THE DAMAGING EFFECTS OF MINING ON VERTICAL SHAFTS AND ANCILLARY EXCAVATIONS

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A dissertation submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Science in Engineering

Johannesburg, 1988
DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

(Signature of candidate)

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ABSTRACT

In order to assess the damaging effects of mining on vertical shafts and ancillary excavations the consequences of the formation of shaft pillars, the early extraction of the orebody in the shaft pillar area and the extraction of established shaft pillars must be quantified. The applicability of analytical as well as numerical models is examined against previous experience with vertical shafts and ancillary excavations. Damage criteria suggested and used in the South African mining industry are reviewed.

The analytical method for the design of circular shaft pillars at shallow depth is shown to produce safe and economical shaft pillar sizes. After a case study of early extraction of the orebody in the shaft pillar area, an elastic model is found to give an acceptable description of the response of rock masses beyond the loosened up hangingwall. Damage criteria pertaining to the early mining of the reef in the shaft pillar area are suggested. Finally the mechanisms of damage associated with different methods of extracting shaft pillars are discussed.
ACKNOWLEDGEMENTS

The writer is grateful to Professor S. Budavari for the assistance he offered during the preparation of this dissertation. Acknowledgement is given to the Chamber of Mines of South Africa for financial support of this project. The computer equipment made available to the writer by the Department of Mining Engineering at the University of the Witwatersrand was invaluable.
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LIST OF SYMBOLS

E  Young's modulus

GPa  Giga Pascal

H  Depth of origin of coordinate system below surface

kg/m³  Kilogram per cubic metre

l_c  Critical halfspan

mm/m  Millimetre per metre

MPa  Mega Pascal

q  Virgin vertical stress

r  Radial component

R  Radius of shaft pillar

s_m  Effective stoping width

sw  Stoping width

τ_x  Induced tilt

τ_x max (i)  Maximum induced tilt

z  Distance above (negative) or below (positive) the reef

ε_max  Maximum strain

ε_z max (i)  Maximum induced vertical strain

σ_max  Maximum stress

σ_CR  Critical stress

σ_z  Resultant vertical stress

ν  Poisson's ratio
1. INTRODUCTION

Since shaft systems provide access to underground workings and are part of the main ventilation route, they are the most important facilities of a mine. Their trouble-free operation must be maintained throughout the economic life of the mine. While mining is carried out at a sufficiently large horizontal distance from a shaft system, it is relatively easy to maintain it in a safe and trouble-free condition. However, extensive mining, carried out in the vicinity of the shaft, can damage the shaft which may result in the reduction of its effectiveness in serving the mine.

Only a limited number of methods are available to protect shaft systems from the damaging effects of mining or to minimise the disturbing influence of strata movement on shafts. The most common method is to leave the ore unmined in an area surrounding the shaft, that is to leave a shaft pillar. At great depth the protection of vertical shafts through the action of pillars becomes uneconomical and impractical. An alternative option to leaving a shaft pillar is to extract the reef in the shaft pillar area as soon as the shaft intersects the reef. In this case, the vertical shaft is situated in a zone of inevitable rock movement. While a shaft pillar affords a high degree of
protection it also presents the problem of the eventual extraction of the pillar as a remnant.

In all three cases mentioned above, deformation of the shaft can be expected. When a shaft pillar is formed, high vertical stresses, compressive in nature, and corresponding compressive, vertical strains will be generated along the shaft and across the shaft pillar. In the case of early mining of the reef in the shaft pillar area, tensile vertical stresses and strains will result in a part of the rock mass around the shaft. The extraction of the shaft pillar as a remnant involves all the mechanisms of strata movement associated with both creating a shaft pillar and early extraction of the reef in the pillar area. Here, asymmetrical mining carried out around the vertical shaft can induce a tilt component that has the effect of bending the shaft.

If the vertical shaft passes through soft layers, high compressive stresses along the shaft could squeeze these strata into the shaft, resulting in damage to the shaft lining or equipment. Uncontrolled compressive stresses could damage auxiliary excavations in the shaft pillar area, while punching of the pillar into the hanging- or footwall and even complete col-
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If the vertical shaft passes through soft layers, high compressive stresses along the shaft could squeeze these strata into the shaft, resulting in damage to the shaft lining or equipment. Uncontrolled compressive stresses could damage auxiliary excavations in the shaft pillar area, while punching of the pillar into the hanging- or footwall and even complete col-
lapse of the pillar is possible.

Excessive compressive strain values can be manifested in buckled guides, buntons and pipe columns. Damage to the shaft lining is also a symptom of high strain. Tensile strain could lead to similar damage to shaft equipment such as is caused by compressive strain, but damage to the shaft lining will be seen as cracks rather than slabbing. Bed separation and cracks in the shaft lining associated with excessive tensile strain values could also cause inflows of water. A large tilt of a section of the shaft axis can make high-speed winding dangerous or even impossible.

As rock mechanics theory has advanced in the last twenty years, the design of shaft systems has changed since 1886 when the outcrop of the Witwatersrand gold reef was discovered. The early shaft systems were rectangular and unlined, with rope guides and wooden buntons for securing pipe columns. Later, shaft systems had timber guides and buntons and were for the most part still rectangular and unlined. Presently the majority of shafts are circular and equipped with steel buntons and guides while the lining is usually unreinforced concrete.
In each of these general groups of shaft systems the effects of mining will be different. A great deal of information is available on the effects that mining had on vertical shafts, but the relevance of this knowledge must be weighed against the particular shaft system serving the mine and the prevalent geological conditions.

The mechanisms of damage discussed above can be related to expected conditions through quantitative analysis. The lowest values that give rise to unacceptable conditions or permanent damage are referred to as critical values and constitute the basic quantities of the appropriate damage criteria. The availability of these damage criteria is essential from both safety and planning points of view.

The establishment of adequate damage criteria consists of several interrelated steps. The first one is the identification of conditions resulting in damage to the shaft system. This is followed by the selection of parameters that can be used to gauge the effects of these conditions on the various structural elements of the shaft. The final step consists of specifying values for the critical parameters. The methodology utilised in the development of such damage criteria
usually involves the correlation of measured parameters to the degree of damage observed. The writer will adhere as far as possible to the procedure as laid out here.

Study of the literature indicates that considerable effort has been expended on activities related to the effects of mining on vertical shafts. The consequences of the formation of shaft pillars, the early extraction of the orebody in the shaft pillar area and the extraction of established shaft pillars have all been looked at to various degrees. Some damage criteria were developed from in situ observations and subsequently used in planning. Most of these attempts were restricted to conditions associated with the extraction of tabular deposits.

In spite of its great practical importance and the effort expended, this field of study has not advanced far enough. There remains a lack of empirically established critical parameters and some of those being used have been suggested without sufficient justification.

Irrespective of whether a shaft pillar is to be created or not, the planning of the course of action aims to
limit the damaging effects of mining in the vertical shaft. The exact limits which determine the design of shaft pillars or extraction of the reef in the pillar area, are not well defined. Even though a wealth of information is available on this subject, there is a need to update some quantities while some observations have never been compared with modern theory.

The available theory will be evaluated and tested against practical measurements or observed data. Damage criteria in present use will be assessed in an attempt to establish the most suitable applications of these values and where possible new values will be suggested. This study is primarily concerned with vertical shafts and ancillary excavations in hard-rock mines extracting tabular deposits. However, some experiences in European coal mine practice will be considered and used.

The next chapter includes a review of previous work in this field. A chapter is devoted to methods for determining the effects of creating shaft pillars. A number of methods are discussed and evaluated and some results are presented. A further chapter deals with the design of a shaft pillar. Other chapters include a comparison of actual shaft pillars with calculated pillar sizes,
mining of the reef in the shaft pillar area and the extraction of the shaft pillar as a remnant. Conclusions are presented in the final chapter.
2. REVIEW OF PREVIOUS WORK

Introduction
The three basic sets of conditions that can occur in the mining of a mineral deposit around a vertical shaft are as follows:

i) a vertical shaft protected by a shaft pillar,
ii) a vertical shaft surrounded by a shaft pillar area being mined out early in the life of the shaft and
iii) a vertical shaft surrounded by a remnant shaft pillar that is in the process of being extracted.

Literature will be divided into and discussed according to these categories.

2.1 The Effects of Leaving a Shaft Protection Pillar

2.1.1 The theoretical analysis of stresses and differential movement around the shaft and in the shaft pillar area

Salamon (1965) developed a mathematical method for calculating stresses and strains induced by mining. This method is based on the face element principle and superposition of the effects of many infinitesimal mining steps. An understanding of the equations arrived at by Salamon requires a thorough knowledge of calculus and principles of rock mechanics theory.

Wagner and Salamon (1975) simplified the process of
calculating stresses along a vertical shaft and also across a circular shaft pillar. However, the approach advocated in this paper only applies to deep level mining. This method can be used to calculate shaft pillar sizes for deep-level mines using a number of simple equations.

Budavari (1986) used the face element principle to develop equations for the calculation of stresses and differential movement. These equations apply to shallow and moderately deep hard-rock mines, and complete the set of applied equations for the analysis of conditions around circular shaft pillars.

It is now possible to calculate the induced effects of mining on any point along the shaft or in a circular shaft pillar. To calculate the maximum possible values of vertical stress, vertical strain and tilt, the most severe mining configuration is considered i.e. the mining plan which maximises the stress, strain or tilt, Figure 2.1.

A disadvantage of the face element principle is that it becomes too involved when applied to a complex geometry. This shortcoming is in fact true for all the above-mentioned analytical methods. These methods can
give a first approximation of the size of the shaft pillar, which can then be altered to accommodate complex geology and local needs. Assumptions that are made to simplify analytical solutions distract from the value of these methods.

With the advent of more powerful digital computers it was practicable to design a system which would directly calculate the convergence distributions and provide certain other facilities in one program.

Figure 2.1 Most adverse geometries with respect to vertical strain and stress (a), tilt (b).

Plewman, Deist and Ortlepp (1969) describe the original development of the MINSIM (mining simulation) program
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![Figure 2.1](image)

Figure 2.1 Most adverse geometries with respect to vertical strain and stress (a), tilt (b).

Plewman, Deist and Ortlepp (1969) describe the original development of the MINSIM (mining simulation) program
and its application to a shaft pillar problem. MINSIM was developed by the Chamber of Mines of South Africa and has the capability of modelling complex mining layouts.

Budavari (1983) traces the further development of MINSIM. In its latest form this program can be used to do a stress analysis for multiseam configurations and also faulted ore bodies. For regular layouts, such as circular shaft pillars or parallel-sided longwall panels mathematical methods can be used to check MINSIM outputs.

Wagner (1983) describes a method of arriving at the final stress acting around a tunnel. The field stress is calculated using MINSIM, or any other method, and then the stress concentration is calculated using a factor for the shape of the excavation. This method could be used to arrive at the horizontal stresses around a shaft and the stresses around service excavations.

Kratzsch (1983) has developed a graphical method for determining the actual displacements in the shaft. This method is based on the assumption that the shaft barrel breaks away from the surrounding rock virtually as
soon as the influence of mining is felt. This assumption based on experience in European coal mines might not necessarily apply in South African hard-rock mines.

All of the above methods are based on idealised material behaviour. To a greater or lesser extent these methods rely on idealised geometries to make their applications feasible. The value of these methods can be greatly enhanced by testing their results against practical observations.

2.1.2 The design of a shaft pillar

Wilson (1971) suggested that the following factors should be considered when designing a shaft pillar:

- geological composition of rock, especially the phyllosilicate content of the rock,
- the extent and configuration of mined out areas surrounding the shaft system,
- the mechanical properties of the rocks,
- the location of service excavations in the pillar area and
- the mining height.

Wilson shows graphically stress and displacement dis-
tributions around shaft pillars at three different depths using the face element principle. Wilson concludes that the induced vertical field stress along the axis of the shaft can be used to design the dimensions of the shaft pillar. However, this conclusion is based on the analysis of shafts that are mostly shallow to moderately deep. To ignore the natural stresses at great depth could be disastrous.

Wagner and Salamon (1975) pointed out that the maximum values of differential movement are less than or equal to \(0.28s_m/R\) for strain and \(0.13s_m/R\) for tilt, where \(s_m\) is the effective stoping width and \(R\) is the radius of the shaft pillar. These values apply if the rock is considered to be an homogeneous linearly elastic continuum.

At depth greater than 1000 m the shaft pillar radius necessary to satisfy a maximum vertical stress of 100 MPa is in excess of 250 m. If this value and a stoping width of 1 m is substituted into either of these equations, it will be seen that the maximum vertical strain and tilt will not exceed 1 mm/m. This value is given by a number of authors as being acceptable. At greater depth the shaft pillar needs to be larger to ensure an acceptable stress distribution. The result is that the
maximum strain and tilt that can be expected will become smaller as mining becomes deeper.

Kratzsch (1983) suggests that vertical stress is only important in the effect it has on weak beds. Vertical compression of weak beds can result in compaction of weak layers which can lead to horizontal pressure on the shaft.

Kratzsch concerns himself with vertical displacement in the shaft and protection of the shaft against subsidence. Kratzsch's design procedure is then an approach aimed at the analysis of vertical displacements so that the shaft can be protected against the effects of these movements.

2.1.3 Observations of damage that occurred in shafts protected by pillars

Cazalet (1942) recalled a case where a shaft pillar collapsed, resulting in the underground hoist being buried at the bottom of the shaft. In this case two reefs had been extensively mined out around the shaft pillar. This is the worst case of damage to a vertical shaft on record.

In the case of No. 2 Shaft, Harmony Gold Mining Co.
Ltd., the service excavations started collapsing. This case was the subject of one of the first studies in which the MINSIM program was used. The results of this study can still be further analysed.

At West Rand Consolidated Mines the South Shaft pillar showed signs of taking stress. Van der Wal and Coetsee (1972) indicated areas of high stress in this shaft pillar but it is not shown to what extent the area around the pillar had been mined out.

Two more cases are discussed in the papers of the Association of Mine Managers. These are by Crocker (1981) and Williams (1978).

2.2 Early Extraction of the Reef in the Shaft Pillar Area

2.2.1 The theoretical analysis of stresses and differential movement associated with this process

Kratzsch (1983) explains the application of the graphical method of determining actual shaft deformation and how it applies to the early extraction of the reef in the shaft pillar area. A very clear distinction is made between a shaft with an unslit lining and a shaft with a slit at the reef horizon.
More O'Ferrall (1983) advises against the early removal of the reef if the dip is more than 30 degrees. If the reef is to be mined early in the life of the shaft the inner pillar must be stoped during the sinking phase and equipped so as to permit the anticipated displacement.

Daemen (1972) simulated the effects of various closure distributions in the shaft pillar area. The method used is also applicable at shallow and moderate depth. For each of the four cases considered, values of vertical strain, vertical displacement, distortion, compression, tension, radius of curvature and vertical stress were calculated. It appears that with a very symmetrical mining pattern, damage can be kept to a minimum. The results tabulated by Daemen are not quantitative, but are only given in comparison with a reference case. Although this study could point out most suitable arrangements for protecting vertical shafts, he did not publish the equations used and did not advance a design method.

If the reef around the vertical shaft is extracted as a parallel-sided longwall it is possible to apply plane strain solutions as proposed by Salamon (1968). In this way the expected mining conditions can be pre-
dicted. This method is restricted to a single panel of uniform geometry. As a first estimation, this method can be very valuable, giving immediate results. In an actual mining situation it is essential to be able to model complex geometries and this is where two-dimensional solutions have limited application.

Budavari (1983) explains the application of the three dimensional MINSIM program. This method is most useful when complex layouts need to be modelled in an attempt to predict future mining conditions around a vertical shaft. It is possible, with this program, to calculate a large number of different functions at any position required. The ability that a computer program has to simulate a mining layout within a reasonably short time, makes it possible to contrast different mining methods so as to arrive at the method that would result in the least amount of damage.

Solms (1985) mentions the fact that many mining engineers have previously advocated the early removal of the shaft pillar. The fact that the early extraction of the shaft pillar area around Harmony Ventilation Shaft was not without problems has prevented mine management from attempting the same method again. A careful study of the mechanisms of damage related with this process,
could give the mining world more confidence when early extraction of the shaft pillar becomes unavoidable in deeper mines. Hill (1962) suggested that the water hazard is the only reason why the early mining of shaft pillars should not be carried out.

2.2.2 Observations of the early extraction of reef in the pillar area

Two cases are discussed in the papers of the Association of Mine Managers where the reef around the vertical shaft was extracted early in the life of the mine.

Barcza and von Willich (1960) recorded a series of measurements taken down the Harmony Gold Mine Ventilation shaft and in the stopes while the reef around the shaft was being mined. This paper has great scientific value owing to the quality of measurements taken and the corresponding mining layouts given. A further reason why this paper is of importance is the fact that mining around the Harmony Ventilation Shaft was carried out using a series of longwall faces. This method is very popular on South African gold mines and therefore a study involving this mining method would be well justified.

The second case where no shaft protection pillar was
left is recorded by Mort (1956). This paper contains more practical information. The scatter mining method employed during the mining of the reef around this shaft is seldom employed in modern mines and for this reason very little gain can result from a study of this paper.

2.3 Extraction of the Pillar After its Formation

2.3.1 Planning of the extraction of the pillar

This case deals with the extraction of the pillar as a remnant. This process is extremely difficult and hazardous, since the pillar is already highly stressed before extraction starts. A wealth of information on the subject is given by More O’Ferrall (1983) and Kratzsch (1983).

More O’Ferrall points out that this process will be assisted by taking steps, during the planning and sinking of the shaft, aimed at making pillar removal more successful. Both More O’Ferrall and Kratzsch give practical advice on the planning of shaft linings and equipment. Although Kratzsch also advances theories on how to calculate stresses and strains in the vertical shaft during pillar extraction, these methods are merely suggested and not discussed in such a manner as to be of practical use.
2.3.2 Observations during the extraction of pillars

A large number of cases are recorded in the papers of the Association of Mine Managers of South Africa. The vertical shafts discussed in these papers are shallow to moderately deep.

Van Emmenis and More O’Ferral (1972) described the extraction of the Toni Shaft pillar. During this extraction a large number of strain measurements were taken and compared with equivalent values calculated with the electrical resistance analogue. To arrive at a reasonable correlation between measured and calculated values, a Young’s modulus value of 16.15 GPa was used. After calculating strain values, at a point where damage to steelwork resulted, Van Emmenis and More O’Ferral decided on a critical strain value, relating to shaft steelwork. The value of 0.4 mm/m, describes the maximum vertical strain that shaft steelwork can withstand without buckling or without being adjusted.

2.4 Damage Criteria

In a study of damage suffered by vertical shafts it is necessary to establish criteria that have been accepted in the past. The extent of damage that can be tolerated in a vertical shaft is very small, since the vertical shaft is the main access route into and out of the
mine. High speed winding in vertical shafts also mean that a high degree of protection is needed in vertical shafts.

2.4.1 Differential movement criteria

Strain

Daemen (1972) accepted a value of 1,5 mm/m as the maximum allowable strain. This value was suggested by Mohr. Although the study carried out by Mohr was done in a very thorough manner, the type of shaft lining and equipment used in Germany during the 1950's differs greatly from concrete and brick lined shafts in South African mines.

Wilson (1971) after a study of a number of damaged and undamaged shafts arrived at the value 0,2 mm/m as the maximum allowable strain. This value was arrived at after vertical strains in shafts were calculated using the MINSIM method and a Young's modulus of approximately 70 GPa. No mention is made of the type and extent of damage that occurred in damaged shafts.

Wilson and More O'Ferrall (1970) mentioned that the maximum vertical strain that steelwork can withstand is 0,12 mm/m. This is correlated with calculated values corresponding to a point where damage to shaft steel-
work did result. Van Emmenis and More O'Ferrall (1972), after doing a back calculation at the same shaft, concluded that the Young's modulus should be reduced 5 times to arrive at a correlation between measured and theoretical values. This means that the strain when damage occurred, was actually 0,6 mm/m. Van Emmenis and More O'Ferrall also measured strains as large as 0,6 mm/m in sections of this shaft where no damage occurred. This value is then a calculated and measured value and van Emmenis and More O'Ferrall concluded that 0,4 mm/m should be used as a design criterion for steelwork in vertical shafts.

Salamon (1965) quotes values suggested by Polish engineers. In this case four categories of protection are given. However, these values are aimed at the protection of surface structures.

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<th>Acceptable strain (mm/m)</th>
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<td>ii</td>
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Kratzsch (1983) gives maximum allowable values for lengthening and shortening of vertical shafts as 3 mm/m and 1 mm/m respectively.
Wagner and Salamon (1975) concluded that the maximum value of strain is given as $0.28s_m/R$ where $s_m$ is the effective stoping width and $R$ is the radius of the shaft pillar. For a large shaft pillar the maximum value of strain remains small. This is an important conclusion but does not apply at shallow depth where $R$ is also small.

**Tilt**

Excessively high values of tilt will only occur if the mining configuration around the shaft is not balanced. It is therefore a simple matter to guard against extreme values of tilt. Economic pressures might result in mining layouts that cause damage to vertical shafts. It is therefore important to keep this criterion in mind. According to Budavari (1986) large values of tilt can be expected at shallower depths and above the reef intersection.

Daemen (1972) accepts the value 1.5 mm/m as the maximum allowable tilt. Salamon (1965) quotes the Polish values according to different levels of protection (table below). These values are aimed at protection of surface structures but have been employed in mining. Wagner (1983) writes that experience has shown that the maximum tolerable value of tilt is 1 mm/m.
### 2.4.2 Stress criterion

The amount of stress that rock can withstand can be expressed in different ways. Some authors choose to consider only the stress induced by mining. At greater depth it is essential to consider the natural stress in the rock as well as the stress induced by mining. The relationship between total stress applied and uniaxial compressive strength of the rock can also be used to estimate damage.

Wiid and Arnold (1981) considered 20 MPa to be the maximum allowable induced stress at any point in the shaft. Wilson (1971), after a study of damaged and undamaged shafts in South Africa, concluded that the maximum allowable induced stress is approximately 17 MPa. Wilson also provides generalised geological sections with the uniaxial compressive strengths of different layers. These rock strengths vary from 138 MPa to 330 MPa.

Wilson and More O’Ferrall (1970) concluded that 55 MPa can be regarded as the stress value where support be-

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</tbody>
</table>
comes necessary for tunnels. This value is based on experience gained on seven mines in the Orange Free State. This value is based on studies carried out with the electrical resistance analogue and can claim to have scientific backing.

Wilson and More O’Ferrall(1970) used the vertical applied stress level against the uniaxial compressive strength as a measure of stability of tunnels.

<table>
<thead>
<tr>
<th>Vertical applied stress/</th>
<th>Tunnel condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>uniaxial compressive strength</td>
<td>---------</td>
</tr>
<tr>
<td>0.1</td>
<td>Stable unsupported</td>
</tr>
<tr>
<td>0.2</td>
<td>Minor sidewall spalling</td>
</tr>
<tr>
<td>0.3</td>
<td>Severe sidewall spalling</td>
</tr>
<tr>
<td>0.4</td>
<td>Heavy support required</td>
</tr>
<tr>
<td>&gt;0.5</td>
<td>Possible rockburst conditions</td>
</tr>
</tbody>
</table>

These ratios were later quoted by Hoek and Brown (1980). These experiences indicate that it is unadvisable to apply a vertical stress in excess of half the uniaxial strength of the rock.

It is important to note that apart from Wilson and Wiid and Arnold all the above criteria apply to horizontal tunnels where the stress will be perpendicular to the tunnel. Wilson and Wiid and Arnold do not consider the natural stress in the rock. This assumption can lead to
serious errors in the estimation of the overall stress on an excavation.

Kratzsch (1983) states that the vertical stress is only important in the influence it will have on soft layers encountered down the shaft. The horizontal component of stress due to the Poisson's ratio effect of vertical induced stress is negligible and damage from high vertical stress can only damage the shaft when soft layers are pushed or squeezed into the shaft.

After Wagner and Salamon (1975) it is appropriate to note that at great depth the maximum values of tilt and strain are not excessive and therefore stress is the criterion to consider.

CONCLUSIONS

It has been shown that a number of theoretical models and computer systems are available for the purpose of studying the effects of mining on vertical shafts and ancillary excavations. Some parameters to be used with these methods have been identified. These methods are aimed at quantifying the effects of mining on shaft systems in terms of stress, strain and tilt.

Some aspects of the available theoretical work are
still outstanding. It is not always clear which is the critical criterion, the effects of variability of material parameters are not defined and the effects of creating a shaft pillar on an inclined reef is not known.

When it comes to limiting values a number of approaches to the application of these figures exist. The magnitudes of critical values suggested also show a large variation. Only a few cases are on record where critical values were obtained by correlating theory with observations. Most of the critical values obtained in this manner are the results of studies at relatively shallow depth. It is not known whether these values apply at great depth.
3. THE EFFECTS OF LEAVING A SHAFT PILLAR

Introduction

Traditionally vertical shafts and their ancillary excavations are protected by shaft pillars. Shaft pillars cannot prevent damage entirely but can control the effects of mining on shaft systems. A number of methods that can be used to quantify the expected conditions are examined here. The most important parameters that describe the effects of mining are identified. Some aspects of the mathematical methods are studied.

3.1 Theoretical Analysis of Stress and Differential Movement along the Shaft and in the Shaft Protection Pillar

In the case of a vertical shaft protected by a shaft pillar, the simplest case is that of a circular pillar on a horizontal reef. In such a case, the stresses and differential movements along the shaft and at any depth in a horizontal direction, can be determined with a closed form solution.

Under these circumstances, where stresses will be compressive, it is fair to consider the rock to behave as a linearly elastic continuum.

The analytical method developed by Salamon (1965) and
applied by Budavari (1986) is based on the face element principle and superposition of the effects of many infinitesimal mining steps. Stresses and strains are calculated with the circular pillar considered as intact, while the area around the pillar is assumed to be mined extensively or to infinity. The closure in the mined out area is considered to be equal to the stoping width right up to the pillar edge.

Salamon (1965) arrived at simple equations, aimed at quantifying the effects of mining where a circular protection pillar is left. These equations only applied at infinite depth.

The equation for calculating the total vertical stress along a vertical shaft protected by a circular shaft pillar is given by Salamon as

$$
\sigma_z = \frac{S_m E}{4R(1-v^2)} \frac{(1+\beta^2)}{(1+\beta^2)^2,5} + q \left[ 1 + \frac{z}{H} \right],
$$

(1)

where \( R \) is the radius of the shaft pillar, \( E \) is the modulus of elasticity, \( v \) is the Poisson's ratio of the rock mass and \( q \) is the vertical component of the pre-mining stress. \( \beta = z/R \) where \( z \) is the distance above or below reef (negative above and positive below). \( H \) is the depth below surface of the reef intersection. Since
the convergence, \( s_m \), is assumed to be equal to the width of the extracted reef, it is also referred to as the effective stoping width. It is possible that the convergence can be less than the mining height eg. if backfill is used.

The largest stresses do not occur at the shaft-reef intersection so that it is important to calculate stresses above and below the reef.

Another equation derived by Salamon can be used to determine the maximum induced vertical strain that can be expected when a circular shaft pillar of a given radius is created:

\[
\varepsilon_{z \text{ max}}^{(i)} = \frac{0.28 \ s_m}{R}.
\]

The variables in this equation have already been defined.

To calculate the maximum tilt that can be expected with a given shaft pillar size, the following equation can be employed

\[
\theta_x \text{ max}^{(i)} = \frac{0.13 \ s_m}{R}.
\]

At infinite depth it is a fairly simple matter then to
determine the stress at any point along the shaft and the maximum strain and tilt that can be expected with a given pillar radius. In fact, these equations are so simple that the worst possible conditions expected in terms of any one of the parameters can be calculated with a hand-held calculator.

An incorrect assumption as to which of these criteria is the limiting one could lead to dangerously high values of the other variables. It is also good practice to determine distributions of stress and strain along the shaft axis and across the shaft pillar, so that the location and magnitude of large values are known.

Budavari (1986) deduced equations, using the face element principle, so that stresses and differential movements could be calculated in a shallow and moderately deep situation. These equations are very involved and the solution of these equations by integration is only possible with the aid of a powerful computer. Three programs, developed by the writer and using these equations are included in appendix A. These programs are written in PASCAL and run on an IBM mainframe computer. With these programs vertical stress, vertical strain and tilt can be calculated at any point along the shaft, while vertical stress across
the shaft pillar can also be calculated.

Figure 3.1 Distribution of induced vertical stress along the shaft

3.2 Distributions of Stress, Strain and Tilt
The consequences of leaving a shaft pillar will be to bring about a redistribution of stresses in the rock surrounding the shaft system. By using mathematical models based on elastic theory the effects of mining can be quantified. In the specific case of a circular
pillar surrounded by concentric mining Budavari's method can be used. Some results will be shown here.

Distributions of induced vertical stresses are shown in Figure 3.1. It can be seen that the maximum values occur at distances of approximately 0.5 times the pillar radius above and below the reef level. At shallow depth the stresses below the reef level tend to be larger and as one goes deeper the distribution tends to become more symmetrical.

Figure 3.2 Distribution of vertical induced strain along the shaft
A few distributions of vertical induced strain are given in Figure 3.2. It is not surprising to see that this distribution looks very much like that of the vertical induced stress, with the significant difference that the maximum values of strain are considerably larger than the values calculated at the reef level. Once again the distributions tend to become more symmetrical at greater depth.

![Figure 3.3 Distribution of tilt along the shaft](image)

Figure 3.3 Distribution of tilt along the shaft
The distributions of tilt shown in Figure 3.3 indicate that the largest tilt can be expected a distance of approximately 0.5 times the shaft pillar radius above reef level. There is once again a tendency for the distribution to become more symmetrical at greater depth.

3.3 A Brief Comparison of the Theoretical Methods

Three methods are available for determining the effects of mining on a vertical shaft and its ancillary excavations. All three methods are based on elastic theory. The closed form solutions proposed by Salamon (1965) and Budavari (1986) are essentially the same method. Salamon's infinite depth solution is in fact a special case of Budavari's method. Iterative methods, such as that used in the MINSIM program, are meant for general application in rock mechanics, but can be used to determine the effects of mining on vertical shafts.

Salamon's solution comprises a set of simple equations applicable only at great depth. The short equations were achieved by ignoring the influence of the surface. By retaining this effect Budavari derived a closed form solution valid for any depth. The complexity of these equations means that it is only possible to solve them through the use of a powerful computer. Both closed
form solutions simulate circular shaft pillars with mining around the pillar considered to be in concentric circles. It is important to remember that these methods were derived for horizontal reefs and results do not apply in cases where the reef dips more than 20 degrees. The fact that these equations are for a specific application means that data input is kept to a minimum.

The MINSIM model is a versatile tool in the hands of the rock mechanics engineer. Circular pillars, square pillars or satellite pillars can be simulated on any dip. The data input necessary for a MINSIM run is however quite substantial. Although it is possible to simulate total closure with the MINSIM model, the area outside the MINSIM "window" is always considered as intact and from this point of view the MINSIM model cannot simulate a worst possible case.

The methods described here can also be used in combination eg. Salamon's method can give maximum values and Budavari's method or the MINSIM program could be used to calculate stress, strain and tilt distributions. In fact, quantitative values of the important parameters are so easy to arrive at that it is not necessary to devote a large part of this study to the selection of one parameter as the critical one.
3.4 Some Aspects of the Closed Form Solutions

A number of aspects of the closed form solutions mentioned above have been investigated. It is of interest to this study to know where convergence between the finite depth and infinite depth solutions occur, what is the influence of the extent of mining on the shaft pillar and what is the implication of the assumption that closure equals stoping width.

3.4.1 Convergence of finite depth and infinite depth solutions

It is generally assumed that the infinite depth solution applies at depths greater than 1000 m. However this is not necessarily correct. The theoretical definitions of shallow, moderate and great depth were given by Salamon (1968). These definitions are based on the ratio of $H/l_c$ i.e. depth divided by critical halfspan. The following table gives the ratios of $H/l_c$ for shallow, moderate and great depth.

<table>
<thead>
<tr>
<th>DEPTH DEFINITION</th>
<th>RATIO OF $H/l_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>0 - 1</td>
</tr>
<tr>
<td>Moderate</td>
<td>1 - 5</td>
</tr>
<tr>
<td>Great</td>
<td>greater than 5</td>
</tr>
</tbody>
</table>

According to this classification system it is possible
to calculate quantitative values for shallow, moderate and great depth, if some standard values for the elastic constants, stoping width and density of host rock are used.

<table>
<thead>
<tr>
<th>DEPTH DEFINITION</th>
<th>QUANTITATIVE DEPTH IN METRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>0 - 830</td>
</tr>
<tr>
<td>Moderate</td>
<td>830 - 1855</td>
</tr>
<tr>
<td>Great</td>
<td>1855 - Infinity</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Young's modulus: 70 GPa
Poisson's ratio: 0,2
Stoping width: 1 m
Density of rock: 2700 kg/m³

Below are given three tables of corresponding values calculated according to infinite depth and finite depth solutions. The depths at which these values were calculated were chosen as examples of particular cases in the moderate and great depth ranges.

TABLE 3.3
VERTICAL INDUCED STRAINS DOWN THE SHAFT CALCULATED ACCORDING TO INFINITE DEPTH AND FINITE DEPTH SOLUTIONS

<table>
<thead>
<tr>
<th>Z (m)</th>
<th>INFINITE DEPTH</th>
<th>FINITE DEPTH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Strain (mm/m)</td>
<td>Strain (mm/m)</td>
</tr>
<tr>
<td>-800</td>
<td>0,15</td>
<td>0,13</td>
</tr>
<tr>
<td>-600</td>
<td>0,28</td>
<td>0,27</td>
</tr>
<tr>
<td>-400</td>
<td>0,57</td>
<td>0,55</td>
</tr>
<tr>
<td>-200</td>
<td>0,91</td>
<td>0,90</td>
</tr>
<tr>
<td>0</td>
<td>0,63</td>
<td>0,62</td>
</tr>
<tr>
<td>200</td>
<td>0,91</td>
<td>0,90</td>
</tr>
<tr>
<td>400</td>
<td>0,57</td>
<td>0,56</td>
</tr>
<tr>
<td>600</td>
<td>0,28</td>
<td>0,28</td>
</tr>
<tr>
<td>800</td>
<td>0,15</td>
<td>0,14</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Depth: 1500 m
Young's modulus: 70 GPa
Poisson's ratio: 0,2
Shaft pillar radius: 700 m
Stoping width: 1 m
to calculate quantitative values for shallow, moderate and great depth, if some standard values for the elastic constants, stoping width and density of host rock are used.

**TABLE 3.2**

<table>
<thead>
<tr>
<th>DEPTH DEFINITION</th>
<th>QUANTITATIVE DEPTH IN METRES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shallow</td>
<td>0 - 830</td>
</tr>
<tr>
<td>Moderate</td>
<td>830 - 1855</td>
</tr>
<tr>
<td>Great</td>
<td>1855 - Infinity</td>
</tr>
</tbody>
</table>

**INPUT DATA FOR TABLE ABOVE:**

- Young's modulus: 70 GPa
- Poisson's ratio: 0.2
- Stoping width: 1 m
- Density of rock: 2700 kg/m³

Below are given three tables of corresponding values calculated according to infinite depth and finite depth solutions. The depths at which these values were calculated were chosen as examples of particular cases in the moderate and great depth ranges.

**TABLE 3.3**

**VERTICAL INDUCED STRAINS DOWN THE SHAFT CALCULATED ACCORDING TO INFINITE DEPTH AND FINITE DEPTH SOLUTIONS**

<table>
<thead>
<tr>
<th>Z (m)</th>
<th>INFINITE DEPTH Strain (mm/m)</th>
<th>FINITE DEPTH Strain (mm/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-800</td>
<td>0.15</td>
<td>0.13</td>
</tr>
<tr>
<td>-600</td>
<td>0.28</td>
<td>0.27</td>
</tr>
<tr>
<td>-400</td>
<td>0.57</td>
<td>0.55</td>
</tr>
<tr>
<td>-200</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>0</td>
<td>0.63</td>
<td>0.62</td>
</tr>
<tr>
<td>200</td>
<td>0.91</td>
<td>0.90</td>
</tr>
<tr>
<td>400</td>
<td>0.57</td>
<td>0.56</td>
</tr>
<tr>
<td>600</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>800</td>
<td>0.15</td>
<td>0.14</td>
</tr>
</tbody>
</table>

**INPUT DATA FOR TABLE ABOVE:**

- Depth: 1500 m
- Young's modulus: 70 GPa
- Poisson's ratio: 0.2
- Shaft pillar radius: 300 m
- Stoping width: 1 m
TABLE 3.4
VERTICAL INDUCED STRESSES DOWN THE SHAFT CALCULATED ACCORDING TO INFINITE DEPTH AND FINITE DEPTH SOLUTIONS

<table>
<thead>
<tr>
<th>Z (m)</th>
<th>INFINITE DEPTH (Stress in MPa)</th>
<th>FINITE DEPTH (Stress in MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-800</td>
<td>13.06</td>
<td>12.01</td>
</tr>
<tr>
<td>-600</td>
<td>23.08</td>
<td>22.15</td>
</tr>
<tr>
<td>-400</td>
<td>40.39</td>
<td>39.42</td>
</tr>
<tr>
<td>-200</td>
<td>54.88</td>
<td>53.99</td>
</tr>
<tr>
<td>0</td>
<td>47.97</td>
<td>47.35</td>
</tr>
<tr>
<td>200</td>
<td>54.88</td>
<td>54.15</td>
</tr>
<tr>
<td>400</td>
<td>40.39</td>
<td>39.75</td>
</tr>
<tr>
<td>600</td>
<td>23.08</td>
<td>22.66</td>
</tr>
<tr>
<td>800</td>
<td>13.06</td>
<td>12.71</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Depth: 2000 m
Young's modulus: 70 GPa
Poisson's ratio: 0.2
Shaft pillar radius: 380 m
Stoping width: 1 m

In the tables above, Z is the vertical distance above or below the shaft-reef intersection in metres. Z is negative above the reef and positive below.

TABLE 3.5
VERTICAL INDUCED STRESS ACROSS SHAFT PILLAR CALCULATED ACCORDING TO INFINITE DEPTH AND FINITE DEPTH SOLUTIONS

<table>
<thead>
<tr>
<th>ALPHA (r/R)</th>
<th>INFINITE DEPTH (Stress in MPa)</th>
<th>FINITE DEPTH (Stress in MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>34.39</td>
<td>33.74</td>
</tr>
<tr>
<td>0.1</td>
<td>34.66</td>
<td>33.99</td>
</tr>
<tr>
<td>0.2</td>
<td>35.47</td>
<td>34.79</td>
</tr>
<tr>
<td>0.3</td>
<td>36.93</td>
<td>36.25</td>
</tr>
<tr>
<td>0.4</td>
<td>39.26</td>
<td>38.58</td>
</tr>
<tr>
<td>0.5</td>
<td>42.84</td>
<td>42.19</td>
</tr>
<tr>
<td>0.6</td>
<td>48.54</td>
<td>47.98</td>
</tr>
<tr>
<td>0.7</td>
<td>57.18</td>
<td>58.14</td>
</tr>
<tr>
<td>0.8</td>
<td>75.74</td>
<td>79.97</td>
</tr>
<tr>
<td>0.9</td>
<td>133.51</td>
<td>162.29</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Depth: 2500 m
Young's modulus: 70 GPa
Poisson's ratio: 0.2
Shaft pillar radius: 530 m
Stoping width: 1 m
In Table 3.5 alpha represents the ratio of the radial distance from the centre of the circular pillar (r), to the radius of the pillar (R).

After a comparison of corresponding values tabulated above it seems fair to claim good correlation between the infinite depth and the finite depth solutions at depths as shallow as 1500 metres. This value is in the moderate depth range according to the variables used. The convergence of values calculated according to the two applications is also shown to be good at a depth of 1000 m in Figure 4.3. Finally, it must be stated once again that the actual depth at which a reasonable correlation occurs, depends on the particular situation. The infinite depth solution should definitely not be used at the theoretically defined shallow depth, since the effects of the surface are not taken into account.

With the programs developed by the writer, it is so convenient to use the complete solution that there is in fact no reason for using the infinite depth solutions.

3.4.2 Influence of the extent of mining on the shaft pillar

If it was known to what extent mining around a circular
shaft pillar has an influence on the shaft system, future analysis of this type of problem could be greatly simplified. As an example the influence of the first 1000 m of mining beyond the pillar edge (carried out in a concentric circle around the pillar) was compared with mining to infinity (taken as 30000 m from the pillar edge). In two other cases the influence of mining on the shaft-reef intersection was calculated in steps of 1000 m away from the pillar edge.

### TABLE 3.6
PERCENTAGE INFLUENCE OF THE FIRST 1000 M OF MINING BEYOND THE SHAFT PILLAR EDGE

<table>
<thead>
<tr>
<th>Depth in metres</th>
<th>Radius of pillar (m)</th>
<th>Percentage of total stress at pillar centre contributed by mining within 1000 m of pillar edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>110</td>
<td>95,14</td>
</tr>
<tr>
<td>300</td>
<td>160</td>
<td>99,64</td>
</tr>
<tr>
<td>600</td>
<td>210</td>
<td>100,00</td>
</tr>
<tr>
<td>1000</td>
<td>246</td>
<td>100,00</td>
</tr>
<tr>
<td>1500</td>
<td>300</td>
<td>94,77</td>
</tr>
<tr>
<td>2000</td>
<td>380</td>
<td>91,99</td>
</tr>
<tr>
<td>2500</td>
<td>530</td>
<td>91,65</td>
</tr>
<tr>
<td>3000</td>
<td>870</td>
<td>93,30</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Young's modulus: 70 GPa  
Poisson's ratio: 0,2  
Stoping width: 1 m

In the table above pillar sizes are first calculated according to the assumption that closure equals stoping width. Critical stress is taken as 100 MPa and critical strain as 1 mm/m. Values of 100 percent merely indicate
that the calculated values for the influence of the first 1000 m actually exceed the value calculated for the influence up to infinity (taken as 30000 m).

TABLE 3.7
INFLUENCE ON THE SHAFT-REEF INTERSECTION OF MINING BEYOND THE SHAFT PILLAR EDGE

<table>
<thead>
<tr>
<th>Radius of mining beyond the pillar (m)</th>
<th>Induced stress at the shaft-reef intersection (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth 1000 m</td>
<td>Depth 2000 m</td>
</tr>
<tr>
<td>0 - 1000</td>
<td>74,3609</td>
</tr>
<tr>
<td>1000 - 2000</td>
<td>1,9270</td>
</tr>
<tr>
<td>2000 - 3000</td>
<td>-2,6620</td>
</tr>
<tr>
<td>3000 - 4000</td>
<td>-1,1187</td>
</tr>
<tr>
<td>4000 - 5000</td>
<td>-0,3724</td>
</tr>
<tr>
<td>5000 - 6000</td>
<td>-0,1179</td>
</tr>
<tr>
<td>6000 - 7000</td>
<td>-0,0336</td>
</tr>
<tr>
<td>7000 - 8000</td>
<td>-0,0055</td>
</tr>
<tr>
<td>8000 - 9000</td>
<td>0,0034</td>
</tr>
<tr>
<td>9000 - 10000</td>
<td>0,0057</td>
</tr>
</tbody>
</table>

INPUT DATA FOR TABLE ABOVE:
Young's modulus: 70 GPa
Poisson's ratio: 0,2
Shaft pillar radii: 246 m and 380 m
Stoping width: 1 m

The information tabulated above clearly shows the large influence of mining close to the edge of the shaft pillar. In most cases the area within a 1000 m distance from the shaft pillar edge will be extensively mined out. It is then in the area immediately surrounding the shaft pillar that the influence of backfill and stabilising pillars will be felt most strongly. The
removal of stabilising pillars in the close vicinity of the shaft pillar could have far reaching effects on the shaft pillar and vertical shaft. Computer simulation could give an indication of the expected conditions after such a step.

3.4.3 Implication of the assumption that closure equals stoping width

Comparisons will be made between the theoretical condition of full closure up to the pillar edge and gradual closure from the pillar edge up to the point of full closure. The point at which full closure occurs can be estimated fairly accurately by assuming the shaft pillar to be an isolated strip pillar and using the closed form solution for this situation. Closure up to this point can be assumed to be in a straight line. However, this is not a very accurate assumption and it would be more correct to calculate the closure distribution with the MINSIM program. The influence of mining is then calculated in five steps. The assumed closure for the purposes of this calculation is such that it is always greater than the closure along the distribution line calculated with the MINSIM program.

The use of the MINSIM program in this case would be in line with the philosophy that a shaft pillar should be
designed for the worst possible case. Two-dimensional programs that can simulate non-elastic rock behaviour, might also have an application in this case. However, it will not be possible, with these programs, to model the effects of creating a circular pillar. The writer does not suggest that inelastic phenomena such as back-break will not have an effect on a shaft pillar, but an investigation into these phenomena, is not in the scope of this study.

Figure 3.4 Distribution of closure around circular shaft pillars at depths of 800 m, 1000 m and 1500 m.

The closure distributions arrived at with the MINSIM program are given in Figures 3.4 and 3.5. The pillar
sizes used here were determined according to the 100 MPa and 1 mm/m criteria.

![Graph showing distribution of closure around a circular shaft pillar at depths of 2200 m and 2300 m.](image)

Figure 3.5 Distribution of closure around a circular shaft pillar at depths of 2200 m and 2300 m.

No examination of the effects of this assumption will be done for shallow depths. At shallow depth, the calculated shaft pillar size will already be small. In the shallow depth case the pillar size would rather be decided by the size of auxiliary excavations to be placed in the pillar. The influence of the constant closure assumption will then be examined at moderate and great depth only.
Tables in which the stress distributions calculated according to the full closure (i.e. full closure up to the pillar edge) and the gradual closure assumptions, are contrasted below.

\textbf{TABLE 3.8}
\textbf{INDUCED STRESSES ACROSS PILLAR CALCULATED WITH GRADUAL CLOSURE AND FULL CLOSURE}

<table>
<thead>
<tr>
<th>Alpha (r/R)</th>
<th>r (m)</th>
<th>Gradual closure Stress in MPa</th>
<th>Full closure Stress in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0</td>
<td>56.67</td>
<td>60.02</td>
</tr>
<tr>
<td>0.07</td>
<td>20</td>
<td>57.05</td>
<td>60.22</td>
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<td>58.26</td>
<td>60.85</td>
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<td>60</td>
<td>60.35</td>
<td>61.95</td>
</tr>
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<td>80</td>
<td>63.41</td>
<td>63.58</td>
</tr>
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<td>100</td>
<td>67.58</td>
<td>65.86</td>
</tr>
<tr>
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<td>73.06</td>
<td>68.97</td>
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<td>80.14</td>
<td>73.17</td>
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<td>78.93</td>
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<td>180</td>
<td>101.24</td>
<td>87.10</td>
</tr>
<tr>
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<td>200</td>
<td>117.20</td>
<td>99.32</td>
</tr>
<tr>
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<td>220</td>
<td>139.39</td>
<td>119.27</td>
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<tr>
<td>0.80</td>
<td>240</td>
<td>172.56</td>
<td>157.11</td>
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<tr>
<td>0.87</td>
<td>260</td>
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</tr>
<tr>
<td>0.93</td>
<td>280</td>
<td>368.52</td>
<td>708.71</td>
</tr>
</tbody>
</table>

\textbf{INPUT DATA FOR TABLE ABOVE:}

- Depth: 1500 m
- Radius of pillar: 300 m
- Stoping width: 1 m
- Young's modulus: 70 GPa
- Poisson's ratio: 0.2

\textbf{CLOSURE ASUMMED:}

A) In 4 steps up to 30000 m beyond the pillar edge.
- 0-50 m from pillar edge, stoping width = 0.74 m,
- 50-100 m from pillar edge, stoping width = 0.96 m,
- 100-150 m from pillar edge, stoping width = 0.98 m,
- 150-29700 m from pillar edge, stoping width =1 m,

B) Full closure i.e. 1 m up to the pillar edge.

In the table above r is the radial distance from the
centre of the circular pillar (in metres), while alpha is the ratio of r over the radius of the shaft pillar.

**TABLE 3.9**

**INDUCED STRESSES DOWN SHAFT CALCULATED WITH GRADUAL CLOSURE AND FULL CLOSURE**

<table>
<thead>
<tr>
<th>RATIO Z/R</th>
<th>Z (m)</th>
<th>Gradual closure Stress in MPa</th>
<th>Full closure Stress in MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.67</td>
<td>-800</td>
<td>8.28</td>
<td>7.70</td>
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<tr>
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<td>-600</td>
<td>17.79</td>
<td>16.98</td>
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<td>-400</td>
<td>37.21</td>
<td>36.96</td>
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<td>-200</td>
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<td>65.70</td>
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<td>0</td>
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<td>200</td>
<td>63.41</td>
<td>66.02</td>
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<td>37.63</td>
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<tr>
<td>2.00</td>
<td>600</td>
<td>18.95</td>
<td>18.01</td>
</tr>
<tr>
<td>2.67</td>
<td>800</td>
<td>9.90</td>
<td>9.13</td>
</tr>
</tbody>
</table>

**INPUT DATA FOR TABLE ABOVE:**
- Depth: 1500 m
- Radius of pillar: 300 m
- Stoping width: 1 m
- Young's modulus: 70 GPa
- Poisson's ratio: 0.2

**SAME CLOSURE AS TABLE 3.8.**

In Table 3.9, Z once again represents the vertical distance above (negative) or below (positive) the shaft-reef intersection. So Z/R will represent the ratio of distance from the shaft-reef intersection to the radius of the pillar.

It is clear from Table 3.8 that the stress at the pillar edge is approximately two times greater with the constant closure assumption. However, this is only true
in the outer 10 percent of the pillar, while across the rest of the pillar the correlation is good. The values given show that in the particular case cited, the constant closure assumption is very reasonable indeed.

Stresses calculated at the shaft-reef intersection show a smaller difference than those at the pillar edge. When these stresses are compared, the smaller difference is between the total stresses. Since the total stresses will be those to affect the rock, it is more realistic to compare these. The differences between the total stresses, calculated according to the two different methods, varies from approximately 12 percent at the upper limit of shallow depth to approximately 2 percent at the lower limit of great depth. The distribution of differences with depth is plotted in Figure 3.6.

In conclusion it can be said that at shallow depth the safety factor will be larger with the constant closure assumption. At great depth the difference between total stresses, calculated according to the two methods, decreases to as little as 2 percent. It is safe to say that at great depth the difference between the two assumptions is negligible. The constant closure assumption can then not be expected to lead to the design of