NEW APPROACHES TO SECONDARY MATHEMATICS

By

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I WOULD like to talk about some of the experimentation that is going on in England, although it is difficult to pass comment on something on which there are not as yet any results. I do not think anyone is in a position to say what the results will be. I think it is quite easy for someone to carry out an experiment under the best possible conditions, but "by their fruits ye shall know them" and I do not think we have lived long enough to see what the fruits are.

In the primary schools, the main way in which the experiments have been carried out has been the training of teachers—special courses, in-service training and so on. In all the teachers' courses that we run in mathematics, it is believed that if you have a complete morning devoted to mathematics, that is enough for you to digest for one day. You spend the rest of the day thinking about it, writing about it, if you like, working on it. Then you are ready for another stint the next day. We give the teachers in their courses sufficient information only to stimulate them, to make them read and to think out their own ideas from what we have said. This has proved highly successful.

The great changes that have taken place have come about through the teachers' courses in the primary schools, where there are rooms full of all kinds of equipment, where the teachers may spend three days discovering for themselves the kind of situations that they are going to present to the children. When teachers discover that they are going to have a morning's practical work, they go white because they think that they are going to expose all their weaknesses and that we shall find out they cannot do some of the things that we expect the children to do. But once they have done the practical work and once they have carried out these experiments, their faces light up and it is a joy to see them. One thing is certain—whatever they have done at that course, they are going to do in their school In fact at times this worries me because if they have been doing a series of discoveries about curves, I guarantee that whatever else they are going to do next week at their school, they will be having curves.

We have left it to the teachers—it does not matter what experiments we are going to run in England or anywhere else, it does not matter what we are going to say from the rostrum, we are powerless—it is entirely in the hands of the teachers themselves to look with open minds at all that is offered them, sieve it out and accept or reject it, but most of all to think about it.

When we talk about experimentation we owe an enormous amount to people who are concentrating on the psychological aspect of this work, but it is the teacher who will know whether it has succeeded or not. We can say anything we like here, and all of you can go back and can make it fail. I do not think that any teacher would legitimately do that, but it is quite possible actually to make it fail if you want to. This puts a great responsibility on you. Do the teachers realise how important they are? Do they realise that in their hands are the children of the next generation, and on the teachers themselves depends what standard of thought, what confidence that child will achieve in himself? If you are doing work that you know is frustrating your children (and if it is frustrating your children, I would imagine it could well be frustrating you as well), something is wrong and something has to be done. To bring about change is a difficult thing. To get teachers to think away from the traditional lines is a very, very difficult thing.

Think about what we have said, think about your position in charge of the children, think about the children, and if your children can face with confidence the new challenges you offer them then all the experiments you are doing are correct. If they cannot face new challenges with confidence, you have got to think again.

I want to make it clear that experimentation is not taking place everywhere in England. Enlightened mathematics teaching is not found everywhere in England. I think the day is coming though when they will all have to change and they will have to start some of the new lines of thought because so many are doing it and it is spreading so much that authorities and teachers will feel

they are behind if they do not do something about it. It is almost a kind of gentle blackmail imposed by the teachers themselves.

The greatest danger to all experimentation in our country is examinations—not that I think examinations are necessarily harmful; it depends on what type of examination it is, whether it is going to assist in teaching on a wide scale or not. Do not get the idea that I am decrying examinations—I may decry the form which some of them take.

When we come to examine the secondary schools, here again experiments are taking place in what I would call certain areas. It is not common throughout the country. I do not know whether any of these experiments that I am going to discuss with you will eventually pass out completely and die. Whereas, in England, the primary schools on the whole are clammering to get rid of examinations, the secondary schools seem to be bringing them back. We have, as you know, the G.C.E. We have coming up next year the C.S.E. (Certificate of Secondary Education) and I should not be surprised if in a few years' time, there is not another one for the next lot down. I think the indroduction of the C.S.E. to the secondary schools of England may well put the clock back ten years or perhaps twenty. The 1944 Act was, in my opinion, the greatest piece of educational thought that has ever been produced. It gave the secondary schools of England the freedom to do whatever they wanted to develop their children into human beings. When we start to bring in these examinations I am a bit worried as to whether or not the experimentation will die, because they will now have to work to a syllabus.

Some teachers here before me now may think out some presentation, some development of mathematics, that can change the whole course of the teaching, with the result that all I have said to you in the time I have been with you may in a few years be considered incorrect because somebody has thought of something else. It may be some teacher just coming out of training college who produces a spark of thought that can change the whole aspect of what we do. If you can think out some ways that are going to make your children thoroughly enjoy the subject you teach, you should declare it to everyone, not keep it to yourself. That is the sign of a great person—one, who having discovered something, offers it for the benefit of everyone else.

Many of our mathematical educationalists in the secondary field are defining modern mathematics as mathematics which is or might be of interest in schools, but which was not present when they were at school. They are thinking about what mathematics is new in our modern life. With this idea in mind they are experimenting on linear programming, things that we have not touched before, such as games theory and numerical techniques with computers. They are also giving consideration to mathematics which is old but which has become important because of modern conditions and changing circumstances. We have found matrices and groups and many of the iterative techniques coming into being. But I think that the greatest experiments that are being carried out are those where consideration has been given to the means by which systems are structured. Modern mathematics is concerned with structure. I do not think it is to be studied axiomatically, rather the discovery of the structure in concrete situations which are part of a pupil's experience. They are really trying to follow up what is best in primary school teaching in mathematics in order that we do not build up an attitude in the primary school and then kill it in the secondary school. This is always a danger, that you can get the primary schools full of life, all the experiment in the world going on, all the children free to discover, then along they go to a secondary school, and the whole attitude and the whole presentation completely vanish.

Thinkers are distinguishing between set theory and set language. No one is suggesting that set *theory* should be taught in schools, but set *language* ought to begin with the reception class and the set algebra can begin in primary schools with great confidence once any operations on sets take place. It is going to be very difficult if you try a crash programme called "sets" as a little compartment fitted in somewhere in your scheme after you have done everything else. This could well happen where you get a syllabus in which the work is all detailed and then at the end comes "the language of sets", and so we have half a term on that. That would be an entire waste of time—it must start in the reception class. Set language, which involves set membership, which involves inclusion of sets, can clarify incidental work and simplify vocabulary. The concepts of a set will not involve changing your traditional work. In all the experiments in England, sets have been introduced at the beginning, but it has never been intended to alter the kind of work that they do traditionally; but by using that language, we may find integration of mathematics, which I consider essential.

All the experiments in England are concerned with integrating mathematics. If we are going to have a lesson on arithmetic, a lesson on algebra and a lesson on geometry, we have not made the best use of mathematics, neither have we presented the subject as a whole.

The whole purpose behind the introduction of modern mathematics in schools can be summed up in this way: *all the experiments take notice of contemporary mathematics.* They include contemporary usage of mathematics in industry and in science. They have looked at industry and science, they have seen what is wanted and they are considering in what way mathematics in schools can assist those people. Some of them have concentrated on being of assistance to the university, so that when pupils go there, they will not be lost because they do not know the language. Many students cannot somehow get into the language of modern mathematics. They cannot wipe out the traditional terms and they cannot think for themselves. The mathematics that they have been doing in England has not allowed them to think for themselves as much as the professors would have liked.

All the people who have run experiments in England have given full consideration to the fact that they are trying to put mathematics into a setting which the pupils recognise as within the experience of the twentieth century. All of them throughout have the twentieth century in mind. What can we do in mathematics that is going to be helpful to these people as far as this century is concerned? All that they have put into their experiments is taught in the light of educational developments of the past thirty years, paying due attention to providing background experience aiming at insight into structure and encouraging people to recognise the patterns into which mathematical ideas fall. It is recognised that any new syllabus must leave the pupils with techniques adequate to solve all the traditional problems they meet in the world at large.

However, reasoning must play an important part. Emphasis is placed on discussion from which some originality based on sound thinking and accompanied by reason will result. It is fair to state that all experiments in progress are the result of deep thinking by groups of people interested in reforming the teaching of mathematics. The presentation, the order of presenting material, the standards to which the material is presented, naturally vary with each experiment.

Now a word about the Midlands Experiment: Mr. Cyril Hope of Worcester is the brain behind this experiment. The Midlands Experiment is concerned with a cross section of normal children in normal schools, and the material is suitable for the average child even in the secondary modern school: in fact, some of the experiments in it are so lively and so well explained that I have seen primary school children working on certain suggestions of the "O" level Midlands Experiment book. This experiment is the sum total of all that has been tried in the last twenty years. It is not a continuous development of mathematics—it is a project book. They take something that is appropriate, develop it in a lively and interesting manner, and out of each project, children do their addition, substraction and multiplication and learn something else as well. The experiment itself has not been going very long, but it is spreading rapidly and it is spreading into the best top classes of primary schools. It lays great emphasis on sets and brings them in in the various forms in which I have introduced them to you.

The second experiment is one that was carried out in Leicestershire by Professor Skemp of Manchester. Skemp is a psychologist and was previously a teacher of mathematics. His project in Leicestershire will, I believe, have a wide influence in presenting mathematics both to primary, secondary modern and grammar schools. In his scheme, three schools agreed to take part—a grammar school, using two of its three unstreamed first forms and two unselected high schools, one using its top stream, the other its second stream. Thus Dr. Skemp was in a position to judge his experiment by comparing the results of children taught by his presentation and those who were given the presentation in what could be described as traditional form. He is the one man who has set out to try his experiment in schools with experimental and control groups. He stated very emphatically that if results were not satisfactory and satisfactory reasonably soon, he was prepared to drop the whole thing. In order to ensure that the children were taught on the lines he wanted, Skemp wrote the text himself, commencing with trial chapters which he revised eventually. Since then he has been writing chapters as fast as he can because I understand the children have been covering the work faster than he could write the material. Dr. Skemp has produced a work that can be used in any school you like. This is the one experiment that I think you could really go to town on because this is an experiment in which the whole mathematics syllabus evolves. Skemp has developed this scheme on very simple lines with a follow through from number and it is developed for decimals, the rational numbers, indices, logarithms and we go right through, developing a common thought. I would recommend unreservedly that you read Dr. Skemp's book, *Understanding Mathematics.*

Both these experiments require the discussion approach. Another point about Skemp's work is that it is the kind of book you could give to a child and he could work through it on his own.

The third experiment is the St. Dunstan series, which is some work that was done by Dr. Matthews. I think Matthews is a man who has vision, but has not as yet been understood by many people in England. The St. Dunstan experiment is one in which again we have the modern mathematics, but this is done in project form. Dr. Matthews is an authority on matrices and on set language. Mathematical development, mathematical thought and the power to discuss mathematically have resulted from children using this series. Matthews includes some logarithms which is rather interesting. He puts them in "for the sake of the morale of the third and fourth streams". One thing about the Matthews plan that I like is that he has issued his work in booklets on the project lines, but they are books that the children could just wade through once they have had a certain amount of instruction on them. But you cannot give them the books as you can Skemp's and allow them to get on on their own without preliminary discussion. Only Skemp has written in the simple language that allows a child to get on on his own.

The last project that I want to mention is the Southampton Mathematics Project directed by Professor Brian Thwaites. Again we have the experimental topics. It is interesting to read the names of the schools that have been taking part in this experiment—Battersea Grammar School, Charterhouse, Exeter School, Holloway School, Marlborough College, Sherbourne School, Winchester College and Winchester County High School for Girls. This is what the author of this project says: "At the outset, it is important to stress the experimental nature of this project. We have felt that it was only realistic to work within the framework of existing examination arrangements, so the trial syllabuses and specimen papers have received a wide circulation". (They knew this was going to be included in the examinations, so they got out the scheme and selected their projects and their development around the very things that were going to be placed in that examination.) "It should not be thought however that these represent more than a basis for investigation. It is less than two years since the project was launched and we expect these syllabuses to be substantially revised in the light of experience. We regard it as our function to try out any idea or any new topic which appears to us to be reasonable and practicable." (If any of you are going to try to run experiments here, I think you might well take their ideas into consideration.) "Where we have not done so, it may be for a variety of reasons—the lack of imagination on our part, a belief that the topic in question does not offer sufficient return in terms of mathematical ideas for the time spent on it. As an example, we have decided on these grounds for the present not to go as far as developing the technique of differentiation in an elementary mathematical syllabus at ordinary level." I would suggest that every teacher here could well do with the whole set of books of the Southampton Project as their reference books. "After a year on the new syllabus, there have been signs of mathematical indigestion in our pupils due to the necessity of introducing a large number of new concepts and new vocabulary early in the course. This is a special problem of starting with a 13-year-old group who have to readjust their mathematical outlook already well established along traditional lines."

If you give it to a child in its raw form, he will have indigestion. Here it is admitted that this can happen, and it is pointed out that they feel this is due to starting with 13-year-old children, who have not been able to get rid of some of the traditional mathematics, and the new stuff has really caused trouble. I think it will anywhere if it is introduced at 13 and this is where I am going back to what I said in the beginning—the earlier the better.

"We have not always found it easy to ensure that pupils achieve an adequate measure of success. New material places greater stress on verbal facility, and the simple pleasure of getting sums right has not always been provided for." I think from that we can learn a great mathematical truth. Whatever experiments we are going to do, we must have some success in the beginning for the children. I would imagine that you as teachers will ensure that all cases that whatever work your children do, the first few are right. I think there is nothing more horrible than to set them problems — they get them wrong at the beginning and the crosses carry on all through.

"There is a shortage of carefully graded problem material. This is a temporary problem, but such material can only be built up slowly as experience is gained with teaching along the new lines."

May I sum up thus: in all the experiments that are taking place in England, they are trying to introduce a new language that will integrate the whole of mathematics. They are trying to get the language of sets to be the means of introducing this integration of subjects. But they have not forgotten that discussion is an important part of mathematical training. They have not forgotten that it is a good thing to put some of these concepts in very simple form. What are the results to be? I do not know; no one knows. I only know this—that in the schools where all these experiments are carried out, some progress is being made and

some joy is present because in every case, they are being carried out by enthusiastic teachers. That is half the battle with any experiment. We must have people who will "have a go". If you who take counsel at this stage in the presentation and development of this subject feel that the time is ripe, that things in the garden are not what they should be, now is the time to take courage and see what experimentation can do; experimentation introduced with wisdom and judgement is not a haphazard thing. These experiments have been very carefully thought out. You will have to get the books if you want to know more about them, but I want you to understand this: they have thought about all the best ways of integrating mathematics to make children happy. I can only hope that perhaps there may be people today who will themselves put on paper some experimental work that may integrate the subject as you would like it to be integrated.