determined exogenously. When $\lambda_0$ is assumed to be six per cent, then Roll and Ross (1980:263) find the following (the group percentage shows those groups where the corresponding number of factors were significant):

<table>
<thead>
<tr>
<th>NUMBER OF FACTORS</th>
<th>ONE</th>
<th>TWO</th>
<th>THREE</th>
<th>FOUR</th>
<th>FIVE</th>
</tr>
</thead>
<tbody>
<tr>
<td>GROUP %</td>
<td>88.1</td>
<td>57.1</td>
<td>33.3</td>
<td>16.7</td>
<td>4.8</td>
</tr>
</tbody>
</table>

At the $\alpha = 0.05$ level, the fifth factor is not significant. Similarly, Roll and Ross (1980:263) argue that the 16.7 per cent statistic for four factors exceeds 9.75 per cent that this has occurred purely by chance. They are able to conclude that at least three factors are sufficient for pricing, but that it was unlikely that more than four factors were priced.

When $\lambda_0$ is estimated from the data, Roll and Ross (1980:264) report that two factors are significant in 47.6 per cent of the groups, while three or more factors are significant in 7.1 per cent of the groups.

The issue of interest is whether or not the three to five factors found in the forty two groups are the same or whether each portfolio had three to five unique factors (this last idea, while conceivable is unlikely). Roll and Ross (1980:270) report that due to the mathematics of factor analysis it is impossible to determine whether these factors are the same or not. It is possible, however, to determine whether or not $\lambda_0$ differs across groups. Roll and Ross (1980:271) are able to show that the $\lambda_0$ are not significantly different from each other. This is evidence that the other factors are probably the same. Roll and Ross (1980:271) concede that this is a very weak test.

The evidence cited above broadly supports the APT. Roll and Ross (1980:266) also report some evidence contra to the APT. They demonstrate that the "own variance" of returns is a priced factor. This violates the principle of arbitrage that underlies the APT. They argue that this result may be as a result of skewness in the (daily) data.
From these (humble) beginnings, the empirical research into the APT has exploded. Unfortunately, as we shall see, this line of research has not been as successful as was hoped. Conway and Reinganum (1988:1) state that "determining the number of factors underlying security returns has proved to be an elusive and difficult endeavour."

Kryzanowski and To (1983) also used factor analysis to test the APT. They argue (Kryzanowski and To 1983:35) that the alpha factor analysis and the Rao (canonical) factor analysis are the better variants in the classical factor analysis to use in this situation.

Kryzanowski and To (1983:38) drew eleven fifty-security portfolios from the CRSP tapes at random. The sample consisted of the monthly returns for the period 1948-1977. The empirical results of the alpha and the Rao factor analysis techniques were very similar. Kryzanowski and To (1983:39) find that not only do both techniques identify a similar number of factors, but that each technique is able to explain approximately the same percentage of variance. It appears from their table one and two (Kryzanowski and To 1983:40-41) that there may be up to ten factors. They argue, however that factors beyond the fifth are either trivial or non-general (Kryzanowski and To 1983:42).

Kryzanowski and To (1983:44) also test whether or not the number of factors that are "found" are a function of the size of the factor model under consideration. In order to test this hypothesis, they create four portfolios with ten, twenty, thirty and forty securities and apply the alpha and Rao techniques to them. They are able to conclude that the number of factors discovered is a function of the size of the portfolio under consideration. They state that the this finding does not necessarily mean that the number of nontrivial factors is a function of the number of securities in the portfolio (Kryzanowski and To 1983:45).

Given the low explanatory power of the factors beyond the fifth, Kryzanowski
and To (1983:48) are able to conclude that "it seems reasonable to hypothesize that a factor structure of five factors is sufficient from an economic perspective". They go on to argue that perhaps a one or two factor model is sufficient to explain returns in the market. This is in keeping with Roll and Ross (1980:264) who find some evidence for two factors (when $\lambda_0$ is estimated). Generally, this paper can be interpreted as being support for the APT, like Roll and Ross (1980) they are unable to determine whether or not the factors remain constant across the various portfolios.

Cho, Elton and Gruber (1984) simulate a return generating process using $\beta$'s calculated by the Wilshire group. The reason for this is to examine the robustness of the Roll and Ross (1980) technique. They also examine actual data drawn from the daily return CRSP file for the period 1973-1980. Fifty eight portfolios with thirty securities each were formed on an alphabetical basis and the risk free rate is taken as daily quotations on ninety day certificate of deposit (Cho, Elton and Gruber 1984:2).

By using the same (maximum likelihood) factor analysis technique as did Roll and Ross (1980), Cho, Elton and Gruber (1984:6) are able to show that there are more factors present in the real data than in their simulated data (based on the zero-$\beta$ CAPM). They report (Cho, Elton and Gruber 1984:2) that the Roll and Ross (1984) technique does have a slight tendency to overstate the number of factors. Their findings lend support to the APT in general. Cho, Elton and Gruber (1984:7) find that more factors (five to six) than the number identified by Roll and Ross (1980) are necessary to explain returns. This, they ascribe to a possible change in the covariance structure of returns (Cho, Elton and Gruber 1984:7).

Dhrymes, Friend and Gultekin (1984:323) argue that many of the problems associated with the CAPM are also prevalent in the tests of the APT. The "major part of the problem", they argue, is as a result of limitations in computer software - it appears that their criticisms arise from this point and not from
any inherent defect in the theory. They (Dhrymes, Friend and Gultekin 1984:324) introduce three criticisms of the Roll and Ross (1980) study:
- in order to test the APT one must first be able to specify the underlying factor structure a priori;
- testing thirty security portfolios is not equivalent to testing the entire market as sources of covariance between securities may be eliminated and consequently not identified in the subsequent analysis; and,
- there is a positive relationship between the number of factors discovered and the size of the sample analyzed.

Dhrymes, Friend and Gultekin (1984:325) note that the theoretical proof of the APT requires a "strong" law of large numbers, given this it is reasonable to question, whether all factors would be priced in a subset or whether the a subset of factors would be priced within an abbreviated environment

Dhrymes, Friend and Gultekin (1984:337-339) replicate the Roll and Ross (1980) study using the same data set as the original study (provided to them by Roll). They use the data set (which they claim has missing observations) to replicate the factor analysis of Roll and Ross (1980), but using larger portfolios. They are able to show, as did Kryzanowski and To (1983) that the number of factors is a positive function of the size of the sub-sample. Dhrymes, Friend and Gultekin's (1984:340) results are as follows:

<table>
<thead>
<tr>
<th>Size of portfolio</th>
<th>15</th>
<th>30</th>
<th>45</th>
<th>60</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of factors</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>9</td>
</tr>
</tbody>
</table>

Roll and Ross (1984:347) respond to the Dhrymes, Friend and Gultekin (1984) paper by observing that their first criticism is really a value judgement and cannot be counted on rational grounds. They respond only to the second and third criticisms in a non-technical manner.

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62 This is the Chamberlain (1983) argument.
The second and third criticisms levelled by Dhrymes, Friend and Gultekin (1984) are closely related to each other. The two can be combined into the argument that factor analysis is not appropriate for testing the APT. Roll and Ross (1984:348) argue that at present, using subsets of data is the only feasible method of testing the APT. In any event, they also argue that factor analysis is only one method of testing the APT. They also make the argument that the total number of factors that are "found" by the analysis is not important - a positive relationship between the size of the portfolio and the factors is to be expected. It is the number of priced factors that is of interest (Roll and Ross 1984:349).

In an attempt to circumvent some of the criticisms of Dhrymes, Friend and Gultekin (1984), Cho (1984) employs a technique known as inter-battery factor analysis. In this technique the factor analyses is performed on the inter group correlation matrix of two portfolios. In this way the factors that are extracted are constrained to be the same between groups.

Using daily data for the period 1962-1972 drawn from the CRSF tapes, Cho (1984:1492-1493) forms twenty two industry portfolios on the basis of standard industrial classification codes. The reason for forming industry codes is to make the portfolios as different from each other as possible. This method also ensures that the factors that are extracted are not industry specific factors.

Using this methodology and data base, Cho (1984:1490) investigates five hypotheses:
- the risk-free rate is the same between groups;
- the risk premia is the same between any two groups;
- the risk-free rate and the risk premia are the same between groups;
- the risk-free rate ≠ 0, while the risk premia are the same between groups;
- the risk-free rate ≠ 0, while the risk premia are not constrained to be equal.
Cho (1984:1495) reports that depending on which of the groups are being compared there are between two and nine factors. Contrary to Dhrymes, Friend and Gultekin (1984) there appears to be no relationship between the number of factors found and the size of the sample size.

Cho (1984:1497-1498) reports that at the $\alpha = 0.05$ level, the results of the first three hypotheses seem to violate the APT. Yet, Cho (1984) argues that this is support for the APT due to the fact that the samples are not strictly independent and the possibility of the daily data violating the assumption of normality distorting the significance tests. The results for hypotheses four and five support the validity of the APT.

Cho (1984:1499) concluded that the APT can be tested adequately if were possible to use this analysis for more than two groups at a time. This he states should be a topic for further research.

Pari and Chen (1984) also make use of industry portfolios - in an attempt to identify the nature of the factors. They use monthly returns data for the period 1975-1980 taken from the COMPUSTAT files. The reason for the use of monthly data being that daily return data does not conform to the multivariate normal assumption of the testing methodology (Pari and Chen 1984:122).

Pari and Chen (1984:123) form industry portfolios, which allows them to investigate the factor structure underlying the major industries in the economy and to estimate the systematic factors, using canonical factor analysis. Pari and Chen (1984:126) find evidence of a three factor model (significant at $\alpha = 0.01$) that explain eighty four per cent of the total variability in returns. They also calculated estimates for the sensitivities to each factor by regressing share returns on the corresponding factor scores.

They report that the majority of industries are most highly correlated to the first factor which is identified as a "general market index or indicator of overall..."
economic activity" (Pari and Chen 1984: 126-127). The second factor appears to be an "energy factor" i.e. the price of energy or the variability of the that price may be a source of systematic risk. The third factor is identified as being "a measure of interest rate risk" (Pari and Chen 1984:127).

This paper represents the high point of the factor analysis approach to APT. Subsequent papers took on a more subtle approach to calculating the number of factors inherent within the data. Trzcinka (1986:347) was able to argue that previous studies had been joint tests of APT pricing and the number of factors assumed or discovered by the researcher. Trzcinka (1986) focuses specifically on the number of factors within the data.

Trzcinka (1986:348) argues that there is a relationship between the number of factors and the number of "exploding" eigenvalues as the number of securities in the covariance matrix increases. This presents a method of estimating the number of factors necessary in the APT - compute the eigenvalues for ever increasing covariance matrices and determine if the number of eigenvalues is increasing or if the number is constant but exploding. If the number is constant (and exploding), then this is the number of factors necessary for the APT, but if the number of eigenvalues is not constant it violates the assumption of linearity in the return generating process (Trzcinka 1986:348).

Trzcinka (1986:349-350) makes use of weekly data for the twenty year period ending 1983 (taken from the 1983 CRSP tapes). He constructs ten portfolios beginning with a fifty stock portfolio, the second portfolio includes the first but has an additional fifty and so on until ten portfolios have been formed. The sequence of portfolio sizes is as follows: n = [50, 100, 200, 300, 400, 500, 600,

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63 This argument is based on Chamberlain and Rothschild (1983:1284), where it is noted that when performing a factor analysis that "eigenvectors corresponding to the exploding eigenvalues converge to factor loadings".

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By investigating the eigenvalues of the subsequent portfolios, Trzcinka (1986:350) is able to examine the argument made by Dhrymes, Friend and Gultekin (1984) that the factors discovered increase with portfolio size.

Trzcinka (1986:354) reports that there is an "obvious relationship" between the number of factors found and portfolio size. From his table III, Trzcinka (1986:359) shows that while the eigenvalues increase as the portfolio size increases, the first eigenvalue appears to explode. This indicates that the APT is not an "empty" theory - if there were no dominant eigenvalues then the validity of the theory could be questioned (Trzcinka 1986:348 and 366).

Trzcinka (1986:358) also argues that it is possible for the second largest to the fifth largest eigenvalues to be of economic significance. The eigenvalues below that in the fifth rank are not likely to be significant. This result, Trzcinka (1986:358-360) argues adds plausibility to those studies that have found up to five factors.

Although Trzcinka (1986:367) finds only one eigenvalue to be statistically important, he argues that it is not correct to argue that the APT should be a one factor model. He gives three reasons for this:
- his method is a test of the assumptions of the model, it does not predict the number of factors necessary to price securities;
- he has assumed that the underlying factor structure is constant over a twenty year period; and,
- the term "eigenvalue" used in the Chamberlain and Rothschild (1983) article is not mathematically equivalent to the term used in factor analysis.

He concludes by arguing that his test can be interpreted broadly as meaning that there is at least one or more factors defined by an exploding eigenvalue. In corroboration of this point, Brown (1989:1247) is able to show that the larger the number of equally important factors, the more likely it is that "a casual" investigator would argue that only one is responsible for returns.
5.3.2.1 IS THE APT TESTABLE?

Shanken (1982) has posed the question of whether the APT is testable or not. By making use of a two security example he answers in the negative. The reasons for this answer are as follows:

- equivalent sets of securities may have different factor structures; and,
- the usual empirical formulation\(^{64}\) of the APT may lead to differing implications for any given set of securities (Shanken 1982:1134).

In order for the implications of the APT to be consistent across differing sets of securities, all securities must have the same expected return. This observation eliminates the purpose of the theory. Based on these observations, Shanken (1982:1135-1136) argues that in order to test the APT we need to specify a priori the underlying factor structure of returns i.e. we are faced by a pseudo-Roll problem.

Almost three years later, a reply was forthcoming from Dybvig and Ross (1985c). They begin their discussion with the following comment:

"In the social sciences, as in the physical sciences, the question of testability of a theory is independent of the question of the accuracy of the approximations used in its derivation" (Dybvig and Ross 1985c:1176).

Dybvig and Ross (1985c:1177) also make the point that as the APT relies on the law of large numbers that it cannot be captured in a two security example, indeed such a portfolio is unlikely to be M-V efficient. They further argue that the APT can only make sense under the following conditions:

- there are many assets;
- all factors are significant;

\(^{64}\) Shan ken (1982:1132) argues that the usual empirical formulation of the APT is, if: \( r_i = E_i + \beta_i \delta_i + \ldots + \beta_k \delta_k \) then: \( E_i = \gamma_0 + \sum \beta_i \gamma_i. \) Acceptance or rejection of this formulation does not represent acceptance or rejection of the APT per se.
- idiosyncratic variances are not too large; and,
- the number of securities is at least twice as large as the number of factors (Dybvig and Ross 1985c:1178 and 1179).

Based on these comments, they conclude that Shanken's (1982) analysis has little relevance in actual tests of the theory.

In response to the problem of testing the APT on subsets of data, Dybvig and Ross (1985c:1183-1184) argue that, "[i]f the factors are estimated from exogenous economic data ..., the APT is always testable on subsets, and pooling assets will still yield an unbiased ... test". It is to these studies that we now turn.

5.3.3 **EXOGENOUS APPROACHES TO THE APT**

These approaches are necessary because as Brown (1989:1261) has pointed out, "mechanical application of purely statistical approaches to determining the number of pervasive factors ... may lead to false inferences ... [I]t seems that economic analysis and intuition are essential ingredients to this process."

The exogenous approach to testing the APT flows from the economic rationale for the APT. Given that investors buy future cashflows and that information drives the market, it seems reasonable to establish the nature and identity of that information. Chen, Roll and Ross (1986:384) stated that while the structure of asset returns seemed to indicate that share returns responded to "exogenous influences", that "a rather embarrassing gap exists between the theoretically exclusive importance of systematic "state variables" and our complete ignorance of their identity". The advantages of using this approach to the APT is two-fold:

- factors and their prices can be given an economic meaning; and,
- stock price behaviour can be linked to macroeconomic events (Burmeister and McElroy 1988:721).

Berry, Burmeister and McElroy (1988:30) argue that there are three properties
that an economic variable must contain before it can be considered within the APT framework:

- the factors must be unpredictable;
- the factors must have a pervasive effect on returns; and.
- the factors must influence expected returns.

These properties indicate that the factors would be of a macro-economic nature.

The first property implies that it is unexpected changes in economic variables that will cause prices to change, as is obvious from the EMH. Roll and Ross (1984:15) argue that expected changes in these variables will have included in the expected return portion of (5.1). The second property implies that it is factors that impact on either the denominator and numerator of (5.7) in all firms that affect share returns. The third factor is obvious from a theoretical perspective, but leads to econometric problems - as no two sets of researchers have agreed on the "common pervasive factors" as yet.

The first published paper to use the exogenous methodology was that of Chen, Roll and Ross (1986). They used a variation of the Fama and MacBeth (1973) methodology to test a number of macro-variables for economic significance. Share return exposures to economic variables were estimated using regression analysis. Returns were then regressed on these exposures ($\beta$'s) to establish whether the factor was priced.

Chen, Roll and Ross (1986:395) report that unexpected changes in industrial production, the risk premium and the yield curve; unanticipated inflation and changes in expected inflation are significant in pricing returns. These factors are economically plausible. The factor which is surprising in its absence is oil prices. They (Chen, Roll and Ross 1986:401) explain this by arguing that there is no a priori reason for the view that oil prices are more important in pricing

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65 That methodology will not be discussed in great detail here, as it is discussed in the next chapter.
stocks than, say, industrial production.

In a similar study, Burmeister and Wall (1986:4 and 9) report that the following factors are priced:
- unexpected changes in risk premia or default risk;
- unexpected changes in the term structure of interest rates;
- unexpected changes in inflation;
- unanticipated changes in the growth rate of final sales; and,
- any unanticipated change in the market index not explained by the first four factors.

These factors are similar to those found by Chen, Roll and Ross (1986). Berry, Burmeister and McElroy (1988:30-31) have argued that there is no "correct" set of factors. Any set of factors that "qualify" i.e. meet the requirements for being an APT factor, can be included in the APT. Any concern that not all the factors are included can be overcome by including a "residual market factor" such as unanticipated changes in the market index not explained by any of the other factors, as Burmeister and Wall (1986:9) did.

5.3.3.1 ECONOMIC THEORY AND EXOGENOUS FACTORS

It is not sufficient to merely list the factors that have been "discovered". It is necessary to investigate each, in order to determine a priori any impact they can have on share returns and shed some light on the factor loading coefficients. The purpose of this section is not to provide a comprehensive analysis of the variables and survey of the literature. It merely serves to illustrate that potential relationship between the variable in question and share returns.

5.3.3.1.1 INDUSTRIAL PRODUCTION/SALES

Zarnowitz (1985:531) lists share returns as being a leading indicator in business cycles. This fact is obvious, following from our discussion on the
EMH. This has been empirically substantiated by researchers, such as Fama (1990:1089) who argues that large fractions of annual share return variances can be explained by macro-variables such as real GNP, industrial production, and investment. It is these variables that either affect or proxy for cash flows to firms.

Fama (1990:1089-1090) reports that growth rates in industrial production explains more than forty three per cent of annual share return variances. The most obvious explanation of the phenomena is that of consumption smoothing. Balvers, Cosimano and McDonald (1990:1110) explain that consumption opportunities are linked to industrial output. Investors will smooth their consumption by adjusting their required rate of return on financial assets.

Using this insight, Balvers, Cosimano and McDonald (1990:1110) argue that as industrial production is serially correlated, that it is also predictable. Given the theoretical linkage between production and share returns, there must also be a predictable component to share returns. Balvers, Cosimano and McDonald (1990:1118-1119) provide empirical evidence in favour of this hypothesis, (but argue that this predictability is not a violation of the EMH, as in order to exploit this phenomena investors would have to forego smoothing their consumption). Fama (1990:1106) also provides evidence in favour of this hypothesis, that when the marginal return on capital falls with the level of investment that consumption smoothing will lead to declining share returns when income is high and increasing share returns when income is low.

5.3.3.1.2 INFLATION

Geske and Roll (1983:1) state that the relationship between stock returns and inflation is "puzzling" - the empirical results are in conflict with "both economic..."

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66 It can be argued that final sales and industrial production are proxies for the same economic factor, as such they are considered in the same section.