Modern Mathematics at St. Andrew's School (Bloemfontein)

by

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AS a teacher of secondary school mathematics I was, for several years, disturbed by the fact that so many of my pupils who were reasonably good at arithmetic found algebra difficult and I concluded that there must be something radically wrong in our approach to the teaching of arithmetic in the primary schools. For seven years these children had been extremely well drilled in the computational aspects of arithmetic but the conceptual aspect appeared to have been ignored. It almost seemed that they were scared of algebra as they expected difficulties and inconsistencies that did not exist. Having accepted, for many years, that the number represented by the numeral 123 had very little to do with those represented by 312, 213, etc. they were suspicious of the equality relationship of xyz, zxy, yxz, etc. resulting from the commutative law for multiplication of which they had never heard. Furthermore, well learnt facts such as $2\frac{1}{2}=2+\frac{1}{2}$ whereas $25=2\times10+5$ seemed to have destroyed their confidence in the logical basis of the subject.

At about this stage in my thoughts on the matter I read *Modernising School Mathematics* by Merydydu G. Hughes, and I became extremely interested in the new approaches to mathematics teaching overseas as described by him. I was even inspired, without being requested to do so, to write an article on *Recent Trends in the Teaching of Mathematics* which appeared in the January 1963 issue of *The Teacher* (O.F.S.T.A.), but my article appeared to have very little impact. I was then offered the wonderful opportunity to spend a year in the U.S.A. studying *Modern Mathematics* at an Academic Year Institute on a grant from the National Science Foundation. We had just introduced the Cuisenaire—Gattegno method of teaching arithmetic in our Sub A class and I left instructions with my Sub A teacher to experiment with the introduction of elementary algebra at that stage — which she did with remarkable success. In July 1964 I returned from the U.S.A. determined to play a part in the modernising of mathematics teaching in the Republic. I addressed meetings of school principals and meetings of mathematics teachers in Natal and the Transvaal and conducted a course for my own primary school teachers during the latter half of that year so that we were prepared to introduce *modern mathematics* at St. Andrew's as from the beginning of 1965.

With teachers only partly trained in the new approach it was essential that they be provided with adequate textbooks for which teacher's guides were readily available and that the course should be one which had passed the preliminary experimental stage. I was firmly convinced, and still am, that modern mathematics in South Africa should first be introduced in the primary school. In the United States, many of the programmes were started at the middle school level and then extended to the elementary schools with the result that the middle school programmes had to be considerably revised at a later stage. It was therefore essential to start on a course which had been successful overseas, which catered for our needs and for which a complete set of textbooks with teacher's guide was available, and the S.M.S.G. course was the only one which, at the time, fulfilled all these requirements. This course started at the Grade 4 level (Std II) — Grades 1, 2, 3 and Book K have since been published — and we introduced S.M.S.G. grade 4 at both the Std I and Std II levels and S.M.S.G. grade 5 at the Std III level. The Std I teacher, who had inherited the class that had been trained in Cuisenaire, adapted the S.M.S.G. course with considerable enthusiasm but was only able to complete half of the Grade 4 (Std II) work. The Std II and Std III teachers were more conservative and, being reluctant to abandon their traditional methods, taught mathematics as an extra subject while persisting, to a certain extent, with the old fashioned arithmetic teaching — just to make sure! The Standard III teacher found that she had to revert to S.M.S.G. grade 4 for a time as she and the pupils were not able to cope with grade V without some grade IV grounding.

This year the Std II class will easily complete Grade IV and make a start on Grade V while the Std III class should experience little difficulty in completing Grade V. In other words our Std III pupils this year will be working on a par with pupils in the U.S.A. who are following one of their most advanced mathematics programmes, and our Std II pupils will be ahead of their American counterparts.

The S.M.S.G. course at these levels emphasizes the difference between numerals and numbers, introduces elementary set theory, inequalities, operations, mathematical sentences, the number line, multibase arithmetic etc. and develops the

axioms of a number field. A considerable amount of congruence work on elementary geometry — rays, lines, planes, congruence and geometrical figures is included. The reactions of both pupils and teachers were most favourable but, partly due to the teachers' inexperience in teaching modern mathematics and their tentativeness in adopting it, progress was slow. The texts themselves appear to be rather laborious, but they are a considerable advance on texts previously available for the primary school.

During 1965 in Standard IV and V we were reluctant to change the course, but made some progress in teaching set theory and multibase arithmetic. This year, however, we are following the Scottish Mathematics Project in Std V and it is hoped to continue with this course through to matriculation. To date only Book I is available, but if the succeeding volumes match Book I, this course, which is less revolutionary than most other modern courses, should prove to be more than adequate for the situation in South Africa, and is recommended for use until we produce our own modern mathematics texts. (Some of these texts have already appeared, but only a few are satisfactory and others should be banned!)

Our one year experience of *modern mathematics* in primary school (and here I refer to a complete course and not just "bits and pieces" as in our Stds. IV and V) has resulted in our support for the oft proclaimed viewpoint that children now enjoy mathematics and no longer regard it as a subject beyond their comprehension. The teachers have also welcomed the approach as being one which requires less drill work and more understanding and we are at present also experimenting with desk calculators in the primary school in order to cut down further on the more boring aspects of elementary arithmetic.

In the Senior School we introduced modern Mathematics last year at two levels — Standard VI and post matriculation. The latter form continued its normal course in pure and applied mathematics but worked on the St. Dunstan's booklets in addition. These were not very suitable for this purpose as much of the material is too elementary, whereas other portions are very advanced and require to be supplemented. In Standard VI we used Pearson Allen's Modern Algebra Book I (published by Ginn & Co.). This book was selected as it covers the traditional material from a very modern approach. In common with most modern mathematics textbooks it starts with set theory, the number line and mathematical sentences. This is followed by a long and interesting chapter on logic which, however, enters into far more detail than is really required for subsequent study. Operations with the numbers of arithmetic follows and, at this stage, pupils have really acquired the language of *modern mathematics*. The set of real numbers is then studied, the axioms for a number field are enumerated and rigid proofs are then supplied for algebraic processess. Several of my colleagues who teach mathematics at the Std. VI level and who had been influenced by the British intuitive approach were of the opinion that the axiomatic approach of the American programmes was unsuitable for school children. My one year experience in teaching modern mathematics to Std. VI confirms my belief that the pupils find the axiomatic approach far more satisfactory than the intuitive approach, and this applied particularly to the conscientious pupils. Unfortunately there was only one boy in the class with an I.Q. in the 120–125 range so that I was unable to judge the reactions of really intelligent pupils. To illustrate my contention I quote one example. After being led up to the axioms for a number field, i.e. the closure, commutative and associative properties for addition and multiplication, the existence of additive and multiplicative identities and inverses, the distributive property of multiplication of addition, they learnt them with enthusiasm, and when we discussed the proposition that $-(-a)=a$, they accepted it with understanding. They knew that there exists an additive inverse for every real number, and that if a is a real number its additive inverse is $(-a)$, and hence the additive inverse of $(-a)$ viz $-(-a)$ is the same number as a. Other propositions i.e. If a and b are real numbers and $ab=0$ then $a=0$ or $b=0$ followed without difficulty, and I found it most exhilarating to teach these pupils. There were, however, a few pupils in the class who were not prepared to learn their axioms or the results of the theorems and they floundered badly. The class attitude to the symbolism of modern mathematics was most refreshing. They enjoyed being able to read mathematical sentences which confounded matriculation pupils and flabbergasted their parents.

Our geometry course in Std. VI was less satisfactory. I tried to strike a compromise between the S.M.S.G. and the traditional approaches. I believe that this is possible but it would require months of work on what amounts to writing a textbook, and for this no conscientious schoolmaster can find the time. The American approach is too fussy and the traditional approach too illogical for them to be easily reconciled. Furthermore I consider that our future approach to geometry should be based upon vectors, but this has yet to be accepted by our examining authorities.

To sum up, I would say that our experiments at St. Andrew's have met with qualified success. We shall never revert to the traditional approach, but we are not yet convinced that we have found the ideal modern approach. We do, however, believe that we have made real progress and we entertain high hopes for the future.