CHAPTER IX.

REVIEW AND CONCLUDING REMARKS

9.1 The principle of effective stress.

In chapters I and IV the definition of the principle of effective stress was discussed for fully and partly saturated soils. The generalised principle of effective stress, applicable to saturated and partly saturated soils, may be stated as follows:

(i) The volume change and shearing characteristics of a soil are dependent purely on the effective stress.

(ii) The effective stress \( \sigma \) in a soil is defined as the excess of the total applied stress \( \sigma_0 \) over the pore pressure \( u \).

The definition of pore pressure, as it is used in the above context, was defined in chapter III as that component of effective stress in a soil which is derived from pressures exerted by the fluids within the pores of the soil.

9.2 The validity of the principle of effective stress in partly saturated soils.

A great deal of experimental work is available to show that the principle of effective stress is valid in a saturated soil. This is not the case for a partly saturated soil. A detailed literature survey has revealed that little or no work has been undertaken with the specific object of testing the validity of the effective stress principle for partly saturated soils. Its unquestionable validity for fully saturated soils coupled with an apparent but deceptive self-evidence has led workers to assume its validity for partly saturated soils.
In chapter IV some available experimental data on partly saturated soils was analysed with the purpose of testing the validity of the principle of effective stress. It was found that in the majority of cases the soils exhibited behaviour under changes in applied load and water content which could not be explained on the basis of the principle of effective stress. As a result of the rather unexpected findings of chapter IV a series of tests were run on three different soils ranging from a silty sand to a silty clay. These tests consisted of comparing the volume change characteristics, in confined and all round compression, of partly saturated specimens with those of equivalent fully saturated specimens. The results of these tests together with results obtained by analysing some existing data on shear strength and volume change characteristics of connected soils, confirm the findings of chapter IV.

The investigation as a whole indicates that most soils, from sands right through to clays, exhibit behaviour which, below a critical degree of saturation, cannot be accounted for by the effective stress principle. This critical degree of saturation appears to be closely related to the grain size characteristics of the soil. In granular materials it is probably not greater than about 45% while in clays it appears to be upwards of 85%. Knight (1961) performed a large number of oedometer tests on collapsing sands at different degrees of saturation. The results of his tests also pointed to the existence of a critical degree of saturation at which the soil will consolidate as if it were completely saturated*. This critical degree of saturation discovered by Knight is clearly identical to the critical degree of saturation discussed previously. Preliminary indications are that the critical degree of saturation is the same for volume and shear strength effects.

9.3 The structural behaviour of partly saturated soils.

In chapter IV the structure of partly saturated
soils and the mechanism of intergranular stress application was considered. The brief investigation furnished an explanation for the observed behaviour of these soils.

It appears that the structure of most granular soils is very different from the traditional spherical grained open pack and closed pack structure so often assumed. In fact the grains tend to form local bridges or arches. Application of external loads under drained conditions cause both shear and normal forces to develop at each grain contact point and the grains take up a closer pack by rolling and sliding. However the notion of drying out the soil causes high curvature needed to form at the grain contact points which, far from causing the grains to move relative to each other, actually 'glues' them together thereby increasing the rigidity of the soil structure. The 'bonded' particles offer considerable resistance to local shear forces induced by additions of applied load. If the partly saturated soil is wetted while under load these 'bonds' are removed and the soil structure 'collapses' to the equivalent saturated condition.

In the case of clayey soils evidence is presented which shows that the structure tends to form into 'packets' of clay particles when the soil dries. In its desiccated condition the soil structure may be considered as a granular structure in which each grain is composed of numerous clay particles tightly bonded together. As in the case of a granular soil there is little tendency for these 'particles' to slip relative to each other during the application of external loads. Unlike the granular soils, however, there are two aspects of behaviour to consider on wetting under load. In the first instance the removal of 'bonds' from between each 'grain' will tend to induce collapse. On the other hand each 'grain' will take up water and expand. The overall behaviour of the soil will depend on the magnitude of the applied loads and the change in water content. At low applied loads the volume of the soil
will increase and at large applied loads a decrease of volume on wetting can be expected. It is quite possible that soaking of the soil under load will result first in a rapid reduction in volume due to removal of intergranular 'bonds' and then a slow increase in volume due to particles taking up water.

9.4 Intergranular stresses in partly saturated soils.

In chapter IV a test performed by Bishop and Donald (1961) was described which showed that the equation used to define effective stress was of the correct form. However the work of this thesis has shown that in many instances this so-called 'effective stress' is not uniquely related to soil behaviour. In chapter VII it was suggested that the term 'intergranular stress' ($\sigma'_i$) should be used until the soil had been shown to obey the principle of effective stress.

\[ \sigma'_i = \sigma - P_a + y (P_a - P_w) \]  \hspace{1cm} (9.1)

Present methods of measuring $\chi$ depend on the assumption that the principle of effective stress is valid for the soil at the specific degree of saturation considered. Since, in many cases, this assumption is not valid, the measured values of $\chi$ have no physical significance but are merely convenient empirical parameters useful in relating the behaviour of an unsaturated soil to that of the equivalent fully saturated soil.

The direct measurement of true intergranular stress in a partly saturated soil will not be possible until a method of measuring true values of $\chi$ has been developed. However the results presented in chapter VI and the analysis of shear strength data in chapter VII indicate that in the special case of a partly saturated soil which is compressed under externally applied loads the $\sigma'_i \log \sigma'_i$ relationship is a straight line which joins up with the straight line portion of the $\sigma'_i \log$ applied load curve for the
material. The significance of this very important concept will be discussed later in this chapter.

9.5 Practical considerations.

In the introduction to this thesis it was pointed out that the principle of effective stress had played a fundamental role in the prediction of the behaviour of fully saturated soils. The discovery that the principle of effective stress is not valid for many partly saturated soils will necessitate a certain amount of readjustment along lines which do not follow the traditional effective stress approach. It is clear from the results of this investigation that the prediction of the behaviour of partly saturated soils on the basis of tests involving changes in intergranular stresses due to applied loads only could lead to gross errors, particularly for conditions where collapse or wetting occur. Although a rather negative contribution, this is probably the most important practical consideration arising out of the investigation.

The implication of the previous paragraph is that, in order to predict accurately the behaviour of unsaturated soils, it is necessary to test the soil over the range of applied pressures and pore pressures likely to be encountered in the field. Such a procedure is obviously far more complex and time consuming than the traditional approach in which it is necessary only to simulate the field effective stress conditions, independent of absolute magnitudes of \( \sigma \) and \( u \), in the laboratory.

In South Africa, where the soils are usually partly saturated and often very desiccated, it has become standard practice to test the soil at its natural moisture content and in a saturated condition. For the prediction of volume changes the double oedometer test, which was described in chapter IV, has proved both simple and reliable.

By making use of some of the results derived in
this investigation the Author has proposed a revised
method of analysing the double oedometer test, (see
Appendix G). This method can be used to estimate
insitu pore pressures and total heave for expansive
desiccated clays. The method relies fundamentally on
the concept of a linear e: log ε relationship for a
partly saturated soil which is compressed under
externally applied loads. It must be mentioned here
that the method of heave prediction is only applicable
when the change in applied loads are small in
comparison with the initial pore pressures. A large
increase in applied load might result in settlement
on wetting due to the collapse of structure mentioned
earlier. It is improbable that applied loads, which
are sufficiently large to cause settlement of a
desiccated clay on wetting, could be encountered in
practice. Nevertheless the proposed revised method
is likely to over-estimate heave in conditions where
large applied loads are involved. The whole problem
of the behaviour of partly saturated clayey soils on
wetting under load is one requiring urgent attention.
In the case of partly saturated granular materials the
work of Knight (1961) and the results presented in
this investigation offer convincing evidence that, on
wetting under applied load, the soil settles to a
unique saturated compression curve.

From the point of view of shear strength prediction
the two tests usually employed are the standard
unconsolidated undrained test for the soil at its
natural moisture content and the consolidated soaked
undrained test for the soil in a saturated condition.

The volume change and shear strength tests described
enable the behaviour of the soil to be estimated for
the two extremes of possible moisture content conditions
likely to be encountered in the field. In cases where
more exact procedures are warranted the techniques
developed by workers like Donald (1961) and Blight
(1961) should prove very valuable.
9.6 Apparatus improvement.

In chapter V was described the high pressure all-round compression apparatus used in some of the tests. The performance of the apparatus was, in general, satisfactory. However diffusion of air through the rubber sheath surrounding the sample proved serious since it prevented the accurate measurement of volume changes during raised atmosphere tests. Bishop and Donald (1961) described some triaxial apparatus in which the sample and rubber sheath were surrounded by a mercury bath so as to prevent air from leaving the sample. A scheme for modifying the present apparatus is depicted in Fig. 9.1. The modification consists simply of a 3¼" I.D. perspex cylinder three inches in length which is fitted with a flange at one end. An 'O' ring is recessed into the flange and the cylinder is fastened to the 'top hat' base plate by means of four 3/16" diameter bolts. When the sample has been set up in the normal manner the perspex cylinder is clamped in position and mercury run into it until the required depth is reached. The 'top hat' is then bolted in place and the test performed in the normal manner. It is important to note that the incorporation of mercury will require that the 'top hat' base plate, sample base and sample end plate be chromium plated.

Blight (1961) has succeeded in making ceramic discs having air entry values as high as 75 p.s.i. The permeability of such discs is likely to be very much greater than an osmotic membrane. In order to reduce the length of time involved for the samples to reach equilibrium it is suggested that below pressures of about 70 p.s.i. ceramic discs be used instead of pressure membranes. The discs could be bonded into the standard sample bases using epoxy resin.

2.7 Scope for further investigation.

The investigation just described has indicated certain deficiencies in the present approach to the
problem of the behaviour of partly saturated soils. It has also indicated lines along which further research is urgently required. The following is a brief list of suggestions, arising directly out of the work of this thesis, which might serve as a guide in planning future work.

1. The discovery that the principle of effective stress is not always valid for partly saturated soils makes it imperative that the behaviour of such soils, on soaking under various confining pressures, should be studied. The apparatus developed for the present investigation would be suitable for the volume change considerations of such a study.

2. The preliminary evidence for a linear $e: \log s$ relation for partly saturated soils compressed under applied loads requires further investigation. The establishment of this concept might contribute significantly to the understanding of the behaviour of these soils.

3. The work of this thesis indicates that present methods of measuring $\lambda$ are unreliable. It would be of great value to evolve a method for measuring true $\lambda$ which does not depend on the validity of the principle of effective stress.

4. The measurement of pressure deficiency in desiccated Free State clays under their in-situ confining pressures would be of value. The apparatus developed in this thesis could be used for this purpose. A suggested method, which is similar to the 'rate of creep method' proposed by Blight (1961), is as follows. The sample is placed in the raised atmosphere apparatus at a given confining pressure. The pressure deficiency is estimated and the air pressure within the sample is adjusted to this value. The level in the null balance indicator is brought into view by manipulating the compensating plunger. This level is observed to see whether the sample is swelling or compressing. If the soil is compressing the air pressure is too high and a lower pressure is tried and vice-versa. The process is repeated until no movement of the meniscus is observed over a reasonable length of time.
References to Chapter IX.

Proc. 5th Int. Conf. S.Y. and F.E. Paris.


Donald, I. B. (1961) "The mechanical properties of saturated and partly saturated soils with special reference to the influence of negative pore water pressures."

Knight, K. (1961) "The collapse of structure of sandy sub-soils on wetting."
DETAILS OF PROPOSED MERCURY RESERVOIR

SECTIONAL ELEVATION

PLAN
The derivation of expressions for $B$ and $\varphi u$ from considerations of $\sigma'f$

Bishop and Eldin (1950) have shown that the friction mobilized between soil grains during shearing results from a stress $\sigma'f$ given by

$$\sigma'f = (\sigma - u) + \varphi u.$$

There $\sigma$ is the total normal stress
$u$ is the pore pressure
$\varphi$ is the effective contact area of the soil particles projected onto a plane through the soil per unit area of the plane.

If it is assumed that compression of the soil results only from slipping and rolling of the grains relative to each other and that distortion of the grains is negligible then the compressibility of the soil $C_u$ must result from the action of $\sigma'f$.

$$i.e. \quad C_u = -\frac{1}{V} \frac{\partial V}{\partial \sigma'f}.$$

If $C_u$ is the compressibility of the pore water and the grains are taken to be incompressible then, for a sample whose initial volume is $V$ and whose initial porosity is $n$, the volume changes can be written

(a) Decrease in volume of the pore water = $C_u n V \partial u$
(b) Decrease in volume of the soil structure = $C_v V \partial \sigma'f$

If no drainage is permitted these must be equal and

$$C_u n V \partial u = C_v V \partial \sigma'f \quad \cdots \quad (A.1)$$
Now \( \sigma' = (\sigma - u) + au \) and if \( u \) is assumed to remain constant then \( \sigma' \) is assumed to remain constant then

Equation (A.1) may therefore be written

\[
 n \cdot \frac{C_y}{C_1} \cdot \Delta u = \Delta \sigma = \Delta \sigma (1-a)
\]

and thus

\[
\Delta u = \left[ \frac{1}{n \cdot \frac{C_y}{C_1} - 1-a} \right] \Delta \sigma
\]

i.e. \( \frac{\Delta u}{\Delta \sigma} = n = \left[ \frac{1}{n \cdot \frac{C_y}{C_1} - 1-a} \right] \ldots \ldots \ldots \ldots (A.2)

From equation (A.1)

\[
\Delta \sigma' = n \cdot \frac{C_y}{C_1} \cdot \Delta u
\]

\[
= \left[ \frac{C_y}{C_1} \right] \frac{1}{n \cdot \frac{C_y}{C_1} + 1-a} \Delta \sigma \ldots \ldots \ldots \ldots (A.3)
\]

From the geometry of the Mohr diagram (see fig. A.1) it can be seen that if a change in the minor principal stress \( \Delta \sigma \) produces a change in the deviator stress \( \Delta \sigma_d \) at failure then

\[
\sin \theta = \frac{\Delta \sigma_d}{\Delta \sigma + \frac{1}{3} \Delta \sigma_d} \ldots \ldots \ldots \ldots (A.4)
\]

Now the effect, in an undrained triaxial test, of an increase in cell pressure \( \Delta \sigma_3 \) is to cause a slight increase in effective stress \( \Delta \sigma' \) in the sample (given by equation A.3). Hence the angle of undrained shearing resistance is given by

\[
\sin \theta_{uu} = \frac{\Delta \sigma_3}{\Delta \sigma + \frac{1}{3} \Delta \sigma_3} \ldots \ldots \ldots (A.5)
\]

If the angle of shearing resistance for consolidated
Now \( \sigma' = \sigma - u + au \) is assumed to remain constant then \( \sigma' \) and \( \sigma \\) are written

\[ \sigma' = 1 + a u (1-a) \]

Equation (A,1) may therefore be written

\[ \frac{C\mu}{9} \cdot u = C_0 r u (1-a) \]

and thus

\[ \frac{u}{C_0} = \left[ \frac{1}{n \cdot C\mu / C_0 + 1-a} \right] \sigma \]

i.e.

\[ \frac{u}{C_0} = \left[ \frac{1}{n \cdot C\mu / C_0 + 1-a} \right] \sigma \quad \ldots \ldots \quad (A.2) \]

From equation (A,1)

\[ \frac{C\mu}{C_0} \cdot u = C_0 \cdot a u \]

\[ = \left[ \frac{1}{n \cdot C\mu / C_0 + 1-a} \right] \sigma \quad \ldots \ldots \quad (A.3) \]

From the geometry of the Mohr diagram (see fig. A,1) it can be seen that if a change in the minor principal stress \( \sigma_3 \) produces a change in the deviator stress \( \sigma \) at failure then

\[ \sin \theta = \frac{A \cdot \delta \sigma}{\delta \sigma_3 + \frac{1}{2} \delta \sigma_d} \quad \ldots \ldots \quad (A.4) \]

Now the effect, in an undrained triaxial test, of an increase in cell pressure \( \delta \sigma_3 \) is to cause a slight increase in effective stress \( \delta \sigma \) in the sample (given by equation A,3). Hence the angle of undrained shearing resistance is given by

\[ \sin \theta_{uu} = \frac{A \cdot \delta \sigma}{\delta \sigma_3 + \frac{1}{2} \delta \sigma_d} \quad \ldots \ldots \quad (A.5) \]

If the angle of shearing resistance for consolidated
undrained tests $\phi_{cu}$ is known then the change in strength is given by

$$\Delta \sigma_d = \frac{2 \sin \phi_{cu}}{1 - \sin \phi_{cu}} \Delta \sigma \quad \ldots \ldots \quad (A.6)$$

![Mohr diagram for stress increments](image)

Substituting equations (A.3) and (A.6) in equation (A.5) gives an expression for the angle of undrained shearing resistance as

$$\sin \phi_{um} = \frac{1}{1 - \sin \phi_{cu} \left[ 1 + \left( \frac{1 - a}{n, \phi_{cu}} \right) \right]} \quad (A.7)$$

Reference: Bishop, A.W. and Eldin, G. (1950) "Undrained triaxial tests on saturated sands and their significance in the general theory of shear strength."

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APPENDIX B

Brief description of the forces between clay particles.

The forces between clay particles only become important when they are large relative to the forces related to the mass, i.e. they are only significant when the specific area is high.

Clay particles have been found, in nearly all cases, to carry net negative charges. In order to neutralize this negative charge cations from the surrounding medium cluster round the particle. If electrostatic forces were acting alone these cations would form a film on the surface of the particle approximately one ion in thickness. However the electrostatic forces which tend to produce concentration changes in the surrounding solution are opposed by osmotic forces which tend to equalize the concentration of every ion everywhere in the solution. As a result of these rival forces a diffuse double layer is produced. The concentration of cations is high next to the solid surface but not so high as to neutralize the whole charge on the solid and the remainder of the cations required for neutralization are spread in the solution at a distance from the solid-liquid interface. When two colloids in a suspension approach each other so that their diffuse double layers interact a net repulsive force between the two colloids results. The repulsion is due to excess positive electrical charges in the double layers.

Besides the repulsive forces discussed above there are also attractive forces between clay particles. These forces arise from electrical momenta existing within the units composing the solid. The forces are similar to those acting between two short bar magnets in certain relative positions.
the magnets repel each other and in others they attract. Since there are more attractive positions than there are repulsive positions the net effect is attraction. These attractive forces are known as 'van der Waals forces'.

Frequently the charges on the surface of a clay particle are reversed at its edges. This can result in attraction between adjacent particles. These electrical attractive forces together with the electrical repulsive forces mentioned previously are known collectively as Coulombic forces.

Bibliography.


In section 9.8 it was shown that Henry’s law could be written

\[ C = K_h \cdot P \quad \ldots \ldots \quad (C.1) \]

where:
- \( C \) is the molecular fraction of dissolved gas.
- \( P \) is the pressure of the gas in contact with the liquid.
- \( K_h \) is Henry’s constant in atmosphere per mol of gas.

Now \( C \) is given by the following expression

\[ C = \frac{W_{gd}}{W_g \cdot W_l} \quad \ldots \ldots \quad (C.2) \]

where:
- \( W_{gd} \) is the weight of gas dissolved.
- \( W_g \) is the molecular weight of the gas.
- \( W_l \) is the weight of pure liquid.
- \( W_l \) is the molecular weight of the liquid.

Substituting (C.2) in (C.1)

\[ K_h \cdot P = \frac{W_{gd} \cdot M_l}{W_g \cdot W_l} \quad \ldots \ldots \quad (C.3) \]

\[ \frac{W_{gd} \cdot M_l}{W_g \cdot W_l} \]

\( ^\wedge \) After Wilf (1956)
Now \( \frac{W_{gd} \cdot V_1}{M_g \cdot W_1} \) is very small (approximately 0.0001 for 100 psi air pressure)

Therefore equation (C.3) can be written

\[
K_h \cdot p = \frac{W_{gd} \cdot V_1}{M_g \cdot W_1} \quad \ldots \ldots \ldots (C.4)
\]

From Boyle's law it is known that

\[
\frac{W_g}{P_g} = \frac{P \cdot V_g}{R \cdot T}, \quad \ldots \ldots \ldots \ldots \ldots (C.5)
\]

where

- \( W_g \) is the weight of gas
- \( M_g \) is the molecular weight of the gas
- \( p \) is the pressure in the gas
- \( V_g \) is the volume of the gas
- \( R \) is the universal gas constant
- \( T \) is the absolute temperature.

The weight of gas dissolved in a liquid can be converted to an equivalent volume \( V_{gd} \) measured in the gas phase at a pressure \( p \) as follows:

From equations (C.4) and (C.5)

\[
K_h \cdot p \cdot \frac{W_1}{M_1} = \frac{W_{gd}}{M_g} = \frac{P \cdot V_{gd}}{R \cdot T}.
\]
	herefore

\[
V_{gd} = \frac{R \cdot T \cdot W_1 \cdot K_h}{M_1}
\]

\[
= \frac{R \cdot T \cdot V_1 \cdot V \cdot K_h}{M_1}
\]
therefore

\[
\frac{V_{gd}}{V_1} = \frac{R.T. \gamma_1 KH}{M_1} = H
\]

where \( \gamma_1 \) is the density of the liquid and \( H \) is the well-known Henry's coefficient of solubility for a gas in a liquid and is seen to be independent of pressure but varies with temperature.
therefore

\[ \frac{V_{gd}}{V_1} = \frac{R.T. \frac{Y_1}{M_1} K_h}{M_1} = H \]

where \( Y_1 \) is the density of the liquid and \( H \) is the well known Henry's coefficient of solubility for a gas in a liquid and is seen to be independent of pressure but varies with temperature.
Appendix D

Electron microscope study.

The purpose of this study was to examine a typical desiccated expansive clay under the electron microscope so as to obtain some idea of the particle sizes, their orientation and areas of contact. Two methods are available: the one is to obtain thin sections of the clay and the other is to make replicas of the clay surface. It was decided to use the thin section method since the techniques are less involved and sections are thought to give a clearer picture of the clay structure.

Before a thin section of the clay can be obtained the particles must be fixed relative to each other by impregnating the clay with an embedding medium which is harder than the particles. This impregnation presents many difficulties. The distances between the individual clay particles are exceedingly small and the embedding medium must be capable of penetrating between them.

Even in very dry clays the particles are surrounded by adsorbed layers of water and the embedding medium must replace this water. Needless to say impregnation should be accompanied by as little disturbance of the structure as possible.

J.H. Gibbons (1959) succeeded in producing an embedding medium which is completely miscible with water. The embedding medium is a resin named "Aquan" and is prepared by extracting the completely miscible fraction of a partly miscible commercial resin "Spon 912". This resin is cured by mixing 10 ml. of Aquan with 25 ml. undecenyl succinic anhydride and 0.35 ml. benzyl dimethylamine and heating for four days at 60°C. Attempts to obtain sections using this type of embedding medium failed completely due to poor penetration.

Sections of plant and animal tissue have successfully been obtained using methacrylate embedding
plastic (D.H. Moore and P.M. Grimley 1957). This plastic is not miscible with water and it is necessary to dehydrate the specimens in alcohol before impregnating. n-Butyl methacrylate is usually used for soft tissues but a harder plastic can be obtained by mixing n-methyl methacrylate with the butyl.

The plastic is cured in the following manner. 2% by weight of Luperco CDB* is added to the plastic and mixed for 15 minutes. The catalysed plastic is then dehydrated with 150°C CaCl₂ and filtered. The plastic is then polymerized by incubating at 55°C for about an hour, after which time it has become fairly viscous. The specimen, which has already been impregnated with the catalyzed unpolymerized plastic, is then placed in the pre-polymerized plastic and incubated at 55°C for twelve hours. The specimen is then kept at room temperature for at least 24 hours after which time it may be sectioned.

Small specimens of clay found at a site near Witry were soaked in pure alcohol for three days. The specimens were about 1mm thick and 4mm square. After three days the alcohol was replaced by increasingly concentrated solutions of n-butyl and n-methyl (1:4) in alcohol until finally the alcohol was completely replaced by the plastic. After standing in this plastic for a few days the samples were transferred to gelatin capsules which were then filled with the pre-polymerized plastic. After curing they were sectioned in a diamond microtome.

This method has yielded fairly satisfactory results and two electron micrographs are shown in chapter V. The microshattering due to desiccation is clearly visible. Also the arrangement of the clay particles within each packet can be seen. However

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* Luperco CDB is the commercial name for 2,4-dichlobenzoyl peroxide preparation as manufactured by Novolacous Corp., Buffalo.
The relationship between void ratio and initial confining pressure for saturated and partly saturated samples of Mangla shale compacted at 180% moisture content

Derived from results obtained by Blight (1966)