of existing exchange control regulations" (Bhana 1987:77).

Diversification involves trade-offs. Elton and Gruber (1977:415) write that when an investor increases the number of securities in his portfolio he is decreasing his risk, but also increases his transaction costs. This increase in transaction costs has the effect of decreasing the expected return of the portfolio, as these costs have to be covered before profits are earned. Similarly, Statman (1987:354) argues that the investor should continue to diversify his portfolio as long as the marginal benefits of doing so outweigh the marginal costs of diversification. The interesting question is, "How many securities are required to derive the result shown in (3.5)?".

The classic article in answer to this question, is Evans and Archer (1968). They determined that there is no "economic justification [for] increasing portfolio sizes beyond ten or so securities" (Evans and Archer 1968:767). Wagner and Lau (1971:51) also found that the gains from diversification tended to be small beyond ten securities. This appears to have become generally accepted in the literature (Statman 1987:353-354).

In a 1974 survey into the characteristics of the individual (American) investor, Lease, Lewellen and Schlarbaum (1974:214-215) found that the average investors portfolio consisted of between ten and fifteen different securities. In a similar study conducted in South Africa, Firer (1988) found that the mean value for South African investors was ten different securities. Thus he concludes (Firer 1988:27) that, "the benefits of diversification are not entirely lost on the individual investor in S.A.". It would appear that investors are vindicating the studies done by Evans and Archer (1968) and Wagner and Lau (1971).

There is, however, at least one dissenting voice: Statman (1987:362) argues that a well-diversified portfolio should include between thirty and forty securities. If he is correct, then the question of why investors hold
undiversified portfolios presents itself. Statman (1987:362) offers three potential explanations:
- investors are ignorant of the benefits of diversification;
- there is an additional variable affecting investors' choice of portfolio size; and,
- investors do not consider all their assets to be part of an integrated portfolio.
Of these three explanations the first is unlikely and no evidence of the third is to be found in Statman's (1987) paper, although he does refer to an unpublished paper by Black (1982) who argued that people keep their money in different pockets and display risk aversion-seeking in some pockets and not others. The second reason seems most promising.

Levy and Sarnat (1990:275-276) argue that in practice the degree of diversification is limited by two factors:
- investors may not have the resources necessary to hold a fully diversified portfolio; and,
- there are information costs associated with holding a large portfolio, i.e. it is expensive to keep track of all the shares in a portfolio.
It is these two reasons that is the answer to Statman (1987) the costs associated with holding a diversified portfolio restrict the number of shares that the individual can hold.

3.1.2.1 GAINS FROM DIVERSIFICATION

Recall that the variance of a portfolio is:
\[ \sigma_p^2 = \Sigma X_i \sigma_i^2 + \Sigma X_i \Sigma X_j \sigma_{ij} \]
Recall also that covariance can defined as follows:
\[ \sigma_{ij} = \sigma_i \sigma_j \rho_{ij} \]
Thus, we can rewrite the variance of a portfolio as such:
\[ \sigma_p^2 = \Sigma X_i \sigma_i^2 + \Sigma X_i \Sigma X_j \sigma_i \sigma_j \rho_{ij} \]

Here we can see that the value taken on by \( \rho_{ij} \) can have an important effect on the variance of the portfolio. Since the correlation coefficient can vary between
positive and negative one, its value can have a large effect on the total variance of the portfolio. To further examine the effect of correlation coefficients upon diversification, consider the following example.\(^{19}\)

Consider two securities, A and B, characterised by the following properties:

**TABLE 3.6**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_A = 10)</td>
<td>(\mu_B = 20)</td>
</tr>
<tr>
<td>(\sigma_A^2 = 100)</td>
<td>(\sigma_B^2 = 900)</td>
</tr>
</tbody>
</table>

By substituting these variables into equation (3.6), and by making various assumptions regarding the value of \(\rho\), we are able to see the impact of \(\rho\) on diversification.

**TABLE 3.7**

<table>
<thead>
<tr>
<th>(X_A)</th>
<th>(E(R_p))</th>
<th>(\sigma_p^2), if (\rho = +1)</th>
<th>(\sigma_p^2), if (\rho = +.5)</th>
<th>(\sigma_p^2), if (\rho = 0)</th>
<th>(\sigma_p^2), if (\rho = -.5)</th>
<th>(\sigma_p^2), if (\rho = -1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
<td>900</td>
</tr>
<tr>
<td>0.2</td>
<td>18</td>
<td>676</td>
<td>628</td>
<td>580</td>
<td>532</td>
<td>484</td>
</tr>
<tr>
<td>0.4</td>
<td>16</td>
<td>484</td>
<td>412</td>
<td>340</td>
<td>268</td>
<td>196</td>
</tr>
<tr>
<td>0.6</td>
<td>14</td>
<td>324</td>
<td>252</td>
<td>180</td>
<td>108</td>
<td>36</td>
</tr>
<tr>
<td>0.8</td>
<td>12</td>
<td>196</td>
<td>148</td>
<td>100</td>
<td>52</td>
<td>4</td>
</tr>
<tr>
<td>1.0</td>
<td>10</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

**SOURCE:** Levy and Sarnat (1990: 273)

We can see from table three that there is an inverse relationship between the correlation coefficient and the degree of risk reduction.

\(^{19}\) This example is taken from Levy and Sarnat (1990: 272-273).
3.1.2.2 DIVERSIFICATION AND RETURN

By holding a diversified portfolio, the investor is able to substantially reduce the risk that he would be subject to if he only held one share. The ability to reduce total risk implies that the investor should not be rewarded for incurring the total risk. According to Modigliani and Pogue (1974:74) there is, "no economic requirement for the return earned to be in line with the total risk. Instead, we should expect realised returns to be related to that portion of security risk which cannot be eliminated by portfolio combination".

Thus, it would appear that only some of the risk inherent within a portfolio can be reduced, but not all risk. If we consider equation 3.5, we can deduce this, because as the number of stocks in the portfolio increases, the variance of the portfolio tends towards the average covariance of the stocks in the portfolio. Copeland and Weston (1963:191) write that "investors can always diversify away all risk except the covariance of an asset with the market portfolio....the only risk which investors will pay a premium to avoid is covariance risk". That risk that can be diversified away is referred to "unsystematic" risk and that risk that cannot be diversified away is referred to "systematic" risk (Sharpe 1964:138). This can be summarised as follows:

TOTAL RISK = SYSTEMATIC RISK + UNSYSTEMATIC RISK

and,

TOTAL RETURN = SYSTEMATIC RETURN + UNSYSTEMATIC RETURN.

If the investor held the market portfolio, that risk that was not diversifiable would be the risk associated with the market, i.e. the market risk. Sharpe (1964:143) refers to this risk as, "the risk resulting from swings in economic activity". As this is the only risk that the investor is unable to diversify, it is the only risk that should have a return associated with it.
3.1.2.3 DIVERSIFICATION ON THE JSE

The purpose of this section is to investigate the effects of diversification on portfolio standard deviation using data from the Johannesburg Stock Exchange (JSE). The classic Wagner and Lau (1971) methodology is employed and in the process a methodological shortcoming of these type of studies will be highlighted. We show that the number of securities required for total diversification of unsystematic risk is far higher than is generally thought. Given the costs associated with diversification it is unlikely that the average individual investor could hold a fully diversified portfolio.

In order to determine how many shares are required to derive (3.5), a sample of shares was drawn from the I-Net service. All those shares, where a total returns file (capital gains plus dividends) could be compiled for the period 1987 to 1992, were included (two hundred and forty one shares in total).

Shares were assigned at random to form equally weighted portfolios of the order $N = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50\}$. For every value of $N$ four hundred portfolios were formed. The standard deviations for each portfolio was calculated and an average calculated. The average standard deviation for an investor who holds a one share portfolio was also calculated (he would face a standard deviation of 0.12 on average).
The usual Wagner and Lau type diversification curve is shown in figure 3.1. It shows that the (mean) standard deviation of the portfolio returns are small beyond portfolio sizes of ten to fifteen securities. A comparison of figure 3.1 to the Wagner and Lau (1972:49) results (shown in table 3.8) reveals that while the absolute level of (mean) risk is higher for corresponding portfolio sizes, that the rate of risk reduction brought about by diversification is greater than that in the US. The comparison of portfolio standard deviations to those found by Wagner and Lau (1971:49) indicate that to face the same level of absolute risk that a NYSE investor faces with just one share, the JSE investor must hold five shares on average. Given that the NYSE investor is able to diversify internationally, the risk estimates calculated by Wagner and Lau (1971) are overstated. Correspondingly, our results reflect the high costs that South African investors bear due to exchange control (for more evidence, see Bhana 1987).
<table>
<thead>
<tr>
<th>N</th>
<th>WAGNER &amp; LAU σ</th>
<th>Δ%</th>
<th>JSE σ</th>
<th>Δ%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.070</td>
<td>100</td>
<td>0.119</td>
<td>100</td>
</tr>
<tr>
<td>5</td>
<td>0.046</td>
<td>65.7</td>
<td>0.071</td>
<td>59.2</td>
</tr>
<tr>
<td>10</td>
<td>0.042</td>
<td>60.0</td>
<td>0.062</td>
<td>51.9</td>
</tr>
<tr>
<td>15</td>
<td>0.040</td>
<td>57.1</td>
<td>0.059</td>
<td>48.9</td>
</tr>
<tr>
<td>20</td>
<td>0.039</td>
<td>55.7</td>
<td>0.057</td>
<td>47.5</td>
</tr>
<tr>
<td>25</td>
<td>N/A</td>
<td>N/A</td>
<td>0.056</td>
<td>46.9</td>
</tr>
<tr>
<td>30</td>
<td>N/A</td>
<td>N/A</td>
<td>0.055</td>
<td>46.4</td>
</tr>
<tr>
<td>35</td>
<td>N/A</td>
<td>N/A</td>
<td>0.055</td>
<td>46.0</td>
</tr>
<tr>
<td>40</td>
<td>N/A</td>
<td>N/A</td>
<td>0.054</td>
<td>45.7</td>
</tr>
<tr>
<td>45</td>
<td>N/A</td>
<td>N/A</td>
<td>0.054</td>
<td>45.5</td>
</tr>
<tr>
<td>50</td>
<td>N/A</td>
<td>N/A</td>
<td>0.054</td>
<td>45.3</td>
</tr>
</tbody>
</table>

The Wagner and Lau (1971) approach assumes that the (mean) portfolio standard deviations can proxy for population standard deviations. There is nothing wrong with this in principle, as population parameters nearly always have to be estimated using some type of sampling technique. It ignores the fact that estimators are statistics which are variable in themselves. With this in mind, ninety per cent confidence intervals on the standard deviations for the various portfolios were calculated. The results of this exercise can be seen in figure 3.2.
Here we see that as the number of shares in the portfolio increases so the standard deviation of the portfolio decreases on average (the standard Wagner and Lau result). We also see that the variability of the standard deviation declining (for five shares the ninety per cent confidence interval is 0.0399 as opposed to 0.0098 for fifty shares).

The inference to be drawn from figure 3.2 is that since equally sized portfolios have differing standard deviations they by definition have highly variable risk. The effect of diversification is to reduce risk, but via foresight and investigation it is possible, for a given portfolio size, to reduce risk even further. In figure 3.3, we see that the average return on the various portfolios is relatively constant, but that the variability of the returns for a given portfolio size is quite large. This variability declines up to portfolios of around fifty shares. We refer to this (diversifiable) risk as "second order risk".
It seems that on average the investor is not rewarded for the variability in portfolio standard deviations. This indicates that second order risk is also susceptible to diversification, but the number of securities necessary for this is far larger than a naive reading of Wagner and Lau (1971) would suggest, yet is substantially less than the entire market.

From our results several observations can be made. The first is that (almost) half the mean standard deviation is diversified away when ten shares are held in a portfolio as opposed to holding just one share. Thus there are distinct advantages to holding a portfolio. Yet, we see that reductions from diversification are small after about ten to fifteen shares. Given the transactions and the (portfolio) monitoring costs associated with a portfolio of that size, it is unlikely that the average individual could profitably hold a larger portfolio. Given Firer's (1988) results it appears that individual investors do not hold portfolios much larger than ten shares. Yet at this portfolio size second order risk is high. It appears from table 3.9 that the investor with a ten share portfolio could be subject to twenty three per cent more risk than is implied by mean standard deviation, but for which he receives no return. Investors are thus constrained to hold portfolios that are not fully diversified.
TABLE 3.9: SECOND ORDER RISK AS A % OF FIRST ORDER RISK

<table>
<thead>
<tr>
<th>SIZE</th>
<th>SECOND ORDER RISK %</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>56.12</td>
</tr>
<tr>
<td>10</td>
<td>45.56</td>
</tr>
<tr>
<td>15</td>
<td>35.72</td>
</tr>
<tr>
<td>20</td>
<td>31.16</td>
</tr>
<tr>
<td>25</td>
<td>30.27</td>
</tr>
<tr>
<td>30</td>
<td>24.95</td>
</tr>
<tr>
<td>35</td>
<td>23.60</td>
</tr>
<tr>
<td>40</td>
<td>22.29</td>
</tr>
<tr>
<td>45</td>
<td>19.33</td>
</tr>
<tr>
<td>50</td>
<td>18.17</td>
</tr>
</tbody>
</table>

A disturbing implication of this research is that were the investor to increase his portfolio from (say) five shares to ten (hoping to benefit from additional diversification), he could find that the standard deviation on his portfolio increases. This indicates that once the decision has been taken to hold more than one share, the choice of shares becomes vital. This in turn implies that the investor will need to incur search costs when considering adding additional shares to his portfolio. It is not enough simply to throw a dart at a newspaper.

The individual investor can, via diversification, reduce the first order risk, i.e. achieve the Wagner and Lau (1971) result, but would still bear the second order risk - which is diversifiable only by incurring the high cost of large portfolios or expert advice for smaller portfolios. We can conclude then that the market is biased against independent small investors, which explains why they either avoid the market or are entering it via unit trusts, etc. In addition it explains why unit trusts hold so many stocks. The large institutional investors are able to diversify away second order risk. This then is the valuable service that they provide to society. Instead of decrying their domination of the market, we should welcome their existence as they provide a market-related
mechanism whereby second order risk is diversified.

3.2 PORTFOLIO SELECTION

In section 3.1 we demonstrated Markowitz's (1952) definitions of risk and return, we can now turn to how portfolios are constructed using those expressions. He argued that in order for this method to be used that two conditions must exist, viz.:
- the investor must want to maximise return and minimise risk; and,
- "reasonable" $\mu_i$ and $\sigma_i$ must be established (Markowitz 1952:93).

These two conditions are quite onerous, Witt and Dobbins (1979:159) have expanded them into four assumptions that underlie the Markowitz model:
- the return of the investment summarises the outcome of the investment, where investors conceptualise a probability of returns;
- the variance (or standard deviation) of the probability distribution is an adequate measure of risk;
- investors are only interested in risk and return as defined by Markowitz; and
- investors are risk averse.

Once these conditions are met, we are able to generate "efficient frontiers". These are a locus of points representing those portfolios that fulfil at both of the following criteria:
- the maximum level of return given a certain level of risk; and,
- the minimum risk given a certain level of return.

The rational investor will not consider the entire opportunity set, but will only consider those portfolios that dominate all others. Those portfolios that dominate all others are known as the efficient set and they lie along the efficient frontier. Copeiand and Weston (1983:165) define the efficient set as follows: "The efficient set is the set of mean-variance choices from the investment opportunity set where for a given variance (or standard deviation) no other investment opportunity offers a higher mean return."

The investor will wish to choose that portfolio that meets his risk-return
preferences. As noted by Copeland and Weston (1983:162) this will occur where the investors marginal rate of substitution (for risk and return) is equal to the marginal rate of transformation (for risk and return).20

It would appear that given the Markowitz contribution the problem of portfolio selection is solved. This is far from the truth. In order to generate an efficient frontier a vast amount of information is necessary. Levy and Sarnat (1984:356) state that there are both estimation and technical problems associated with the Markowitz model. The inputs to the model are as follows:

- $\mu_i$, the expected rate of return on stock $i$;
- $\sigma^2_i$, the variance of the return of $i$;
- $\sigma_{ij}$, the covariance between the return on $i$ and $j$; and,
- $R_f$, the risk free rate of interest.

Levy and Sarnat (1984:356) point out that these parameters are unknown and need to be estimated. They demonstrate (Levy and Sarnat 1984: 358) that the general formula for determining the number of calculation in order to make use of the Markowitz model is:

$$N = \left( n^2 + 3n + 2n + 2 \right)/2$$

where, $N = \text{number of estimators required}$; and,

$$n = \text{number of risky assets under consideration}.21$$

Markowitz (1952:100-101) wrote that, "[t]o use the [M-V] rule in the selection of securities we must have procedures for finding reasonable $\mu_i$ and $\sigma_{ij}$. These procedures, I believe, should combine statistical techniques and the judgement of practical men". Later, Markowitz (1991:469) had this to say, "It is theory which can be used to direct practice, at least by large (usually institutional) investors with sufficient computer and database resources" (emphasis added).

---

20 Graphically this implies that an indifference curve is tangential to the efficient frontier.

21 According to the JSE Handbook (August 1991), there are 838 companies listed on the board. If an investor wished to include all these shares in his calculation, it would be necessary to perform 352 380 calculations.
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Name of thesis  The capital asset pricing model and arbitrage pricing theory on the Johannesburg Stock Exchange  1993

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