Search for a heavy Higgs boson in multi-Higgs doublet models

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Declaration

I, the undersigned, hereby testify that the research presented in this work is my own original effort and I declare that I have acknowledged all material obtained from other resources. The material presented in this dissertation have not been submitted to any university for a degree.

Amir Abouelrous, March 15, 2019

This dissertation has produced two proceedings:

- Analysis of a heavy Higgs boson of mass around 270 GeV in left-right symmetric models. Proceedings submitted to SAIP Conference 2017; Contribution: Studied global and discrete symmetries and their roles in suppressing flavor changing neutral currents mediated by heavy bosons in left-right symmetric models. Investigated the lower mass bound of the heavy bosons in left-right symmetric models.

- The heavy boson searches and the phenomenology of the Two-Higgs Doublet Model type II extended with one real scalar singlet. Proceedings submitted to HEPP Workshop 2018; Contribution: Studied the mass hierarchy of the CP-even bosons in two-Higgs doublet model plus scalar singlet in the framework of the Madala hypothesis.
First I have to express my total gratitude towards God Almighty for giving me the strength and wisdom to accomplish this task.

I would like to express my gratefulness to my supervisor Prof. Bruce Mellado. He offered me the opportunity to join his research group where I learned physics as well as interpersonal skills. I would like to express my appreciation and acknowledgement to the School of physics and the professors at the University of the Witwatersrand. I would like to thank Prof. Daniel Joubert for accepting me in the postgraduate physics program even though I had a BSc in geophysics. If he hadn’t given me the opportunity, I wouldn’t have written this dissertation.

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Abstract

A number of excesses (around 3\(\sigma\)) in the proton-proton collision data collected during Run 1 by the ATLAS and CMS experiments at the Large Hadron Collider point to the possibility of extensions to the Standard Model Higgs sector. In Run 2, the excesses are much stronger which confirms the observed Run 1 excesses. Of particular importance are the excesses in the production of multiple-leptons. While present in Run 1 data, these have become very significant in Run 2. We introduce the Madala hypothesis in order to explain the excesses observed in the data. This hypothesis postulates a heavy boson \(H\) (Madala boson) and a lighter scalar \(S\) which can decay into both dark matter and Standard Model particles. To accommodate the excesses in the data, the dominant decay channel of the heavy boson \(H\) is: \(H \rightarrow Sh\), where \(h\) is the Standard Model Higgs boson.

In this project we study two models which have an extended Higgs sector as compared to the Standard Model. First, we study Left-Right Symmetric Models and analyze the Higgs spectrum. Flavor Changing Neutral Currents constrain the mediating heavy boson to be of \(O(\text{TeV})\). We apply global and discrete symmetries to suppress Flavor Changing Neutral Currents and to constrain the mass of the heavy boson mediating them to \(O(\text{GeV})\). Then we analyzed the mass spectrum of the neutral boson and place a lower limit on the mass of the heavy boson \(H\). Our analysis of global and discrete symmetries in Left-Right Symmetric Models constrained the heavy boson \(H\) that mediate Flavor Changing Neutral Currents to \(O(\text{TeV})\). We conclude that Left-Right Symmetric Models cannot accomodate the heavy boson \(H\).

The second model we study is the Two-Higgs Doublet Model Plus a Scalar Singlet. We will analyze the branching ratios of the heavy boson \(H\) and the \(Z\) gauge boson using the program N2HDECAY. Then we discuss the results in the framework of the Madala hypothesis. Finally, we study the mass spectrum of the neutral heavy boson in Two-Higgs Doublet Model Plus a Scalar Singlet and determine if we can fit the heavy boson \(H\) in the model. The results obtained in this research indicate that these models could accomodate the Madala boson. Further studies are required to perform off-shell decays of the heavy boson \(H\).
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### Acronyms

- **vev**: Vacuum expectation value
- **2HDM**: Two-Higgs Doublet Model
- **2HDM+S**: Two-Higgs doublet model plus scalar singlet
- **AGUS**: Approximate global $U(1)$ symmetries
- **ATLAS**: A Toroidal LHC Apparatus
- **BSM**: Beyond the Standard Model
- **CKM**: Cabibbo-Kobayashi-Maskawa matrix
- **CMS**: Compact Muon Solenoid
- **CP**: Charge conjugation and parity
- **DM**: Dark matter
- **EM**: Electromagnetic
- **eV**: electron-Volt
- **EWSB**: Electroweak symmetry breaking
- **FCNCs**: Flavor changing neutral currents
- **GeV**: Giga electron-Volt
- **LHC**: The Large Hadron Collider
- **LRSMs**: Left-right symmetric model
- **MeV**: Mega electron-Volt
- **MLRSMs**: Minimal Left-right symmetric model
- **N2HDECAY**: HDECAY program extension for 2HDM+S
- **QCD**: Quantum Chromodynamics
QED  Quantum Electrodynamics

Run 1  The LHC operating period from 2009 till 2012

Run 2  The LHC operating period from 2015 till 2017

SM  Standard Model

SSP  SARAH scan and plot

TeV  Tera electron-Volt

TME  Transverse missing energy

Symbols

$\chi$  Dark matter candidate

$\Delta_{R,L}$  Right and left-handed triplet fields

$\gamma$  Photon

$\Phi$  Complex scalar field

$\phi$  General scalar field

$\Psi$  General fermion field

$\sqrt{s}$  Centre of mass energy

$\theta_W$  Weinberg angle

$B_s$  $B_s$ meson

$g$  Gluon

$H$  CP-even heavy Higgs boson

$h$  Standard Model Higgs boson

$K^0$  Neutral Kaon meson

$L$  Lagrangian

$p_T$  Transverse momentum distributions

$q$  Quarks

$S$  Higgs-like scalar

$W^\pm$  W-bosons

$Z$  Z-boson
The Standard Model (SM) is an experimentally established model of elementary particles and their interactions \cite{1, 2, 3}. In 2012, the Higgs boson \cite{4, 5}, the last missing particle in the SM, was discovered at the ATLAS and CMS experiments at the large Hadron Collider (LHC) at CERN \cite{6, 7}. We give a brief introduction to the SM and a short description of the Madala hypothesis and how it can be embedded in models beyond the SM (BSM).

The structure of this chapter is as follows: In section 1.1 we give a brief overview of the SM. In section 1.2 we briefly explain the Brout-Englert-Higgs (BEH) mechanism. In section 1.4 we will introduce the Madala hypothesis. In section 1.5 we discuss the connection between the Madala hypothesis and theories BSM.

1.1 Overall Overview of the Standard Model

The SM is a model that describes the interactions of high energy particles. It has been verified by many experiments, such as the LHC and the Tevatron. A new era for high energy particle physics began when the Higgs boson was discovered by ATLAS and CMS experiments at the LHC \cite{6, 7}. In June 2012, the LHC announced the discovery of the last missing piece of the SM, the SM Higgs boson $h$ with a mass $m_h \sim 125$ GeV, at the center of mass energy $\sqrt{s} = 7$ and 8 TeV. The universe is composed of few fundamental building blocks called elementary particles. The interactions of these elementary particles are governed by four fundamental forces. Our current best physical and mathematical model that describes the interactions of these elementary particles with three of the fundamental forces is explained in the SM. It has successfully explained many experimental results and made precise predictions to a wide variety of phenomena \cite{8}. Even though the SM is the best description of elementary particle physics we know today, there are several shortcomings in the SM that make us consider it as the low energy limit of a more fundamental theory. The theory describes only three out of the four fundamental forces, excluding gravity. If we just add gravity to the SM we will not reproduce the experimental results without modifying the SM. The SM is considered incompatible with the successful theory of general relativity.

The SM explains only 5% of the energy present in the universe. It also does not
answer the nature of dark matter (DM). DM does not (or weakly) interacts with the SM fields yet it constitutes about 26% of the energy in the universe. In the SM the Higgs boson gets very large quantum corrections due to the presence of virtual particles. These corrections are much larger than the actual mass of the Higgs. Thus, inorder to obtain the observed mass of the Higgs, we have to fine tune the bare mass parameter to cancel most of the quantum corrections. The fine-tuning is considered unnatural by many theoretical physicists. It also predicts massless neutrinos, but experiments have proven that they have very tiny masses and mixing occurs between them. We can add neutrino mass terms in the SM but this will lead to new theoretical problems. The SM only contains left-handed neutrinos since no right-handed neutrinos are observed.

Our current knowledge of the universe is that it’s governed by four fundamental forces: the gravitational force, the electromagnetic force, the weak force and the strong force. These forces have varying strengths and different interaction ranges. Gravity is the weakest force and it has an infinite interaction range. The electromagnetic force is much stronger than gravity even though it has an infinite interaction range. The weak and strong forces are much stronger than the gravitational and electromagnetic forces but dominate only at the level of subatomic particles.

The interactions between particles and three of the fundamental forces are mediated by particles that act as force-carriers. These particles are called “gauge bosons” and they are responsible for interactions between the matter particles. Each fundamental force has its unique gauge bosons. The strong force is mediated by the “gluon”, the electromagnetic force is mediated by the “photon”, and the “W and Z bosons” mediate the weak force. The mediating particle for gravity is not included in the SM, but physicists have postulated the “graviton” as the force mediator. All the particles occurring in the SM have been experimentally observed including the 125 GeV Higgs boson $h$. The SM describes how the electromagnetic, weak and strong forces interact with matter particles via the mediating particles, Fig. 1.1.

The $Z$ and the $W$ are massive, while the photon and the gluons are massless. This causes weak interactions to be “weak” at low energy $O(100 \text{ GeV})$. The SM is an effective theory which operates at the low energy regime. At the low energy regime (electroweak scale $O(100 \text{ GeV})$), the weak and electromagnetic interactions become unified and indistinguishable in the framework of the electroweak interaction. Above
the electroweak scale (≈ 246 GeV) nature exhibits a higher degree of symmetry. Very high energy scales (O(TeV) and above) allow the possibility of unifying the strong force with the weak and electromagnetic forces [8]. The SM is based on the gauge group:

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y. \]  

(1.1)

It is a product of the electroweak group \( SU(2)_L \otimes U(1)_Y \) with the Quantum Chromodynamics (QCD) group \( SU(3)_C \). There are 8 gluons \( G^a_\mu \) of \( SU(3)_C \) color, 3 weak bosons \( W^i_\mu \) of \( SU(2)_L \) and \( B_\mu \) boson of \( U(1)_Y \) hypercharge. The SM Lagrangian is given by:

\[ L_{SM} = L_{\text{kineic}} + L_{\text{gauge}} + L_{\text{Higgs}} + L_{\text{Yukawa}}. \]

(1.2)

The \( L_{\text{kineic}} \) term describes fermion and gauge boson interactions which are invariant under the SM gauge group. The kinetic term for fermions have the following form:

\[ L^f_{\text{kineic}} = i \sum \tilde{\Psi} \gamma^\mu D_\mu \Psi, \]

where \( \Psi \) and \( \tilde{\Psi} \) are fermion fields and \( D_\mu \) is the covariant derivative associated to \( SU(2)_L \otimes U(1)_Y \):

\[ D_\mu = \partial_\mu + ig W^i_\mu \tau^i + g B_\mu Y, \]

(1.3)

where \( g_W \) and \( g_B \) are the respective couplings for the vector fields \( W^i_\mu \) and \( B_\mu \).

The \( L_{\text{gauge}} \) term contains the kinetic terms for the gauge fields and the interactions between them:

\[ L_{\text{gauge}} = -\frac{1}{4} W^a_\mu W^a_\nu - \frac{1}{4} G^a_\mu G^a_\nu - \frac{1}{4} B^\mu B^\nu, \]

(1.4)

where \( W^i_\mu, G^a_\mu \) and \( B^\mu \) are the field strength tensors of the \( SU(2)_L, SU(3)_C \) and \( U(1)_Y \) gauge fields respectively. They can be defined as follows:

\[
\begin{align*}
G^a_\mu & = \partial^\mu G^a_a - \partial^a G^a_\mu - g f^{abc} G^a_b G^c_\mu, \\
W^a_\mu & = \partial^\mu W^a_i - \partial^i W^a_\mu + g \varepsilon^{ijk} W^a_j W^c_k, \\
B^\mu & = \partial^\mu B^\nu - \partial^\nu B^\mu,
\end{align*}
\]

(1.5)

where \( f^{abc} \) and \( \varepsilon^{ijk} \) are the structure constants of the \( SU(3)_C \) and \( SU(2)_L \) groups, respectively. The \( L_{\text{Yukawa}} \) term describes the Yukawa interactions, which consist of the most general possible couplings of the Higgs scalars to the bilinear fermion fields:

\[ L_{\text{Yukawa}} = -\sum_{i=1}^{3} y_{ij} \tilde{\Psi} \Phi \Psi + h.c. \]

where \( \Phi = \tau_2 \Phi^* \tau_2 \) is the Higgs field and \( y_L, y_Q, y_R, y_L, y_Q \) are \( 3 \times 3 \) Yukawa matrices in flavor space. The SM Higgs Lagrangian term \( L_{\text{Higgs}} \) is given by:

\[ L_{\text{Higgs}} = Tr|D_\mu \Phi|^2 - V_{\text{Higgs}}, \]

(1.6)

where \( V_{\Phi} \) is the potential for the Higgs field:

\[ V_{\Phi} = \mu^2 (\Phi^\dagger \Phi) + \lambda (\Phi^\dagger \Phi)^2, \quad \mu^2 < 0. \]

There are seventeen named particles in the SM; see Fig. 1.2. They can be categorized into two groups: the first group consists of building blocks of matter, called fermions,
while the second group consists of mediators of interactions, called bosons. Both
groups are defined by their mass, their spin and the quantum numbers that determine
their interactions. There are twelve fermions and five gauge bosons in the SM. The
fermion fields are represented as left-handed doublets:

\[ L^i_L = (\nu^i_L, e^i_L), \quad Q^i_L = (u^i_L, d^i_L), \] (1.7)

where \( e_L \) and \( \nu_L \) are the left-handed electrons and their corresponding neutrinos
while \( u_L \) and \( d_L \) are the left-handed up and down quarks, respectively. We also have
the right-handed singlets:

\[ \nu_R, \quad e_R, \quad u_R, \quad d_R, \] (1.8)

where \( e_R \) and \( \nu_R \) are the right-handed electron and their corresponding neutrino
while \( u_R \) and \( d_R \) are the right-handed up and down quarks, respectively. The repre-
sentations of the multiplets with respect to \( SU(3)_C, SU(2)_L \) and \( U(1)_Y \), respectively
are:

\[ L_L = (1, 2, -1), \quad \nu_R = (1, 1, 0), \quad Q_L = (3, 2, \frac{1}{3}), \] (1.9)
\[ e_R = (1, 1, -2), \quad u_R = (3, 1, \frac{4}{3}), \quad d_R = (3, 1, -\frac{2}{3}). \]

The representations describe how each field transforms under the SM group. For
example \( L_L \) does not transform under \( SU(3)_C \) since it is a singlet under \( SU(3)_C \)
and has a hypercharge value of \(-1\). The fermion particles are classified into three
families with different masses and different electromagnetic charge. Since heavier
particles are unstable, they decay into lighter particles which constitute most of the
ordinary matter we see every day. In each family, the fermions are characterized by
their charges defined under the strong and electromagnetic interactions. Leptons
are neutral under the strong interactions while quarks are charged. Three quarks
carry the electromagnetic charge \( \pm \frac{2}{3} \) (up, charm, top) while the other three carry
the electromagnetic charge \(-\frac{1}{3}\) (down, strange , bottom). Three leptons carry the
electromagnetic charge -1 (electron, muon, tau) and their corresponding neutrinos
are neutral under the electromagnetic charge. Neutrinos are hard to detect since
they are neutral under both the electromagnetic and strong interactions. They have
a very small mass as compared to other SM fermions. The masses of the SM fermions
cover a wide range from the 170 GeV top quark down to the \( O(eV) \) neutrinos.

The concept of chirality describes whether a particle is left-handed or right-handed.
The chirality of a massless particle is equivalent to the helicity which is the dot
product between the spin and momentum of the particle. If a particle travels in
the same direction as its spin, then helicity = 1 and the particle is identified as
right-handed. Conversely, a left-handed particle travels in the opposite direction of
its spin and has helicity = -1. An anti-particle has the opposite sign of helicity when
compared to its corresponding particle.

Chirality for massive particles is more complex, since the dot product between mo-
mentum and spin depends on the reference frame and is not a simple Lorentz-
invariant property. In the SM, left-handed particles are arranged into an isospin
doublet, which can be rotated by three \( SU(2)_L \) transformations. Since the \( SU(2)_R \)
doesn’t exist, this means that right-handed particles have a trivial isospin repre-
sentation and don’t transform under isospin rotations. In the SM, the left and
right-handed particles have different group representations. Such theories are called chiral theories.

The SM is a non-Abelian gauge theory where the symmetry group: $SU(2)_L \otimes SU(3)_C$ is non-commutative. It is based on a gauge principle in which the exchanged gauge bosons are gauge fields of corresponding symmetry groups. The gauge transformations of the fields in the SM form a Lie group which refers to the symmetry group (gauge group) of the theory. For a quantum field theory to be gauge invariant, we add gauge fields to the Lagrangian to ensure that it is gauge invariant under the local group transformations. When the theory is quantized, gauge bosons are created from the quanta of the gauge fields. A gauge transformation of a general quantum field $\Psi$ is represented by:

$$\Psi \rightarrow e^{i\epsilon_a T^a} \Psi \simeq (1 + i\epsilon_a T^a)\Psi,$$

where $T^a$ are the generators of the gauge group and the parameter $\epsilon_a = \epsilon_a(x)$ depends on spacetime. An important consequence of the dependence of the quantum field $\Psi$ on spacetime is that the derivative of the field is no longer gauge invariant. To obtain a gauge invariant derivative of a field, we introduce gauge covariant derivatives:

$$D_\mu = \partial_\mu + igA^a_\mu,$$

where a vector field $A^a_\mu$ and a coupling constant $g$ have been introduced (provided that this new field transforms as: $A^a_\mu \rightarrow A^a_\mu - \frac{1}{g} \partial_\mu \epsilon(x)$). This allows the covariant derivative $D_\mu \Psi$ to remain invariant under gauge transformations. The vector fields $A^a_\mu$, which were introduced, are known as vector bosons, or gauge bosons. For each gauge group in the SM we need to introduce a vector field to ensure the gauge invariance of the whole theory. In addition, each group transformation has an associated coupling constant, which allows the gauge fields to mediate forces between the fields. The vector bosons can then be written in terms of the generators ($T^a$) of the group as: $A^a_\mu = A^a_\mu T^a$. The SM contains the following vector bosons and coupling constants:

$$SU(3)_C : G^a_\mu, \quad a = 1, 2, \ldots, 8,$$
$$SU(2)_L : W^a_\mu, \quad a = 1, 2, 3,$$
$$U(1)_Y : B_\mu.$$  \hspace{1cm} (1.10)
The coupling constant for the strong interactions is $\alpha_S$ while for the electromagnetic and weak interactions is given by $\alpha_{EM}$ and $\alpha_W$, respectively. The vector bosons which mediate the strong interactions are the eight gluons $G^a_\mu$. The vector bosons responsible for electroweak interactions are the three $W$ bosons $W^a_\mu$ and the hypercharge boson $B_\mu$.

1.2 BEH Mechanism and symmetry breaking

The BEH mechanism [4, 5] is the process by which all particles acquire mass. It describes a mechanism where a field with a non-zero ground state becomes the source for all particles to gain mass. The mechanism assumes that the spin-zero Higgs field permeates space-time, which is a doublet in $SU(2)$ and with a non-zero $U(1)$ hypercharge. The BEH mechanism introduces $\Phi$ with a non-zero vacuum expectation value ($vev$) $v$. $\Phi$ is a complex scalar field which has the form:

$$\Phi = \left[ \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right] = \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{array} \right].$$

The BEH mechanism describes how the electroweak symmetry in the SM is broken to the electromagnetic symmetry. This mechanism is known experimentally as Electroweak Symmetry Breaking (EWSB). This mechanism allows the fermions and the massive gauge bosons to acquire mass. In the SM, the mechanism for electroweak symmetry breaking postulates the existence of the (spin = 0) Higgs field $\Phi$. The Higgs field permeates space-time where it has a non-zero value even in its ground state. Before EWSB, the mass terms for leptons, quarks and vector bosons are forbidden since they are not invariant under gauge transformations. The SM massive particles acquire their masses when they interact with the Higgs field.

Both the gauge bosons and fermions can interact with this field, and due to that interaction, they acquire mass. The mechanism relies on the non-zero ground state value of the Higgs field which breaks the local gauge invariant $SU(2)_L \otimes U(1)_Y$ symmetry to the electromagnetic gauge group $U(1)_{EM}$. To retain the symmetries of the Lagrangian, we can only add $SU(2)_L \otimes U(1)_Y$ multiplets. To ensure that the hypercharge $Y = +1$, the upper and lower components of the doublet must have
specific values for the electric charge. We then add the potential \( V(\Phi) \) for the Higgs field so that the symmetry of the Lagrangian will be spontaneously broken; see Fig. 1.3. The Lagrangian for the scalar field is given by:

\[
L_{\text{scalar}} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - V(\Phi),
\]  

(1.12)

The symmetry of the potential \( V(\Phi) \) results in an infinite number of degenerate states satisfying \( \Phi^\dagger \Phi = v^2 \) at the minimum energy of the potential [8]. Since \( V(\Phi) \) depends only on the term \( \Phi^\dagger \Phi \), we can choose the unitary gauge: \( \phi_1 = \phi_2 = \phi_4 = 0 \) and \( \phi_3 > 0 \). In the unitary gauge we set \( \phi_3 = h \), where \( h \) is the physical Higgs scalar. We shift \( \phi_3 \) by the vev \((v)\) to give us: \( \phi_3 = v + h \). In the unitary gauge the scalar doublet can be written as follows:

\[
\text{Vacuum} = \langle \phi_0 \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix}.
\]  

(1.13)

In order to conserve electric charge, only a neutral scalar field can acquire a vev. The vacuum we defined above conserves electric charge since \( I = \frac{1}{2} \), \( I_3 = -\frac{1}{2} \) and if we choose \( Y = +1 \), we find \( Q = I_3 + \frac{1}{2} Y = 0 \). Choosing this vacuum will break the \( SU(2)_L \otimes U(1)_Y \) gauge group but leave \( U(1)_{EM} \) invariant. This leads to massive gauge bosons \( W^\pm \) and \( Z \) and a massless photon \( A \). The vev of the Higgs field breaks the electroweak gauge group down to the electromagnetic gauge group:

\[
SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{EM}.
\]  

(1.14)

We can check which symmetries associated to the gauge bosons are broken by analysing the action of the generators on the SM gauge group \( SU(2)_L \):

\[
\begin{align*}
\tau_1 \phi_0 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} = + \frac{1}{\sqrt{2}} \begin{bmatrix} v + h \\ 0 \end{bmatrix} \neq 0 \longrightarrow \text{broken}, \\
\tau_2 \phi_0 &= \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} = -i \frac{1}{\sqrt{2}} \begin{bmatrix} v + h \\ 0 \end{bmatrix} \neq 0 \longrightarrow \text{broken}, \\
\tau_3 \phi_0 &= \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} = -\frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} \neq 0 \longrightarrow \text{broken}, \\
Y \phi_0 &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix} \neq 0 \longrightarrow \text{broken}.
\end{align*}
\]  

(1.15)

The Goldstone theorem states that the spontaneous breaking of a Lagrangian symmetry will produce massless excitations corresponding to the broken symmetry [8]. The invariance of a Lagrangian under a continuous symmetry group \( O \) while it’s vacuum is only invariant under a subgroup of \( O \) will result in massless particles (as many as the number of the broken generators). Each broken generator causes one degree of freedom to be removed. In the SM, we had in total four generators for the direct product \( SU(2)_L \otimes U(1)_Y \). After EWSB, four degrees of freedom were removed and we are only left with one unbroken generator. The degrees of freedom removed become longitudinal components (Goldstone scalars) which are absorbed by the massless gauge bosons to become massive. This results in all the four gauge bosons \((W_1, W_2, W_3 \text{ and } B^{\mu \nu})\) acquiring mass through the BEH mechanism. After rotation, the four gauge bosons \((W_1, W_2, W_3 \text{ and } B^{\mu \nu})\) become the massive vector bosons \( W^\pm, Z \) and the massless photon \( A \). The requirement of a massless photon demands the \( U(1)_{EM} \) symmetry conserve the invariance of the vacuum. The
Section 1.3. Mass terms for the Higgs and gauge bosons

\(U(1)_{EM}\) symmetry is conserved because the vacuum is neutral as can be shown:

\[
U(1)_{EM} : Q\phi_0 = \frac{1}{2}(\tau_3 + Y) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ v + h \end{bmatrix} = 0 \rightarrow \text{unbroken.} \tag{1.16}
\]

The generator of \(U(1)_{EM}\) symmetry relates the electric charge of the resulting \(U(1)_{EM}\) group to the third generator of \(SU(2)_L\) and the charge of \(U(1)_Y\).

1.3 Mass terms for the Higgs and gauge bosons

We can derive the gauge bosons masses from the kinetic term of the scalar Lagrangian: \(L_{\text{scalar}} = (D^\mu \Phi)\dagger (D_\mu \Phi) - V(\Phi)\). The Higgs boson mass and the Higgs self-interactions are derived from the potential term: \(V(\Phi)\). The covariant derivative of the complex doublet is given by:

\[
D_\mu \Phi = [\partial_\mu + ig_2 \frac{1}{2} \tau_i W_i^{\mu\nu} + i g_B \frac{1}{2} Y B^{\mu\nu}] \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix}. \tag{1.17}
\]

The masses of the gauge bosons and their interaction with the Higgs boson are derived from the \((D^\mu \Phi)\dagger (D_\mu \Phi)\) terms. Substituting the Higgs field \(vev\) into the kinetic term gives us the masses of the gauge bosons, where

\[
D_\mu \Phi = \frac{i}{\sqrt{8}} [g W (W_1^{\mu\nu} + W_2^{\mu\nu}) + g B Y B^{\mu\nu}] \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + h \end{bmatrix}. \tag{1.18}
\]

After simplifying the above expressions, we get:

\[
(D^\mu \Phi)\dagger = \frac{i}{\sqrt{8}} (g W (W_1 + i W_2), (-g W W_3 + g B Y B^{\mu\nu})). \tag{1.19}
\]

This gives us the following expression for the kinetic term:

\[
(D^\mu \Phi)\dagger (D_\mu \Phi) = \frac{1}{8} \mu^2 [g_W^2 (W_1^2 + W_2^2) + (-g_W W_3 + g_B Y B^{\mu\nu})^2]. \tag{1.21}
\]

We now rewrite \(W_1, W_2, W_3\) and \(B^{\mu\nu}\) in terms of the physical gauge bosons \(W^\pm, Z\) and the photon \(A\), since these are the gauge bosons that are observed in experiments. We can rewrite \(W_1\) and \(W_2\) using \(W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp i W_2)\). If we look at the terms containing \(W_1\) and \(W_2\) in the Lagrangian, we find that:

\[
g_W^2 (W_1^2 + W_2^2) = g_W^2 (W^+^2 + W^-^2). \tag{1.22}
\]

The vector bosons \(W_3\) and \(B^{\mu\nu}\) combine to form the neutral gauge bosons \(Z\) and the photon \(A\). Expanding the \((-g_W W_3 + g_B Y B^{\mu\nu})^2\) term, we obtain:

\[
(-g_W W_3 + g_B Y B^{\mu\nu})^2 = (W_3, B^{\mu\nu}) \left( \begin{array}{cc} g_W^2 & -g_W g_B Y \\ -g_W g_B Y & g_B^2 \end{array} \right) \left( \begin{array}{c} W_3 \\ B^{\mu\nu} \end{array} \right). \tag{1.23}
\]
In the following calculations we use $Y = 1$. We can relate the photon and the $Z$ gauge boson to the vector bosons, $W_3$ and $B^{\mu\nu}$, by the relations:

$$A = \cos \theta_W B^{\mu\nu} + \sin \theta_W W_3,$$

$$Z = -\sin \theta_W B^{\mu\nu} + \cos \theta_W W_3,$$

where $\theta_W$ is the Weinberg angle (which describes the mixing of the $W_3$ and $B^{\mu\nu}$ gauge fields to produce the $Z$ boson and photon $A$):

$$\sin \theta_W = \frac{g_B}{\sqrt{g_W^2 + g_B^2}}, \quad \cos \theta_W = \frac{g_W}{\sqrt{g_W^2 + g_B^2}}.$$  \hspace{1cm} (1.24)

The mass terms of the gauge bosons can be obtained by rewriting the kinetic term in terms of the physical fields:

$$(D^\mu \Phi)^\dagger (D_\mu \Phi) = \frac{1}{8} v^2 [g_W^2 (W^+)^2 + g_W^2 (W^-)^2 + (g_W^2 + g_B^2) Z^2].$$

Thus, the mass terms for the massive gauge bosons are:

$$M_{W^\pm} = \frac{1}{2} v g,$$

$$M_Z = \frac{1}{2} v \sqrt{(g_W^2 + g_B^2)},$$

$$M_A = 0.$$  \hspace{1cm} (1.27)

We cannot predict the Higgs boson mass in the SM since $\lambda$ is a free parameter. The mass term for the Higgs boson is given by:

$$M_h = \sqrt{2 \lambda v^2}.$$  \hspace{1cm} (1.28)

### 1.4 The Madala hypothesis

Although the SM particle spectrum is now complete, there are continuous research efforts for new Higgs bosons at the LHC. The Madala hypothesis extends the SM Higgs sector by introducing two new scalars that are heavier than the SM Higgs boson $h$. A new heavy scalar $H$ (the Madala boson) was proposed to explain several anomalous features in the LHC Run 1 data [10, 11] and Run 2 data [12, 13]. The motivation to introduce a new boson stemmed from the four groups of excesses that included the Higgs boson $p_T$, the limits on the production of $hh$, the production of multiple leptons in the search for $tth$ and the $VV$ invariant mass spectrum [10, 11, 12, 13, 14, 15, 16]. The full Lagrangian for the model consists of the SM Lagrangian $L_{SM}$ in addition to the BSM Lagrangian $L_{BSM}$:

$$L = L_{SM} + L_{BSM},$$

where all of the new interactions and states are described by the BSM Lagrangian: $L_{BSM} = L_H + L_Y + L_T + L_Q$, where $L_H, L_Y, L_T$ and $L_Q$ are the Higgs, Yukawa, trilinear and quartic interactions, respectively [10]. The postulated heavy boson, $H$, was considered to have couplings to the SM particles. The dominant decay of $H$ is: $H \rightarrow Sh$, where $S$ is a new Higgs-like scalar. Therefore, it was only considered in
Section 1.4. The Madala hypothesis

Figure 1.4: Feynman diagrams representing the decay of $H$. The left diagram is due to the quartic coupling $\lambda_{hH\chi\chi}$ while the right diagram represents the addition of the scalar singlet $S$ [11].

The mass range $2m_h < m_H < 2m_t$, since if it were heavier it would be dominated by $H \rightarrow t\bar{t}$ decays due to the strong Yukawa coupling and if it was lighter the resonant di-Higgs decays would be forbidden. In [10, 11, 15], the formalism was expanded in order to include the phenomenology of an intermediate scalar that would explain the coupling of the heavy boson to DM and the SM Higgs boson. In this extended framework, the heavy boson would also decay into the SM Higgs boson and the intermediate scalar, as well as into two intermediate scalars.

A scalar DM mediator $S$ was introduced to explain the nature of the effective vertex [11], and the decays of $H$ were such that the dominant channels are $H \rightarrow Sh, hh$. This results in the production of a Higgs boson with missing energy through an effective vertex Fig. 1.4.

The parameters of the model were constrained further in order to calculate the minimised $\chi^2$ of the anomalous data in the LHC run 1 against $\chi^2$ of the postulated heavy boson model, Fig. 1.5. The best fit to the heavy Higgs boson obtained was $m_H = 272^{+12}_{-9}$ GeV [10]. The scalar singlet $S$ has a mass in the range $m_h < m_S < m_H - m_h$, such that it is more kinematically accessible through the decays of $H$ as mentioned above. It was also considered that $S$ is allowed to couple with all of the SM particles as well as a dark matter candidate $\chi$, since it has Higgs-like couplings to the SM [10, 11, 15]. Since $S$ has Higgs-like decay modes, it would then decay predominantly to the vector bosons $Z$ and $W^\pm$ if it has a mass of around 150 GeV or higher.

The Madala hypothesis can investigate, with regards to several ATLAS and CMS measurements, the di-lepton invariant mass spectra. In [10] an excess in di-lepton plus jets and missing transverse energy was predicted, see Fig. 1.6. The available data from the ATLAS and CMS experiments display this excess. In [12], the focus of the study was the decay of the scalar boson $S$ to di-lepton final states. In order to understand the excess in the di-lepton results, the BSM di-leptons signal was compared to the ATLAS and CMS data. The best fit event yield was calculated by varying the BSM normalisation and fitting the SM+BSM prediction to the data, using $\chi^2$ defined in [12]. The scale factor (the combined production strength of $H$: $\beta^2_\eta$ for different datasets) is used to measure the performance of the BSM signal. The minimised value of $\chi^2$ has the lowest value of 0.72 per degree of freedom in the fit. The combined $\beta_\eta^2$ calculated from the di-leptons invariant mass of $H \rightarrow Sh$ signal, with ATLAS and CMS data, is found to be $\beta_\eta^2 = 1.22 \pm 0.38$ [12]. A combined fit was performed with a common value of $\beta_\eta^2$. This is done in order to test the ability of the simplified model to describe the data. This corresponds to a significance of $3.2\sigma$.
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Figure 1.5: Plot of the minimised $\chi^2$ values as a function of $m_H$ [10].

Figure 1.6: Distributions of the ATLAS data and SM background comparing to the BSM signal for the di-lepton invariant mass with at least one b-tagged jet [12].
for $m_H = 270$ GeV and $m_S = 150$ GeV \cite{12}. This result is in good agreement with the scale factor extracted from the collected data so far. In \cite{12}, the combined $\beta_g^2$ values for the Higgs boson signal strength and leptons with b-tagged jets are $\beta_g^2 = 1.69 \pm 0.54$ and $\beta_g^2 = 1.22 \pm 0.38$, respectively. Combining these results together corresponds to $6.3\sigma$.

1.5 The Madala hypothesis in theories beyond the Standard Model

The Madala hypothesis introduces new Higgs bosons that do not exist in the SM. There are many possible theoretical models which could accommodate additional scalars. The excesses seen in the ATLAS and CMS data could be explained by adding a heavier boson $H$ and a scalar singlet $S$ in a model with an extended Higgs sector. In chapter 2, we will study Left-Right Symmetric Models (LRSMs) and investigate if they can accommodate the new postulated heavy bosons. We have chosen this model because it has an attractive left-right symmetry where we have mirror gauge bosons $W_{LR}^\pm$ and $Z_R$, as well as additional Higgs bosons in the Higgs sector. The next model we will study is the Two-Higgs Doublet Model plus scalar singlet (2HDM+S). The scalar singlet field is a singlet under the SM gauge group \cite{11}. This model contains five neutral Higgs bosons, which may include the heavy boson $H$. 
In this chapter we study the roles of global and discrete symmetries to suppress Flavor Changing Neutral Currents (FCNCs) in LRSMs. We study the roles of global and discrete symmetries in constraining the mass of the heavy boson $H$ to $m_H \sim 270$ GeV, to fit the Madala hypothesis.

We structure this chapter as follows: In section 2.1 we give an overview of LRSMs. In section 2.3 we discuss tree-level FCNCs in LRSMs. In section 2.5 we discuss the role of global symmetries in suppressing FCNCs. In section 2.6 we discuss discrete symmetries and explain their relation to the FCNC problem. In section 2.7 we summarise the results of the study.

### 2.1 Overview of Left-Right Symmetric Models

A good candidate BSM to search for the postulated Higgs bosons $H$ and $S$ is LRSMs [17]. The LRSMs gauge group is an extension of the SM gauge group by adding an $SU(2)$ right handed doublet field given as: $G_{LRSMs} \equiv SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$. The right handed doublet field $SU(2)_R$ is introduced to restore the parity symmetry [18]. This left-right symmetry is then broken spontaneously to the SM gauge group [19].

A FCNC allows a fermion current to change its flavor without changing its electric charge. In the SM, FCNCs do not exist at tree-level and are suppressed at loop-level by the Glashow–Iliopoulos–Maiani (GIM) mechanism [20]. The GIM mechanism explains why processes that change strangeness by 2 units ($\Delta S = 2$ transitions) are suppressed. Due to the enlarged Higgs sector of LRSMs, neutral Higgs bosons can mediate FCNCs at tree-level and give contributions to neutral Kaon meson mixings (neutral Kaon and neutral anti-Kaon oscillation) larger than the observed values [19]. The experimentally observed neutral Kaon meson mixing, $\Delta^{exp}_{m_K}$, places a constraint on neutral Higgs-Kaon meson mixing $\Delta^{H_0}_{m_K}$. This constrains the mediating Higgs bosons masses to be of the $O$(TeV) [17]. LRSMs were motivated to address some of the shortcomings in the SM, such as the asymmetry between left-right representations and the source of parity violation in weak interactions. This has led physicists to introduce a theory that has left-right chiral symmetry. The left-right
chiral symmetry extends the SM gauge group by introducing the right-handed doublet $SU(2)_R$ [18]. In LRSMs, parity violation occurs when the left-right symmetry is broken at a very high energy scale, which results in the parity asymmetry observed at and below the electroweak scale.

LRSMs offer the possibility of restoring left-right symmetry, which is then spontaneously broken to the SM gauge group. The extended symmetry in the electroweak sector introduces new phenomenology and several interesting theoretical features. The original hypercharge quantum number $Y$ of the SM is modified to the baryon minus lepton quantum number ($B-L$), which is conserved at a high energy scale and breaks down to the SM hypercharge gauge symmetry at the electroweak scale. The electromagnetic charge $Q$ is described by the Gell-Mann-Nishijima formula:

$$Q = T_3^L + T_3^R + \frac{Y_{B-L}}{2}, \quad (2.1)$$

where $T_3^L$ and $T_3^R$ are the third generators of $SU(2)_L$ and $SU(2)_R$ respectively. The generators of $SU(2)_L$ and $SU(2)_R$ are given by $T_i = \frac{1}{2} \tau_i$, where $\tau_i$ are the Pauli matrices. The action of the electromagnetic charge matrix $(Q)$ on the LRSMs multiplets gives the electric charges of the fields. Since fermions have known values of electric charge, we can use Eq. 2.1 to find the values of $Y_{B-L}$.

Leptons have $Y_{B-L} = -1$ and quarks have $Y_{B-L} = \frac{1}{3}$. In LRSMs, the $U(1)$ generator has a physical interpretation as the $B-L$ quantum number. This introduces an attractive physical interpretation for the charge as compared to the SM, which lacked any physical meaning for the hypercharge $U(1)$. The non-zero small neutrino mass can also have a physical explanation in the framework of LRSMs. In LRSM, the seesaw mechanism [19] introduces a very heavy right-handed neutrino, which produces a very light left-handed neutrino.

In order to achieve a left-right symmetry, the LRSMs Lagrangian must be invariant under the (discrete) left-right symmetry:

$$\Psi_L \leftrightarrow \Psi_R, \quad \Delta_R \leftrightarrow \Delta_L, \quad \Phi \leftrightarrow \Phi, \quad (2.2)$$

where $\Delta_L$ and $\Delta_R$ are the left-handed and right-handed triplet fields, respectively. The LRSMs Lagrangian consists of four parts:

$$L_{LRSMs} = L_{kinetic} + L_{gauge} + L_{Higgs} + L_{Yukawa}. \quad (2.3)$$

The $L_{kinetic}$ term describes fermion and gauge boson interactions which are invariant under the LRSMs gauge group. The kinetic term for fermions have the following form:

$$L_{kinetic} = i \sum \bar{\Psi} \gamma^\mu D_\mu \Psi,$$

$$= L_L \gamma^\mu (i\partial_\mu + ig_L \frac{1}{2} \tau_i W^{\mu \nu}_{Li} - ig_B \frac{1}{2} B^{\mu \nu}) L_L$$

$$+ L_R \gamma^\mu (i\partial_\mu + ig_R \frac{1}{2} \tau_i W^{\mu \nu}_{Ri} - ig_B \frac{1}{2} B^{\mu \nu}) L_R$$

$$+ Q_L^\alpha \gamma^\mu (i\partial_\mu + ig_L \frac{1}{2} \tau_i W^{\mu \nu}_{Li} + ig_B \frac{1}{6} B^{\mu \nu})_{\alpha \beta} + gG \frac{1}{2} G^{\mu \nu}_{\alpha} Q_L^\beta$$

$$+ Q_R^\alpha \gamma^\mu (i\partial_\mu + ig_R \frac{1}{2} \tau_i W^{\mu \nu}_{Ri} + ig_B \frac{1}{6} B^{\mu \nu})_{\alpha \beta} + gG \frac{1}{2} G^{\mu \nu}_{\alpha} Q_R^\beta, \quad (2.4)$$
where $L_{L,R}$ and $Q_{L,R}$ are the left-handed and right-handed leptons and quarks fields, respectively. The $L_{\text{gauge}}$ term contains the kinetic terms for the gauge fields and the interactions between them:

$$L_{\text{gauge}} = - \frac{1}{4} W^{\mu
u}_{Li} W^{\mu
u}_{Li} - \frac{1}{4} W^{\mu
u}_{Ri} W^{\mu
u}_{Ri} - \frac{1}{4} G^{\mu\nu}_{a} G_{\mu\nu}^{a} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu}, \quad (2.6)$$

where $W^{\mu}$, $G^{\mu\nu}$ and $B^{\mu\nu}$ are the field strength tensors of the SU(2)$_{L,R}$, SU(3)$_{C}$ and U(1)$_{B-L}$ gauge fields respectively. They can be defined as follows:

$$G^{\mu\nu}_{a} = \partial^{\mu} G^{\nu}_{a} - \partial^{\nu} G^{\mu}_{a} - g_{G} f^{abc} G^{\mu}_{b} G^{\nu}_{c},$$

$$W^{\mu\nu}_{Li} = \partial^{\mu} W^{\nu}_{Li} - \partial^{\nu} W^{\mu}_{Li} + g_{L} \varepsilon^{ijk} W^{\mu}_{Lj} W^{\nu}_{Lk},$$

$$W^{\mu\nu}_{Ri} = \partial^{\mu} W^{\nu}_{Ri} - \partial^{\nu} W^{\mu}_{Ri} + g_{R} \varepsilon^{ijk} W^{\mu}_{Rj} W^{\nu}_{Rk},$$

$$B^{\mu\nu} = \partial^{\mu} B^{\nu} - \partial^{\nu} B^{\mu}, \quad (2.7)$$

where $f^{abc}$ and $\varepsilon^{ijk}$ are the structure constants of the SU(3)$_{C}$ and SU(2) groups, respectively. The $L_{\text{Yukawa}}$ term describes the Yukawa interactions, which consist of the most general possible couplings of the Higgs scalars to the bilinear fermion fields:

$$L_{\text{Yukawa}} = - \sum_{i=1}^{3} c(H_{i} f f) y_{ij} \bar{\psi}_{j} \Phi_{i} \psi_{f} + h.c.,$$

$$= - c(H_{i} f f) \left[ y_{ij}^{L} L_{i} \Phi_{1} L_{R} + y_{ij}^{Q} Q_{L} \Phi_{1} Q_{R}^{i} + y_{ij}^{d} Q_{L} \Phi_{1} Q_{R}^{i} \right] \quad (2.8)$$

$$+ \left[ y_{ij}^{L} Q_{L} \Phi_{2} L_{R} + y_{ij}^{Q} Q_{L} \Phi_{2} Q_{R}^{i} + y_{ij}^{d} Q_{L} \Phi_{2} Q_{R}^{i} \right] + \left[ y_{ij}^{Q} L_{i} \tau_{2} \Delta_{R} L_{R} \right] + h.c,$$

where $\Phi = \tau_{2} \Phi^{*} \tau_{2}$ and $y_{ij}^{i,u,ui,d}$ are $3 \times 3$ Yukawa matrices in flavor space. The Yukawa Lagrangian couples the scalar field to a left-handed fermion field and a right-handed fermion field. If the scalar field acquires a non-zero vev, then when this vev is substituted into the Yukawa Lagrangian, it results in mass terms for the fermions. The SM massive particles only have Dirac mass terms, since all the charged fermions are distinct from their anti-particles. The Higgs Lagrangian term $L_{\text{Higgs}}$ contains the potential term as well as the Higgs kinetic terms of the Higgs fields [21]:

$$L_{\text{Higgs}} = \sum_{i} [\text{Tr}[D_{\mu} X_{i}]^{2}] - V_{LRSMs}, \quad (2.9)$$

where $V_{LRSMs}$ is the LRSMs scalar potential and $X_{i} = \Phi, \Delta_{L}, \Delta_{R}$. It consists of the covariant derivatives of the three scalar fields, which give the kinetic terms for the scalars and interactions between the scalar and gauge fields. When the scalar vevs are substituted into $L_{\text{Higgs}}$, the gauge bosons acquire mass terms. Under the SU(2)$_{L} \otimes SU(2)_{R} \otimes U(1)_{B-L}$ symmetry, the covariant derivatives for the Higgs fields are given by:

$$D_{\mu} \Phi = \partial_{\mu} \Phi - i \frac{g_{L}}{2} (\sigma_{\nu} W^{\nu}_{Li}) \Phi + i \frac{g_{R}}{2} \Phi (\sigma_{\nu} W^{\nu}_{Ri}),$$

$$D_{\mu} \Delta_{L,R} = \partial_{\mu} \Delta_{L,R} - i \frac{g_{L,R}}{2} W^{\nu}_{L,R} \left[ \sigma_{\nu}, \Delta_{L,R} \right] - i g_{B} B^{\mu\nu} \Delta_{L,R}, \quad (2.10)$$

Quarks and leptons are left-right symmetric in LRSMs [22], as they are represented by left and right handed doublets:

$$Q_{L} = \begin{pmatrix} u \\ d \end{pmatrix}_{L} \sim [3, 2, 1, \frac{1}{3}], \quad Q_{R} = \begin{pmatrix} u \\ d \end{pmatrix}_{R} \sim [3, 1, 2, \frac{1}{3}],$$

$$L_{L} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{L} \sim [1, 2, 1, -1], \quad L_{R} = \begin{pmatrix} \nu \\ e \end{pmatrix}_{R} \sim [1, 1, 2, -1].$$
The numbers in the square brackets denote the quantum numbers under the groups $SU(3)_C, SU(2)_L, SU(2)_R$ and $U(1)_{B-L}$, respectively. LRSMs have an extended Higgs sector which consists of the bi-doublet $\Phi$ and left and right triplets $\Delta_{L,R}$:

$$\Phi = \begin{bmatrix} \Phi_1^0 & \Phi_1^+ \\ \Phi_2^- & \Phi_2^0 \end{bmatrix} \equiv [1, 2, 2, 0], \quad \Delta_{L,R} = \begin{bmatrix} \frac{\Delta_{L,R}^+}{\sqrt{2}} & \Delta_{L,R}^{++} \\ \Delta_{L,R}^0 & -\frac{\Delta_{L,R}^+}{\sqrt{2}} \end{bmatrix} \equiv [1, 3(1), (1)3, 2].$$  

(2.11)

To spontaneously break the symmetries, we must choose the appropriate vacuum structure of $\Phi$ and $\Delta_{L,R}$. Since our goal is to conserve the electric charge $Q$ of $U(1)_{EM}$, the neutral Higgs field’s components must gain the vev of the bi-doublet and triplet fields given by $\langle \Phi_1^0 \rangle = v_1$, $\langle \Phi_2^0 \rangle = v_2$ and $\langle \Delta_{L,R}^0 \rangle = v_{L,R}$, respectively:

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}, \quad \langle \Delta_{L,R} \rangle = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 \\ v_{L,R} & 0 \end{bmatrix}.$$  

(2.12)

When the bidoublet acquires vevs for the neutral fields, the fermions acquire their masses via their couplings to the fermion bilinears $\bar{\Psi}_L \Psi_R$ and $\bar{\Psi}_R \Psi_L$, ($\Psi =$ quarks and leptons fields):

$$L_{fermions} = -m \bar{\Psi} \Psi = -m (\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L).$$

LRSMs include right handed neutrinos, which are added as a result of the parity symmetry between left and right particles [14].

### 2.2 BEH mechanism and symmetry breaking

The bidoublet is not sufficient to break the left-right symmetry, as it has the quantum number $B - L = 0$. To break the left-right symmetry of LRSMs, we require the two Higgs triplets $\Delta_{L,R}$ (since the $B - L$ quantum number of $\Delta_{L,R}$ equals two), to break the $B - L$ symmetry:

$$B - L < \Delta_{L,R} >= 2 \lessgtr 2 < \Delta_{L,R} > \neq 0.$$  

(2.13)

Unlike the SM, symmetry breaking occurs in two stages. The first stage takes place at a high energy scale above the electroweak scale, where the LRSMs gauge group is broken to the SM gauge group by the vev of the neutral component of the Higgs triplet $\Delta_R$. At the second stage of EWSB, the vevs of the bi-doublet Higgs field $\Phi$ breaks the SM electroweak gauge group to $U(1)_{EM}$:

$$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \rightarrow SU(2)_L \otimes U(1)_Y,$$

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{EM}.$$  

(2.14)
The most general scalar potential which is invariant under the left-right symmetry $\Delta_L \leftrightarrow \Delta_L$ and $\Phi \leftrightarrow \Phi^\dagger$ of the Higgs fields is:

\[
V_{LRSM_s} = - \mu_2^2 (Tr[\Phi^\dagger \Phi]) - \mu_2^2 (Tr[\Phi^\dagger \Phi]) + Tr[\Phi^\dagger \Phi])
- \mu_3^2 (Tr[\Delta_L \Delta_L^\dagger] + Tr[\Delta_R \Delta_R^\dagger])
+ \lambda_1 ((Tr[\Phi^\dagger \Phi])^2 + \lambda_2 ((Tr[\Phi^\dagger \Phi])^2 + (Tr[\Phi^\dagger \Phi])^2)
+ \lambda_3 (Tr[\Phi^\dagger \Phi])Tr[\Phi^\dagger \Phi])
+ \rho_1 ((Tr[\Delta_L \Delta_L^\dagger])^2 + (Tr[\Delta_R \Delta_R^\dagger])^2)
+ \rho_2 (Tr[\Delta_L \Delta_L]Tr[\Delta_L^\dagger \Delta_L^\dagger] + Tr[\Delta_R \Delta_R]Tr[\Delta_R^\dagger \Delta_R^\dagger])
+ \rho_3 (Tr[\Delta_L \Delta_L]Tr[\Delta_R^\dagger \Delta_R^\dagger]
+ \rho_4 (Tr[\Delta_L \Delta_L]Tr[\Delta_R \Delta_R^\dagger] + Tr[\Delta_R^\dagger \Delta_R^\dagger]Tr[\Delta_R \Delta_R])
+ \alpha_1 (Tr[\Phi^\dagger \Phi]) (Tr[\Delta_L \Delta_L^\dagger] + Tr[\Delta_R \Delta_R^\dagger])
+ \alpha_2 (Tr[\Phi^\dagger \Phi]) (Tr[\Delta_R \Delta_R^\dagger] + Tr[\Delta_R \Delta_R^\dagger]Tr[\Delta_L \Delta_L])
+ \alpha_3 (Tr[\Phi^\dagger \Phi] (Tr[\Delta_L \Delta_L^\dagger] + Tr[\Phi^\dagger \Phi] (Tr[\Delta_L \Delta_L^\dagger])
+ \beta_1 (Tr[\Phi^\dagger \Phi]) (Tr[\Phi^\dagger \Phi] (Tr[\Delta_R \Delta_R^\dagger] + Tr[\Phi^\dagger \Phi]) (Tr[\Delta_L \Delta_L^\dagger])
+ \beta_2 (Tr[\Phi^\dagger \Phi] (Tr[\Phi^\dagger \Phi] (Tr[\Delta_R \Delta_R^\dagger] + Tr[\Phi^\dagger \Phi] (Tr[\Delta_R \Delta_R^\dagger]),
\]

where $\mu_i^2$ are mass parameters and $\lambda_i, \rho_i, \alpha_i, \beta_i$ are dimensionless couplings. All the parameters in the LRSMs potential are real due to the left-right symmetry, with the exception of $\alpha_2$. In our study, we assume that CP is explicitly conserved in the potential so that $\alpha_2$ is real. In the see-saw picture of neutrino mass, we can safely ignore $v_L$, which corresponds to $\beta = 0$. Small values for $v_L$ result in a light left-handed neutrino (which are proportional to $v_L$) less than $O(1)\text{eV}$. This results in $v_R$ to be at least of the $O(10^8)\text{GeV}$. A very large $v_R$ will result in large masses for the additional Higgs and gauge bosons of the $O(10^8)\text{GeV}$. In the limit $v_L = 0$ (corresponding to $\beta = 0$), the potential will have more symmetry, such as $\Delta_L \rightarrow -\Delta_L$. In this case the vev see-saw relation becomes:

\[
(2\rho_1\rho_3)v_L v_R = 0.
\]

We require $v_R$ to be non-zero, since the mass of the additional Higgs and gauge bosons is proportional to $v_R$. In addition, $v_R$ is required to be non-zero in order to break the $SU(2)_R$ gauge symmetry [19]. This leads us to set $v_L = 0$. When spontaneous symmetry breaking occurs, the potential will be at the minimum when we evaluate the Higgs fields using their vev. There are six minimization conditions, where two of the vevs are considered, a priori, to be complex ($v_2$ and $v_L$), see [23]:

\[
\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial v_L} = \frac{\partial V}{\partial v_R} = \frac{\partial V}{\partial Im v_2} = \frac{\partial V}{\partial Im v_L} = 0.
\]
Section 2.2. BEH mechanism and symmetry breaking

The first derivative constraints for the $\Phi_{1,2}$ are:

$$
\frac{\partial V}{\partial \phi_1} = v_1^2 \lambda_1 + 3 v_1^2 v_2 \lambda_4 + v_2^3 \lambda_4 + v_1 v_2^2 (\lambda_1 + 4 \lambda_2 + 2 \lambda_3) \\
+ v_1 \left[ -\mu_1^2 + \frac{(\alpha_1 v_2^2)}{2} + \beta_1 v_1 v_2 \right] \\
+ v_2 \left[ -2 \mu_2^2 + \alpha_2 v_1^2 + \frac{\beta_1 v_2 v_3}{2} + \alpha_2 v_2^2 \right],
$$

$$
\frac{\partial V}{\partial \phi_2} = v_2^3 \lambda_1 + 3 v_1 v_2^2 \lambda_4 + v_1^3 \lambda_4 + v_1^2 v_2 (\lambda_1 + 4 \lambda_2 + 2 \lambda_3) \\
+ v_1 \left[ -2 \mu_2^2 + \alpha_2 v_2^2 + \frac{\beta_1 v_1 v_2}{2} + \alpha_2 v_3^2 \right] \\
+ v_2 \left[ -\mu_1^2 + \frac{(\alpha_1 v_2^2)}{2} + \frac{(\alpha_3 v_2^2)}{2} + \beta_3 v_1 v_2 + \frac{(\alpha_1 v_2^2)}{2} + \frac{(\alpha_3 v_2^2)}{2} \right],
$$

(2.18)

where $\phi_{1,2}^c$ are the real components of $\Phi_{1,2}$. Using the relations above, we can solve for $\mu_1^2$ and $\mu_2^2$:

$$
\mu_1^2 = \frac{2 v_1 v_2 (\beta_2 v_1^2 - \beta_2 v_2^2) + (v_2^2 + v_1^2) (\alpha_1 k_- - \alpha_3 v_2^2)}{2 k_-^2} + (k_+^2 \lambda_1 + 2 v_1 v_2 \lambda_4),
$$

$$
\mu_2^2 = \frac{v_1 v_2 [\beta_1 k_+^2 - 2 v_1 v_2 (\beta_2 - \beta_3)] + (v_2^2 + v_1^2) (2 \alpha_2 k_- + \alpha_3 v_1 v_2)}{4 k_-^2}
$$

+ $v_1 v_2 (2 \lambda_3 + \lambda_3) + \lambda_4 k_+^2$, (2.19)

where $k_\pm^2 = (v_1^2 \pm v_2^2)$. The first derivative constraints for the triplet fields $\Delta_{L,R}$ are:

$$
\frac{\partial V}{\partial \Delta_R} = \rho_3 v_2^2 v_R + \rho_1 v_1^2 + v_L \left[ \beta_2 v_1^2 + \frac{\beta_1 v_1 v_2}{2} + \frac{\beta_3 v_2^2}{2} \right] \\
+ v_R \left[ \frac{\alpha_1 v_2^2}{2} + 2 \alpha_2 v_1 v_2 + \frac{\alpha_1 v_2^2}{2} + \frac{\alpha_3 v_2^2}{2} - \mu_3^2 \right],
$$

(2.20)

$$
\frac{\partial V}{\partial \Delta_L} = \rho_3 v_2^2 v_L + \rho_1 v_1^2 + v_L \left[ \beta_2 v_1^2 + \frac{\beta_1 v_1 v_2}{2} + \frac{\beta_3 v_2^2}{2} \right] \\
+ v_L \left[ \frac{\alpha_1 v_2^2}{2} + 2 \alpha_2 v_1 v_2 + \frac{\alpha_1 v_2^2}{2} + \frac{\alpha_3 v_2^2}{2} - \mu_3^2 \right],
$$

where $\Delta_{L,R}^c$ are the real components of $\Delta_{L,R}$. Using the above relations, we can solve for $\mu_3^2$ and $\beta_2$ [18]:

$$
\mu_3^2 = \frac{\alpha_1 k_+^2 + 4 \alpha_2 v_1 v_2 + \alpha_3 v_2^2 + 2 \rho_1 (v_2^2 + v_1^2)}{2},
$$

(2.21)

$$
\beta_2 = \frac{-\beta_1 v_1 v_2 - \beta_3 v_2^2 + (2 \rho_1 - \rho_3) v_1 v_2}{v_1^2}.
$$

We construct the Higgs mass matrices from the bilinear terms obtained from expanding the potential about the vevs of the Higgs fields. We can determine the Higgs mass matrix $M_{ij}$ by:

$$
\frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \bigg|_{\Phi_i = \Phi_j = 0} = M_{ij}^2.
$$

(2.22)
First, we evaluate the real mass matrix components in the $\phi_1', \phi_2', \Delta_L', \Delta_R'$ basis. In order for the Higgs bosons to acquire positive mass, the mass matrices are required to have positive eigenvalues:

\[
M_{11}^r = \lambda_1(3v_1^2 + v_2^2) + 2v_2^2(2\lambda_2 + \lambda_3) + 6v_1v_2\lambda_4 - \mu_1^2 + \alpha_1 \frac{(v_L^2 + v_R^2)}{2} + \beta_2 v_L v_R, \\
M_{12}^r = M_{21}^r = 2v_1v_2(\lambda_1 + 4\lambda_2 + 2\lambda_3) + 3\lambda_4 k_+^2 - 2\mu_2^2 + \alpha_2 (v_L^2 + v_R^2) + \frac{(\beta_1 v_L + v_R)}{2}, \\
M_{13}^r = M_{31}^r = \frac{v_L(2\beta_2 v_1 + \beta_1 v_2)}{2} + v_R(\alpha_1 v_1 + 2\alpha_2 v_2), \\
M_{14}^r = M_{41}^r = \frac{v_R(2\beta_2 v_1 + \beta_1 v_2)}{2} + v_L(\alpha_1 v_1 + 2\alpha_2 v_2), \\
M_{22}^r = \lambda_1(v_1^2 + 3v_2^2) + 2v_1^2(2\lambda_2 + \lambda_3) + 6v_1v_2\lambda_4 - \mu_1^2 + (\alpha_1 + \alpha_2)(\frac{v_L^2 + v_R^2}{2}) + \beta_3 v_L v_R, \\
M_{23}^r = M_{32}^r = \frac{v_L(\beta_1 v_1 + 2\beta_3 v_2)}{2} + v_R[2\alpha_2 v_1 + v_2(\alpha_1 + \alpha_3)], \\
M_{24}^r = M_{42}^r = \frac{v_R(\beta_1 v_1 + 2\beta_3 v_2)}{2} + v_L[2\alpha_2 v_1 + v_2(\alpha_1 + \alpha_3)], \\
M_{33}^r = \alpha_1 k_+^2 + 4\alpha_2 v_1 v_2 + \frac{\alpha_3 v_2^2}{2} - \mu_3^2 + \rho_3 v_L^2 + 3\rho_1 v_R^2, \\
M_{34}^r = M_{43}^r = \beta_2 v_T^2 + \beta_1 v_1 v_2 + \frac{\beta_3 v_2^2}{2} + \rho_3 v_L v_R, \\
M_{44}^r = \alpha_1 k_+^2 + 4\alpha_2 v_1 v_2 + \frac{\alpha_3 v_2^2}{2} - \mu_3^2 + \frac{\rho_3 v_R^2}{2} + 3\rho_1 v_L^2, \\
\tag{2.23}
\]

where $M_{i,j}^r$ are the real components of $M_{i,j}$. We have rotated the mass matrices into the flavor-diagonal bases to enable us to change from the $(\phi_1', \phi_2', \Delta_L', \Delta_R')$ basis to the $(\phi_L', \phi_R', \Delta_L', \Delta_R')$ basis. We can accomplish this change of basis using the rotation matrix $R$:

\[
R = \begin{bmatrix}
v_1 & v_2 & 0 & 0 \\
-k_1 & k_2 & 0 & 0 \\
k_2 & -k_1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

To compute the real components of the mass matrix in the flavor-diagonal basis $(\phi_L', \phi_R', \Delta_L', \Delta_R')$, we substitute the first-derivative conditions and use the condition: $v_L = 0$. We rotate the mass matrix to the flavor-diagonal basis as: $\tilde{M}^r = R M^r R^T$, where $\tilde{M}^r$ is the mass matrix in the $(\phi_L', \phi_R', \Delta_L', \Delta_R')$ basis.
where $\tilde{M}^r$ is in the flavor-diagonal basis.

\[
\begin{align*}
\tilde{M}^r_{11} &= 2\lambda_1 k^2 \frac{8\nu_1 \nu_2 (2\lambda_2 + \lambda_3)}{k^2} + 8\nu_1 \nu_2 \lambda_4, \\
\tilde{M}^r_{12} &= \tilde{M}^r_{21} = 4\nu_1 \nu_2 \frac{k^2 (2\lambda_2 + \lambda_3)}{k^2} + 2\nu_1 \nu_2 \lambda_4, \\
\tilde{M}^r_{13} &= \tilde{M}^r_{31} = \alpha_1 \nu_R k^2 + \nu_2 \nu_R \frac{(4\alpha_2 \nu_1 + \alpha_3 \nu_2)}{k_+}, \\
\tilde{M}^r_{14} &= \tilde{M}^r_{41} = 0, \\
\tilde{M}^r_{22} &= (4\lambda_2 + 2\lambda_3) k^2 + \alpha_3 \nu_1 \nu_2 k^2, \\
\tilde{M}^r_{23} &= \tilde{M}^r_{32} = \nu_R (2\alpha_2 k^2 + \alpha_3 \nu_1 \nu_2), \\
\tilde{M}^r_{24} &= \tilde{M}^r_{42} = \nu_R k^2 (\beta_1 \nu_1 + 2\beta_3 \nu_2), \\
\tilde{M}^r_{33} &= 2\rho_1 \nu_R, \\
\tilde{M}^r_{34} &= \tilde{M}^r_{43} = 0, \\
\tilde{M}^r_{44} &= \frac{-\nu_1 \nu_2 (2\rho_1 - \rho_3)}{2}.
\end{align*}
\]  

(2.24)

After diagonalizing these matrices, we can obtain the physical Higgs masses. Diagonalizing the mass matrix $\tilde{M}^r$ (which consists of the real parts of the fields) in the $v_R \gg v_1, v_2 \gg v_1$ limit, gives four non-zero eigenvalues. The first eigenvalue gives us the physical mass of the SM Higgs boson $h$, which is the only Higgs boson that is not proportional to $v_R$. For the case $v_R \gg v_{1,2}$, the physical Higgs masses are:

\[
\begin{align*}
M^2_{H^0_0} &\approx \frac{4\nu_1 \nu_2 (2\lambda_1 + \lambda_3) + 2\nu_1 \nu_2 \lambda_4}{v^2}, & M^2_{H^0_1} &\approx \frac{1}{2} \alpha_3 \nu^2_R v_1^2 - v_1^2, \\
M^2_{H^0_2} &\approx 2\rho_1 \nu_R, & M^2_{H^0_3} &= \frac{1}{2} v^2_R (\rho_3 - 2\rho_1), \\
M^2_{A^0_1} &\approx \frac{\alpha_3 \nu^2_R}{2} v_1^2 - v_2^2, & M^2_{A^0_2} &= \frac{1}{2} v^2_R (\rho_3 - 2\rho_1), \\
M^2_{H^{\pm}_1} &\approx \frac{1}{4} \alpha_3 (v_1^2 - v_2^2), & M^2_{H^{\pm}_2} &\approx \frac{1}{4} \alpha_3 [(v_1^2 - v_2^2) + 2 \frac{v^2}{(v_1^2 - v_2^2)} v_R^2], \\
M^2_{H^{\pm}_1} &\approx \rho_2 v_R, & M^2_{H^{\pm}_2} &= \frac{v^2_R}{2} (\rho_3 - 2\rho_1). \\
\end{align*}
\]  

(2.25)

where $v$ is the SM vev. The scalar spectrum in the LRSMs consists of four singly charged ($H^+_1, H^+_2$), four doubly charged bosons ($H^{\pm\pm}$), and six neutral bosons ($H^0_0, H^0_1, H^0_2, H^0_3$) and ($A^0_1, A^0_2$). We consider $H^0_0$ as the SM Higgs boson, while the other neutral Higgs bosons are heavier bosons found in LRSMs. All the physical Higgs states, except $H^0_0$, are proportional to $v_R$ [23]. In LRSMs, we have additional right-handed gauge bosons $W_R^\pm$ and $Z_R$: 

\[
W_L^i = \begin{pmatrix} W^1_L \\ W^2_L \\ W^3_L \end{pmatrix}, \quad W_R^i = \begin{pmatrix} W^1_R \\ W^2_R \\ W^3_R \end{pmatrix}.
\]

$W_L^i$ transforms as a singlet under $SU(2)_{L,R}$ and vice versa for $W_R^i$. In LRSMs, $B^{\mu\nu}$ is the gauge boson corresponding to the $U(1)_{B-L}$ gauge group. The left-right
Section 2.3. Tree-level flavor changing neutral currents

In the SM, tree-level FCNCs are absent, since the $Z$ boson exchange is flavor-conserving. At loop-level, charged gauge bosons can mediate FCNCs in the SM. In LRSMs, if the two Higgs bi-doublets couple to the same quark field, they can mediate tree-level FCNCs. BSM contributions to FCNCs come from the extra gauge and Higgs bosons, Fig. 2.1. The charged and neutral Higgs bosons contribute to neutral meson mixings [24]. Neutral mesons such as Kaons and B mesons are known to oscillate between particle and anti-particle states, Figs. [2.2, 2.3]. Kaons carry strangeness of unit 1, while anti-Kaons carry strangeness of $-1$. The oscillations lead to a difference in strangeness by two units ($\Delta S = 2$). These oscillations are known as neutral meson mixing.

However, in LRSMs, we have a serious problem. Tree-level contributions to neutral meson mixings can occur via heavy neutral Higgs bosons [19]. These interactions violate flavor by two units and can bring unacceptably large contributions to Kaon meson mixing. In this work we will focus our attention on neutral Higgs boson contributions to Kaon meson mixing in LRSMs. This requires us to study the Higgs bi-doublet couplings to quarks. The most generic Yukawa interaction that is invariant separately under the $SU(2)_L$ and $SU(2)_R$ gauge groups is:

$$L_{Yukawa} = \bar{\Psi}^i_L \left( (Y)_{ij} \Phi + (\tilde{Y})_{ij} \tilde{\Phi} \right) \Psi^j_R + \text{h.c.}$$

(2.28)
The bi-doublet field $\Phi$, and its conjugate $\bar{\Phi}$, both couple to the quark doublet fields. When the second Higgs doublet couples to quark doublets, it can result in tree-level FCNCs, since its neutral component have off-diagonal couplings. Substituting the $v_1$ of the Higgs fields in Eq. 2.28, we obtain the quark mass matrices in the physical basis $M^u$ and $M^d$:

$$\frac{1}{\sqrt{2}}u_L^* V_L^u \left( Y v_1 + \bar{Y} v_2 \right) V_R^u u_R = u_L^* M^u u_R,$$
$$\frac{1}{\sqrt{2}} d_L^* V_L^d \left( \bar{Y} v_2 + \bar{Y} v_1 \right) V_R^d d_R = d_L^* M^d d_R. \quad (2.29)$$

We define the mass matrices in the physical basis, $M^u$ and $M^d$, by diagonalizing the quark mass matrices in the weak basis, $m^u$ and $m^d$ using the unitary matrices $V_{L,R}^u$ and $V_{L,R}^d$. We can solve $Y_1$ and $Y_2$ in terms of the physical masses of the up and down quarks and the unitary matrices $V_{L,R}^{u,d} [27]$:

$$Y = \sqrt{2} \left( v_1^* V_L^u M^u V_R^{u\dagger} - v_2^* V_L^d M^d V_R^{d\dagger} \right), \quad \bar{Y} = \sqrt{2} \left( -v_2 V_L^u M^u V_R^{u\dagger} + v_1 V_L^d M^d V_R^{d\dagger} \right),$$

where $v_\pm = |v_1|^2 \pm |v_2|^2$. To define the flavor changing and flavor conserving terms, we introduce the orthogonal neutral fields $\phi_0^+$ and $\phi_0^-$:

$$\phi_+^0 = \frac{1}{v_+^2} (-v_2^* \phi_1^0 + v_1 \phi_2^0), \quad \phi_-^0 = \frac{1}{v_+^2} (v_1^* \phi_1^0 + v_2 \phi_2^0). \quad (2.31)$$

The Higgs bi-doublet field couplings to quarks, and can be written in terms of $\phi_+^0$ and $\phi_-^0$:

$$\frac{\sqrt{2}}{v_+^2} u_L \left[ \phi_-^0 \frac{v_+^2}{v_+} M^u + \phi_+^0 \left( -\frac{2v_1^* v_2}{v_+} - M^u + v_+ V_L^{CKM} M^d V_R^{CKM} \right) \right] u_R, \quad (2.32)$$
$$\frac{\sqrt{2}}{v_+^2} d_L \left[ \phi_-^0 \frac{v_+^2}{v_+} M^d + \phi_+^0 \left( -\frac{2v_1^* v_2}{v_+} - M^u + v_+ V_L^{CKM} M^d V_R^{CKM} \right) \right] d_R. \quad (2.33)$$

The quark couplings in the second term are non-diagonal since the CKM matrix is non-diagonal. This leads to flavor mixing effective between the quark families, which results in tree-level FCNC. The first term quark couplings are flavor diagonal, since it is proportional to the diagonalized quark mass matrices $M^u$ and $M^d$. We could suppress tree-level FCNCs mediated by the $\phi_0^i$ couplings by applying Approximate Global $U(1)$ Symmetries (AGUS) to the Yukawa Lagrangian [28]. AGUS assume that the off-diagonal terms of $Y_1$ and $Y_2$ have very small values, which results in small quark mixings. In the next section we calculate the lower bound on the neutral Higgs boson mass from Kaon meson mixing constraints.
2.4 Neutral Kaon mass difference

In this section we derive the constraining relation of the mass of the Higgs boson mediating tree-level FCNCs. We will only study the Higgs bi-doublet field, since neutral scalars from the triplet fields have weak coupling to the quarks due to $B - L$ conservation [29]. The neutral Higgs boson contribution to the $\Delta S = 2$ Hamiltonian is given by [22]:

$$H_{H_0^0}^{\Delta S=2} = -\frac{4G_F}{\sqrt{2}M_{H_0^0}^2} \sum_{s,s' = d,s,b} \bar{s}' P_L s \bar{s}' P_R \sum_{i,j = u,c,t} \lambda_i^{LR} \lambda_j^{RL} m_i m_j,$$  \tag{2.34}

where $G_F$ is the Fermi constant that describes the interaction strength. $\lambda_i^{LR} = V_{is}^L V_{id}^R$, $\lambda_i^{RL} = V_{is}^R V_{id}^L$, and $m_i, m_j$ are the masses for up, charm and top quarks: $m_s \approx 2.4$ MeV, $m_c \approx 1.3$ GeV and $m_t \approx 172$ GeV [30]. $O_s = \bar{s}' P_L s \bar{s}' P_R$, is local 4-quark operator. The lower limit on $m_{H_0^0}$ is constrained by the experimental value for neutral Kaon mass difference $\Delta_{\text{exp}} m_K$:

$$\Delta_{\text{exp}} m_K = 2 \text{Re} \left( M_{H_0^0}^{12} \right) \approx 3.483 \times 10^{-12} \text{ MeV}. \tag{2.35}$$

The additional neutral Higgs bosons in LRSMs contribute to neutral Kaon meson mixing $\Delta_{m_k}^{H_0^0}$. We can calculate $\Delta_{m_k}^{H_0^0}$ using $\Delta_{m_k}^{H_0^0} = 2 \text{Re} \left( M_{H_0^0}^{12} \right)$, where the mixing matrix element $M_{H_0^0}^{12}$ is related to the Hamiltonian $H_{H_0^0}^{\Delta S=2}$ by:

$$M_{H_0^0}^{12} = \frac{1}{2m_K} \langle K^0 | H_{H_0^0}^{\Delta S=2} | \bar{K}^0 \rangle. \tag{2.36}$$

We can calculate the right hand side of the above equation by computing the matrix element of $H_{H_0^0}^{\Delta S=2}$. The matrix elements of the local 4-quark operator $O_s = \bar{s}' P_L s \bar{s}' P_R$, is [22]:

$$\langle K^0 | \bar{s}' P_L s \bar{s}' P_R | \bar{K}^0 \rangle = 59 \text{ MeV}. \tag{2.37}$$

Substituting the value of the Fermi constant and the mass of the neutral Kaon meson $m_K \approx 497$ MeV, we obtain the relation:

$$\frac{\sum_{i,j = u,c,t} \lambda_i^{LR} \lambda_j^{RL} m_i m_j}{M_{H_0^0}^2} < 3.56 \times 10^{-9}. \tag{2.38}$$

Since we require $\Delta_{m_k}^{\text{exp}} > \Delta_{m_k}^{H_0^0}$, this gives a constraint on $\Delta_{m_k}^{H_0^0}$. The lower limit on $M_{H_0^0}^2$ will depend on the term $\sum_{i,j = u,c,t} \lambda_i^{LR} \lambda_j^{RL} m_i m_j$.

2.5 Approximate Global $U(1)$ Symmetries

In this section we study AGUS and their role in suppressing masses of Higgs bosons that mediate tree-level FCNCs in neutral Kaon system. AGUS assume that the off-diagonal elements of the Yukawa couplings $Y$ and $\tilde{Y}$ have small values. They can generally be formulated as:

$$(u_i, d_i) \mapsto e^{-i\theta_i} (u_i, d_i), \tag{2.39}$$
where \( u_i \) and \( d_i \) are the up and down quark families, respectively, and \( e^{-i\theta_i} \) is a phase factor. The angle \( \theta \) is taken as a constant, which makes it a global transformation. The Yukawa couplings are related to the CKM matrices, Eq. 2.29. If we can approximately diagonalize the Yukawa couplings, then we can suppress tree-level FCNCs [28]. This can be achieved by fine tuning the off-diagonal terms of the CKM matrix, \( V_{cd} \) and \( V_{ts} \). To see the fine tuning of the off-diagonal terms in the CKM matrix, we expand the term:

\[
\sum_{i,j=u,c,t} \lambda_{i}^{LR} \lambda_{j}^{RL} m_{i} m_{j} = V_{us} V_{ud} V_{us} m_{u} m_{u} + V_{us} V_{ud} V_{cd} m_{u} m_{c} \\
+ V_{us} V_{td} V_{ts} m_{t} m_{t} + V_{cs} V_{cd} V_{us} m_{c} m_{u} \\
+ V_{ts} V_{td} V_{ts} m_{t} m_{t} + V_{ts} V_{td} V_{cd} m_{t} m_{c} \\
+ V_{ts} V_{td} V_{ts} m_{t} m_{l}. 
\]

The experimental CKM matrix elements are [31]:

\[
V_{CKM}^{LR} = \begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{bmatrix} = \begin{bmatrix}
0.9750 & 0.22 & 0.0032 \\
0.22 & 0.974 & 0.044 \\
0.004 - 0.014 & 0.04 & 0.999
\end{bmatrix}. \tag{2.40}
\]

In our calculations we assume \( V_{cs} = V_{ad} \approx 1 \). Using the above CKM matrix elements and the quark masses, we find that \( \sum_{i,j=u,c,t} \lambda_{i}^{LR} \lambda_{j}^{RL} m_{i} m_{j} \approx 86 \text{ GeV} \). To satisfy experimental constraints on \( \Delta_{m_{k}}^{H^0} \), the mass bound on \( M_{H^0}^{LR} \) is:

\[
M_{H^0}^{LR} > \frac{86000 MeV}{3.558 \times 10^{-5}}. \tag{2.41}
\]

The above relation implies that \( M_{H^0}^{LR} > 4.9 \text{ TeV} \). Using the following fine-tuned values for \( V_{cd} \) and \( V_{ts} \), we can suppress \( M_{H^0}^{LR} \) to about 270 GeV:

\[
V_{CKM}^{LR} = \begin{bmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{bmatrix} = \begin{bmatrix}
0.9750 & 0.22 & 0.0032 \\
0.004 & 0.974 & 0.044 \\
0.004 & 0.016 & 0.999
\end{bmatrix}. \tag{2.42}
\]

Thus, in LRSMs, if we fine-tune \( V_{cd} \) to 0.004 and \( V_{ts} \) to 0.016, we can obtain a heavy boson around 270 GeV. Unfortunately, this fine-tuning contradicts experimental results [28] and we conclude that we cannot accomodate a heavy boson of \( \mathcal{O}(10^3) \text{GeV} \) in LRSMs.

### 2.6 Discrete symmetries

The source of the tree-level FCNCs problem is the presence of two Higgs doublets in the Higgs bi-doublet field. When the Higgs bi-doublet field couples to the same quark field, it will result in tree-level FCNCs [19]:

\[
L_Y = \Psi_{L}^{i} \left( (Y)_{ij} \Phi + (\tilde{Y})_{ij} \tilde{\Phi} \right) \Psi_{R}^{j} + h.c.. \tag{2.43}
\]

Our goal is to suppress tree-level FCNCs by forbidding the \( \tilde{Y} \) coupling to the quark fields. We define a \( \mathbb{Z}_2 \) symmetry [19] to forbid simultaneous coupling of \( Y \) and \( \tilde{Y} \) to the SM quark fields:

\[
\Phi \mapsto i \Phi, \quad Q_{R} \mapsto -i Q_{R}, \quad \tilde{\Phi} \mapsto -i \tilde{\Phi},
\]
\[ L_L \mapsto -iL_L, \quad \Delta_L \mapsto -\Delta_L, \quad (2.44) \]

with all other fields unchanged. Under the \( Z_2 \) symmetry, the Yukawa Lagrangian is invariant only if the \( \bar{Y} \) coupling is zero:

\[ L_Y = \bar{\Psi}_L^i \left( (Y)_{ij} \Phi + (\bar{Y})_{ij} \bar{\Phi} \right) \Psi_R^j + h.c. \mapsto \bar{\Psi}_L^i \left( (Y)_{ij} \Phi - (\bar{Y})_{ij} \bar{\Phi} \right) (-i\Psi_R^j) + h.c.. \quad (2.45) \]

In this case only the the \( Y \) matrix couples to quark fields. This leads to diagonal mass matrices of the form:

\[ M^u = \frac{1}{\sqrt{2}} V^u_L [Y v_1] V^u_R, \quad M^d = \frac{1}{\sqrt{2}} V^d_L [e^{i\alpha} v_2 Y] V^d_R. \quad (2.46) \]

The proportionality between \( M^d \) and \( M^u \) would not allow presence of a non-trivial CKM matrix. Furthermore, it does not explain the mass patterns between the up and down quarks:

\[ M_d = \frac{v_2}{v_1} M_u, \quad \frac{v_2}{v_1} \ll 1. \quad (2.47) \]

One suggestion to solve the above problems is to add a new bi-doublet field, \( \rho \), with \( B - L = 2 \) [19]:

\[ \rho = \begin{bmatrix} \rho_1^+ & \rho_2^+ \\ \rho_1^- & \rho_2^- \end{bmatrix}, \quad \rho \mapsto -i\rho. \quad (2.48) \]

For symmetry breaking to occur, we require the vevs of the Higgs bi-doublets:

\[ \langle \phi_0 \rangle = \begin{bmatrix} v_1 & 0 \\ 0 & v_2 \end{bmatrix}, \quad \langle \rho_0 \rangle = \begin{bmatrix} 0 & 0 \\ v_\rho & 0 \end{bmatrix}. \quad (2.49) \]

The vevs must satisfy the constraint: \( v_1^2 + v_2^2 + v_\rho^2 = v^2 = (246\text{GeV})^2 \) [32]. The heavy boson has a mass \( m_{H^0} \sim \sqrt{v R} \). To determine \( m_{H^0} \), we need to know \( v_R \).

First, we need to calculate the lower limit on the \( W^\pm_R \) coupling constant \( g_R \). In the SM, the mass of \( W^\pm_L \) is proportional to \( v \):

\[ m_{W^\pm} = \frac{1}{2} v g_L. \quad (2.50) \]

This gives \( g_L \approx 0.65 \), where \( g_L \) is the \( W^\pm_L \) coupling constant. Now we want to find the lower limit on \( g_R \). It is theoretically required in LRSMs that [29]:

\[ \frac{g_R}{g_L} \geq \tan \theta_W \approx 0.55 \implies g_R \approx 0.357. \quad (2.51) \]

The strongest limit on \( m_{W^\pm_R} \) comes from the neutral Kaon meson system: \( m_{W^\pm_R} > 2.5\text{TeV} \) [22]. Using the above result in \( m_{W^\pm_R} = g_R v_R \) gives \( v_R \approx 7 \text{ TeV} \). This gives the lower limit \( m_{H^0} \approx 1.3 \text{ TeV} \). If we fine tune \( v_R \) to about 330 GeV, then we could have a heavy boson around 270 GeV. This fine tuning contradicts phenomenology which demands that \( v_R \) be of \( \mathcal{O}(\text{TeV}) \) [29, 33]. Based on the above analysis, we conclude that the heavy boson in LRSMs lies in the TeV range. In [32], it was shown that a lower limit can be obtained: \( m_{H^0} = 812 \text{ GeV} \), which is inconsistent with the Madala hypothesis.
2.7 Results

In summary, our goal was to consider a neutral heavy boson around 270 GeV in LRSMs. FCNCs in Kaon meson mixings constrained the heavy boson $H$ to be in the TeV range. We then studied global and discrete symmetries and their role in suppressing FCNCs in neutral Kaon meson mixings. AGUS resulted in a heavy boson of about 5 TeV. To obtain a heavy boson in the GeV range, we must fine tune the CKM matrix elements $V_{cd}$ to 0.004 and $V_{ts}$ to 0.016 which contradicts experimental observations. Next we analysed discrete symmetries and our calculations showed that the lowest mass for the heavy boson will be of $\mathcal{O}(\text{TeV})$. These results indicate that the neutral Higgs bosons in LRSMs are heavier than the postulated heavy boson $H$. 
We propose a model with an extended Higgs sector to accommodate the heavy boson $H$ and the scalar singlet $S$. The 2HDM+S is based on the extension of the 2HDM by a real scalar singlet field [11]. Due to its extended Higgs sector, 2HDM+S could accommodate the heavy boson decays: $H \rightarrow Sh, hh, SS$. We impose the model constraints and analyze the branching ratios and decay widths of the heavy boson $H$.

This chapter is structured as follows: In section 3.1 we give an overview of 2HDM+S. In section 3.2, we briefly discuss symmetry breaking in the model. In section 3.4 we discuss the 2HDM+S model constraints. In section 3.5 we explain the methodology and tools used in the analysis. In section 3.6 we discuss the results.

3.1 Two-Higgs-Doublet Models plus scalar singlet

We introduce 2HDM with an extra real scalar $S$ that is neutral under the SM gauge group. In this work, we will only study extensions of the type II 2HDM models by including a scalar gauge-singlet with mass $m_S$. If we choose a complex $S$, it will contain both scalar and pseudoscalar fields, whereas a real scalar singlet will only contain a scalar field [34]. In this work we consider the singlet field $\Phi_S$ to acquire a real non-zero vev, which allows mixing between all the CP-even neutral states. The 2HDM+S Higgs sector consists of six Higgs bosons: three are neutral CP-even, one is a CP-odd and two are singly charged. In this work we will study the branching ratios of the heavy Higgs $H$ and determine if we can reproduce the branching ratios predicted by the Madala hypothesis. The possible final decay states of the Higgs bosons in 2HDM+S are shown in Table. 3.1. If the Madala hypothesis is embedded into 2HDM+S, it has the potential to explain the excesses observed in the data [14].

The mass eigenstates for the Higgs bosons are constructed from superpositions of the doublet and singlet fields. The most general CP-conserving 2HDM+S potential
Section 3.1. Two-Higgs-Doublet Models plus scalar singlet

Table 3.1: Decay modes of the Higgs bosons in 2HDM+S [11].

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Scalars</th>
<th>Decay modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>D.1</td>
<td>$h$</td>
<td>$bb, \tau^+\tau^-, \mu^+\mu^-$, $ss, cc, gg, \gamma\gamma, ZZ, W^+W^-$, $ZZ$</td>
</tr>
<tr>
<td>D.2</td>
<td>$H$</td>
<td>D.1, $hh, SS, Sh$</td>
</tr>
<tr>
<td>D.3</td>
<td>$A$</td>
<td>D.1, $ii, Zh, ZH, ZS, W^\pm H^\mp$</td>
</tr>
<tr>
<td>D.4</td>
<td>$H^\pm$</td>
<td>$W^\pm h, W^\pm H, W^\pm S$</td>
</tr>
<tr>
<td>D.5</td>
<td>$S$</td>
<td>D.1, $\chi\chi$</td>
</tr>
</tbody>
</table>

Table 3.1: Decay modes of the Higgs bosons in 2HDM+S [11].

$V_{2HDM+S}$ is given by:

$$
V_{2HDM+S} = V_{2HDM} + V_{\text{Singlet}},
$$

$$
V_{2HDM} = m^2_{11} |\Phi_1|^2 + m^2_{22} |\Phi_2|^2 - m^2_{12}(\Phi_1^\dagger \Phi_2 + h.c) + \frac{\lambda_1}{2}(\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_1}{2}(\Phi_2^\dagger \Phi_2)^2 + \lambda_2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{\lambda_5}{2}[(\Phi_1^\dagger \Phi_2)^2 + h.c],
$$

$$
V_{\text{Singlet}} = \frac{1}{2} m^2_{S} |\Phi_S|^2 + \frac{\lambda_6}{8} m^2_S |\Phi_1|^4 + \frac{\lambda_7}{2}(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{\lambda_8}{2}(\Phi_2^\dagger \Phi_2)|\Phi_S|^2,
$$

where $\lambda_S$ is the singlet quartic coupling. We set the term $m^2_{12} \neq 0$ in the 2HDM+S potential, which corresponds to a soft breaking of the $Z_2$ symmetry. The 2HDM+S potential consists of 2HDM terms in addition to terms resulting from contributions of the singlet field $\Phi_S$. We consider the $\lambda_i$ to be real, which corresponds to a model without explicit CP violation. We can obtain this potential by imposing the $Z_2$ symmetry:

$$
\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2, \quad \Phi_S \rightarrow \Phi_S.
$$

(3.2)

To allow mixing among the CP-even neutral particles, we will need another $Z_2^*$ symmetry [35]:

$$
\Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow \Phi_2, \quad \Phi_S \rightarrow -\Phi_S.
$$

(3.3)

Symmetry breaking occurs when the two Higgs doublet fields and the singlet field acquire the real vevs: $v_1$, $v_2$ and $v_S$, respectively. They can be formulated as:

$$
\Phi_1 = \left( \frac{\phi_1^+}{\sqrt{2}} (v_1 + H_1 + i\eta_1) \right), \quad \Phi_2 = \left( \frac{\phi_2^+}{\sqrt{2}} (v_2 + H_2 + i\eta_2) \right), \quad \Phi_S = v_S + H_3,
$$

(3.4)

where $\phi^+_{1,2}$ are complex charged fields, $H_{1,2,3}$ are real neutral CP-even fields and $\eta_{1,2}$ are CP-odd fields, respectively. The two Higgs doublets vevs must satisfy the relation: $v^2 = v_1^2 + v_2^2$ where $v = 246$GeV. Under the 2HDM+S gauge group, the gauge representations for the scalar fields are:

$$
\Phi_1 \sim (1, 2, \frac{1}{2}), \quad \Phi_2 \sim (1, 2, \frac{1}{2}), \quad \Phi_S \sim (1, 1, 0),
$$

(3.5)

where the quantum numbers represent the gauge groups $SU(3), SU(2)$ and $U(1)_Y$ respectively. The scalar singlet $S$ could be a DM candidate if it doesn’t acquire a vev [35, 36]. In this work we consider the singlet $S$ to acquire a vev which excludes it from being a DM candidate. The 2HDM+S Lagrangian consists of four terms:

$$
L = L_{\text{kinetic}} + L_{\text{gauge}} + L_{\text{Higgs}} + L_{\text{Yukawa}}.
$$

(3.6)
The $L_{\text{kinetic}}$ term contains the interactions between fermions and gauge bosons which are invariant under the 2HDM+S gauge group. The fermionic kinetic terms have the following form:

$$L_{\text{kinetic}} = i \sum \bar{\Psi} \gamma^\mu D^\mu \Psi,$$

$$= i \left( L_L \gamma^\mu (i \partial_\mu + i g_L \frac{1}{2} \sigma^i W^\mu_{Li} + i g_B \frac{1}{2} B^\mu) L_L ight. \right.$$ \hspace{1cm}

$$+ \left. Q_L \gamma^\mu (i \partial_\mu + i g_L \frac{1}{2} \sigma^i W^\mu_{Li} + i g_B \frac{1}{6} B^\mu + g_G \tau^\mu \frac{1}{2} G^\mu \right) Q_L \right),$$

where $\Psi = L_{L,R}$ and $Q^{u,d}_{L,R}$ are the leptons and quarks fields. The term $L_{\text{gauge}}$ contains the kinetic terms for the gauge fields and the interactions between them. It is given by:

$$L_{\text{Gauge}} = -\frac{1}{4} W^\mu_{Li} W^\mu_{Li} - \frac{1}{4} G^\mu_{a} G^\mu_{a} - \frac{1}{4} B^\mu B^\mu,$$  

where $W^\mu_{Li}$, $G^\mu_{a}$ and $B^\mu$ are the field strength tensors of the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge fields, respectively. They are defined as follows:

$$G^\mu_{a} = \partial^\mu G^\mu_{a} - \partial^\nu G^\mu_{a} - g G f^{abc} G^\mu_{b} G^\mu_{c},$$

$$W^\mu_{Li} = \partial^\mu W^\mu_{Li} - \partial^\nu W^\mu_{Li} + g_{L} \varepsilon^{ijk} W^\mu_{Lj} W^\nu_{Lk},$$

$$B^\mu = \partial^\mu B^\mu - \partial^\nu B^\nu,$$

where $f^{abc}$ and $\varepsilon^{ijk}$ are the structure constants of the $SU(3)_C$ and $SU(2)_L$ groups, respectively. The $L_{\text{Yukawa}}$ term consists of the Yukawa interactions, which describe the most general possible couplings of $\Psi$ and $\tilde{\Psi}$ to the Higgs fields:

$$L_{\text{Yukawa}} = -\sum_{i,j=1}^{3} \sum_{k=1,2,3} c(H_i f f) y_{lj}^{\mu_{i} \mu_{j} \mu_{k}} \bar{\Psi}_k \Psi + \text{h.c.},$$

$$= -c(H_i f f) \left( y_{l1j}^{\mu} L_j \Phi_1 + y_{l2j}^{\mu} L_j \Phi_2 + y_{l3j}^{\mu} L_j \Phi_3 \right) \right.$$ \hspace{1cm}

$$+ \left[ y_{l1j}^{\mu} R_j \Phi_1 + y_{l2j}^{\mu} R_j \Phi_2 + y_{l3j}^{\mu} R_j \Phi_3 \right] \right.$$ \hspace{1cm}

$$+ \left[ y_{l1j}^{\mu} S_j \Phi_1 + y_{l2j}^{\mu} S_j \Phi_2 + y_{l3j}^{\mu} S_j \Phi_3 \right] \right) + \text{h.c.,}$$

where $\Phi_k = \Phi_1, \Phi_2, \Phi_3$ and $y_{lj}^{\mu}$ are $3 \times 3$ Yukawa matrices in flavor space. The terms $c(H_i f f)$, describe the effective coupling between the CP-even Higgs states. The Higgs Lagrangian term $L_{\text{Higgs}}$ consists of the kinetic terms of the Higgs fields as well as their potential:

$$L_{\text{Higgs}} = \sum_{i} |D^\mu \Phi_i|^2 - V_{2HDM+S}.\left(3.11\right)$$

Under the $SU(2)_L \otimes U(1)_Y$ symmetry, the covariant derivatives for the Higgs fields are given by:

$$D^\mu \Phi_1,2 = \left[ \partial^\mu + i \frac{g_L}{2} \left( \sigma^i W^{\mu}_{Li} + i \frac{g_B}{2} B^{\mu} \right) \right] \Phi_1,2,$$

$$D^\mu \Phi_S = \left[ \partial^\mu - i \frac{g_B}{2} B^{\mu} \right] \Phi_S.\left(3.12\right)$$
3.2 Symmetry breaking and minimization conditions

Minimizing the potential at the vevs of the three Higgs fields leads to three minimum conditions:

\[
\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial v_S} = 0. \tag{3.13}
\]

The first derivative conditions for \(\Phi\) are:

\[
\begin{align*}
\frac{\partial V}{\partial \Phi_1} &= 0 \rightarrow \quad m_{11}^2 = -\frac{1}{2}(v_1^2\lambda_1 + v_2^2\lambda_{345} + v_S^2\lambda_7) + \frac{v_2}{v_1}m_{12}^2, \\
\frac{\partial V}{\partial \Phi_2} &= 0 \rightarrow \quad m_{22}^2 = -\frac{1}{2}(v_2^2\lambda_2 + v_1^2\lambda_{345} + v_S^2\lambda_8) + \frac{v_2}{v_1}m_{12}^2, \\
\frac{\partial V}{\partial \Phi_S} &= 0 \rightarrow \quad m_S^2 = -\frac{1}{2}(v_1^2\lambda_7 + v_2^2\lambda_8 + v_S^2\lambda_6),
\end{align*} \tag{3.14-3.16}
\]

where \(\lambda_{345} = \lambda_1 + \lambda_2 + \lambda_3\). EWSB occurs as in the SM, where the gauge group \(SU(2)_L \otimes U(1)_Y\) is broken by the vevs of the neutral components of the Higgs fields to \(U(1)_{EM}\). This ensures that the potential is at the global minimum. We can obtain the global minimum of the potential by using the stationary point that leads to the lowest value for the scalar potential. The minimizations conditions for the Higgs fields are:

\[
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right), \quad \langle \Phi_S \rangle = \frac{1}{\sqrt{2}}v_S. \tag{3.17}
\]

We want the global minimum of the potential to conserve both the gauge symmetry of electromagnetism and CP, as well as to produce three CP-even massive scalars. To conserve the gauge symmetry of electromagnetism, both of the Higgs bi-doublet vevs: \(v_1\) and \(v_2\) must lie in the neutral component. We will assume that \(v_1, v_2\) and \(v_S\) are real, to prevent CP violation in the scalar sector [35]. This results in a global minimum that allows the singlet field to mix with the CP-even scalars from the Higgs bi-doublets.

3.3 CP-even neutral Higgs boson masses

The 2HDM+S Higgs sector consists of additional Higgs bosons with respect to 2HDM, due to the addition of the real scalar singlet. The CP-even neutral Higgs mass matrix is enlarged to a \(3 \times 3\) matrix. In the interaction basis \((\chi_1, \chi_2, \chi_3)\) it can be written as:

\[
M_{\text{scalar}}^2 = \begin{pmatrix}
\lambda_1 c_\beta^2 v^2 + t_\beta m_{12}^2 & \lambda_{345} c_\beta s_\beta v^2 - m_{12}^2 & \lambda_7 c_\beta v_S \\
\lambda_{345} c_\beta s_\beta v^2 - m_{12}^2 & \lambda_8 s_\beta v_S & \lambda_{23} c_\beta s_\beta v_S \\
\lambda_7 c_\beta v_S & \lambda_{23} c_\beta s_\beta v_S & \lambda_6 v_S^2
\end{pmatrix}, \tag{3.18}
\]

We can diagonalize the CP-even neutral Higgs mass matrix using the orthogonal matrix \(R\):

\[
R = \begin{pmatrix}
c_{\alpha_1} c_{\alpha_2} & s_{\alpha_1} c_{\alpha_2} & s_{\alpha_2} \\
-c_{\alpha_1} s_{\alpha_2} s_{\alpha_3} + s_{\alpha_1} c_{\alpha_3} & c_{\alpha_1} c_{\alpha_3} - s_{\alpha_1} s_{\alpha_2} s_{\alpha_3} & c_{\alpha_2} s_{\alpha_3} \\
-c_{\alpha_1} s_{\alpha_2} c_{\alpha_3} + s_{\alpha_1} s_{\alpha_3} & -(c_{\alpha_1} s_{\alpha_3} + s_{\alpha_1} s_{\alpha_2} c_{\alpha_3}) & c_{\alpha_2} c_{\alpha_3}
\end{pmatrix}, \tag{3.19}
\]

Section 3.4. Constraints on 2HDM+S

where $\alpha_{1,2,3}$ are the mixing angles for the CP-even Higgs states and $\beta$ is the mixing angle for the CP-odd Higgs states. $c_{\alpha_{1,2,3}} = \cos\alpha_{1,2,3}$, $s_{\alpha_{1,2,3}} = \sin\alpha_{1,2,3}$, $c_\beta = \cos\beta$, $s_\beta = \sin\beta$ and $t_\beta = \tan\beta$. The mixing angles for the CP-even Higgs states can be chosen in the range [35]:

$$-\frac{\pi}{2} < \alpha_{1,2,3} < \frac{\pi}{2}.$$  

To rotate the interaction basis $(\chi_1, \chi_2, \chi_3)$ into the physical mass eigenstates: $H_1, H_2$ and $H_3$, we use the matrix $R$:

$$
\begin{pmatrix}
H_1 \\
H_2 \\
H_3
\end{pmatrix} = 
R
\begin{pmatrix}
\chi_1 \\
\chi_2 \\
\chi_3
\end{pmatrix}.
$$

(3.20)

The mass squared matrix $M^{2\text{ scalar}}$ can be diagonalized using the orthogonal matrix $R$:

$$
RM^{2\text{ scalar}} R^T = \text{diag}(m_{H_1}^2, m_{H_2}^2, m_{H_3}^2).
$$

(3.21)

The physical mass eigenstates of the CP-even Higgs bosons, obtained after diagonalizing the mass matrix, are:

$$
H_1 = \sqrt{s_{\alpha_3}(s_{\alpha_2}v_S^2\lambda_6 + v_c\beta c_{\alpha_1} c_{\alpha_2} v_S \lambda_7 + v_c\alpha_2 s_\beta s_{\alpha_1} v_S \lambda_5)},
$$

$$
H_2 = \sqrt{(c_{\alpha_1}c_{\alpha_3} - s_{\alpha_1}(s_{\alpha_2} s_{\alpha_3}) (c_{\alpha_2}s_{\alpha_1}(m_{12}^2 c_\beta + v_2^2 s_\beta^2 \lambda_2)) + \sqrt{s_\beta s_{\alpha_2} v_S \lambda_8 + c_{\alpha_1}(c_{\alpha_2}(-m_{12}^2 + v_2^2 c_\beta s_{\alpha_1}(\lambda_1 + \lambda_2 + \lambda_3))},
$$

(3.22)

$$
H_3 = \sqrt{(-c_{\alpha_1}c_{\alpha_3} s_{\alpha_2} + s_{\alpha_1} s_{\alpha_3}) (v_c\beta s_{\alpha_2} v_S \lambda_7 + c_{\alpha_2}s_{\alpha_1}(-m_{12}^2 + v_2^2 s_\beta c_\beta)) \times (\lambda_1 + \lambda_2 + \lambda_3) + c_{\alpha_1}c_{\alpha_2}(v_2^2 c_\beta^2 \lambda_1 + m_{12}^2 t_\beta)},
$$

where $ct_\beta = \cot\alpha_3$. In our study we consider the mass hierarchy: $H_1 < H_2 < H_3$, which corresponds to the physical scalars: $h < S < H$. This mass hierarchy is introduced in order to explain several anomalous features in the LHC Run 1 and Run 2 data [16]. Based on the previous studies, see [11], the CP-even neutral Higgs bosons have the following mass ranges:

$$
h : = 125\text{GeV},
$$

$$
S : m_h < m_S < m_H - m_h,
$$

$$
H : 2m_h < m_H < 2m_t.
$$

(3.23)

In 2HDM+S, the scalar singlet $S$ will acquire a $v\text{ev}$ and mix with $h$ and $H$. We assume that the mixing of $S$ with $h$ is small enough such that (by varying the parameters of the potential) it will not spoil any experimental bounds [11].

3.4 Constraints on 2HDM+S

In analysing 2HDM+S there are theoretical and experimental constraints that must be satisfied. To verify experimental observations, the 2HDM+S must comply with the Higgs data from the LHC. This requires the Higgs sector of 2HDM+S to include the observed 125 GeV Higgs boson. The model must also comply with the theoretical
constraints of tree-level perturbative unitarity and vacuum stability [35]. In addition, the scalar potential is required to be at the global minimum. The 2HDM+S is described by the following set of input parameters:

\[ \alpha_1, \alpha_2, \alpha_3, v, v_S, m_{H_{1,2,3}}, m_{H_A}, m_{H^\pm}, \tan\beta, m_{12}^2. \] (3.24)

For 2HDM+S to be consistent with LHC data, it must comply with the exclusion bounds, especially from the Run 1 data. The experimental constraints on the charged Higgs bosons \( m_\pm \) in 2HDM imposes a mass of at least 480 GeV [35]. We choose the following mass for the CP-odd Higgs bosons \( m_A \):

\[ 480 \text{GeV} \leq m_A < 1 \text{TeV}. \] (3.25)

We also set the following constraints on \( v_S \) and \( m_{12}^2 \) to satisfy the required branching ratios of \( H \):

\[ 1 \text{GeV} \leq v_S < 1.5 \text{TeV}, \]
\[ 0 \leq m_{12}^2 < 500 \text{TeV}. \] (3.26)

We choose \( t_\beta \approx \sqrt{0.8} \) in order to scale the SM cross-section to match the data [37]. To satisfy tree-level perturbative unitarity constraints, the eigenvalues of the scalar scattering matrix must be below the upper value of \( 8\pi \). The Feynman rules of the \( S^4 \) interactions impose \( \lambda_S \) in the range: \( 0 < \lambda_S \leq 4\pi \) [38]. We require the following conditions for tree-level perturbativity:

\[ |\lambda_3 - \lambda_4| < 8\pi, \quad |\lambda_3 + 2\lambda_4 \pm 3\lambda_5| < 8\pi, \]
\[ \left| \frac{1}{2}(\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_3^2}) \right| < 8\pi, \]
\[ \left| \frac{1}{2}(\lambda_1 + \lambda_2 + \sqrt{(\lambda_1 - \lambda_2)^2 + 4\lambda_5^2}) \right| < 8\pi, \quad (\lambda_7, \lambda_8) < 4\pi. \] (3.27)

The vacuum is required to be stable at tree-level. This means that the 2HDM+S potential has to be bounded from below. In addition, the 2HDM+S potential must be positive in the limit where the fields go to infinity. The necessary and sufficient conditions for vacuum stability are [35]:

\[ \lambda_{1,2,S} > 0, \quad \lambda_3 > -\sqrt{\lambda_1\lambda_2}, \quad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}. \] (3.28)

If \( \lambda_7 \) and \( \lambda_8 \) are greater than zero:

\[ \lambda_7 > -\sqrt{\frac{1}{12}\lambda_S\lambda_1} \quad \text{and} \quad \lambda_8 > -\sqrt{\frac{1}{12}\lambda_S\lambda_2}. \] (3.29)

If \( \lambda_7 \) or \( \lambda_8 \) is less than zero, we also require:

\[ -2\lambda_7\lambda_8 + \frac{1}{6}\lambda_S\lambda_3 > -\sqrt{4 \left( \frac{1}{12}\lambda_S\lambda_1 - \lambda_7^2 \right) \left( \frac{1}{12}\lambda_S\lambda_2 - \lambda_8^2 \right)}, \]
\[ -2\lambda_7\lambda_8 + \frac{1}{6}\lambda_S(\lambda_3 + \lambda_4 - |\lambda_5|) > -\sqrt{4 \left( \frac{1}{12}\lambda_S\lambda_1 - \lambda_7^2 \right) \left( \frac{1}{12}\lambda_S\lambda_2 - \lambda_8^2 \right)}. \] (3.30)
3.5 Methodology and tools

We ran simulations of the $2\text{HDM}+\text{S}$ model using $2\text{NHDECAY}$. This program allowed us to calculate branching ratios of the neutral Higgs bosons in $2\text{HDM}+\text{S}$. We then analyzed the branching ratios of the heavy Higgs $H$ and the $Z$ gauge boson. The $2\text{NHDECAY}$ code is the HDECAY program extension of $2\text{HDM}+\text{S}$ [35]. It is a Fortran program used for the calculation of the branching ratios and decay widths of the Higgs bosons in $2\text{HDM}+\text{S}$. We used the mixing angles and the masses of the CP-even Higgs bosons as inputs, and imposed the constraints from tree-level unitarity, vacuum stability and perturbativity. We then performed a parameter scan in $2\text{HDM}+\text{S}$ while satisfying all the experimental and theoretical constraints. We then studied the heavy Higgs $H$ and the $Z$ gauge boson branching ratios. The input parameters of $2\text{HDM}+\text{S}$ are specified in the n2hdecay.in input file, which is an extension of the hdecay.in input file [35]. The user can calculate the $2\text{HDM}+\text{S}$ branching ratios and total decay widths by setting the input value $N2\text{HDM}=1$ in n2hdecay.in input file. In the first block we set $\text{PARAM}=1$, as well as type of the fermion sector symmetry. In the 2 Higgs Doublet Model block, we set the type of $2\text{HDM}$ used, as well as the parameter basis. We also set the values of $\tan\beta$, $m_{12}^2$ and the masses of the charged and pseudoscalar Higgs bosons, $m_{H^\pm}$ and $m_A$. In the $N2\text{HDM}$ block, we set the masses of the three neutral bosons $H_{1,2,3}$ in addition to the the three mixing angles $\alpha_{1,2,3}$ and the $vev$ of the singlet field $v_S$. 

Figure 3.1: Madala boson $H$ decays produced by $N2\text{HDECAY}$. 

$$H_3 \rightarrow H_1 H_1$$
$$H_3 \rightarrow H_1 H_2$$
$$H_3 \rightarrow H_2 H_2$$
$$H_3 \rightarrow t A$$
$$H_3 \rightarrow c A$$
$$H_3 \rightarrow b A$$
$$H_3 \rightarrow H_1 H_2$$
$$H_3 \rightarrow H_2 H_2$$
$$H_3 \rightarrow H_3 \tau^+ \tau^-$$
$$H_3 \rightarrow s A$$
$$H_3 \rightarrow c A$$
$$H_3 \rightarrow b A$$
$$H_3 \rightarrow H_1 H_2$$
$$H_3 \rightarrow H_2 H_2$$
$$H_3 \rightarrow H_3 \gamma$$
$$H_3 \rightarrow H_3 g g$$
$$H_3 \rightarrow H_3 Z A$$
$$H_3 \rightarrow H_3 W^+ W^-$$
$$H_3 \rightarrow H_3 Z Z$$
Figure 3.2: SM Higgs boson $h$ decays produced by N2HDECAY.

Figure 3.3: SM Higgs boson $h$ decays produced at the LHC [39].
3.6 Results

We performed scans of the 2HDM+S parameter space using N2HDECAY. We then scanned the 2HDM+S parameters in the range allowed by theoretical and experimental constraints, obtaining branching ratios for the heavy boson $H$ and the $Z$ gauge boson. There are cutoffs in the $H$ branching ratios, since N2HDECAY does not include off-shell Higgs-to-Higgs decays, Fig. 3.1. The dominant decays of $H$ are to $hh$ and $Sh$. The decay $H \to SS$ was suppressed to about 10%, as predicted by the Madala hypothesis [37]. In order to comply with experimental observations, we need to reproduce the decays of $h$ at the LHC. We imposed a deviation limit of <10% on the decays of $h$ produced by N2HDECAY. There were deviations above 10% in some of the $h$ decays produced by N2HDECAY, Fig. 3.2. The decay channel $h \to W^+W^-$ produced by N2HDECAY was about 20% larger than $h \to W^+W^-$ produced at the LHC, Fig. 3.3. Nevertheless, the $h$ decays produced by N2HDECAY comply with the LHC results.

To accommodate the Madala hypothesis in 2HDM+S, we require the following branching ratios of the heavy boson $H$: $H \to Sh \gg H \to hh$ and $H \to Sh + H \to hh \approx 80\%$ [12]. The $H$ decays produced by N2HDECAY agree with the Madala hypothesis. The decay $H \to Sh$ was about 80%, while $H \to hh$ was about 50%. The decay $H \to SS$ was suppressed to about 10% since it’s kinematically off-shell. We obtained parameter points that satisfy the model constraints as well as the decays of $H$ as predicted by the Madala hypothesis. The analysis presented in this dissertation is considered as a first attempt to investigate the Madala hypothesis in 2HDM+S.
4.1 Conclusion

This dissertation discusses the search for new bosons $H$ and $S$, which are introduced in the Madala hypothesis to explain certain anomalous features in the LHC Run 1 and Run 2 data. In order to explain the excesses in the data, the heavy boson $H$ and the scalar singlet $S$ must have masses around 270 GeV and 150 GeV, respectively [12].

Two different extensions to the SM are discussed in this work. First, we discussed LRSMs and investigated the CP-even Higgs sector. Models with more than one Higgs doublet can mediate FCNCs at tree-level. In order to suppress tree-level FCNCs, we applied global and discrete symmetries to the Yukawa coupling terms. Next, we studied tree-level FCNCs in Kaon meson mixing and concluded that the mediating boson has to be at least in the TeV range. Our results indicate that the Madala hypothesis cannot be accommodated in LRSMs.

We then studied the 2HDM+S and analyzed the Higgs sector. Using N2HDECAY, we performed scans of the parameter space constrained by the theoretical and experimental requirements. The results obtained in this work seem to be consistent with the Madala hypothesis. We could not study the off-shell decays of the heavy boson $H$ since N2HDECAY doesn’t allow these processes.

This study is considered as a first investigation of the Madala hypothesis in LRSMs and 2HDM+S. Future work could include performing a full parameter scan in 2HDM+S using SSP (SARAH Scan and Plot) interphased with Spheno (Supersymmetric Phenomenology) [40]. SSP is a Mathematica package that allows off-shell decays of the heavy boson $H$ which could modify the results obtained by N2HDECAY.


