Connecting Metacognition and Mathematical Proficiency: A Case Study of South African Matriculants

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A Dissertation submitted to the Faculty of Science, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Science.

Signed on 10 September 2016 in Austin, Texas, USA
DECLARATION

I declare that this Dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

[Signature]
(Signature of candidate)

10th day of September 2016 in Austin, TX, USA
ABSTRACT

This dissertation investigated the possible connection between the two constructs of metacognition and the strands of mathematical proficiency. Using a small convenience sample of Grade 12 learners at a public all-girls high school in Johannesburg, South Africa, the teacher/researcher examined if training in metacognitive knowledge influenced the learners’ conceptual understanding, procedural fluency, strategic competence and heuristic strategy use. Two instruments (pre-test and post-test) developed by the researcher and completed by the students were on-line assessments of metacognition utilising written self-reports of strategy use that occurred simultaneously with mathematical problem solving. Qualitative analyses were applied to the data and learners demonstrated declarative, procedural and conditional metacognitive knowledge in their explicit heuristic and content-specific strategies. The learners also demonstrated strategies which could be inferred from their working. The study showed evidence of mixed transferability of metacognitive training to mathematical proficiency. Questions for further research are provided.
In memory of my mother and best friend

Connie Wakefield

1924 - 2014
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# TABLE OF CONTENTS

## CHAPTER 1: INTRODUCTION TO THE STUDY

1.1 Introduction ........................................ 1  
1.2 Context of the study .................................. 2  
1.3 Rationale .............................................. 3  
1.4 Research aims ........................................... 4  
   1.4.1 Assessing knowledge of cognition vs. regulation of cognition ...... 6  
1.5 Research questions ...................................... 7  

## CHAPTER 2: LITERATURE REVIEW

2.1 Introduction ........................................... 8  
2.2 Theoretical foundations of metacognition .................. 8  
2.3 Additional conceptualizations of metacognition ................. 9  
2.4 Issues, problematic areas and needs for further research in metacognition .... 10  
2.5 Assessing metacognition .................................. 12  
2.6 Metacognition and mathematics education .................... 13  
   2.6.1 Schoenfeld’s mathematical problem solving and metacognition ........ 13  
   2.6.2 Empirical research on metacognition and mathematics .................. 15  
   2.6.3 Metacognition training and the IMPROVE system ....................... 18  
2.7 Cognitive Science, Mathematics and Mathematics Education ............... 20  
2.8 Mathematical Proficiency .................................. 22  
   2.8.1 The Strands of Mathematical Proficiency ............................... 22  
   2.8.2 Schoenfeld’s conception of mathematical proficiency ................. 25  
2.9 Pólya’s problem solving heuristics .......................... 27  
2.10 Conceptual Framework .................................... 30
CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

3.1 Philosophical foundations of research 33
   3.1.1 Introduction 33
   3.1.2 Ontological and epistemological positioning as researcher 34
   3.1.3 Ontological positioning as mathematics teacher 35
   3.1.4 Epistemological positioning as mathematics teacher 36

3.2 Research Methodology 37

3.3 Participants and context 38

3.4 Research method and instruments 40
   3.4.1 Research method 40
   3.4.2 Instruments 40
   3.4.3 Intervention 42
   3.4.4 Training procedure 42

3.5 Description of Data Analysis 46
   3.5.1 Introduction 46
   3.5.2 Data sets 47
   3.5.3 Method of analysis: Instruments 1 and 2 47
   3.5.4 Operationalising the three strands of mathematical proficiency 49
   3.5.5 Operationalising strategy use 50

3.6 Validity, Reliability and Ethics 53
   3.6.1 Validity and Reliability 53
   3.6.2 Ethical Considerations 53

CHAPTER 4: ANALYSIS AND RESULTS

4.1 Introduction 54

4.2 Question 1: Data Handling 55
4.2.1  Question 1: Mathematical Proficiency  55
4.2.2  Question 1: Strategy Use  63
4.3  Question 2: Measurement  71
  4.3.1  Question 2: Mathematical Proficiency  71
  4.3.2  Question 2: Strategy Use  81
4.4  Question 3: Sequences  93
  4.4.1  Question 3: Mathematical Proficiency  94
  4.4.2  Question 3: Strategy Use  100
4.5  Question 4: Applications of Quadratic Functions  108
  4.5.1  Question 4: Mathematical Proficiency  109
  4.5.2  Question 4: Strategy Use  111
4.6  Results  117
  4.6.1  Concise summary of trends  117
  4.6.2  Results  120

CHAPTER 5: CONCLUSION  124

REFERENCES  130
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Diagram of Data Analysis Steps</td>
<td>48</td>
</tr>
<tr>
<td>3.2</td>
<td>Steps to coding Mathematical Proficiency</td>
<td>50</td>
</tr>
<tr>
<td>3.3</td>
<td>Steps to coding Strategy Use</td>
<td>52</td>
</tr>
<tr>
<td>4.1</td>
<td>Memorandum: Question 1, Instrument 2</td>
<td>59</td>
</tr>
<tr>
<td>4.2</td>
<td>Figure 4.2 I1.Q2 Flow diagram of steps</td>
<td>74</td>
</tr>
<tr>
<td>4.3</td>
<td>Memorandum for I1.Q2</td>
<td>75</td>
</tr>
<tr>
<td>4.4</td>
<td>Memorandum for I2.Q2</td>
<td>78</td>
</tr>
<tr>
<td>4.5</td>
<td>Memorandum for I1.Q3 Compared to Learner Response</td>
<td>93</td>
</tr>
<tr>
<td>4.6</td>
<td>Memorandum for I1.Q4</td>
<td>109</td>
</tr>
<tr>
<td>4.7</td>
<td>Learner Responses to I1.Q4</td>
<td>109</td>
</tr>
<tr>
<td>4.8</td>
<td>Memorandum for I2.Q4</td>
<td>110</td>
</tr>
<tr>
<td>4.9</td>
<td>Two-way contingency tables connecting mathematical working and strategy use</td>
<td>123</td>
</tr>
<tr>
<td>5.1</td>
<td>A procedural network for subtraction</td>
<td>125</td>
</tr>
</tbody>
</table>
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Conceptual Framework combining the strands of mathematical proficiency and strategies</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Frequency of CU Codes for I1.Q1.2</td>
<td>57</td>
</tr>
<tr>
<td>4.2</td>
<td>Frequency of Conceptual Understanding Codes for I2.Q1</td>
<td>60</td>
</tr>
<tr>
<td>4.3</td>
<td>Explicit Strategies: Question 1.1, Instrument 1</td>
<td>64</td>
</tr>
<tr>
<td>4.4</td>
<td>Explicit Strategies: Question 1.2, Instrument 1</td>
<td>65</td>
</tr>
<tr>
<td>4.5</td>
<td>Strategy Use Question 1: Instruments 1 and 2 Compared</td>
<td>67</td>
</tr>
<tr>
<td>4.6</td>
<td>Representative Explicit Strategies: Question 1, Instrument 2</td>
<td>68</td>
</tr>
<tr>
<td>4.7</td>
<td>Frequency of Mathematical Proficiency Rating Codes: I1. Q2</td>
<td>72</td>
</tr>
<tr>
<td>4.8</td>
<td>Frequency of Steps Achieved and “Points of Derailment”</td>
<td>77</td>
</tr>
<tr>
<td>4.9</td>
<td>Representative Explicit Strategies: Question 2, Instrument 1</td>
<td>82</td>
</tr>
<tr>
<td>4.10</td>
<td>Frequency of Strategy Use by Categories: I1.Q2</td>
<td>83</td>
</tr>
<tr>
<td>4.11</td>
<td>Implied Strategy Use: I1.Q2</td>
<td>86</td>
</tr>
<tr>
<td>4.12</td>
<td>Strategy Use Question 2: Instruments 1 and 2 Compared</td>
<td>88</td>
</tr>
<tr>
<td>4.13</td>
<td>Representative Explicit Strategies: Question 2, Instrument 2</td>
<td>89</td>
</tr>
<tr>
<td>4.14</td>
<td>Frequency of Strategy Use by Categories: I1.Q3</td>
<td>100</td>
</tr>
<tr>
<td>4.15</td>
<td>Representative Explicit Strategies: Question 3, Instrument 1</td>
<td>101</td>
</tr>
<tr>
<td>4.16</td>
<td>Strategy Use Question 3: I1 and I2 Compared</td>
<td>105</td>
</tr>
<tr>
<td>4.17</td>
<td>Representative Explicit Strategies: Question 3, Instrument 2</td>
<td>106</td>
</tr>
<tr>
<td>4.18</td>
<td>Heuristic Strategy Use I1.Q4</td>
<td>112</td>
</tr>
<tr>
<td>4.19</td>
<td>Heuristic Strategy Use (Ineffective): I2.Q4</td>
<td>114</td>
</tr>
<tr>
<td>4.20</td>
<td>Heuristic Strategy Use (Partially effective): I2.Q4</td>
<td>115</td>
</tr>
<tr>
<td>4.21</td>
<td>Heuristic Strategy Use (Effective): I2.Q4</td>
<td>116</td>
</tr>
<tr>
<td>4.22</td>
<td>Strategy Use Question 4: I1 and I2 Compared</td>
<td>117</td>
</tr>
</tbody>
</table>
APPENDICES

Appendix A – 1: Instrument 1 (Pre-test)  I
Appendix A – 2: Instrument 1 Memorandum  IV
Appendix B – 1: Instrument 2 (Post-test)  VII
Appendix B – 2: Instrument 2 Memorandum  IX
Appendix C – 1: Metacognitive questioning: Comprehension questions  XIII
Appendix C – 2: Metacognitive questioning: Connection questions  XIV
Appendix C – 3: Metacognitive questioning: Strategy questions  XV
Appendix D – 1: 2/D and 3/D Trigonometry revision assignment  XVI
Appendix D – 2: 2/D and 3/D Trigonometry revision assignment: Memorandum  XVIII
Appendix E – 1: Grade 11/12 Trigonometry Revision  XX
Appendix E – 2: Grade 11/12 Trigonometry Revision: Memorandum  XXIV
Appendix F – 1: Grade 11/12 Euclidean Geometry Revision  XXXI
Appendix F – 2: Grade 11/12 Euclidean Geometry Revision: Memorandum  XXXII
Appendix G – 1: Grade 12 Paper 2 Challenge  XXXVI
Appendix G – 2: Grade 12 Paper 2 Challenge: Memorandum  XXXVII
Appendix G – 3: List of Content-specific strategies for Trigonometry  XL
Appendix H – 1: Exemplar: Learner Responses with codes (Before coding)  XLI
Appendix I – 1: Instrument 1: Criteria for coding values per question  XLIX
Appendix I – 2: Instrument 2: Criteria for coding values per question  LII
Appendix J – 1: Exemplar: Learner examples to determine codes  LVI
Appendix K – 1: Exemplar: Learner Responses with codes (After coding)  LX
Appendix L – 1: Tally of Codes (whole group): Instrument 1  LXVII
Appendix L – 2: Tally of Codes (whole group): Instrument 2  LXX
Appendix M – 1: Tally of Mathematical Proficiency ratings (per learner): Instrument 1  LXXIII
Appendix M – 2: Tally of Mathematical Proficiency ratings (per learner): Instrument 2

Appendix N – 1: Tally of Strategy Use ratings (per learner): Instrument 1

Appendix N – 2: Tally of Strategy Use ratings (per learner): Instrument 2

Appendix O – 1: Learner Strategy Use: Instrument 1

Appendix O – 2: Learner Strategy Use: Instrument 2

Appendix P – 1: “Notes” Page for Instrument 2

Appendix Q – 1: Title Approval