inclusive and adjusting for dividends in a rough and ready way\textsuperscript{55}, mean returns for each day of the week were calculated. It is not possible to discern any Monday effect on the JSE.

<table>
<thead>
<tr>
<th>WEEKDAY RETURN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1986-1991</td>
</tr>
<tr>
<td>Monday</td>
</tr>
<tr>
<td>Tuesday</td>
</tr>
<tr>
<td>Wednesday</td>
</tr>
<tr>
<td>Thursday</td>
</tr>
<tr>
<td>Friday</td>
</tr>
<tr>
<td>Mean Returns</td>
</tr>
<tr>
<td>0.057</td>
</tr>
<tr>
<td>0.040</td>
</tr>
<tr>
<td>0.218</td>
</tr>
<tr>
<td>0.082</td>
</tr>
<tr>
<td>0.037</td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>295</td>
</tr>
<tr>
<td>305</td>
</tr>
<tr>
<td>305</td>
</tr>
<tr>
<td>301</td>
</tr>
<tr>
<td>295</td>
</tr>
</tbody>
</table>

Returns are calculated as \((P_1 - P_0 + D_1)/P_0\)

As can be seen, the mean return for Monday is not even the lowest for the week. The positive return on Monday indicates that there should be no Monday effect - nonetheless, an ANOVA was conducted to determine whether or not a Monday effect was noticeable. The result was unsurprising, the hypothesis that Monday is no different from any other day could not be rejected \((F = 1.13; p = .34\%)\).

In a similar vein, Bradfield (1990a) investigating seasonality on the JSE has found no January effect. He does find evidence in favour of December and July effects. Bradfield (1990a:8) argues that the July effect is an artifact of the sample period. The December effect he hypothesises is as a result of lacklustre trading as that month is traditionally a holiday month in SA\textsuperscript{56}.

Using monthly ALSI returns (excluding dividends) for the period 1970-1991, the average return and standard deviation for the twelve months of the year.

\textsuperscript{55} In common with most markets, the last day to register tends to fall on the last trading day of the week. Dividends on the shares making up the index were assumed to accrue evenly to the shareholders on the last trading day of the week. The additional gain from dividends was added to the return for the first trading day of the next week.

\textsuperscript{56} This suggests that there may be a holiday effect in SA. As yet this hypothesis has not been tested using JSE data. Bhana is said to have submitted such a study for publication. I have been unable to contact him regarding the results.
were calculated. The results are shown below in table 4.3. It can be seen that the returns for four months of the year, January, April, August and October are negative. The negative average return for January is inconsistent with the hypothesis of a January effect. This result contradicts Bradfield's (1990a:8) view that the standard deviation in December is low (due the fact that December is characterised by thin trading).57 Potential reasons for this include the different period under consideration, Bradfield (1990a:7) considers the period 1974-1984, and the use of different indices (Bradfield 1990a:7 constructs his own equally weighted indices).

TABLE 4.5: MONTHLY RETURNS AND STANDARD DEVIATIONS FOR 1970-1991

<table>
<thead>
<tr>
<th>MONTH</th>
<th>AVERAGE RETURN</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>JANUARY</td>
<td>-0.00988</td>
<td>0.1205502</td>
</tr>
<tr>
<td>FEBRUARY</td>
<td>0.005293</td>
<td>0.0832409</td>
</tr>
<tr>
<td>MARCH</td>
<td>0.001882</td>
<td>0.1097082</td>
</tr>
<tr>
<td>APRIL</td>
<td>-0.00514</td>
<td>0.0851519</td>
</tr>
<tr>
<td>MAY</td>
<td>0.010873</td>
<td>0.0739358</td>
</tr>
<tr>
<td>JUNE</td>
<td>0.013536</td>
<td>0.0895928</td>
</tr>
<tr>
<td>JULY</td>
<td>0.028291</td>
<td>0.0988342</td>
</tr>
<tr>
<td>AUGUST</td>
<td>-0.00603</td>
<td>0.1309521</td>
</tr>
<tr>
<td>SEPTEMBER</td>
<td>-0.046231</td>
<td>0.0865948</td>
</tr>
<tr>
<td>OCTOBER</td>
<td>-0.00347</td>
<td>0.0951873</td>
</tr>
<tr>
<td>NOVEMBER</td>
<td>0.009806</td>
<td>0.0997560</td>
</tr>
<tr>
<td>DECEMBER</td>
<td>0.032586</td>
<td>0.1138849</td>
</tr>
</tbody>
</table>

In order to put to rest any arguments as to whether any month of the year is different from any other, a t-test and a Kolmogorov-Smirnov two-sample test were performed. For the t-tests the null hypothesis was of no difference

57 In addition, our estimates of standard deviation differ radically from those of Bradfield (1990a:9).
between the months. The results revealed no difference at the $\alpha = 5$ per cent level of significance. At the 10 per cent level, the month of September was found to be different from January, April and October. The Kolmogorov-Smirnov null hypothesis was that the distributions of the months are identical. The alternate hypothesis was that the distributions for each of the months were different to each other. In no case, was it possible to reject the null hypothesis at the $\alpha = five$ per cent level. While this result differs from that of Bradfield's (1990a) study, the net effect is the same - no evidence of a January effect can be found.

As Bradfield (1989a:6) states, "not all of the aspects of [the CAPM] are directly translatable from the canon of established empirical findings on the NYSE to ... the JSE". It would appear that the various anomalies that characterise the NYSE are lacking the JSE. At present there is no explanation for this phenomena.\textsuperscript{59}

4.4 CONCLUSION

The words of Reinganum (1981a:45) seem to capture the essence of this chapter and it is fitting to quote them at this point:

"One must surely conclude that alternative models of capital market equilibrium ought to be seriously considered and tested. ...[T]he simple one-period capital asset pricing model is an inadequate empirical representation of capital market equilibrium".

In this chapter we have investigated the internal and external consistencies of

\textsuperscript{58} The nature of the difference was not specified.

\textsuperscript{59} In private conversation, David Bradfield has indicated that he thinks that the statistical tests that we have been using are too insensitive to detect these anomalies, especially in light of the thin trading problems on the JSE. These criticisms are valid for this paper. No account has been taken of the problem of thin trading and in comparison to US tests, the data base is small.
the CAPM. We have found that the CAPM is not internally consistent. As far as the external consistency is concerned we have seen that in the US, there are various phenomena that the theory is unable to explain. Within the SA context, these anomalies do not exist (with one exception).

Given the fact that the CAPM does seem to be flawed, a lot of research has been conducted in the US into a new theory. The conditions for a new theory have certainly been met. In the SA context, the evidence on the CAPM appears to be mixed and inconclusive, however at the end of the day the CAPM will always suffer from the restrictiveness of its assumptions and the fact that it cannot be tested. This provides one with the raison d'être to investigate one of those theories. In the next chapter we will begin our investigation of the arbitrage pricing theory.
"...it takes a theory to beat a theory," George Stigler (1982:145).
"Neither phlogiston nor the ether drift died quiet deaths, and the Capital Asset Pricing Model’s (CAPM’s) demise isn’t going to be any easier." Stephen Ross (1984:54)

Academic disenchantment with the CAPM has led to research into other models and theories that could explain returns in capital markets. The theory that shows the most promise is the arbitrage pricing theory (APT) of Ross (1976, 1977). Ross (1985:73) has argued that the APT has its base within common sense - "[t]he APT is based on the notion that the market prices assets by looking at central systematic risks... That is about as close to common sense as economics gets".

In this chapter, the APT is derived, both algebraically and theoretically. The evidence for and against the "common sense" theory of Ross (1976,1977) will be reviewed. The APT will be compared to the CAPM to determine whether or not it is superior to that model. Finally, that meagre research that has been conducted on the APT in SA will be discussed.

5.1 APT - A RIGOROUS DERIVATION

The APT is based on the law of one price. Ross (1977:190) points out that the CAPM is based on one of two assumptions:
- that share returns are multivariate normally distributed; or,
- economic agents have quadratic preferences.

He states that while neither of these assumptions are well founded, restrictions should be made on probability structures and not on investor preferences. These restrictions are justified by the fact that in the investment process there is one ex post result and a large amount of information that is common
knowledge.

The assumptions that underlie the APT are as follows:

- investors believe that share returns are generated by a model in the form:

  \[ x_i = E_i + \beta_{i1}\delta_1 + \ldots + \beta_{ik}\delta_k + \varepsilon_i; \]

  \[ x_i = \text{expected return on security } i; \]
  \[ E_i = \text{ex ante expected return}; \]
  \[ \beta_{ik} = \text{sensitivity to the risk factor}; \]
  \[ \delta_k = \text{mean zero ex post risk factor}; \]
  \[ \varepsilon_i = \text{random noise}. \]

and,


This latter assumption is not as stringent as it first appears. As long as investors are agreed on the impact that an economic variable has on share returns, the assumption is valid. They may have differing opinions as to the value that the factor may take on.

Harrington (1987:193) states that there are five assumptions underlying the APT:

- investors are risk averse and try to maximise their wealth;
- investors can borrow and lend at a risk-free rate;
- markets are frictionless;
- investors agree to the number and nature of common factors; and,
- there are no riskless arbitrage opportunities in the market.

The first three of these assumptions are common to the CAPM, but the last two are not. While these assumptions are "reasonable", the APT does not specify what the common factors are, nor does it give any indication of what they might possibly be. In order to discover what the factors might be, we need to investigate an economic rationale for the APT, but first we will show Ross's (1976,1977) original derivation.

Ross (1977:194-199) derives his model by making use of a special case of (5.1),
viz.: \( x_i = E_i + \beta_i \delta + \varepsilon_i; \ i = 1, \ldots, n. \)

Fogler (1982:22) argues that the APT includes elements that are both \textit{ex ante} in nature and \textit{ex post}. In essence, the APT states that investors expect a certain return, but that \textit{ex post} "factors" lead to returns that differ from those that were expected.

Ross (1977:195) shows that (5.1) "constitutes a far more satisfactory basis for a capital market theory without the additional baggage of mean variance theory". Consider the following situation: A portfolio that uses no wealth is constructed, i.e. some assets are sold short (or money is borrowed) and the proceeds are invested in other assets. This situation can be characterised as follows:

\[ e_\eta = 0 \quad (5.2) \]

where \( e \) = wealth invested in portfolio \( \eta \).

The return of this portfolio is as follows:

\[ R = x_\eta = E_\eta + \beta_\eta \delta + \varepsilon_\eta \]

If the portfolio is diversified: \( \varepsilon_\eta = 0. \)

Therefore \( R = E_\eta + \beta_\eta \delta \)

In addition to (5.2) above, assume the investor has been able to structure the portfolio so as to have no systematic risk:

\[ \beta_\eta = 0 \quad (5.3) \]

\[ \therefore R = E_\eta. \]

The investor has been able to manipulate the market and prices in order to provide himself with a money machine i.e. wealth has become a function of the investors preferences. Arbitrage within the market should preclude this from occurring. Thus, we can conclude:

\[ E_\eta = 0. \quad (5.4) \]

Ross (1977:197) argues that any portfolio that meets the requirements of (5.2) and (5.3) must also meet (5.4). Competition drives the market to equilibrium. One of the conditions of this equilibrium is that an investor cannot earn a
positive return, without making a non-zero investment or incurring additional risk.

On the basis of these formulae, Ross (1976:343 and 1977:197) shows that:
\[ E_t = E_o + \alpha \beta_i \]  
(5.5)

In this instance, it can be argued that \( E_o \) is the risk-free rate. In order to see this, consider a non-arbitrage zero-\( \beta \) portfolio i.e. \( \varepsilon_a = 1 \), but \( \beta_a = 0 \). In order to avoid arbitrage opportunities, \( E_t = E_o = R_t \).

Ross (1977:343) states that if \( \alpha \) is a particular portfolio of interest, eg. the market portfolio then (5.5) becomes:
\[ E_t = E_o + (E_m - E_o)\beta_i \]  
(5.6)

This is the equivalent of the SML equation, however, no assumptions have been made as to the mean-variance nature of \( \alpha_m \) or that the market was in equilibrium. There is no reason why the market portfolio should play an important role in this theory. It is only one of a number of common factors that could influence returns (but see section 5.4.2 below).

It is possible to generalise a 1-factor situation:
\[ x_i = E_t + \beta_i \delta + ... + \beta_k \delta_k + \varepsilon_i \]

where: \( E_t = E_o + (E_m - E_o)\gamma_1 \beta_1 + ... + \gamma_k \beta_k \)

such that \( \sum \gamma_i = 1 \).

This can be interpreted as meaning that the risk premium on the \( i \)th asset is a convex combination of its \( \beta \)-weight times the risk premium on the common factors.

It has been argued by Chamberlain (1983:1317) and others (eg. Lehmann and Modest 1988) that this representation of the APT requires an infinite number of assets. Lehmann and Modest (1988:215) argue that the effect of this is that the APT may not necessarily price the stocks in any particular portfolio correctly. Thus it is necessary to make additional assumptions to the model in order to convert it to a finite market theory. Chamberlain (1983:1312-1315)
provides the "missing link" between the Ross (1976; 1977) version and a finite market version. The necessary and sufficient criteria for APT pricing in a finite economy is a risky, well diversified portfolio on the M-V frontier. Chamberlain (1983:1318) adds that when this condition is met, the APT may be tested on a subset of assets.

For empirical purposes, this latter condition creates a pseudo-Roll (1977) problem. Lehmann and Modest (1988:217) argue that a rejection of the APT reflects the following joint hypothesis:
- the theory is rejected;
- the researcher has been unable to construct reliable estimates of the factors; or,
- both of the above.

The APT has been seen as an alternative to the CAPM's inherent problems. It seems that the Roll (1977) problem may persist.

5.2 AN ECONOMIC RATIONALE FOR APT

It is possible to approach the APT from both an algebraic or economic angle. In the last section we saw the algebraic derivation of the APT. In this section, we show the economic underpinnings of the APT. It is necessary to do so because in the final analysis all (economic) theories must be based on economics and not mathematical manipulation.

When an investor buys a share or a portfolio of shares, he is buying future cash flows from that share. Future cash flows can arise from one of two sources (hopefully cash flows will arise from both sources, however this cannot be guaranteed):
- future dividends; and,
- capital gains.

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This is the basis for an exchange between Shanken (1982;1985) and Dybvig and Ross (1985), but see below.
Gordon (1962:45) developed a model where the price of a share is the discounted value of dividends. A modern version of that model is taken from Burmeister and Wall (1986:1):

$$P_t = E_t \Sigma \frac{D_{t+k}}{(1 + \delta)^{k+1}}$$

(5.7)

where $P_t =$ price at time $t$;

$E_t =$ a conditional expectations operator given the information known at time $t$;

$D_{t+k} =$ the net cash flow available to shareholders at time $t + k$; and,

$\delta =$ a discount rate.

This model has given rise to some controversy in the past decade. Shiller (1981) argued that stock price indices appear to be excessively volatile compared to ex post dividends. He states (Shiller 1981:421) that, "the movements in stock price indexes could not realistically be attributed to any objective new information". Marsh and Merton (1986:485) countered that Shiller (1981) had misspecified the process that described the evolution of dividends and stock prices. The lack of volatility in dividends came about because of "smoothing". In any event, this model has been used in the APT literature.

If the investor holds a diversified portfolio, he would only have to consider systematic risk. The "core" of the APT is that there are a few systematic factors that affect systematic risk (Roll and Ross 1984a:15). As a result, returns of diversified portfolios are a function of those systematic factors. Given (5.7), above, it seems that it is those factors that will affect either cash flows or the discount rate, that could be sources of systematic risk.

When constructing a portfolio, the investor could consider the expected values of these "systematic factors" and plan accordingly. It follows that returns will be a function of both the expected value of these variables and any unexpected
changes in the expected value. Roll and Ross (1984a:15) argue that as it is impossible to forecast unexpected changes (by definition) we need to calculate sensitivities of asset returns to changes in "systematic factors".

What the APT states then, is that in order to explain share returns, we must calculate co-variances with a number of economic variables and not just with a single market portfolio.

5.3 **EMPIRICAL TESTS OF THE APT**

Dhrymes (1984:37) argues that there are four implications of the APT which may be falsified by empirical investigation. These are as follows:
- Are the number of common factors "small" and manageable? (if so then this is evidence in favour of the APT);
- Is there a reliable method of estimating the underlying factor structure of returns? (if no, then the theory is not falsifiable and suffers from the same defect as the CAPM);
- Is there a reliable technique for estimating the risk premia?; and,
- How much of the return on the market does the APT "explain"?

This list is by no means conclusive. There are other, more practical considerations, that should be added. What is the economic identity of the factors; are these factors stable over time and across markets? These are only two issues that Dhrymes (1984) has not considered. In this section, we will consider the US evidence and that (little) research that has been conducted in SA. Extensive empirical testing will be left for the next chapter.

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61 These issues, while important, are not the empirical questions that this paper will consider. Before looking at these issues, we need to determine whether or not the APT is a valid theory or an empty set.
5.3.1 PROBLEMS WITH EMPIRICAL TESTING

The major advantage of the APT is that it is a relative pricing theory that relies on a no-arbitrage condition, i.e. it can be tested on a sub-set of assets and does not require the entire investment universe as does the CAPM (Roll and Ross, 1980: 251). The major disadvantage of the APT is that it does not (indeed cannot) identify the systematic factors. Indeed the model does not even specify the number of factors. The economic rationale for the APT does give us a clue as to the nature of the factors and the rigorous derivation of the theory indicates that there are only a few factors. Beyond these two considerations the theory is silent as to how it can be tested and implemented.

Kryzanowski and To (1983:33) have argued that there are two questions of interest raised by the APT, viz.:
- How many factors are there that impact on security returns?; and,
- What is the nature or economic identity of these factors?

It is these two questions that the empirical literature has largely addressed itself. It is these two questions to which the bulk of the rest of this chapter shall consider.

5.3.2 ENDOGENOUS APPROACHES TO THE APT

The first test of the APT that was published has been largely ignored. Most articles refer to it as an historical curiosity and then proceed to follow or criticize the second published paper on the APT. This paper, by Roll and Ross (1980) quickly became established as the classic study in the field.

Roll and Ross (1980) employ a methodology that is equivalent to the early CAPM tests of Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). Due to the different nature of the APT and its tests, it is not possible to use regression analysis. The "equivalent" technique is called factor analysis.
Roll and Ross (1980:257) use daily data from mid 1962 to 1972 to calculate the factor structure. One thousand two hundred and sixty securities are used in total. Due to computational limitations, they are unable to calculate the factor structure for all one thousand two hundred and sixty securities simultaneously and are forced to form portfolios (forty two portfolios of thirty securities each). The securities were assigned to portfolios alphabetically.

For each portfolio a two-step approach was taken. Firstly, a covariance matrix of the returns (capital gain and dividends) was estimated and a maximum likelihood factor analysis performed in order to calculate the matrix of loadings. This loadings matrix was then used in the second stage to explain the cross section of returns. (This second stage is akin to using regression analysis to calculate the SML.) This step estimates the values of the sensitivities of the returns to the factor loadings.

Any factors that are identified in the first pass that are not necessary for pricing, will be "unpriced" in the second round. Due to this, Roll and Ross (1980:258) decide to specify more factors than what seems necessary from the data - the second test will discard unnecessary factors but will not indicate whether additional factors are necessary. Roll and Ross (1980:259) argue that the same number of factors should be present in all forty two portfolios.

Using the first portfolio only, Roll and Ross (1980:259) estimate a two per cent probability that at least six factors are present in the data. They then specify five factors and calculate the likelihood ratios that this number of factors is sufficient in each of the forty two portfolios. They report that, "over three-quarters of the groups had at least an even chance that five [factors] were enough" (Roll and Ross 1980:259).

These five factors need not be priced, i.e. have a $\beta \neq 0$. In order to test this, the second step is followed. This, however, requires that a value for $\lambda_0$ (an intercept term that should proxy the risk free rate) be estimated from the data or be