do provide evidence in favour of the CAPM. Certainly, the traditional approaches are at best inconclusive.
CHAPTER FOUR
CAPM: FACT OR FICTION

"There is a big story breaking in money management. The capital asset pricing model is dead" ("Deep Quant", quoted in Wallace 1980:23).

Friedman (1953:27) has argued that theory can only be judged by its predictive power, i.e., how well does it explain "the class of phenomena which it is intended to 'explain'". In the previous chapter, we saw that the CAPM did not entirely live up to its predictions. This is to be expected. However, a body of literature has arisen that firstly casts doubts on the CAPM's ability to explain security returns and secondly that there are various anomalies that it cannot explain at all. In this chapter, we will consider both arguments. Firstly, is the CAPM internally consistent? Roll (1977, 1978, 1980) has argued that it is impossible to know. Fama and French (1992) have argued that it is not. Secondly, is the CAPM externally consistent? The chapter will conclude by considering the SA evidence in this regard.

4.1 THE INTERNAL CONSISTENCY OF THE CAPM


The gist of his (Roll 1977) argument is simple. Not only has there been no

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Some of the material in this chapter has been accepted for publication. The paper is entitled "The Monday Effect on the Johannesburg Stock Exchange: Revisited" by Sinclair Davidson and Steven Meyer, South African Journal of Business Management 24(3) (1993 forthcoming).
unambiguous tests of the CAPM. It is unlikely that there will ever be such a
test. The effect of this is that it is not possible to state that the CAPM has been
tested and proved either way.

The usual method used to calculate β's is to use a market index of some sort.
This leads to a problem that can be referred to as "index unique-β's". Each
security has a unique β vis-a-vis the market portfolio, but when a proxy for the
market is used, the calculated β is index unique. This has important
consequences for portfolio performance evaluation. In his 1978 and 1980
articles, Roll argued this point. We see that these arguments are interrelated
as one flows from the other.

4.1.1 DOES THE COMPOSITION OF THE MARKET PORTFOLIO MATTER?\footnote{This title is taken from Brown and Brown (1987).}

In order to demonstrate that the composition of the market portfolio does
matter, we will need to review the theory that was shown in the previous
chapter. Recall (3.11):

\[ \beta_i = \frac{\sigma_{m}}{\sigma_{m}^2} \]

β is a relative measure of risk, risk relative to the market portfolio. However,
only a proxy for the market portfolio is being used, β measures risk relative to
that proxy.

Tests of the CAPM consist largely of a two-pass regression. The second pass
regression is examined to determine whether or not it conforms to the
theoretical SML. Roll (1979:394) states that the linearity of the SML has been
considered to be "a second and testable implication" of the CAPM. However,
Roll (1977:130) argues that a perfect linear relationship between return and
β will exist, if and only if the proxy portfolio is mean-variance efficient. The
proof of this may be seen from our derivation of the SML (section 3.3.3 above).
If we replace population parameters with sample parameters and if the proxy
market portfolio is M-V efficient, then there will be a perfect linear relationship as per the SML (Levy 1983:147). The SML is a technical result, and not necessarily proof of the CAPM.

Roll (1977:138) argues that tests of the CAPM have merely tested the efficiency of the proxy, they have not tested the validity of the CAPM. He states that there are many ex-post portfolios that are M-V efficient. It can happen that such a portfolio is chosen by chance as the proxy portfolio. In the resulting test it would appear that the CAPM is valid. This problem can be overcome if the composition of the market portfolio were known. We are only able to test the CAPM if we are able to include the true market portfolio in the test. Any other test of the CAPM, tests the M-V efficiency of the proxy.

Studies such as Black, Jensen and Scholes (1972), have tested a joint hypothesis:
- the CAPM is valid; and,
- the proxy is the market portfolio.
As we saw, they rejected the traditional form of the CAPM in favour of Black’s (1972) zero-β version. However, in light of the joint hypothesis, their methodology could lead to one of three conclusions:
- the CAPM is invalid;
- the proxy is not the market portfolio; or
- both of the above (Roll 1977:144).
Roll (1977:150-151) does argue that if the proxy were perfectly correlated to the market portfolio, it would be possible to test the CAPM. However, it is beyond our knowledge to determine whether or not any proxy is perfectly correlated with the market portfolio (but see Shanken 1987 discussed below).

We may conclude that the first step towards testing the validity of the CAPM is to determine the composition of the market portfolio. This may appear, on first glance, to be an academic quibble. However, it has vital practical implications, as Roll (1978, 1980) demonstrated.
4.1.2 ROLL'S CRITIQUE AND THE JENSEN ALPHA

One of the practical consequences of the CAPM has been to use the SMI to evaluate the performance of portfolio managers. Academics have used a similar methodology in event studies. The Jensen-\(\alpha\) technique was developed as a method to evaluate the performance of mutual funds. The argument being that when the market is in equilibrium, any positive deviation from the SML will be as a result of superior management.

Jensen (1969:212) has defined portfolio performance as a measure of a manager's ability to select "winners" from the population of securities. The measure that Jensen (1969:188) advocates is the difference between the expected return and the actual return given the \(\beta\) of that security, i.e. whether or not the security plots above, on or below the SML.

The demonstration (below) of the Jensen measure follows Morris and Pope (1981:238):

The CAPM may be stated in the following form (see above section 3.4.3)

\[
R_i = (\mu - \beta)R_f + \beta R_m
\]

The Jensen-\(\alpha\) measure measures any abnormal return over the SML for a given \(\beta\), i.e.

\[
R_i = \alpha_i + (1 - \beta)R_f + \beta R_m
\]

\[
\alpha_i = (R_i - R_f) - \beta(R_m - R_f)
\]

If the security plots above the SML, i.e. has \(\alpha > 0\), it is a "winner"; if \(\alpha < 0\), the security is a "loser" and if \(\alpha = 0\), it is neutral.

Roll (1978, 1980), based on his earlier work (1977), showed that the Jensen-\(\alpha\) was not what it purported to be. His criticisms have been supported by others, e.g. Dybvig and Ross (1985a,b). Roll (1978:223) argues that in order to use the Jensen-\(\alpha\), that two steps are necessary:
- an index must be chosen; and,
- \(\beta\)'s must be calculated for this index.
Roll (1978) then proceeds to demonstrate, based on the unique index β problem, that this approach will yield different answers based on different indices. He states (Roll 1978:224) that the results can be reversed based on "innocuous changes in [the indexes] computation". He further argues (Roll 1978:233) that if the chosen index is M-V efficient that the Jensen approach will be unable to differentiate between "winners" and "losers", i.e. $\alpha_1 = 0$.

Roll (1977:132) argues that if the Jensen approach cannot differentiate between winners and losers unless the index is M-V inefficient, what justification can be made for using this performance evaluation criteria? It seems that Roll feels that there is no justification for using the criteria.

The conclusion that can be drawn is that once we are unable to identify the market portfolio, the theory is firstly untestable and secondly unusable. Mayers and Rice (1979:4) have stated that Roll's (1977) analysis is "most damning because it implies that the theory has little operational usefulness, even if the theory is valid".

4.1.3 IS ROLL OVERSTATING THE DEFICIENCY OF THE CAPM?

Mayers and Rice (1979:3) state that Roll's (1977) analysis of the CAPM "presents an exciting intellectual challenge". The conclusions of his analysis are very wide reaching. It creates a challenge on two grounds:
- if Roll is wrong, this must be shown; and,
- if he is right, where do we go from here?

In this section, we will consider whether or not Roll is overstating his case or not. It would appear that no academic has challenged Roll's (1977) argument concerning the technical derivation of the SML. Indeed, Mayers and Rice (1979:4) state that, "his conclusions based on the mathematics of the efficient set are unassailable". There have, however, been some rather innovative attempts to circumvent the fact that the global market portfolio is required to
test the CAPM. These include studies by Stambaugh (1982), Gibbons (1982), Shanken (1987) and Gibbons, Ross and Shanken (1989). Other studies have investigated the operational usefulness of the CAPM, e.g. Dybvig and Ross (1985a,b) and Grinblatt and Titman (1989).

One of the principles that came out of Roll's (1977) critique is that the composition of the proxy market portfolio does matter. Stambaugh (1982:237) argues that the impact of this critique comes not from the fact that assets are not included in the proxy portfolio, but that the tests of the theory are sensitive to the specification of the proxy. Stambaugh (1982:238) tests the relationship:

$$E(r_i) = \gamma_1 + \gamma_2 \beta_i$$

for three hypotheses: linearity, whether the intercept term is equal to a risk free rate and whether the risk premium is positive. He uses various indices in order to test the relationship.

Stambaugh (1982:239) compiled returns (for the period 1953-1976) for seven categories of assets: NYSE stocks, corporate bonds, US bonds, Treasury bills, residential real estate, house furnishings and automobiles. He then combines these assets into portfolio's to be used as proxy market portfolio's (Stambaugh 1982:244):

Index one: NYSE stocks (value weighted);

Index two: index one plus corporate bonds, government bonds and T-bills (value weighted);

Index three: index two plus house furnishings, real estate and automobiles (value weighted); and,

Index four: index three adjusted to form a proxy for all other assets excluded from the sample (NYSE stock = 0.1 and all other assets are assigned a weighting of 0.9).

Using these indices as market portfolio's, Stambaugh (1982:251) proceeds to test the three hypotheses, using a fixed set of assets (nineteen common stock portfolio's; four preferred stocks and five bond portfolio's.)
His results are in favour of the first and the third hypotheses, i.e. in favour of linearity and positive risk premia. The intercept term, however, is not equal to the risk free rate. The results are insensitive to the choice of market proxy. Stambaugh (1982:257) interprets these results as rejecting the standard form of the CAPM, but supporting the Black (1972) version.

These results should not be construed as general evidence in favour of the CAPM. Haugen (1990:250) makes the following point concerning this study:

"Beginning a mile away, it appears [that the] results don't change when we move a foot closer to our destination".

A possible explanation of the relative insensitivity of the various indices could be that they are so small relative to the global portfolio, that tests of the CAPM cannot differentiate between them. This study, while interesting in itself, is not an unambiguous test of the CAPM. Stambaugh (1982) does not invalidate the essence of Roll's (1977) critique.

Gibbons (1982) suggests the use of multivariate analysis in order to test the CAPM. This approach offers the possibility of an unambiguous test of the CAPM. The multivariate technique measures the strength of the restrictions that the CAPM places on the parameters of the market model. The validity of the CAPM can be judged by whether or not this restriction holds or not (Gibbons 1982:22).

Recall (3.20): \( \alpha = \gamma(1 - \beta_i) \); where \( \gamma = R_f \).

It is possible to test the CAPM by determining whether or not this relationship holds across a whole series of market model regressions. In this case, the null hypothesis is as follows:

\( H_0: \alpha = \gamma(1 - \beta_i) \); and the alternate hypothesis is,

\( H_A: \alpha \neq \gamma(1 - \beta_i) \).

Gibbons (1982:6) argues that this relationship has generally been assumed to be true and has been used to calculate point estimates for \( \gamma \). These point estimates are then compared to the return on the market portfolio and a risk
free rate as a test of the CAPM. This approach actually tests the relationship.

Gibbons (1982) tests for (3.20) using data taken from the (monthly) CRSP tapes for the period 1926-1975. This period is divided into ten five year periods. The return on the equally weighted CRSP portfolio is taken as a proxy for the market portfolio. Gibbons (1982:12) reports that the CAPM is (significantly) rejected for five of the periods and is (insignificantly) rejected for three of the remaining five periods. Gibbons (1982:12) offers two possible explanations:
- the equally weighted CRSP portfolio is not M-V efficient; and,
- to the extent that the proxy is efficient, the traditional and the Black versions of the CAPM are rejected.

Of course, we have returned to the joint hypothesis problem - the proxy is more than likely to be M-V inefficient. Thus the first explanation has a lot of validity, but what can we say about the second? To what extent is the proxy efficient? What does this tell us about the CAPM? These philosophical questions bring us straight back to the "Roll problem". While this study strengthens the arguments against the CAPM, it is not an unambiguous test of that theory.

While Gibbons (1982) relies on the proxy being reasonably efficient in order to reject the CAPM, Shanken (1987:92) develops a technique where, "a prior belief about the correlation \( \rho \) between the proxy and the true market portfolio can be explicitly incorporated" into the test. The test becomes a joint hypothesis that firstly the pre-specified correlation is correct and that the CAPM is correct. Roll (1977:150-151) makes the point that it is possible to test the CAPM using a proxy for the market, if \( \rho = 1 \). Shanken (1987) demonstrates that it is possible to test the CAPM using a proxy as long as we know what \( \rho \) is.

Shanken (1987:94) develops a concept "\( d \)" which largely resembles the Jensen-\( \alpha \):
\[
d = E(R_p) - R_f - B\gamma
\]

(4.1)
where $\gamma$ = the price of risk; and,

$B = \hat{\beta}$ estimate from the function; $R = a + BP + e$ (this is Shanken's specification of the market model).

The relationship between $P$ (the proxy portfolio) and the true market portfolio $(m)$ is as follows: $m = a_m + b_mP + e_m$.

Shanken (1987:94) reproduces the Roll (1977) result by stating that:
when $\rho = 1; \ d = 0$.

However, it is generally the case that $\rho \neq 1$. Shanken (1987:93-94) states that it is interesting to 'bound' pricing deviations when $\rho = 1$. This bound takes on the following form:

$$|E(R_p) - R_t - \gamma y| \leq \sigma_p \sigma_m (1 - \rho^2)^{1/2} \quad (4.2)$$

Thus the absolute value of the deviation must be less than or equal to the product of the assets residual risk, the market risk and the correlation between the proxy and the market. Shanken (1987:95-96) then demonstrates that $\rho^2 = \theta_p^2 / \sigma_m^2$

Where: $\theta_p^2 = |E(R_p) - R_t| / \sigma_m$

Thus, (4.3) can be reduced to the following:

$$d \leq \theta_p^2 (\rho^2 - 1)$$

The significance of this observation is that if $R_t$ is known or observable, then all we need to test the CAPM is a knowledge of $\rho$.

Shanken (1987:99) tests the hypothesis that the $\rho$ between the market portfolio and the CRSP equally weighted portfolio is 0.7 and that the CAPM is a valid description of the return generating process. This is rejected at the 0.05 level. If the CAPM is true, this implies that the CRSP portfolio explains less than forty nine per cent of the market portfolio's variation. Shanken (1987:102) repeats this test using a proxy for the market that includes US government bonds. The hypothesis that $\rho = 0.7$ is rejected at the 0.1 level. This, Shanken (1987:105) interprets as meaning that the CAPM is false or that the $\rho$ value is misspecified.
Despite Shanken's (1987) complex statement and test of the CAPM, we are no further down the road to an unambiguous test of the CAPM. Unless knowledge of \( \rho \) is obtainable via divine intervention, a knowledge of the global market portfolio will be necessary to test the CAPM.

Gibbons, Ross and Shanken (1989) make use of the multivariate technology to develop a test of the efficiency of a given portfolio. They formulate their test along the lines of Black, Jensen and Scholes (1972). The excess returns model implies that \( H_0: \alpha_i = 0, \forall i = 1..N. \)

They argue (Gibbons, Ross and Shanken 1989:1124) that it is possible to test this hypothesis by making use of the Hotelling \( T^2 \) test - a multivariate equivalent to the \( t \)-test. However, there is a geometric explanation of the test that highlights the economic interpretation of the approach.

Gibbons, Ross and Shanken (1989:1149-1150) show that the relevant statistic \( W \), is calculated as follows:

\[
W = \left( \frac{1}{\sqrt{1 + \theta_p^*}} \right)^2 - 1 = \psi^2 - 1
\]

where \( \theta^* \) = the maximum ex post price of risk (average return divided by standard deviation);
\( \theta_p \) = the ex post price of risk for the portfolio in question (Sharpe reward to variability ratio).

Gibbons, Ross and Shanken (1989:1126) argue that \( \psi^2 \) should be close to one if the null hypothesis is true (the significance of \( W \) is calculated from \( F \) tables).

Gibbons, Ross and Shanken (1989:1129) make use of monthly returns for the period 1931-1965 to test the ex ante efficiency of the CRSP equally weighted index. They calculate \( W = 0.02333 \), as such the efficiency of that portfolio...
cannot be rejected. They write that this is the equivalent of saying that, "if this index is taken as the true market portfolio, then the Sharpe-Lintner version of the CAPM cannot be rejected" (Gibbons, Ross and Shanken 1989:1129).

Gibbons, Ross and Shanken (1989:1130) also show the expected value and standard deviation of $W$ to be:

$$E(W) = \frac{N}{T - N - 3}; \text{ and,}$$

$$\sigma_w = \left( T - N - 3 \right)^{-\frac{1}{2}} \left[ 2N(T - 3)/T - N - 5 \right]^{\frac{1}{2}}$$

where $N = \text{the number of portfolios included in the test; and,}$

$$T = \text{the number of observations per portfolio.}$$

For the Gibbons, Ross and Shanken (1989:1130) data, $E(W) = 0.02$ and $\sigma_w = 0.011$. They note that as $E(W) > W$, it is not surprising that they are unable to reject the null hypothesis.

Bradfield and Affleck-Graves (1991:32) replicate this study using data drawn from the JSE. Twelve $\beta$-sorted portfolios (containing one hundred shares) for the twelve year period 1973-1984 were used to calculate $W$. The ALSI was used as a proxy for the market portfolio. Bradfield and Affleck-Graves (1991:35) do not indicate whether this index was adjusted for dividends or not, but conclude that the \textit{ex ante} efficiency of the CAPM cannot be rejected.

Using data from section 3.6, the Gibbons, Ross and Shanken (1989) study was replicated for the ALSI and weighted index (both including dividends). The data for the period 1981-1985 was grouped alphabetically into ten fifteen share portfolios (the final seven shares in the sample of one hundred and fifty seven were dropped). The reward to variability ratios were calculated and $W$ statistics calculated.

$$W_{\text{ALSI}} = 0.007121043$$

$$W_{\text{weighted}} = 0.007974225$$

These $F$-statistics are insignificant at any standard level of significance.
For the period 1981-1985, we are unable to reject the null hypothesis, i.e. if we made use of these portfolios to test the CAPM, we would be unable to reject the CAPM. When we examine a SML using the portfolios that generated the W-statistic, we see the following:

TABLE 4.1 SECURITY MARKET LINE 1981-1985

<table>
<thead>
<tr>
<th>PARA. EST</th>
<th>T-STAT</th>
<th>P-LEVEL</th>
<th>R^2</th>
<th>ADJ-R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>α_0</td>
<td>-.000552</td>
<td>-.16</td>
<td>0.8747</td>
<td></td>
</tr>
<tr>
<td>β_0</td>
<td>.003659</td>
<td>.35</td>
<td>0.7388</td>
<td>0.0147</td>
</tr>
</tbody>
</table>

We see that the central hypothesis of the CAPM is rejected. Despite Gibbons, Ross and Shanken (1989:1129) who argue that the CAPM cannot be rejected under these conditions, we find that the SML is not statistically significant. This is a clear contradiction of the model.

The data for the second period was divided into nine twenty share portfolios and the tenth portfolio contained eighteen shares (accounting for the one hundred and ninety eight shares). The procedure was repeated, resulting in the following:

W_{\text{ALSI}} = 0.00796983

W_{\text{weighted}} = 0.010085039

Again we should be unable to reject validity of the CAPM. However, using the portfolios that generated the W statistics, the SML does not bear out that prediction (the SML is negatively sloped, but not significant).
TABLE 4.2 SECURITY MARKET LINE 1986-1990

<table>
<thead>
<tr>
<th></th>
<th>PARA. EST</th>
<th>T-STAT</th>
<th>P-LEVEL</th>
<th>R²</th>
<th>ADJ-R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0.006081</td>
<td>1.48</td>
<td>0.1764</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_A )</td>
<td>-0.00452</td>
<td>-0.84</td>
<td>0.4258</td>
<td>0.0809</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Thus it appears that the univariate and multivariate tests contradict each other. Power considerations may be at the source of this contradiction. This is particularly troublesome as Bradfield (1989c:6-34) has found that the power of univariate tests, while lower on the JSE than on the NYSE, is "substantially" higher than that of multivariate tests.

We now leave the "cannot be tested" part of the Roll (1977) critique and turn our attention to the "cannot be used" aspect of that critique. Roll (1977:132) hints that the use of the SML is never justified. Dybvig and Ross (1985a,b) investigate this issue in order to determine whether Roll is correct or not. In their first paper, Dybvig and Ross (1985a) investigate deviations from the SML that are caused by superior information (leading to superior performance). They find that even in the absence of statistical measurement problems, that the SML is not a good tool of analysis in the evaluation of performance.

There is only one instance when a manager will be correctly ranked by the SML, when a riskless asset exists and the manager has no knowledge of the return or variance of the evaluators index (Dybvig and Ross 1985a:393). This latter assumption is unusual and it is difficult to ascribe economic meaning to it. Dybvig and Ross (1985a:397) state that these results are "slim pickings" and that they do not justify the extensive use made of the SML to evaluate portfolio performance.

In their second paper, Dybvig and Ross (1985b:401) examine the use of SML analysis when the index portfolio is inefficient, but there are no informational differences. They find that SML analysis "does not provide a reliable global tool