e}{dE} = \nabla \left(D(E) \nabla \frac{dn_e}{dE} \right) + \frac{\partial}{\partial E} \left(b(E) \frac{dn_e}{dE} \right) + Q_e(E, \mathbf{x}) \quad (3.41)$$

We can then implement a method used in Baltz & Wai (2004) and Colafrancesco et al. (2006). We define the variable u as follows:

$$b(E) \frac{dn_e}{dE} = - \frac{dn_e}{du} \quad (3.42)$$

Taking the integral over E we then find

$$u = \int_E^{E_{max}} \frac{dE'}{b(E')} \quad (3.43)$$

$$\Rightarrow b(E) = \frac{E}{\tau_{loss}} \quad (3.44)$$

If $E_{max} \rightarrow \infty$ then we find $u = \tau$. This allows us to rearrange Equation 3.41 as Equation 3.45.

$$b(E)Q_e(E, \mathbf{x}) = \left(-\frac{\partial}{\partial t} + D(E)\Delta - \frac{\partial}{\partial u} \frac{dn_e}{du} \right) \quad (3.45)$$

This equation requires us to search for a Green's function for the operator on the LHS. We must first use a Fourier transform in four dimensions. We then have for the transform $t \rightarrow \omega, \mathbf{x} \rightarrow \mathbf{k}$:

$$\left(-i\omega + D(E)k^2 - \frac{\partial}{\partial u} \right) \tilde{G} = \frac{1}{(2\pi)^2} e^{-i(\omega t' + \mathbf{k} \cdot \mathbf{x}')} \cdot \delta(u - u') \quad (3.46)$$

The solution to the above Green's equation is

$$\tilde{G} = -\frac{1}{(2\pi)^2} \exp \left(-i(\omega t' + \mathbf{k} \cdot \mathbf{x}) - i\omega(u - u') - k^2 \int_{u'}^u d\tilde{u} D(\tilde{u}) \right) \quad (3.47)$$

We must then transform this back from the Fourier space to find Equation 3.48. This is the free Green's function solution as we have yet to apply our appropriate

boundary conditions.

$$G_{free} = -\frac{1}{(4\pi(\nu - \nu'))^{\frac{3}{2}}} \exp\left(\frac{|\mathbf{x} - \mathbf{x}'|^2}{4(\nu - \nu')}\right) \times \delta((t - t') - (u - u')) \quad (3.48)$$

$$\text{where } d\nu = D(u)du \quad (3.49)$$

As we stated before, we assume spherical symmetry for dSph. We will assume that the Green's function disappears at the characteristic length, r_h . Let us label the non-spatial coordinates of the Green's function as $\mathbf{Y}$. We can now use the image charge method, so we introduce a set of image charges as in Equation 3.50 to obtain Equation 3.51. This then leads us to the source function given in Equation 3.52.

$$(r_n, \theta_n, \phi_n) = ((-1)^n r + 2nr_h, \theta, \phi) \quad (3.50)$$

$$G(\mathbf{r}, \mathbf{Y}) = \sum_{n=-\infty}^{\infty} (-1)^n G_{free}(\mathbf{r}_n, Y) \quad (3.51)$$

$$\begin{aligned} \frac{dn_e}{dE} &= \frac{1}{b(E)} \int_E^{M_\chi} dE' \frac{1}{\sqrt{4\pi(\nu - \nu')}} \sum_{n=-\infty}^{\infty} (-1)^n \int_0^{r_h} dr' \frac{r'}{r_n} \\ &\times \left(\exp\left(-\frac{(r' - r_n)^2}{4(\nu - \nu')}\right) - \exp\left(-\frac{(r' + r_n)^2}{4(\nu - \nu')}\right) \right) Q_e(r', E', t') \end{aligned} \quad (3.52)$$

If the time scale for diffusion is much longer than the time scale in which the electrons and positrons lose energy, we can simplify Equation 3.52 and write the number density at equilibrium as in Equation 3.53.

$$\left(\frac{dn_e}{dE}\right)(r, E) = \frac{1}{b(E)} \int_E^{M_\chi} Q_e(r, E') dE' \quad (3.53)$$

Chapter 4

Theoretical Predictions of Emission

Electrons produced during $\chi\chi$ annihilation will produce synchrotron radiation when moving through magnetic fields, they will interact with protons and ions, causing thermal bremsstrahlung and they will cause inverse Compton scattering of CMB photons. These radiative processes allow us to probe the electron spectrum thereby obtaining information on the nature of the Dark Matter particles which underwent annihilation.

As we discussed in Chapter 2, synchrotron radiation is the process of greatest interest to us and so we will now explore this mechanism in detail. The synchrotron radiation process relies strongly on the presence of a magnetic field within the dSph in order for emission to occur. We will thus explore the magnetic fields of dSph in general as well as listing some parameters specific to the six dSph, of which we have taken radio observations. These dSph are Carina, Fornax, Sculptor, Bootes II, Hercules and Segue 2.

We then go on to discuss other parameters needed to calculate the emission from dSph, as well as some further assumptions we make. Further discussion of this emission process is found in the appendix along with some graphs illustrating the synchrotron emission and dependence on the various parameters.

4.1 Synchrotron Radiation

When charged particles pass through a magnetic field, $\vec{B}$, they will be accelerated by that field. This acceleration is perpendicular to the magnetic field and will cause the particles to spiral about the magnetic field - the particles are accelerated radially. As a result of this acceleration, the gyrating particles will emit radiation and for high energy electrons, this emission is known as synchrotron radiation.

During WIMP annihilations, electrons are produced and the synchrotron emission resulting from interaction with a magnetic field forms part of a diffuse emission, detectable through deep radio imaging. (Natarajan et al., 2013) Synchrotron radiation is the dominant form of Dark Matter induced emission at radio frequencies. The synchrotron radiation predicted is sensitive to the distribution of Dark Matter in the halo, the diffusion coefficient $D(E = 0)$, the magnetic field strength B , the neutralino mass M_χ , the annihilation rate $\langle \sigma \nu \rangle_\chi$ and the annihilation channel. (Colafrancesco et al., 2006) In addition these parameters are sensitive to the Dark Matter density profile. Some of these parameters such as the magnetic field are observationally uncertain and this has an effect on the strength of our constraints on Dark Matter. The following derivation parallels that given in Longair (1992).

We have electrons and positrons with energy E_e , which can be written as in Equation 4.1.

$$E_e = \gamma m_e c^2 \quad (4.1)$$

The magnetic field within our dSph is spherically symmetric and is given by $B(r)$. The thermal electron density of background plasma is given by $n(r)$. We consider the limit in which the frequency of the emitted photons is much larger than the non-relativistic gyro-frequency ν_g (Equation 4.2) and the plasma frequency ν_p (Equation 4.3).

$$\nu_g = \frac{eB}{2\pi mc} \quad (4.2)$$

$$\nu_p = 8980 \sqrt{n(r) \text{ cm}^{-3}} \quad [\text{s}^{-1}] \quad (4.3)$$

We introduce the classical electron radius r_0 (Equation 4.4). In addition we will introduce the quantities x (Equation 4.5) and $F(t)$ (Equation 4.7) for ease of notation.

$$r_0 = \frac{e^2}{mc^2} \quad (4.4)$$

$$x \equiv \frac{2\nu}{3\nu_g \gamma^2} \left(1 + \left(\frac{\gamma \nu_p}{\nu} \right)^2 \right)^{\frac{3}{2}} \quad (4.5)$$

$$F(t) \equiv \int_t^\infty K_{5/3}(z) dz \quad (4.6)$$

$$\simeq 1.25 t^{\frac{1}{3}} \exp(-t) (648 + t^2)^{\frac{1}{12}} \quad (4.7)$$

We can now use our notation to write an equation for the synchrotron power spontaneously emitted at frequency ν and averaged over the direction of emission. This equation is given in 4.8.

$$P_{synch}(\nu, E, r) = \int_0^\pi \sqrt{3}\pi r_0 m c \nu_g \sin^2 \theta F\left(\frac{x}{\sin \theta}\right) d\theta \quad (4.8)$$

In order to compare our results to observations we must first find the local synchrotron emissivity at the frequency ν . We can calculate the emissivity by integrating the synchrotron power over the equilibrium number density of electrons and positrons as in Equation 4.9.

$$j_{synch}(\nu, r) = \int_{m_e}^{M_x} \left(\frac{dn_{e^-}}{dE} + \frac{dn_{e^+}}{dE} \right) P_{synch}(\nu, E, r) dE \quad (4.9)$$

During neutralino annihilation, electrons and positrons are produced in pairs. This means we can write:

$$\frac{dn_{e^+}}{dE} + \frac{dn_{e^-}}{dE} = 2 \frac{dn_{e^+}}{dE} \quad (4.10)$$

We wish to compare our predictions with measurements of the flux density spectrum integrated over the radio halo size, S_{synch} (Equation 4.11). D_H is the luminosity distance of the structure.

$$S_{synch}(\nu) = \int \frac{j_{synch}(\nu, r)}{4\pi D_H^2} \quad (4.11)$$

In addition, we may wish to consider the azimuthally averaged surface brightness distribution $I_{synch}(\nu, \Theta, \Delta\Omega)$ (Equation 4.12) at frequency ν , within a beam of angular size $\Delta\omega$. $I_{synch}(\nu, \Theta, \Delta\Omega)$ is found by integrating along the line of sight l over a cone of size $\Delta\Omega$ centered in a direction forming an angle Θ with the direction of the structure center.

$$I_{synch}(\nu, \Theta, \Delta\Omega) = \int_{\Delta\Omega} \int_l \frac{j_{synch}(\nu, l)}{4\pi} dl d\Omega \quad (4.12)$$

4.2 Magnetic Fields

The electrons and positrons emitted during the annihilation of two neutralinos must move through a magnetic field in order for synchrotron emission to be released. WIMP annihilations from dSph will only be detectable within the radio band if there exists a large scale magnetic field within the dSph. The existence of this magnetic field is critical as it affects both the emission due to synchrotron processes as

well as the propagation and diffusion of the particles (Spekkens et al., 2013). This means also that the strength of the magnetic field determines whether the predicted radio emission from secondary electrons and positrons is detectable by current instrumentation.

The standard method for measuring the magnetic field within a celestial body is to study polarized radio emission resulting from synchrotron emissions. It is then possible to use the equipartition theorem to estimate the strength of the magnetic field. We can achieve this by combining measurements from the polarisation as well as measurements of the synchrotron spectral index and then using some simple assumptions, we can obtain the strength of the magnetic field. This method however requires the presence of an interstellar medium (ISM) within the body and to date, no such ISM has been found for a dSph. (Regis et al., 2014b)

A second technique is to use Faraday rotation measures to estimate the magnetic field in dSph. Measurements of the Faraday rotation of the polarisation angle of the polarised emission from background galaxies along the line of sight can be used to estimate the integral over the line of sight of the magnetic field and electron densities in dSph. This method relies on carefully calibrated measurements of a frequencies which are widely separated. (Regis et al., 2014b) Current instrumentation such as LOFAR and PAPER, amongst others possess the required bandwidth, however, they do not currently reach the sensitivities required for such measurements.

We must thus turn to using the postulated evolutionary link between dSph and irregular dwarf galaxies (dIrr). There is evidence which suggests the star formation history of dIrr and dSph was similar until fairly recently. In addition it has been shown through numerical simulations that tidal stripping caused by the Milky Way halo can effectively transform a dIrr to a dSph. This implies that these two classes of galaxies share a formation history. This would allow us to assume that the magnetic field within dSph is not dissimilar from that of dIrr.

dIrr have been found to possess an ISM and the first technique discussed has been used to determine the turbulent magnetic field strength of these galaxies. Magnetic fields of the order of $\sim 3 - 10\mu G$ have been measured in dIrr. The common origin of dIrr and dSph then suggests that dSph must once have had magnetic fields of similar strength. We have no information with which we can determine the evolution of the magnetic field within dSph since this time, however we feel it is not unreasonable to assume that dSph should still harbour a weak magnetic field on the order of $1\mu G$ (Regis et al., 2014b). This assumption is strengthened since we know that only a very small plasma density would be needed to maintain such a weak magnetic field strength.

We follow the example of Regis et al. (2014b) and assume a spherically symmetric magnetic field, as in Equation 4.14. The value r_* is the stellar radius, which contains half the light of the galaxy. The values of r_* are listed in Table 4.1 and are taken from (McConnachie, 2012) and the references therein.

We will base our calculation of the magnetic field strength B_0 on a phenomenological argument. The magnetic field of star forming dSph is typically found to be of the order of a few μG . The generation of magnetic fields within galaxies is highly dependent on dynamo processes. These dynamo processes are sustained by turbulent energy which is predominantly caused by supernovae explosions and thus we can see that the magnetic field should be linked to the star formation rate of the galaxy. We can thus see that the magnetic fields of star forming dSph should form an upper limit on the magnetic field of our dSph.

Further to this argument, it has been noted that there is a strong correlation between the star formation rate, Σ_{SFR} and the magnetic field within local group galaxies ranging in size from the Milky Way to irregular dwarf galaxies. The findings for more massive spiral galaxies and irregular galaxies continue this correlation. We then assume that this correlation law can be extrapolated down from these larger, gas rich systems to our smaller dSph targets. This assumption is backed by the significant star formation history of classical dSph which should have allowed for the formation of a relevant magnetic field.

We consider the magnetic field to be induced only by star formation in the most recent Gyr. We assume that 1% of the total stellar mass of the dSph is produced during this period. We use the star formation rates discussed in Regis et al. (2014b). Other possibilities regarding the star formation are discussed in Regis et al. (2014b) but the difference in the magnetic field is minimal and so we will not discuss it here.

In order to normalise the magnetic field we use data from the Large Magellanic Cloud found in Gaensler et al. (2005), which leads us to equation 4.13. We can then calculate the value of B_0 for each of our dSph of interest and these values are listed in Table 4.1.

$$B = 0.35\mu G \left(\frac{\Sigma_{SFR}}{M_\odot kpc^{-2} Myr^{-1}} \right)^{0.3} \quad (4.13)$$

$$B(r) = B_0 e^{-\frac{r}{r_*}} \quad (4.14)$$

4.2.1 Dark Matter Halo

The final factor in determining the synchrotron emission is the shape of the Dark Matter halo's profile. We will first briefly discuss substructure and then we will move on to discuss the three halo shapes we introduced in section 3.1.3.

dSph	$B_0(\mu G)$	$r_*(arcminutes)$
Carina	0.7	8.2
Fornax	1.2	16.6
Sculptor	1.2	11.3
BootesII	0.4	4.2
Hercules	0.4	8.6
Segue2	0.4	3.4

Table 4.1: A Table Displaying the Magnetic Field Strength and Characteristic Radius for the Six DSph of Interest

NFW Profile

The NFW profile is the profile given in Equation 3.11. We thus have the density of our Dark Matter halo as

$$\rho_{NFW}(r) = \frac{\rho_0}{\frac{r}{a}(1 + \frac{r}{a})^2} \quad (4.15)$$

EIN Profile

The EIN profile is the profile given in Equation 3.10. We thus have the density of our Dark Matter halo as

$$\rho_{EIN}(x) = \rho_0 \exp\left(\frac{-2}{\alpha(\frac{r}{a}^\alpha - 1)}\right) \quad \alpha \simeq 0.15 \quad (4.16)$$

BUR Profile

The NFW profile is the profile given in Equation 3.9. We thus have the density of our Dark Matter halo as

$$\rho_{BUR}(x) = \rho_0 \frac{1}{(1 + \frac{r}{a})(1 + (\frac{r}{a})^2)} \quad (4.17)$$

Each of these equations for the halo density profile requires a characteristic length and a density normalization factor. We obtained the distance of the dSph from McConnachie (2012) and these are listed in Table 4.2. The values of the characteristic length and halo density normalisation are taken from Martinez (2013).

4.2.2 Dark Matter Halo Substructure

As discussed in Equation 3.1.4, subhalos are of interest mainly when we are dealing with a source of neutralinos which is spatially extended (Colafrancesco et al., 2006). We are considering dSph, systems which are compact and do not have an extended source. For this reason, we may neglect subhalos in our calculations.

dSph	Distance (kpc)	a_{NFW} (kpc)	$\rho_0 \text{ } NFW$ ($M_\odot pc^{-3}$)	a_{EIN} (kpc)	$\rho_0 \text{ } EIN$ ($M_\odot pc^{-3}$)	a_{BUR} (kpc)	$\rho_0 \text{ } BUR$ ($M_\odot pc^{-3}$)
Carina	105	0.21	0.30	0.063	3.3	0.30	3.7×10^{-2}
Fornax	147	0.47	0.13	0.19	0.91	0.58	2.2×10^{-2}
Sculptor	86	0.39	0.17	0.13	1.4	0.47	2.8×10^{-2}
Bootes II	42	0.17	0.42	0.052	4.0	0.23	5.3×10^{-2}
Hercules	132	0.20	0.36	0.060	3.4	0.26	4.9×10^{-2}
Segue 2	35	0.16	0.44	0.048	4.3	0.22	5.4×10^{-2}

Table 4.2: Halo Density Normalization and Characteristic Length for the Three Halo Profile Shapes of Interest for the Six DSph Studied

4.2.3 Velocity Averaged Annihilation Rate

In order to determine the concentration of electrons and positrons we must have an understanding of the Dark Matter annihilation rate. Our Dark Matter particles have weak interactions. We can thus write the annihilation rate as $\langle \sigma \nu \rangle_\chi$ where σ is our annihilation cross section and ν is the WIMP relative velocity. This quantity is an average over the momentum distribution. We can rewrite this as in Equation 4.18. Using the simplest model we can assume the velocity independent term in Equation 4.18 will dominate and we can approximate $\langle \sigma \nu \rangle_\chi$ as an approximately velocity independent value.

$$\sigma \nu = a + b\nu^2 + \mathcal{O}(\nu^4) \quad (4.18)$$

$$\simeq a$$

$$\begin{aligned} \Rightarrow \langle \sigma \nu \rangle_\chi &\approx \langle \sigma \nu \rangle_{\chi,0} \\ &= 2.18 \times 10^{-26} cm^3 s^{-1} \end{aligned} \quad (4.19)$$

4.2.4 Source Function

We must also be able to determine the source function of our dSph in order to calculate the synchrotron emission. Let us consider a region of volume δV . Then for a number density of particles within the region, n_χ , in a time interval δt , the probability of a neutralino annihilation is given by $\langle \sigma \nu \rangle_\chi n_\chi \delta t$. The total number of neutralinos in the region is given by $n_\chi \delta V$. We can thus see that the number of annihilations per unit time, per unit volume is given by $\langle \sigma \nu \rangle_\chi n_\chi^2$ (Natarajan et al., 2013). We can now write an equation for the energy released in the form of electrons and positrons per unit time, per unit volume, per unit energy as in Equation 4.20 where $dN_{e^+e^-}/dE$ is the number of electrons and positrons per energy.

$$Q(r, E) = \frac{\langle \sigma \nu \rangle_\chi \rho_\chi^2}{M_\chi} \frac{dN_{e^+e^-}}{dE} \quad (4.20)$$

The number of electrons and positrons produced per unit energy must be nor-

malised. We know that electron-positron pairs are not the only product of neutralino annihilation. This means that our number of particles must be normalised to less than one, as in Equation 4.21. For hard channels, the majority of the energy remains in the form of electron positron pairs while for soft channels, the composition may be as little as 15-30% of the energy going to electron positron pairs.

$$\int \frac{dN_{e^+e^-}}{dE} dE \leq 1 \quad (4.21)$$

4.2.5 Diffusion

As discussed in section 3.1.5, we must consider also the effect of diffusion in calculating our expected emissions. In Equation 3.40 we referred to the diffusion coefficient, $D(E)$ and the energy loss term $b(E)$. We will now discuss these parameters further as applicable to dSph.

$$\begin{aligned} \frac{\partial}{\partial t} \frac{dn_e}{dE} &= \nabla \left(D(E, \mathbf{x}) \nabla \frac{dn_e}{dE} \right) + \frac{\partial}{\partial E} \left(b(E, \mathbf{x}) \frac{dn_e}{dE} \right) + Q_e(E, \mathbf{x}) \\ \Rightarrow \frac{dn_e}{dE} &= \frac{1}{b(E)} \int_E^{M_\chi} dE' \frac{1}{\sqrt{4\pi(\nu - \nu')}} \sum_{-\infty}^{\infty} (-1)^n \int_0^{r_h} dr' \frac{r'}{r_n} \\ &\times \left(\exp \left(-\frac{(r' - r_n)^2}{4(\nu - \nu')} \right) - \exp \left(-\frac{(r' + r_n)^2}{4(\nu - \nu')} \right) \right) Q_e(r', E', t') \end{aligned}$$

$D(E)$ is the diffusion parameter, which we assumed to be positionally independent. We can write $D(E)$ as in Equation 4.22.

$$\begin{aligned} D(E) &= D_0 \left(\frac{E}{E_0} \right)^\gamma \\ D_0 &= 10^{-3} kpc^2 Myr^{-1} \\ \gamma &= 0.7 \end{aligned} \quad (4.22)$$

D_0 is the diffusion coefficient. We know that for galaxy clusters, D_0 is found to be $D_{0,GC} \sim 0.3 kpc^2 Myr^{-1}$. The Milky Way is one order of magnitude smaller than a galaxy cluster and it has been found that for the Milky Way, $D_{0,MW} \sim 0.01 kpc^2 Myr^{-1}$. This is one order of magnitude smaller than that for a galaxy cluster. We assume a similar trend should occur for dSph which are an order of magnitude smaller than the Milky Way and so we use $D_{0,dSph} = 10^{-3} kpc^2 Myr^{-1}$ (Natarajan et al., 2013). The index γ is considered to be in agreement with the index for the Milky Way and so we use $\gamma = 0.7$.

The energy loss term, $b(E)$ contains the energy loss due to ICS, synchrotron radiation, thermal bremsstrahlung and coulomb processes. However, since the coulomb and bremsstrahlung processes are very weak for dSph at energies above 1GeV (which are the energies of interest to us), we neglect their contributions. (Natarajan et al., 2013). We can thus write our energy loss term as in Equation 4.23 with the parameters listed in Equation 4.24.

$$b(E) = b_0 \left(\frac{E}{E_0} \right)^2 \quad (4.23)$$

$$\begin{aligned} b_0 &= 0.788 \left(1 + 0.102 \left(\frac{B}{B_0} \right)^2 \right) \\ B_0 &= 1\mu G \\ E_0 &= 1GeV \end{aligned} \quad (4.24)$$

Finally, in order to be able to determine the diffusion we must note that the distance r_h at which we have no more diffusion, is assumed to be twice the luminous extent of the dSph. We can write r_n in terms of r_h as in Equation 4.25. $\nu - \nu'$ is a characteristic diffusion length and we compute $\nu(E)$ as in Equation 4.26.

$$r_n = (-1)^n r + 2nr_h \quad (4.25)$$

$$\nu(E) = \frac{D_0 E_0}{b_0(1-\gamma)} \left(\left(\frac{E_0}{E} \right)^{1-\gamma} - \left(\frac{E_0}{M_\chi} \right)^{1-\gamma} \right) \quad (4.26)$$

In the case of high turbulence, we can consider the particles to be essentially confined at their location of injection. This allows us to neglect the diffusion term and we can write the diffusion equation as Equation 4.27.

$$n_e(E, r) = \frac{1}{b(E, r)} \int_E^\infty Q_e(r, t) dE' \quad (4.27)$$

Observational Aims

We are searching for signals from secondary electrons and protons undergoing synchrotron processes within dSph. The amount of radiation produced in the synchrotron processes is determined by the magnetic field. We have discussed the magnetic fields of dSph, which are given by Equation 4.14 and the relevant parameters for each dSph of interest are listed in Table 4.1.

The amount of synchrotron radiation produced is also dependent on the number of electrons and positrons produced in the neutralino annihilation as well as their distribution. This factor is more complex and relies on the Dark Matter distribution within the dSph halo, the neutralino annihilation rate (source function for $e^\pm$ production) as well as the amount of diffusion occurring within the dSph halo.

We have discussed three possible distributions of the Dark Matter particles within the halo, namely the NFW, EIN and BUR halo profiles given in Equations 4.15, 4.16 and 4.17. The characteristic length and halo density normalisation parameters for each profile for each dSph are listed in Table 4.2.

The diffusion is discussed in Section 4.2.5 and the number of particles at a particular point with a particular energy is given by Equation 4.27. Equation 4.23 describes the diffusion itself. It can be seen however that the number of particles still depends on the source function, as given in Equation 4.20. As discussed, we are able to estimate ρ_χ^2 and $\frac{dN_{e^+e^-}}{dE}$. In order to obtain an estimate on $\langle \sigma\nu \rangle_\chi$ and the neutralino mass, M_χ , we will observe the radio signal from each dSph and after appropriate data reduction to determine the radio emission attributable to synchrotron radiation, allowing us to place bounds on $\langle \sigma\nu \rangle_c$ *hi* and M_χ .

Chapter 5

Observations and Data Analysis

We wish to study the diffuse emission resulting from the electrons and positrons produced during the annihilation of two neutralinos. We have discussed possible structure types to study and we have concluded that dSph are a very good candidate. The following parallels the discussion in Regis et al. (2014a), a paper which I co-authored.

We have performed observations using the ATCA at 16cm in order to obtain information about the diffuse emissions of six of these structures. We use radio frequency observations as the expected emission mechanisms are synchrotron emission, inverse Compton scattering and thermal bremsstrahlung and emissions from secondary electrons and protons through synchrotron emission will be dominant at radio frequencies.

We present here observations of six dSph, performed with the ATCA. Of these, three were “classical” dSph, namely Carina, Fornax and Sculptor and three were “ultra-faint” dSph, namely BootesII, Segue2 and Hercules. We produced a map of approximately one degree around each classical dSph and about half a degree around each ultra-faint dSph. These fields were covered by means of a mosaic strategy, as discussed in section 5.1.

These observations were performed during July/August 2011 and comprised a total of 123 observing hours. We will be focusing on the continuum spectrum obtained through observations at 2100MHz (16cm) in a 2GHz wide band with a resolution of 1MHz. All four polarization signals were recorded. Complete technical details of our observations can be found in Regis et al. (2014a).

The observational setup was designed towards obtaining a diffuse radio continuum on the scale of a few arc minutes, corresponding to scales in the order of dSph size. The maps produced all have an rms noise level below $0.05 mJy$. This low level of noise was obtained as a result of low levels of Galactic contamination around our six dSph. In addition, our ATCA observations have very good sensitivity and spatial

resolution.

Our observations also allow the detection of radio emissions with a sensitivity of $\sim 50\mu Jy$ on a scale of a few arcseconds up to 15 arcminutes at 2GHz. This sensitivity allows us to discuss small scale sources detected within our six dSph. This is important as one critical issue in the study of the diffuse emission of dSph towards indirect Dark Matter detection is the contamination of diffuse emission by the presence of unresolved background sources.

We produce a catalog of 1392 sources detected within the six dSph. We will discuss the analysis of these sources, their removal from our maps and their comparison to sources found within the FIRST, NVSS and SUMSS observations where these observations overlap the observed dSph in section 5.3. We also discuss the type of these smaller scale sources and determine the associated source number counts.

5.1 Observational Setup

The ATCA array comprises six 22m antenna. Five of ATCA dishes formed the core array for each observation. The sixth antenna was located 4.5km away from the core in order to obtain high sensitivity at small angular scales.

Fornax and Sculptor, as well as part of Hercules were observed using a hybrid configuration with a maximum core baseline of 214m (H214). In order to image the remainder of Hercules as well as Carina, Segue2 and BootesII we utilised a hybrid configuration with a maximum core baseline of 168m (H168).

The use of these configurations gives us a primary beam which covers $42'$ at 1.1GHz and $15'$ at 3.1GHz. The synthesized beam is $\sim 3.5'$ to $\sim 1'$ over the frequency range for the core configuration and becomes $\sim 12''$ to $\sim 4''$ if we exclude the long baseline from antenna six. The imaging was completed using a mosaic technique, the details of which can be found in Regis et al. (2014a). We obtain images of $\sim 1^\circ$ for the classical dSph and $\sim 0.5^\circ$ for the ultra-faint dSph.

The three classical dSph (Carina, Fornax and Sculptor) were mapped using a nineteen field mosaic. Each field of the mosaic had an on source integration time of one hour. For two of the ultra-faint dSph, Hercules and BootesII, we produced our maps through a mosaic of seven fields, each with an on source integration time of about two hours.

Segue2 is smaller than the other five dSph studied and thus we used a mosaic with just three fields. In order to maximise the sensitivity of the observations, we

used an on source integration time of three hours per field. Complete details of the mosaicking patterns used are found in our paper, Regis et al. (2014a).

5.2 Data Reduction

The *Miriad* data reduction package was used to reduce the data. The procedures discussed in the *Miriad* user guide were used to account for instrumental bandpass, phase, gain and flux density calibration. Further details of the set up and calibration are discussed in Regis et al. (2014a).

Approximately one third of the data had to be removed due to contamination resulting from radio frequency interference. The identification of bad data was achieved through combined use of the automated flagging routines provided by *Miriad* and flagging by hand. The *Miriad* task MFCLEAN was iterated four times on each mosaic panel. The thermal noise was calculated using the assumption that 33% of each data set was identified to be a result of contamination and the final images were cleaned to a cutoff sensitivity at five times this thermal noise.

Our data was affected by a correlator bug, which resulted in all the mosaic panels being correlated at the position of the first panel for that dSph. We have corrected for this by correcting the information in the image headers, thus correcting the error in the image plane.

The images of our dSph displayed effects from the non-coplanar array. W-term effects include a systematic offset in source position across the dSph field. This offset increases as the distance from the phase center increases. If z is the zenith angle and Θ is the distance of the source center from the phase center (see Equation 5.1), then for a coplanar array, the positional shift is given by Equation 5.2. (Taylor et al., 1999)

$$\Theta \equiv \sqrt{l^2 + m^2} \quad (5.1)$$

$$\text{Position error} \simeq \frac{\Theta^2 \sin z}{4.12 \times 10^5} \quad (5.2)$$

Our maximal zenith angles are $50 - 68^\circ$ and correspond to Segue2, which is most distant from the latitude of ATCA. Towards the edge of the observational beam, the positional error is an appreciable factor of the restoring beam for Segue2. This resulted in the appearance of multiple slightly offset components in bright sources in the mosaicked image, however the arcs associated with wide field effects are not present since the zenith angle range is insignificant in comparison with the dimen-

sions of the restoring beam. The positional offset is largest at the cutoff point and here the difference between the two zenith angles is $< 1''$.

This positioning problem was addressed through enforcing the NCP projection within the *Miriad* imager, thus reducing the problem to a 2D Fourier Transform. This removed the need for w-term approximations. The NCP projection does not however remove the w-component of the visibility data. This leads to data artifacts which are most significant in Segue2 and Hercules as these two dSph are furthest from the ATCA latitude. If we neglect the data from the sixth antenna at a long baseline, the beam size becomes sufficiently large to combat the position offset between baselines.

We thus produce two maps for each target. The first map is a high resolution map with down-weighting of the short baselines and we minimize the offset issues by enforcing an NCP projection. These high resolution maps are shown for Fornax in Figure 5.5 and for Segue2 in Figure 5.17.

The second set of maps were generated by applying a Gaussian taper of $15''$ to the data prior to performing the Fourier transform. This increases the beam sufficiently to remove the presence of w-term effects within the image and provides a lower level of RMS noise off-source. However, since this method down-weights the long baseline data, these maps are not sensitive to scales above a few tens of arcseconds, can underestimate the flux of extended sources and have a lower resolution. We show these maps for all six dSph in Figures 5.1, 5.6, 5.18, 5.15, 5.20 and 5.22. We use these maps in conjunction to determine total source fluxes and to extract other important data. A third map is produced to maximise the sensitivity for large scale emissions by following the same procedure outlined above but tapering with a full-width half-maximum (FWHM) value of $FWHM = 60''$. The three sets of maps are imaged with a Briggs robustness parameter of -1, (Briggs, 1995).

We wish to compare the data obtained with emissions from neutralino annihilations. These emissions will reflect in the diffuse emission from our dSph so our next step is to catalog and extract the point sources from our maps.

5.3 Source Subtraction

Two automated routines were tested for source extraction and cataloging - namely the task SFIND in *Miriad* and the SExtractor package, (Huynh et al., 2012; Massardi et al., 2008). Details of our use of these routines are found in Regis et al. (2014a). These two packages gave quite different photometric results for some sources. This was found to be due to the optimisation of SExtractor for optical images. This caused incorrect interpretation of certain signal and noise structures, such as large

scale noise correlation. SFIND is much better suited to handling sidelobes and artifacts as it is designed to analyse radio images. We thus use the results from SFIND in the following analysis.

These two above mentioned routines give very similar results for the number of sources with an average discrepancy in source position of $\sim 1''$, which we take to be an indication of our positional accuracy. This estimate of the error in the position of the faintest sources compares favourably with the ICRF catalog to within this error. Theoretically, using Equation 5.3, a degradation of one order of magnitude is expected for the faintest sources so we conservatively assume the positional error to be $1''$.

$$\text{Positional error} = \frac{\text{FWHM}}{2} \frac{\sigma_{\text{rms}}}{S_{\text{peak}}} \quad (5.3)$$

If we include the long baseline data from antenna six, our synthesized beam is $\sim 8''$ giving a confusion limit of $\simeq 3\mu Jy$. Neglecting dish six, our synthesized beam is $\sim 2'$ giving a confusion limit of $\simeq 500\mu Jy$. This tells us confusion is not an issue for our map.

As discussed above, we have two maps for each dSph. The higher resolution map with the NCP projection enforced down-weights the short baselines and this can lead to poor reconstruction of the extended diffuse flux density. The maps in which we use Gaussian tapering strongly down-weight the long baseline of antenna six, allowing us to recover the extended flux density. The lower resolution of the tapered map can lead a multi-component source in the untapered map to appear as a single source.

As a result, when there is one to one correspondence of the sources in the tapered and untapered maps, we use the flux density of the tapered map as our primary estimate of the flux. If however there appear multiple sources in the untapered map corresponding to a single source in the tapered map, we find the source in the untapered map closest to the source in the tapered map and associate the flux density with that source. The total flux density for the associated source is then estimated by taking the total flux density in the tapered map minus the flux density of the companion sources to the associated source. For sources seen in the untapered map and not in the tapered map, the flux density remains unchanged. If a source is seen in the tapered map and not in the untapered map, it is added to the catalogue for completeness. This process revealed 1835 sources from the six dSph.

Our fields were visually inspected to determine which sources are multi-component sources. For θ_d the distance between sources and S_{peak} the peak flux density, it was

determined that a source is a multi-component source if it satisfies either criteria in Equation 5.4, (Magliocchetti et al., 1998) . Of the 1835 sources detected, 178 appear to be multi-component sources. See Table 5.1.

$$\begin{aligned}\theta_d &< 1' \\ \theta_d &< 100'' \sqrt{\frac{S_{peak}}{10mJy}}\end{aligned}\tag{5.4}$$

dSph	Number of sources	Number of multiple component sources
Carina	225	32
Fornax	225	51
Sculptor	316	44
BootesII	173	20
Hercules	169	16
Segue 2	147	15

Table 5.1: Number of Sources in the FoV of Each DSph Galaxy as well as the Number of Multi-component Sources Therein

We quote the source size from the untapered map, remembering that if the total flux density estimate is significantly below that of the tapered map, the source size is underestimated. We wish to define a criterium for the deconvolution of low flux density sources. This criterium will help us correct our number counts of sources for resolution bias.

The tapered map is limited by confusion so we obtain sources predominantly from the untapered map at these low fluxes. We thus consider S_{tot} from the untapered map. We assume sources with $S_{tot} < S_{peak}$ are due to noise. We thus define a lower envelope as in Equation 5.5. $\mathcal{A}$ is defined for each dSph by setting 90% of sources with $S_{tot} < S_{peak}$ to lie within this envelope. We then reflect this such that $S_{peak} < S_{tot}$ and consider all sources to lie above this upper envelope to be successfully deconvoluted.

$$\frac{S_{tot}}{S_{peak}} = 1 - \frac{\mathcal{A}}{(1 + \frac{S_{peak}}{\sigma_{rms}})^{1.5}}\tag{5.5}$$

We find $\mathcal{A} = (17, 4.1, 17, 6.7, 6.7, 5.4)$ for the Carina, Fornax, Sculptor, BootesII, Segue2 and Hercules dSph respectively (in the order listed in the first paragraph of this chapter). Carina and Hercules have too few sources with $S_{tot} < S_{peak}$ to effectively determine $\mathcal{A}$ so we set this to be the same as Sculptor and BootesII,

respectively, which have properties similar to Carina and Hercules. A significant fraction of our sources are considered to be resolved based on this analysis.

Radio bandwidth smearing is another effect which was taken into account. This effect occurs since bandwidth is not infinitesimally small and causes a reduction in the peak flux of a source corresponding to an increase in source size. Total integrated flux density is conserved. If we write d as the distance from the center of the pointing and θ_B as the synthesized FWHM, then the bandwidth smearing, A , can be written as in Equation 5.6 for a single pointing. For a mosaic however, the smearing is more complex and the attenuation averaging over the primary beam of each pointing can be estimated as in Equation 5.7. $P(x)$ (Equation 5.8) is the primary beam pattern and r_i^c is the center of the pointing i .

$$A = \frac{S_{peak}}{S_{peak}^0} = \frac{1}{\sqrt{1 + \frac{2 \log 2}{3} \left(\frac{\Delta \nu}{\nu} \frac{d}{\theta_B} \right)^2}} \quad (5.6)$$

$$\bar{A} = \frac{\sum_{i=1}^{N_p} P(r - r_i^c) A(r - r_i^c)}{\sum_{i=1}^{N_p} P(r - r_i^c)} \quad (5.7)$$

$$P(x) = \exp(-4 \log 2 \left(\frac{x}{FWHM} \right)^2) \quad (5.8)$$

Our channel width is small with $\Delta \nu = 1 MHz$, thus the bandwidth smearing as given by Equation 5.7 is of the order of 1%. This has been verified empirically by comparison of peak flux density of single bright sources near the center of the observations with the corresponding peak flux value of the same source within the mosaic. We neglect bandwidth smearing effects since these two values are in agreement within their error bars.

Our maps suffer from incomplete UV coverage. The UV coverage of a radio map indicates the ability of the telescope to resolve objects in the sky. The incomplete UV coverage obtained results in an effect called clean bias whereby flux from sources can be redistributed to noise peaks. This process was alleviated by stopping the cleaning process with a maximum residual flux well above the theoretical noise. We stopped our cleaning process at five times this limit and thus we did not apply a correction to our fluxes due to clean bias.

In order to get the cleanest diffusion maps possible we wish to make sure that we have detected all the point sources. We do this through comparisons with findings in existing radio catalogs, namely SUMSS, FIRST and NVSS. Carina is contained in the SUMSS field of view (FoV), Fornax and Sculptor fall into both the SUMSS and NVSS FoV, Segue2 is covered by the NVSS FoV, whilst BootesII and Hercules

dSph	NVSS	SUMSS	FIRST
Carina		✓	
Fornax	✓	✓	
Sculptor	✓	✓	
BootesII	✓		✓
Hercules	✓		✓
Segue 2	✓		

Table 5.2: A Table Showing the Overlap between each DSph FoV and the NVSS, SUMSS and FIRST catalogues

are covered by the FIRST FoV and the NVSS (refer to Table 5.2). In order to make these comparisons we have utilised two databases, namely NASA/IPAC Extragalactic Database (NED) at <http://ned.ipac.caltech.edu/> and ASI Science Data Center (ASDC) at <http://www.asdc.asi.it/>.

A source by source look up was completed. For each dSph it was determined which point like sources matched with known sources in existing catalogues. A comparison of flux and size was made, as well as a check for multiple components where the resolution was available.

In addition, for each source we obtained a spectral energy distribution (SED), where such data was available. The SED's were obtained from the ASDC sky explorer found on www.asdc.asi.it. The sky explorer tool data from several missions and experiments, both ground and space-based, together with catalogs, archival data, covering frequencies in the ranges of infrared, radio, optical, UV, x-ray (both soft and hard) and γ -ray. The aim of obtaining the SEDs was to facilitate predictions of radio flux from multi-frequency sources and to thus refine the cleaning of the background diffuse emission. Spectral indices of sources in our catalog were obtained through the above comparison. Comparison was restricted to sources greater than $10'$ from the image boundaries to avoid effects from highly non-uniform rms and the primary beam.

The flux densities were summed for multicomponent sources. Our catalog has a small loss of diffuse flux for sources close to one another not forming a multicomponent source. This results from them being seen as a single source in the tapered image and our choice to associate the corresponding diffuse flux to the peak of the source closest to the peak of the combined source in the tapered image. This results in neglecting some portion of the diffuse emission of further sources. This occurs for only about 3% of all our sources. These results and comparisons will be discussed separately for each dSph below.

5.3.1 Carina

We show below an image of the Carina Dwarf Spheroidal galaxy (see Figure 5.1) obtained by creating a mosaic of 19 panels, each with an on source integration time of one hour per field in the H214 configuration. This data was then processed as discussed above and we show in Figure 5.1 the image obtained by applying a Gaussian taper of $15''$.

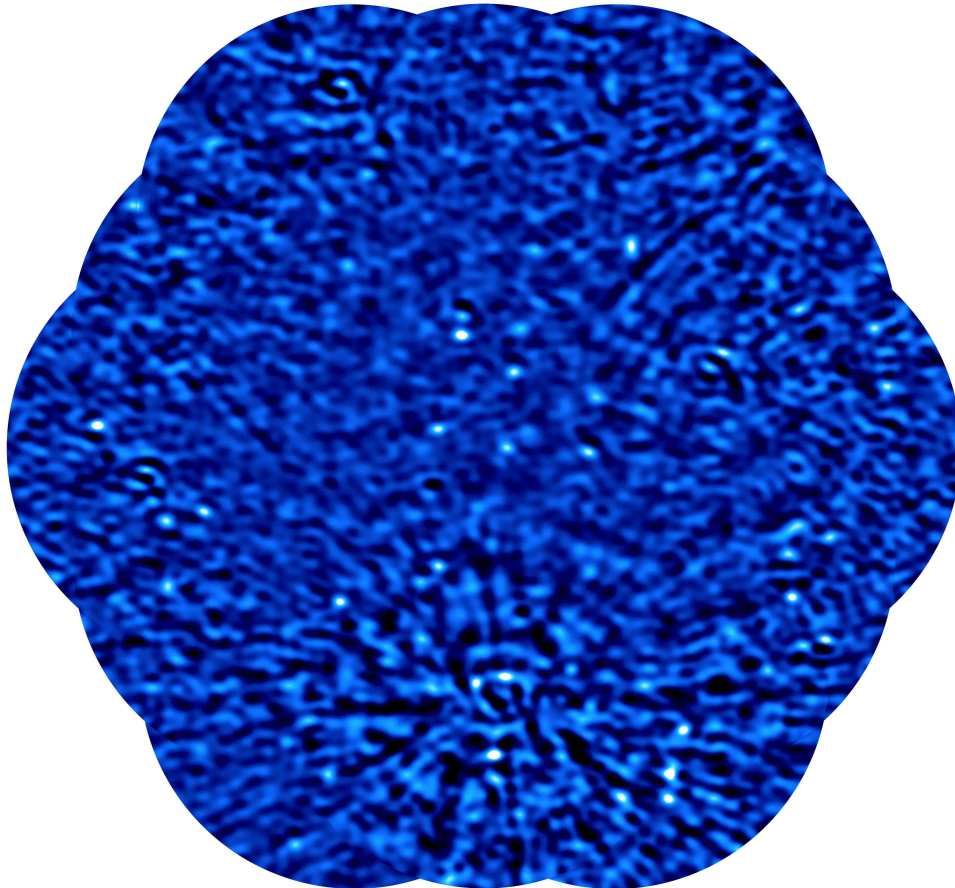


Figure 5.1: ATCA radio image of the Carina dSph at 16cm, obtained by forming a mosaic of 19 panels and applying a Gaussian Taper of $15''$

We then use the methods discussed above to obtain a list of point source positions within the image, the error in those positions and the flux coming from each source. Figure 5.2 shows the spatial distribution of point sources within the Carina dSph with the colour representing the $\log(\text{flux})$, thus giving an indication of source brightness.

We have performed a search for known sources, in all frequency ranges, near each of the listed sources in our image. For each known source found to be near a source in our image, we have obtained a flux, a source name and classification, and a spectral energy distribution, where this information was available. The search

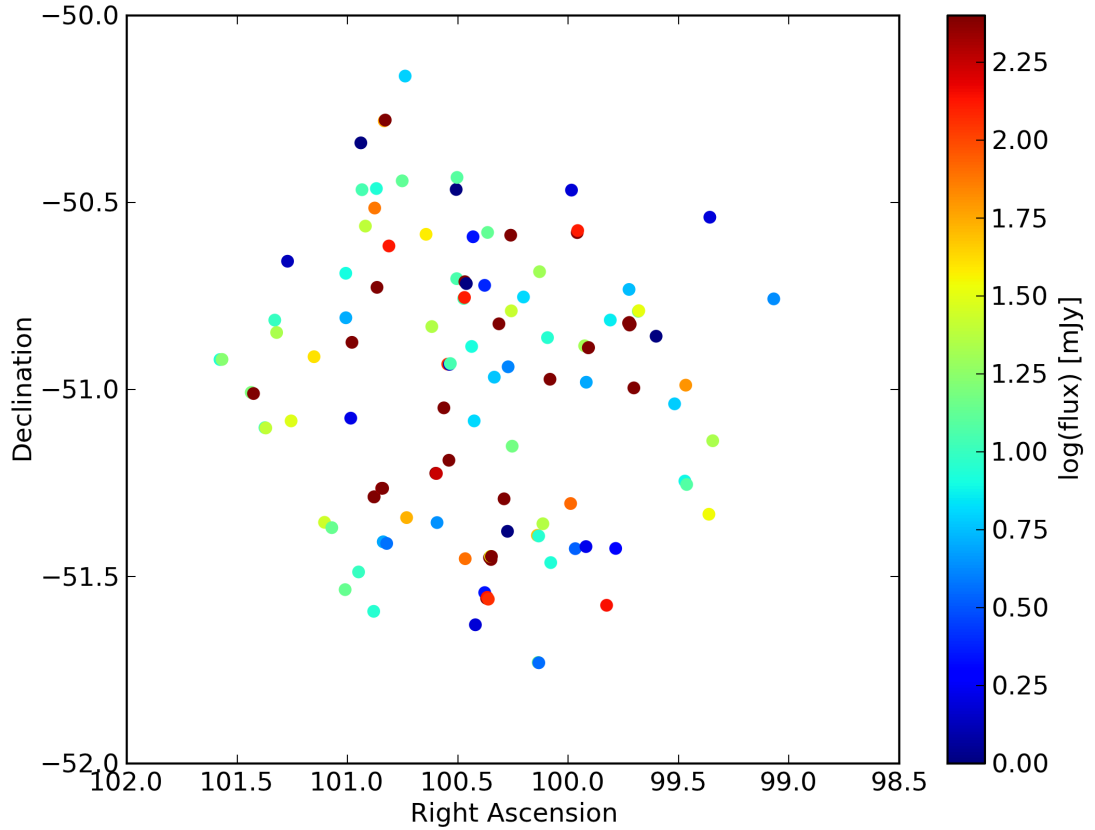


Figure 5.2: A plot showing the position and brightness of all sources in the Carina dSph FoV

was completed using two databases. The first is the ASDC, or ASI (Italian Space Agency) Science Data Center at www.asdc.asi.it. The second is the NASA/IPAC Extragalactic database at ned.ipac.caltech.edu.

We find that the Carina FoV has an rms of $40\mu Jy$ in the inner region and an rms of $50\mu Jy$ when including the outer region. The field contains 225 sources, of which 32 are multicomponent sources. The only catalog overlapping the Carina FoV is the SUMSS survey and for sources greater than $10'$ from the boundary of our image we find 39 of our sources lie within the SUMSS FoV. Of these sources all 39 have SUMSS matches. The average spectral index $\beta = -0.9 \pm 0.1$. These spectral indices are consistent with an prevalence of synchrotron sources. The average positional offset $\Delta\theta = 2.8''$ is consistent with SUMSS errors. The SUMSS positional errors are larger than our positional errors as a result of their larger beam size.

For the Carina FoV we have also created a database of multi-frequency sources. For each source located we have also obtained a spectral energy distribution (SED). The aim of this data collection was to enable further refinements in the point source extraction process by predicting any contributions to the radio emission of sources

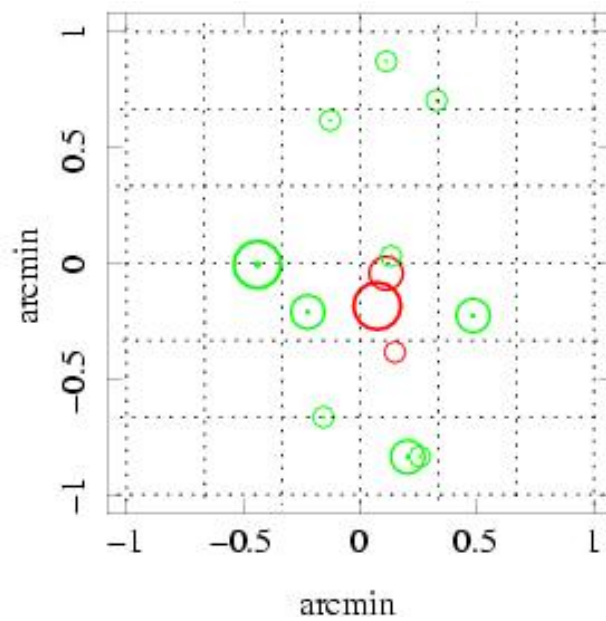


Figure 5.3: Multi-frequency sources in Carina FoV

This image is centred at 99.72301° right ascension, -50.82068° declination, within the Carina FoV showing multi-frequency sources.

Red = radio frequency source, green = optical source.

Image taken from www.asdc.asi.it

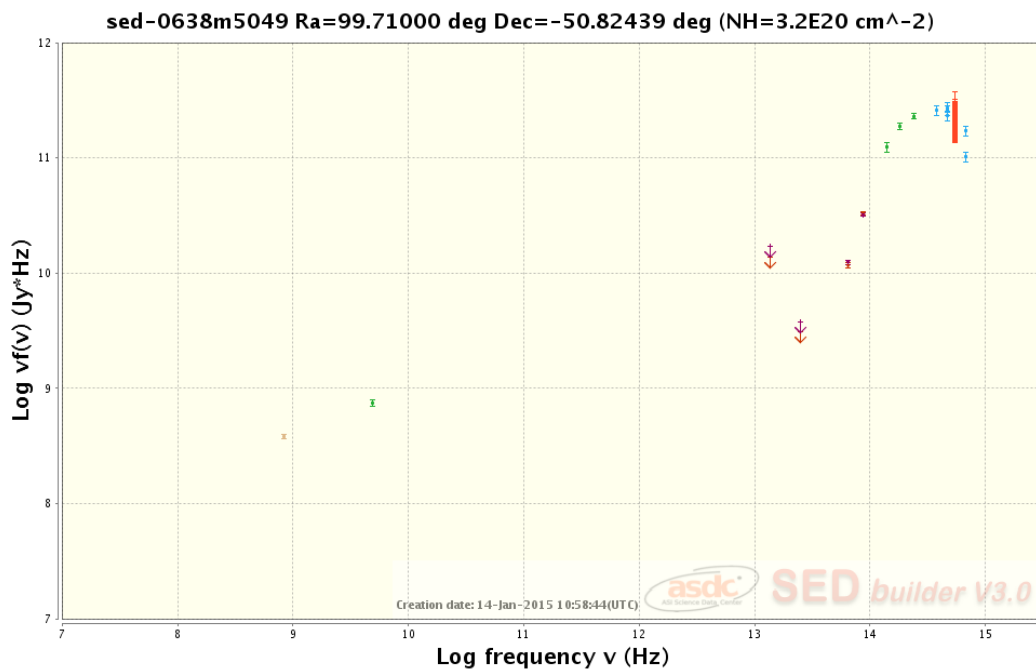


Figure 5.4: A SED for the optical source located at 99.71000° right ascension, -50.82439° declination

Image taken from www.asdc.asi.it

not dominant at radio frequencies. This database is extensive and is available from me on request. Such potential refinements will be particularly useful to future studies, which have greater sensitivity but lower resolution.

For example, in Figure 5.3 we show a area of 2 arcminute by 2 arcminutes centered at 99.72301° right ascension, -50.82068° declination. This image lies in the Carina FoV and shows 3 radio sources (red) as well as a number of optical sources. For each such source we then obtain an SED. A sample SED is shown in Figure 5.4 for the optical source located at 99.71000° right ascension, -50.82439° declination.

5.3.2 Fornax

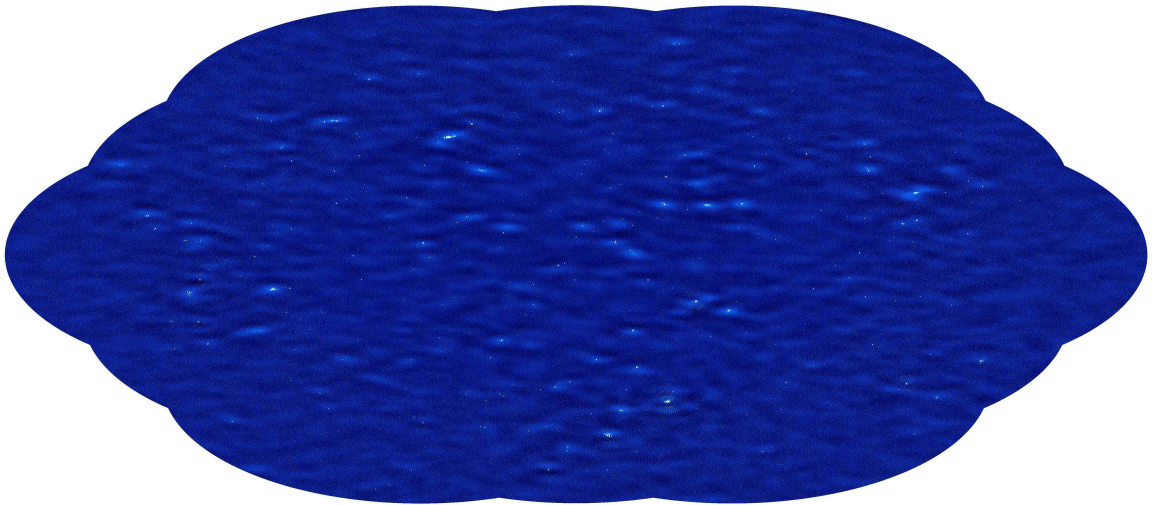


Figure 5.5: ATCA radio image of the Fornax dSph at 16cm, obtained by forming a mosaic of 19 panels and enforcing an NCP projection

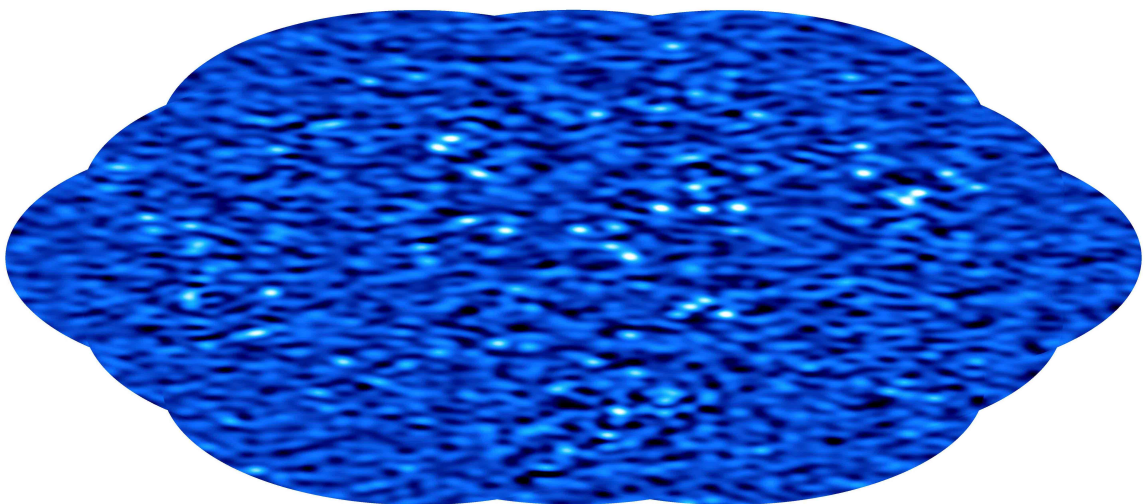


Figure 5.6: ATCA radio image of the Fornax dSph at 16cm, obtained by forming a mosaic of 19 panels and applying a Gaussian Taper of $15''$

We show above an image of the Fornax Dwarf Spheroidal galaxy (see Figure 5.5,

5.6) obtained by creating a mosaic of 19 panels, each with an on source integration time of one hour per field in the H168 configuration. This data was then processed as discussed above and we show in Figure 5.5 the image obtained by enforcing an NCP projection, whilst Figure 5.6 shows the same image applying a Gaussian taper of $15''$.

This allows us to again create a catalog of sources within the image using the methods discussed, the error in those positions and the flux coming from each source see Figure 5.7. Figure 5.8 shows the spatial distribution of point sources within the Fornax dSph with the colour representing the $\log(\text{flux})$, giving an indication of source brightness. We have performed a search for known sources, in all frequency ranges, near each of the listed sources in our image. We have obtained a flux, a source name and classification, and a spectral energy distribution for each of these corresponding sources where possible. Our information was obtained from the NED and ASDC databases.

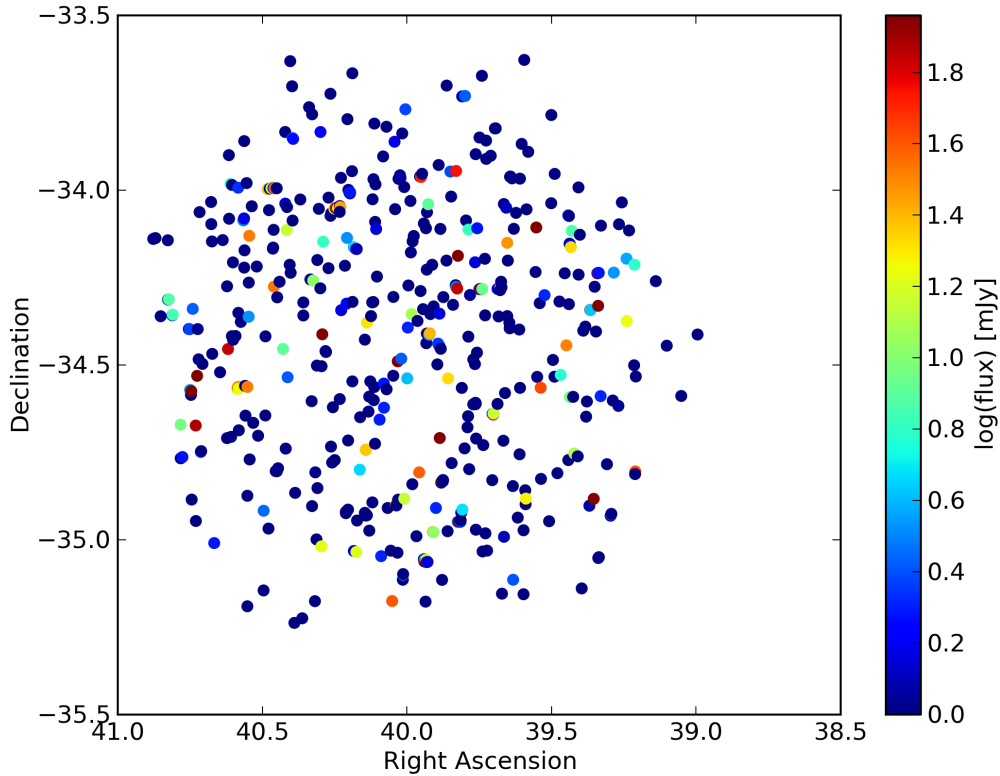


Figure 5.7: A plot showing the position and brightness of all sources in the Fornax dSph FoV

For Fornax we have performed extensive analysis in determining, which sources were multi-component sources, applying the criteria in Equation 5.4 as well as performing a visual inspection and comparison to the images from the SUMSS and NVSS catalogs. We have looked for known sources near each of these extended

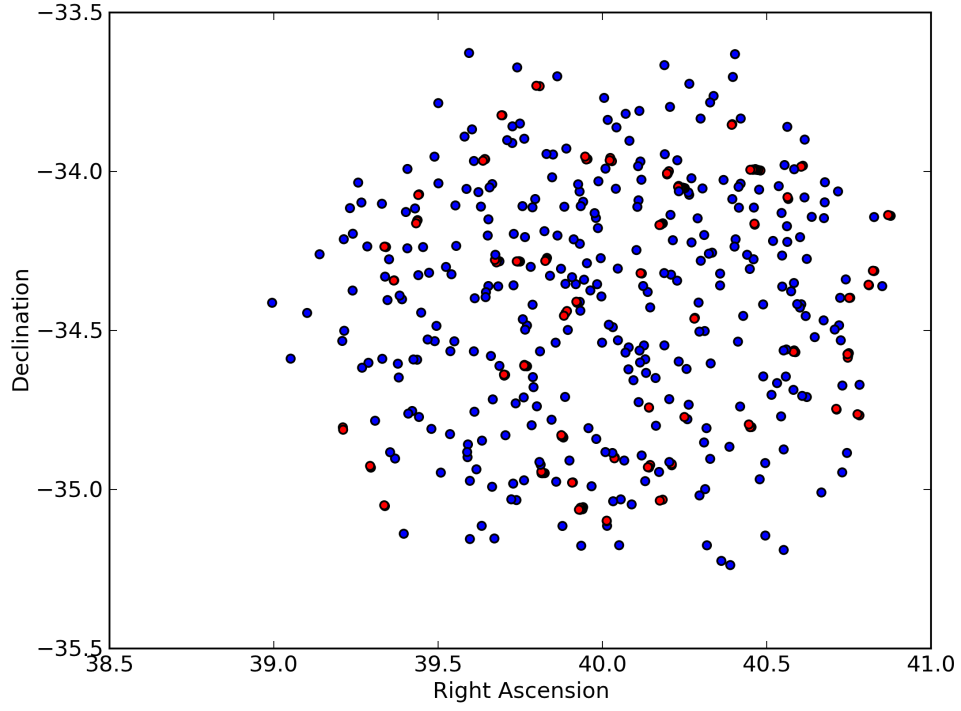


Figure 5.8: A plot showing the distribution of single and multicomponent sources in the Fornax dSph FoV

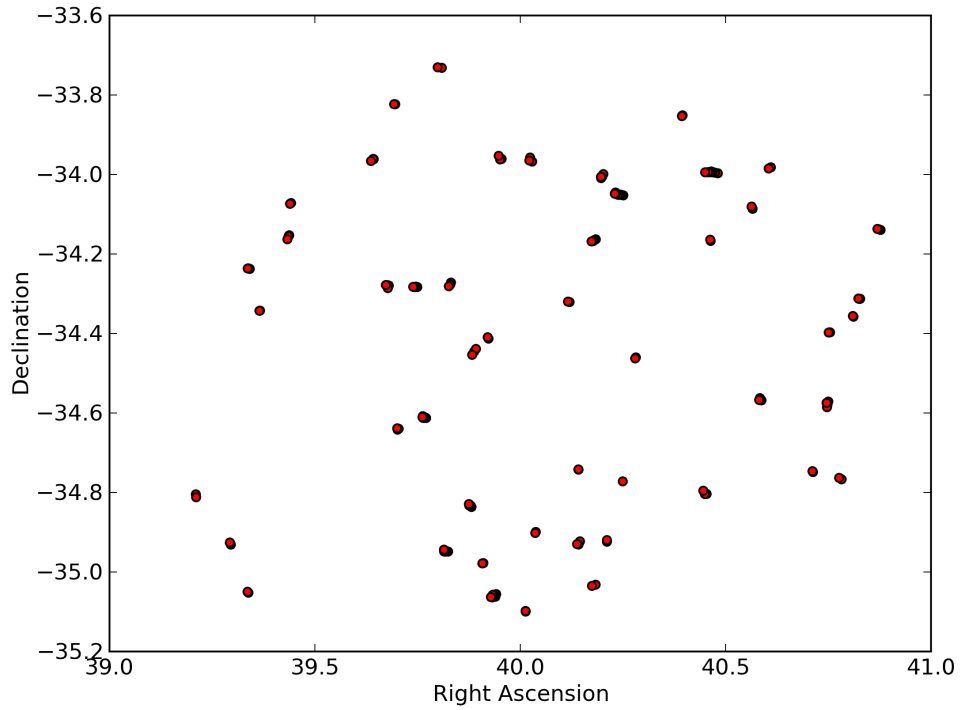


Figure 5.9: A plot showing the multicomponent sources in the Fornax dSph FoV

sources, again obtaining a source classification and SED where this was available as well as the corresponding radio and optical images using the ASDC and NED databases. Around 20% of the sources in the Fornax FoV are multi-component sources. Figure 5.8 shows the distribution of the multi-component sources as compared to the single component sources while Figure 5.9 shows only the components of the extended sources in the Fornax FoV.

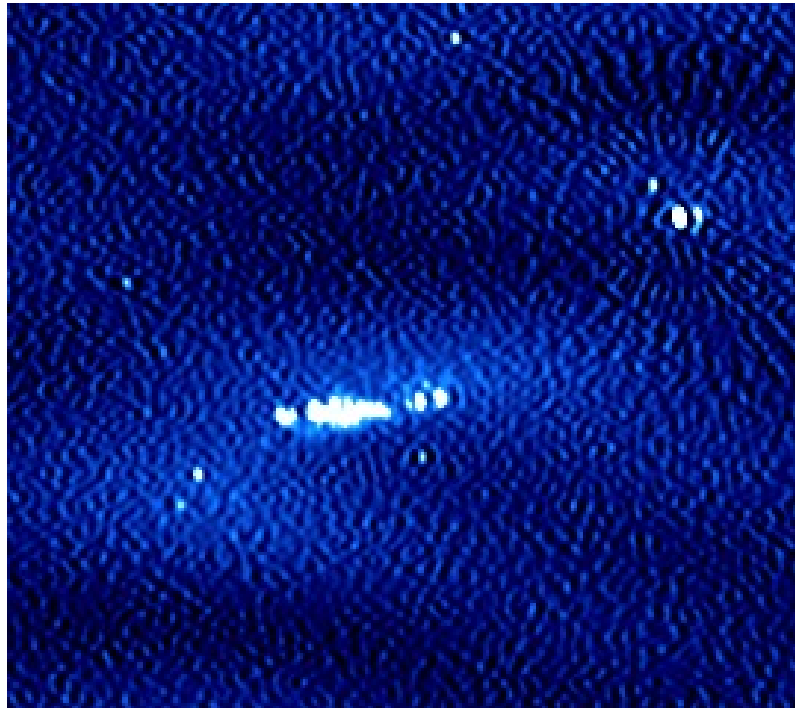


Figure 5.10: A multicomponent source within the Fornax FoV - Image is 3' diameter to facilitate comparison with other images. Image taken from untapered data

The variation in the resolution of the tapered and untapered maps is easily seen by comparing the images of multi-component sources within the Fornax FoV. We illustrate this in Figures 5.10 and 5.11, which each show a cropped image of the same area of a multicomponent source. Figure 5.10 and 5.11 are shown at the same scale. We also show in Figure 5.12 the high quality of the resolution available in our data as compared to previous surveys. Figure 5.13 shows the sources within the region at multifrequencies and Figure 5.14 shows the same region at optical wavelengths.

We find that the Fornax FoV has an rms of $36\mu Jy$ in the inner region and an rms of $43\mu Jy$ when including the outer region. The field contains 362 sources, of which 51 are multi-component sources ($\sim 50\%$). The Fornax FoV is overlapped by both the SUMSS survey and the NVSS catalog.

For sources greater than $10'$ from the boundary of our image we find 80 of our sources lie within the NVSS FoV with 79 of these having matches in the NVSS cat-

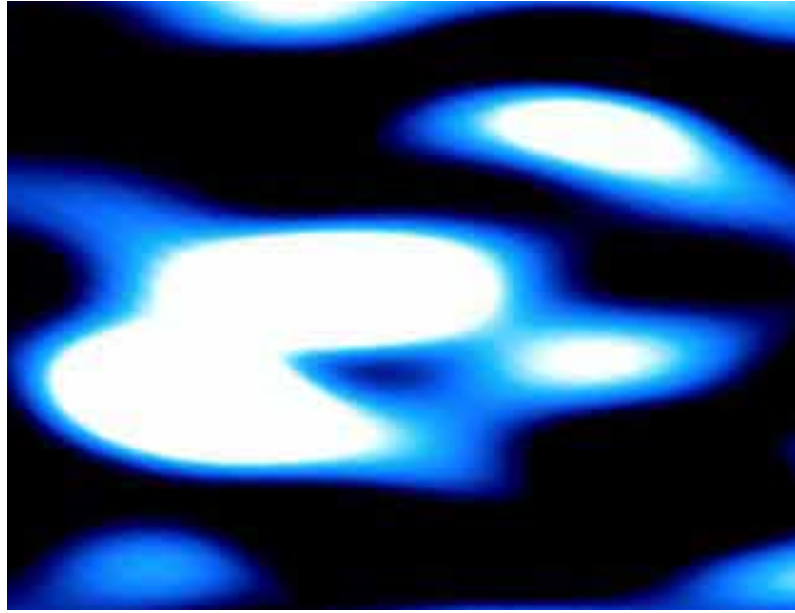


Figure 5.11: The same multicomponent source as shown in Figure 5.10 showing the loss of resolution in the tapered maps, which exclude long baseline data from dish 6.

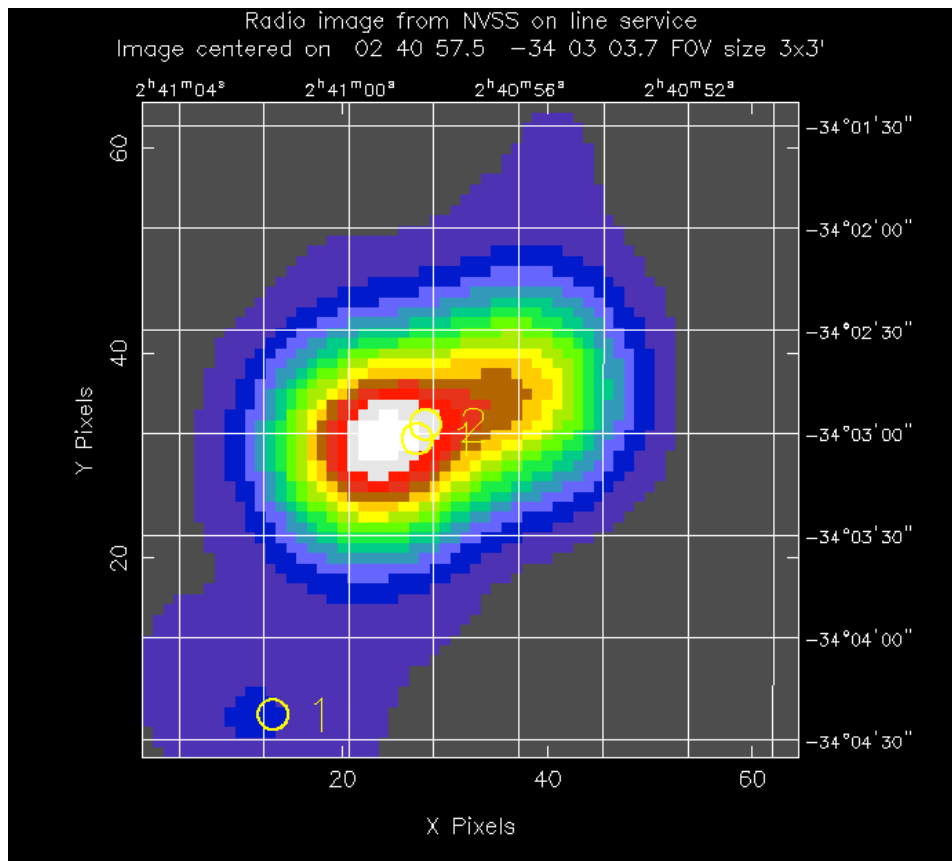


Figure 5.12: A look at a radio frequency image of the region shown in Figure 5.10 taken from www.asdc.asi.it

alog. The single unassociated source has no corresponding C.L. peak, suggesting a strongly variable source.

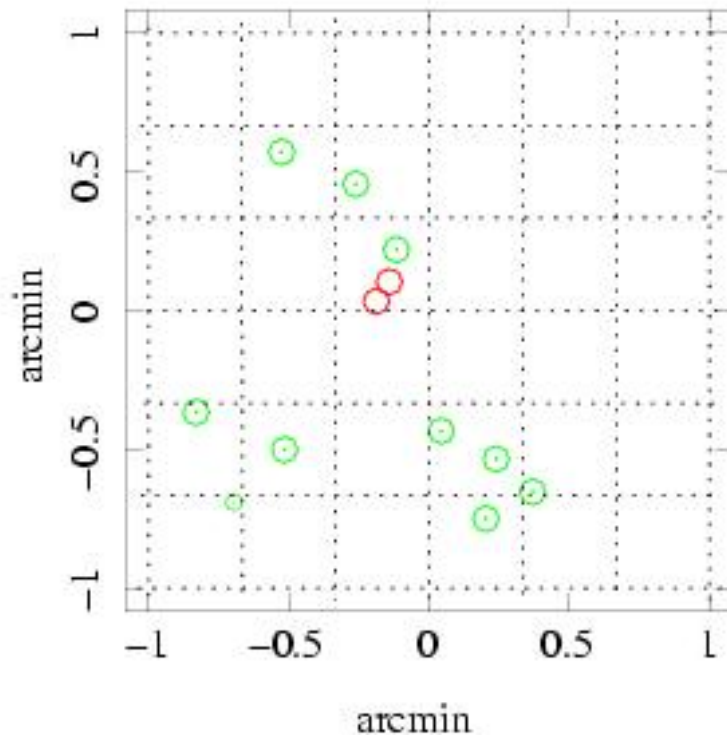


Figure 5.13: Multi-frequency sources in the same region as shown in Figure 5.10 taken from www.asdc.asi.it

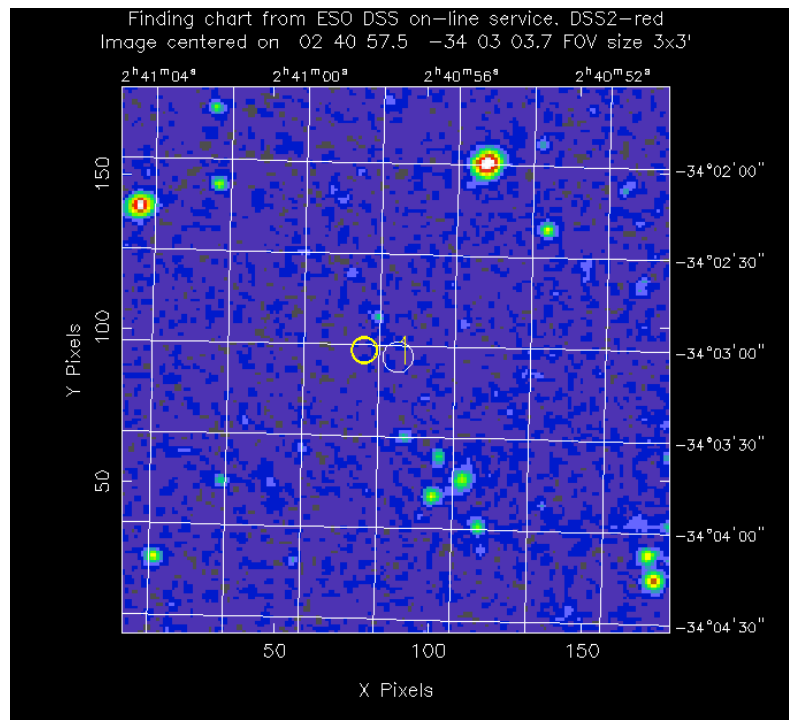


Figure 5.14: A look at an optical image of the same region as in Figure 5.10 taken from www.asdc.asi.it

Five of the sources, which have matches in the NVSS catalog have a significant mismatch in flux. Three of these sources are close to brighter sources and the mis-

match may result from the loss of diffuse flux near bright sources discussed above. There is no apparent reason for the discrepancy in the two remaining sources, suggesting they may be moderately variable sources.

The average spectral index in the NVSS FoV was $\beta = -0.9 \pm 0.1$, indicating a prevalence of synchrotron sources. The average positional offset of $3.5''$ agrees with the NVSS errors due to their larger beam width.

The SUMSS catalog contains 46 sources from the Fornax FoV and all 46 have SUMSS matches. The average spectral index $\beta = -0.8 \pm 0.1$. These spectral indices are again consistent with a prevalence of synchrotron sources. The average positional offset $\Delta\theta = 1.9''$ is consistent with SUMSS errors.

5.3.3 Sculptor

We show above an image of the Sculptor Dwarf Spheroidal galaxy (see Figure 5.15) obtained by creating a mosaic of 19 panels, each with an on source integration time of one hour per field in the H168 configuration. This data was then processed as discussed above and we show in Figure 5.18 shows the Sculptor applying a Gaussian taper of $15''$.

We then produce a catalog of sources within the image using the methods discussed, the error in those positions and the flux coming from each source. Figure 5.16 shows the spatial distribution of point sources within the Sculptor dSph with the colour representing the $\log(\text{flux})$, indicating relative source brightness.

We have performed a search for known sources, in all frequency ranges, near each of the listed sources in our image and for each of these we have obtained a flux, a source name and classification, and a spectral energy distribution, where this information was available. The search was again done using the NED and ASDC databases.

We find that the Sculptor FoV has an rms of $31\mu Jy$ in the inner region and an rms of $53\mu Jy$ when including the outer region. The field contains 316 sources, of which 44 are multicomponent sources. The Sculptor FoV is overlapped by the SUMSS survey and the NVSS and for sources greater than $10'$ from the boundary of our image we find 40 of our sources lie within the SUMSS FoV, all of which are associated. The average spectral index $\beta = -0.6 \pm 0.1$ is consistent with a predominance of synchrotron sources. The average positional offset is $\Delta\theta = 2.2''$, consistent with SUMSS errors.

67 sources lie within the NVSS FoV, of which 59 are associated. The average spectral index is $\beta = -0.4 \pm 0.1$. These spectral indices are consistent with an prevalence of synchrotron sources. The average positional offset $\Delta\theta = 4.1''$ is consistent with NVSS errors. Of the eight unassociated sources, one has low C.L. pointing

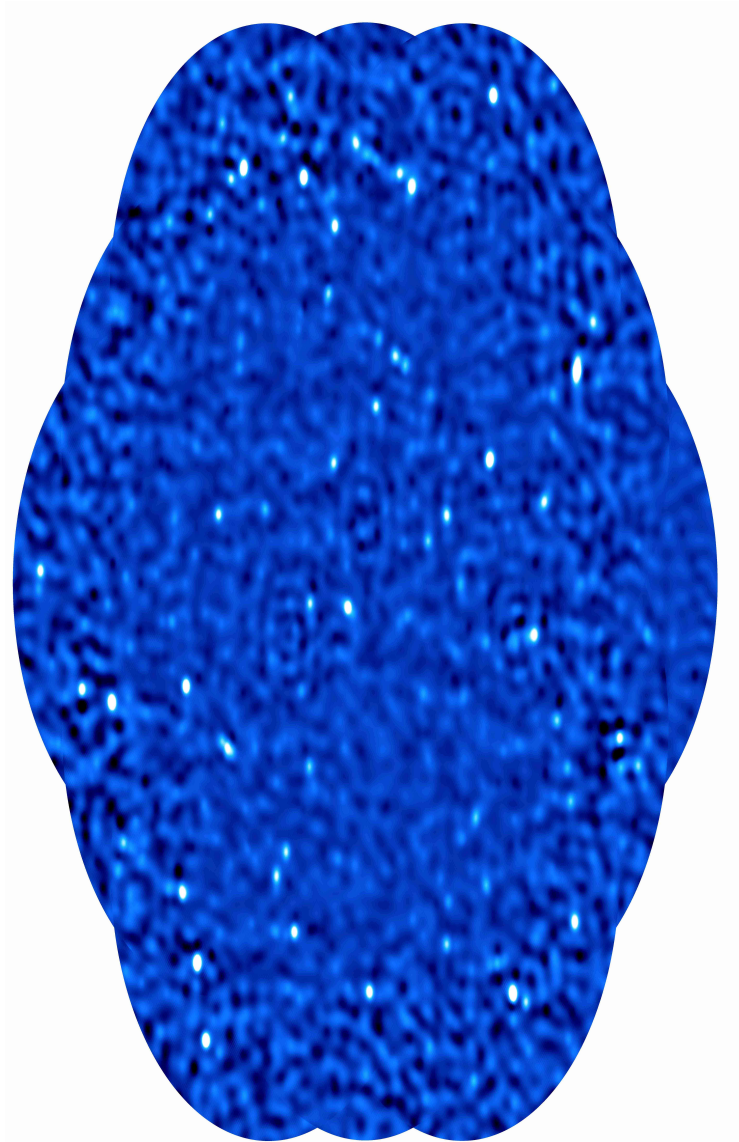


Figure 5.15: ATCA radio image of the Sculptor dSph at 16cm, obtained by forming a mosaic of 19 panels and applying a Gaussian Taper of 15''

towards some variability. Four of the unassociated sources lie close to bright sources and may be sidelobes in the NVSS catalog or could form part of multicomponent sources. Two sources have no apparent reason for the mismatch and could be variable sources. The remaining unassociated source lies in a noisy region close to the boundary of our image.

5.3.4 Segue2

We show above an image of the Segue2 Dwarf Spheroidal galaxy (see Figures 5.17, 5.18) obtained by creating a mosaic of 3 panels, each with an on source integration time of four hour per field in the H214 configuration. This was done to maximise sensitivity as Segue2 is a smaller dSph. This data was then processed as discussed above and we show in Figure 5.17 the image obtained by enforcing an NCP projec-

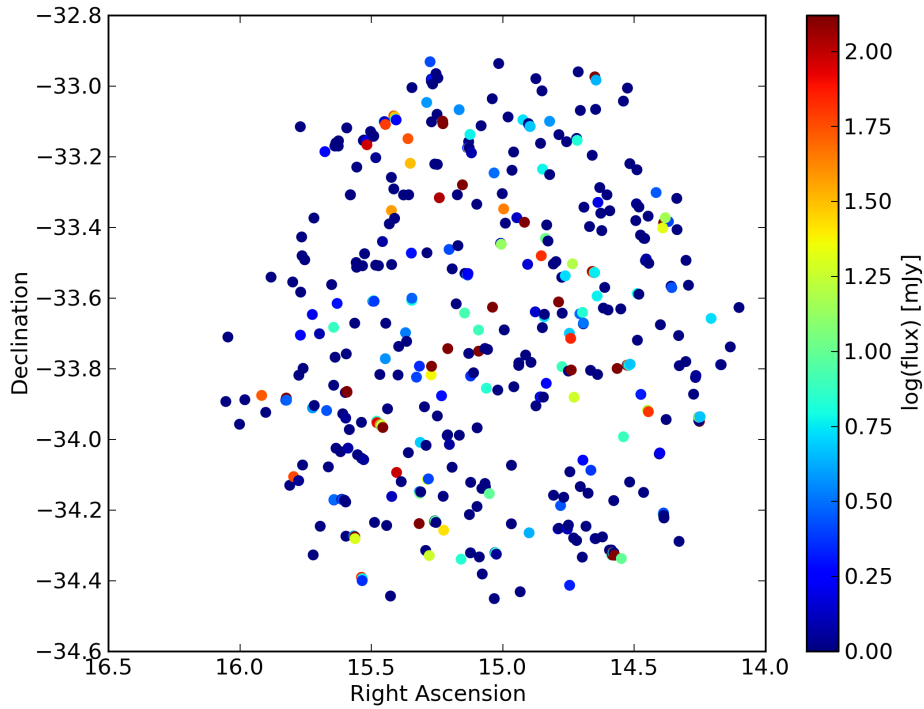


Figure 5.16: A plot showing the position and brightness of all sources in the Sculptor dSph FoV

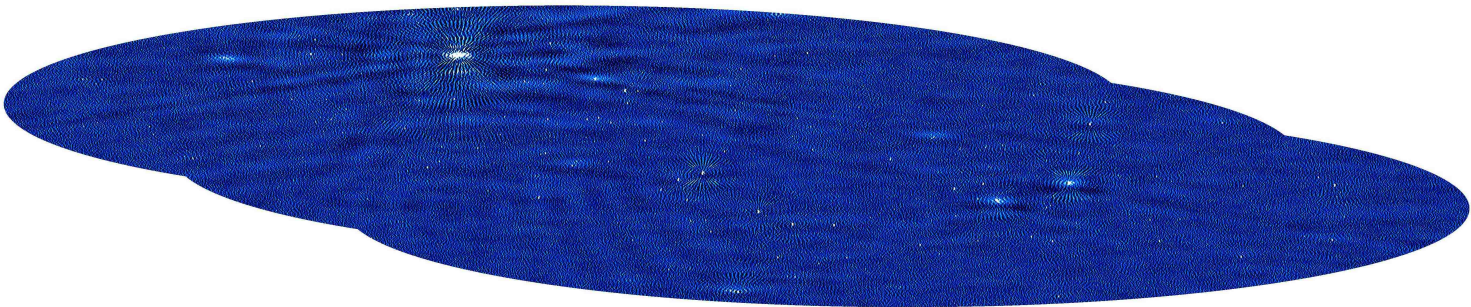


Figure 5.17: ATCA radio image of the Segue2 dSph at 16cm, obtained by forming a mosaic of 3 panels and obtained by forming a mosaic of 19 panels and enforcing an NCP projection

tion whilst Figure 5.18 shows the same image applying a Gaussian taper of $15''$.

We then produce a catalog of sources within the image using the methods discussed, the error in those positions and the flux coming from each source. Figure 5.19 shows the spatial distribution of point sources within the Segue2 dSph with the colour representing the $\log(\text{flux})$, indicating relative source brightness.

We have performed a search for known sources, in all frequency ranges, near each of the listed sources in our image and for each of these we have obtained a flux, a source name and classification, and a spectral energy distribution, where this

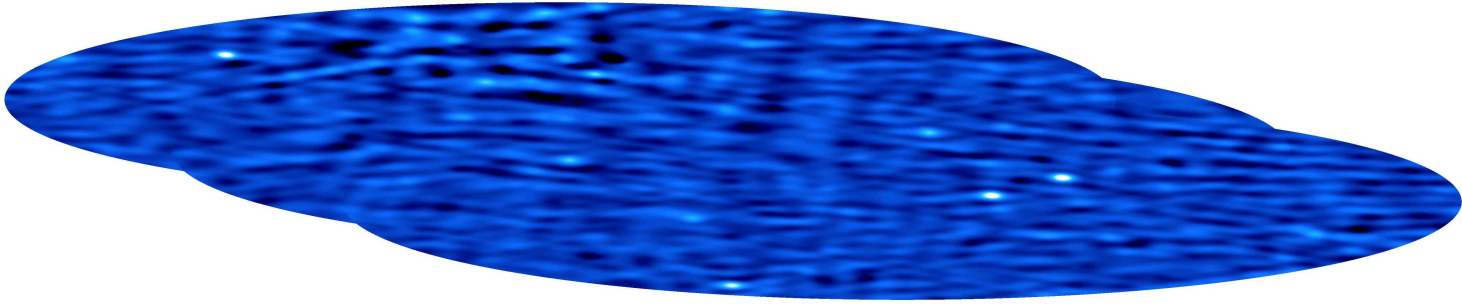


Figure 5.18: ATCA radio image of the Segue2 dSph at 16cm, obtained by forming a mosaic of 3 panels and applying a Gaussian Taper of $15''$

information was available. The search was again done using the NED and ASDC databases.

We find that the Segue2 FoV has an rms of $25\mu Jy$ in the inner region and an rms of $29\mu Jy$ when including the outer region. The field contains 147 sources, of which 15 are multicomponent sources ($\sim 10\%$). The Segue2 FoV overlaps the NVSS catalog.

For sources greater than $10'$ from the boundary of our image we find 18 of our sources lie within the NVSS FoV with 17 of these having matches in the NVSS catalog. The average spectral index in the NVSS FoV was $\beta = -2.0 \pm 0.3$. The average positional offset of $5''$ agrees with the NVSS errors.

We have problems with the Segue2 map as a result of two factors. The high declination (DEC) as well as a very bright source (4C +20.10). The high DEC creates a more significant effect from positional offset. The bright source causes an issue with dynamic range within our image. This leads to a significant loss of diffuse flux in the tapered image for sources surrounding the 4C source, which in turn leads to a low spectral index of $\beta = -2.0 \pm 0.3$. The issue could be alleviated through the consideration of the total flux from the maximum between the flux of the long and short baseline maps. For the sake of consistency however, we continue to use the method outlined thus far. If we neglect sources within $30'$ from the 4C source, we can reduce the spectral index to $\beta = -1.1 \pm 0.3$.

Six of the sources within the NVSS field have a very low spectral index. Four of these lie close to the 4C source reducing the diffuse flux observed. One of the other sources with low spectral index lies within a crowded, poorly reconstructed region whilst the remaining source could be a truly variable source. The one NVSS source, which is unassociated, lies in a region with no apparent issues leading to the conclusion that it may have an intrinsically low spectral index or it may be strongly variable.

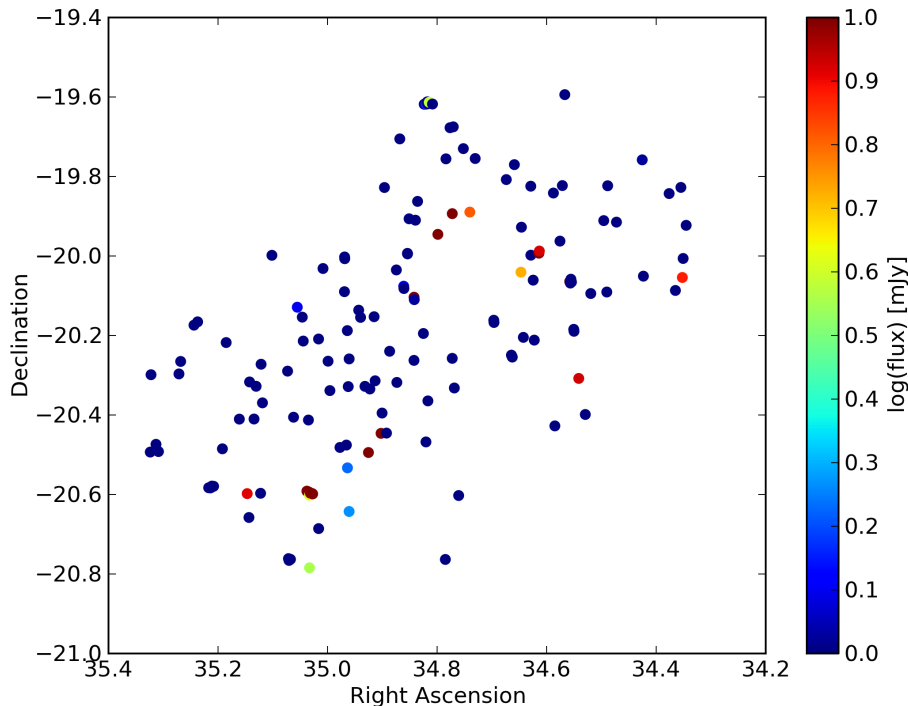


Figure 5.19: A plot showing the position and brightness of all sources in the Segue2 dSph FoV

5.3.5 Bootes II

The BootesII dSph was imaged by ATCA at 16cm in the H214 configuration. The image was created from a mosaic of 7 fields, each with an on source integration time of 2 hours. The data was reduced as discussed above. Our image of the BootesII dSph can be seen in Figure 5.20 and was created by enforcing a Gaussian taper of $15''$ prior to performing the Fourier Transform. We use SFIND as discussed above to generate a list of point sources, which are illustrated in Figure 5.21, showing the positions and brightness of the point sources within the BootesII FoV.

The BootesII dSph overlaps the NVSS and FIRST catalogs. The rms of BootesII was found to be $34\mu Jy$ in the inner region of the dSph and $41\mu Jy$ including the outer region of the dSph. There are 173 point sources, of which 20 are multicomponent sources. Inspection of the data through the NED and ASDC databases reveal that 39 of the sources within the BootesII FoV lie within the NVSS catalog, all of which are associated. The average spectral index $\beta = -1.0 \pm 0.1$ is consistent with a prevalence of synchrotron sources. The average positional offset was found to be $4.5''$, which is again consistent with the larger error in the NVSS catalog due to their beam size.

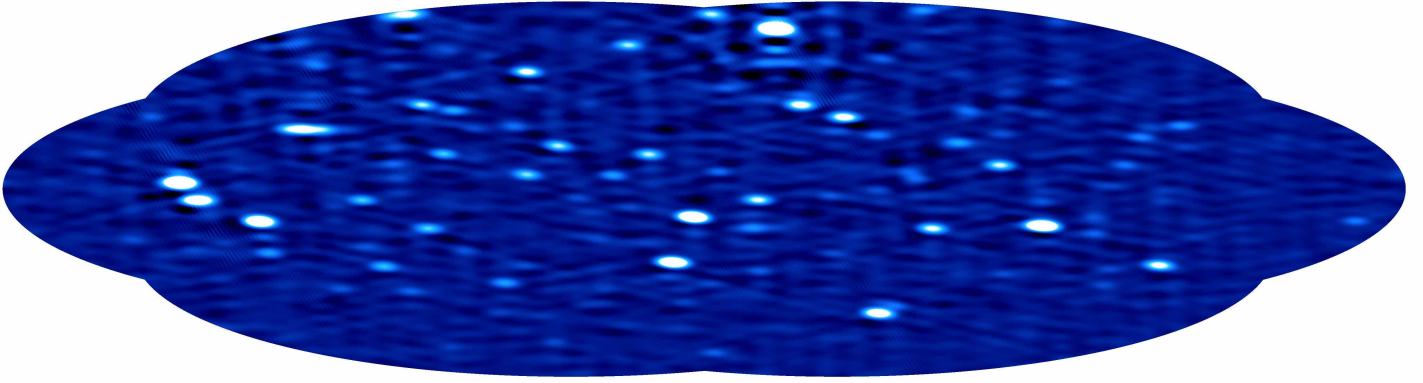


Figure 5.20: ATCA radio image of the BootesII dSph at 16cm, obtained by forming a mosaic of 7 panels and applying a Gaussian Taper of 15''

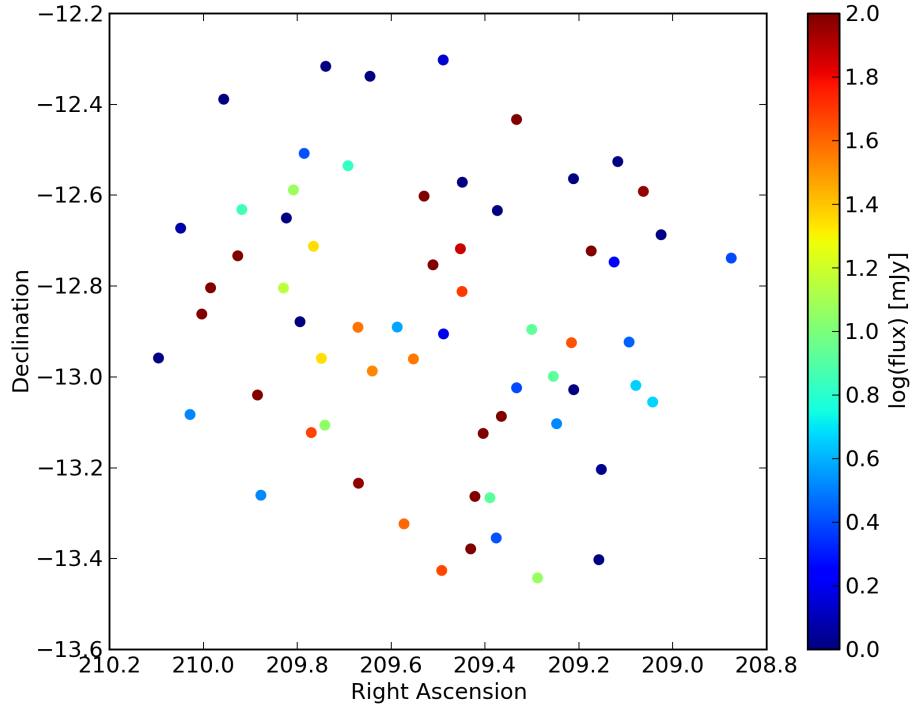


Figure 5.21: A plot showing the position and brightness of all sources in the BootesII dSph FoV

Similarly, the inspection revealed that for the FIRST catalog, 68 of the sources in the BootesII FoV lie within the field covered by the catalog. 65 of these sources are associated with sources in FIRST. Two of these unassociated sources lie below the FIRST detection limit. The third unassociated source has no explanation, suggesting it may be a variable source. The FIRST catalog can distinguish sources or structures at a scale of 2'' to 30'', making it ideal to compare to the long baseline maps. The average spectral index $\beta = -0.7 \pm 0.2$ is slightly decreased with respect to the average of the full catalog, yet this can be attributed to the lack of diffuse flux in the long baseline maps and this spectral index is still consistent with a prevalence

of synchrotron sources. The average positional offset is $\Delta\theta = 1.3''$, consistent with our estimated positional error.

5.3.6 Hercules

We show below an image of the Hercules dSph galaxy at 16cm created by using ATCA by mosaicking a set of 7 panels, each with an on source integration time of 2 hours per field and applying a Gaussian Taper of $15''$ as discussed above, see Figure 5.22. A list of sources was extracted from this map using the SFind tool in *Miriad* and the methods outlined above. Hercules was found to have an rms value of $30\mu Jy$ in the inner region and $37\mu Jy$ including the outer region. There are 169 sources in our catalog, of which 16 are multicomponent sources. (Regis et al., 2014a)

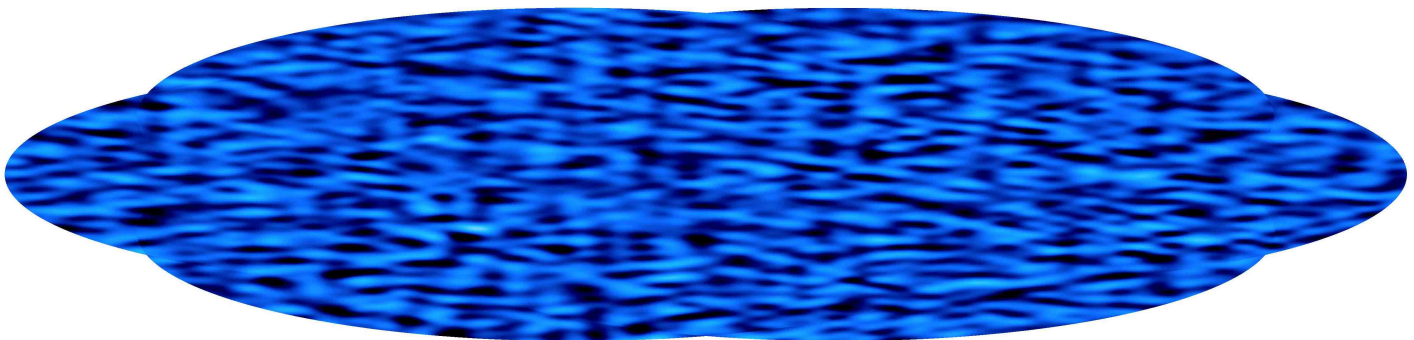


Figure 5.22: ATCA radio image of the Hercules dSph at 16cm, obtained by forming a mosaic of 7 panels and applying a Gaussian Taper of $15''$

The list of sources for the Hercules FoV was then inspected and compared to the NVSS, SUMMS and FIRST catalogs using the ASDC and NED databases. It was found that the Hercules FoV overlaps with the NVSS and FIRST catalogs. Within the NVSS catalog, we find 24 sources, of which 23 are associated. The single unassociated source lies close to a bright source and lies just above the detection threshold for the NVSS. This implies that the source is either missed in our map since it lies in a noisy region or is a sidelobe in NVSS. This source is not seen in the FIRST catalog, suggesting that it is indeed a sidelobe in NVSS. The average spectral index is found to be $\beta = -1.1 \pm 0.3$, consistent with synchrotron source being prevalent and the average positional offset of $4.5''$ agrees with the NVSS errors.

The Hercules FoV has 58 sources within the FIRST catalog, of which 57 are associated. The unassociated source is close to a bright source and near to the detection limit of FIRST. The average spectral index was found to be $\beta = -1.0 \pm 0.1$, which is slightly lower than expected but can likely be attributed to the long baselines, which as discussed above, lead to a decrease in diffuse flux. The average positional offset of $\Delta\theta = 1.7''$ is in agreement with our estimate of the positional

offset.

5.4 Non-thermal Diffuse Emissions

In the previous sections we discussed observations of six dSph taken with the ATCA. We then went on to discuss the reduction of this data as well as the analysis of point sources within these dSph. The issue of point sources is critical to the detection of Dark Matter. This is because there is significant contamination of the maps by background sources. The following discussion parallels the work discussed in Regis et al. (2014b), to which I was a contributor.

The expected signal from Dark Matter is diffuse and thus requires a large beam to be observed. The predicted signal is also a weak one. This means we require high levels of sensitivity. As we increase the sensitivity of a large beam, the presence of background sources becomes an issue as confusion limits are rapidly reached. This means we must identify and extract all background sources to the best of our ability in order to obtain a map that can be analysed for a diffuse emission attributable to emission for Dark Matter. We have discussed this analysis in depth and we will now examine the limits on Dark Matter it is possible to achieve using our radio observations.

Diffuse emissions generally comprise both a thermal and a non-thermal component. The thermal component results from processes associated with the presence of gas within the structure whilst the emissions from Dark Matter are non-thermal in nature. In the case of dSph, the non-thermal emissions will dominate as the gas density is thought to be very low. The diffuse emission should thus predominantly comprise emissions from synchrotron processes associated with the interaction of high energy electrons with the interstellar magnetic field. (Regis et al., 2014b)

There are several factors to consider in determining the size of the area to consider when exploring the diffuse emission. The largest object, which can be imaged using our mosaicking technique is less than $30'$. In addition, the region beyond $30'$ suffers from uneven coverage and rms. Finally, we expect the diffuse emission from sources associated with the stellar component of the dSph as well as from the Dark Matter halo to be contained within the half-light radius and halo scale radius. For the classical dSph, this radius is $\leq 20'$ whilst for ultra-faint dSph, the radius is $\leq 10'$. For these reasons we consider only the inner region of our images. We use data within $30'$ from the center of the dSph for the classical dSph whilst for the ultra-faint dSph, we consider data which lies within $20'$. Although the possibility exists that the diffuse signal is influenced by the potential presence of clouds or Dark Matter subhalos, we only consider the case where our diffuse emissions are centered on the optical centre of the dSph.

After source subtraction and statistical analysis of the observations, we do not attain any detection of a spherical diffuse emission. We can however still use our data to obtain upper limits on the diffuse emission. As mentioned above, we consider only emissions centered on the dSph optical centre. We estimate the uncertainty of this positioning to be at the level of arcminutes. Our data exhibits a homogeneous sensitivity on larger scales and so we do not expect this potential misalignment to have a significant effect. Similarly, we continue to assume spherical symmetry. The homogeneity of our data implies that the possibility that our data has a slight ellipticity should have minimal effect on the bounds derived. We also find that the impact of the halo profile, while locally significant, has minimal impact on the spatially averaged emissivity bounds.

5.5 Dark Matter Point Sources

If the spatial distribution of the Dark Matter within our dSph follows a cuspy profile with a core radius $\ll 10pc$, then the emission from Dark Matter will appear as a point like source. This source is likely to be located at the center of our image, however the uncertainty with regards to the exact position of the Dark Matter source means that it could lie anywhere towards the center of the image. The possibility of such a scenario is best examined using the untapered maps.

The positional uncertainty in our maps is $1'$. In addition we are uncertain as to the exact position of the Dark Matter halo. We thus choose to inspect point like sources within $2'$ of the center of our image (centered on the optical center of the dSph) as possible Dark Matter emissions. We find within our catalogue of point sources, eight which could potentially be point like Dark Matter sources. We find two such sources with $2'$ of the center of the Carina FoV as well as two within $2'$ of the center of the Segue 2 FoV. We find also one each of such sources within $2'$ of the center Fornax, Sculptor, BootesII and Hercules dSph FoV respectively. The details of these sources are listed in Table 5.3.

Of the point sources listed in Table 5.3, only those of Carina and the second source of Segue 2 (last line of Table) have not appeared in the source catalog discussed previously as they are too weak. The first Carina source can be ruled out as a Dark Matter source as it is seen at other frequencies and is a known source type. The remaining seven sources are however uncategorised and could theoretically be Dark Matter sources. While this conclusion does not yet have much evidence to support it, it has not yet been ruled out.

The Dark Matter distribution within dSph has been predicted by numerical N-body simulations to be clumpy. This implies that some of our point sources could indeed be the result of Dark Matter clumps. Further research would be required

dSph	RA (J2000)	DEC (J2000)	Distance (arcmin)	Flux density mJy	$\langle \sigma \nu \rangle_\chi$ $10^{-26} \text{cm}^3 \text{s}^{-1}$
*Carina	06 41 33.5	-50 58 11.7	0.6	0.28 ± 0.05	11.9
Carina	06 41 27.6	-50 59 09.5	1.9	0.30 ± 0.05	12.7
Fornax	02 40 00.3	-34 25 07.6	1.8	0.16 ± 0.04	1.1
Sculptor	01 00 15.0	-33 44 00.3	1.9	0.28 ± 0.06	1.8
Bootes II	13 58 04.2	12 52 53.6	2.2	0.17 ± 0.05	0.51
Hercules	16 31 00.2	12 46 48.1	0.8	0.11 ± 0.04	11.7
Segue 2	02 19 18.7	20 09 13.1	1.4	0.09 ± 0.03	0.27
Segue 2	02 19 18.0	20 11 39.1	1.2	0.22 ± 0.03	0.66

Table 5.3: A List of Point-like Sources Within 2' of the Optical Center of each of the Six DSph

The values of $\langle \sigma \nu \rangle_\chi$ are calculated for $M_\chi = 100 \text{GeV}$

* This is not a Dark matter source, it is a known source type visible at other frequencies

(using our database of SEDs for each source) to determine which individual sources are known source types, with radiation seen at other frequencies as these sources could be discarded as not being a result of Dark Matter annihilation. This removal of radio emitting sources could reduce the observed number of observed clumps. In addition such an analysis would remove radio emissions from non Dark Matter sources. Alternately such clumps could lead to an enhancement of the overall diffuse emission as discussed in Chapter 3.

5.6 WIMP Constraints

The major uncertainties in our upper bound constraints of the WIMP particles arise from uncertainties in the shape of the Dark Matter halo profile and the strength of the Magnetic field. We will briefly discuss these two factors below.

We have considered three possible shapes for our Dark Matter halo, NFW, EIN and BUR as described in Section 3.1.3. We find that the BUR profile has the weakest constraints on the Dark Matter properties, probably due to the lack of a core. The EIN profile provides the strongest constraints. The shape of the Dark Matter halo however has a smaller impact on the constraints than does the magnetic field strength.

The magnetic field directly affects both the rate of emission and the diffusion of the electrons and positrons. The uncertainty in the strength of the magnetic field is thus one of the largest obstacles in predicting Dark Matter emissions from dSph, (Regis et al., 2014c). This problem can be addressed in the future by obtaining Faraday rotation measures and utilising these to better constrain the magnetic field strength. This would in turn allow us to improve our constraints on the Dark Matter

annihilation rate and particle mass.

5.7 Diffuse Emissions, Dark Matter Point Sources and WIMP Constraints

We expect the dominant diffuse emissions in the dSph to be due to synchrotron emissions from Dark Matter. We considered the region extending 30' from the center of the classical dSph images and 20' from the ultra-faint dSph. We considered only the case where the diffuse emissions were centered on the optical center of the dSph. We found our positional uncertainty to be arcminutes with the data displaying homogeneity on larger scales leading us to conclude potential misalignment to be of minimal consequence. The homogeneity of the data also implies that any deviation from spherical symmetry to have little effect. The shape of the Dark Matter halo profile was found to be locally significant but of little consequence to the spatially averaged emissivity bounds.

After a source subtraction was performed and after statistical analysis of the observations, no spherical emission was detected. These observations can however be used to place upper limits on the diffuse emission.

It is possible that the emission from Dark Matter could appear as a point like source located towards the center of our observations if the Dark Matter follows a cuspy halo profile with a core radius of $a \ll 10pc$. Due to the positional uncertainty discussed above we examined point like sources within 2' of the center of our image and found eight such potential Dark Matter sources. One of these can be discarded since it is a known source type seen outside the radio band (not a Dark matter source). These sources are listed in Table 5.3.

The values of $\langle \sigma \nu \rangle$ were derived using the information contained in Chapter 4. We assume the full radio flux density is due to synchrotron emissions resulting from the secondary $e^\pm$ produced in neutralino decay. The flux density for each point source within 2' of the optical center of the respective dSph is listed in Table 5.3.

We first calculate our magnetic field which we assume to be spherically symmetrical using Equation 4.14 and the values listed in Table 4.1.

$$B(r) = B_0 e^{\frac{r}{r_*}}$$

We also calculate the Dark Matter halo profile density distributions as per Equations 4.15, 4.16 and 4.17 using the parameters given in Table 4.2.

$$\begin{aligned}
\rho_{NFW} &= \frac{\rho_0}{\frac{r}{a}(1 + \frac{r}{a})^2} \\
\rho_{EIN} &= \rho_0 \exp\left(\frac{-2}{\alpha(\frac{r}{a} - 1)}\right) \quad \alpha \simeq 0.15 \\
\rho_{BUR} &= \rho_0 \frac{1}{(1 + \frac{r}{a})(1 + \frac{r^2}{a})}
\end{aligned}$$

We then calculate the diffusion as given by Equations 4.23 and 4.24.

$$\begin{aligned}
B(E) &= b_0 \left(\frac{e}{E_0}\right) \\
b_0 &= 0.788(1 + 0.102(\frac{B^2}{B_0})) \\
B_0 &= 1\mu G \\
E_0 &= 1GeV
\end{aligned}$$

We now need to consider our source function as given in Equation 4.20.

$$Q(r, t) = \frac{<\sigma\nu>_{\chi} \rho_{\chi}^2}{M_{\chi}} \frac{dN_{e^+e^-}}{dE}$$

We can calculate ρ_{χ} . We assume $M_{\chi} = 100GeV$ and we know $dN_{e^+e^-}/dE$ from Equation 4.21. So our only unknown is $<\sigma\nu>_{100GeV}$. As mentioned above however, we have the density flux $j_{synch}(\nu, r)$ which is given by Equation 4.9.

$$j_{synch}(\nu, r) = \int_{m_e}^{M_{\chi}} \left(2 \frac{dn_{e^+}}{dE}\right) P_{synch} dE$$

In order to calculate this we need Equation 4.8.

$$P_{synch}(\nu, r, E) = \int_0^{\pi} \sqrt{3} \pi r_0 m_e c \nu_g \sin^2 \theta F\left(\frac{x}{\sin \theta}\right) d\theta$$

In order to calculate this we need the functions given by Equations 4.7, 4.5, 4.2 and 4.3.

$$\begin{aligned}
F(t) &\simeq 1.25 t^{1/3} \exp(-t) (648 + t^2)^{1/12} \\
x &= \frac{2\nu}{3\nu_g \gamma^2} \left(1 + \left(\frac{\gamma \nu_p}{\nu}\right)^2\right)^{3/2} \\
\nu_g &= \frac{eB}{2\pi m c} \\
\nu_p &= 8980 \sqrt{n_e(E, r)}
\end{aligned}$$

As you can see the plasma frequency relies on the number of electrons and positrons and thus the source function as given in Equation 4.27.

$$n_e(r, E) = \frac{1}{b(E, r)} \int_E^\infty Q(r, t) dE$$

This then allows us to calculate the value of $\langle \sigma \nu \rangle_{100 \text{ GeV}}$. The calculated values are given in Table 5.3.

Another possible description of Dark Matter by N-body simulations describes the Dark Matter distribution as clumpy. Such a distribution could cause the appearance of point like sources due to these Dark matter clumps. This possibility would require further research using the database of SEDs collected by myself to individually analyse each point source and explore whether it is a known source type (thus not Dark Matter) or whether the point source could potentially be a Dark Matter clump.

The greatest uncertainties in our WIMP constraints result from the fact that while we have made many simplifying assumptions, we do not know either the Dark Matter halo profile shape, nor the magnetic field strength.

We have found that the BUR halo profile gives the weakest constraints on Dark Matter. This we believe is due to the lack of a core in this model. The EIN profile however gives the strongest Dark Matter constraints. The effect of the halo profile is however less than that of the magnetic field strength which controls both the emission of synchrotron radiation and the diffusion of the synchrotron source - the electrons and positrons. This limitation can be improved on in future work by obtaining Faraday rotation measures which can be used to constrain the magnetic field. This will be possible with the next generation of radio telescopes.

Chapter 6

Conclusions and Perspectives

6.1 Comparison to Other Radio Observations

dSph galaxies have been used as an indirect probe of Dark Matter by Spekkens et al. (2013) and Natarajan et al. (2013). These studies utilised radio data from the Green Bank Telescope (GBT). Spekkens et al. (2013) considers observations of four dSph galaxies whilst Natarajan et al. (2013) presents only data from Ursa MajorII.

Spekkens et al. (2013) use deep radio observations, designed to detect the annihilation of WIMP particles within the Dark Matter halo. These annihilation emissions will take the form of extended synchrotron emissions. The four dSph observed are Coma Bernices, Draco, Ursa MajorII and Willman1. These dSph lie in the local galaxy and are classical dSph. Natarajan et al. (2013) uses this same data but focuses only on Ursa MajorII, testing the effects of Dark Matter annihilation to various primary channels. Spekkens et al. (2013) focuses on annihilation to $b\bar{b}$ states. We thus focus on the results from Spekkens et al. (2013) in this discussion.

Discrete source confusion in the work by Spekkens et al. (2013) was combated through the use of the NVSS catalogue. The sensitivity achieved in the background source subtracted maps of Spekkens et al. (2013) is $\lesssim 7mJy/beam$. We compare this to the radio maps produced from our observations where a sensitivity of $\sim 0.05mJy/beam$ was achieved, which is an increase in strength of about two orders of magnitude. In addition, Spekkens et al. (2013) has a resolution of $\sim 10'$ whilst our data from ATCA has a resolution of $\sim 1'$. This means the data analysed by Spekkens et al. (2013) can integrate more signal. These two factors result in our constraints on the upper bound on the annihilation rate of Dark Matter being around one order of magnitude stronger than those by Spekkens et al. (2013).

Spekkens et al. (2013) also used observations of the Draco dSph from the Very Large Array (VLA). These observations of Draco were roughly concurrent to the GBT observations and were used to quantify the variability of the discrete source background. This effect was found to be negligible.

For the dSph, Ursa MajorII and Willman1, Spekkens et al. (2013) found no significant emission whilst emissions from the Coma and Draco dSph were dominated by the foreground. These observations however can produce an upper limit on the velocity averaged annihilation cross section of Dark Matter. This limit is found to be $\langle \sigma \nu \rangle_\chi \lesssim 10^{-25} \text{cm}^3 \text{s}^{-1}$ for one set of charged particle propagation parameters adopted by Colafrancesco et al. (2007). These constraints were found by using the fiducial Colafrancesco et al. (2007) models and setting $M_\chi = 100 \text{GeV}$, annihilating to the final states $b\bar{b}$. The magnetic field was assumed to be $B = 1\mu\text{G}$.

Spekkens et al. (2013) note that these limits can be improved through several methods. The first of these improvements is achievable by obtaining observations of a larger sample of dSph. This improvement is reflected in the results of our observations of six dSph with the ATCA. An additional improvement of the constraints can be made by mapping a larger area around each dSph, as Spekkens et al. (2013) encountered problems resulting from the predicted halo size being comparable to the size of the map. We have combated this by using a mosaicking strategy, mapping an area of $\sim 1^\circ$ around the classical dSph observed and $\sim 0.5^\circ$ around the ultra-faint dSph observed.

Further improvements to these constraints can be made once we have more information on the magnetic fields within dSph as well as on the shape of the Dark Matter halo as this would allow us to make better predictions.

6.2 Comparison to γ -ray Observations

WIMP Dark Matter annihilations and decays produce γ -ray emission. Many studies of dSph have been made at γ -ray frequencies. Until now, the strongest constraints on Dark Matter from indirect astrophysical searches are those from γ -ray searches, however none of these studies has achieved a detection of Dark Matter. The strength of such γ -ray constraints results from the dearth of astrophysical sources, which forces us to turn to the more robust predictions of signals of Dark Matter annihilation at high energies.

One such study of the γ -ray emissions from dSph has been made using Fermi-LAT (Large Area Telescope on Fermi). The resulting constraints on the annihilation rate of Dark Matter as a function of its mass are the strongest upper limits on the annihilation rate of Dark Matter, $\langle \sigma \nu \rangle_\chi$, for WIMP Dark Matter particles with mass $M_\chi \lesssim 500 \text{GeV}$. (Atwood et al., 2009)

Assuming the mass of the WIMP to be $M_\chi = 100 \text{GeV}$, for annihilations into $b\bar{b}$, then the Fermi-LAT two year data gives $\langle \sigma \nu \rangle_\chi \lesssim 10^{-25} \text{cm}^3 \text{s}^{-1}$ for a single dSph. Combining the Fermi-LAT observations of 10 such dSph gives $\langle \sigma \nu \rangle_\chi \lesssim$

$7 \times 10^{-26} \text{cm}^3 \text{s}^{-1}$. These results from the two year Fermi-LAT data for individual dSph are comparable to those of Spekkens et al. (2013). (Abdo et al., 2010; Atwood et al., 2009)

More recently four-year observations have been released from Fermi-LAT for 25 dSph, which are satellites of the Milky Way galaxy. A subset of 15 of these dSph were analysed and for a confidence level of 95% they obtain an upper limit of $\langle \sigma \nu \rangle_{\chi} \lesssim 3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}$. This result is obtained assuming $M_{\chi} \lesssim 10 \text{GeV}$, a NFW Dark Matter distribution and where the WIMPs decay to $\tau^+ \tau^-$ and hadronic final states, Abdo et al. (2010). This constraint would be an order of magnitude less for a leptonic final state.

Fermi-LAT looks at γ -ray flux between 500MeV and 500GeV . Higher energy γ -ray observations at TeV level using ground based Cherenkov telescopes have been made for the most promising dSph. Observations at such high energies are most sensitive to WIMPs with $M_{\chi} \gtrsim 100 \text{GeV}$. For Dark Matter annihilating into quarks and $\tau^+ \tau^-$, searches, including HESS and MAGIC, have reached a limit on the annihilation rate of $\langle \sigma \nu \rangle_{\chi} \approx 10^{-23} - 10^{-24} \text{cm}^3 \text{s}^{-1}$. (Aharonian et al., 2009; Aliu et al., 2009)

The radio signals from Dark Matter are more uncertain than those from γ -ray emissions. This is because γ -rays are produced through prompt emissions whilst the radio waves are produced from secondary emissions. These secondary emissions require us to make assumptions about quantities that are still observationally uncertain, such as the magnetic field within a dSph as well as the spatial diffusion the secondary electrons and positrons will undergo.

The upper limit of the annihilation rate of Dark Matter WIMPs obtained through our analysis of the ATCA data is comparable to that of Fermi-LAT as well as MAGIC (at higher energies). For annihilation into $b\bar{b}$ the ATCA data is slightly more constraining than the Fermi-LAT data, whilst for annihilation into $\tau^+ \tau^-$, the ATCA data is of the same order of magnitude, Regis et al. (2014c) The MAGIC data provides stronger constraints in the high energy regime.

6.3 Comparison to X-Ray Observations

Inverse Compton scattering occurs as a secondary emission mechanism resulting from WIMP annihilation. This process occurs when electrons and positrons are deflected by charged particles. In particular we can test the population of non-thermal $e^{\pm}$ produced during Dark Matter annihilations through inverse Compton scattering on the CMB, Jeltama & Profumo (2008). The resulting radiation is emitted at X-ray frequencies.

Data from the XMM-Newton telescope has been examined where it covers the fields of three dSph, in particular Carina, Fornax and Ursa Minor. Under the assumptions of a Milky Way-like spatial diffusion, an NFW Dark Matter profile and a $M_\chi = 10\text{GeV}$, these analyses have obtained an upper bound on the velocity averaged annihilation rate of $\langle \sigma v \rangle_\chi = 10^{-22} - 10^{-23} \text{cm}^3 \text{s}^{-1}$. Future observations of X-ray emissions from dSph could yield stronger constraints and are highly complementary to the radio search for secondary emissions of Dark Matter.

It is possible to make direct comparisons between constraints from X-ray data and those from radio observations when we use a common diffusion scheme. The assumptions required in both analyses are the same, with the magnetic field strength being the only assumption used in radio analyses, which is not crucial to X-ray analyses. This could potentially lead to the use of comparative data to constrain the magnetic fields within dSph. Comparison of constraints from the ATCA observations discussed show the bounds achieved are four orders of magnitude more constraining on the annihilation rate than those obtained through X-ray observations, assuming a magnetic field strength of $B \gtrsim 0.01\mu\text{G}$.

6.4 Perspectives

Within the next ten years, it will be possible to encroach on the WIMP parameter space using new radio observations of dSph. There are a number of radio telescope arrays under construction, nearing completion or recently constructed, which can be used to probe Dark Matter further. These include ASKAP, MeerKAT, SKA (phase 1 and phase 2), LOFAR and JVLA.

The JVLA could allow us to probe a larger sample of dSph using the same strategies discussed in this thesis, by allowing us to conduct a similar survey in the northern hemisphere. This will be achieved by the use of the SKA as well as its precursors such as MeerKAT, regardless of the astrophysical assumptions.

The Evolutionary Map of the Universe (EMU) is a key science project being undertaken by ASKAP. This project will provide deep continuum data with a sensitivity of $10\mu\text{Jy}$. The survey will encompass a field of view of 30° squared with a $10''$ resolution. This survey will thus have greater sensitivity than the data presented. In addition the survey encompasses 14 known dSph.

The number of known dSph is increasing with each subsequent optical survey and we expect surveys of the southern sky such as SkyMapper and the Dark Energy Survey to reveal more dSph within the field of view of the EMU survey. Such an increase in the number of dSph for which we have deep radio data will allow for

a large gain in the strength of the upper bound on the Dark Matter annihilation rate.

The MeerKAT FoV is smaller than that of ASKAP yet this upcoming telescope will have a much higher surveying speed. Thus this telescope will be best utilised in the Dark Matter search by obtaining deep observations of the most promising dSph. Deep observations with MeerKAT are expected to achieve a noise level of $\lesssim 1\mu Jy$. This represents an increase in sensitivity by a factor of ~ 50 increase in sensitivity compared to the ATCA data discussed.

The SKA phase 1 should offer a further increase in sensitivity of as much as two orders of magnitude, while SKA phase 2 promise to further increase SKA phase 1 sensitivity by a factor of ~ 10 . The total gain in sensitivity expected is around a factor of 10^3 compared to the ATCA data presented here, Regis et al. (2014c). Further improvements will result from the ability to observe Faraday rotations within dSph, allowing us to further constrain the dSph magnetic field strength.

It is important to note that in probing faint flux in extended emission, confusion becomes a significant issue, impacting the data more strongly as the flux becomes fainter. This factor makes the subtraction of point sources critical and could have significant impact on the sensitivity of future studies.

In addition, there exists a very low level contribution to the radio emission from non-thermal emissions. These emissions are associated with the star formation within dSph. As we reach very high observational sensitivities, we will begin to observe these contributions and we will need to detangle them from the secondary emissions resulting from Dark Matter annihilation.

6.5 Conclusions

In this thesis, we explore deep radio probes of Dark Matter. We began by describing the history of the search for Dark Matter and the compelling evidence that Dark Matter exists. We also discussed in a broad fashion, the various techniques for exploring Dark Matter. From many indirect techniques such as neutrino detection or gamma-ray studies, we have chosen to search for emission signals from particles produced during WIMP annihilation, specifically signals from synchrotron emission.

We then continued by describing the annihilation products and their associated emissions. We used this information to determine which astrophysical structures would be good targets for detections of Dark Matter annihilation. We find that dSph are very well suited to this type of detection as they are close to the Milky Way galaxy, numerous, contain large amounts of Dark Matter, contain no sources of diffuse radio, X-ray or γ -ray radiation and are comparatively simple structures.

We then described in detail the nature of dSph.

In order to take our probe of Dark Matter further, we laid out the theoretical framework required to make predictions of the Dark Matter annihilation signals. We laid out our assumptions (specific to dSph) and made predictions of the synchrotron emissivity. We then went on to discuss our radio observations of six dSph using ATCA. This forms the main body of our work.

We obtained data from deep radio observations of six dSph, namely Carina, Forax, Sculptor, Hercules, BootesII and Segue2. These data were reduced and analysed as discussed in Chapter 5. Strong emphasis was placed on the cataloguing and extraction of background point-like sources. This issue is important as the resolution required over the extended emission means that confusion is a critical problem for the analysis of the observation data for signals from each dSph. We produced a catalog of 1392 sources within the six dSph.

We do not achieve a detection of a non-thermal diffuse emission from these dSph. We can however use the data to place an upper bound on the Dark Matter annihilation rate as a function of the neutralino mass. We discuss this issue qualitatively and we note that the bounds can be compared to those of Natarajan et al. (2013); Spekkens et al. (2013) and those of the Fermi-LAT 5-year data. The bounds, which can be obtained from these data, are more constraining than those of Spekkens et al. (2013) and Natarajan et al. (2013). The upper bounds are also comparable with those of the Fermi-LAT collaboration and in the most optimistic combination of halo shape, magnetic field and diffusion scheme, are stronger than the Fermi-LAT upper bounds. The greatest uncertainty in the results comes from the poorly understood strength of the magnetic field within these systems. The results are also affected by the shape of the Dark Matter halo, which is poorly constrained.

These results can be improved on through surveys utilising forthcoming instruments. These include improvements due to better resources for determining and constraining the magnetic field strength (for example instruments with wide bandwidth such as SKA phase 1). In addition, we will be able to improve on these limits by conducting similar surveys in the Northern hemisphere (allowing us to probe a greater number of dSph). These limits will also be improved on by the expected massive increases in sensitivity and survey speed of upcoming instruments.

The confusion problem is expected to be a major issue for future surveys as an increase in sensitivity results reaching confusion limits more rapidly. This work may then form a basis from which future source subtraction procedures can be developed.

Chapter 7

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Appendices

Appendix A

Synchrotron Emissivity - Derivation

Synchrotron Radiation

When charged particles pass through a magnetic field, $\vec{B}$, they will be accelerated by that field. This acceleration is perpendicular to the magnetic field and will cause the particles to spiral about the magnetic field - the particles are accelerated radially. As a result of this acceleration, the gyrating particles will emit radiation, and for high energy electrons, this emission is called synchrotron radiation.

A.1 Volume Emissivity

We wish to calculate the emissivity per unit volume, $J(\nu)$, of the synchrotron emission for an electron spectrum, $N(E)dE$. The volume emissivity is the power emitted per unit volume, per unit frequency from a cloud of atoms or ions.

A.1.1 Approximate Volume Emissivity

The volume emissivity is roughly the energy radiated per unit time in the range ν to $\nu + d\nu$ by electrons with energies between E and $E + dE$. The number of electrons in this energy range is given by $N(E)dE$, where $N(E)$ is the electron energy distribution. We can thus approximate the volume emissivity by

$$J(\nu) = \left(-\frac{dE}{dt} \right) N(E) \frac{dE}{d\nu} \quad (\text{A.1})$$

For this analysis we will consider a general case of a power law distribution of electron energies

$$N(E) = n_0 \left(\frac{E}{E_*} \right)^{-\alpha} h\left(\frac{r}{r_0}\right) \quad (\text{A.2})$$

where $h(\frac{r}{r_0})$ is the spatial distribution of our electron spectrum.

We now need to consider the energy loss rate of synchrotron emission in order to

find the volume emissivity. It can be shown that a high energy electron moves in a spiral path, at a constant pitch angle ϕ , in a uniform magnetic field. The acceleration is perpendicular to the field lines of the magnetic field and so the velocity of the electron is constant along the direction of the magnetic field. The electron will rotate about the magnetic field direction with relativistic gyrofrequency

$$\nu_g = \frac{eB}{2\pi\gamma m_e} \quad (\text{A.3})$$

where γ is the usual Lorentz factor. We also have that the radiation loss rate of a charged particle, in the laboratory frame of reference is given by:

$$-\left(\frac{dE}{dt}\right)_{rad} = \frac{q^2\gamma^4}{6\pi\epsilon_0 c^3} [|a_\perp|^2 + \gamma^2 |a_\parallel|^2] \quad (\text{A.4})$$

As mentioned above, the acceleration is perpendicular to the velocity of the electron so

$$a_\parallel = 0 \quad (\text{A.5})$$

If the magnetic field is uniform, the acceleration leads to circular motion about the magnetic field direction giving

$$a_\perp = \frac{evB\sin\phi}{\gamma m_e} \quad (\text{A.6})$$

Substituting Equations A.5 and A.6 into Equation A.4 we obtain

$$-\left(\frac{dE}{dt}\right)_{rad} = \frac{e^4 B^2}{6\pi\epsilon_0 c m_e^2} \frac{v^2}{c^2} \gamma^2 \sin^2\phi \quad (\text{A.7})$$

The pitch angle ϕ determines the energy loss rate of the electrons. A high energy electron is randomly scattered in pitch angle and so in order to obtain an expression for the average loss rate, we average over the distribution of the pitch angles. Due to irregularities in the magnetic field and streaming instabilities the pitch angle is likely to be randomised and so the distribution is expected to be isotropic. The expected distribution is thus given by

$$p(\phi)d\phi = \frac{1}{2} \sin\phi d\phi \quad (\text{A.8})$$

We thus find the average energy loss rate for synchrotron emission

$$\begin{aligned} -\left(\frac{dE}{dt}\right) &= \frac{e^4 B^2}{6\pi\epsilon_0 c m_e^2} \frac{v^2}{c^2} \gamma^2 \int_0^\pi \sin^2\phi p(\phi) d\phi \\ &= \frac{1}{9} \frac{e^4 B^2}{\pi\epsilon_0 c m_e^2} \frac{v^2}{c^2} \gamma^2 \end{aligned} \quad (\text{A.9})$$

We also know

$$\begin{aligned}
E &= \gamma m_e c^2 \\
&= \left(\frac{\nu}{\nu_g} \right)^{\frac{1}{2}} m_e c^2 \\
&= \sqrt{\frac{2\pi m_e^3 c^4 \nu}{eB}} \quad \text{from Equation A.3}
\end{aligned} \tag{A.10}$$

Thus using Equations A.2, A.9 and A.10 in Equation A.1 and simplifying, we obtain

$$J(\nu) = \frac{n_0 E_*^\alpha}{9\epsilon_0} (2\pi)^{\frac{1-\alpha}{2}} c^{1-2\alpha} e^{\frac{5+\alpha}{2}} B^{\frac{1+\alpha}{2}} \nu^{\frac{1-\alpha}{2}} m_e^{\frac{1-3\alpha}{2}} h\left(\frac{r}{r_0}\right) \tag{A.11}$$

This equation gives us the correct form for the emissivity, however, in order to find the exact volume emissivity we must first find the emissivity of a single electron and then integrate the contributions of electrons of different energies to the intensity.

A.1.2 Exact Volume Emissivity

We begin with the Liénard-Wiechert potentials:

$$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi r} \left[\frac{q \vec{v}}{1 - \frac{\vec{v} \cdot \vec{n}}{c}} \right]_{ret} \tag{A.12}$$

$$\phi(\vec{r}, t) = \frac{1}{4\pi\epsilon_0 r} \left[\frac{q}{1 - \frac{\vec{v} \cdot \vec{n}}{c}} \right]_{ret} \tag{A.13}$$

From these we can write down the expression for the relation between the acceleration and the spectral energy distribution of the radiation for an arbitrarily moving electron.

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2}{16\pi^2\epsilon_0 c} \left| \int_{-\infty}^{\infty} \left\{ \vec{n} \times \left[\left(\vec{n} - \frac{\vec{v}}{c} \right) \times \frac{\dot{\vec{v}}}{c} \right] \left(1 - \frac{\vec{v} \cdot \vec{n}}{c} \right)^{-3} \right\} e^{i\omega t} dt \right|_{ret}^2 \tag{A.14}$$

Using the relations

$$t' = t - \frac{R(t')}{c}$$

$$R(t') = |\vec{r}| - \vec{n} \cdot \vec{r}_0(t')$$

$$\vec{n} \times \left[\left(\vec{n} - \frac{\vec{v}}{c} \right) \times \frac{\dot{\vec{v}}}{c} \right] \left(1 - \frac{\vec{v} \cdot \vec{n}}{c} \right)^{-2} = \frac{d}{dt'} \left\{ \left(1 - \frac{\vec{v} \cdot \vec{n}}{c} \right)^{-1} \left[\vec{n} \times \left(\vec{n} \times \frac{\vec{v}}{c} \right) \right] \right\}$$

we can simplify Equation A.14 to obtain

$$\frac{dI(\omega)}{d\Omega} = \frac{e^2 \omega^2}{16\pi^3 \epsilon_0 c} \left| \int_{-\infty}^{\infty} \vec{n} \times \left(\vec{n} \times \frac{\vec{v}}{c} \right) \exp \left\{ i\omega \left(t' - \frac{\vec{n} \cdot \vec{r}_0(t')}{c} \right) \right\} dt' \right|^2 \tag{A.15}$$

In order to simplify the calculations we make a change of coordinates. Since the electron spirals about the magnetic field lines, at angular frequency $2\pi\nu_g$ and pitch

angle ϕ , at any time the orbit has a certain radius of curvature $\vec{a}$. We take it's instantaneous plane of orbit to be the x-y plane and take the x-axis to have it's origin at the point where the velocity vector $\vec{v}$ lies in the x-z plane which includes the observer. We take the y-axis as the direction of the instantaneous radius vector. We now have $\vec{n}$ parallel to the direction of the magnetic field. We then define our new coordinate system with the same origin as our (x,y,z) coordinates. We define the unit vector $\vec{\epsilon}_{\parallel}$ to be lying in the plane containing $\vec{n}$ and the magnetic field lines and $\vec{\epsilon}_{\perp}$ to be lying along the y-axis so we now have $\vec{\epsilon}_{\parallel} = \vec{n} \times \vec{\epsilon}_{\perp}$

This new coordinate system leads us to integrals for the intensities in the $\vec{\epsilon}_{\parallel}$ and $\vec{\epsilon}_{\perp}$ directions

$$\begin{aligned} \frac{dI_{\perp}(\omega)}{d\Omega} &= \frac{e^2\omega^2}{16\pi^3\epsilon_0 c} \left| \int_{-\infty}^{\infty} \frac{\nu t'}{a} \exp \left\{ \frac{i\omega}{2\gamma^2} \left[t'(1 + \gamma^2\theta^2) + \frac{c^2\gamma^2}{3a^2} t'^3 \right] \right\} dt' \right|^2 \\ \frac{dI_{\parallel}(\omega)}{d\Omega} &= \frac{e^2\omega^2}{16\pi^3\epsilon_0 c} \left| \int_{-\infty}^{\infty} \exp \left\{ \frac{i\omega}{2\gamma^2} \left[t'(1 + \gamma^2\theta^2) + \frac{c^2\gamma^2}{3a^2} t'^3 \right] \right\} dt' \right|^2 \end{aligned} \quad (\text{A.16})$$

We can then use change of variables to reduce our integrals into a standard form. We make the following substitutions:

$$\theta_{\gamma}^2 = (1 + \gamma^2\theta^2) \quad (\text{A.17})$$

$$y = \frac{\gamma ct'}{a\theta_{\gamma}} \quad (\text{A.18})$$

$$\eta = \frac{\omega a \theta_{\gamma}^3}{3c\gamma^3} \quad (\text{A.19})$$

This gives us

$$\frac{dI_{\perp}(\omega)}{d\Omega} = \frac{e^2\omega^2}{16\pi^3\epsilon_0 c} \left(\frac{a\theta_{\gamma}^2}{c\gamma^2} \right) \left| \int_{-\infty}^{\infty} y \exp \left[i\eta \frac{3}{2} \left(y + \frac{y^3}{3} \right) \right] dy \right|^2 \quad (\text{A.20})$$

$$\frac{dI_{\parallel}(\omega)}{d\Omega} = \frac{e^2\omega^2}{16\pi^3\epsilon_0 c} \left(\frac{a\theta_{\gamma}^2}{c\gamma^2} \right) \left| \int_{-\infty}^{\infty} \exp \left[i\eta \frac{3}{2} \left(y + \frac{y^3}{3} \right) \right] dy \right|^2 \quad (\text{A.21})$$

Then using the modified Bessel functions

$$\frac{1}{\sqrt{3}} K_{\frac{1}{3}}(\eta) = \int_0^{\infty} \cos \left[\frac{3\eta}{2} \left(x + \frac{1}{3}x^3 \right) \right] dx \quad (\text{A.22})$$

$$\frac{1}{\sqrt{3}} K_{\frac{2}{3}}(\eta) = \int_0^{\infty} x \sin \left[\frac{3\eta}{2} \left(x + \frac{1}{3}x^3 \right) \right] dx \quad (\text{A.23})$$

We can obtain

$$\frac{dI_{\perp}(\omega)}{d\Omega} = \frac{e^2\omega^2}{12\pi^3\epsilon_0 c} \left(\frac{a\theta_{\gamma}^2}{c\gamma^2} \right)^2 K_{\frac{2}{3}}(\eta) \quad (\text{A.24})$$

$$\frac{dI_{\parallel}(\omega)}{d\Omega} = \frac{e^2\omega^2\theta^2}{12\pi^3\epsilon_0 c} \left(\frac{a\theta_{\gamma}^2}{c\gamma^2} \right)^2 K_{\frac{1}{3}}(\eta) \quad (\text{A.25})$$

Most of the radiation is emitted in small angle θ with respect to ϕ . This means that over the period of one of the electron's gyration about the magnetic field, we take the integral over the angle $2\pi\sin\phi d\theta$ and we obtain

$$I_{\perp}(\omega) = \frac{e^2\omega^2 a^2 \sin\phi}{6\pi^2\epsilon_0 c^3 \gamma^2} \int_{-\infty}^{\infty} \theta_{\gamma}^4 K_{\frac{2}{3}}^2(\eta) d\theta \quad (\text{A.26})$$

$$I_{\parallel}(\omega) = \frac{e^2\omega^2 a^2 \sin\phi}{6\pi^2\epsilon_0 c^3 \gamma^2} \int_{-\infty}^{\infty} \theta_{\gamma}^2 K_{\frac{1}{3}}^2(\eta) d\theta \quad (\text{A.27})$$

Then setting

$$x = \frac{2\omega a}{3c\gamma^3} \quad (\text{A.28})$$

$$F(x) = x \int_x^{\infty} K_{\frac{5}{3}} z dz \quad (\text{A.29})$$

$$G(x) = x K_{\frac{2}{3}}(x) \quad (\text{A.30})$$

We can write

$$I_{\perp}(\omega) = \frac{\sqrt{3}e^2\gamma\sin\phi}{8\pi\epsilon_0 c} [F(x) + G(x)] \quad (\text{A.31})$$

$$I_{\parallel}(\omega) = \frac{\sqrt{3}e^2\gamma\sin\phi}{8\pi\epsilon_0 c} [F(x) - G(x)] \quad (\text{A.32})$$

The above integration over $2\pi\sin\phi d\theta$ represents the energy emitted by a single electron in the two orthogonal polarisations during one period of it's orbit. The time taken for the electron to orbit the magnetic field once is given by

$$\begin{aligned} T_r &= \nu_r^{-1} \\ &= \frac{2\pi\gamma m_e}{eB} \end{aligned} \quad (\text{A.33})$$

The emissivity of the electron is thus given by

$$\begin{aligned} j(\omega) &= \frac{I(\omega)}{T_r} \\ &= \frac{I_{\perp}(\omega) + I_{\parallel}(\omega)}{T_r} \\ &= \frac{\sqrt{3}e^3 B \sin\phi}{8\pi^2\epsilon_0 c m_e} F(x) \end{aligned} \quad (\text{A.34})$$

where $I(\omega)$ is the specific intensity of the emission and T_r is the time taken for a single orbit of the electron in the magnetic field, during which the energy given by the specific intensity is emitted.

We can now find the emissivity per unit volume as described above

$$J(\omega) = \int_0^{\infty} j(x) N(E) dE \quad (\text{A.35})$$

We need to take this integral at fixed ω or equivalently, $x = \frac{\omega}{\omega_c}$

$$\begin{aligned} x &= \frac{\omega}{\omega_c} \\ &= \frac{2\omega m_e^3 c^4}{3eB \sin \phi} \end{aligned} \quad (\text{A.36})$$

$$= \frac{A}{E^2} \quad (\text{A.37})$$

Using Equation A.37 we can write

$$E = \sqrt{\frac{A}{x}} \quad (\text{A.38})$$

$$dE = -\frac{1}{2} \sqrt{\frac{A}{x^3}} dx \quad (\text{A.39})$$

This allows us to rewrite the integral in Equation A.35 as

$$\begin{aligned} J(\omega) &= \int_0^\infty \left(\sqrt{\frac{A}{x}} \right)^{-\alpha} n_0 h \left(\frac{r}{r_0} \right) E_*^\alpha j(x) \left(-\frac{1}{2} \sqrt{\frac{A}{x^3}} dx \right) \\ &= \frac{\sqrt{3} n_0 E_*^\alpha e^3 B \sin \phi A^{\frac{1-\alpha}{2}}}{16\pi^2 \epsilon_0 c m_e} h \left(\frac{r}{r_0} \right) \int_0^\infty F(x) x^{\frac{\alpha-3}{2}} dx \end{aligned} \quad (\text{A.40})$$

It can be shown that

$$\int_0^\infty x^\mu F(x) dx = \frac{2^{\mu+1}}{\mu+2} \Gamma \left(\frac{\mu}{2} + \frac{7}{3} \right) \Gamma \left(\frac{\mu}{2} + \frac{2}{3} \right) \quad (\text{A.41})$$

So replacing μ with $\alpha - 3$ we obtain

$$\begin{aligned} J(\omega) &= \frac{3^\alpha n_0 E_*^\alpha e^{\frac{5+\alpha}{2}} m_e^{\frac{1-3\alpha}{2}} B^{\frac{\alpha+1}{2}} \sin^{\frac{\alpha+1}{2}} \phi \omega^{\frac{1-\alpha}{2}} c^{1-2\alpha} h \left(\frac{r}{r_0} \right)}{8\pi^2 \epsilon_0 (\alpha+1)} \\ &\quad \times \left(\Gamma \left(\frac{\alpha}{4} + \frac{19}{12} \right) \Gamma \left(\frac{\alpha}{4} - \frac{1}{12} \right) \right) \end{aligned} \quad (\text{A.42})$$

Again, the pitch angles will be isotropically distributed and so wish to integrate over the distribution $p(\phi) d\phi = \frac{1}{2} \sin \phi d\phi$. Making use of the integral

$$\frac{1}{2} \int_0^\pi \sin^{\frac{\alpha+3}{2}} \phi d\phi = \frac{\sqrt{\pi}}{2} \frac{\Gamma \left(\frac{\alpha+5}{4} \right)}{\Gamma \left(\frac{\alpha+7}{4} \right)} \quad (\text{A.43})$$

We obtain the volume emissivity for a power spectrum of electrons

$$J(\omega) = \frac{3^\alpha \sqrt{\pi} n_0 E_*^\alpha e^{\frac{5+\alpha}{2}} m_e^{\frac{1-3\alpha}{2}} B^{\frac{\alpha+1}{2}} \omega^{\frac{1-\alpha}{2}} c^{1-2\alpha} h \left(\frac{r}{r_0} \right)}{16\pi^2 \epsilon_0 (\alpha+1)} \frac{\Gamma \left(\frac{\alpha}{4} + \frac{19}{12} \right) \Gamma \left(\frac{\alpha}{4} - \frac{1}{12} \right) \Gamma \left(\frac{\alpha+5}{4} \right)}{\Gamma \left(\frac{\alpha+7}{4} \right)} \quad (\text{A.44})$$

or written in terms of frequency

$$J(\nu) = \frac{3^\alpha (2\pi)^{\frac{1-\alpha}{2}} n_0 E_*^\alpha \Gamma\left(\frac{\alpha}{4} + \frac{19}{12}\right) \Gamma\left(\frac{\alpha}{4} - \frac{1}{12}\right) \Gamma\left(\frac{\alpha+5}{4}\right)}{16\pi^{\frac{3}{2}} \epsilon_0 (\alpha+1) \Gamma\left(\frac{\alpha+7}{4}\right)} e^{\frac{5+\alpha}{2}} m_e^{\frac{1-3\alpha}{2}} B^{\frac{\alpha+1}{2}} \nu^{\frac{1-\alpha}{2}} c^{1-2\alpha} h\left(\frac{r}{r_0}\right) \quad (\text{A.45})$$

Comparing this to the approximate form in Equation A.11

$$J(\nu) = \frac{n_0 E_*^\alpha}{9\epsilon_0} (2\pi)^{\frac{1-\alpha}{2}} c^{1-2\alpha} e^{\frac{5+\alpha}{2}} B^{\frac{1+\alpha}{2}} \nu^{\frac{1-\alpha}{2}} m_e^{\frac{1-3\alpha}{2}} h\left(\frac{r}{r_0}\right)$$

we see that they contain the same functional form and dependence on physical parameters but differ in the constant factors.

A.1.3 Spatial Dependencies

We wish to see how these two functions behave as a function of certain values of n_0 , α and different magnetic fields. Before this can be done we need to first consider the spatial dependences of our magnetic field and electron density distribution. We first note that we can write our magnetic field in terms of a magnitude and a spatial dependence as $B = B_0 g\left(\frac{r}{r_0}\right)$. Then our function $g\left(\frac{r}{r_0}\right)$ could have the form $\left(\frac{r}{r_0}\right)^{-b}$. If we then consider the spatial dependence of the electron density distribution we might think that our function $h\left(\frac{r}{r_0}\right)$ should similarly have the form $\left(\frac{r}{r_0}\right)^{-d}$. However if we think about this in physical terms this would imply a density of electrons that is infinite at the centre of our system. This means that the electron density distribution needs to have a 'core' in it's spatial dependence. From observations we know that for thermal bremsstrahlung the spatial form is

$$h\left(\frac{r}{r_0}\right) = \left[1 + \left(\frac{r}{r_0}\right)^2\right]^{\frac{-3\beta}{2}} \quad \text{where } \beta \in [0.5, 1] \quad (\text{A.46})$$

We used a mock distribution for our Dark Matter where the spatial dependence has the form

$$h\left(\frac{r}{r_0}\right) = \left[1 + \left(\frac{r}{r_0}\right)\right]^{-\chi} \quad (\text{A.47})$$

These two distributions both have a maximum value of 1 for $r = 0$, i.e they do not blow up to infinity at the centre of our system and they decrease radially outward as expected. If we make a plot of these two functions for varying $\frac{r}{r_0}$ (see Figure A.1) we see that there is a steeper drop off in the value of $h\left(\frac{r}{r_0}\right)$ for the thermal bremsstrahlung than for the dark matter. The effect of varying the exponent is that the higher the value of α/β , the faster the function drops to zero.

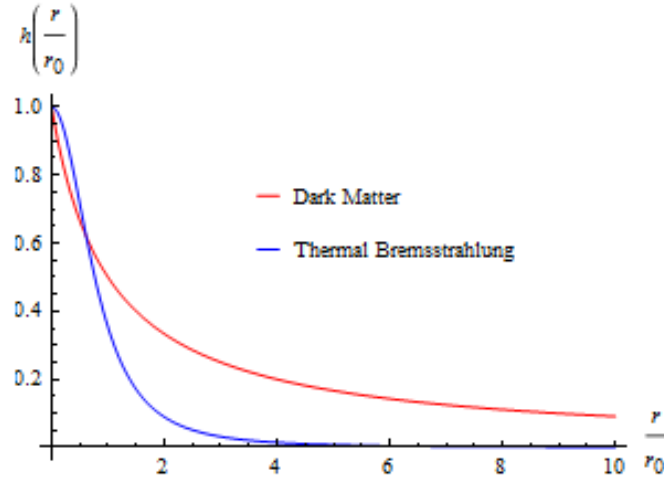


Figure A.1: A plot of the spatial dependence of the electron density distribution, $h\left(\frac{r}{r_0}\right)$ vs radial distance from source for different sources

A.1.4 Variation of Volume Emissivity

We can then plot $J_{approx}(\nu)$ and $J_{exact}(\nu)$ to see how they vary with frequency, magnetic field, n_0 and value of α as well as the spatial form of the magnetic field and the spatial distribution of electrons as well as comparing the exact and approximate forms of the volume emissivity equation. We wish to consider the following values:

$$\begin{aligned}
 E_* &= 1\text{GeV} \\
 n_0 &= 10^{-10} - 10^{-3} e^- cm^{-3} GeV^{-1} \\
 B_0 &= 0.1 - 10\mu G \\
 \alpha &= 2 - 3
 \end{aligned} \tag{A.48}$$

We first look at the difference between the exact and approximate forms of the equation: We can plot our functions at a fixed frequency, magnetic field, electron density spectrum, α and spatial dependence to compare the exact and approximate forms.

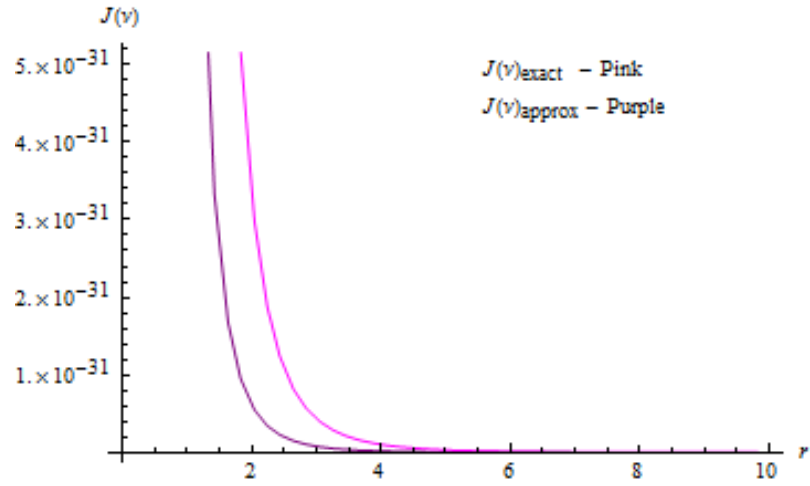


Figure A.2: A plot of $J(\nu)_{\text{exact}}$ and $J(\nu)_{\text{approx}}$ vs radial distance from source

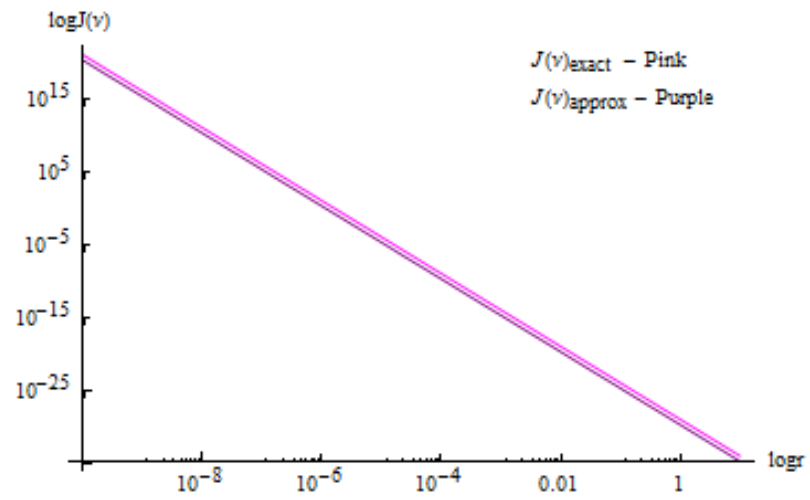


Figure A.3: A plot of $\log J(\nu)_{\text{exact}}$ and $\log J(\nu)_{\text{approx}}$ vs the log of the radial distance from the source

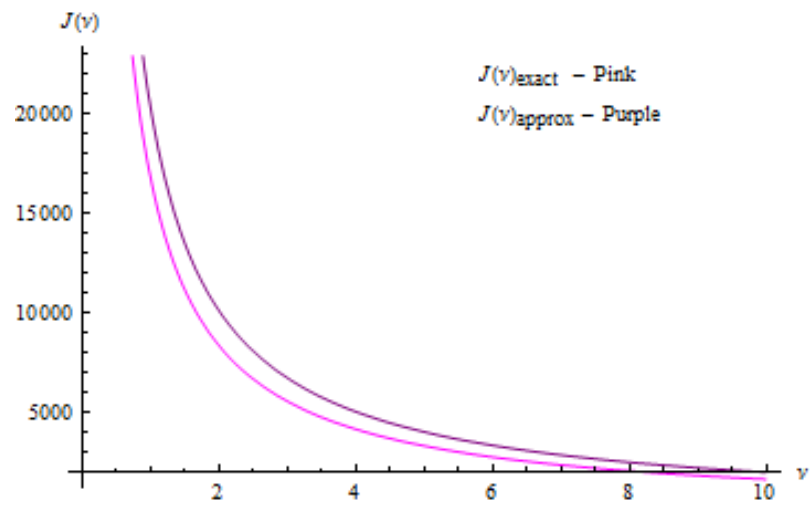


Figure A.4: A plot of $J(\nu)_{\text{exact}}$ and $J(\nu)_{\text{approx}}$ vs source frequency

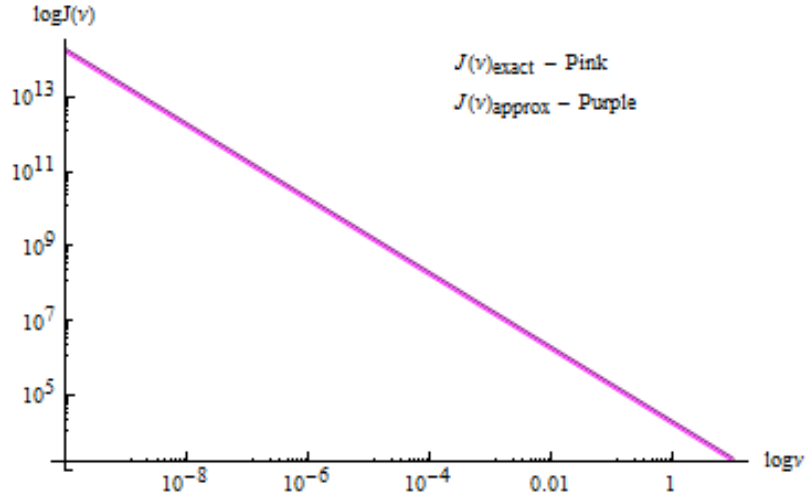


Figure A.5: A plot of $\log J(\nu)_{exact}$ and $\log J(\nu)_{approx}$ vs the log of the source frequency

Figures A.2 – A.5 show us that the functions differ by a small constant. From the loglog plots we can see that our function is a power law with a negative power and the difference between the exact and approximate functions is that of a constant and since the functions are close to one another in the loglog plot, this difference is not a large one.

We can now consider the effect of the value n_0 on the exact function for the emissivity. In order to do this we plot graphs of $J(\nu)$ vs r at constant ν and $J(\nu)$ vs ν at constant r , keeping the magnetic field, α and spatial dependence constant.

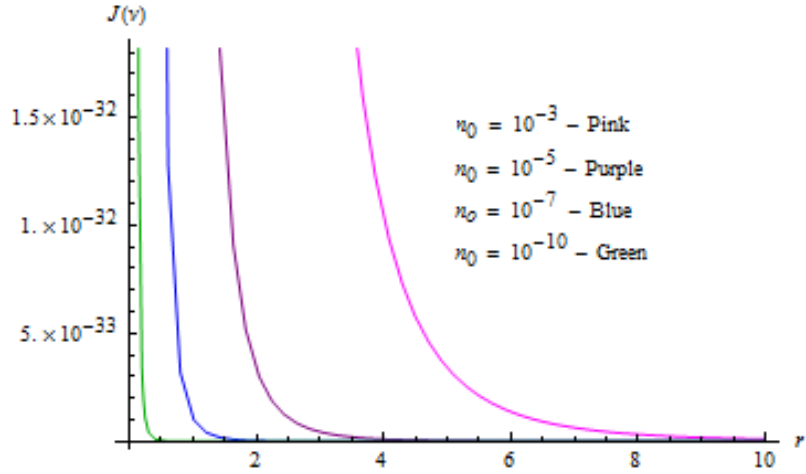


Figure A.6: A plot of $J(\nu)$ vs radial distance from source for different values of n_0

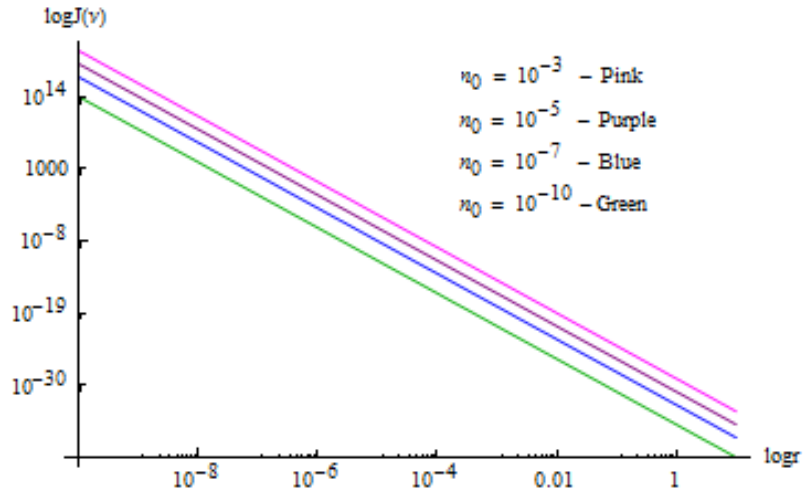


Figure A.7: A plot of $\log J(\nu)$ vs the log of the radial distance from the source for different values of n_0

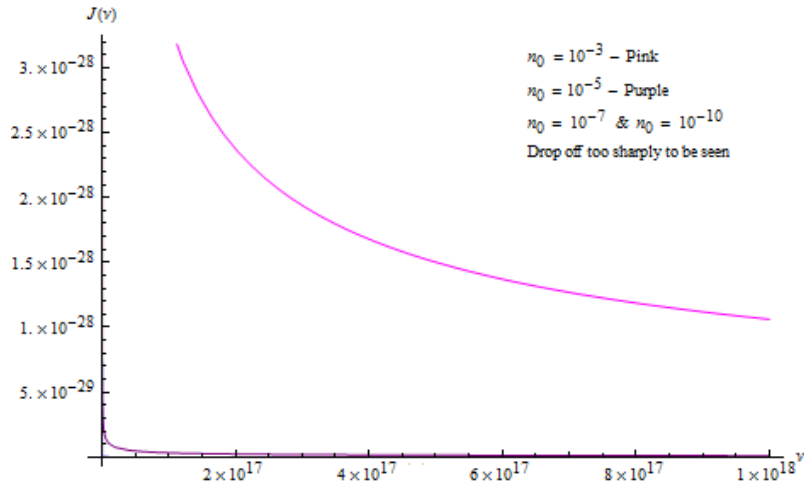


Figure A.8: A plot of $J(\nu)$ vs source frequency for different values on n_0

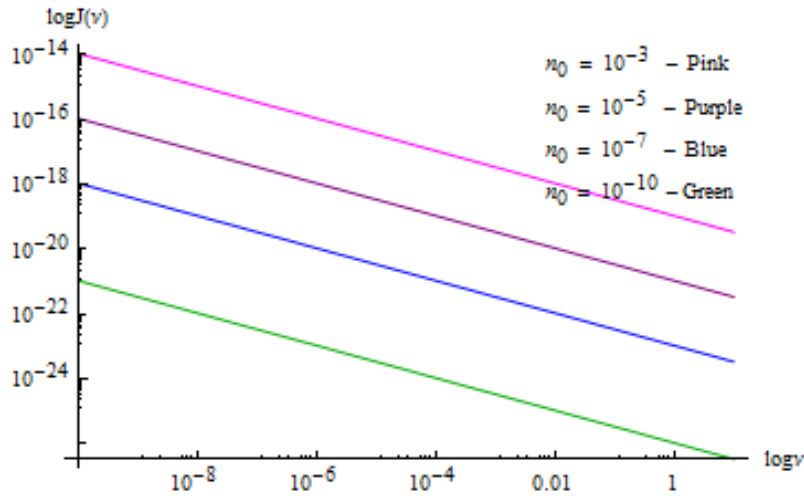


Figure A.9: A plot of $\log J(\nu)$ vs the log of the source frequency for different values of n_0

Figures A.6– A.9 show that the value of n_0 affects the volume emissivity as a constant factor but the effect is greater when we consider the change in the emissivity with frequency than when considering the change with respect to the distance from the source.

We can also consider how our volume emissivity changes with respect to the change of the magnetic field. We plot graphs of the volume emissivity for different magnetic fields at constant distance from the source for changing frequency and for constant frequency as we vary the distance from the source. In all the plots we keep n_0 constant as well as keeping the same spatial distribution and value of α .

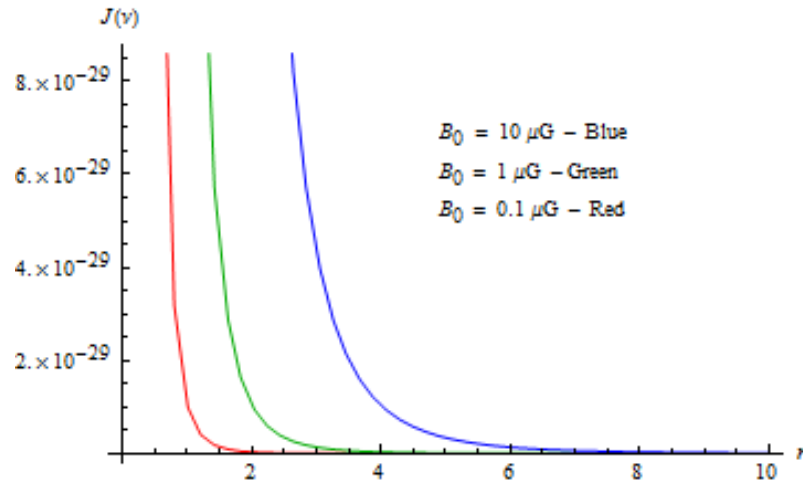


Figure A.10: A plot of $J(\nu)$ vs radial distance from source for different magnetic field strengths

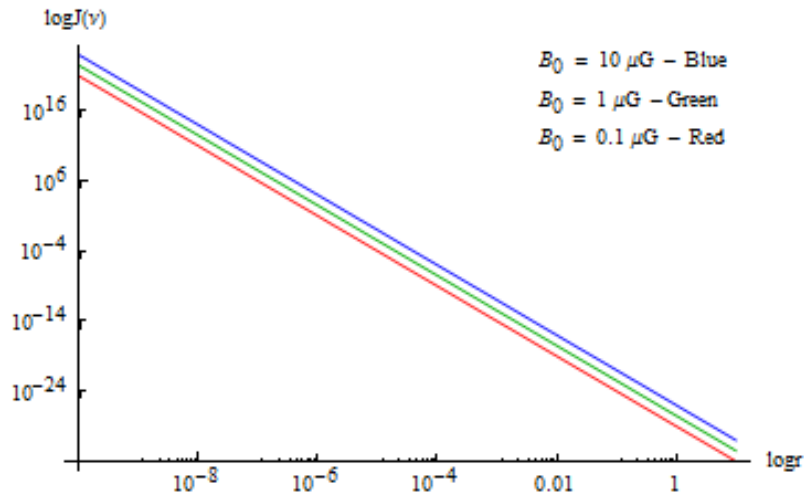


Figure A.11: A plot of $\log J(\nu)$ vs the log of the radial distance from the source for magnetic field strengths

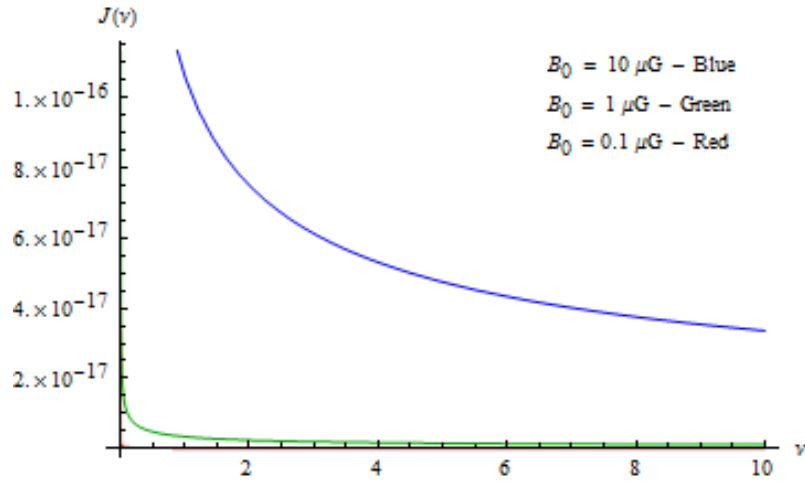


Figure A.12: A plot of $J(\nu)$ vs source frequency for different magnetic field strengths

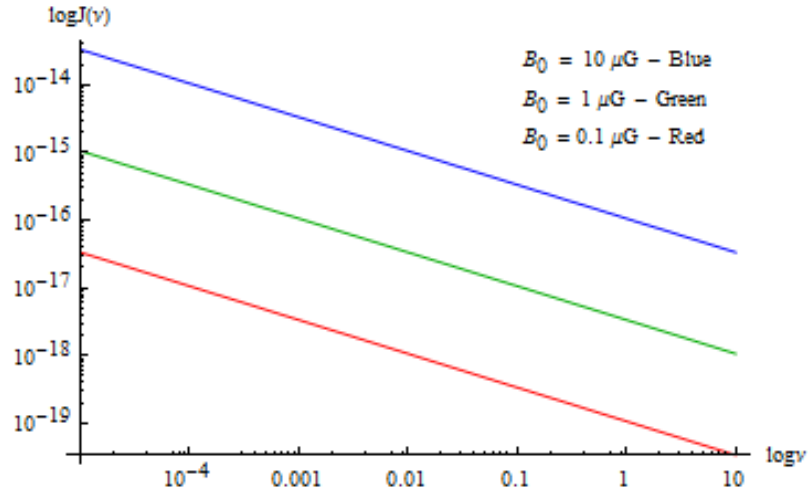


Figure A.13: A plot of $\log J(\nu)$ vs the log of the source frequency for different magnetic field strengths

We see from Figures A.10 – A.13 that the change in magnetic field again changes our volume emissivity by a constant factor, and that this change is larger in the direction of varying ν than in the direction of varying r .

We should now look at how changing the value of α , the power in the power law for the energy distribution of the electrons.

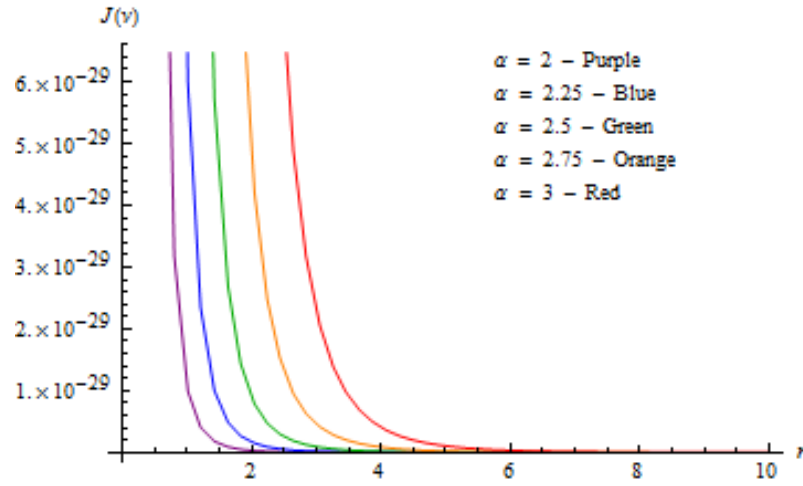


Figure A.14: A plot of $J(\nu)$ vs radial distance from source for different values of α

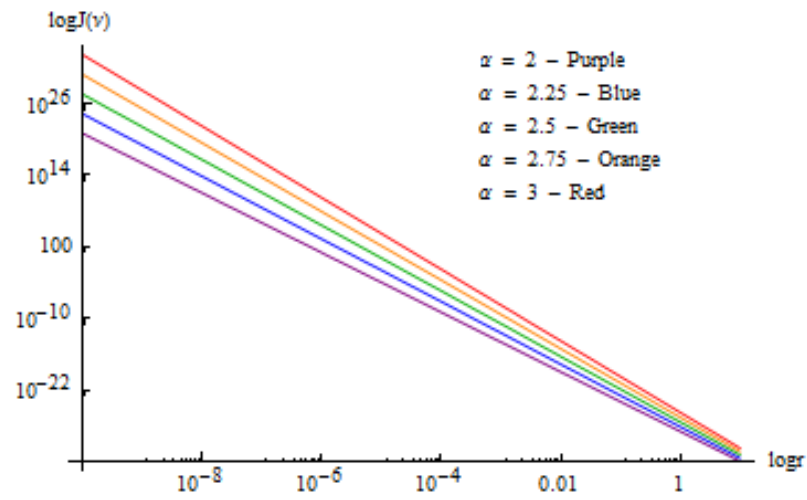


Figure A.15: A plot of $\log J(\nu)$ vs the log of the radial distance from the source for different values of α

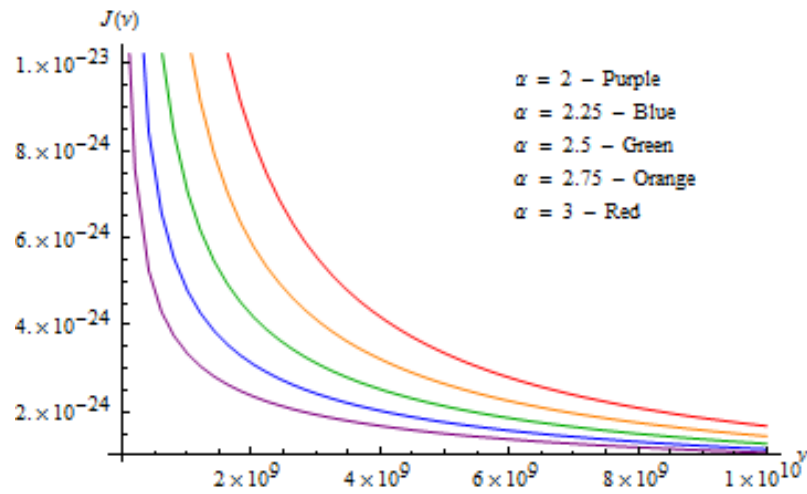


Figure A.16: A plot of $J(\nu)$ vs source frequency for different values of α

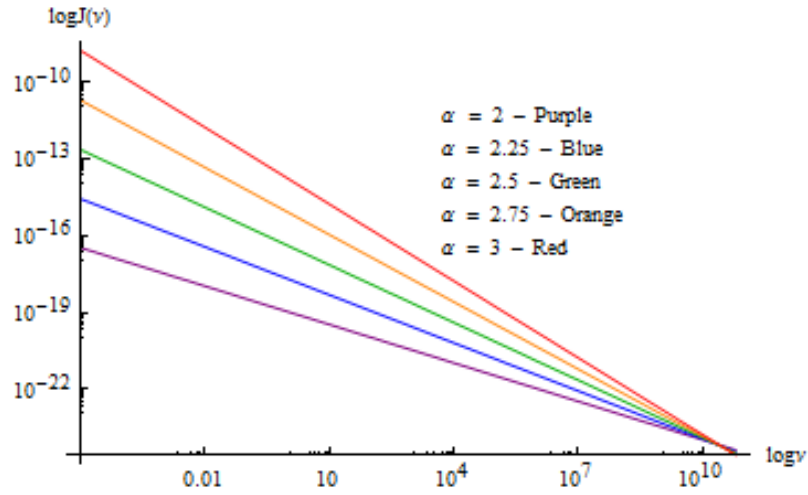


Figure A.17: A plot of $\log J(\nu)$ vs the log of the source frequency for different values of α

We can clearly see from Figures A.14–A.17 that this changes the form of our function. The value of α clearly affects the power in the power law on which the volume emissivity depends.

We can also consider how the volume emissivity is affected by the spatial form of the magnetic field.

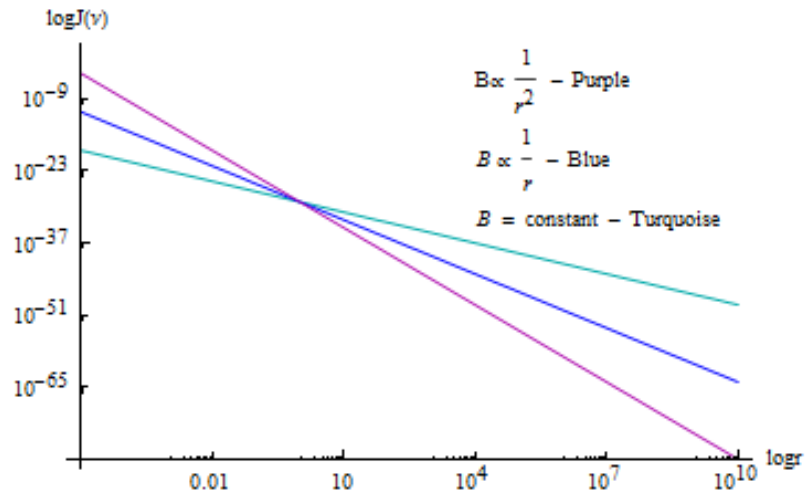


Figure A.18: A plot of $\log J(\nu)$ vs the log of the radial distance from the source for different spatial dependencies of the magnetic field

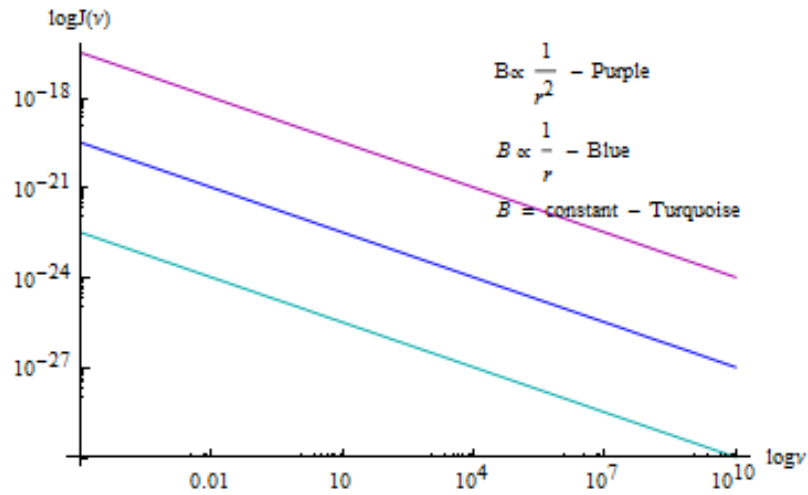


Figure A.19: A plot of $\log J(\nu)$ vs the log of the radial distance from the source for different spatial dependencies of the magnetic field

From Figures A.18–A.19 we can see that the spatial dependency of the magnetic only affects our emission by a constant factor in the direction of changing ν but it changes the form of the dependency in the r direction. We can easily see also that a change in the spatial dependence of the electron energy distribution will likewise influence the form of the volume emissivity in the r dependency but will only change the volume emissivity by a constant factor in the ν dependency.

If we plot a spectrum in three dimensions we can see that the volume emissivity is greatest at low frequencies and small radial distances from the source and that it falls off as a power law as we increase the frequency or as we increase the radial distance from the source. This can be seen in Figure A.20.

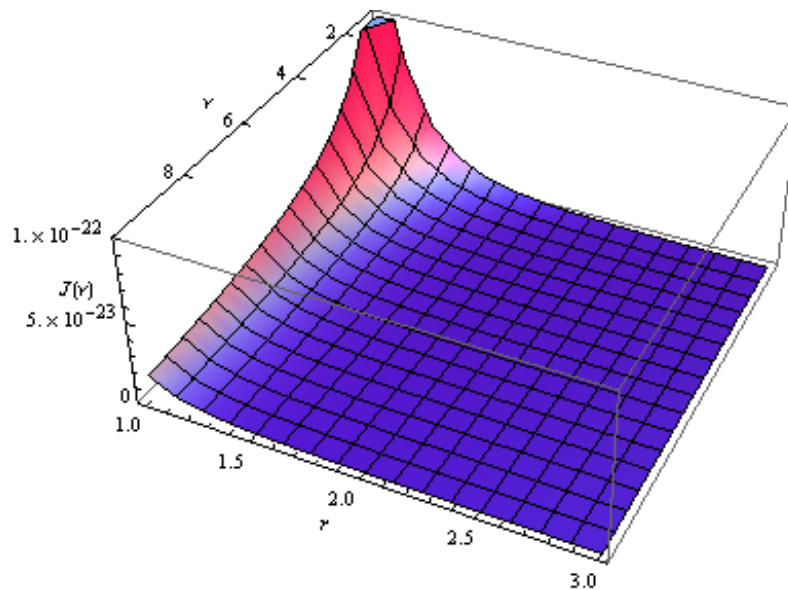


Figure A.20: A plot of $J(\nu)$ vs the radial distance from the source and the frequency