

An Investigation of learners' performance in Algebra from Grades 9 to 11

VASANTHA MOODLEY

A research report submitted to the Wits School of Education, Faculty of Humanities,
University of the Witwatersrand in partial fulfilment of the requirements for the degree
Master of Science by combination of coursework and research

Edenvale
2014

The copyright of this thesis vests in the University of the Witwatersrand, Johannesburg, South Africa, in accordance with the University's Intellectual Property Policy.

No portion of the text may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, including analogue and digital media, without prior written permission from the University. Extracts of or quotations from this thesis may, however, be made in terms of Sections 12 and 13 of the South African Copyright Act No. 98 of 1978 (as amended), for non-commercial or educational purposes. Full acknowledgement must be made to the author and the University.

An electronic version of this thesis may be found via the Library's online catalogue.

For permission requests, please contact the University Legal Office or the University Research Office (www.wits.ac.za).

ABSTRACT

This study is an investigation into the performance of learners in algebra using the levels of understanding as measured by the ICCAMS diagnostic instrument. The study was conducted in two phases. The first phase of the study consisted of an analysis of the scripts of a sample of 29 learners in Grade 9 who had written the test administered by the Wits Maths Connect-Secondary unit at Wits University. The scripts of the same 29 learners in Grade 10 were analysed to determine the progression within the levels of these learners from Grades 9 to 10. Eighteen learners progressed from a lower to a higher level. During the analysis of the tests it was found that the conjoining error was the main obstacle to some learners in progressing from moving from level 1 to level 3.

During phase 2 of the study, a sample of 6 learners was selected from the original 29 learners. These learners completed a written task to investigate errors made in algebra in Grade 11. Interviews were conducted with these learners based on a written task. The analysis of the interviews and written task illustrated the problems learners experienced with level 2 questions, particularly with respect to the conjoining error.

LIST OF ACRONYMS

TIMSS	Trends in International Mathematics and Science Study
UFS	University of the Free State
AARP	Alternative Admissions Research Project
WMC-S	Wits Maths Connect Secondary
ICCAMs	Increasing Competence and Confidence in Algebra and Multiplicative Reasoning
CSMS	Concepts in Secondary Mathematics and Science
VSS	Variable Secondary School

DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.



Vasantha Moodley
6th day of June in the year 2014

DEDICATION

To my beloved parents Shunmugam and Govindammah Moodley who are my most ardent supporters. They live and are guided by the words of Saint Thiruvallavar:

Learning is wealth none could destroy.

Nothing else gives genuine joy

Thiruvallavar

ACKNOWLEDGEMENTS

This report would not have been possible without the support and guidance of many people. My sincere thanks and appreciation must be expressed to the following people:

Thanks to the learners, who so graciously gave up their time and other commitments, to allow me to conduct this study with them. You have made a huge contribution to this study and to the mathematics community in general.

Thank you to the head of the mathematics department at the school in which this study was conducted as well as the deputy principal, both of whom liaised between the learners and me to ensure the smooth progress of the interviews. Thanks also to the principal who granted permission to conduct this study at the school.

Thank you to my supervisor Dr. Craig Pournara for your mentoring and support. Your patience, guidance and suggestions all made in the most gentle manner is much appreciated.

To Vincent whose invaluable assistance throughout this study ensured that it was completed with the minimum of stress.

My fan club made up of Keshnie Padayachee, Meikie Lanyoh and Beth Corns. Your encouragement and belief in me contributed to the completion of this study with my sanity intact.

My fellow masters students Neighbour and Lethu for your ongoing support throughout this study.

The members of the maths department at my school for your consideration and encouragement. A special thanks to Lynn, Heleen and Leigh for assisting in the administration of some of my duties at school.

To Marthie who was always willing to assist, thank you for saving me from many a disaster with the computer.

A very special thank you to Vanitha, Kantha, Mervin and Keith without whose support in so many ways this research would not have been possible.

Table of Contents

ABSTRACT.....	ii
LIST OF ACRONYMS.....	ii
DECLARATION	iii
DEDICATION	iv
ACKNOWLEDGEMENTS.....	v
LIST OF TABLES.....	ix
LIST OF FIGURES.....	x
CHAPTER 1: INTRODUCTION AND RATIONALE	1
1.1. Introduction	1
1.1.1. Problem Statement.....	1
1.1.2. The Study Focus.....	1
1.1.3. Purpose of the Study	2
1.2. The Wits Maths Connect-Secondary project (WMC-S).....	3
1.3. The ICCAMS test.....	3
1.3.1. Description of the levels in the ICCAMS test	5
1.4. Research Questions	6
1.5. Usefulness of the study.....	7
1.6. Conclusion.....	7
CHAPTER 2	9
LITERATURE REVIEW AND THEORETICAL FRAMEWORK.....	9
2.1. Introduction	9
2.2. Algebra in the Curriculum	9
2.3. The Theoretical Framework.....	10
2.3.1. Slips, Mistakes and Errors	13
2.4. Errors in Algebra	14
2.4.1. Classification of errors according to interpretation of letters	14
2.4.2. Other errors in Algebra	17
2.5. Conclusion.....	20
CHAPTER 3	21
RESEARCH DESIGN AND METHODOLOGY.....	21
3.1. Introduction	21
3.2. Overall design of the study	21
3.3. Background to the WMC-S Tests	22

3.4.	The School.....	23
3.5.	The Sample.....	23
3.5.1.	Selection of Sample for Phase One	23
3.5.2.	Selection of Sample for Phase Two	24
3.6.	The Research Instruments	26
	The first phase of my study entailed the analysis of the actual test scripts of the selected 29 learners who had written the WMC-S tests in Grades 9 and 10. The WMC-S tests were conducted towards the end of each year by the WMC-S unit. These tests were conducted along the same lines as the CSMS tests.	26
3.7.	The Pilot Study	27
3.8.	Preparation for interviews	27
3.8.1.	Data Collection Instruments	28
3.9.	Ethical Considerations.....	35
3.10.	Reliability.....	36
3.11.	Validity	37
3.12.	Limitations of the study	38
3.13.	Conclusion.....	39
CHAPTER 4		40
RESULTS AND ANALYSIS OF THE GRADE 9 AND 10 TEST RESPONSES.....		40
4.1.	Introduction	40
4.2.	A Description of Phase 1 of the Study.....	40
4.2.1.	An explanation of the coding used	40
4.2.2.	Challenges experienced in the Re-coding of Scripts	42
4.3.	How the scores were calculated	46
4.4.	Results and Findings.....	47
4.4.1.	Shifts in levels from Grade 9 to Grade 10	47
4.4.2.	Comparison between errors in Grade 9 and Grade 10.....	48
4.4.3.	Level 1 Results.....	51
4.4.4.	Level 2 Results.....	53
4.4.5.	Level 3 Results.....	56
4.4.6.	Level 4 Results.....	57
4.4.7.	Learners who remained at the same level.....	59
4.4.8.	Common Errors at all levels	61
4.5.	Conclusion.....	63
CHAPTER 5		64

RESULTS AND ANALYSES OF GRADE 11 TASK AND INTERVIEWS	64
5.1. Introduction	64
5.2. A Description of the Coding of the Grade 11 Task.....	64
5.3. Results of the Grade 11 Task.....	67
5.4. An analysis of the errors made by learners from Grades 9 to 11	68
5.4.1. Overall results from Grades 9 to 11 as per response to each question.....	70
5.4.2. A discussion of the most persistent errors	71
5.4.3. Errors that have faded	84
5.4.4. Issues relating to level 2 questions	87
5.5. Conclusion	94
CHAPTER 6	95
CONCLUSION AND REFLECTIONS.....	95
6.1. Introduction	95
6.2. Research Questions	96
6.2.1. Has there been a shift in individual learner performance in algebra as evidenced by a change in levels from Grade 9 to Grade 10?.....	96
6.2.2. What errors are made by learners?	97
6.2.3. Which errors continue from Grade 9 to Grade 11?	98
6.2.4. Which errors fade away by Grade 11?.....	99
6.2.5. What are the particular questions on level 2 that posed a challenge to learners?	100
6.3. Reflections.....	101
6.4. Challenges/limitations	102
6.5. Recommendations for teaching.....	103
6.6. Conclusion.....	104
REFERENCES	106
APPENDICES	110
Appendix A ICCAMS sections of the Annual test from WMC-S	110
Appendix B The Grade 11 Task	114
Appendix C Adapted version of ICCAMS marking codes	119
Appendix D Letter to the Principal	121
Appendix E Information Sheet Learners	122
Appendix F Information Sheet Parents/Guardians.....	123
Appendix G Learner Consent Form.....	124
Appendix H Parent's/Guardian's Consent Form	125

LIST OF TABLES

Table 1: A Description of the Hierarchical Levels in the ICCAMS.....	5
Table 2: The Number of Questions in each Hierarchical Level	22
Table 3: A comparison of questions between WMC-S test and Grade 11 task	29
Table 4: Question numbers in WMC-S corresponding to numbers in Grade 11 task.....	33
Table 5: Description of the error codes	40
Table 6: Comparison of error code 9 in 2011 and 2012	41
Table 7: Extract of spread sheet of results of learners in Grade 9.....	46
Table 8: Results of correct responses to level 1 questions.....	51
Table 9: Results of correct responses to level 2 questions.....	53
Table 10: Common response to question 1.4.....	54
Table 11: Results of correct responses to level 3 questions.....	56
Table 12: Results of correct responses to level 4 questions.....	57
Table 13: Errors according to interpretations of letters	61
Table 14: Spread sheet of responses in Grades 9, 10 and 11	66
Table 15: Levels of learners	67
Table 16: Summary of results according to interpretation of letters	70
Table 17: Categories of Conjoining	72
Table 18: Responses by Patricia in each grade	75
Table 19: Questions in Grade 11 task	78
Table 20: Questions 5.4 and 10.4	79
Table 21: Level 2 questions in WMC-S test.....	87
Table 22: Number of correct responses to level 2 questions	90

LIST OF FIGURES

Figure 1: Example of first line coding.....	42
Figure 2: Example of inconsistency in first line coding.....	43
Figure 3: Example of second line coding.....	43
Figure 4: Inconsistency in coding.....	44
Figure 5: An example of the coding of question 11.....	45
Figure 6: Example of error in coding.....	45
Figure 7: The number of learners on each level in Grades 9 and 10.....	47
Figure 8: Brenda's answer to question 2 in WMC-S test in Grade 9.....	49
Figure 9: Brenda's answer to question 2 in Grade 10.....	49
Figure 10: Response to question 2 in Grade 10.....	50
Figure 11: Response to question 2 in Grade 9.....	50
Figure 12: Response to question 4.2.....	50
Figure 13: Grade 9 and 10 results per question on level 1.....	52
Figure 14: Grade 9 and 10 results per question on level 2.....	54
Figure 15: Grade 9 and 10 results per question on level 3.....	56
Figure 16: Grade 9 and 10 results per question on level 4.....	58
Figure 17: Example of response to question 11.....	59
Figure 18: Comparison of common errors between Grades 9 and 10.....	62
Figure 19: Example of recording of errors.....	69
Figure 20: Lettie's response to question 9.2.....	83
Figure 21: Number of correct responses per level 2 question.....	90

CHAPTER 1: INTRODUCTION AND RATIONALE

1.1. Introduction

1.1.1. Problem Statement

“The number of quality Maths and Physical Science passes achieved by the class of 2011 bodes ill for government’s plans to create five million jobs by 2020” (Parker, 2012).

Although newspaper headlines are sometimes exaggerated and made with the aim of sensationalising, this depressing statement was made in response to a statement released by the Department of Education, that less than 20 % of those Grade 12 learners who wrote Mathematics in 2011 obtained a mark higher than 50 % (Parker, 2012). According to the Department of Basic Education, the pass rate in the Grade 12 examination in 2013 is the highest in the last six years. The key issue though, is that the quality of the passes in crucial subjects like mathematics and physical science are “still below desirable levels” (Department of Basic Education, 2014). From a total of 241 509 learners who wrote mathematics, 143 719 learners obtained a mark of below 40 % in the 2013 Grade 12 examination, which translates to 59,5 % of the learners. Although this signifies an improvement by 4,8 percentage points, the Department of Education acknowledges that the low number of learners achieving a mark of above 40% is still cause for concern.

It is no secret therefore that South Africa lags behind the rest of the world in terms of Mathematical attainment. This is verified by the TIMSS study conducted in 1995, in which South African Mathematics learners came last out of 41 countries (Makgato & Mji, 2006). Furthermore, universities lament the lack of sufficient content knowledge of first year learners in Mathematics. This is corroborated by the following statistics provided by Dennis & Murray (2012) from the University of the Free State (UFS). The UFS undertook a battery of Alternative Admissions Research Project (AARP) tests during 2009. According to the AARP Centre, the results indicate that the majority of these learners would require additional support in order to pass Mathematics at university level as the average for the Mathematics tests was 40% (Dennis & Murray, 2012).

1.1.2. The Study Focus

All of the above statistics provide proof of dismal Mathematics results at school leading to poor performance in the subject at university level. The statistics provided, however, only provide results for Mathematics performance in general. There is a very limited amount of literature pertaining to any gains (or lack thereof) made in algebra specifically, in South

Africa. My research is a focus on the Wits Maths Connect Secondary (WMC-S) tests administered to a cohort of learners in a school in Johannesburg in South Africa with a view to investigating their performance in algebra through Grades 9 to 11. The tests comprised of three components: curriculum related questions, questions selected from the TIMSS study and a section specifically devoted to algebra. My focus is on the algebra component of the WMC-S tests. The algebra questions were derived from the Increasing Competence and Confidence in Algebra and Multiplicative Reasoning (ICCAMS), which is a test that originated in England. More detail is provided in section 2.3.

1.1.3. Purpose of the Study

According to Reddy, van der Berg, Janse van Rensburg & Taylor (2012), mathematics results in Grade 8 are a good indicator of who would pass matric. They further add that mathematical skills in the earlier years predicted later mathematical performance and maintain therefore that mathematical performance must be improved by Grade 8 in order to raise exit level outcomes. This finding is also encapsulated by the following comments presented by the Department of Basic Education in their diagnostic assessment of the Grade 12 examination:

The algebraic skills of the learners are poor. They struggle with Mathematics in Grades 11 and 12 because they cannot do the basic mathematics of Grades 8, 9 and 10. If this problem can be rectified, learners will perform much better in the Grade 12 examination (Department of Basic Education, 2014, p.126).

These findings suggest that the study of errors in algebra by learners from Grade 9 to 11 will be a useful endeavour to undertake as it will provide some form of clarification into how results particularly in algebra can be improved. One of the most prevalent errors in algebra is the conjoining of terms. The conjoining of terms is demonstrated, for example, by learners providing a solution of $5x$ when simplifying an expression like $3x + 2$. These findings result from studies in England and Australia (Küchemann, 1981; Booth, 1984; Tahir, Cavanagh & Mitchelmore, 2009). I will show in my study how the conjoining error is an important error to consider in algebra within the South African context.

At the outset, it is necessary to explain the structure of the write-up of this study as well as the reasons for not conforming to the “usual” structure of a research report. As a beginning researcher one expects the study to follow a pre-determined path of firstly defining the study by offering a focus of the particular topic of study through questions which require

some investigation. It is natural to expect that a study follows a linear pattern with the research questions formulated and the rest of the study following a set pattern of providing the theoretical framework that guides the study, a description of the methodology and the resultant analysis of the study. What initially began as an endeavour to analyse the errors of learners in algebra in Grades 9, 10 and 11 and the progress or lack thereof, culminated in uncovering/discovering questions on the instrument used in the testing process. This prompted an additional research question.

In order to understand the further research question it is necessary to provide a background of the origination of this study.

1.2. The Wits Maths Connect-Secondary project (WMC-S)

The WMC-S, working within the ambit of the University of the Witwatersrand (WITS), is an initiative aimed at assisting learners as well as teachers at selected schools. WMC-S is a five-year project (2010-14). The project is working with 11 disadvantaged secondary schools in a district in Johannesburg in collaboration with the district and provincial education department. The aim is to improve the quality of mathematics teaching and learner performance so as to increase access to mathematics-related study at tertiary level. The project began in 2010 and will continue until the end of 2014. The project is facilitated in collaboration with the ICCAMS project and staff at King's College, London. WMC-S is using some of the ICCAMS algebra items in its learner assessments in schools. In 2010, WMC-S administered the tests in all project schools. The same learners wrote the same test in Grades 9 and 10 at the end of each year. The aim was to track the performance of learners over 5 years with the aim of informing teaching. This project is therefore still a work in progress.

1.3. The ICCAMS test

The ICCAMS test originated from the Concepts in Secondary Mathematics and Science (CSMS) study which was a test designed and conducted in England in 1976. During a replication of the results in the science component of the test 30 years later, there were suggestions that learners' understanding of some of the mathematical concepts together with some science related concepts had declined (Hodgen, Küchemann, Brown & Coe, 2008).

The ICCAMS included questions from the CSMS test and focussed on three topics namely algebra, ratio and fractions. The questions were designed to specifically focus on

conceptual understanding and application of the particular topics rather than the testing of mechanical routines, although there were a small number of questions that assessed mathematical procedures (Hart, 1981). The algebra items were specifically designed to address the different meanings that learners attach to letters; for example, letter not used, letter evaluated and letter as object (Hodgen et al, 2008). See section 2.2 for an explanation of the different meanings learners attach to letters.

Not all of the questions from the original CSMS tests were used in the ICCAMS. The questions selected were based on consistent performance across different age-groups in the sample, as well as items within each level having high correlations and strong hierarchical relationships. Hart (1981) explains that the levels of hierarchy were determined by grading the tests of the sample group of children (about ten thousand) and who were subsequently ranked according to the percentage of correct responses obtained. Items that were solved successfully by the same individual children were seen as belonging together and were categorised into 4 different levels (section 3.2. provides a detailed description of the levels). The remaining items were selected for diagnostic purposes in order to provide information on learners' understanding of important ideas. These include substitution, simplifying expressions as well as the constructing, interpreting and solving of equations (Hart, 1981).

Learners were judged to be on a particular level if they had answered at least two-thirds of the questions at that level correctly. A “level 0” was created to accommodate those learners who were not able to correctly answer two thirds of questions at level 1. The questions selected for the ICCAMS from the original CSMS tests were selected to form a hierarchy of four levels (Booth, 1988; Küchemann, 1981).

1.3.1. Description of the levels in the ICCAMS test

The levels in the WMC-S tests were the same as that of the ICCAMS tests and are described in table 1.

Table 1: A Description of the Hierarchical Levels in the ICCAMS

Level	Explanation	Examples
1	The problems can be solved without having to operate on letters as unknowns. The letters could be evaluated, treated as objects or ignored.	1. <i>If $p+q = 41$ then $p+q+5 = \dots$</i> The letters can be ignored as the LHS is increased by 5.
		2. <i>If $p+4 = 7$ then $p = \dots$</i> This is an example typically used in primary school with the use of a place-holder as $\Delta + 4 = 7$ where the place holder represents a letter for example “p”
		3. <i>Write down the perimeter of a square with sides of length g.</i> The variable g can be thought of as a name or label for each of the four sides rather than an unknown.
2	There is a need to use some mathematical syntax. Learners need to use some of the rules of algebraic operations and conventions.	1. <i>If $h=3g + 1$ and $g = 5$, what is the value of h?</i> The learner needs to know that $3g$ means $3 \times g$ and the multiplication needs to be done first before adding 1.
		2. <i>Given a drawing of a pentagon with sides labelled a, a, a, b, b, write an expression for its perimeter.</i> The learner needs to add 3 a 's and 2 b 's to get $3a + 2b$ and must be able to identify that a and b do not necessarily have to stand for the same unknown.
3	The letters have to be treated as generalised numbers.	1. <i>If $p+q = 41$ then $p+q+r = \dots$</i> Learners have to regard “ $41+r$ ” as meaningful even though they may represent numbers and not objects. They also need to cope with a lack of closure of the expression $41+r$.
	Letters treated as specific unknowns	2. <i>Write down the perimeter of a shape with g sides, all of length 4.</i> Learners have to see “ g ” as representing a specific unknown number that cannot be treated as a name or a label.
4	Letters are used explicitly as variables.	1. <i>State whether $a+b+c = a+b+d$ is sometimes, always or never true.</i> Learners have to recognise that this statement is true when $c=d$ and is therefore sometimes true.
	Letters used as specific unknowns but involving a co-ordination of operations	2. <i>Multiply $p+2$ by 5.</i> Learners need to see that both p and 2 must be multiplied by 5.
	Letters represent numbers of objects or their costs rather than the objects themselves.	3. <i>Bananas cost b cents each and pears cost p cents each. If I buy 3 bananas and 5 pears what does $3b + 5p$ stand for?</i> Learners need to see this statement as a cost of the bananas and pears rather than seeing it as objects i.e. 3 bananas and 5 pears.

The levels described in the above table are strictly hierarchical. The questions on both levels 1 and 2 can be solved without using letters as unknowns. The differences in these 2 levels are that level 1 questions consist of numerical values and can be calculated immediately (example: “ $p+5=8$ ”), while level 2 questions (example: “*If $h=3g+1$ and $g=5$, what is the value of h ?*”), contains some ambiguity. Levels 3 and 4 require the letters to be treated as specific unknowns or letters as generalised numbers or variables. The difference in these two levels is that while a level 3 question requires the use of a single operation; (for example “subtract 3 from 5k”), a level 4 question requires the use of more than one operation; for example: “multiply $p+2$ by 5”, (Hart, Brown, Kerslake, Küchemann & Ruddock et al, 1985). It is highly unlikely then for a learner to be categorised as being on level 2 if he/she cannot answer questions on level 1 correctly.

1.4. Research Questions

The research questions are:

1. Has there been a shift in individual learner performance in algebra as evidenced by a change in levels from Grade 9 to Grade 10?
2. What errors are made by learners?
3. Which errors continue from Grade 9 to Grade 11?
4. Which errors fade away by Grade 11?

The first research question addresses the performance of the learners’ in the WMC-S test according to levels as determined by the ICCAMS instrument.

Initial analyses of the tests by the WMC-S unit, which are as yet unpublished, suggest that some errors are persistent while others are not. The second, third and fourth research questions are thus an attempt to investigate this preliminary finding.

During my study, I discovered that there were some learners in the sample group who had written the ICCAMS test and were established to be on level 3 without first obtaining the minimum number of questions on level 2 correct. Furthermore, while analysing the responses of learners in the WMC-S test, the conjoining of terms by the learners in the sample proved to be a crucial factor in some learners not being able to obtain the minimum number of correct responses to be on level 2. In particular some of the questions on level 2 in the WMC-S test seem to hinder the achievement of learners on level 2. I will show how some of the questions on level 2 in the WMC-S test posed a challenge to learners in terms of achieving at level 2. I will address this issue with particular reference to the conjoining error.

1.5. Usefulness of the study

In light of the lack of literature regarding performance in algebra in a South African context, I believe that the results of the study will potentially provide information on the current level of learners' achievement in algebra, with a view to informing teachers, learners and other stakeholders of some of the common problem areas in algebra. The information gleaned from the study may contribute to shedding some light on, not only the prevalent errors made by learners in algebra, but also the recurrence of specific errors since the learners' performance in the WMC-S test was tracked over three years. The study provides insight into the extent to which individual learners improve their performance in algebra. It will also offer insight into some of the persistent errors made by learners and the obstacles that prevent learners from moving to higher hierarchical levels on the WMC-S tests. This study involves the identification of errors on analysis of learner scripts in the WMC-S test as well as interviews conducted in Grade 11. The interviews provide a rich source of information on the thinking of learners in arriving at their incorrect answers. Teaching can thus be informed by not only the errors identified, but also how these errors are arrived at. Thus teaching can be designed to address these errors.

1.6. Conclusion

I have outlined the rationale for the study as well as the focus of the study which are linked to poor mathematics results within the South African Context. In order to provide an explanation of my research questions, it was necessary to provide a background to the WMC-S tests by discussing the features of the ICCAMS test. This description focused on the description of the different levels of understanding inherent in the ICCAMS framework.

Chapter 2 provides a review of pertinent literature in algebra as well as the theoretical framework that guides my study. It also provides brief explanations on the nature of algebra as well as the nature of errors in algebra within a constructivist framework.

In chapter 3, I discuss the selection of the school as well as the sample within which I conducted my study. The rationale for the separation of my study into two phases is explained. The data collection instruments that I have utilised to collect as well as analyse my data is elaborated on. I also focus on issues relating to ethics as well as some of the limitations of my study.

Chapter 4 is a description of the results of the performance of the sample group of 29 learners in the WMC-S test. I provide a detailed description of the re-coding of the scripts of the sample group of learners as well as some of the challenges experienced while re-coding. I also provide the results of the learners in the WMC-S test in Grades 9 and 10 by discussing the progression of learners through the different levels from Grades 9 to 10. Phase 2 of the study entails the analysis of the results of the Grade 11 task. The description of the Grade 11 task is provided in chapter 5. A detailed analysis of the task is provided by discussing the common errors in Grade 11. A discussion of those errors that are persistent in all 3 grades as well as those errors that have faded away by Grade 11 is provided. The particular issues relating to level 2 questions are also discussed. In chapter 6, I conclude my study by elaborating on the key findings of the study in relation to the research questions. I reflect on the study as well as discuss some of the limitations of the study.

CHAPTER 2

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

In order to analyse the learners' performance in the WMC-S tests it is necessary to understand what the purpose of school algebra is as well as how errors arise in terms of the teaching and learning process. Errors in general stem from many sources and are different in nature. Information about the nature of algebraic errors as well as a discussion about the theoretical framework that underpins my study is a necessary exercise. A detailed discussion about the ICCAMS framework that will be used to analyse algebraic errors is essential in understanding how the ensuing data is analysed. All of these issues are explained in detail in this chapter.

2.2. Algebra in the Curriculum

According to Bell (1995), algebra is regarded as a common problem area for learners. Booth (1988) adds that algebra is a source of confusion for learners. This is a serious concern since the South African mathematics curriculum, as in other countries, attaches great importance to algebra and that “Algebra is to mathematics what walking and doing chores is to every-day life” (Wong, 2009). The Curriculum and Assessment Policy Statement (CAPS) specifies in Learning Outcome 2 (Patterns, Functions and Algebra), that “learners' conceptual development progresses from an understanding of number to an understanding of variable, where the variables are numbers of a given type in generalized form” (Department of Basic Education, 2011). Great importance should therefore be attached to progression from numbers to letters, as cited in the CAPS document. Furthermore, according to the Department of Basic Education (2014), “the algebraic skills of the learners are poor” and “Basic algebraic manipulation needs attention”. This comment was in reference to common errors in the first mathematics paper written by Grade 12 learners in the matriculation examination as evidenced by the scrutiny of 100 randomly selected scripts by the examiner of the mathematics paper. This paper tests the skill of learners in algebra and functions, amongst other topics. This was, however, not the only reference to the lack of algebra skills in the Grade 12 examination. One of the comments made in reference to paper 2, which included questions on geometry,

trigonometry and data handling, was that learners “need to exercise caution with algebraic manipulation skills” (Department of Basic Education, 2014). The reference to algebraic manipulation skills in the second mathematics paper which primarily tests various topics in geometry, trigonometry and data handling emphasises the importance of algebra throughout the South African educational curriculum in schools.

There are a number of reasons why algebra is confusing to learners; however, the most common explanation is the gap between arithmetic and algebra. Booth (1988) maintains that one of the reasons for this gap is that the focus in arithmetic is on finding numerical answers while the focus in algebra is the derivation of procedures and expressions in a general and simplified form. This idea of the gap between arithmetic and algebra is also expounded by Linchevski & Herscovics (1996) who maintain that the gap between arithmetic and algebra is characterised by the inability of learners to operate spontaneously with or on the unknown. Moreover, Falle (2007) contends, citing research by MacGregor & Stacey (1997), that the arithmetic-algebra gap is the source of difficulties experienced by learners in algebra because knowledge of algebra is built on already acquired arithmetical knowledge. Arithmetic does not operate at the same level of abstraction as algebra since arithmetic focusses on numerical computations (Sfard & Linchevski, 1994) while algebra involves written symbols as well as an understanding of operations like the order of operations, inverse operations etc. (Herscovics & Linchevski, 1994). Arithmetic and algebra differ fundamentally in that in arithmetic, computational procedures are separated from the object obtained (Linchevski & Herscovics, 1996). This suggests that in algebra it is necessary for students to consider groups of numbers and symbols as objects, whereas this is not the case in arithmetic.

It is important to consider the types of errors made by learners in algebra, as well as their persistence over time, so that appropriate teaching strategies can be devised by teachers. In order to discover how and why learners commit certain errors, it is necessary to understand how learners learn using an appropriate learning theory.

2.3. The Theoretical Framework

Constructivism is a learning theory, originating from Jean Piaget (the father of constructivism), that explains learning as being not simply a passive process of receiving knowledge from the surrounding environment. Rather, learning is viewed as a dynamic

process of an individual which involves the interaction between the existing knowledge of an individual with new ideas which are gained through the social environment (Meadows, 2004). According to Jaworski (1996), constructivism encompasses two important principles:

1. Knowledge is actively constructed by the learner who is not just a passive recipient of the knowledge;
2. The process of “coming to know” is constantly being adapted and modified by the learners’ experiences of the world.

Jaworski (1996) adds that, within a constructivist viewpoint, knowledge fits experience. Knowledge needs to be modified if the experiences of an individual change. This is very succinctly expressed by Jaworski (1996) who explains that if there was some pre-existing mathematical knowledge then we can only know it through what we ourselves have constructed as well as modified through our own experiences. The individual does not only interpret knowledge but also organises this knowledge into units made up of inter-related ideas which are called schemas. These are mental representations of some mental or physical actions that can be performed on an object or phenomenon (Hatano, 1996). Learning therefore entails the interaction between the individual’s schemas and new ideas. This process involves two processes called assimilation and accommodation.

- **Assimilation** occurs when a new idea is interpreted in terms of an existing schema. When an individual perceives new objects or events in terms of existing schemas or operations, these ideas then serve to expand existing concepts thus forming a new concept. Individuals tend to apply any existing mental structure that is available to assimilate a new event, and actively seek to use this newly acquired mental structure. An example to demonstrate this process is when a learner has learnt that a number squared is a number multiplied by itself. When confronted with $(a + b)^2$ this new expression must be assimilated as using the same process of squaring a number.

- **Accommodation** is explained as the incorporation of new ideas which are different from existing schemas. There is an existing schema which may be relevant but not sufficient to assimilate the new schema. It is therefore necessary to re-construct and re-organise existing schemas. Accommodation therefore occurs when existing schemas need to be modified or new schemas must be created in order to account for a new experience. This process can be demonstrated by using the example of x^4 . When the learner first learns the concept of exponents he/she learns it as x multiplied by itself 4 times. When confronted with the example x^{-4} , the learner now cannot use the same reasoning as x multiplied by itself negative 4 times does not make sense. There now needs to be some restructuring to accommodate this new knowledge. The learner must now go beyond the multiplication idea to definitions to make sense of negative exponents.

Powell & Kalina (2009) maintain that, according to Piaget, assimilation occurs when individuals add new knowledge to their own already existing schemas. Accommodation occurs when individuals change their schemas to "accommodate" the new information or knowledge. This adjustment process occurs in learning, since new information is processed to fit into what is already in one's memory.

Both assimilation and accommodation occur simultaneously. Although most often we are assimilating familiar material in the world around us our minds also adjust to accommodate this material. The assimilation of new knowledge must lead to a degree of accommodation of old knowledge in order to achieve a balance between assimilation and accommodation which is equilibration (Meadows, 2004; Melis, Goguadze, Libbrecht, & Ullrich, 2010; Olivier, 1989). It is therefore necessary to integrate new knowledge into an existing schema in order to understand a new idea (Hatano, 1996). It is, however, sometimes impossible to link a new idea to any existing schema which results in a "new" box being created in the mind of the individual. This may result in the individual trying to memorise the idea, learning through rules, since it is isolated knowledge that is not linked to existing knowledge. This causes errors in mathematics as learners try to recall partially remembered rules (Olivier, 1989; Hatano, 1996). Nesher (1987) further maintains that errors can be traced to prior learning. She adds that most of these errors result from the over generalisation of previously learned knowledge that is now incorrectly applied. An

example to demonstrate this would be when a learner has learnt that when solving an equation like $x + 3 = 5$, the 3 “must be taken over to the other side and it becomes -3”. This results in the learner solving the equation $2x = 6$ as $x = 6 - 2 = 4$ or by solving the same equation as $x = \frac{6}{-2} = 3$. The above example also demonstrates the problems associated with the transmission of knowledge from one person to another. Hatano (1996) believes that the transmission of knowledge cannot be perfect as it allows for some ambiguity and different interpretations. If this knowledge that is transmitted is not properly constructed, then this may lead to errors.

Jaworski (1996) relates an example in a classroom of a boy commenting that a geometric shape with angles measuring 45° , 45° and 90° is a triangle while one in which the angles measure 30° , 60° and 90° is not. This suggests that the boy had a particular concept of a triangle in his mind that includes the first set of measurements as forming a triangle while the second does not. The child therefore constructed some meaning for a triangle according to his experiences. The discovery of why the boy believed that the one shape was a triangle and the other was not is an important consideration in determining what experiences led to his assertion. When new knowledge experienced is not compatible with existing knowledge then there is a potential for errors to occur. It is therefore necessary to understand the underlying reasons for errors in mathematics.

Constructivism therefore offers a lens through which we can perceive errors as being necessary to the learning process and within which I will be conducting my research.

2.3.1. Slips, Mistakes and Errors

Olivier (1989) distinguishes between slips and errors. Slips are mistakes made due to carelessness and which can be made by experts as well as novices. Errors are systematic and are symptoms of underlying conceptual structures that cause these errors (Olivier, 1989; Küchemann, 1981; MacGregor & Stacey, 1997; Kieran, 1992). These conceptual errors are referred to as misconceptions and are caused when previously learnt strategies are used incorrectly to solve new problems (Russel, Dwyer & Miranda, 2009). It is therefore necessary to understand the nature of errors in an attempt to correct them. Olivier (1989) provides the following example to demonstrate how the construction of new knowledge develops from existing knowledge and how this may lead to errors.

If $e + f = 8$ then complete $e + f + g =$ _____.

Using previous knowledge a learner will try to retrieve knowledge of the addition process previously learnt. One of the solutions offered then will be 12 (obtained from adding $4+4+4$). This indicates that the learner is attaching values to the letters to arrive at a solution. This type of question may be new to the learner who will try to use previously acquired knowledge to attempt a solution. The learner retrieves previous knowledge of addition of numbers and therefore provides values for these variables. Olivier (1989) refers to this as being a default evaluation as the learner somehow manages to produce replacement numbers for the variables resulting in an incorrect solution. This demonstrates how previously learnt knowledge influences new knowledge presented to learners. Nesher (1987) believes that the fact that the rules of mathematics and an individual's own beliefs are independent allow for discrepancies between them and which may result in errors. A possible example that illustrates this discrepancy is that a child has learnt that when performing the algorithm $2(3) + 4$ the answer of 10 is obtained by multiplying 2 and 3 to obtain 6 and then adding the 4 to obtain an answer of 10. When faced with an example of $2x + 4$, the learner obtains an answer of $6x$ because of his belief of what the answer should look like. Understanding the nature of algebraic errors is therefore crucial in informing the teaching and learning of algebra.

2.4. Errors in Algebra

Usiskin (1999) maintains that school algebra encompasses the understanding of letters or variables and operations. Learners therefore begin with algebra when they first encounter letters in an expression (MacGregor & Stacey, 1997).

2.4.1. Classification of errors according to interpretation of letters

Küchemann (1981) describes the following interpretation of letters used by learners in the ICCAMS test. The examples provided as well as the common errors found are similar to the ones in the ICCAMS test.

2.4.1.1. Letter evaluated:

In this interpretation of letters, learners attach values to letters. In this way they avoid having to operate on a specific unknown. This interpretation also allows for questions where the learner has to find a specific value for an unknown but again without having to operate on the unknown. An example of this category would be “what can you say about h if $h+3=5$ ”. This question can be categorised as a level 1 question in this interpretation of

letters. This same interpretation can lead to the following level 2 question: “what can you say about v if $w+4=v$ and $w=2$ ”. Although both these questions involve the letter having a specific value, the difference in levels is due to the level 2 question consisting of 2 unknowns. A level 3 question using the same interpretation of letters would be: “given $e+f=8$, what is the value of $e+f+g$ ”. This question is more difficult in that it consists of 3 different letters. A common error in this example is an answer of “12” which is found by learners providing each of the letters with the value 4.

2.4.1.2. Letter not used

The letter is not used or is ignored completely. An example of this is “if $s+t=10$ then $s+t+3=\dots$ ”. Although this question involves 2 unknowns, it is regarded as a level 1 question since nothing needs to be done with the unknowns. All that is required is that the unknowns can be eliminated by adding “3” to the value provided i.e. “ $10+3$ ”. A level 2 question in this category can be “if $r-134=542$ then $r-135=\dots$ ”. Although the unknown can be avoided in the same way as the previous question, the level of understanding is now increased because of the larger values as well as the presence of the minus sign. The minus sign prompted some of the learners in the original CSMS test to add 1 to 542 instead of subtracting.

2.4.1.3. Letter as object

Letters are regarded as representing objects. For example the question “simplify $2p+3p$ ” can be regarded as shorthand for 2 pens + 3 pens. While this is considered to be a level 1 question in the ICCAMS, the question “simplify $2p+3r+4p$ ” would be a level 2 question since it involves more than 1 unknown and can be viewed as 2pens, 3 rulers and 4 pens. Viewing letters in this way makes the items easier than viewing the letters as unknowns. While viewing the letters as objects allowed the learners to answer the question more easily since they represented concrete objects, the problem arises when translating the relationship, for example, “1\$ equals R11”. This is sometimes represented as “ $R=11\$$ ” instead of “ $\$=11 \text{ rand}$ ”

2.4.1.4. Letter as specific unknown

Learners regard a letter as a specific but unknown number and they operate on it directly. An example of this is “Add 3 to $m+3$ ” which would be a level 2 question in this category. A level 3 question would be “add 3 to $4m$ ” while a level 4 question would be “multiply $m+2$ by 4”. While it may be surprising to consider the question “add 3 to $4m$ ” to be on level 3, the common error is one of conjoining where learners write the answer as “ $7m$ ” or just “7” by combining the 3 and 4 which were meaningful to them. The answers illustrate that the letter was left as it was ($7m$) or was ignored completely (7). The level 4 question illustrates an increased complexity from the level 3 question in that it requires the 4 to be multiplied by both terms in “ $m+2$ ”. The common error in this question is an ambiguous answer like “ $4 \times m + 2$ ”.

2.4.1.5. Letter as generalised number

In this category the letter is seen to take on more than one value. An example of this interpretation of letters is “What can you say about t if $t+s=6$ and t is less than s ?” This is a level 3 question as categorised by the CSMS test and is more difficult than the items consisting of specific unknowns. A level 4 equivalent of this interpretation would be “ $A+B+C = A+D+C$ is: ”

Always True	Never True	Sometimes True
-------------	------------	----------------

A common answer to the first question would be to find one specific answer or a systematic list of a few values.

2.4.1.6. Letter as variable

In this interpretation of letters, the letter is seen to represent a range of unknown values. There is, however, a specific relationship between two such sets of values. For example the equation $3p+5r = 60$ can be satisfied by a few pairs of values for “ p ” and “ r ” such as (5, 9), (10, 6), (15, 3) or (20, 0). The relationship between them can be that “an increase in p is greater than the corresponding decrease in r ” or that “the increase in p is $\frac{5}{3}$ of the decrease in r ”. An example in the CSMS test was “which is larger n or $n+2$?”. A common error in this example was that learners chose one of the two by substituting a single value for n , like $n=1$ or 2.

Apart from these interpretations of letters, the items in the CSMS study were categorised into four different levels of understanding (these were discussed in Chapter 1). Küchemann (1981) maintains that items on levels 1 and 2 can be solved without having to operate on letters as unknowns. This suggests that questions on levels 1 and 2 can be answered using the first 3 interpretation of letters while questions requiring the latter three interpretation of letters are those that require a greater level of understanding and are therefore on levels 3 and 4.

2.4.2. Other errors in Algebra

Conjoining

In all of the above interpretations of letters one of the most common errors is the inappropriate conjoining of terms (Küchemann, 1981; Booth, 1988; Tirosh, Even & Robin, 1998).

A conjoining error is demonstrated by learners providing the answer **ab** in answer to the question $a + b = \dots$, or providing an answer of $5x$ when simplifying an expression like " $2x + 3$ ". Küchemann (1981) viewed this error as an indication of a "lack of closure". This error is therefore referred to as "premature closure" in the ICCAMS coding instrument. Collis (as cited in Küchemann, 1981) maintained that the inability to accept the "lack of closure" indicates an inability to generalise from given information as well as the inability to make links from new data.

This notion of "conjoining" is explained by Stacey and MacGregor (1994) from a constructivist viewpoint, as being the result of the use of previous knowledge from arithmetic as well as other areas. For example learners may use their knowledge of place value; $53 = 50 + 3$ or from knowledge of fractions where $3\frac{1}{2}$ is found from adding 3 to $\frac{1}{2}$ and from chemistry where CO_2 is derived from the addition of oxygen to carbon. Further to these reasons for conjoining, Tirosh et al (1998) believe that the tendency to conjoin terms inappropriately results from learners wanting to "close" or finish an expression or when learners interpret brackets in an expression by first simplifying the bracket (e.g., when $3(y + 5)$ becomes $15y$). Results of research conducted by Falle (2007) suggest that the tendency to conjoin terms by learners diminish as their ability to deal with

conceptually more advanced problems improves. The tendency to conjoin is more prevalent when the expressions become more challenging to learners. When faced with this challenge, it causes learners to search for something that they have already learnt; to look for an image that matches the expression they are asked to simplify. This results in the learner altering the form of the expression resulting in conjoining.

Booth (1988) maintains that one of the reasons for conjoining, is that in arithmetic the focus is on numerical answers while algebra requires the derivation of relationships and expressing these in a generalised and simplified form. Learners may therefore use any strategy to arrive at a numerical answer; like the answer 7 to the question “simplify $3m+4$ ”. Even those learners who do arrive at a general expression may not view it as a “proper” answer. Learners thus display the need for closure so that the question “simplify $2a+5b$ ” elicits a response of “ $7ab$ ”. This answer may also demonstrate a belief from arithmetic of what “well-formed answers” should look like.

This conjoining of terms is further explained by Malara & Iaderosa (1999) as occurring as a result of the gap between arithmetic and algebra. They maintain that the “dot” to represent multiplication is generally accepted in arithmetic but leads to confusion and ambiguities in algebra. Learners may therefore interpret “ $2ab$ ” as “ $2a+b$ ”. This occurs because learners tend to see the “+” sign in situations where the operation sign is not explicit.

Another explanation provided by Booth (1988) of the need by learners to conjoin terms is that an expression denotes the relationship or procedure through which the answer may be derived as well as the answer itself. The example “ $4 + 3m$ ” can be regarded as the instruction stating “add 3 to $4m$ ” as well as the answer which is the result of the addition. The confusion as explained by Booth (1988) stems from the viewing of the expression as the sum of 3 and $4m$ and in the latter interpretation “that which is 4 bigger than $3m$ ”. Part of the problem stems from the interpretation of operations. In arithmetic the + sign denotes the action of adding while the “=” sign signifies the answer of the operation. The notion of the addition sign representing both an operation as well as the result of the operation or that the equal sign as representing an equivalence relation may not be valued by the learner.

Brackets

Malara and Iaderosa (1999) contend that one of the problems faced by learners in the gap between arithmetic and algebra is one of the differing meanings or multiplicity of the roles of symbols within these 2 realms of mathematics. For example the parentheses (brackets) are used to indicate the priority of an operation over others. In algebra, however, in addition to indicating the priority of operations, they also serve as a barrier between two signs that may not be written one beside the other. An error like $(-1 + 4)y(-2) = 3y - 2$ then completely changes the meaning of the operation required.

Exponents

In a study by MacGregor & Stacey (1997), it was found that students thought of exponents as an instruction to multiply, without having a clear idea of what was being multiplied. When asked to write “x times 4”, some students represented this as “ x^4 ”. However, this problem was found to occur with older students who had been taught the topic on exponents. This then demonstrates the impact of the misinterpretation and misapplication of new knowledge. The interference of new knowledge with old knowledge is a problem that is reflected in this study and is discussed further in the analysis of the results.

A study by Tahir et al (2009) demonstrated that students who considered variables as objects made the following errors relating to exponents:

$x + x = x^2$ and $x + 5x = 7x = x^7$. When students were provided with instruction where the emphasis was on the meaning of a letter, these students avoided the type of errors illustrated by the above examples. The results of the study reinforce the idea that the incorrect interpretation of letters results in a less sound concept of a letter.

General Errors

Christou, Vosniadou and Vamvakoussi (2007) suggest that most errors experienced by learners in algebra with regard to letters result from an inappropriate transfer of prior knowledge of numbers in arithmetic to interpretation of letters in algebra. They posit that learners believe that when a letter changes then the value that it represents also changes. This belief thus results in learners being unable to comprehend that an expression like “ $x+y+z$ can be equal to “ $x+w+z$ ” because of their belief that different letters must correspond to different numbers.

Christou et al (2007) also suggest that learners associate letters of the alphabet with the linear ordering of the number system. For example, the letter “c” is viewed as possessing the value of 3 because it is the third letter in the alphabet. When faced with the question “what is the value of c in $a+b+c=7$ if $a+b=3$?” learners may provide the value of 3 because “a” corresponds to the value 1, “b” corresponds to 2 and therefore “c” must equal to 3. The other issue related to letters is that learners have learnt that in arithmetic if there is no sign in front of a number then it implies that the number is positive. Learners therefore believe that “ x ” represents positive numbers while “ $-x$ ” represents a negative number. (Vlassis, 2004; Christou et al, 2007).

2.5. Conclusion

In this chapter, I discussed the differences between errors, slips and mistakes within a constructivist view of learning. I have also provided the different interpretations of letters that are used by learners and as discussed and used in the ICCAMS instrument. A discussion of other errors, including conjoining, in algebra is also provided. I will show later in the study (chapter 5) how the conjoining error provides a challenge to the learners in their progression through the levels in the WMC-S test. I will now provide an explanation of the design and methodological issues that are crucial to the study.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1. Introduction

This chapter is a focus on the methodology as well as some features of case studies that guide my research. I also discuss the selection of the school within which my study was conducted together with the sample group of learners selected for the collection of my data. Also included are the issues concerning rigour, ethical considerations as well as some of the limitations of the study.

3.2. Overall design of the study

My study focusses on the analysis of the results of the learners who have written the WMC-S tests in Grades 9, 10 and 11. I have divided my analysis into 2 phases. Phase 1 of the study centres around the progression of the learners through the 4 levels of the WMC-S tests using the ICCAMS instrument discussed in Chapter 1 and addresses the first research question i.e. Has there been a shift in individual learner performance in algebra as evidenced by a change in level from Grade 9 to 10?

The second phase of my study emphasises the errors made by learners. This phase was conducted on a sample of the original group of learners who were in grade 11 in 2013.

The study therefore consisted of a mixed method data analysis with phase one being quantitative and phase two being qualitative. According to Marshall (1996), the aim of the quantitative approach to analysing data, is to test pre-determined hypotheses and is useful in answering 'what?' questions. In order to answer the research question regarding the shift in levels of learners in the ICCAMS component of the WMC-S test, it is imperative to analyse the levels of performance of the learners. Quantitative analysis is therefore an appropriate strategy to determine the level of each learner in the WMC-S test. Qualitative studies aim to provide illumination and understanding of pertinent issues and are useful for answering 'why?' and 'how?' questions (Marshall, 1996). Patton (1999) adds that they serve to “illuminate the stories behind the qualitative data”. Qualitative analysis is thus most conducive to investigating the errors made by learners in algebra as it serves to shed light on the results of the quantitative analysis.

3.3. Background to the WMC-S Tests

The WMC-S test consists of thirty three questions from the algebra component of the ICCAMS. The questions for the WMC-S test were selected from questions from the ICCAMS test which were in turn selected from the CSMS tests. According to Hart et al (1985) the questions in the original CSMS test were grouped to form a hierarchy of levels but had to satisfy particular criteria in order to be grouped into a particular level. The 6 criteria that had to be satisfied were:

- the questions should have approximately the same level of difficulty,
- the values of the homogeneity coefficient should be at an acceptable level,
- the items should be linked with the items on easier and harder groups of questions,
- there should be mathematical coherence to the items,
- the groups should be scalable i.e. a child's success on a group should entail success on easier groups,
- there should be no gross discrepancies when each group's results were analysed in the same way.

These questions comprised of a different number of questions at each level as illustrated in the following table:

Table 2: The Number of Questions in each Hierarchical Level

Level	Number of questions
1	6
2	6
3	5
4	5
No levels assigned (to be used for diagnostic purposes)	11

Learners were determined to be on a particular level of understanding if they were able to successfully solve at least two thirds of the questions on that level. For example a learner will be determined to be at level 1 if he/she was able to correctly solve at least 4 out of the 6 questions on level 1. In some cases the nearest fraction was taken; for example a learner will be at level 3 if 3 out of the 5 questions were successfully solved by the learner.

Although the WMC-S tests were conducted with learners at all the project schools, I will be looking at the data collected from a single school.

3.4. The School

For ethical reasons the school cannot be named and will henceforth be called Variable Secondary School (VSS).

Variable Secondary School is situated in a township in Johannesburg. Learners come from a low socio-economic background. The unemployment rate in the area in which the school is situated is high. The school is a no-fee school which means that the learners do not pay any school fees. The home language of the learners is not English although the school is an English medium school.

.

3.5. The Sample

In choosing the school within which my study was conducted, I looked at those schools that were part of the WMC-S project with a large number of learners who had written the test so that I would have an adequate sample to analyse. The other reason for choosing VSS was that it is in close proximity to where I live and work.

My study consists of three sets of data i.e. 29 test scripts for Grade 9 learners, 29 test scripts for the same learners who were in Grade 10 in 2012, as well as interviews with 7 Grade 11 learners. The 7 learners from Grade 11 were selected from the sample of the 29 learners. In 2011 the entire enrolment of Grade 9 learners in the school who were willing to participate wrote the test. In Grade 10, there were Mathematics as well as Mathematical Literacy learners who wrote the tests.

My analysis consists of two phases. The first phase consists of analysing the WMC-S test scripts in an attempt to address the first research question.

The second phase of the study encompasses the analyses of interviews with learners (who were in Grade 11) who were selected from the original sample, in order to address the second, third and fourth research questions which are:

- What errors are made by learners?
- Which errors continue from Grade 9 to Grade 11?
- Which errors fade away by Grade 11?

All 7 of the selected learners continued with Mathematics in Grades 10 and 11.

3.5.1. Selection of Sample for Phase One

Researchers from the WMC-S coded the test scripts according to a uniform coding system (see appendix C). Each of the questions in the algebra component was coded by the

members of the WMC-S unit, using codes for *correct response*, *incomplete response*, *incorrect response* and *unanswered*. These results were entered on a spread sheet. I added on a column to reflect the total number of correct responses. Each learner's total was then divided by the total number of questions in the algebra section (33) to convert the mark to a percentage. Those learners in Grade 10 who had obtained a mark above 40% were then selected for script analysis.

The decision to choose learners who obtained above 40% was made on the basis of having enough answers to analyse. A number of learners left answers blank or incomplete. I therefore felt that those with marks above 40% would provide me with sufficient information from their answers to investigate their errors. The other motivation in choosing those above 40% in Grade 10 was so that I could see if individual learners had progressed through the levels from Grade 9 to Grade 10. Only 30 of the 212 learners in Grade 9 obtained a mark greater than 40% whereas this number was 53 in Grade 10. The marks of those learners who were selected in Grade 10 ranged from 40% to 93%. This wide range of selected learners in terms of their performance in the test would provide me with the necessary data to ascertain which errors were persistent and which were not. It was therefore necessary to consider those learners in Grade 10 who had made some improvement but who still made errors. This would make it possible for me to investigate those errors that are persistent as opposed to those that have disappeared.

3.5.2. Selection of Sample for Phase Two

After analysing the results of the written work of learners in the ICCAMS part of the WMC-S test, it was found that most learners (19 out of a total of 29 i.e. 66% of learners) progressed from a lower level to a higher level from Grade 9 to Grade 10. This is not surprising as the learners had written the same test the year before and had learned more algebra during the course of the year. I decided to focus on problems experienced by the learners with level 2 questions during the interviews in the second phase of my study. My justification for this is that it was unexpected that 9 learners progressed through more than 1 level. For example, there were 2 learners who were on level 0 in Grade 9 but who progressed to level 3 in Grade 10. Even more surprising was that 3 of these 9 learners progressed from level 1 to level 3 but were unable to answer the minimum number of questions at level 2 to be on level 2. In other words, these learners experienced problems with some of the level 2 questions and were not able to correctly answer at least 4 out of

the 6 questions at level 2. This is unexpected as, according to Hart (1981), success at a lower level is a pre-requisite for success on a higher level. In addition, there were another 2 learners who moved down a level. Both of them moved from level 2 in Grade 9 to level 1 in Grade 10. This is also unexpected. The other interesting observation is that there were more learners on level 1 and 3 than on level 2 in Grade 9, while in Grade 10 there was the same number of learners on level 1 and 2 but more than this number on level 3. This is outlined in the results for phase 1 of the study.

I have selected particular learners for the interviews using criterion sampling by incorporating the criteria outlined below:

- One learner who moved down a level i.e. from level 2 to level 1
- Two learners who progressed from level 1 to level 3 without first reaching level 2
- Two learners who remained on level 1 in Grade 9 and 10
- One learner who remained on level 2 in both Grades 9 and 10

All of the above learners were selected since 5 of the learners experienced some form of problem with level 2 questions as is evidenced above. The last learner was selected as she was on level 2 and her answers could be used for comparison purposes between her and those who were unable to answer the minimum number of questions correctly to be on level 2.

According to Sandelowski (2000) criterion sampling is “a type of purposeful sampling of cases based on preconceived criteria such as scores on an instrument”. She maintains that purposeful sampling is used in order to collect more data, using instruments such as interviews or observations, from the participants in order to enhance understanding of “the information rich case” as well as to elaborate on or clarify the results. Criterion sampling is therefore an appropriate method of sampling in my study as the sample selected not only allowed me to investigate the errors made by learners but more importantly to address the problems centred on the answering of level 2 questions.

3.6. The Research Instruments

The first phase of my study entailed the analysis of the actual test scripts of the selected 29 learners who had written the WMC-S tests in Grades 9 and 10. The WMC-S tests were conducted towards the end of each year by the WMC-S unit. These tests were conducted along the same lines as the CSMS tests.

These tests were analysed using the coding for the ICCAMS instrument (see appendix C for more detail on the coding used).

My principal data collection instrument for the second phase of my study is the interviews. The rationale for the interviews is that the tests for the Grade 11 learners were only conducted at the end of 2013. As a result there was no available Grade 11 data for the sample of Grade 11 learners. In order to investigate the progress of the selected learners, it was imperative to identify their responses to some of the test items after 2 years of writing the same test.

The other reason for the interview is as Opie (2004) maintains, that interviews offer the opportunity to ask the question “why”. In merely analysing scripts it may sometimes be unclear as to why a learner is obtaining a particular response. In analysing a response from the learner’s written work, I may infer a reason as to how the answer was obtained. This may, however differ from what the learner actually did to arrive at a particular conclusion. The interview assists in understanding how the learner arrives at a particular answer by asking guiding questions which is explained by Hatch (2002) as the questions prepared in advance of the formal interview and designed to guide the conversation. For example, questions like “How would you go about answering the question?” can be a guiding question. The interview questions were semi-structured as they were based on the test items in the WMC-S tests.

I have used the WMC-S tests to ascertain the level of understanding of the learners in terms of algebra using the levels developed for the ICCAMS test while the interviews will assist in shedding some light as to the reasoning of the learners in arriving at a particular solution. The interviews were based on a written task that learners completed. The task was in turn based on similar questions to those in the WMC-S test. Prior to the interviews a pilot study was conducted.

3.7. The Pilot Study

Cohen & Morrison (2000) assert that the purpose of a pilot study is to inform the main study about the quality of the questions in a task. The pilot will assist in indicating the suitability of the task in terms of the clarity of the instructions or questions, the structure of the task as well as the context of the questions. The pilot study will therefore help to get rid of data that may be irrelevant to the study (Opie, 2004). Cohen & Morrison (2000) add that the sample for the pilot interview should be similar to the sample for the study in order to analyse trends in the pilot that may recur during the main study.

Bearing in mind these important considerations, the 2 learners for the pilot interviews were selected as follows:

- One learner who remained on level 1 in both Grades 9 and 10
- One learner who moved from level 2 in Grade 9 to level 1 in Grade 10.

The first learner was selected as he had not progressed to level 2 while the second was chosen as he had regressed from level 2 to level 1. Both of these learners appear to have experienced some problems with level 2 questions.

3.8. Preparation for interviews

I met with those learners selected to be interviewed in order to explain the rationale as well as the protocol for the interviews. During this meeting, it was explained to the learners that they were not compelled to be interviewed if they were unhappy to do so. They were also assured that confidentiality would be maintained throughout the study. They were provided with information sheets relating to what the study was about and what their contribution to the study was. I also provided them with consent forms for them as well as their parents to sign. These were duly signed and returned prior to the interviews.

Only 7 interviews were eventually conducted since 1 of the original 8 selected learners did not consent to do so prior to the interview. There was thus, only 1 learner from the two, who progressed from level 1 to level 3 without first reaching level 2 who consented to be interviewed.

A pilot interview was conducted with 1 of the learners as the second learner originally selected to complete the pilot interview, was not available for a few days. As a result, only one pilot interview was done although the learner who was not available for the pilot availed herself thereafter for the remainder of the study.

As a result of the pilot, two minor changes were made to the written test. One of them involved a change in variable (see question 11) as one of the learners assumed that the question was related to the question above it because the same variable was used. The other involved adding on that the figure given was incomplete (see question 9.2).

3.8.1. Data Collection Instruments

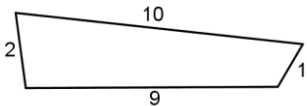
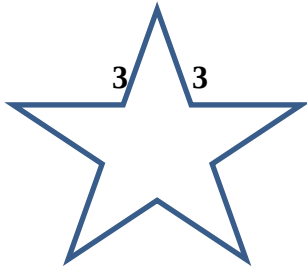
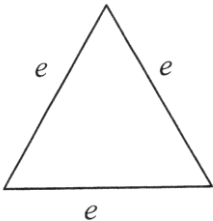
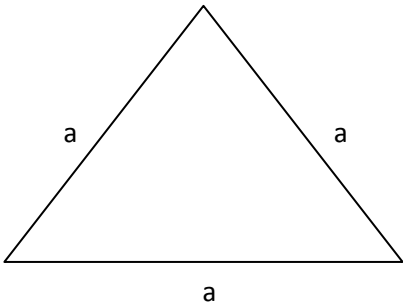
Based on the responses to the test items, learners were selected to complete a written task and thereafter an interview, which was audio-taped and transcribed for later analysis.

3.8.1.1. The Grade 11 Task

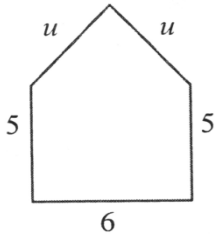
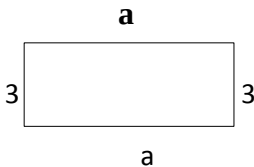
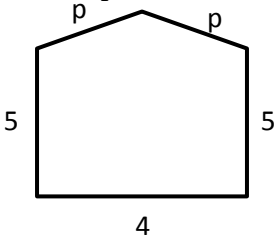
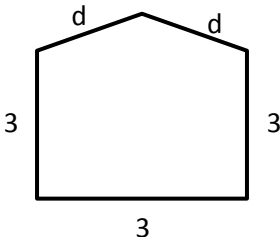
The task was designed to include questions of a similar type to that of the ICCAMS part of the WMC-S test to allow for comparisons between Grades 9, 10 and 11. A comparison between the levels in Grades 9, 10 and 11 will only be possible if the learners had written the same test or questions that are similar to each other in every grade. In Grades 9 and 10 the same test was written. The Grade 11 task contained questions that were very similar to those in the WMC-S test.

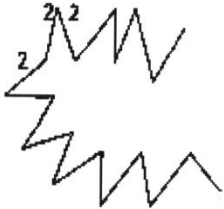
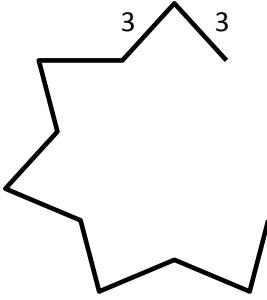
The questions from the WMC-S test as well as the equivalent questions in the Grade 11 task are provided in table 3 below to illustrate the similarity between the questions.

Table 3: A comparison of questions between WMC-S test and Grade 11 task

WMC-S Test		Grade 11 Task	
Question number	Questions on level 1	Question number	Questions on level 1
1.1.	Simplify: $2a+5a$	1.	Simplify $3x+2x$
5.1.	If $a+b=43$, then $a+b+2=$	3.	If $j+k=5$ then $j+k+2=$
6.1.	Find a if $a+5=8$	2.	If $k+2=5$, then $k=$
9.1	Work out the perimeter 	9.1.	Find the perimeter. Each side equals 3 units. 
10.1.	Find the perimeter. 	5.1.	Find the perimeter 

WMC-S Test		Grade 11 Task	
Question number	Questions on Level 2	Question number	Questions on level 2
1.4.	Simplify : $2a + 5b + a =$	6.1.	Simplify: $2p + 3a + p$
		6.2.	$2p + 3a + 3p$
7.1.	If $u = v + 3$ and $v = 1$, find the value of u	7.1.	Find the value of x if $x = y + 5$ and $y = 2$.
		7.2.	Find the value of x if $x = y - 1$ and $y = 3$.
7.2.	If $m = 3n + 1$ and $n = 4$, find the value of m	8.1.	Find the value of p if: $p = 2n + 1$ and $n = 2$
		8.2.	Find the value of p if: $p = 5n - 1$ and $n = 4$
8.3.	What is the area of the shape n m	4.1.	Find the area p q
		4.2.	Find the area $2a$ b

WMC-S Test		Grade 11 Task	
10.3	Find the perimeter	5.2	Find the perimeter
		5.3.	Find the perimeter
		5.4	Find the perimeter
			
			
			

WMC-S Test		Grade 11 Task	
Question Number	Questions on level 3	Question Number	Questions on level 3
1.2.	Simplify: $2a+5b$	10.1	Simplify: $3x+2y$
		10.2	Simplify: $2x + x - y$
5.3.	If $e + f = 8$, then $e + f + g =$	12.	If $a + b = 3$ then what is $a + b + c?$
10.4.	Find the perimeter  Part of this figure is not drawn. There are n-sides altogether all of length 2	9.2.	Find the perimeter.  Part of a figure is drawn. Each side equals 3 units and the figure has n sides.

The questions in the Grade 11 task for phase 2 consisted of 5 questions on level 1, 10 questions on level 2 and 5 questions at level 3.

More level 2 questions were chosen as my focus was on the problems experienced by learners with level 2 questions. (See section 3.5.2. for a detailed explanation). Questions on level 1 were included as there were 3 learners who were on level 1 in Grade 10. It was necessary to determine whether these learners were able to answer questions at level 1 in

Grade 11. They were also included in order to ascertain whether those learners who could not answer the minimum number of questions correctly at level 2, were able to answer questions on level 1. Level 3 questions were included to determine whether those learners who were able to answer the minimum number of questions at level 2 correctly, were able to answer questions at level three. No questions at level 4 were included as the focus was on problems experienced at level 2.

As is evident from table 3, the questions in the WMC-S test and the Grade 11 task were very similar to each other. The similarity of the questions in both provides the opportunity to make comparisons between errors made in Grades 9 and 10 with the errors made by learners in Grade 11. The similarity of the questions also enables the comparison between the levels of the learners in Grades 9 and 10 with their level in Grade 11.

Table 4 reflects the question numbers in the Grade 11 task that correspond to the particular question number in the WMC-S test.

Table 4: Question numbers in WMC-S corresponding to numbers in Grade 11 task

	Level 1 questions					Level 2 questions					Level 3 questions			
Question number in WMC-S test	1.1	5.1	6.1	9.1	10.1	1.4	7.1	7.2	8.3	10.3	1.2	3.3	5.3	10.4
Question numbers in Grade 11 task	1	3	2	9.1	5.1	6.1	7.1	8.1	4.1	5.2; 5.3	10.1	11	12	9.2
						6.2	7.2	8.2	4.2	5.4	10.2			

The task was given to each of the 7 learners before the interview. There was no time limit within which the task had to be completed. All of the learners took a maximum of 20 minutes to complete the task. To ensure anonymity, the name of the learner was not reflected on the task script. Each learner had a code which I indicated on the script. The learners answered the questions on the script itself.

3.8.1.2. The Interview

Each learner was interviewed immediately after answering the task. The interview was audiotaped to ensure the reliability of the information. In addition the learner could also

work out any additional questions, asked during the interview, on the script or, if there was insufficient space, on a separate sheet of paper.

The questions asked during the interview took the form of probing the learner on how he/she had arrived at a particular answer. The probing of how learners had arrived at a particular conclusion also included those questions where learners had obtained a correct answer. This was done to explore whether the learner knew how to solve that particular problem or whether the learner guessed the correct answer; however more time was spent on probing those questions that elicited an incorrect response from the learner.

The probing of questions sometimes took the form of providing an incorrect answer in order to ascertain whether the learner was certain about his/her answer and why it was being done in a specific manner. An example of the type of questions asked is provided by the following transcript. The question was: ***Find the area of a rectangle whose length is “2a” and breadth is “b” (see Q4.1 in task)***

(“I” refers to the interviewer, throughout the report, while Sbu (not his real name) is one of the learners interviewed)

- | |
|---|
| <p>I: <i>Ok. So here what did you do? (refer to Q4.2.)</i></p> <p>Sbu: <i>I did the same thing length times breadth that is 2a times b which is 2ab</i></p> <p>I: <i>Would it be different if b was here and two a (2a) was here? (swopped the 2a and b)</i></p> <p>Sbu: <i>Yes it will not be the same</i></p> <p>I: <i>So what will your answer be?</i></p> <p>Sbu: <i>Here it's going to be 2a times b ... yes it will be the same</i></p> <p>I: <i>So is 2ba the same as 2ab?</i></p> <p>Sbu: <i>No it's not going to be the same.</i></p> <p>I: <i>What's the difference?</i></p> <p>Sbu: <i>a is smaller than b so it comes before the b</i></p> <p>I: <i>So if I wrote it as 2ba, I'll get it wrong?</i></p> <p>Sbu: <i>Yes I think it's wrong</i></p> <p>I: <i>So you say a is smaller than b? Why? How would you know? I know that 2 is smaller than 5 how would I know that a is smaller than b?</i></p> <p>Sbu: <i>Because in the alphabetical order</i></p> |
|---|

I: *So the one that comes first is smaller and the one after is bigger?*

Sbu: *That's how I interpret it*

This probing allowed me the opportunity to ask the learner how he had arrived at a particular answer. The information thus provided me with an understanding of his reasoning in arriving at an answer. The difference between an answer of “2ab” and “2ba” in the mind of the learner could not have been ascertained by this answer in the test. The interviews therefore provided an avenue to explore the reasons behind some of the errors that the learners make.

3.9. Ethical Considerations

The research was carried out with due consideration to ethical issues. Since the tests were conducted by the WMC-S, permission had been granted by the Gauteng Department of Education to conduct the research at the school. However, permission letters (see appendix G) for the conducting of interviews were given to each of the selected learners. These letters outlined the purpose of the research and the reasons for the interviews. The letters also served to assure the learners as well as the parents that their participation in the research was purely voluntary. The letter also explained that should the learners initially accept to participate in the research but decide to opt out, there would be no consequences. The anonymity of the learners was guaranteed at every stage of the project. The original scripts themselves did not reflect the names of the learners. Each learner was given a code and these were used to match results in Grades 9 and 10. During the task- based interview, codes were provided to learners so that their names did not appear on the task. In the research report, pseudonyms are used to protect the identity of the learners as well as the name of the school where the research was conducted. It therefore follows that in this study all names used (learners as well as school) are pseudonyms.

The parents as well as the learners were made aware that should they require the final copies of the research results, this would be made available to them on request. In order to ensure the safe-keeping of the scripts, these were stored in a lock-up cupboard at the WMC-S office which itself can only be entered via a security code. For the purposes of the analysis of the scripts I made copies of them. To ensure the safety of these scripts, I stored them in a lock up cupboard in my room.

3.10. Reliability

Reliability is viewed as being synonymous with dependability and the accuracy of the representation of the total population under study (Opie, 2004; Golafshani, 2003). This indicates the extent to which results can be regarded as stable (Lincoln and Guba cited in Opie, 2004). Replicability of results cannot be guaranteed in qualitative research because of bias inherent in all individuals involved in the study. Any given data may be represented and interpreted differently by different researchers. It is therefore incumbent on the researcher to find measures to enhance the dependability of the study in order for the findings of the study to be consistent. In other words if the research is repeated with the same group under the same conditions at a different time it will produce similar results. My study is based on the same tests (WMC-S test) which were written by learners in 2011 and 2012. The WMC-S tests were conducted in exactly the same manner each year in terms of procedure and instructions.

According to Patton (1999), triangulation is based on the idea that there is no single method that can form the basis of an explanation. Different methods reveal different aspects of empirical reality therefore multiple methods of data collection and analysis provide for a stronger foundation for claims made and in the words of Patton (1999) provides “more grist for the research mill”. In order to enhance the credibility of my research, I used the analysis of a task (Grade 11 task) provided to the learners together with the analysis of interviews conducted with the selected learners.

McMillan & Schumacher (2010), maintain that interviews allow for the probing, clarification and elaboration of responses which allow for more accurate responses than by the use of a test script only. The interview questions were based on a task that learners had to complete (Appendix B). These interviews were audio-recorded in an attempt to capture the learner responses as accurately as possible. In order to ensure that no information could be lost due to technical failure, I made use of two devices, a cell phone as well as an I-pad. The learners were made aware of the recording devices; however both devices were out of sight of the learners in order to reduce feelings of intrusiveness by learners.

The tests were initially coded by a team of professionals under the auspices of the WMC-S unit. All members were provided with the coding criteria. Apart from the coding criteria, members of the coding team were also advised about the possible differing errors that

could be made and how they could be coded. This was done under the supervision of the project leaders of the WMC-S. However, to ensure reliability, I re-coded the test scripts based on the original criteria used as determined by the team within the WMC-S. The same coding was used for Grades 9 and 10. Only those scripts selected for analysis based on the criteria mentioned earlier in the selection of the sample (those who achieved above 40%), were re-coded. If there was a discrepancy between my coding and the original marker's coding, I consulted with the project leaders at WMC-S. A decision was then taken on how to proceed depending on the particular problem. For example there were errors in the original coding which could be categorised as human error. To illustrate this, I provide the following example: Question 7.1 in the WMC-S test was "if $u=v+3$ and $v=1$, find the value of u ". The answer provided was "4" which was correct. The original code was provided as "9" which was for an incorrect answer. This was clearly a mistake by the original coder and after consultation with my supervisor this was amended to a code of "1" which was the code for the correct answer. There were other scripts which displayed human error in coding and were duly amended. There were, however, instances where I disagreed with the original coding but which were not amended after consultation with the project leaders at WMC-S. An example of this is question 10.1 in the WMC-S test. The question was to determine the perimeter of a triangle with each side equal to " e " units. The answer provided was " $e + e + e = e^3$ ". The answer was coded as "1" which was used to indicate a correct answer. Although this to me was incorrect, the reason for coding it as correct was that the coders were asked to code the first line of the answer to ensure consistency among the big team of markers. I therefore coded according to the criteria used by the original team in the WMC-S to ensure that there was no bias in terms of the results. This is discussed in detail in section 4.1.1.

3.11. Validity

According to Schumacher (2010), validity refers to the degree to which the explanations of phenomena match reality. This suggests that validity does not lie in the data but rather in the interpretation of the data. Morse, Barrett, Mayan, Olsen & Spiers (2002) maintain that validity can be enhanced by using different verification techniques, one of these being the constant checking and re-checking of data in order to build a solid foundation. Rather than being considered only at the end or the beginning of the research, this should be deliberated on during every stage. The re-coding of scripts was not only completed at the

beginning of the research. During the course of the research, at every stage, if a discrepancy was observed, I returned to the scripts to ensure that the coding had been done as accurately as possible. For example, it is unusual for a learner to proceed to a higher level of the WMC-S test without first succeeding at a lower level. When this was encountered, the particular script was re-coded to determine any inconsistencies that may have occurred during the coding.

In addition to the coding used, I also used interviews which were audio-taped as this would provide “accurate and relatively complete records” (McMillan & Schumacher, 2010). I also conducted pilot interviews prior to the actual interviews in order to identify any weaknesses in questioning techniques or other areas of weakness in the interview procedure. Furthermore, I sought advice from my supervisor as well as other colleagues from the WMC-S unit at different stages of the project. The ICCAMS instrument itself has also been recognised as a valid instrument from other research conducted. The CSMS instrument was first developed and implemented in the 1970’s and was subsequently published. It was used again as the ICCAMS instrument (Küchemann, 1981; Hart et al., 1985) which has international recognition. It is currently being used in South Africa and has since most recently been used in Germany (Oldenburg, Hodgen & Küchemann, 2013).

3.12. Limitations of the study

One of the limitations of this study is that while I had scripts to analyse for learners in Grades 9 and 10, scripts were not available for learners in Grade 11 as my study was done before the Grade 11 learners wrote the test. My study would have been enhanced as well as served as verification of the results had the test scripts for Grade 11 learners been available for analysis together with the interviews. The difficulty is then that I could not compare 3 years of test data but the test data from Grades 9 and 10 against the Grade 11 task and interviews. While the Grade 11 task was designed to be similar to the WMC-S test, there were limitations in that a question similar to question 10.2 in the WMC-S test was not included in the Grade 11 task. The other inherent problem was that not all questions that were used in the WMC-S test were used in the Grade 11 task as the focus was on questions on level 2. Hence it was not possible to establish whether learners were on level 4 or not. The interview results were not used in the determination of the levels as some of the learners corrected their answers in the Grade 11 task after prodding. The results would therefore not have been valid. While there were limitations in using the interview results, the interviews provided rich data in terms of how learners made sense of the questions.

The interviews provided a sense of how particular answers were arrived at which was not possible by the test answers only.

The other limitation is the teacher's role in the performance of the learner is in the background of the study. My study therefore did not take into consideration how the performance of the learner in algebra is enhanced or inhibited by the teacher's methodology, knowledge and other areas of the teacher's role in the learning process.

3.13. Conclusion

I have explained the background to the WMC-S tests as well as provided details with regard to the different levels of understanding in the tests. An explanation of how the school as well as the learners were selected for both phases of the study was provided. I described the details of the written task that was used to conduct my interviews with the learners. I finally presented information on how issues regarding the validity, reliability and ethical considerations were dealt with in the study as well as some of the limitations of the study.

I now turn to the actual results of the data which was separated into two phases. I will then provide an analysis of the key findings of my study.

CHAPTER 4

RESULTS AND ANALYSIS OF THE GRADE 9 AND 10 TEST RESPONSES

4.1. Introduction

This chapter provides the details of phase 1 of the study which involved the results and analysis of the WMC-S test written by the learners in Grades 9 and 10. I will illustrate the progression of the learners in terms of the four levels of understanding using the ICCAMS instrument. I also provide details of some of the challenges that were experienced during the re-coding of the scripts of the selected learners. I then provide examples of some of the common errors displayed by the learners. Finally I provide a rationale for the deeper analysis of the learners' performance in level 2 questions with particular emphasis on the conjoining error.

4.2. A Description of Phase 1 of the Study

Twenty-nine learners from those who had written the tests in Grades 9 and 10 were selected for script analysis during phase 1 of the study. I re-coded all 29 scripts using the coding of the ICCAMS instrument.

4.2.1. An explanation of the coding used

The codes used were the same as that which was developed by the WMC-S team. This coding was in turn developed from the original ICCAMS tests. Each question in the WMC-S test was coded using the different representation of letters. The following table represents the codes used:

Table 5: Description of the error codes

Description	Missing answer	Correct	Ambiguous	Letter evaluated	Letter as object	Letter not used	Premature closure	Wrong
Code	0	1	2	3 or 4	5 or 6	7	8	9

Each correct response, in some of the questions was coded as either 1, (if there was a single correct answer) or 1(a), 1(b) or 1(c) depending on the number of different ways to represent a solution. As an example “ $5e+10$ ” could also be represented as “ $5(e+2)$ ” which are both correct. The same applied to other categories. For example in the category “premature closure”, learners could have different answers all representing premature closure. In question 1.4 which required learners to simplify “ $2a+5b+a$ ”, the possible answers all denoting premature closure were “ $8ab$ ” which was coded as 8a; “ $7ab$ ” which was coded as 8b; “ $7aba$ ” which was coded as 8c and “ $8aba$ ” which was coded as 8d (refer to Appendix C for a more detailed description of the coding). In addition the code “0” referred to questions which were unanswered by the learners.

The ICCAMS instrument utilised 2 codes (3 and 4) to represent “letter evaluated” and codes 5 and 6 for “letter as object” depending on common answers in the research in England. I have chosen to use one code to represent each of these representations of letters for the purposes of this study. I have therefore used the code 3 to represent “letter evaluated” and the code 5 to represent “letter as object”

The re-coding was completed using the codes by the WMC-S team in 2012 in coding the scripts of the Grade 10 learners and differed slightly from the one used in 2011. There were two differences in the coding used in 2011 and 2012. In 2012, the error codes for wrong answers (code 9) provided for more options than was provided in 2011. The reason for this was that during the coding of scripts in 2011, learners provided some common incorrect answers that were not catered for in the coding used in 2011. These were added on in 2012. An example is provided in the table below.

Table 6: Comparison of error code 9 in 2011 and 2012

Question 1.4	Code in 2011	Option in 2011	Code in 2012	Option in 2012
Simplify $2a+5b+a$	9a)	$7a^2b/8a^2b$	9a)	$10a^2b$
	9b)	$2a^2 + 5b$	9b)	$7a^2b$
	9c)	other	9c)	$8a^2b$
			9d)	$2a^2 + 5b$
			9e)	Other

It is evident from the table that there were 3 codes in code 9 in 2011 while there were 5 options in 2012. This was done in other questions as well by taking into consideration the responses of the learners in 2011, if there were common answers in 2011 that were not catered for in the coding. The other difference was that the number of options in the other categories was reduced to simplify the coding process. For example, there were more options for code 3 catered for in the 2011 coding than provided for in 2012.

4.2.2. Challenges experienced in the Re-coding of Scripts

Changes were made to the original coding i.e. coding by the first marker, (after consultation with my supervisor) because of first line coding, the coding of question 11, coding depending on the previous answer and human error. I now provide a detailed discussion of these coding issues.

4.2.2.1. First line coding

A major issue in the coding was that learners used more than 1 step to provide the answers to various questions. An example of this is question 3.3. "Add 4 to $3n$ ". Some learners provided the following as their response: " $3n+4 = 7n$ ". The original code was "1" which represented a correct response. To me, however this was clearly incorrect. A possible reason for coding this as correct was the coding policy that was stipulated by the WMC-S team. Coders were asked to code the first line of the answer if there was more than one line in the answer. This should only be the case when two options are provided by the learners indicated by the word "or". This is a contentious issue to me as in some questions, learners wrote for example, " $e+e+e=e^3$ ". Using the above method of coding, this is coded as correct even though the answer is clearly incorrect. Although I understand that coding the first line provides more information about the error for e.g. conjoining errors, I also believe that this may have the opposite effect in masking the error as in question 3.3 above. This is demonstrated by the following answer in response to the question "add 4 to $3n$ ".

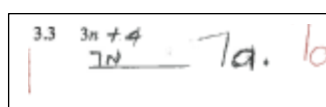


Figure 1: Example of first line coding

The above answer of " $7n$ " was originally coded as correct because the first line " $3n+4$ " represented the correct answer. I re-coded the answer (after consultation with my supervisor) as a conjoining error (code 7a) as I believe that coding it as "correct" masks the error demonstrated by the learner.

Coding an answer as “1” even if an error was made in subsequent steps is masking the error made after the first line. This issue was raised with my supervisor and subsequently with the person in charge of the original coding. The response was that this was done to ensure consistency as there was a team of coders and not just a single individual. The issue about this having the opposite effect in masking the error made by the learner in subsequent steps was considered to be not as important, as those learners who did this were not considered to be a significant number. For example there were learners who were incorrect in the first line of their answer but the final answer was correct as demonstrated by the following part of a script:

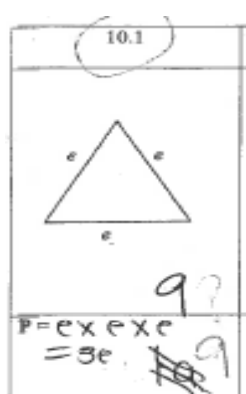


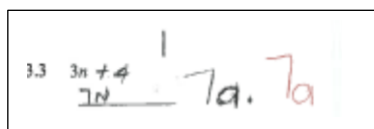
Figure 2: Example of inconsistency in first line coding

This was coded as incorrect (as it should). Although the learner provided the correct answer, his method of working demonstrated a lack of knowledge of what “ $e \times e \times e$ ” should be.

Apart from the above issues, the coding of the scripts by coding the first line of the answer was problematic since the reason for coding the first line of the answer was to ensure consistency. This may, however, have been counterproductive as in some instances the second line was coded. An example of this is when simplifying “ $2a+5b+a$ ” (Question 14. in the WMC-S test), the answer as well as the coding is provided below. This script was coded 3 times to ascertain the accuracy of the coding.

Figure 3: Example of second line coding

As is evidenced, this answer was coded as “premature closure” (code 8a). If the coding was consistent and the first line of the answer was coded then this should have been coded as correct (code “1”). The following answer by the same learner indicates the inconsistency in the coding:



The image shows a handwritten mathematical expression. On the left, there is a small '3.3' followed by the expression $3n + 4$. Below this, the number '20' is written. To the right of this, there is a vertical line, and then the expression $7a. 7a$ is written in red ink.

Figure 4: Inconsistency in coding

The question required the learners to add “4” to “3n”. The first line of the answer was coded as correct (code 1). The second line, however, denotes a conjoining error and I therefore coded it as such.

The above 2 examples illustrate the inconsistency in the coding that proved a challenge to me. A decision was therefore taken by my supervisor and me that for the purposes of this study, the point at which an error occurred was coded accordingly.

4.2.2.2. Coding of question 11

Another issue in the coding that was problematic to me was how question 11 was coded. The question was “*Cakes cost c rand and buns cost b rand each. If I buy 4 cakes and 3 buns, what does $4c + 3b$ stand for*”. In some cases learners provided answers like “4c rand and 3b rand” which was coded as incorrect (code 9). To me this indicated that learners understood that this represented the amount spent on cakes and buns and I therefore coded it as correct after consultation with my supervisor. There were other instances though where the answer was ambiguous as in “The number of cakes and buns and the number of money”. Although I re-coded this as incorrect, I still have doubts about this particular question and the responses provided as I believe that language barriers may prevent learners from expressing their answers accurately. One of the answers that reflect the problem in expressing the answer to this question is illustrated below.

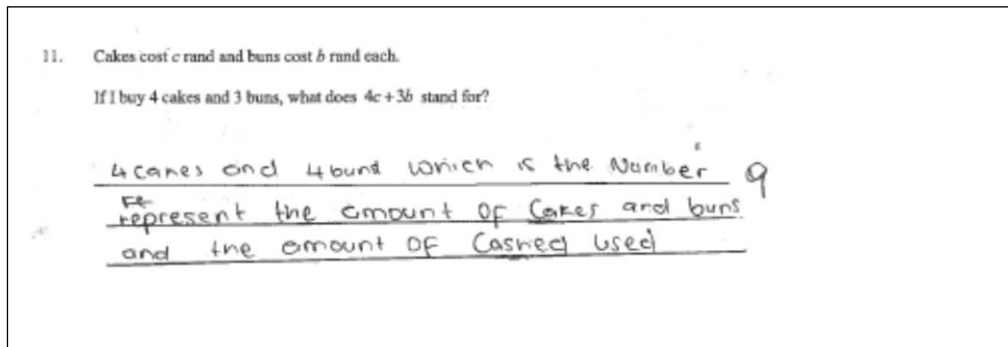


Figure 5: An example of the coding of question 11

The answer that was provided by the learner was, “4 cakes and 4 buns which is the number represent the amount of cakes and buns and the amount of cash used”. The answer was coded as incorrect. I believe, though, that language barriers may have prevented the learner from clearly expressing himself. The learner did mention the words “amount of cash used” indicating that he did have some idea of the cost of the items in his answer.

4.2.2.3. Coding depending on the previous answer

The coding for some of the questions depended on the answer provided for the previous question. An example of this is question 3.3 with the answer being “ $3n+4$ ”. This was coded as 1 if the answer to the previous question (Add 4 to $n+5$) was “ $n+9$ ” which was the correct answer. It was, however, coded as 2 (ambiguous) if the answer to the previous question was “ $n+5+4$ ”. The first marker did not take this into consideration when coding which resulted in incorrect coding in some instances.

4.2.2.4. Human Error

Finally, incorrect coding arose due to human error. This accounted for many of the errors in the coding. An example of this is provided in the following part of an answer by a learner who provided the answer “12” to the question “multiply $3n$ by 4”. This was coded as correct although it should have been “letter evaluated” and therefore coded as “7”.

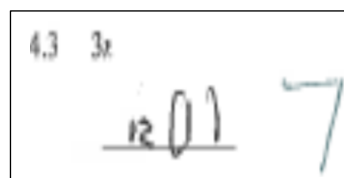


Figure 6: Example of error in coding

4.3. How the scores were calculated

The coded responses were entered onto a spread sheet. All questions of the same level were grouped and recorded together so that the level for each learner could be determined. Each correct response was added to find the total number of correct responses per learner in a particular level. The sum of the correct responses for each question in each level was calculated in order to determine the level of each learner. In addition, other questions which were not levelled, but which were included to be used for diagnostic purposes only, were also totalled. The total number of correct responses was calculated to find the percentage the learner received in the test. A small portion of the excel spread sheet is provided to illustrate how the above was done. The spread sheet indicates (as examples) how the recording was done and the levels determined for all levels.

	Grade	9	9	9	9	9	9	9	9			9	9	9	9	9	9	9	9	9	9	9			
	Level	L1	L1	L1	L1	L1	L1	L2	L2	L2	L2	L2	L2	L3	L3	L3	L3	L3	L4	L4	L4	L4	L4		
	Question no.	1.1	5.1	6.1	8.2	9.1	10.1	1.4	7.1	7.2	8.3	10.2	10.3	1.2	1.8	3.3	5.3	10	1.5	2	4.2	8.4	11	level	
																								%	
4		1	1	1	1	1	1a	8a	1	1	1a	1a	1a	8a	1	7c	4d	1a	9b	9a	7b	9	1	61	level2
5		1	1	1	1	1	9	1	1	1	1a	9	9	8a	1	2	8	9	9f	9e	9	9	9	36	level2
9		1	1	9d	9a	1	9	8a	9b	9d	1a	9b	9	8a	9e	2	8	9	9f	9a	2b	9	8	12	level0

Table 7: Extract of spread sheet of results of learners in Grade 9

Learner 5, for example, obtained correct answers to all but one of the questions on level 1 and could not answer any question on level 4 (all code 9). This learner was determined to be on level 2 as he was able to answer the minimum of two thirds of the questions on level 2 correctly (codes 1 or 1a) but was unable to do the same on level 3 and 4. He obtained 36% for the test in Grade 9.

The final percentage as well as the level of each of the learners in each grade was copied in the final columns in order to compare their marks and levels in each of the grades. On the basis of the comparison of results between Grade 9 and 10, learners were selected for phase 2 of the research i.e. task based interviews with these learners who were in Grade 11 in 2013.

4.4. Results and Findings

The results of the WMC-S test in Grades 9 and 10 were used to compile a summary of the learners' progression from Grade 9 to Grade 10. The results were summarised according to the levels achieved in each grade in order to address the first research question. The majority of the learners displayed a shift from lower to higher levels on the WMC-S test from Grades 9 to 10. There were, however, a few learners who remained at a particular level in both grades and some who moved from a higher to a lower level.

4.4.1. Shifts in levels from Grade 9 to Grade 10

The progression of learners through the levels from Grade 9 to Grade 10 are summarised as follows:

- There were 19 learners who progressed from the lower hierarchical levels to higher levels.
- There were 8 learners who did not change levels. Of these, two remained at level 1, one at level 2 and five at level 3.
- There were 2 learners who moved down a level from level 2 to level 1.

The summary of Grade 9 levels as compared to Grade 10 levels is illustrated by the following graph.

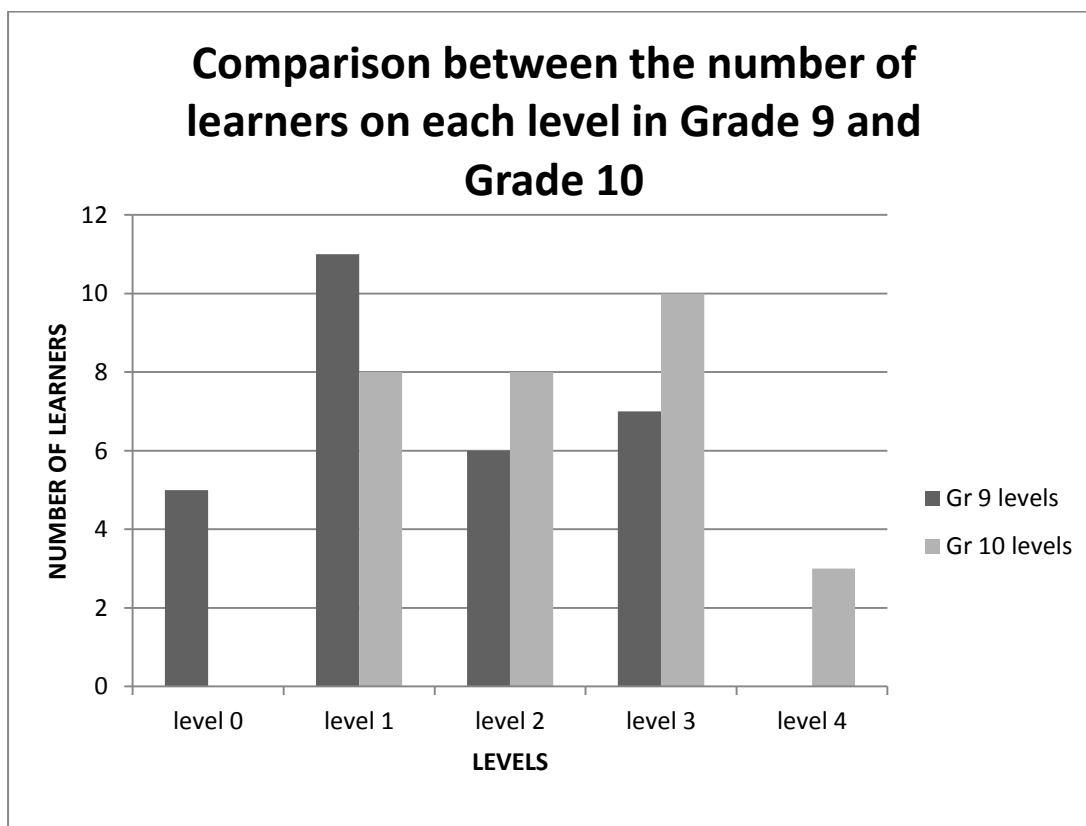


Figure 7: The number of learners on each level in Grades 9 and 10

Four learners progressed through more than a level from Grade 9 to Grade 10. For example, a learner who was on level 1 in Grade 9 was able to achieve the minimum number of correct responses on level 2 as well as on level 3 and has thus progressed from level 1 in Grade 9 to level 3 in Grade 10.

The biggest movement by a single learner is from level 0 to level 3. The other significant move is the movement from level 2 to level 4.

There were 3 learners in Grade 9 and 2 in Grade 10 who progressed from a lower to a higher level without first reaching a lower level. All 5 of them were not able to achieve at level 2. For example, in Grade 9, a learner progressed from level 1 to level 3 without first correctly answering the minimum number of questions to be on level 2. The learner was, however, able to answer two thirds of the questions on level 3 correctly. According to Hart (1981), success at a level is not generally achieved by learners without having succeeded at a lower level. As this represented an anomaly, I re-coded the scripts of the learners (in both Grades 9 and 10) as well as the scripts of the 4 learners who skipped a level from Grade 9 to Grade 10 so that I could verify that the coding was consistent and accurate.

Furthermore, the scripts of 2 of the learners who progressed to level four were also checked with my supervisor to ensure that my coding was consistent with the original. All of the scripts that were re-checked were confirmed as being accurately re-coded.

An unexpected finding that is evident from the graph is that in Grade 9 there were more learners on level 3 than on level 2. There were 6 learners on level 2 and 7 on level 3 in Grade 9 and in Grade 10 there were 8 on level 2 and 10 on level 3. The above graph indicates that there are more learners on level 1 and 3 than on level 2. This is surprising and therefore begs a deeper investigation.

4.4.2. Comparison between errors in Grade 9 and Grade 10

While some errors were persistent in both grades, there were errors in particular questions that were different from Grade 9 to Grade 10. I discuss some of the persistent errors as well as some of the differences in the errors.

The learner who moved up from level 2 to 4 (Brenda) displayed different errors in question 2, which is a question on level 4, from Grade 9 to Grade 10. In Grade 9 she indicated that “ $n+2$ ” is “ $3n$ ”, (by displaying a conjoining error), and is thus greater than “ $2n$ ”. In

Grade 10 she implied that multiplication is greater than addition. The following illustrates Brenda's answers in both grades:

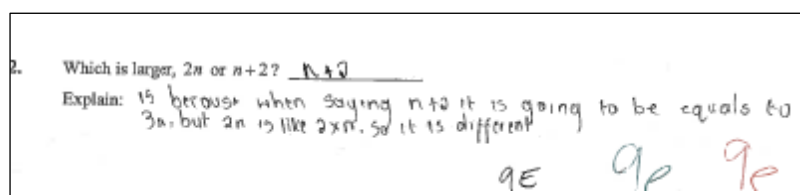


Figure 8: Brenda's answer to question 2 in WMC-S test in Grade 9

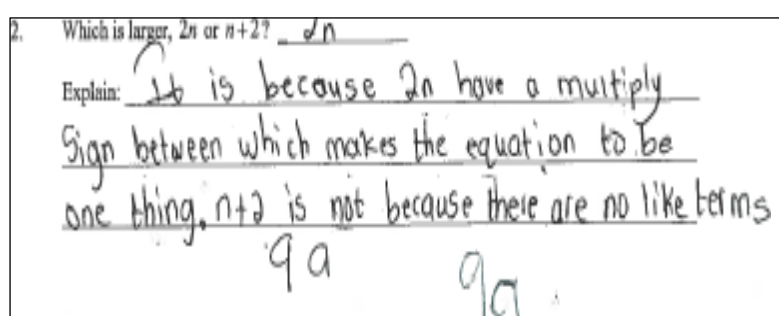


Figure 9: Brenda's answer to question 2 in Grade 10

No learner obtained a correct answer in question 2 in either Grade 9 or 10. The only other error made by Brenda in Grade 10 was in question 4.2. The answer was incorrect in both grades. In Grade 9 the answer to “ $n+4$ multiplied by 5” was “ $n+20$ ” and in Grade 10 for the same question, the answer provided was “ $4n+5$ ”. In Grade 9 the answer could be categorised as “letter not used” and was categorised as ambiguous in Grade 10. It is worth mentioning that only 2 learners obtained a correct answer for question 4.2 in both grades.

The learner who progressed from level 0 to level 3 also displayed some persistent errors e.g. in question 2, the learner maintained, in both grades, that multiplication is bigger than addition and therefore “ $2n$ ” is larger. Both the answers are illustrated below:

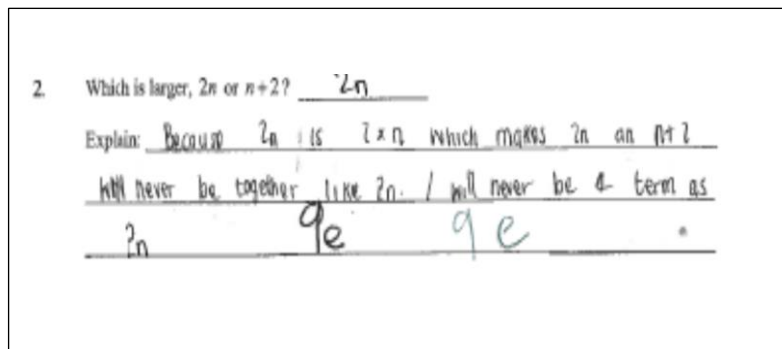


Figure 10: Response to question 2 in Grade 10

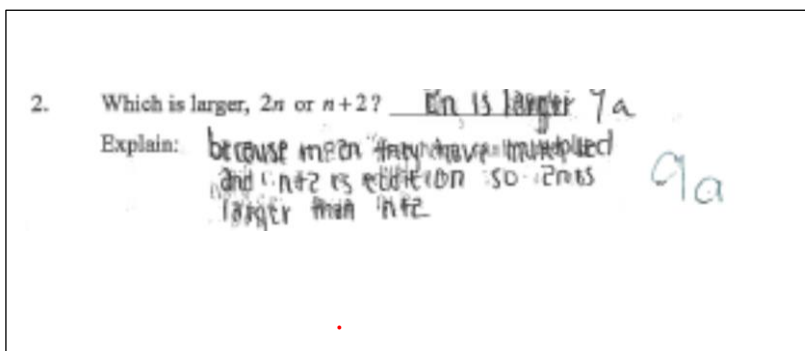


Figure 11: Response to question 2 in Grade 9

What is interesting, however, is that both learners, discussed above, displayed the same error in Grade 9 and Grade 10 i.e. question 4.2 which required learners to multiply “ $n+5$ ” by 4. In Grade 9 the answer was “ $n+20$ ” and in Grade 10 it was “ $4n+5$ ”. In each case 4 was multiplied by either the first term or the second but not by both.

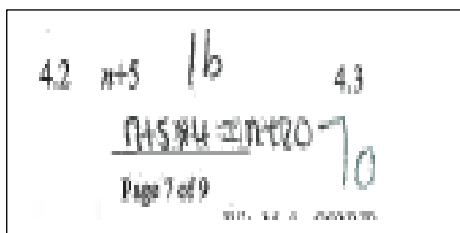


Figure 12: Response to question 4.2

This question was answered correctly by only 2 learners in each of the grades and can therefore be considered to be a challenge to most of the learners. The most common answer to this question was “ $n+20$ ” (provided by 6 learners in Grade 9 and 7 in Grade 10). Other common answers were “ $20n$ ” (conjoining) or either “ $4 \times n + 5$ ” or “ $n + 5 \times 4$ ” which were coded as ambiguous. Since the second phase of the study focussed on errors

experienced in level 2 questions, the errors illustrated in the answers to this question were not explored at the Grade 11 level.

I calculated the percentage obtained by the 29 learners in each of the questions in each year in order to determine the particular questions that were answered incorrectly by the most number of learners. The following were the results on the percentage of correct responses obtained by the learners in each of the questions at each level in Grades 9 and 10.

4.4.3. Level 1 Results

The results of the responses to questions on level 1 indicate that in Grade 9, questions 8.2 and 10.1 elicited the least number of correct responses. The least number of correct responses in Grade 10 were in questions 8.2 and 1.1. There was however a large increase in the number of correct responses to level 1 questions from Grade 9 to Grade 10. The following table indicates the results for the questions on level 1 in both grades.

Table 8: Results of correct responses to level 1 questions

	GRADE 9	GRADE 10
Level 1 Questions	Number correct	Number correct
1.1	24	23
5.1	23	27
6.1	23	25
8.2	16	21
9.1	28	29
10.1	20	26

These results were used to produce bar graphs in order to make comparisons between the results in Grades 9 and 10 more comprehensible at a glance.

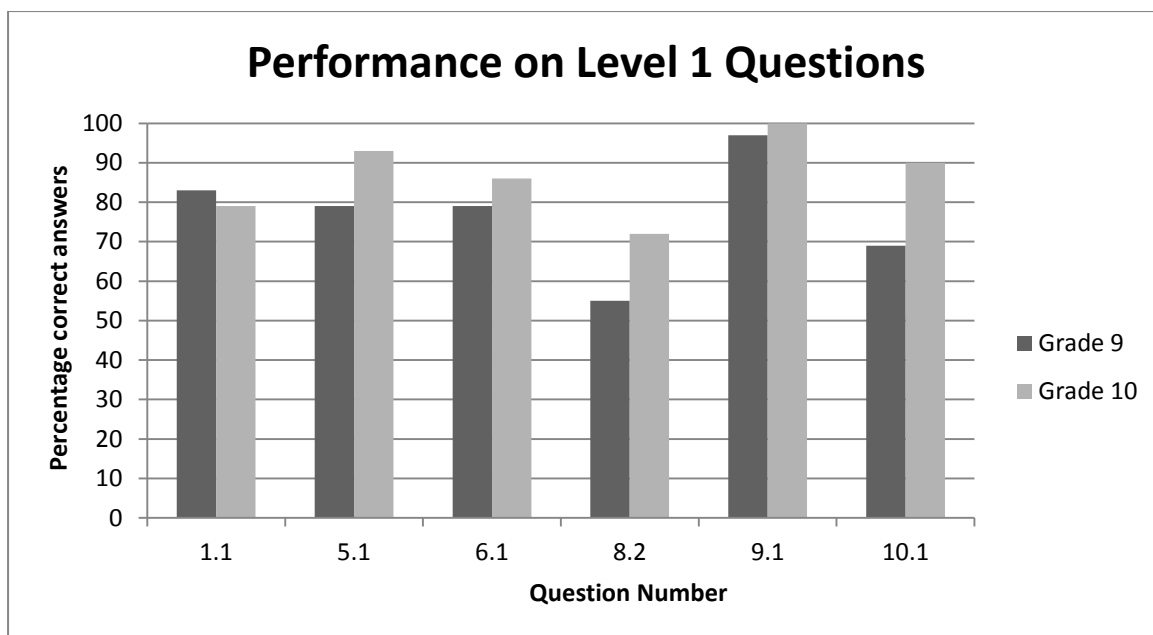


Figure 13: Grade 9 and 10 results per question on level 1

These results indicate that there was 1 additional learner in Grade 10 who obtained an incorrect answer in question 1.1. The other questions indicated an overall increase in the number of learners providing correct answers with the biggest improvement displayed in question 10.1. There was an increase of 6 learners who obtained correct responses to this question in Grade 10 as compared to Grade 9 while there were 5 more learners who obtained correct answers to question 8.2 in Grade 10 than in Grade 9.

The above results also indicate that at the Grade 9 level, the questions at level 1 did not pose much of a challenge to learners except for question 8.2. In Grade 9 only 55% of the learners answered this question correctly. There was, however, a change of 17 percentage points in Grade 10 where 72% of the learners answered it correctly, although this was still the lowest percentage of correct responses at the Grade 10 level. Question 8.2 required the learners to find the area of a rectangle with a length of 10 units and a breadth of 6 units. On analysing the scripts of the learners who provided incorrect responses to this question, it was found that some of them just indicated the formula as being “lb” without substituting values for the length and breadth. Others just added the lengths of all the sides thus seeming to confuse the area with the perimeter of a figure. This error does not indicate an error in algebra and so will not be discussed further.

On analysing the scripts of the learners who obtained incorrect answers to question 1.1, the common error detected was “ $2a+5a=7a^2$ ”. This error was found to occur in both grades (4

in Grade 10 and 2 in Grade 9). This error may be the result of learning exponents (in both grades) where learners are taught that when multiplying powers having the same base, the exponents are added. Olivier (1989) maintains that “rote learning” is the result of learning isolated ideas without linking them to previously learnt knowledge. Rote learning occurs when assimilation and accommodation is not possible and results in distorted rules which cause errors. In addition, the error may have occurred because of the failure to link new knowledge to previously learnt knowledge. Hiebert and Carpenter (as cited by Russel et al, 2009) contend that if new knowledge is not attached to prevailing networks, to solve new problems, students rely on strategies developed through their experience with similar material. Sometimes these strategies are used inappropriately, resulting in an error.

4.4.4. Level 2 Results

Unlike at level 1, there was an increase in the number of correct responses in all questions at level 2. Questions 1.4 and 7.2 proved to be challenging to the learners in both grades as these questions elicited the least number of correct responses in level 2. The results of the responses to questions on level 2 are illustrated by the following table:

Table 9: Results of correct responses to level 2 questions

	GRADE 9	GRADE 10
Level 2 Questions	Number correct	Number correct
1.4	11	14
7.1	16	24
7.2	10	16
8.3	17	24
10.2	14	20
10.3	13	19

The corresponding bar graph is displayed below:

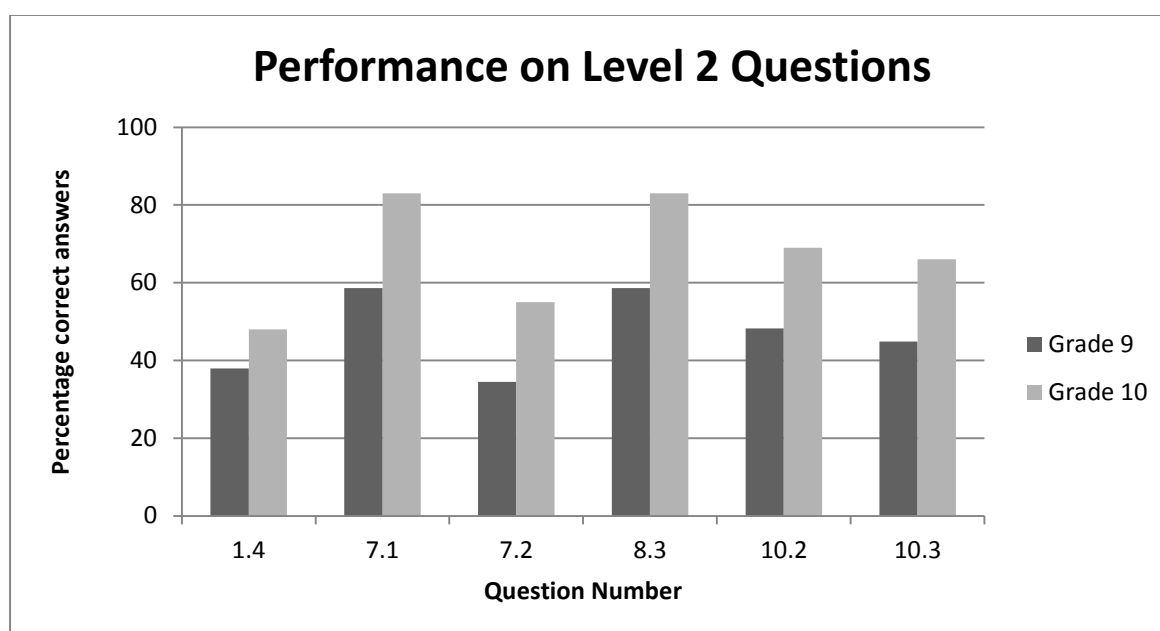


Figure 14: Grade 9 and 10 results per question on level 2

What is clearly observed in this graph is the consistency in the number of correct responses in all questions at this level in both grades as well as those incorrect responses i.e. the peaks and dips are maintained in both grades. The questions that had a high number of correct responses in Grade 9 still had the highest number of correct responses in Grade 10. The increase in the number of correct responses suggests that the learners at Grade 10 now had more exposure to algebra than was the case in Grade 9. The questions that posed the most challenge to learners in Grade 9 still challenged them the most in Grade 10, although there was an increase in the number of learners who answered them correctly. For example, there were only 3 more learners who answered question 1.4 correctly in Grade 10 than in Grade 9 indicating that this question still challenged the learners in spite of them learning more algebra in Grade 10.

The most common error in question 1.4 was one of conjoining. Learners responded to “ $2a+5b+a$ ” with the answer “ $8ab$ ” indicating conjoining.

Question	Response	Number of learners in Grade 9	Number of learners in Grade 10
Simplify $2a+5b+a$	$8ab$	5	10

Table 10: Common response to question 1.4

Table 10 indicates that in Grade 9, five learners displayed this error while in Grade 10, three learners provided the same response. There was only 1 learner who repeated the conjoining error in both grades. Two of the 5 learners were able to correct their error from Grade 9 to Grade 10. The other 2 obtained incorrect answers to the question in Grade 10 but did not display a conjoining error. There was 1 learner who obtained the correct answer to the question in Grade 9 but did not answer it correctly in Grade 10 and displayed a conjoining error. There was 1 learner who displayed a different form of the conjoining error from Grade 9 to Grade 10 i.e. he produced an answer of “7aba” in Grade 9 but “8ab” in Grade 10.

The most common answer to 7.2 which required the learners to find the value of “m” if “ $m=3n+1$ and $n=4$ ”, was “35”. There were 13 learners who obtained an incorrect answer to this question in Grade 9 and 8 in Grade 10. Five of them obtained the answer “35” in Grade 9 and 4 obtained the same answer in Grade 10. The answer “35” was most likely derived from placing the “4” “next to” the “3” thus making it “34” and adding “1” to obtain “35”. Other answers to this question in both the grades were “8”, which was found by adding the “3, 4 and 1” and the answers of “4” or “4n”.

The other common errors in questions on this level relate to the finding of the perimeter of quadrilaterals. The common error, however, stems from conjoining as the lengths of the sides of the figure are given in terms of letters (question 10.2) as well as letters and numbers (question 10.3). The common error in question 10.2 was that the perimeter of the figure with sides given as “h, h, h, h and t” was “4ht” in both Grades 9 and 10 while the common answer for the figure with sides “u, u, 5, 5 and 6” was “16u” in both grades. Both of these answers illustrate the conjoining error.

4.4.5. Level 3 Results

The questions on level 3 posed a challenge to most learners in both grades. In Grade 9 the highest percentage of learners responding correctly to any question was 38% while this number was 52% in Grade 10. The results are indicated on table 11:

Table 11: Results of correct responses to level 3 questions

	GRADE 9	GRADE 10
Level 3 Questions	Number correct	Number correct
1.2	8	14
1.8	11	15
3.3	8	15
5.3	3	13
10.4	5	7

The bar graph displaying these results is illustrated below.

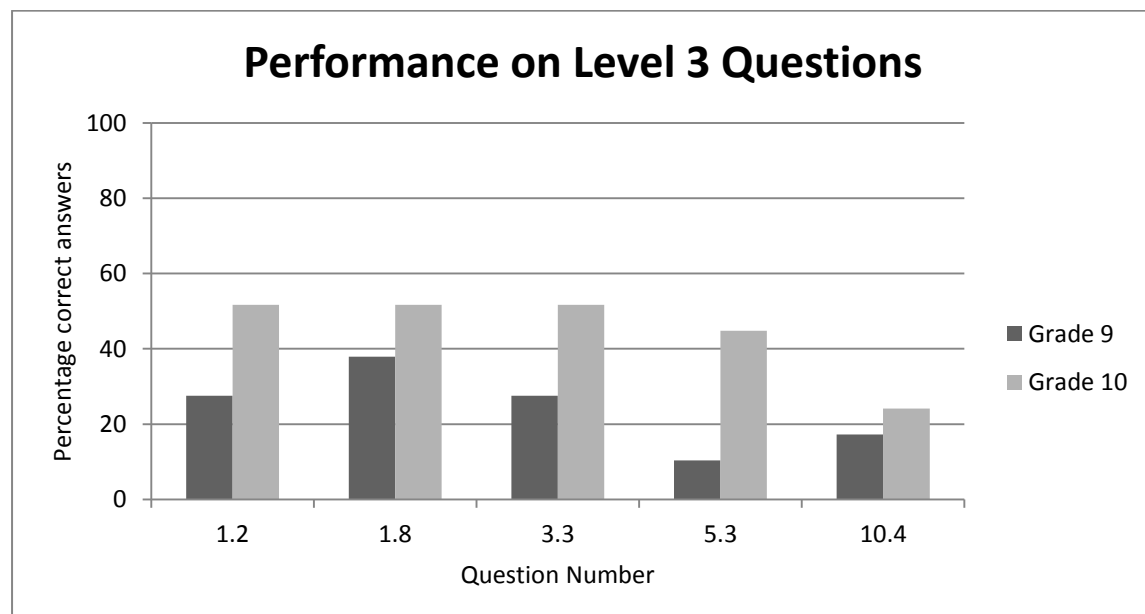


Figure 15: Grade 9 and 10 results per question on level 3

There were only 7 learners who were on level 3 in Grade 9 and 10 in Grade 10. An interesting observation is that all 7 either maintained their level from Grade 9 or progressed up a level. Five learners remained at level 3 while 2 progressed to level 4.

There were a further 3 who progressed from levels below 3 to level 3. An interesting observation at this level, was that every learner who provided an incorrect response in question 1.2, in Grade 10, had the same incorrect answer i.e. $2a + 5b = 7ab$. This is further evidence of the conjoining error.

Another surprising observation is that while only 10% of the learners in Grade 9 obtained a correct response to question 5.3 this percentage increased to 45% in Grade 10. This question required learners to find the value of “ $e+f+g$ ” given that “ $e+f=8$ ”. The common error in this question was “12” which was probably found by learners substituting the value “4” to each of the letters. This error indicates that learners used “letter evaluated” when interpreting the letters in this question. The increase in the number of correct answers may be attributed to an increased emphasis on equations in Grade 10, particularly with the equal sign signifying an equivalence relation rather than viewing it as a sign that produces a result of an operation. Learners have also learnt more algebra in general in Grade 10.

4.4.6. Level 4 Results

All the questions at this level seem to have posed a challenge to the learners with the highest percentage of correct responses being 31% in Grade 9 and 21% in Grade 10. There was, however, a slight improvement in the number of correct responses from Grade 9 to Grade 10. This is expected because Hart (1981) maintains that success at a lower hierarchical level is generally necessary for success at a higher level. The table represents the results of the learners’ answers to questions on level 4:

Table 12: Results of correct responses to level 4 questions

	GRADE 9	GRADE 10
Level 4 Questions	Number correct	Number correct
1.5	2	4
2	0	5
4.2	2	5
8.4	0	5
11	9	6

The corresponding bar graph is displayed below.

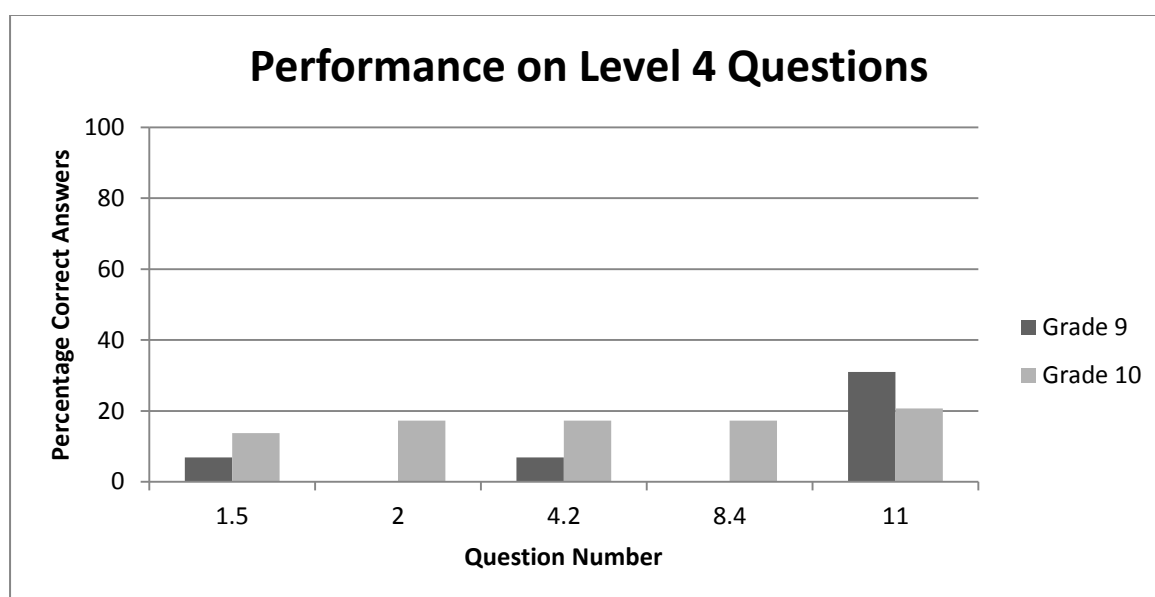


Figure 16: Grade 9 and 10 results per question on level 4

The most interesting observation in this level was that question 11 showed a decrease in the number of learners who answered it correctly from Grades 9 to 10. The question was: *“Cakes cost c rand and buns cost b rand each. If I buy 4 cakes and 3 buns, what does $4c+3b$ stand for”*. The most common answer to this question was “4 cakes and 3 buns”. There were 7 learners who provided the above answer to this question in Grade 9 while this number rose to 8 learners in Grade 10. This error illustrates that the learners regarded the “letter as object”. It is possible that the learners viewed the letter “ c ” as representing “cakes” with the letter “ b ” representing “buns” rather than the cost of these 2 items. MacGregor & Stacey (1997) contend that this error may be caused by teachers in the classroom frequently using words like “Let C denote the circumference” or “we will use c to stand for the cost”. Learners therefore think that these letters represent the words or objects themselves.

It was previously mentioned that this question posed a challenge during the re-coding of the scripts. It is possible that the coding of the responses had an impact on the decrease in the number of learners who answered this question from Grade 9 to Grade 10. To demonstrate this, the following is the answer provided by one of the learners in Grade 10.

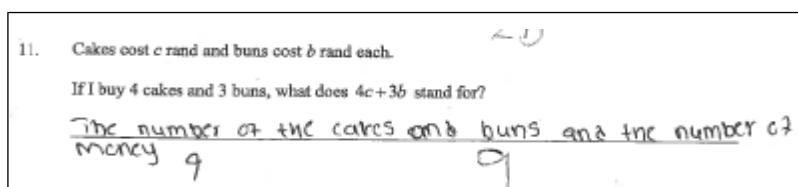


Figure 17: Example of response to question 11

Her answer reflects that the expression denotes the number of cakes and buns and the number of money thus demonstrating her understanding that the expression represents the cost. An answer from another learner was “it stand for the price and the product name” which also indicates that she had some idea that the expression represented cost by mentioning the word “price”. It was mentioned earlier that the school within which the research was conducted was a school where English is not the home language of the learners but it is an English medium school. It is possible that language barriers prevent learners from expressing their answers clearly. Since I did not include this question as part of the Grade 11 task, I could not establish that it was indeed a language barrier that posed a challenge to learners in the answering of this question.

I now proceed to examine some of the errors made by learners who did not move levels from Grade 9 to Grade 10 in the answering of questions on the different levels.

4.4.7. Learners who remained at the same level

In order to address a part of the first research question it is necessary to note that there were 8 learners out of the 29 learners who indicated no shift in levels from Grade 9 to Grade 10. Two of the 8 learners remained on level 1, one at level 2 and 5 at level 3. In order to determine the reasons for the lack of progress in levels from Grade 9 to 10, it is necessary to investigate some of the errors made by these learners that have contributed to the lack of progress through the levels. Some of the errors that have proved to be enduring during grades 9 and 10 are the simplifying of expressions involving brackets and the conjoining of terms.

Sally remained at level 3 in both years. There were 3 questions that she answered incorrectly in both grades. They were, however, answered differently in the different years. As an example, question 1.5 “simplify $(a-b) + b$ ” was answered as “ $a-b$ ” in grade 9 but as “ $ab-b^2$ ” in Grade 10. This may be the result of the topic “products” (involving brackets) which is covered in Grade 10. One of the topics in the Grade 10 syllabus specified by the curriculum documents (NCS and presently CAPS) is “products”. This topic relates to the

simplification of expressions by the removal of brackets. It includes examples of the type “simplify $(a - b)(a + b)$; $(2a + b)(a - b)$ or $b(a - b)$. The error made by the learner is possibly one where the learner treats “ $(a-b)+b$ ” as “ $(a-b)b$ ”. One reason may be that as soon as brackets are encountered they may signify multiplication to learners. Another reason could be that learners may have learnt that the “+” sign is sometimes not written in front of a number as the absence of the “+” sign indicates that the number is positive. Learners therefore ignore the positive sign and thus multiply out the brackets. This may illustrate confusion in the minds of the learners between the “+” sign as signifying an operation of adding numbers or indicating a positive number.

The issue about multiplying out the brackets is also highlighted by the answer provided to question 1.9 i.e. “simplify $(a+b) + (a - b)$ ”. The correct answer was provided in Grade 9 but in Grade 10 was answered as “ $a^2 - b^2$ ”. The learner most likely used FOIL (which indicates the order of multiplying the terms in the first bracket with the terms in the second bracket) or sum and difference to obtain the answer even though there was an addition sign between the brackets.

The multiplying of the brackets as soon as brackets are used in a question is also illustrated by the other learner, Fred, who also remained at level 3. Question 1.3 “simplify $(a+b)+a$ ” was answered correctly in Grade 9 but in Grade 10 the answer provided was $a^2 + ab$ suggesting that Fred multiplied “a” into the bracket. Question 1.9 “simplify $(a+b) + (a-b)$ ” was also answered correctly in Grade 9 but the answer in Grade 10 was $a^2 - 2ab - b^2$. This again indicates the learner multiplying brackets instead of adding. The above issues concerning brackets seem to suggest that new knowledge learnt in Grade 10 i.e. finding products seem to interfere with prior knowledge. This may be a possible reason for the learner obtaining correct answers to the above questions in Grade 9 but not in Grade 10.

Question 3.3 “add 4 to $3n$ ” was answered correctly by Fred, in Grade 9 but in Grade 10, “ $3n + 4 = 7n$ ” was provided as the answer. His answer indicated the error of “letter not used” as he seems to have ignored the letter and added the numbers. He probably then placed the letter next to his answer of 7.

The other interesting point is how question 4.2 was answered by Fred. In Grades 9 and 10 the answer to “ $n+5$ ” multiplied by 4 was “ $n+ 20$ ” indicating a persistent error. The learner only multiplied the “5” by “4” to obtain the answer “ $n+20$ ” in both Grades 9 and 10. The striking observation in the answer displayed in this question is the contrast in answers to

questions 4.2 and either 1.3 and 1.9. Irrespective of the fact that there was a “+” sign between the brackets in 1.9 and between the bracket and a term in 1.3 Fred multiplied out the bracket when all the question required was to add like terms. In question 4.2 however, the question required the learners to multiply “ $n+5$ ” by “4” without the use of brackets. This suggests that the learner multiplied out as soon as brackets were encountered in a question but seemed not to have grasped that multiplication of each term by “4” was required in question 4.2, because there was an absence of brackets.

The other persistent error by Fred was indicated in question 2. In both grades the answer was that “ $2n$ ” and “ $n+2$ ” were the same. In Grade 9 Fred wrote that “ $n+2$ ” was not added, but was the same as “ $2n$ ” and in Grade 10, he mentioned that “ $n+2 = 2n$ ” and is therefore the same. This indicates conjoining and seems to be a persistent error by Fred.

4.4.8. Common Errors at all levels

The errors in the answering of all the questions on all levels were summarised according to the different interpretations of letters that learners use. The errors and their occurrences are summarised in the following table:

Table 13: Errors according to interpretations of letters

	Grade 9	Grade 10
Letter Evaluated (code 3)	16	6
Letter as object (code 5)	11	8
Letter not used (code 7)	44	39
Premature closure(code 8)	68	42

The bar graph illustrating the results in both grades is depicted below:

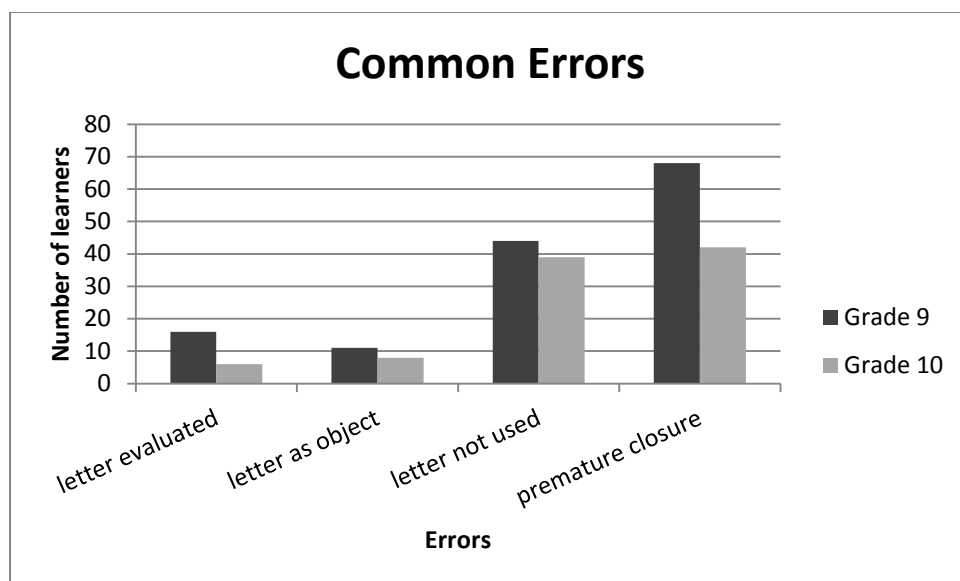


Figure 18: Comparison of common errors between Grades 9 and 10

The occurrence of most of the errors has decreased from Grade 9 to Grade 10 except for “letter as object” which has increased in Grade 10.

The two most common errors according to the different interpretation of letters are “premature closure” (conjoining) and “letter not used”, in both the grades, although the frequency of these errors have declined from Grade 9 to Grade 10. Some errors are unique to particular questions. For example, in question 11, the most common error is “letter as object” which is not evident in any other question. I will not focus on this in the remainder of the study as there was only 1 question in the WMC-S test that this error can be attributed to. The conjoining error though will be a focus of the study as a prominent error especially in questions on level 2.

Apart from errors relating to the incorrect interpretation of errors, other common errors related to exponents, substitution errors and an error that I have termed “HCF (highest common factor) error”. One of the common errors relating to exponents in Grade 9 was the answering of a question like “simplify $2a+5b+a$ ” as “ $7a^2b$ ”. This error indicates that some learners confused the adding of like terms with multiplying them. This error changed to one of conjoining in Grade 10 by some of the learners. The same question was answered in Grade 10 as “ $7ab$ ” or “ $8ab$ ”.

The substitution error was most common in question 7.2 which was “if $m=3n+1$ and $n=4$, find the value of m ”. The most common answers to this question were “35” and 8. In order to obtain the first answer learners may have placed the “4” next to the “3” and obtained “34” and then added “1” to obtain “35”. This error was found to occur in both grades 9 and 10. The answer of 8 to the same question may have been derived from adding “3” and “4” to obtain “7” and then adding “1” to obtain “8”. Instead of multiplying “3” and “4” learners thus added them. This may also indicate the conjoining error as learners may have considered “3n” to be the result of “3+n”.

The other error related to adding “like” and “unlike” terms. When adding terms like “h, h, h, h and t” some learners obtained an answer of “h+h+h+h+t = ht”. I have termed this error as finding the HCF as learners have considered only one of the repeated letters to obtain their answer which is similar to finding the HCF of numbers.

4.5. Conclusion

In this chapter, I discussed the coding used in the WMC-S test as well as the challenges posed by the re-coding of the scripts in phase 1 of the study. The results of the learners in terms of the levels of achievement in the WMC-S test were provided in this chapter. I drew a comparison between the results of the learners in Grades 9 and 10. One of the main revelations in the results was that 2 learners in each grade were discovered to move on to a higher level without first satisfying the minimum requirements of the lower level. What was expected though was the low level of correct responses on the level 4 questions of the WMC-S test. Some of the main errors that learners made in each level were also discussed. Finally I illustrated the prominence of the conjoining error and to a lesser extent, the error of “letter not used”. In the next chapter, I provide details of phase 2 of the study which describes the qualitative analysis of the results of the interviews with Grade 11 learners.

CHAPTER 5

RESULTS AND ANALYSES OF GRADE 11 TASK AND INTERVIEWS

5.1. Introduction

Phase 2 of the study entailed the sample group of 6 learners completing a written task as well as the conducting of interviews with each of the learners based on the task. I provide the details of how the task was constructed together with the results of the task and how I determined the current level of the learner in Grade 11. The results of the written task are analysed according to the most persistent errors which have been identified as conjoining and “letter evaluated”. Errors like exponents and substitution into an expression are also discussed as errors that have faded away from Grades 9 to Grade 11. I also provide details of particular problems learners experienced with some of the questions on level 2 which lend themselves to conjoining, in both the WMC-S test as well as in the Grade 11 task and which posed an obstacle to learners answering the minimum number of questions to be on level 2.

5.2. A Description of the Coding of the Grade 11 Task

Level 2 questions were a focus in the Grade 11 task although questions on level 1 and level 3 were also included. The main reason that level 2 questions were a focus in the Grade 11 task was that some learners had progressed to level 3 without first obtaining the minimum number of level 2 questions correct to achieve on level 2. This was discussed in detail in section 3.8.1. There were a total of 21 questions divided into the 3 levels. There were 5 questions each on levels 1 and 3 while there were 11 questions on level 2. All of the questions were based on questions of a similar type to those in the WMC-S test. The similarity between the questions in the WMC-S test and the Grade 11 task is illustrated in Table 3 in section 3.8.1.

I could not use the results of every question on level 2, in order to determine the level of the learner in Grade 11, as this would have resulted in an incorrect weighting. In the WMC-S test there were 5 questions on level 2 while the Grade 11 task contained 11 questions on level 2. There were 3 questions in the Grade 11 task that were of the same type as question 10.3 in the WMC-S task. The other questions in the Grade 11 task, on

level 2, had 2 questions each which were of the same type as the equivalent questions in the WMC-S test. I therefore, did not use question 5.2 from the Grade 11 task. Questions 5.2, 5.3 and 5.4 were all of the same type. The results for question 5.2 were not used as one of the questions to determine the level of the learners because it consisted of a four sided figure while the other questions (5.3 and 5.4) were 5 sided figures which were what the WMC-S test questions of a similar type consisted of. Only the results of 10 out of the 11 questions from the Grade 11 task were considered in order to determine the level of learners in Grade 11.

A learner was determined to be on level 2 in the WMC-S test if he/she had obtained correct responses for at least two thirds of the questions on level 2 i.e. $\frac{2}{3} \times 5$. I therefore determined a learner in Grade 11 to be on level 2 if he/she had obtained at least 2 thirds of the questions on level 2 correct i.e. $\frac{2}{3} \times 10 = 6,7$ questions correct. A learner would therefore be required to obtain at least 7 correct answers to be on level 3. I have chosen to round up to 7 and not down to 6 because I wanted to be certain that a learner was definitely on level 2 and not on the border between levels.

The responses in the Grade 11 task were coded using the same codes as the WMC-S test. The results of the responses from the task were entered onto a spread-sheet according to the different levels. The same criteria, as in the WMC-S test, were used to determine the level of the learner in Grade 11. The results of the responses to the questions in the Grade 11 task are depicted in table 14.

Reading vertically
provides an indication of
the result per question

Name	Grade	L1						L2										L3					Level of learner
		1	2	3	5.1	9.1	4.1	4.2	5.2	5.3	5.4	6.1	6.2	7.1	7.2	8.1	8.2.	9.2	10.1	10.2	11	12	
Promise	9	1	1	1	1a	1	1a			1a		8a		1		1		1a	8a	1a	7c	4d	2
	10	9b	9d	1	1a	1	1a			1a		1b		9d		9b		9	1a	1	1	1	3
	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	4	1	1	1	7	3
Lettie	9	1	1	1	9	1	1a			9		1		1		1		9	8a	1	2	8	2
	10	1	9d	1	1a	1	1a			8a		9		1		1		9	8a	1	2	8	1
	11	1	1	1	9	9	1	1	1	1	1	1	1	1	1	1	1	3	1	1	1	9	3
Patricia	9	1	1	1	1a	1	1a			1a		9c		1		9d		9	9b	8a	1	9	2
	10	1	1	1	1a	1	1a			9		8a		1a		9a		0	8a	8a	7a	4d	1
	11	1	1	1	1	0	9	9	8	8c	8c	1	1	1	1	1	1	0	1	1	7	7	1
Sbu	9	1	1	1	9	1	9			9		8c		8		9d		3	9b	9e	7a	3	1
	10	1	1	1	1a	1	1a			8c		8a		1		1		1a	8a	9d	9a	3	1
	11	1	1	1	1	1	1	1	8	8c	8c	8	8c	1	1	1	1	0	8	8	7	4	1
Tembi	9	1	1	1	9	1	1a			9		8a		1		9a		3	8a	9d	7a	3	1
	10	1	1	1	9	1	1a			9		9c		1		1		9	1a	9e	1	8	1
	11	1	1	1	1	1	9	9	1	1	1	1	1	1	1	1	1	3	1	1	1	1	3
Ottie	9	1	1	1	1a	1	1a			1a		1		1		9d		0	8a	1	1	4b	2
	10	1	1	1	1a	1	1a			1a		9a		1		9d		8	8a	8b	1	8	2
	11	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	3	1	1	9	1	3

Table 14: Spread sheet of responses in Grades 9, 10 and 11

Horizontal reading
provides a learner's
performance in a
particular year including
level achieved

The first 2 rows indicate the question numbers on each level from the Grade 11 task. The results of each of the 6 learners selected for the interviews are indicated per grade. The first 2 rows for each learner indicate the results of the learner in the equivalent questions in the WMC-S test while the third row indicates the results for the questions in the Grade 11 task. The last column indicates the final level of the learner in each of the grades from Grade 9 to Grade 11. For example Patricia was determined to be on level 1 on the basis of the results of the Grade 11 task as she had obtained 4 correct responses out of the 5

questions on level 1. She was not able to answer the minimum number of questions on levels 2 and 3 and therefore remained on level 1 in Grade 11. She was on level 2 in Grade 9 but on level 1 in both grades 10 and 11.

The spread-sheet contains the codes 0 to 9 as used in the WMC-S test and subsequently in the Grade 11 task.

5.3. Results of the Grade 11 Task

The results for Grade 11 were determined by coding the task which was written by the learners prior to the interview. The results were based purely on the learners' written responses and not on the answers provided by the learners during the interview. Table 15 illustrates the levels of each of the learners in the sample from Grades 9 to 11.

Table 15: Levels of learners

Learner Name	LEVELS		
	Grade 9	Grade 10	Grade 11
Promise	2	3	3
Lettie	2	1	3
Patricia	2	1	1
Ottie	2	2	3
Sbu	1	1	1
Thembi	1	1	3

These results indicate that 2 of the 6 learners in Grade 11 were not able to obtain the minimum number of correct answers to be placed on level 2. There are, however, some questions on level 2 that posed problems to some of the learners who have reached levels 2 and 3.

The table also indicates an anomaly in the results for Lettie. She was on level 2 in Grade 9 but on level 1 in Grade 10 and on level 3 in Grade 11. I therefore re-coded the scripts of this learner, in all grades, to ensure that there were no discrepancies in coding. What I did find, was that this learner made errors which I believe were due to carelessness which Olivier (1989) describes as “slips”. An example of this is that when solving for “ a ” in the equation “ $a+5=8$ ” (question 6.1 in the WMC-S test) in Grade 10, the learner wrote “ $a= 8-5=2$ ” and when simplifying “ $2a+5b+a$ ” (question 1.4 in the WMC-S test) the learner

wrote " $3a+5a$ " in the first line of her answer. Although the first line may be considered a "slip" she did go on to write her answer as " $8ab$ " which might now indicate a conjoining error. If she was given the benefit of her response in question 6.1 being a "slip" she would have been on level 2 in Grade 10. She would thus have been on level 2 in Grades 9 and 10 and on level 3 in Grade 11.

5.4. An analysis of the errors made by learners from Grades 9 to 11

In order to determine the errors that persisted during the 3 grades and those that have faded away, it is necessary to analyse each of the questions in each level. I have analysed the results of the written tasks as well as the interviews with individual learners from the 6 selected learners. In order to analyse the errors in the Grade 11 task, I made a list of the answers of each of the questions on each level by every one of the 6 learners in Grades 9, 10 and 11. The following is an illustration of how this was done using the responses of Promise.

Question	GR9	GR10	GR11
11. $2a+5a$. 51. $a+b=43$ $a+b+2=4a$ 61. $a+5=8$ 9.1. $2 \frac{10}{9}$ 10.1. $\frac{9}{e}$ Level 1	7a 45 a=3 p=22 p=3e	6a 45 a=4 p=22 p=ere+e=3e	1. $3a+2x=5x$ 3. $j+k=5$ then $j+k+2=7$ 2. $k+2=5$ then $k=3$ 9. 3^3 p=30. 5.1. $\frac{9}{e}$ p=0.010=3
Level 2: 1.4. $2a+5b+a$ 7.1. If $u=v+3$ & $v=1$ 7.2. $m=3m$ & $n=4$ 8.3. $\frac{m}{n}$ 10.3. $\frac{u}{5}$ Level 3	$3a+5b=8cb$ $u=v+3=4$ $m=12+1=13$ $A=mn$ $p=2u+1b$	3a+5b $v+3=4$ $3m+1=4$ $A=mn$ $p=2u+1b$	61. $2p+3a+4p=3p+3a$ $2p+3a+3p=5p+3a$ 7.1. $x+y+8$ y=2 then $x=7$ 7.2. $x+y-1$ y=3 then $x=2$ 8.1. $p=2m+1$, $n=2$, $p=5$ 8.2. $p=5n-1$, $n=4$, $p=19$ 4.1. $\frac{9}{e}$ A=pq 4.2. $\frac{9}{e}$ A=2ab 5.2. $\frac{9}{e}$ A=2ab 5.3. $\frac{9}{e}$ p=2p+14 5.4. $\frac{9}{e}$ p=2d+q 10.1. $3x+4y=3x+22y$ All correct Add 6 to 6: 6+6 12. $a+b=3$ then $a+b+2=5$ 9.2. $\frac{9}{e}$ p=24n

Figure 19: Example of recording of errors

The figure represents the responses of a learner in all questions in the WMC-S test and the equivalent questions in the Grade 11 task. For example, the question “simplify $2a+5a$ ” in the WMC-S test was answered as “7a” in Grade 9, as “6a” in Grade 10 and the equivalent question in the Grade 11 task i.e. “simplify $2x+3x$ ” was answered as “5x”. The incorrect answers in every grade were highlighted so as to compare the errors in each grade.

5.4.1. Overall results from Grades 9 to 11 as per response to each question

Table 15 indicates the errors per grade in all the questions in the 3 grades. These errors are categorised according to the different interpretation of letters as used in the ICCAMS and subsequently in the WMC-S test. It must be noted that not all of the questions in the Grade 11 task were used to ascertain the number of errors according to the different interpretations of letters. The Grade 11 task consisted of questions from levels 1 to 3 which were similar to the questions from the WMC-S test. Since my focus was on questions on level 2, the Grade 11 task consisted of more questions on level 2. Some of the questions on level 2 consisted of more than one question of a similar type to the WMC-S test. For example questions 5.2, 5.3 and 5.4 in the Grade 11 task were similar to question 10.3 in the WMC-S test. If I included results for all 3 questions from the Grade 11 task which were of the same type in the WMC-S test, it would result in a skewed data set because if an error occurred in one of the questions it would be repeated in all questions of the same type. Therefore only the result for question 5.2 was included in my analysis. This question most closely resembled the question in the WMC-S test.

The Grade 11 results were obtained on the basis of the Grade 11 task that learners had written before the interviews. The interviews were conducted to obtain a sense of how learners had arrived at their answers in the Grade 11 task and were not used to determine the levels of the learners in Grade 11. The Grade 11 task was coded using the same coding as the WMC-S test. These codes were entered onto a spread sheet for the sample of 6 learners who had written the test. The total number of errors that were made according to the different interpretation of letters was counted and the results are provided in the following table together with the results of the 6 learners in the WMC-S test in Grades 9 and 10.

Table 16: Summary of results according to interpretation of letters

	Answer missing (code 0)	Letter evaluated (code 3)	Letter as object (code 5)	Letter not used (code 7)	Premature closure (code 8)	Wrong (code 9)	Total
Gr 9	1	6	Nil	4	8	19	38
Gr 10	1	2	Nil	1	14	20	38
Gr 11	3	5	Nil	4	4	8	24
Total	5	13		9	26	47	100

The above table indicates that apart from a general incorrect answer (code 9), the most common error in Grade 9 was “premature closure”(conjoining, code 8) followed by “letter not used” (code 7) and “letter evaluated” (code 3). The number of errors associated with “premature closure” in Grade 10 (14), however, surpassed the number in Grade 9. In Grade 11, this figure was reduced to 4 errors, although it still accounted for the most number of errors in the different interpretation of letters.

5.4.2. A discussion of the most persistent errors

During the process of analysing the errors committed by learners from Grades 9 to 11, it was found that 2 errors were persistent. These were *conjoining* and *letter evaluated*. It was also found that the manner in which conjoining manifested by different learners differed. I have used the terms “**ignoring letter**” and “**adding co-efficients and constants**” to indicate the different manifestations of conjoining.

5.4.2.1. Conjoining

Conjoining refers to the tendency of learners to finish an algebraic expression. An example of this is when simplifying an expression like “ $2a + 5$ ”; learners provide the answer “ $7a$ ”. Tirosch et al (1998) maintain that students have difficulty in accepting a lack of closure and therefore complete or finish the expression.

The conjoining error, according to the results indicated by table 15, is the most prevalent error in all 3 grades. It must be noted however that in Grade 11, this error was prevalent with two of the 6 learners in the sample while the other 4 learners displayed no evidence of conjoining. What is noteworthy is that both the learners who displayed the conjoining error remained on level 1 from Grade 10 to Grade 11 while the others were determined to be at least on level 3 in Grade 11.

The other interesting point is that this error was at its peak during Grade 10. In Grade 10, 14 out of a total of 38 errors were due to the conjoining error. This translates to 37% of the errors in Grade 10. This was reduced to 4 out of a total of 24 errors in Grade 11 which accounts for 16.7% of the total number of errors in the Grade 11 tasks. Only the questions

from the WMC-S test that were selected to be included in the Grade 11 task were used to make this comparison in Grades 9, 10 and 11.

Both the learners in this study, who remained on level 1 in Grade 11, revealed conjoining errors. Although they displayed conjoining errors, the manner in which conjoining was manifested differed. Literature relating to conjoining refers to this error in general without any differences in the way that conjoining is used. During the analysis of learners' responses in the Grade 11 task, I categorised these different manifestations using the terms “**ignoring letter**” and “**adding co-efficients and constants**”. These terms are demonstrated by the following table with pertinent examples:

Table 17: Categories of Conjoining

Description of item	Example	Ignoring letter	Adding co-efficients and constants
Letters and numbers	$p+p +5+5+4$	$14p$	$16p$
Letters only	$2x+x+y$	$3xy$	$4xy$

The examples provided are 2 examples from the task provided to Grade 11 learners. The first example consists of letters as well as numbers while the second consists of letters only. The last 2 columns indicate how the given examples were answered using different manifestations of conjoining.

I have used the term “**ignoring letter**” to describe learners' responses where they have only added the numbers and placed the variable next to the numbers, as indicated by example 1. The distinction between my term of “ignoring letter” as opposed to “letter not used” Küchemann (1981) is that “ignoring letter” refers to learners acknowledging the letter but not using it to obtain their answer. The interpretation of “letter not used” includes the above explanation but also includes examples where learners sometimes do not write down the letter in their answer. An example of this is when learners write “5” in response to a question to simplify “ $3a+2b$ ”. In example 2, learners added “like terms” to arrive at “ $3x$ ” and again just placed the “y” next to “ $3x$ ”. In order to obtain a sense of how learners think about and provide a solution to the particular question in the second example, Sbu explained during the interview that: *“I added 2x and x and got 3x then I didn’t know what to do with the y so I just put it here”*.

The second form of conjoining, “**adding co-efficients and constants**” describes the tendency of learners to provide a letter like “p” with the value of “1”. The learners therefore added the numbers in example 1, to arrive at “14” and then added “p+p” to obtain “2”, which they added to “14”. They then placed a “p” next to this to obtain “16p”. This is illustrated by Sbu in the following statement in response to the question on finding the perimeter of a figure with sides “3”, “3”, “a” and “a”.

Sbu: so I took p plus p and got 2p and 5 plus 5 plus 4 and got 14. So 2p plus 14 is 16p.

Conjoining errors made by Patricia

Patricia is one of the learners who remained on level 1 in Grade 11 and displayed conjoining errors in her answers to some of the level 2 questions.

In questions 5.2, 5.3 and 5.4 in the Grade 11 task, Patricia displayed “conjoining” errors when finding the perimeter of a rectangle (question 5.2) and 2 pentagons (questions 5.3 and 5.4). When finding the area of a rectangle with length “a” units and breadth of 3 units, her original answer was “ $3a+3a=6a$ ”. When asked to show detailed working, during the interview, she wrote “ $3+3+a+a=6+2a=8a$ ”. She then struck off her original answer and maintained that the answer was “8a”. This conjoining error reflects the category “adding co-efficients and constants. Her answers to the next 2 questions also reflect errors of “conjoining”, however these were not consistent. Her answer to the perimeter of pentagon with sides “p, p, 5, 5 and 4” was “14p” and the one with sides “d, d, 3, 3 and 3” was “9d”. While the latter 2 answers illustrated “conjoining” with letter ignored”, the answer to 5.2 illustrates conjoining by “adding letters and constants”. When asked to explain her inconsistency, Patricia put her head in her hands and complained “***This is what I don’t get. How do you infuse numbers with the alphabet?***”

This lament by Patricia echoes a claim by Watson (2009), who maintains that students are often confused by expressions that combine numbers and letters. Patricia’s inconsistency seems to originate from her confusion on the adding of “like and unlike terms”.

Surprisingly her response to the equivalent question in the WMC-S test in Grade 9 was correct while her answers in Grade 10 were not. The questions in the WMC-S tests were to find the perimeter of figures with sides “h, h, h, h and t” and “u, u, 5, 5 and 6”. Her answers in Grade 10 were “5ht” and “16u” respectively. Her answers again reflected an inconsistency in that in the first answer she wrote “ $4h+1t = 5ht$ ” while in her second answer she only added the numbers, and not the co-efficients of the variables, and placed

the variable next to it. The differences in the 2 answers may be the result of the first one consisting of only letters while the second consisted of both letters and numbers. I could not establish if this was indeed the case as the Grade 11 task did not contain an example with letters only. The differences in answers in Grade 9 and 10 could be the result of Patricia believing that she would rather leave her answer incomplete than obtain an incorrect answer as she explained during the interview.

I: So you are not sure how to do these with the numbers and letters?

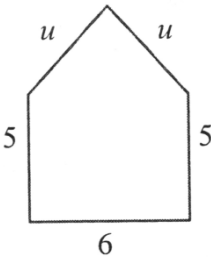
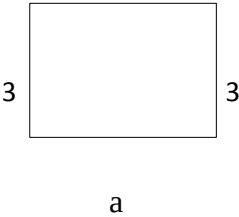
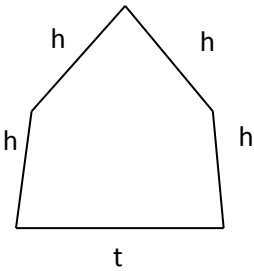
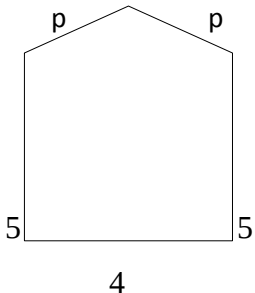
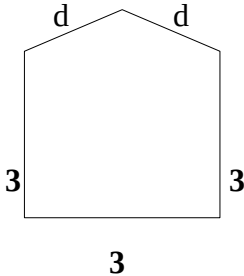
L13: So I leave my answer halfway because I'm not sure what to do next

I: So rather leave it halfway than get it wrong?

L13: Yes

Another surprising issue was that question 6.1 (Simplify $2a+3p+a$) in the Grade 11 task was correctly answered by Patricia. The following table illustrates the questions in the Grade 11 task as well as the equivalent questions in the WMC-S test and the responses by Patricia in each grade relating to conjoining:

Table 18: Responses by Patricia in each grade

WMC-S test	Grade 11 Task	Responses by Patricia		
		Grade 9	Grade 10	Grade 11
1.4. Simplify: $2a+5b+a$	6.1. Simplify: $2a+3p+a$	$3a5b$	$8ab$	$3a+3p$
10.3 Find perimeter 	5.2 Find perimeter 	$2u+16$	$16u$	$3a+3a=6a$
10.2. 	5.3. 	$4h+t$	$4h+1t=5ht$	$14p$
	5.4. 			9d
1.2. Simplify $2a+5b$	10.1 Simplify $3x+2y$	$2a5b$	$7ab$	$3x+2y$
	10.2 Simplify $2x+x-y$			$3x-y$
3.3. Add 4 to $3n$	6. Add 6 to t	$7n$	$7n$	$6t$

Her responses to question 1.4 in both grades 9 and 10 illustrate “conjoining” errors although they were illustrated differently. There was no evidence of “conjoining” in her response in Grade 11 (question 6.1). When asked to explain the discrepancy between previous answers and this one, she maintained that the questions were in different forms or phrased differently. It can be gathered from the answers to all her questions in the Grade 11 task that she was able to answer questions which were of the form “simplify” an expression correctly. If it appeared in a different form from this then she used “conjoining”. This is evidenced from her answers to 5.2, 5.3 and 5.4, as discussed above as well as to question 11 (level 3) which required her to “Add 6 to t” and to which she responded “6t”.

Although Patricia answered questions 10.1 and 10.2 in the Grade 11 test correctly, I am not convinced that she is able to add “like and unlike” terms correctly. Her response to “Simplify: $3x+2y$ ” was to leave it as it was, however, during the interview she changed this to “ $5xy$ ”. She was not sure though and subsequently struck off the “ $5xy$ ” and left her answer as “ $3x+2y$ ”. She was provided with an additional example of “ $3x+y$ ” to simplify and promptly declared the answer to be “ $3xy$ ”. When questioned about the discrepancy between the 2 answers, she replied as follows:

Patricia:	<i>Well the psychology of this ... if it comes in a different form is different</i>
I:	<i>So this is $2y$ (in Question 10.1) and what’s the number for the y here? (points to $3x+y$)</i>
Patricia:	<i>There’s no number. That’s why I’m saying it completely erases</i>
I:	<i>So do you do this differently?</i>
Patricia:	<i>Ja because it doesn’t have a number in front of it.</i>

She thus explained the discrepancy by mentioning that if a letter does not have a number in front of it then the answer is different from those examples with a co-efficient that is not “1”. Thus “ $3x+2y$ ” remains as it is but “ $3x+y$ ” is added as “ $3xy$ ”.

Her answer to the corresponding question in the WMC-S test, “Simplify $2a+5b$ ”, was “ $2a5b$ ” in Grade 9 while it was “ $7ab$ ” in Grade 10. Both represent some form of conjoining although in different forms.

In question 10.2 in the Grade 11 task, she provided the answer “ $3x-y$ ” to “simplify $2x+x-y$ ”, although her answer to another question provided during the interview i.e. “simplify $2x+x+y$ ” was “ $3xy$ ”. This again demonstrated an inconsistency in her answers. When asked to explain the difference, her reply was that the presence of the “minus sign” made it different. Her answer to the question, “simplify $2a+5b+a$ ” was “ $3a5b$ ” in Grade 9, but was “ $8ab$ ” in Grade 10. Her responses to the 2 questions were consistent in Grade 9 as well as in Grade 10 although both illustrated conjoining errors.

Her responses to the last 2 questions in the Grade 11 task also demonstrated conjoining errors. Her answer to “add 6 to t ” was “ $6t$ ” while her answer to “if $a+b=3$ ” then what is “ $a+b+c$ ” was “ $3c$ ”. Her answer to “add 4 to $3n+4$ ” was correct in Grade 9 but was written as “ $7n$ ” in Grade 10 indicating a conjoining error. Her answer in Grade 9 may be attributed to her tendency to leave her answer halfway instead of obtaining an incorrect answer as she explained in the interview in Grade 11 rather than her understanding of the addition of “like” and “unlike terms”.

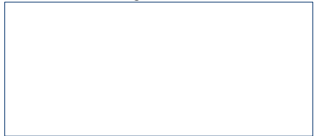
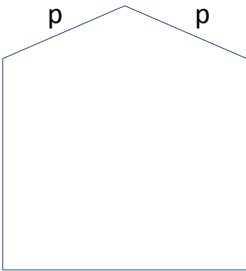
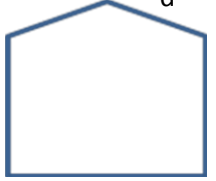
Her responses to the question “if $e+f=8$ then what is $e+f+g$ ” was “16” in Grade 9 but was “10” in Grade 10. Both these responses indicate that she attached values to the letters (letter evaluated) to obtain her answers. In Grade 9 she most likely provided the value of “8” to “ g ” but in Grade 10 the value provided was “2”. In Grade 11, her answer to the corresponding question indicated a “conjoining” error unlike the errors in Grade 9 and 10.

Conjoining errors made by Sbu

Sbu was the other learner who remained on level 1 in Grade 11 and was found to display conjoining errors in all three grades.

Sbu obtained incorrect answers to questions 5.2, 5.3 and 5.4 in the Grade 11 task as well as in the corresponding questions in the WMC-S test in Grades 9 and 10. The following are the questions in the Grade 11 task.

Table 19: Questions in Grade 11 task

5.2.		5.3.	
5.3.			

He demonstrated conjoining errors in all 3 questions by stating that “ $a+a+3+3=9a$ ”, “ $p+p+5+5+6=16p$ ” and “ $d+d+3+3+3=11d$ ”. In particular, his conjoining error reflected “adding co-efficients and constants”. The corresponding questions in the WMC-S test in Grades 9 and 10 though illustrated inconsistencies. In Grade 9, his answer to finding the perimeter of a pentagon with sides “ h, h, h, h and t ” was “ $h^4 + t = ht$ ” while his answer to the same question in Grade 10 was “ $4ht$ ”. While he displayed an error with exponents together with conjoining error in the former answer, his answer in Grade 10 reflected the same error as in Grade 11 i.e. conjoining error.

The other questions that challenged Sbu in the Grade 11 task were to simplify “ $2p+3a+p$ ” and “ $2p+3a+3p$ ”. His answers were “ $6ap$ ” and “ $8ap$ ” respectively; both reflecting a consistent conjoining error. It was consistent because in both cases and throughout the Grade 11 task, Sbu added the co-efficients and constants. In the corresponding questions in the WMC-S test, his answer to “simplify $2a+5b$ ” was “ $7a^2b^5$ ” in Grade 9 but in Grade 10 it was “ $7ab$ ”. His answer in Grade 9 reveals some confusion with exponents and is not unique to this question. As was discussed above, there were other answers in Grade 9 that exposed his confusion with exponents. His answer to the question “simplify $2a+5b+a$ ” was “ $7aba$ ” in Grade 9 and “ $8ab$ ”. Although both answers reflected conjoining, they displayed different forms of the conjoining error. His answer in Grade 9, suggests that he added the coefficients of the first 2 terms and just placed the extra “ a ” next to his answer. His answer

in Grade 10 though suggests that he “added the co-efficients and constants” and is consistent with his answers in the Grade 11 task.

The other questions in level 3 in the Grade 11 task that were incorrectly answered, involved the adding of “like” and “unlike” terms. Sbu displayed conjoining errors in all of these answers as discussed above. For example the question “add 6 to t” produced the answer “6t” in Grade 11 and the corresponding questions in the WMC-S test “add 4to 3n” elicited the answer “12n”. His answer to the same question in Grade 9 was not classified as conjoining as it was “n+7”.

In Grade 11, 4 out of a total of 7 errors made by Sbu consisted of conjoining errors. This translates to 57% of conjoining errors. Furthermore, all his errors on level 2 consisted of conjoining errors and there were 2 conjoining errors on level 3. If these errors were corrected then Sbu would have been on level 3 in Grade 11. The conjoining errors produced by Sbu seems to be enduring as it accounted for 3 out of his 6 errors in Grade 10 which translates to 50% and in Grade 9 translated to 25%. This suggests that this error committed by Sbu is more pronounced in Grade 11 than in the other grades.

5.4.2.2. Letter Evaluated

The other most persistent error from Grades 9 to 11 was “letter evaluated”. There were 6 errors on this interpretation of letters in Grade 9, 2 in Grade 10 and 5 in Grade 11. This error was exhibited in questions 9.2 and 12 in the Grade 11 task and in the equivalent questions (questions 10.4 and 5.3) in the WMC-S test.

Table 20: Questions 5.4 and 10.4

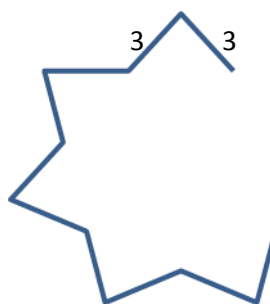
Question in WMC-S test	Question in Grade 11 task
5.3.If $e+f = 8$ then $e+f+g =$	12. If $a+b= 3$ then $a+b+c=$

10.4. Find the perimeter:



Part of this figure is not drawn. There are n -sides altogether all of length 2

9.2. Find the perimeter



Part of the figure is drawn. Each side equals 3 units and the figure has " n " sides

In questions 5.3 and 12 in the WMC-S test and the Grade 11 task respectively, 3 out of the six learners substituted values for e , f and g in the WMC-S test in both grades 9 and 10. Sbu provided an answer of 12 in both Grades 9 and 10. He thus seems to have assigned the value 4 to each of the letters " e " and " f " to obtain "8" and therefore attached the same value of "4" to the variable " g " to determine his answer of "12". This error seems to be persistent as he used the same interpretation of letters to evaluate the equivalent question in the Grade 11 task. His answer to question 12 in the Grade 11 task was "6". My initial explanation for an answer of "6" was that since he had been provided with " $a+b$ " as being equal to 3, he provided the letter " c " with the same value of 3. His justification for the response provided proved to be quite different as is illustrated in the following extract from the interview with him:

I: *So what did you do here?* (referring to question12)

Sbu: *So I get 6*

I: *How?*

Sbu: *I took this as 1*(pointing to “a”) *and this as 2* (referring to “b”) *and then this is 3* (referring to “c”) *and got 6*

I: *Explain why “a” would be 1 and “b” would be 2?*

Sbu: *Because “a” is before “b” and “b” is before “c”. Because it’s a, b, c it’s the alphabet*

I: *So if I had a plus b equals 4 then what’s a plus b plus c?*

Sbu: *It’s 6*

I: *How?*

Sbu : *Because it’s 2 and 2* (referring to “a” and “b”) *and then this is 2* (referring to the “c”)

I: *So you would give it any value? So the 2 values may be equal or 1 maybe bigger than the other? You will give it any values*

Sbu: *No I just look at each one to get the answer. So here* (referring to $a+b=3$) *then if “a” is 1 then “b” must be 2 then “c” must be 3*

I: *So you are looking for a pattern?*

Sbu: *Yes*

I: *So what would you do if a plus b equals5?*

Sbu: (took some time but got to answer 8) *so I took this “a” as 2* (referring to letter “a”) *and this* (referring to letter “b”) *as 3 and took this as 3* (referring to letter “c”)

I: *Why not 4?* (referring to “c”) *Because remember here you took “a” as 1, “b” as 2 and “c” as 3 because they follow each other?*

Sbu: *Oh so here I made a mistake. I should have made it 4* (pointing to “c” value) *and I should have got 9*

The above extract reveals Sbu’s reasoning of how he substituted values for the letters. He did not just take random values but looked for a pattern that would make sense in each example. MacGregor & Stacey (1997) assert that the alphabetical interpretation of letters may originate from 2 sources. The first source is that the Greek numeration system represents the letter “ α ” as possessing the value “1” and “ β ” the value of “2” etc. Secondly they maintain that learners continually see labels in textbooks as 1(a), 1(b), 1(c) etc. thus supporting the view of a letter having a fixed value and order.

In the question “if $a+b=4$ then what is $a+b+c$ ”, he provided each letter with the value 2 because “ $2+2=4$ ” and if “a” and “b” both had the value “4” then so would “c”. The next problem though posed a challenge to him (“ $a+b=5$ then what is $a+b+c$ ”) since “5” is an odd number and his method of working out the previous answer would therefore not work in this example. He however, used “a” as 2 and “b” as 3 and provided the answer of “5”. The challenge though was to arrive at a value for “c”. Although he changed his mind when challenged about why he was inconsistent with this example and the previous one, he seemed to be making up values without attention to the consistency of his approach.

Promise is another learner who represented the letters, in 5.3 in the WMC-S test, by using values in Grade 10. This is surprising because in the equivalent question in the Grade 11 task, she produced an answer of “6” by substituting the letters with values. Her reasoning in producing an answer of “6” is provided below:

Promise: its 6 because I think it's like 3, 6, 9, 12 so I think its 3 again

I: another 3?

Promise: Another 3?

I: I don't understand... you said it goes in 3's. So if I had a plus b equals 4 what is a plus b plus c?

Promise: it would be 6

I: Why 6?

Promise: So a plus b equals 4 so it must be 2 plus 2 so a plus b plus c will be another 2

I: so you using different things here and with the other one.

I: so x minus y equals 5 what is x minus y plus q?

Promise: (thinks for a while). I think it is 10

I: why?

Promise: I would use the 5 so 5 plus 5 equals 10

Both of the learners Sbu and Promise demonstrated that they would use different values for the letters depending on the particular question. They would use values that would suit the given expression.

Patricia provided values to the letters and arrived at answers of 16 and 10 in Grades 9 and 10 respectively. In the Grade 11 task though, she provided the answer “3c”. While she did not provide values for the letters, she did display a conjoining error.

The other question that elicited the error of “evaluating letter” was question 10.4 in the WMC-S test. The most common answer in Grades 9 and 10 was “36”. This answer suggests that learners counted the number of sides that could be seen (18) and calculated the perimeter by multiplying “2” by “18” to provide the answer of “36”. They therefore seem to have used the value of “n” to be “18” to arrive at their response. Other answers that were provided were “12” and “2”. These answers were most likely found by using the sides that were labelled and being equal to 2 units each. The most common answers to the equivalent question in the Grade 11 task were “38” and “30”. These answers were found by either counting the number of sides that were drawn (10), multiplying this by “3” or by completing the figure and adding all the sides as is illustrated by the following extract from an interview with Lettie:

I: So what did u do here in question 9.2?

Lettie: I want to complete the shape. So I looked at the shape and divided it into half. So this side (points to left of the fig) has 2 sides so this side must have 2 sides so I must finish up this because it's not complete, so I add the sides

I: So if I say it has n sides does it not make a difference to anything? Can you make “n” into anything?

Lettie: We can if we are not given the instructions but here we are given the instructions. Each and every side given as 3 so we use n as 3. We cannot use it as any number so although we not given how many sides we know each side is 3

The above extract reveals that Lettie used symmetry to complete the given figure and then calculated the perimeter using the number of sides in her completed figure.

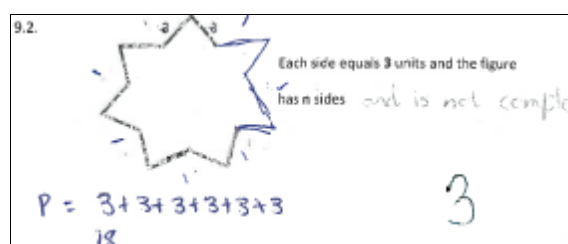


Figure 20: Lettie’s response to question 9.2.

Letter evaluated accounted for 15.8% of the errors in Grade 9, 5.3% of the errors in Grade 10 and 20.8% of the errors in Grade 11. One reason that may account for the large percentage of this error in Grade 11 is the possibility that the phrasing of the question 9.2 in the Grade 11 task was slightly different to the phrasing of the question in the WMC-S test. In the Grade 11 task, the information provided was that “part of this figure is not drawn” while the equivalent question in the WMC-S test was phrased as “part of this figure is drawn”. The question in the Grade 11 task seems to emphasise the part that is not drawn thus possibly prompting the learners to complete the figure to provide a solution to this question. This can therefore be considered as one of the limitations of the study.

5.4.3. Errors that have faded

There were other errors made by learners in Grades 9 and 10 which were not evident in Grade 11. These include exponents, substitution errors and the error of finding the highest common factor of letters.

5.4.3.1. Exponents

There were 4 learners out of the sample of 6 learners who displayed errors associated with exponents in the WMC-S test. Two of the learners presented this error when adding like terms. For example, “ $2a+5b$ ” elicited the response “ $7a^2b^5$ ” in Grade 9 from one of the learners while the question “ $2a+5b+a$ ” was answered as “ $7a^2b$ ” in Grade 10 by the other learner. These were the only two questions which were answered using exponential laws inappropriately by these 2 learners. This error was not repeated in Grade 11 by either of these 2 learners. The third learner (Lettie) displayed an error relating to exponents when finding the perimeter of a figure with sides given as “u, u, 5, 5 and 6” units. The answer that was provided was “ $60 \times u^2$ ”. This suggests that Lettie multiplied the letter “u” by itself to arrive at “ u^2 ”. This error was again demonstrated by Lettie when finding the perimeter of a triangle with each side equal to “a” units in the Grade 11 task. The answer provided by the learner was “ $a \times a \times a = a^3$ ”. Although the exponential law relating to multiplication of exponents was used appropriately in Grade 11, the question was answered incorrectly as it required the perimeter of the triangle. In Grade 9, Lettie’s answer to question 10.1 in the WMC-S test (*Find the perimeter of a triangle with each side equal to “e” units*) was “ $3e$ ”. The same question in Grade 10 elicited the answer “ $e \times e \times e = 3e$ ”. It is impossible to establish whether the response “ $3e$ ” in Grade 9 was determined by Lettie from “ $e \times e \times$

e " or " $e + e + e$ ". It therefore may be possible that, given her response in Grade 10, she obtained the answer using incorrect working but since no working was presented, her answer was coded as correct. She, however, demonstrated her ability to discriminate between the addition and multiplication of like terms during the interview in Grade 11 as is evident by the following excerpt:

Lettie: the difference is here I timesed ($e \times e \times e$) and here I added ($e + e + e$). Remember if the letters are the same we say a times a times a equals a cubed but here we add the numbers in front so we get 3a because they are alike. We are only adding the numbers.

This error was not evident in the answering of any question by this learner in Grade 11. It is possible that Lettie only committed this error when answering questions related to finding the perimeter of a figure and not to algebra in general. It is not possible to establish this though as there were no other algebra questions in the WMC-S test or the Grade 11 task, of the form "simplify" which consisted of letters only where the co-efficients were "1".

Finally, Sbu was the other learner who displayed an error relating to exponents in Grades 9 and 10. Question 10.1 in the WMC-S test involved finding the perimeter of a triangle with each side equal to " e " units. In Grade 9, he added the sides to provide the answer " e^3 ".

This error with using laws of exponents when adding terms was displayed in other questions as well. For example, in the very next question he found the perimeter of a pentagon with sides " h, h, h, h, t " to be " $h^4 + t = ht$ " in Grade 9. He obtained the correct answer but applied an incorrect method in the WMC-S test in Grade 9. He did obtain the correct answer to 10.1 in the WMC-S test in Grade 10 and the equivalent question in the Grade 11 task.

The errors relating to exponents were also not demonstrated by any other learner in Grade 11 and thus can be considered as an error that has faded away.

5.4.3.2. Substitution error

The other error that seems to have faded in Grade 11 pertains to question 7.2 in the WMC-S test. The question was "find m if $m=3n+1$ and $n=4$ ". The most common response to this question was "35". A possible explanation for this answer is that learners "placed" the 4 next to the 3 to obtain 34 and then added the 1 to provide the answer "35". Among the

other answers for this question was “8”. This answer may have been found by adding together the “3” and “4” to obtain “7” and then adding “1”. Both of these answers may stem from a lack of knowledge of what “3n” represents. Those learners, who provided the answer “8”, may have perceived “3n” to be the same as “3+n” thus possibly displaying a conjoining error. Those who arrived at an answer of “35” may possibly be demonstrating confusion with “35” as being the result of “30+5” or from their knowledge of fractions where $3\frac{1}{2}$ as being the sum of “3” and “ $\frac{1}{2}$ ” as espoused by Stacey and MacGregor (1994). This reasoning may suggest that the gap between arithmetic and algebra still poses a problem to learners. This error though is not demonstrated by any of the sample of 6 learners in Grade 11.

5.4.3.3. Finding the Highest Common Factor (HCF)

I have used the term highest common factor to describe the tendency of 2 of the learners in the sample to use only one of the letters when adding terms where more than 1 letter is repeated.

Tembi’s response to the question “find the perimeter of a triangle with each side measuring “e” units” was “e+e+e=e” in both Grades 9 and 10. Further to this, her answer to the question on finding the perimeter of a figure with measurements “h, h, h, h and t” units was “ht” and the perimeter of a figure with sides measuring “u, u, 5, 5 and 6” units was “16+u”. In all of these examples, she has only used one of the letters in her answer suggesting that she ignores the number of times that a letter is repeated. This implies that if she was finding the perimeter of a figure with sides “u, u, u, 5, 5 and 6” i.e. an extra “u” in the question, her answer would be the same as her answer to the latter question. Sbu was another learner who demonstrated this error when answering the question on finding the perimeter of a figure with sides “h, h, h, h and t”. His response was “ $h^4 + t = ht$ ”. Both of these learners seem to reflect only one of the letters that is repeated in their answers. The only possible explanation I can find is that they somehow relate this to finding the Highest Common Factor (HCF) of numbers. Some teachers explain the finding of the HCF as “take only one number from those that are repeated”.

5.4.4. Issues relating to level 2 questions

When analysing the results for the sample of 29 learners in the WMC-S test, it was found that 5 learners were able to succeed at a higher level without first achieving at a lower level. This represented an anomaly since Hart (1981) contends that this is highly unlikely. Since 5 learners out of a sample of 29 learners represents a fairly large number of learners, although it was a small sample, it seemed appropriate to investigate this further.

The WMC-S test consisted of 6 questions on level 2. A learner would be deemed to be on level 2 if he/she was able to answer at least 4 out of the total of 6 questions on level 2. The questions on level 2 are such that if a learner demonstrates errors relating to conjoining, then it is very difficult for that learner to be able to achieve the necessary requirements to be considered to be on level 2. In order to demonstrate this, all 6 of the questions in the

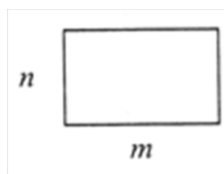
WMC-S test on level 2 are illustrated below:

1.4 $2a + 5b + a =$ _____

7.1 If $u = v + 3$ and $v = 1$, find the value of u _____

7.2 If $m = 3n + 1$ and $n = 4$, find the value of m _____

8.3. What is the area of the following shape:



10. Find the perimeter for each of the shapes:

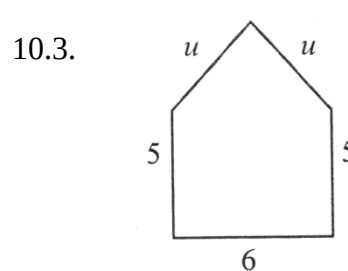
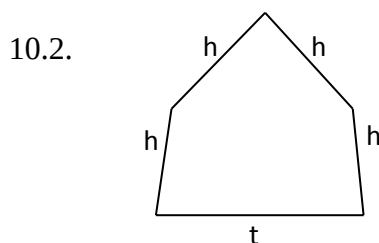


Table 21: Level 2 questions in WMC-S test

Question 1.4 requires the learners to be able to add like terms. A learner who displays conjoining errors will most likely not be able to answer this correctly. Responses to this question in Grades 9 and 10 ranged from “7aba”, “3a5b”, “7ab” and the most common answer from the sample of 29 learners from phase 1 of the study was “8ab”. Only 50% of the 29 learners were able to answer this question correctly in Grade 10 and in Grade 9 this percentage was 42%. Learners who obtained an incorrect answer to this question due to a conjoining error in Grade 10 remained on level 1. This suggests that these learners are unable to answer at least 3 other questions correctly on this level in order to move to a higher level. Questions 10.2 and 10.3 both relate to finding the perimeter of figures with sides given in terms of letters and in the case of question 10.3, letters and numbers. This means that if a learner is unable to add “like” and “unlike” terms then the learner has obtained incorrect answers to 3 out of the 6 questions on this level. It is therefore not possible for the learner to progress from a lower level to level 2 since he/she cannot answer the minimum of 4 questions correctly.

It must be clearly stated though, that this statement is only true for those learners who obtained an incorrect answer to question 1.4 due to a conjoining error. There were learners who were unable to answer question 1.4 correctly due to other errors. For example, the answers “3a+b” or “4a+5b” may be considered to be a “slip”. The learners may therefore, have written “b” instead of “5b” in the former answer and added the “a’s” incorrectly in the latter answer.

Having said this though, it is possible for learners to have answered question 1.4 correctly but not questions 10.2 and 10.3. There were three learners in each of the Grades 9 and 10 who were able to answer question 1.4 correctly but obtained incorrect answers to questions 10.3 and 10.4. A possible reason for this may be the manner in which the information is provided. While question 1.4 required learners to simplify an expression, in questions 10.2 and 10.3 the information was provided on a diagram and required learners to find the perimeter of two figures.

Patricia is one of the learners who demonstrated that she answered questions differently depending on the phrasing of the question. She provided the answer “3p+3a” when answering the question “2p+3a+p”, indicating an understanding of “like” and “unlike”

terms. Question 5.4 in the Grade 11 task (*Find the perimeter of figure with sides d , d , 3 , 3 and 3*), however, was answered as “ $9d$ ”.

I: *So you saying you can't do d plus d plus 3 plus 3 plus 3*

Patricia: *I think I would get $2d$ plus 9 but I'm not sure whether I should add it?*

I: *But you left this as $9d$* (referring to her answer on script)

Patricia: *I left it as $9d$*

I: *But you not sure what to do with these separately?*

Patricia: *No*

I: *But how 9 ? Look at what you did here?* (referring to 5.2. where her answer was $6+2a$ is $8a$)
 Shouldn't you have got $11d$ here (5.4)

Patricia: (laughs) *Don't know what to do*

Her answer of “ $8a$ ” when adding “ a , a , 3 and 3 ” was calculated by adding both “ a ” and “ a ” to obtain “ $2a$ ” and adding the 3 's to obtain “ 6 ”. She then added “ $2a$ ” and “ 6 ” to obtain “ $8a$ ”. Her answer to the question involving the perimeter indicates that she added the 3 's to obtain 9 but just “placed” the “ d ” next to her answer. If she was consistent in her calculation then she would have obtained the answer “ $11d$ ” from “ $2d+9$ ”. The following excerpt is provided to illustrate her reasoning about the inconsistencies in her answers:

I: *Okay so what did u do here?* (question 6.1 Simplify $2p+3a+p$)

Patricia: *So if it's like terms then we add so $2p$ plus p is $3p$ and $3a$ so $3p$ plus $3a$*

I: *So you leave your answer like this?*

Patricia: *mmmm. Ja leave it like this*

I: *So what's the difference between this and the previous question (question 5)*

Patricia: *There's no difference. Let me just say the way it is given is different. Different approach*

Her response that the way it was “given is different” may stem from the fact that one question was asked using a diagram (Question 5.4) while the other (Question 6.1) was provided as an expression to simplify. The questions were also posed differently as the

latter question asked for the perimeter of the figure while the former required the learner to simplify.

The table below reflects the number of learners who obtained correct responses to each of the questions on level 2:

Table 22: Number of correct responses to level 2 questions

Question number in WMC-S test	Grade 9	Grade 10	Question number in Grade 11 task	Grade 11
1.4	2	1	6.1/6.2	5
7.1	5	5	7.1/7.2	6
7.2	2	3	8.1/8.2	6
8.3	5	6	4.1/4.2	4
10.3	3	2	5.3/5.4	4

The bar graph displaying the correct responses to each question on level 2 is displayed below:

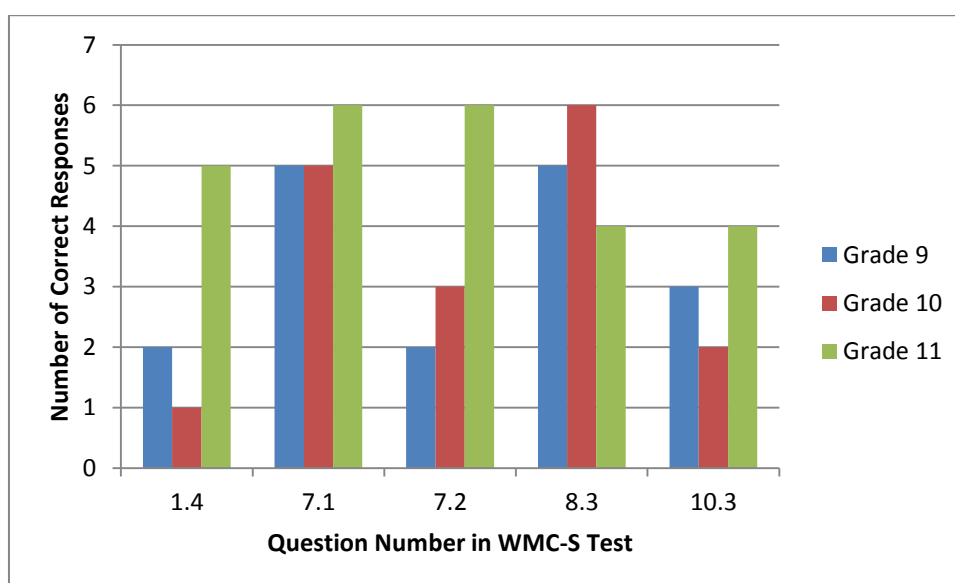


Figure 21: Number of correct responses per level 2 question

In Grade 9 there was only 1 learner (Promise) in the sample of 29 learners who answered question 1.4 in the WMC-S test incorrectly but was able to satisfy the requirement to move on to level 2. Promise answered this question (Simplify $2a+5b+a$) as “ $8ab$ ” and the question “Simplify $2a+5b$ ” as “ $7ab$ ” thus displaying a conjoining error in answering both the questions. She also displayed a conjoining error in answering the question “add 4 to

3n” by providing the answer “12n”. She did not demonstrate a conjoining error in any other question in Grade 9. She, however inexplicably, was able to answer questions 10.2 and 10.3 correctly. This represents an anomaly from the other learners who may have been able to obtain a correct response to question 1.4 but not to questions 10.2 and 10.3.

In Grade 10, 50% of the learners answered each of the questions 1.4 and 10.2 correctly, while the percentage for question 10.3 was 54%. In Grade 9, 42% of the learners answered each of the questions 1.4 and 10.2 correctly and 38% of the learners answered question 10.3 correctly. If these three questions were answered incorrectly then these learners would have remained on level 1 since they would have been unable to obtain at least 4 correct answers on level 2. It is therefore possible that those learners who display conjoining errors are unable to progress to level 2 as they are unable to answer questions 1.4, 10.2 and 10.3 correctly.

The error of conjoining thus poses an obstacle to learners answering the minimum number of questions on level 2 correctly to be considered to be on level 2. It was illustrated earlier that there were 2 learners in Grade 10 and 3 in Grade 9 who were unable to reach level 2 but were able to answer the minimum number of questions on level 3 correctly. Hart (1981) contends that it is unlikely for a learner to be categorised as being on a higher level without first achieving the minimum number of correct responses on a lower level. The burning question then is: Was the composition of the questions in the WMC-S test problematic? Were the questions on level 2 such that it prevented those learners who displayed conjoining errors from achieving at level 2?

One may argue that the sample size is too small to make this generalisation. While I agree that the sample size is an issue, I nevertheless believe that a total of 5 learners (in both Grades 9 and 10) out of a total of 29 learners (17.2%) who display this incongruity is large enough to warrant further investigation of this problem. A possible explanation may be that the CSMS and subsequently the ICCAMS instrument and the coding of the instrument was developed in England. This problem therefore may only be present in the South African context. Furthermore, this project was only conducted in underperforming schools and may not be large enough to generalise. A further investigation may therefore be necessary within the South African context and encompassing a larger area with a larger number of learners.

The results from table 16 suggest that the questions that still posed a problem to some of the 6 learners are questions 8.3 and 10.3 in the WMC-S test and the equivalent questions in the Grade 11 task (4.1, 4.2, 5.3 and 5.4). Question 8.3 in the WMC-S test was answered correctly by more of the 6 learners when they were in Grades 9 and 10 than when they were in Grade 11. On scrutinising the incorrect answers in Grade 11 for this question, it seemed that learners may have forgotten the formula to find the area of a rectangle. This is evidenced by answers of $\sqrt{p+q}$ and $\frac{1}{2}pq$ to the question “find the area of a rectangle with sides “p” and “q”. Since this is not simply an algebraic error I will not dwell on this issue.

Question 10.3 still posed a problem to 2 of the learners in Grade 11. The error displayed by the learners in this question is one of conjoining. Both however displayed different forms of conjoining as discussed in section 5.3.1. Sbu, counted all the letters when adding the sides to find the perimeter of the given figure. His answer to the perimeter of the figure with sides “p, p, 5, 5 and 4” in the Grade 11 task was “16p” which was found by adding the numbers to obtain “14” and counting each of the letters to obtain “2p”. This was added to “14” to obtain “16p”. He was consistent with the manner in which he used conjoining in that his answer to the question “add $2p+3a+p$ ” was “6ap”.

Patricia, however, used a different form of conjoining by “ignoring letter” (which is discussed in detail in section 5.3.1). She obtained the answer “14” when adding the sides “p, p, 5, 5 and 4”. She was, however, able to answer the question “ $2p+3a+p$ ” correctly as “ $3p+3a$ ” (question 6.1 in the Grade 11 task). She thus answered the questions on simplifying expressions correctly but displayed the conjoining error when adding “like” and “unlike terms” in questions involving the perimeter of figures.

The conjoining error then can be viewed as a reason to explain why some learners progress to higher levels while others do not. The two learners who did not progress from level 1 to level 2 both displayed conjoining errors on level 2 questions. While it can safely be said that Sbu did not progress to level 2 because of the conjoining error, the same cannot be mentioned about Patricia. All the errors committed by Sbu in Grade 11 were errors with regard to conjoining in questions 5.3, 5.4, 6.1 and 6.2 as discussed in the section 5.4.2.1.

Patricia, however, was able to answer questions 6.1 and 6.2 correctly because the questions appeared in a different form from questions 5.3 and 5.4. She provided incorrect answers to these 2 questions. Both these errors related to conjoining. She did, however, remain on level one because of errors in the answering of questions 4.1 and 4.2 in the Grade 11 task. Both of these questions required learners to find the area of rectangles with sides “p” and “q” and the other with sides “2a” and “b”. She provided answers of “ $\frac{1}{2} pq$ ” and “ $\frac{1}{2} 2a \times b$ ” respectively. These errors may not relate to algebra because it is quite possible that she had forgotten the formula to determine the area of a rectangle.

Both the learners who were on level 1 in Grade 10 but who had progressed to level 2 in Grade 11 obtained incorrect answers to questions 10.3 and 1.4 in the WMC-S test. In the question (4.1) “simplify $2a+5b+a$ ” in the WMC-S test, Lettie obtained the answer “8ab” while Tembi indicated her answer as “ $2a+5b$ ”. While Lettie’s answer indicated a conjoining error, at first glance Tembi seems to have made a “slip” by seeming to forget about the last “a” value. Further investigation of her other error in level 2, indicated that it may not be just a “slip”. In question 10.3 in the WMC-S test, Tembi’s answer to the perimeter of a pentagon with sides “u, u, 5, 5 and 6” was “ $u+16$ ”. In both questions, she seems to have ignored one of the letters that were repeated i.e. she ignored one of the “a’s” in question 1.4 and one of the “u’s” in question 10.3. It may be possible that she had used only one of the letters that were repeated to obtain her answer. Lettie though was consistent in her use of conjoining and her answer to question 10.3 was “ $16u$ ”.

Questions 1.4 and 10.3 were thus the 2 main questions that proved to be an obstacle to both the learners not being able to reach level 2 in Grade 10. These errors were corrected in Grade 11.

The other 2 learners were both on level 3 in Grades 10 and 11. What is significant is that both these learners did not display any signs of conjoining from Grades 9 to 11 suggesting that their ability to add “like” and “unlike terms” assisted in them progressing to level 3 in Grade 11.

5.5. Conclusion

I began my discussion on the Grade 11 task by providing the details of the composition of the task. The results of the answering of the questions on levels 1 to 3 in the Grade 11 task as well as the equivalent questions in the WMC-S test were illustrated using a bar graph. These results provided the basis for a description of those errors that were persistent throughout the 3 grades as well as those errors that seem to have faded away by Grade 11. The main error that seems to persist through the grades, even though to a lesser extent in Grade 11 than the previous grades, is the conjoining error. This error mainly manifested itself in the questions on level 2. This error therefore seems to provide a barrier to some learners progressing to higher levels. This issue therefore warrants further investigation in the South African context on a wider scale than the limited scale of this study.

CHAPTER 6

CONCLUSION AND REFLECTIONS

6.1. Introduction

The purpose of the study was to investigate the performance of learners in algebra by analysing the results of the ICCAMS section of the tests compiled by the WMC-S. The problems experienced by learners in the transition from arithmetic to algebra are acknowledged by researchers like Stacey and MacGregor (1994), Küchemann (1981) and Booth (1984) amongst others. In addition, within the South African context, the Department of Basic Education (2014, p.126) revealed that the “algebraic skills of learners are poor” in the 2013 Grade 12 examination. The problems related to the poor transition of learners from arithmetic, the problems that learners experience in interpreting letters and the poor performance of learners in algebra in the South African context suggest that research into the errors made in early algebra is a useful undertaking.

My study focussed on the performance of a sample of 29 learners in the WMC-S test. I started out by outlining the rationale for the research within the poor performance of learners in mathematics. The focus was on algebra considering the dearth of literature particularly pertaining to algebra. I discussed the important role of algebra in the curriculum. A discussion of errors within the constructivist framework as well as an explanation of errors related to the interpretation of letters was discussed. In addition some errors related to conjoining, the use of brackets and exponents were illustrated. The overall design of the study, the selection of the sample and the research instruments utilised in both phases of the study was then discussed. I also provided details of the data analysis process. The analysis of phase 1 of the study provided details of the performance of learners in the WMC-S test. The analysis of phase 2 was completed using the results of the Grade 11 task which was based on the WMC-S test as well as interviews based on the Grade 11 task. The analysis culminated in the emergence of common errors as well as those that were not so common and those that disappeared by Grade 11. I conclude the report by discussing the main findings of the study as well as reflecting on some recommendations for teaching.

6.2. Research Questions

This study was guided by the following research questions:

1. Has there been a shift in individual learner performance in algebra as evidenced by a change in levels from Grade 9 to Grade 10?
2. What errors are made by learners?
3. Which errors continue from Grade 9 to Grade 11?
4. Which errors fade away by Grade 11?

In addition, according to Hart (1981) learners will generally progress to a higher level in the ICCAMS test if they obtain the minimum number of correct answers at a lower level. Success at a higher level signifies success at a lower level. During the analysis of phase 1 of the study it was found that there were a total of 5 learners (3 in Grade 9 and 2 in Grade 10) who were unable to answer a minimum of two thirds of the questions on level 2 correctly. These learners were, however, able to answer the minimum number of questions correctly on level 3. It is unusual for 17.2% of learners to progress to a higher level without first succeeding at a lower level. Furthermore, it was discovered that 2 of the learners in the sample, moved from level 2 down to level 1 from Grade 9 to Grade 10. These results prompted an addition to the original research questions and thereby an investigation into the particular problems linked to the answering of questions on level 2. The results of this investigation were detailed in phase 2 of the study.

This study was analysed in 2 phases. Phase 1 of the study sought to address research question 1 while phase 2 addressed research questions 2 and 3.

6.2.1. Has there been a shift in individual learner performance in algebra as evidenced by a change in levels from Grade 9 to Grade 10?

Phase 1 of the study was the analysis of the scripts of the learners of the WMC-S test. After the re-coding of these scripts, the level of each of the learners in the sample was determined. There were a total of 29 scripts of learners from the WMC-S test that were analysed during phase 1 of the study. The results in terms of their levels are outlined below:

There were 19 learners who progressed from a lower to a higher level from Grade 9 to Grade 10 while there were 8 learners who did not change levels from Grade 9 to Grade 10. The surprising finding was that 2 learners moved from level 2 to level 1 from Grade 9 to

Grade 10. This represents an anomaly since there was more algebra learnt in Grade 10 and learners should either remain at the same level or progress to a higher level. On further investigation it was found that both these learners obtained incorrect answers for the same questions on level 2.

These results indicate that 65.5% of the learners progressed from a lower to a higher level using the ICCAMS instrument. During the analysis in phase 1 of the study, it was found that 27.5% of the learners displayed no progression in levels from Grade 9 to Grade 10 and even more surprising that learners dropped a level. This suggests that a large proportion of the learners are still battling to grasp algebraic concepts even though more algebra was completed in the Grade 10 year. One of the reasons for the drop or the stagnation in levels may be the interference of new learning. This is evidenced in the inappropriate use of exponential laws as well as the process of multiplying out brackets even in the presence of an addition sign between brackets. The other reason may be the presence of the conjoining error in specific questions on level 2 that may hinder the progress of learners to higher levels.

6.2.2. What errors are made by learners?

Some of the errors made by learners in the WMC-S test are letter evaluated, conjoining, errors relating to exponents, letter as an object and errors relating to brackets. Examples of questions in which these errors were prevalent as well as some of the errors made in the answering of these questions are discussed below. Letter evaluated and conjoining errors seem to be the most persistent errors while errors relating to exponents seem to have faded away by Grade 11.

The error relating to brackets was prominent during phase 1 of the study. While in Grade 9 some of those learners who obtained incorrect answers relating to brackets conjoined the terms inside the brackets, in Grade 10 they multiplied brackets even in the presence of a " + " sign between the brackets. An example by one of the learners in Grade 9 is " $(a-b) + b = ab-b$ " while the response in Grade 10 was " $(ab - b^2)$ ". This may reflect the interference of new knowledge with prior knowledge as the multiplication of brackets is a topic covered in Grade 10.

The error "Letter evaluated" was illustrated in questions 5.3 and 10.4 in the WMC-S test. The question "if $e+f=8$, what is $e+f+g$?" elicited the most common answer of "8". This response indicated that learners substituted values for "e, f and g" to find the answer. The

values substituted depended on the information provided and with the absence of a consistent method, as indicated by the results of the interview.

Errors involving exponents were common in Grades 9 and 10 but not in Grade 11. A common error was the use of exponents when adding like terms. For example, learners simplified “ $2a+5a$ ” as “ $7a^2$ ”, thus displaying confusion between the addition of terms and multiplication.

The interpretation of “letter as object” was a common error in question 11 of the WMC-S test. The question was “Cakes cost c rand and buns cost b rand each. If I buy 4 cakes and 3 buns, what does $4c+3b$ stand for?” The common answer in this question was “4 cakes and 3 buns” which denote that the learners perceived the letters as objects rather than as a letter representing a number which in this case was the price of the cakes and buns. This was the only question where learners viewed the letters as objects. It must be noted though, that this was the only question in the WMC-S test which lent itself to being viewed as “letter as object”.

6.2.3. Which errors continue from Grade 9 to Grade 11?

During phase 2 of the study an analysis of the Grade 11 task together with results of the interviews provided a rich source of the thinking of the learners in responding to the questions in the Grade 11 task. The most persistent errors that emerged during this stage were conjoining and letter evaluated.

6.2.3.1. Conjoining

Conjoining accounted for 26% of the errors of the sample group of 6 learners from Grades 9 to 11. In Grade 10, it accounted for 36.8% of the errors. Even though this was reduced to 16.7% in Grade 11 it was still the most persistent error from Grades 9 to 11. While research in algebra alludes to the prominence of this error (Herscovics & Linchevski, 1994; Booth, 1988; Falle, 2007) none of them distinguish between different forms of conjoining. I have introduced the terms “**ignoring letter**” and “**adding co-efficients and constants**” to distinguish between different ways of conjoining resulting from the Grade 11 task and the resulting interviews with learners. Although only 2 out of the 6 learners in my sample for phase 2 displayed conjoining errors, they manifested different forms of the error. While Patricia added “ $p, p, 5, 5$ and 4” to obtain “ $14p$ ”, Sbu obtained “ $16p$ ”. Patricia displayed conjoining in determining her answer by “ignoring letter” and Sbu added the

numbers to obtain “14” and then added each of the letters to obtain “2p”. He then added “14” and “2p” to obtain the result of “16p”. Furthermore, Patricia distinguished between how she answered questions with only 2 letters and 2 terms and those with more than 1 letter and more than 1 term. For example, she maintained in her interview that she could not add “ $3x+2y$ ” because they were unlike terms. She, however, obtained an answer of “ $3xy$ ” to the question “simplify $2x+x+y$ ” thus demonstrating an inability to distinguish between “like” and “unlike terms” in different examples. She added “ $2x$ ” and “ x ” to obtain “ $3x$ ” and then placed the “ y ” next to her answer. She was thus consistent in “ignoring letter” when she did conjoin but discriminated between when to conjoin and when not to.

6.2.3.2. Letter Evaluated

This error was visible in 2 questions in the Grade 11 task and accounted for 20.8% of the errors in Grade 11. It was at its lowest in Grade 10, accounting for 5.3% of the errors while this figure was 15.8% in Grade 10. It was shown that learners evaluated letters differently depending on the particular question. For example, the question “if $e+f=8$ then $e+f+g=---$ ”, in the WMC-S task elicited the most common answer of “12” in Grade 9. In Grade 10, the most common error in this question was “8g” illustrating conjoining. In Grade 11, the equivalent question was “if $a+b=3$ then what is $a+b+c$?” The most common answer in Grade 11 was “6” which was explained by Sbu as being the result of providing the values of “a, b and c” with the values “1, 2 and 3” respectively. Thus evaluating letter seems to be a persistent problem in this particular type of question.

6.2.4. Which errors fade away by Grade 11?

While some errors were persistent through grades 9 to 11 there were other errors that seem to have disappeared by Grade 11. These are exponents and errors in substitution.

6.2.4.1. Exponents

A common answer to question 1.4 (“simplify $2a+5b+a$ ”) in the WMC-S test was “ $7a^2b$.” This occurred in other questions similar to this in Grades 9 and 10 but most especially occurred in questions consisting of brackets. For example a common answer to “Simplify $(a-b)+b$ ” was “ ab^2 ”. Four out of the 6 learners in the sample displayed errors relating to exponents in Grades 9 and 10. MacGregor & Stacey (1997) maintain that younger students

do not exhibit this error because they had not learnt the notation for powers. This suggests the interference of new knowledge with previous knowledge.

The only occurrence of the use of exponents in Grade 11 was when a learner indicated two answers to a question relating to perimeter of a figure. She indicated 2 answers to the question “find the perimeter of a triangle with sides measuring “a” units each”. Her answers were “ $a + a + a = 3a$ ” and “ $a \times a \times a = a^3$ ”. This error seems to have faded away as she used the exponential law correctly although in an inappropriate example. She was able to distinguish between adding “like” terms and multiplying them. No other learner in the sample displayed this error. The results therefore suggest that the confusion with the exponential laws seems to have faded away in Grade 11.

6.2.4.2. Substitution errors

Three out of the sample of 6 learners displayed an error in question 7.2 of the WMC-S test in Grades 9 and 10. The question “if $m=3n+1$, find m if $n=4$ ” elicited answers of either “8” or “35”. The answer of “35” was most likely derived by placing the 4 next to the 3 to obtain “34” and then adding 1 to obtain “35”. The value “8” was most likely found by adding the “3” to the “4” and then adding “1”. The answer of “35” was not only exhibited by the 6 learners but was also a common answer with the original sample of 29 learners. While the response of “8” to the question was catered for in the ICCAMS coding scheme, the answer of “35” was not. This may indicate that the response of “35” to this question was unique to the South African context as the coding was derived from common answers to the ICCAMS test in England. None of the 6 learners displayed this error in the Grade 11 task. All of them, during the interviews, were able to explain that the “2” should be multiplied by “2” first and then added to “1” to obtain the answer in the equivalent question “if $p=2n+1$ and $n=2$, find p”.

6.2.5. What are the particular questions on level 2 that posed a challenge to learners?

The most important finding in this level is that those learners who displayed conjoining errors are unable to answer the minimum of at least two thirds of the questions correctly to satisfy the criteria for level 2. There were 3 out of the 6 questions on level 2 in the WMC-S test that posed a challenge to learners who displayed a conjoining error. If learners are prone to conjoining then these 3 questions would most likely be answered incorrectly. This means that these learners would not be able to obtain the minimum number of correct

answers on level 2. There were 5 learners in total who did not achieve level 2 in both Grades 9 and 10, but were able to obtain the minimum number of level 3 questions correct. This represents an anomaly.

The ICCAMS instrument was designed and implemented in England and was thereafter used in other countries. It may be possible that within the South African education landscape this issue with the level 2 questions requires further investigation with a larger sample of learners. It must be stated however, that only 2 of the 6 learners in the sample demonstrated problems with the level 2 questions in the Grade 11 task. It may be entirely possible that this error of conjoining may be unique to this school. The reverse, however, may also be true. It is possible that this problem of conjoining may be more widespread than the results of study display.

6.3. Reflections

My initial misgiving on embarking on this study was on the role of the teacher being backgrounded. It seemed pointless to me to investigate the errors made by learners without having an idea of the role of the teacher in entrenching the error or assisting in the dispelling of the error. During the study, however, it became clear to me that in order to assist in reducing the errors of learners, it is important to understand what the most common and persistent errors are and what the learner's thinking is when answering questions. The study has enhanced my understanding of the errors that learners make. The interviews in particular provided me with an increased understanding of how learners arrive at a particular answer which just viewing answers from scripts do not provide. The understanding of how learners arrive at particular answers provides me with the necessary knowledge to find ways of attempting to reduce these errors.

Although as a teacher of mathematics I expected the conjoining error to be evident during the research, I was nonetheless taken aback by the prominence of this error in the sample group. One would expect that the large amount of time spent on algebra from Grade 8 to Grade 11 would have provided learners with sufficient knowledge so that this error would not have persisted in Grade 11. The prevalence of this error albeit in 2 of the sample of 6 learners indicates that the teaching of algebra needs revisiting. Some recommendations for teaching are provided in section 6.6.

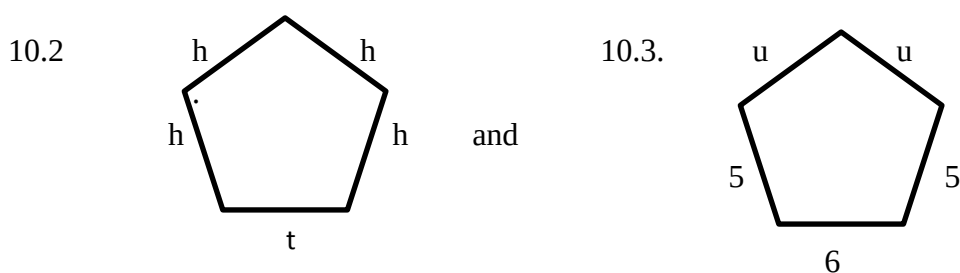
6.4. Challenges/limitations

There were many challenges during this study. Some of them were the time taken to re-code the scripts of the WMC-S test, timing of the interviews during examinations and identifying the level of the learner in the Grade 11 task.

One of the biggest challenges faced in this study was the re-coding of the scripts in the sample for phase 1 of the study. I needed clarity from members of the WMC-S project team on the many discrepancies with the original coding. I could not proceed with the analysis until these coding problems were clarified. This posed a challenge to me in terms of time. A detailed discussion of this was provided in section 4.2.2.

The other challenge was that the interviews were conducted immediately after learners wrote their final examination. The results may thus have been affected by the learners being tired. I tried to motivate them to write on days when they were not involved with examinations but that meant that they had to walk long distances from home just to participate in the interview. They were understandably not too keen to do this.

Identifying the level of learner after writing the Grade 11 task was problematic. The test included questions that were of a similar type to those in the original test (WMC-S test). In order to ensure that a learner was not guessing at an answer, I included more than 1 question of a specific type in the test. For example questions 5.2, 5.3 and 5.4 were all questions of a similar type to questions 10.2 and 10.3 in the Wits test i.e. find the perimeter of the following figures:



Since the WMC-S test included 2 questions of the same type I included 2 of the 3 questions in my test to determine the level of the learners in Grade 11. I chose the questions that were of the same type as 10.3. In addition I did not include, in the Grade 11 task, an example in the calculation of perimeter with 2 different variables and no values like question 10.2 (see diagram above). I therefore was unable to explore the responses of

learners in Grade 11 in questions consisting of adding terms with letters only when provided in diagrammatic form. One of the important findings in the study was the different forms of conjoining learners use when adding “like” and “unlike terms” and the different ways of answering questions when asked to simplify expressions as opposed to adding “like” and “unlike terms” when adding the given sides of a figure. I believe that the presence of a question like 10.2 would have enhanced the study and enriched my knowledge of the different ways that learners conjoin terms.

Finally, there were other questions in the WMC-S test that posed a challenge to learners in Grades 9 and 10 that I would have liked to explore. Some of these were the responses to question 2 (*Which is larger, $2n$ or $n+2$*), question 4.2 (*Multiply $n+5$ by 4*) and question 11 (*Cakes cost c rand and buns cost b rand each. If I buy 4 cakes and 3 buns, what does $4c+3b$ stand for?*). The limited scope of this study prevented further investigation of these questions and the responses of the learners to these questions in Grade 11.

6.5. Recommendations for teaching

Any study is fruitless if the findings of the study are not useful for future gain. Learners in this study displayed many errors relating to the use of letters in algebra. Teachers of mathematics must firstly be aware of the types of algebraic errors produced by learners in order to find methods to attempt to correct them. It is hoped that this study highlights some of these errors.

Herscovics & Linchevski (1994) contend that teachers can assist in reducing the conjoining error by ensuring that learners encounter arithmetic expressions in different equivalent and unclosed (“unfinished”) forms. This is supported by Falle (2007) who adds that a variety of expressions must be experienced by the learner where they can be expressed in different ways without altering the meaning of the expression. Falle (2007) adds that the instruction “simplify” may be too limiting and therefore learners must be encouraged to write and re-write expressions in different ways and then discuss the usefulness of the different representations. Statements like “Put like terms together” and “Get rid of the brackets” by teachers in the classroom also contribute to conjoining as these types of statements do not convey an exact mathematical message. Another recommendation for teaching in assisting to reduce the conjoining error is to show learners using values for letters to test results e.g. $5a+2b + 3a=8a + 2b$ as opposed to the answer “ $10ab$ ” using values for “ a ” and “ b ”.

MacGregor & Stacey (1997) recommend that teachers use algebraic notation more often and in a variety of topics in mathematics when generalising and writing formulae. They add that teachers should emphasise that letters represent numbers and not the names of things (objects). Some teachers promote the use of letters to stand for objects when they use examples like “let L represent the length of the rectangle” rather than stating “Let L stand for the number of metres in the length”.

In order to dispel some of the errors associated with exponents, MacGregor & Stacey (1997) suggest that when teaching the concept “ x^y ”, teachers should stress that this means “the product of y factors each having the value of x ”. This statement may assist in reducing the ambiguity of a statement like “multiply x by itself y times”.

MacGregor & Stacey (1997) also highlight the importance of ensuring that learners do not associate the values of the letters with their order in the alphabet.

In light of the information about how learners arrive at their answers obtained during the interviews, I believe that discussions are a necessity in the classroom to identify and correct errors. Pirie & Schwarzenberger (1988) emphasise that when thoughts are expressed into words students are forced “to organise their thinking and to confront their incomplete understanding”. In addition a discussion affords learners the opportunity to listen to others and to contemplate on the thinking of others. This enables learners to consider the implications of the thinking of others in order to extend their own understanding.

6.6. Conclusion

The most important finding in this study is that the inappropriate conjoining of terms is a stumbling block to some learners progressing to higher levels in the WMC-S test.

Although my sample is small, 5 learners were able to achieve at level 3 without first obtaining the minimum number of questions correct to be on level 2. According to Hart (1981) it is unlikely for this to occur. While this may not have been an issue in other countries in which this instrument (ICCAMS) was used, this study proved otherwise in the South African context. This issue with the level 2 questions is thus worth further investigation with a larger sample.

The Grade 11 task was written by learners at the end of the Grade 11 year. The prevalence of conjoining at this stage by 2 of the 6 learners seems to have been undetected by the teacher/s of these learners. It is therefore possible that, this error will feature just as prominently in Grade 12 as in Grade 11. This may be one of the reasons for the comment by the examiner of the 2013 Grade 12 examination that the “algebraic skills of the learners are poor” (p.126) and that “basic algebraic manipulation needs attention” (Department of Basic Education, 2014, p.136). It is highly unlikely that the Grade 12 teacher focuses on these skills given the large class sizes in the South African education landscape. The basic algebra skills, with particular reference to conjoining, must therefore be consolidated in the earlier grades in order for an improved performance by learners in the Grade 12 examination in South Africa.

REFERENCES

- Bell, A. (1995). Purpose in school algebra. *Journal of mathematical behavior* , 14, 41-73.
- Booth, L. (1984). Algebra. Children's strategies and errors, a report of the strategies and errors in the secondary school project, NFER-Nelson, London.
- Booth, L. (1988). Children's Difficulties in Beginning Algebra. In A.F. Coxford & A.P. Shulte (Eds.), *The Ideas of Algebra, K-12*. Reston: The National Council of Teachers of Mathematics.
- Christou, K., Vosniadou, S., & Vamvakoussi, X. (2007). Students' interpretations of literal symbols in algebra. In S. Vosniadou, Baltas, A., & Vamvakoussi, X. (Ed.), *Reframing the conceptual change approach in learning and instruction*. New York: Elsevier.
- Cohen, L. M., L; and Morrison, K. (2000). *Research methods in education* (5 ed.). London: RoutledgePalmer.
- Dennis, C. R., & Murray, D.M. (2012). Success in first-year mathematics: School-leaving examinations and first-year performance. *South African Journal of Science*, <http://dx.doi.org/10.4102/sajs.v108i7/8.1325>.
- Department of Basic Education. (2011). *Curriculum and Assessment Policy Statement (CAPS). Mathematics. FET*.
- Department of Basic Education. (2014). *2013 National Senior Certificate. National Diagnostic Report* .
- Falle, J. (2007). Students' Tendency to Conjoin Terms: An Inhibition to their Development of Algebra. *Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia*. In J. Watson & K. Beswick (Eds.), *Mathematics: Essential Research, Essential Practice*, 1, pp. 285-294.
- Golafshani, N. (2003). Understanding Reliability and Validity in Qualitative Research. *The Qualitative Report*. 8(4) 597-607.
- Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In P. Steffe, P. Nesher, P. Cobb, G. Goldin & B. Greer (Eds.), *Theories of mathematical learning* (pp. 197-217). New Jersey: Lawrence Erlbaum.
- Hatch, J. (2002). Doing Qualitative research in education settings. *Analysing qualitative data* (pp. 147-191). Albany: Suny Press.
- Hart, K. M. (1981). *Children's Understanding of Mathematics: 11 - 16*. London: John Murray, London.
- Hart, K., Brown, M., Kerslake, D., Küchemann, D., & Ruddock, G., (1985) *Chelsea Diagnostic Mathematics Tests: Teacher's Guide*. Nfer-Nelson.

- Herscovics, N. & Linchevski, L. (1994). A Cognitive Gap Between Arithmetic And Algebra. *Educational Studies in Mathematics*, 27, (pp. 59-78). Netherlands: Kluwer Academic Publishers.
- Hodgen, J., Küchemann, D., Brown, M., & Coe, R. (2008). Children's Understanding of Algebra 30 years on. *Proceedings of the British Society for Research into Learning Mathematics*, 28(3), 36-41.
- Jaworski, B. (1996). Constructivism and teaching-the socio-cultural context. Retrieved October, 26, 2011.
- Kieran, C. (1992). The learning and teaching of algebra, in D.A. Grouws (Ed.), *Handbook of research on Mathematics Teaching and Learning*, Macmillan, New York, pp. 390-419.
- Küchemann, D. (1981). Algebra. In K.M. Hart (Ed.), *Children's understanding of mathematics*: 11-16. London: John Murray., pp. 102-119.
- Linchevski, L., and Herscovics, N. (1996). Crossing the cognitive gap between arithmetic and algebra: Operating on the unknown in the context of equations. *Educational Studies in Mathematics*, 30, 1, 39-65.
- MacGregor, M., & Stacey, K. (1997). Students understanding of algebraic notation: 11-15. *Educational Studies in Mathematics*, 33, 1-19.
- Makgato, M., & Mji, A. (2006). Factors associated with high school learners' poor performance: A spotlight on mathematics and physical science. *South African Journal of Education*, 26(2), 253-266.
- Malara, N and Iaderosa, R. (1999) The interweaving of arithmetic and algebra: some questions about syntactic and structural aspects and their teaching and learning. In Inge Schwank (Ed.), *Proceedings of the first conference of the European Society for Research in Mathematics Education*. (Vol. 2, pp. 159-171). Forschungsinstitut fuer Mathematikdidaktik, Osnabrueck.
- Marshall, M.N., (1996). Sampling for qualitative research. *Family Practice*, 13(6), 522-525.
- McMillan, J.H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry*. (7th ed). Upper Saddle River, New Jersey: Pearson Education, Inc.
- Meadows, S. (2004). Models of cognition in childhood: metaphors, achievements and problems. In H. Daniels, H. and A. Edwards. (Eds.) *The Routledge Falmer Reader in Psychology of Education* (pp. 136-144). London: Routledge/Falmer.
- Melis, E., Gogvadze, G., Libbrecht, P. & Ullrich, C. (2010). "Culturally Aware Mathematics Education Technology", In Handbook of Research in Culturally-Aware Information Technology: Perspectives and Models, pp. 543-557. IGI Global.

- Morse J., Barrett M., Mayan M., Olson K. & Spiers J. (2002). Verification strategies for establishing reliability and validity in qualitative research. *International Journal of Qualitative Methods*, 1(2), 1-19.
- Nesher, P. (1987). Towards an instructional theory: The role of students' misconceptions *For the learning of Mathematics*, 7(3), 33-39.
- Nunes, T., Bryant, P., & Watson, A. (2009). Key understandings in mathematics learning. *London: Nuffield Foundation*. Retrieved on, 22, 2010.
- Oldenburg, R., Hodgen, J., & Küchemann, D. (2013). Syntactic and semantic items in algebra tests – a conceptual and empirical view. In B. Ubuz, Ç. Haser, M-A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education (CERME8)*, (pp. 501-510). Ankara, Turkey: Middle East Technical University on behalf of the European Society for Research in Mathematics.
- Olivier, A. (1989). Handling pupils' misconceptions. *Pythagoras* , 10-19.
- Opie, C. (2004). *Doing educational research: A guide to first time researchers*. London: Sage.
- Parker, F. (2012, January 10). Do the Maths: Results not in line with SA's ambitions. Mail & Guardian Online. Retrieved from <http://mg.co.za/article/2012-01-10-do-the-maths-matric-results-not-in-line-with-sa-s-ambitions>
- Patton M. (1999). Enhancing the Quality and Credibility of Qualitative Analysis. *HSR: Health Services Research*, 34(5) pp. 1189-1208.
- Pirie, S., & Schwarzenberger, R. (1988). Mathematical discussion and mathematical understanding. *Educational Studies in Mathematics*, 19(4), 459-470. Kluwer Academic Publishers.
- Powell, K.C. & Kalina, C.J., (2009). Cognitive and Social Constructivism: Developing tools for an effective classroom. *Education*, 130(2). 241-250.
- Reddy, V.; van der Berg, S.; Janse van Rensburg, D; & Taylor, S (2012). *Educational outcomes: Pathways and performance in South African high schools*. S Afr J Sci 2012; 108(3/4). <http://www.sajs.co.za>
- Russel, M., O'Dwyer, L.M. & Miranda, H. (2009). Diagnosing students' misconceptions in algebra: Results from an experimental pilot study. *Behavior Research Methods*, 41(2), 414-424.
- Sandelowski, M. (2000). Focus on Research Methods. Combining Qualitative and Quantitative Sampling, Data Collection, and Analysis Techniques in Mixed-Method Studies. *Research in Nursing and Health*, 23, 246-255.
- Sfard, A. and Linchevski, L. (1994). The gains and the pitfalls of reification: The case of algebra. *Educational Studies in Mathematics*, 26, 191-228.

- Stacey, K., & MacGregor, M. (1994). 'Algebraic sums and products: Students' concepts and symbolism', in J.P. da Ponte and J.F. Matos (eds.), *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education*, (Vol 1V), University of Lisbon, Portugal: PME, pp. 289–296.
- Tahir, S., Cavanagh, M & Mitchelmore, M. (2009). A multifaceted approach to teaching algebra: Students' understanding of variable. In Tzekaki, M., Kaldrimidou, M. & Sakonidis, H. (Eds.). *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education*, Vol. 1, pp. 217-224. Thessaloniki, Greece: PME.
- Tirosh, D., & Even, R. & Robin, N. (1998). Simplifying Algebraic Expressions: Teacher Awareness and Teaching Approaches. *Educational Studies in Mathematics* , 35, 51-64.
- Usiskin, Z. (1999). Conceptions of School Algebra and Uses of Variables." In *Algebraic Thinking, Grades K–12: Readings from NCTM's School-Based Journals and Other Publications*. Reston, Va. pp7-13.
- Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and Instruction*, 14, pp. 469-484.
- Wong, T. (2009). *Remedies for Common Algebra Mistakes*. Department of Mathematics. College of Southern Nevada.

APPENDICES

Appendix A ICCAMS sections of the Annual test from WMC-S

Section B

ICCAMs

1. $a + 3a$ can be written more simply as $4a$.

Simplify each of the following, where possible:

1.1 $2a + 5a =$ _____ 1.2 $2a + 5b =$ _____

1.3 $(a + b) + a =$ _____ 1.4 $m = 3n + 1 =$ _____

1.5 $(a - b) + b =$ _____ 1.6 $3a - (b + a) =$ _____

1.7 $a + 4 + a - 4 =$ _____ 1.8 $3a - b + a =$ _____

1.9 $(a + b) + (a - b) =$ _____

2. Which is larger, $2n$ or $n + 2$? _____

Explain:

3. **4 added to n** can be written as **$n+4$** .

Add 4 to:

3.1 8

3.2 $n+5$

3.3 $3n$

4. **n multiplied by 4** can be written as **$4n$** .

Multiply each of these by 4:

4.1 8

4.2 $n+5$

4.3 $3n$

5.1 If $a + b = 43$,

then $a + b + 2 =$ _____

5.2 If $n - 246 = 762$,

then $n - 247 =$ _____

5.3 If $e + f = 8$,

then $e + f + g =$ _____

6.1 Find a if $a + 5 = 8$ _____

6.2 Find b if $b + 2 = 2b$ _____

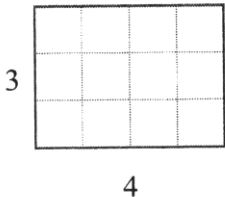
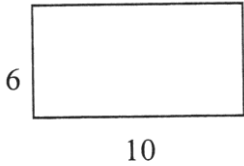
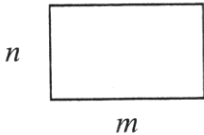
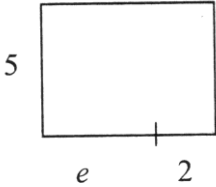
7.1 If $u = v + 3$ and

$v = 1$, find the value of u _____

7.2 If $n = 4$ and

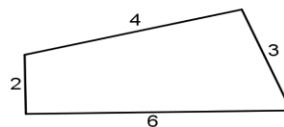
$n = 4$, find the value of m _____

8. What are the areas of the following shapes?

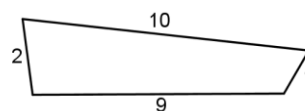
8.1	8.2	8.3	8.4
			
A =	A =	A =	A =

9. The perimeter of this shape is equal to

$6 + 3 + 4 + 2$, which equals 15.

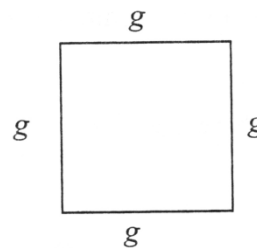


9.1 Work out the perimeter of this shape:



10. This square has sides of length g .

So, for its perimeter, we can write $p = 4g$.



Find the perimeter for each of the shapes:

10.1	10.2	10.3	10.4
P =	P =	P =	P =

11. Cakes cost c rand and buns cost b rand each.

If I buy 4 cakes and 3 buns, what does $4c + 3b$ stand for?

The End

Thank you

Appendix B The Grade 11 Task

1. If $2a + 5a = 7a$ then simplify $3x + 2x$

2. If $k + 2 = 5$, then $k =$

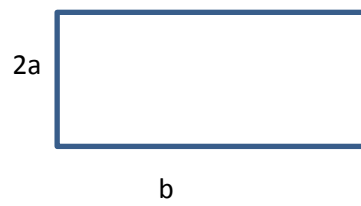
3. If $j + k = 5$ then $j + k + 2 =$

4. Find the area of the following rectangles:

4.1.

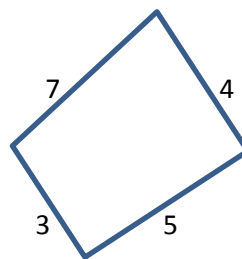


4.2.



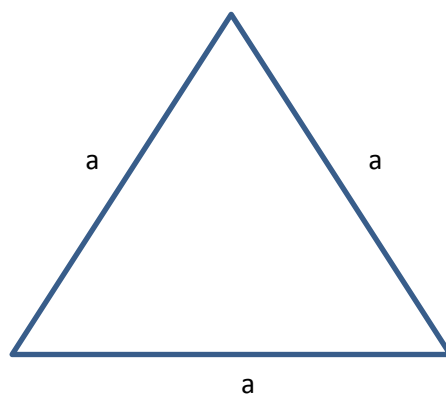
5. The perimeter of this shape is

$$7 + 4 + 5 + 3 = 19$$

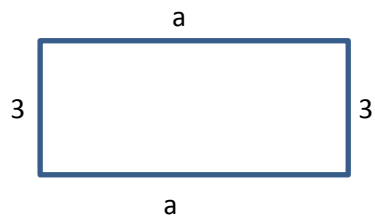


Find the perimeter of each of the following figures:

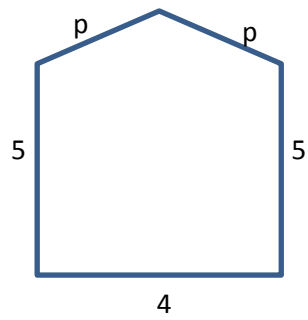
5.1.



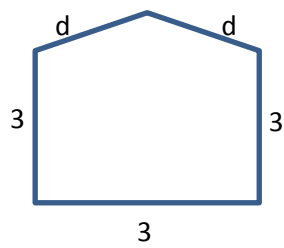
5.2.



5.3.



5.4.



6. If $x + 3x$ can be written more simply as $4x$ then simplify the following , where possible:

6.1. $2p + 3a + p$

6.2. $2p + 3a + 3p$

7. Find the value of x if :

7.1. $x = y + 5$ and $y = 2$.

7.2. $x = y - 1$ and $y = 3$.

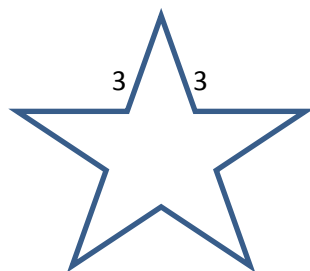
8. Find the value of p if:

8.1. $p = 2n + 1$ and $n = 2$

8.2. $p = 5n - 1$ and $n = 4$

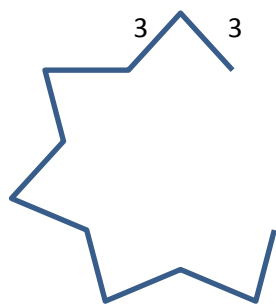
9. Find the perimeter of the following figures:

9.1.



Each side equals 3 units.

9.2.



Part of a figure is drawn.

Each side equals **3** units and the figure

has **n** sides.

10. If $2a + a = 3a$ then simplify the following, where possible :

10.1. $3x + 2y$

10.2. $2x + x - y$

11. Add: 6 to y

12. If $a + b = 3$ then what is $a + b + c$?

Appendix C Adapted version of ICCAMS marking codes

ICCAMS

Ques	Code 0 Missing	Code 1 Correct	Code 2 Ambiguous	Code 3	Code 4	Code 5	Code 6	Code 7 Letter not used	Code 8 Premature Closure	Code 9 Wrong
				Letter Evaluated		Letter as Object				
1	☐	<u>8q</u>		8					9q	9a) $8a^2$ 9b) Other
2	☐	<u>4q+6v</u>							8a) <u>10qy</u> 8b) 9qy 8c) 9qyq	9a) $7a^2b / 8a^2b$ 9b) $2a^2 + 5b$ 9c) Other
3	☐	<u>6+p</u>								☐
4	✓	<u>6p</u>								9a) 6xp 9b) Other
5.1	☐	<u>48</u> ; 4x12 (ignore insertion of units ² or numbers ²)								9a) 16; 4+12 9b) Other
5.2	☐	1a) <u>mn, mxn</u> (ignore insertion of units ²) 1b) <u>m+n</u> ; if answer to 5.1 was 16								9a) m+n (if 5.1 was not 16) 9b) $2(m+n) / 2m+2n /$ $m+m+n+n$ 9c) Other
6	✓	<u>20</u> ; 18; <u>19</u> ; <u>21</u>								☐
7.1	☐	1a) <u>3k</u> 1b) <u>k+k+k</u>	3g	9, 12, 15 ... (any number added 3 times)						☐
7.2	☐	1a) <u>4c+d</u> 1b) <u>4c+1d</u> 1c) <u>c+c+c+c+d</u>	4c, d operation missing						8a) <u>4cd</u> / 4c1d 8b) ccccd	9a) 5cd 9b) Other
7.3	☐	1a) <u>2e+23</u> 1b) <u>2.e+2.7+1.9</u>	2a) 2e,23 2b) 49 2e 23						8a) <u>2e23</u> 8b) <u>ee779</u>	☐

Ques	Code 0 Missing	Code 1 Correct	Code 2 Ambiguous	Code 3	Code 4	Code 5	Code 6	Code 7 Letter not used	Code 8 Premature Closure	Code 9 Wrong
				Letter Evaluated		Letter as Object				
		or 2e+2(7)+9 1c) 2e+14+9 or 2e+7+7+9							8c) 25e	
8	☐	<u>h</u> = 14 / 6+8							<u>2</u>	9a) 6 9b) Other
9	☐	<u>f</u> = 42 / 5(8)+2								9a) 15 (8+5+2) 9b) ±1 9c) 10 9d) Other

Appendix D Letter to the Principal

Protocol number: 2013ECE122M

DATE:

Dear Mr. Kunene

My name is Vasantha Moodley. I am a student in the School of Education at the University of the Witwatersrand.

I am doing research on **“An investigation into learners’ performance in Algebra from Grade 9 to Grade 11”**

Since 2010 Wits Maths Connect Secondary has been tracking learner performance through tests at the end of each year. This research is led by Professor Jill Adler.

My study is a follow-up of this data. I would like to invite some of your learners to participate in interviews in this regard. My research involves interviews with about 8 learners in Grade 11. The interviews will not be done during instruction time at the school and will therefore not disrupt teaching and learning at your school. The interviews will be conducted after school hours at a time suitable for the selected learners.

The reason that I have chosen your school is because the WMC-S tests were conducted at your school from 2010 and your school is one of the most active schools within the WMC-S project. I was wondering whether you would mind if I interviewed 8 of your Grade 11 learners in November.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed 5 years after completion of the project.

Please let me know if you require any further information or have any enquiries. You can also contact my supervisor. I look forward to your response as soon as is convenient.

Yours sincerely

SIGNATURE:

Vasantha Moodley
vasmoodley1@gmail.com
0846944128

Supervisor :
Dr. Craig Pournara
Craig.Pournara@wits.ac.za
011 717 3253

Date:

Dear Learner

My name is Vasantha Moodley and I am a student in the School of Education at the University of the Witwatersrand.

I am doing research on **“An investigation into learners’ performance in Algebra from Grade 9 to Grade 11”**

Data has been collected from the WMC-S tests written at the end of 2011 and 2012. This research is a follow-up of these. My investigation involves interviews with 8 learners in Grade 11. The interviews will not be done during instruction time at the school and will therefore not disrupt teaching and learning at your school. The interviews will be conducted after school hours at a time suitable for you.

I was wondering whether you would mind participating in an interview in order to assist me with my research. The interview will involve you having to answer questions in both written and verbal form. These questions are similar to the questions that you answered in tests written by you in Grades 9 and 10. These interviews will be audio-taped to ensure that I have a correct version of the interviews and your comments.

Remember, this is not a test, it is not for marks and it is voluntary, which means that you don’t have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

The results will be published in journals and presented at conferences. I will not be using your own name but I will make one up so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

Please feel free to contact me or my supervisor if you have any questions.

Thank you

SIGNATURE

Vasantha Mooldey
vasmoodley1@gmail.com
0846944128

Supervisor :
Dr. Craig Pournara
Craig.Pournara@wits.ac.za
011 717 3253

Protocol number: 2013ECE122M

DATE:

Dear Parent/Guardian

My name is Vasantha Moodley and I am a student in the School of Education at the University of the Witwatersrand.

I am doing research on **“An investigation into learners’ performance in Algebra from Grade 9 to Grade 11”**

Data has been collected from the Wits Maths Connect Secondary tests written by your child at the end of 2011 and 2012. This research is a follow-up of these results.

The reason why I have invited your child is because he/she has written the WMC-S tests in Grades 9 and 10. I would therefore like to investigate the progress made by your child in Algebra from the last two years to now. I was wondering whether you would mind if I invited your child for an interview. The interview will not be done during instruction time at the school and will therefore not disrupt teaching and learning at the school. The interview will be conducted after school hours at a time suitable for you and your child. The interview will be audio-taped to ensure that the information I obtain will be an accurate version of the interview.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

The results of the study will be presented at conferences and published in journals. Your child’s name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me or my supervisor know if you require any further information.
Thank you very much for your help.

Yours sincerely,

SIGNATURE

Vasantha Moodley
vasmoodley1@gmail.com
0846944128

Supervisor :
Dr. Craig Pournara
Craig.Pournara@wits.ac.za
011 717 3253

Appendix G **Learner Consent Form**

Please fill in the reply slip below if you agree to participate in my study called: **An investigation into learners' performance in Algebra from Grade 9 to Grade 11**

My name is:

Permission for interview

I would like to be interviewed for this study. YES/NO

I know that I can stop the interview at any time and don't have to answer all the questions asked. YES/NO

I understand that my written response from the interview will be collected at the end of the interview. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project and the wider Wits Maths Connect Secondary research. YES/NO

I know that Vasantha Moodley will keep my information confidential and safe and that my name and the name of my school will not be revealed. YES/NO

I know that I do not have to answer every question and can withdraw from the study at any time. YES/NO

I know that I can ask not to be audiotaped YES/NO

I know that all the data collected during this study will be destroyed after 5 years of completion of my project. YES/NO

Sign _____ Date _____

Appendix H

Parent's/Guardian's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in my voluntary research project called: **"An investigation into learners' performance in Algebra from Grade 9 to Grade 11"**

I, _____ the parent/guardian of _____

Permission for interview

I agree that my child may be interviewed for this project. YES/NO

I know that he/she can stop the interview at any time and does not have to answer all the questions asked. YES/NO

I understand that my written response from the interview will be collected at the end of the interview. YES/NO

Permission to be audiotaped

I agree that my child may be audiotaped during the interview YES/NO

I know that the audiotapes will be used for this project and for the broader Wits Maths Connect Secondary Project. YES/NO

I know that Vasantha Moodley will keep my information confidential and safe and that my child's name and the name of his/her school will not be revealed. YES/NO

I know that he/she does not have to answer every question and can withdraw from the study at any time. YES/NO

I know that he/she can ask not to be audiotaped YES/NO

I know that all the data collected during this study will be destroyed after 5 years after the completion of the project. YES/NO

Parent Signature: _____ Date: _____