\[ x = S_r + 0.3 \times (1-S_r). \] In clayey soils this relationship is no longer valid since the measured value of \( S_r \) is not representative of the conditions at grain contact points. The experimental determination of true \( x \) requires a method which does not assume the validity of the principle of effective stress. To the Author's knowledge no such method has yet been developed.
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THE RELATIONSHIP BETWEEN PARAMETER $\chi$
AND PRESSURE DEFICIENCY FOR SILT

FIG. 7.1
DEGREE OF SATURATION

RELATIONSHIP BETWEEN DEGREE OF SATURATION AND $pF$ FOR SILT

FIG 7.2
RELATION BETWEEN DEGREE OF SATURATION AND PARAMETER X FOR SILT

FIG 7.3
The relationship between parameter $x$ and pressure deficiency for silty clay.

**FIG. 7.4**
RELATION BETWEEN DEGREE OF SATURATION AND PARAMETER $\chi$ FOR SILTY CLAY

FIG. 7.5
Relation between Parameter \( \gamma \) and Degree of Saturation for Various Soils

- Breahed Silt (Donald)
- Vaich Moraine - 2\( \gamma \) = 0% (Blight)
- Silt - 2\( \gamma \) = 3%
- Talybond Clay - 2\( \gamma \) = 4% (Blight)
- Selset Clay - 2\( \gamma \) = 20% (Blight)
- Mangla Shale - 2\( \gamma \) = 21% (Blight)
- Silty Clay - 2\( \gamma \) = 23%
CHAPTER VIII

PRELIMINARY INVESTIGATION INTO THE SHEAR STRENGTH CHARACTERISTIC OF PARTLY SATURATED SOILS

8.1 Introduction

So far the work of this thesis has been devoted exclusively to considerations of volume change in partly saturated soils. A brief section dealing with the validity of the principle of effective stress in relation to the shear strength characteristics of partly saturated soils is presented in this chapter as it is thought to be of some value, if only as a guide to future work on the problem.

The experimental work described previously consisted of comparing the volume change characteristics of a partly saturated soil under changes in applied load to those of an identical fully saturated soil under the same changes in applied load. It was then possible to assess whether the soil was behaving in accordance with the principle of effective stress or not. Exactly the same approach can be adopted with regard to shear strength of partly saturated soils. Fortunately there is a limited amount of experimental data available which is suitable for analysis along the proposed lines.

8.2 The relation between shear strength and void ratio in saturated soils.

The strength $S$ of a soil can be expressed by the Coulomb equation:

$$ S = c + \sigma \tan \phi $$

where $S$ is the shear strength
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3.2 The relation between shear strength and void ratio in saturated soils.

The strength $S$ of a soil can be expressed by the Coulomb equation:

$$ S = c + \sigma \tan \phi \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.1) $$

where $c$ is the cohesion,
c is the apparent cohesion

\( \sigma \) is the total pressure normal to the shear plane.

\( \phi \) is the angle of shearing resistance.

The values of c and \( \phi \) as given in equation (5.1) have been found to depend upon the conditions of test and are therefore not constant for a soil in a given condition. Hvorslev (1937) performed a number of shear tests on two re-moulded clays and found, by assuming that the true angle of friction is a constant, that there was a direct relationship between cohesion and the equivalent consolidation pressure \( p_e \) at failure. Hvorslev then modified the Coulomb equation to

\[ S = \sigma' \tan \phi_e + c_e \ldots \ldots \ldots \ldots (6.2) \]

where \( \sigma' \) is the effective normal stress on the failure plane

\( \phi_e \) is the true angle of internal friction

\( c_e \) is the true cohesion given by the expression \( c_e = k \cdot p_o \)

where \( k \) is a constant for the soil and \( p_o \) is the pressure on the virgin consolidation curve which corresponds to the void ratio of the soil at failure.

It is well known that there is a linear relationship between void ratio and log effective stress in a normally consolidated clay. Hence if a number of identical samples of soil are consolidated under different effective pressures and then sheared the void ratio at failure will be directly related to log \( c' \) and log \( c_e \) in equation (6.2) i.e. there will be a linear relationship between the void ratio at failure and the log of the strength. This fact was pointed out in the Triaxial Shear Report (1947) which stated that for normally consolidated undisturbed clays there was a unique relationship between void ratio and compression strength irrespective of the method of test. This has
been confirmed by Bjerrum (1954) and Henkel (1959) for undisturbed and remoulded soils. Henkel (1959) has shown that a similar, though different, relationship exists for samples of a particular soil which have the same maximum consolidation pressure. In Fig. 8.1 is shown a typical curve of shear strength against void ratio obtained by Bjerrum for Zurich talus clay. Also shown is the virgin consolidation curve for the material. In general it can be said that for a remoulded soil, with a given initial void ratio, the void ratio at failure is directly related to the shear strength of the soil.

8.7 The relation between shear strength and void ratio in partly saturated soils.

Consider first the case of a partly saturated soil which conformed in accordance with the principle of effective stress. In this case the $c : \log \sigma'$ curve for the soil should coincide with the $c : \log \sigma''$ curve for the equivalent fully saturated soil which has been consolidated from the same initial void ratio as the partly saturated soil. If it is now assumed that $\sigma''$ and $\sigma'$ are not affected by changes in the degree of saturation then, following the lines of the argument presented in the previous section, there will be a unique linear relationship between void ratio at failure and log strength for initially identical samples consolidated under different effective stresses. These effective stresses can be derived either from externally applied loads or pressure deficiencies or both.

The work of this thesis has shown that the large majority of partly saturated soils analysed do not obey the principle of effective stress. This means that there is no unique $c : \log \sigma'$ relationship for such a soil over the unsaturated range and hence there is unlikely to be a unique relationship between void ratio and shear strength over this range. The conclusions reached on the behaviour of partly
saturated soils with regard to volume changes makes it
possible to predict approximately the shear strength
characteristics of such soils.

It has been observed that all partly saturated soils
which do not obey the principle of effective stress are
more compressible than their saturated counterparts.
This means that for any two samples consolidated from
initially identical conditions, the one by means of
smaller loads and the other by means of applied
pressure in addition, the intergranular pressure in
the partly saturated soil will always be larger than
the intergranular pressure in the saturated sample at
any specific void ratio. Since the shear strength of
a soil is largely dependent on the intergranular
pressure in the soil at failure it can be anticipated
that the shear strength of the partly saturated
specimen will be greater than that of the saturated
specimen for any specific void ratio at failure.

In chapter VII it was shown that the $e: \log d'_1$
curves for a partly saturated soil which is compressed
under externally applied loads is approximately a
straight line which is displaced somewhat from the
saturated virgin compression curve. Two such curves
are shown diagrammatically in Fig. 8.2.

The work of Uddin (1954) and Henkel (1959)
mentioned earlier establishes beyond all doubt that
a linear relationship exists between the void ratio
and log strength for a saturated soil. This curve is
eventually parallel to the saturated virgin
compression curve and is shown in Fig. 8.2 as a
broken line labelled $\left( d'_1 = d_1 \right)$ etc. If it can be
assumed that, as in the case of fully saturated soils,
the shear strength of a partly saturated soil is related
to the intergranular consolidation pressure then, since
the relationship between $e$ and $\log d'_1$ in Fig. 8.2 is
linear, there should be a linear relationship between
$e$ and $\log \left( d'_1 \right)$ for the partly saturated soil.
This relationship is shown in Fig. 8.2 as a full line.
and is displaced somewhat from the $a: \log \frac{a_1 - a_3}{a_2}$ straight line. The magnitude of this displacement can only be determined by experiment and is likely to be largely influenced by the magnitude of the volume changes during the test.

![Diagram](image)

**Fig. 8.2.**

Idealized relationships between void ratio and strength for saturated and unsaturated samples of the same soil.

8.4 Existing experimental data.

In a recent investigation into the strength and consolidation characteristics of compacted soils, El'zhet (1961) conducted a number of triaxial compression tests on fully saturated and partly saturated compacted soils. Alpert (1959) also conducted a limited number of such tests on a single compacted soil. The salient features of the tests which are relevant to this investigation will be discussed briefly in this section.

The three soils under investigation are Mengla...
and is displaced somewhat from the line: \( \log \left( \frac{\alpha_1 - \alpha_3}{\alpha_2 - \alpha_3} \right) \) sat. The magnitude of this displacement can only be determined by experiment and is likely to be largely influenced by the magnitude of the volume changes during shearing.

![Diagram showing void ratio vs. strength for different soil conditions](image)

### Fig. 8.2

Idealized relationship between void ratio and strength for saturated and unsaturated samples of the same soil.

#### 8.4. Existing experimental data

In a recent investigation into the strength and consolidation characteristics of compacted soils, Night (1961) conducted a number of triaxial compression tests on fully saturated and partly saturated compacted soils. Alpin (1959) also conducted a limited number of such tests on a single compacted soil. The salient features of the tests which are relevant to this investigation will be discussed briefly in this section.

The three soils under investigation are Mangla
and is displaced somewhat from the \( e: \log \frac{1}{2}(\sigma_1 - \sigma_3) \) sat.
line. The magnitude of this displacement can only be
determined by experiment and is likely to be largely
influenced by the magnitude of the volume changes
during shearing.

![Diagram](image)

**Fig. 8.2.**

Idealized relationships between void ratio and
strength for saturated and unsaturated samples
of the same soil.

### 3.4 Existing experimental data.

In a recent investigation into the strength and
consolidation characteristics of compacted soils,
Blight (1961) conducted a number of triaxial compression
tests on fully saturated and partly saturated compacted
soils. Alpan (1959) also conducted a limited number of
such tests on a single compacted soil. The salient
features of the tests which are relevant to this
investigation will be discussed briefly in this section.

The three soils under investigation are Mangla
shale, Talybond clay and Selset clay. The relevant properties of these three soils are listed in Table 8.1.

**Table 8.1**

Properties of soils used in shear strength analysis

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Mangla shale</th>
<th>Talybond clay</th>
<th>Selset clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>LI.</td>
<td>38.2</td>
<td></td>
<td>33.1</td>
</tr>
<tr>
<td>PI.</td>
<td>21.6</td>
<td>16.6</td>
<td></td>
</tr>
<tr>
<td>S.G</td>
<td>2.73</td>
<td>2.69</td>
<td>2.70</td>
</tr>
<tr>
<td>-2 μ</td>
<td>21</td>
<td>4</td>
<td>20</td>
</tr>
</tbody>
</table>

The samples were prepared by compacting the soil in a standard Proctor mould and jacking thin walled brass sampling tubes into the compacted soil. The samples were then extruded from the tubes and trimmed to size.

Two types of test were performed on the saturated specimens: consolidated undrained and consolidated drained. The samples were saturated by consolidating them with high controlled positive pore pressures (60-100 p.s.i.). A set of 'over consolidated' tests was run on Mangla shale by first consolidating the samples at an effective stress of 110 p.s.i. and then swelling them back to the confining pressure at the start of shearing. After equilibrium had been reached at the required confining pressure shearing took place.

All the tests on partly saturated soils analysed in this chapter were run at constant moisture content. The samples, at their compaction moisture contents, were allowed to come to equilibrium under the required confining pressure (65-Pa). They were then sheared with measured pore air and pore water pressures.
The results (relevant to this investigation) for the saturated and partly saturated tests are tabulated in Appendices E and F respectively. For the saturated consolidated undrained tests the overall water contents and the water contents in the failure zone are given together with the confining pressure and the deviator stress at failure. Using the values of specific gravity given in Table 6.1 the overall void ratio at equilibrium under the initial confining pressure and the void ratio on the failure plane have been calculated for each specimen. In the case of the confined tests on Talybont clay only the water contents in the failure zones were measured so that it was not possible to calculate the initial void ratios.

For the tests on the partly saturated specimens the degree of saturation at equilibrium under the initial confining pressure and at failure are given. Since the moisture content remained constant throughout the test it has been possible to calculate the initial void ratio and the final overall void ratio at failure. Blight (1961) has given good reasons for believing that the void ratios on the failure planes were rather higher than the overall void ratios at failure. It would, however, have been extremely difficult to measure these void ratios accurately.

8.5 The analysis and discussion of results.

In fig. 8.3 is a plot of void ratio against log initial confining pressure for the Mangla shale compacted at approximately 16.3% moisture content. It can be seen that the e: log p curve for the partly saturated soil crosses the saturated e: log p curve at a confining pressure of approximately 5.5 p.s.i. and thereafter lies above it. There is a marked similarity between the curves shown in fig. 8.3 and the curves obtained from the tests described in chapter VI. It is apparent that the partly saturated Mangla shale at 16.3% compaction moisture content does not obey the principle of effective stress.
Curves of void ratio at failure against log of compressive strength for the Mangla shale compacted at 18.3\% moisture content are shown in fig. 8.4. The values obtained for the saturated specimens can be seen to lie on a straight line. Of very great significance is the fact that the values of compressive strength obtained for the partly saturated soil also lie on a straight line. This line is displaced from the saturated line as predicted in section 8.3 and indicates that the soil does not behave in accordance with the principle of effective stress.

The e: log confining pressure curves for the Mangla shale compacted at 18.0\% moisture content are given in fig. 8.5. The results for the normally consolidated and 'over' consolidated saturated samples lie on smooth curves. The values for the partly saturated samples are very scattered and it is difficult to decide on the position of the compression curve. The important observation is, however, that the curve will lie to the left of the saturated normal compression curve showing that the soil probably behaves in accordance with the principle of effective stress. The curve of void ratio against log compressive strength for the saturated Mangla shale is again a straight line (see fig. 8.5). This line appears to be unique for the soil in a 'normally' consolidated and 'over' consolidated condition. The values of compressive strength for the partly saturated samples also lie fairly closely grouped around the saturated compressive strength line. It therefore appears that the soil obeys the principle of effective stress with regard to shear and volume change.

The results for compression and shear in Talybont clay, compacted at approximately 7\% moisture content, are given in figs. 8.7 and 8.8 respectively. The void ratio log confining pressure curve for the partly saturated samples lies above that for the saturated samples. Following the line of reasoning presented previously it must be concluded that the soil does not obey the principle of effective stress for volume change. The relationship between void ratio and log compressive strength is seen, once again, to be linear for the saturated and the partly saturated.
specimens. The observation that the two curves are widely displaced confirms the above conclusion that the soil behaviour is not in accordance with the principle of effective stress.

The results of the tests on Selaet clay are plotted in Figs. 8.1 and 8.10. It can be seen that the soil obeys the principle of effective stress in both volume change and shear. The values of compressive strength for the partly saturated specimens are rather less than for the saturated specimens. This might be due to the fact, mentioned previously, that the measured void ratios are lower than the values on the failure plane.

It is of considerable interest to relate the test results discussed in this chapter with the shapes of the \( x : S_r \) curves, derived from the same tests, given in Fig. 7.7. The analysis of the tests on Mangla shale show that at initial degrees of saturation of approximately 95\% the soil behaves in accordance with the principle of effective stress while at initial degrees of saturation of approximately 82\% this is not the case. It can be seen from Fig. 7.7 that down to a degree of saturation of about 92\% the \( x : S_r \) curve for the Mangla shale follows the theoretical relationship quite well. Below this value, however, it plunges down steeply. The average values of initial degree of saturation for the Talybont clay is about 49\%. The soil at this degree of saturation was shown not to obey the principle of effective stress and this is reflected in the relevant \( x : S_r \) curve which, at \( S_r = 49\% \), is well displaced from the theoretical relationship. The Selaet clay, which had an average initial degree of saturation of about 82\%, was shown to behave in accordance with the principle of effective stress. In this instance, the soil behaviour is not reflected by the \( x : S_r \) curve which is plunging steeply at a degree of saturation of 82\%. In general, however, the postulate that the shape of the \( x : S_r \) curve is a good indication of the range over which the principle of effective stress applies (see Section 7.7) is
The results presented in this chapter show that the strength of a partly saturated soil is not a unique function of void ratio at failure (as is the case for a fully saturated soil). However, in all the cases analysed, the relationship between void ratio and log compressive strength is a straight line for a partly saturated soil which has been compressed from a given initial moisture content. This gives further weight to the belief that the $e: \log \sigma'$ curve is a straight line during compression of a partly saturated soil.

Another very important conclusion to be drawn from the work of this chapter is that the critical degree of saturation at which the soil begins to depart from normal effective stress behaviour is approximately the same for volume change and shear. This critical degree of saturation seems to be fairly well defined by the point at which the $x:S_r$ curve for the material begins to dip down sharply.
correct.

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The relationships between deviator stress and void ratio at failure for samples of Mangla shale compacted at 16.3% moisture content.

Derived from results obtained by Blight (1961).
The relationship between deviator stress and void ratio at failure for samples of Mangla Shale compacted at 18.0% moisture content derived from results obtained by Slight (1961).
THE RELATIONSHIP BETWEEN VOID RATIO AND INITIAL CONFINING PRESSURE FOR SATURATED AND PARTLY SAT SAMPLES OF TALYBOND CLAY COMPACTED AT APPROXIMATELY 7% MOISTURE CONTENT DERIVED FROM RESULTS OBTAINED BY BLIGHT (1966) AND ALPAN (1959)

**Fig. 8.7**
THE RELATIONSHIP BETWEEN DEVIATOR STRESS AND VOID RATIO AT FAILURE FOR SAMPLES OF TALYBOND CLAY COMPACTED AT APPROXIMATELY 7% MOISTURE CONTENT

DERIVED FROM RESULTS OBTAINED BY BLIGHT (1961) AND ALPAN (1959)
The relationship between void ratio and initial confining pressure for saturated and partly saturated samples of Selsel Clay compacted at 11.6% moisture content. Derived from results obtained by Blight (1969).
**THE RELATIONSHIP BETWEEN DEVIATOR STRESS AND VOID RATIO AT FAILURE FOR SAMPLES OF SELSET CLAY COMPACTED AT 11.6% MOISTURE CONTENT**

DERIVED FROM RESULTS OBTAINED BY BLIGHT (1961)

---

FIG. 8.10
Author  Burland J B
Name of thesis  The Concept Of Effective Stress In Partly Saturated Soils. 1961

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