ABSTRACT

The aim of this research project is evaluating the application of the Crum-based transformation in solving engineering systems modelled as two-point boundary value problems. The boundary value problems were subjected to the various combinations of Dirichlet, Non-Dirichlet and Affine boundary conditions. The engineering systems that were modelled were in the fields of electrostatics, heat conduction and longitudinal vibrations. Other methods such as the Z-transforms and iterative methods have been discussed. An attractive property of the Crum-based transformation is that it can be applied to cases where the eigenparameters (function of eigenvalues) generated in the discrete case are negative and was therefore chosen to be explored further in this dissertation. An alternative matrix method was proposed and used instead of the algebraic method in the Crum-based transformation. The matrix method was tested against the algebraic method using three unit intervals. The analysis revealed, that as the number of unit intervals increase, there is a general increase in the accuracy of the approximated continuous-case eigenvalues generated for the discrete case. The other observed general trend was that the accuracy of the approximated continuous-case eigenvalues decrease as one ascends the continuous-case eigenvalue spectrum. Three cases: (Affine, Dirichlet), (Affine, Non-Dirichlet) and (Affine, Affine) generated negative eigenparameters. The approximated continuous-case eigenvalues, derived from the negative eigenparameters, were shown not to represent true physical natural frequencies since the discrete eigenvalues, derived from negative eigenparameters, do not satisfy the condition for purely oscillatory behaviour. The research has also shown that the Crum-based transformation method was useful in approximating the shifted eigenvalues of the continuous case, in cases where the generated eigenparameters were negative: since, as the number of unit intervals increase, the post-transformed approximated eigenvalues improved in accuracy. The accuracy was also found to be better in the post-transformed case than in the pre-transformed case. Furthermore, the approximated non-shifted and shifted continuous-case eigenvalues (except the approximated continuous-case eigenvalues generated from negative eigenparameters) satisfied the condition for purely oscillatory behaviour.