CHAPTER 1

BACKGROUND TO THE STUDY

The aim of this study was to explore instructional strategies that teachers in multilingual classrooms use in order to support their learners' development of mathematical proficiency. Barwell and Setati explain multilingualism as follows, 'Multilingualism is not just about the number of languages you can speak as an individual, it is very much about participating in a multilingual society' (Barwell and Setati, 2005:20). They go on to add that for many African people in South Africa being multilingual is normal. Multilingualism is therefore a prevalent and an important feature of the South African society. There are now 11 official languages in South Africa and the majority of South African citizens can speak two or more languages fluently. It can also be argued that the majority of South African classrooms are multilingual. I see a classroom as multilingual if any of the participants in the classroom, i.e. learners, teachers or helpers in classrooms, is potentially able to draw on more than one language as they learn, teach or help learners.

The complexity, however, in the majority of these classrooms, is that learners are learning mathematics in a language that is not their first language. Setati (2002) reports that even though the language-in-education policy encourages and supports the use of any of the 11 official languages as a language of learning and teaching (LoLT), research has shown that the LoLT is still either English or Afrikaans. So even though multilingualism is encouraged and respected, the African languages are still not chosen as LoLT in multilingual classrooms. In some of these multilingual classrooms the teacher shares the same main language with the majority of the learners or with all the learners. In other classrooms the teacher may share a main language with only a few of the learners. Yet another category is where the teacher does not share a main language with any of the learners. The complexities in these different multilingual classrooms will differ and teachers may use different instructional strategies.

The purpose of this study was to explore the instructional strategies that mathematics teachers in different multilingual classrooms use. This study focused on two teachers in different multilingual classrooms teaching algebra: one in which the teacher shared a main language with learners and another classroom in which the teacher did not share a main language with the majority of the learners. The study was guided by the following research questions:

- What instructional strategies do teachers in two different multilingual classrooms use in order to support learners' development of mathematical proficiency in algebra?
- How do they use these instructional strategies to support their learners' mathematical proficiency in algebra?

In what follows I give a rationale for the study, which draws from research, policy and practice. The rationale will also give a background of my interest in a study like this.

Why this study?

In my teaching career which spans over 35 years in schools in the United States of America, Zimbabwe and South Africa, I have noticed the difference in achievement in certain areas of work between learners who struggled with English language and those who did not. In cases where there was a minimum demand for the English language (example: Solve for $x : 2x^2 - 3x + 1 = 0$), the achievement was the same for both groups. In Zimbabwe, where I shared a home language with learners, I would often explain concepts to them in our common language and the learners would exclaim, "Is that all there is to this problem? Then it is easy." To these learners it was as if the big hurdle or obstacle had been removed. This resulted in learners nick-naming difficult or challenging problems 'chirungu', meaning it is the English that is causing the difficulty.

In the other countries where I did not share a home language with learners, I realised that I had to find other ways of explaining concepts. I would find other words to use to make the explanation clearer. I also found myself using diagrams to explain concepts that were confusing to learners. On one occasion after trying very unsuccessfully to explain the concept of relative velocity, I decided to demonstrate with another student how such a concept is used when they play netball or soccer. I actually used a soccer ball and had one student kick the ball to some other player who was running. Synonyms, diagrams and demonstrations have been some of the ways I have used to help explain concepts when I could not use learners' home language. Sometimes there would be a learner who would have understood the concept and this learner would explain to other learners. The only problem was that sometimes the learners laughed when one of them was explaining and I was not sure of the basis for the laughter --- whether it was from just a learner being funny or if some other phrases unrelated to mathematics had been included.

As a teacher I have a passion for helping learners succeed in mathematics, especially if they come from previously disadvantaged backgrounds. I believe this comes from the fact that it was the help from other people that got me to my present position. Realising the language position of most South African learners and the results of my Honours Project[1] rekindled the passion for helping the (previously) disadvantaged learners. Here I have bracketed the word previously because one could argue that all schools can choose their own LoLT, so why do they not choose their own main language. As will be shown later, for the majority of South African learners the choice for LoLT is rather very slippery.

Why now?

There are several ideologies that impact on the education system in South Africa at this time such as new theories in education that are accepted universally, language-in-education policy and global human rights concerns.

<u>Curriculum Policy</u>

The introduction of Curriculum 2005 (C2005) brought with it a different emphasis for the

[1] Honours Project explored 'The Appropriacy of language used in the Grade 9 Mathematics Common Tasks for Assessment (CTA) Examination to Additional Language learners'.

mathematics classrooms as well as all other subjects. The National Curriculum Statement for Mathematics states that:

- *Learners must acquire a functional knowledge of mathematics that empowers them to make sense of society.*

- *Learners should be able to communicate appropriately by using the descriptions in words, and solve problems creatively and critically.*

- *Learners work collaboratively in teams and groups to enhance mathematical understanding, and to engage responsibly with quantitative arguments relating to local, national and global issues.* (Department of Education, 2000:9 and 10).

The description above emphasises the use of language in the classroom. Learners are no longer just expected to perform procedural calculations, but they must be able to explain and justify their thinking. For multilingual learners, what and how they communicate, discuss, explain and justify can be affected by their proficiency in the LoLT.

<u>Language-in-education Policy</u>

The South African language-in-education policy promotes and respects multilingualism. The Curriculum 2005 document reports that:

a) A learner in a public school shall have the right to instruction in the language of his or her choice where this is reasonably practicable

b) The governing body of a public school may determine the language policy of the school subject to

i. the national policy determined by the minister under the National Education Policy Act 1996

ii. *the provincial policy determined by the Member of the Executive Council, provided that no racial discrimination may be practiced in exercising this policy* (p 23).

Setati (2002) argues that while the new language-in-education policy is widely perceived as good, it has met with difficulties. English and Afrikaans are still the main LoLT chosen in the schools. She further argues that English and Afrikaans are still the only languages used in tertiary institutions and English is still considered a language of economic power. Thus, even though the language policy encourages and respects multilingualism, the other ten official languages do not enjoy the same status as English because of its dominance.

Human rights and Equity

Some of the principles on which the new curriculum was established were that it should promote personal and social development, social goals of justice, equity, and challenge imbalances of the apartheid era. The apartheid era had produced a great divide in the peoples of South Africa especially in the quality as well as quantity of educational facilities available. There were many African schools that did not teach mathematics at all, let alone teaching mathematics at higher grade. This characteristic thinking of the apartheid era is clearly depicted by the speech of the then Minister of Native Affairs, Dr. H.F Verwoerd, “What is the use of teaching the Bantu mathematics when it cannot use it in practice?” (1953:3585). It is clear from this statement that black South Africans had been denied equal opportunities to learning especially in mathematics and science.

Equity issues involve equal opportunities in educational facilities, equal access to jobs and careers. Kahn (2005) argues that mathematics is a gateway subject to further education and for preparation of careers. The Rand Mathematics Study Panel (2002) argues the same point when they report that the access to college and other career opportunities are closed to those who do not have mathematics courses. According to this report, those who do not have access to further education and career opportunities are mostly from impoverished environments and those who learn in a language that is not their home or first language. For equity issues to be met, all learners must be provided with the necessary stepping stones, in this case mathematics, to progress to further education and career training. However, it must be remembered that the success of learning mathematics is now greatly influenced by the LoLT. In South Africa, meeting the equity issues means providing access to successful mathematics learning which is directly related to language issues.

The number of black South African students who have matriculated with mathematics at higher grade highlights the disproportionate state of education and career opportunities that have prevailed in the country. Even after apartheid, very few black students completed matriculation with mathematics at higher grade. In a newspaper report, The Sunday Times (April 16, 2006), it was reported that of about 25 000 chartered accountants in South Africa, there were only about 600 black chartered accountants. A pass in mathematics is one of the entry qualifications to training in higher education ending up as a chartered accountant. This information supports Kahn's (2005) argument that mathematics is a gateway subject. He argues that passing mathematics opens the doors to further education and to prepare for careers. Equity, be it in economic terms or in terms of social goods or education, depends on the educational opportunities that are available and in many cases success in mathematics will open the doors.

Why a focus on teachers and teaching?

The role of the mathematics teacher in the classroom has changed over the past two decades. The reason for this change has been because of the change in the focus of the mathematics curriculum. The teacher is no longer just expected to transmit knowledge, concepts, formulae and algorithms. The teacher has a much more complex role in the mathematics classroom. The Government Gazette of South Africa describes the role of an educator as:

> *A learning initiator, teaching through negotiation, coach, facilitator, stimulator, mediator, supporter and listener in a manner which is sensitive to diverse interests and needs of learners including those with special educational needs. A leader, organizer, administrator and manager of a teaching learning situation, which supports learners, increases the desire for inquiry; explanation and responsibility for own learning.*
>
> (Government Gazette 22 September 2000 No 215:15)

The role of the mathematics teacher becomes even more complex in a multilingual classroom. Not only does a teacher have to fulfill all the roles stated above, but the teacher must cater for the language needs of the learners. In a South African classroom it is possible to have as many as six different home languages. Setati and Adler have both separately and together, researched teacher practices in these multilingual classrooms (Setati 1998, 2000, 2004; Adler, 1998, 1999; Setati and Adler 2000). The research by these authors has drawn attention onto what teachers are doing in South African multilingual classrooms as well teachers' dilemmas they face in these classrooms.

Clarkson, in his keynote conference paper in 2004 at the 28th Conference of the International Group for the Psychology of Mathematics Education in Bergen, Norway, points to the very small number of articles that focus on the teacher's role. He scanned through four international research journals (Journal for Research in Mathematics Education, Educational Studies in Mathematics, For the Learning of Mathematics, and Mathematics Education Research Journal) between the years 2000 and 2003 and found that out of about 300 articles written in English a reasonable number of the articles focused on learners and very few on the teacher's role. This indicates the need for more research on the teacher's role.

This research focused on strategies that teachers in multilingual classroom use to promote learners' understanding in algebra. The research neither looked at nor compared learners' performances as an indication of effective strategies by teachers. It is hoped that the research results can add to the research on what teachers in multilingual classrooms are doing to support learning of their multilingual learners and help to point to 'initiatives that can be implemented to ensure success in multilingual mathematics classrooms' (Setati and Barwell, 2008:2).

Why instructional strategies?

The Webster Encyclopedic dictionary (1976) describes a strategy as a method or plan to achieve some goal. Merell and Tennyson (1981) describe an instructional strategy as a specified sequence of presentation forms which include attribute isolation, attribute matching and a range of difficulty levels. Other educational definitions include a systematic plan, consciously adapted and monitored, to improve one's performance in learning, a planned, deliberate goal-oriented procedure achieved with a sequence of steps that are subject to monitoring and modification (Whole School Literacy Planning Guide). From these definitions it is clear that one of the main characteristics of a strategy is that it is planned, it just does not happen. A strategy is well thought about before it is implemented. It can be argued that in planning a strategy prior experience is brought to the front so that the best possible plan is organized. A strategy is intentionally goal-oriented. It is planned with a specific goal in mind and achieved with a sequence of steps. While there can be a sequence of steps to be followed to carry out the strategy, keeping the strategy flexible to allow for monitoring and modification of the strategy can be a strength.

Several researchers have documented the complexities of teaching and learning in multilingual mathematics classrooms (Mestre, 1981; Cummins, 1984, 1986; Spanos, Rhodes, Dale and Crandall J 1988; Cuevas, 1990; Khisty, 1995; Ndiyakupfamiye, 1994; Moschkovich 1996, 1999; Setati 1996, 2002, 2005; Olivares 1996; Rubenstein 1996; Adler 1997, 1998, 1999). Past research, while it has illuminated some of the complexities of teaching and learning in multilingual classrooms, has not focused much on the strategies that teachers can use to ensure that learners have access to a specific area of mathematics. Knowing what to do and what instructional strategies to use and when to use a particular strategy is one of the main challenges faced by teachers in multilingual classrooms in South Africa.

Some researchers in mathematics education have claimed that the way teachers teach makes a difference in the performance of their learners (Good, 1983; Wang, Haertel and Walberg 1993). Other researchers go on further to say that the choice of instructional strategies can have an even bigger impact on learning and the performance of learners (Marzano, Gaddy and Dean 2000). It can be argued that these research results point to the complex role of mathematics teachers in multilingual classrooms. As mentioned earlier, research in South Africa indicates that it is mainly those learners who learn in a language that is not their home language who do not succeed in Grade 12 mathematics. Exploring strategies that could be used in multilingual classrooms to ensure access to mathematics would be beneficial to both teachers and learners in such classrooms. It is because of this that I decided to explore the strategies that teachers in multilingual classrooms were using to support learners' development of mathematical proficiency.

CONCLUSION

This chapter has given a brief description of the rationale for the study and why it is relevant to mathematics education in South Africa at this time. The chapter has also outlined how the study at this time relates to the curriculum and education policy in South Africa and how it relates to issues of access to mathematics for multilingual learners. Furthermore, the chapter explained why the focus of the research was on teachers, teaching as well as instructional strategies.

Chapter 2 explains the reasons for the choice of a focus on algebra and in particular exponents. The chapter explains what algebra is and its importance in creating opportunities for learners in different careers.

The theoretical framework that guided this study and the literature that relates to the study are discussed in Chapter 3.

Chapter 4 describes the sample selection, how data was collected and the categories used to analyse data. The chapter also describes ethical issues that the study took into consideration and how issues of validity and reliability were dealt with.

Chapter 5 provides the first part of the analysis of data. This chapter describes the strategies used by the teachers in the study.

The second part of data analysis is dealt with in chapter 6. This chapter examines the instructional strategies that the teachers used and analyses when and why the strategies were used.

Chapter 7 discusses the findings of the study, suggests recommendations from the study and discusses limitations of the study.

CHAPTER 2

RATIONALE

Chapter 1 gave an explanation of the overall rationale for doing this study at this time. This chapter gives further rationale for the research. It explains the interest in doing research that focuses on algebra and in particular on the sections of exponents that relate to algebra.

WHY ALGEBRA?

The decision to focus the study on the strategies that teachers are using in teaching algebra was based on its importance in several areas. Considering the different ways in which different researchers have described what algebra is will highlight the importance of algebra. Teresa Rojano, cited in Wheeler (1996), says that algebra is to some sense an extension of arithmetic and it picks up some of the questions that arithmetic cannot handle. Wheeler (1996) adds to this by arguing that algebra is a completion of arithmetic. Wu (2001) and Usiskin (1998) also add to the descriptions by arguing that algebra is generalized arithmetic. Usiskin (1998) says that there are other different conceptions of what algebra is. He adds three other conceptions of algebra: algebra is a means to solving certain problems; algebra is a study of relationships and algebra is the study of structure. To Usiskin, it is impossible to adequately study arithmetic without implicitly or explicitly dealing with variables. He adds that algebra is superior to English in describing number relations like saying $n.0 = 0$ and comparing it to the statement '*the product of any number and zero is zero'*.

The above descriptions also help to show the importance of algebra. The interest in exploring strategies that teachers are using in multilingual classrooms stems from the importance of algebra in several areas. Algebra is important for further studies in mathematics and science, for developing mathematical thinking and reasoning and for developing and providing tools and models for solving problems in real-life situations. Wu elaborates on the importance of algebra when he says "… algebra has come to be regarded as a gatekeeper course … those who successfully pass through will keep going while those who don't will be permanently left behind" (Wu, 2001: 1). This statement refers to the importance of passing elementary algebra courses as prerequisites to continuing education in some tertiary courses. The elementary algebra courses provide the basic understanding of mathematical structures that are a necessary foundation for further courses. The importance of algebra is not only for those who need it for further studies but for everyone. Wu says there is so much emphasis on algebra in the United States of America that there have been cries for "Algebra for All" (Wu, 2001:1).

In America, Moses and Cobb (2001) bring in a political tone to the importance of algebra. They argue that algebra is a civil right. Moses, a civil rights veteran in the United States of America, argues that the ongoing struggle for citizenship and equality for minority people is linked to an issue of mathematics and science literacy. Moses believes that one way to attend to the plight of education of poor black children is to create a culture of literacy that involves algebra. This belief led Moses and Cobb to start the 'Algebra Project' for children from poor families in the United States of America in 1982. They further argue that it is the algebraic thinking that is used in many occupations. They believe that the skills gained from algebra are not only important for gaining access to college mathematics and science related careers, but that the skills are necessary for full participation of any person in the economic life of society. They argue that the economy of the world is becoming increasingly computer based and that to fully understand and master computers students need to be comfortable in manipulating symbolic representations that underlie the mathematical concept. They consider algebra as a necessary stepping-stone to college mathematics and any other opportunities in society. They add that people without algebra are like people who could not read and write in the industrial age.

One of the major universal changes in education is the emphases on the development of thinking and reasoning. This is even more so in the mathematics curriculum. Mathematical thinking and reasoning has become one of the main emphases in mathematics education. In developing mathematical thinking, processes of algebra such as finding patterns and relationships between variables, generalizing and formulating mathematical models are used to enable solutions to real life problems to be found. The National Association of Elementary School Principles in the United States of America argues that algebra is not just an abstract and difficult school subject that is needed by scientists and engineers, but a fundamental and basic education that is required by all students. The importance of algebra to life and mathematics is recognized to such an extent that the United States National Council for Teachers of Mathematics (NCTM) has recommended that algebra should begin in kindergarten where children are led to recognize simple number patterns to develop algebraic thinking. They argue that it is one of the important content areas that students must learn throughout their school years.

In South Africa, the new C2005 emphasises the need for the mathematics curriculum to prepare learners for the real world in which they live. It encourages developing learners' ability to recognize mathematics in their world and know how to apply this knowledge to their benefit. The FET curriculum describes the algebra learning outcome as:

> *The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.----Demonstrate the ability to work with various types of functions. Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).*
> (FET Curriculum. Department of Education, September 2005:14.)

The FET curriculum also emphasizes the need for learners to develop the ability to recognize and understand the concept of functions rather than just solving functions. Learners are expected to investigate, analyse and describe functions. All these are processes that lead to deeper understanding and not just memorizing the rules and formulae for solutions. Learners are expected to be able to explain their ideas, and thoughts, to justify their work and thoughts, to use different representations, to generalize and solve unfamiliar problems. The curriculum states the importance of providing the type of mathematical knowledge that learners can transfer, modify and expand on in their further academic development or their careers. Ability to transfer implies to some extent their ability to generalize.

Algebra is essentially about generalizations (Wheeler, 1996). In human society, all people generalize, so it is something that all people do, even children can generalize. Generalization and formulation of patterns are some of the aspects of algebra that are emphasized internationally. In life it is not possible and it is impractical to develop a specific solution to all individual problems. It is therefore necessary to recognize similarities and differences among problems and thus use similar methods of solutions but modified according to specifics. It is the generalization aspect of algebra that the new curriculum in South Africa is emphasizing and hoping will be developed by all its citizens so that they can better understand events in their lives and be able to manage them. Emphasis in algebra is not only in learning formulae procedures and algorithms, but must include investigating, analyzing, interpreting, describing and generalizing situations.

All this generalization requires use of more language, the lack of which may hamper the progress of learners. New levels of demands on the language of learning and teaching have therefore been placed on both the learner and the teacher. The development of the ability to generalise in mathematics can be greatly affected by the learners' proficiency in the LoLT. Thus language plays an important part in learning algebra. Learners must translate the ordinary English into mathematical expressions. If the learner's understanding of the ordinary English is poor, it will automatically result in a wrong mathematical expression. I have noticed in my own teaching experience that when you allow learners to verbalise what they want to represent algebraically it assists in their thinking process. Schoen (1998) argues that history and research support that verbal grounding is essential for the development of symbolisation which algebra is mostly associated with. I concur with Schoen when he says that students should not be rushed into using symbols but rather that they should be led gradually from verbalization to algebraic symbolism and usually it is through the use of arithmetic rules. Setati et al (2002) argue that when learners are allowed to use their own language for discussion, misconceptions can be revealed in peer discussions and be corrected, otherwise wrong concepts will remain unexposed.

Algebra also involves modelling. Modelling usually deals with finding a representation of real world meanings in mathematical forms, usually in the form of a function. The process of transforming an English statement into a mathematical statement is difficult for many learners, more so for those who are still learning English. In this transformation, learners need to choose variables, describe the relationship in their own words, then write a mathematical expression or equation. Recently I gave this real world situation to my grade 10 class:

> *Two friends Amos and Batho hiked to two different camp sites. The camp sites are 12 km apart. They decide to meet for the afternoon. Amos is camped at the base of a hill. He starts hiking up at 08:00 and hikes at a speed of 1,5 km/h. Batho is at the top of the hill. He starts hiking down at the same time but hikes at a speed of 2,5 km/h. At what time will they meet and how far from the base of the hill will they meet?*

The real world problem above does not have any words or terms that learners would not be familiar with. However, my learners could not start the modelling of this problem. When we enacted the movement of the two friends hour by hour, they were able to solve the problem. It is the movement from ordinary words to mathematical representation that learners struggle with. Using a more arithmetic approach made the transition easier. This shows why a gradual transition from arithmetic helps to develop the algebraic concepts.

The New Jersey Mathematics Curriculum Framework describes algebra as a language that is used to express mathematical relationships, a language of patterns through which much of mathematics is communicated. They further argue that it is a tool that people can use and do use to model real situations and answer questions about these situations. Their framework for algebra emphasizes the need to prepare students for a world that is rapidly changing because of the numerous ever changing technological advances. They believe algebra should be for all students. In a conference in 1993, they put forward the following answers in response to the question, ‘why study algebra’:

- *Algebra provides methods for moving from the specific to the general. It involves the patterns among items in a set and develops the language needed to think about and communicate it to others.*
- *Algebra provides procedures for manipulating symbols to allow for understanding of the world around us.*
- *Algebra provides a vehicle for understanding our world through mathematical models.*
- *Algebra is a science of variables. It enables us to deal with large bodies of data by identifying variables (quantities which change in value) and by imposing or finding structures within data.*
- *Algebra is the basic set of ideas and techniques for describing and reasoning about relationships between variable quantities.*

(New Jersey mathematics Curriculum Framework-Standard 13-Algebra http://dimacs.rutgers.edu/nj_math_coalition/framework.html. Downloaded on 20-08-08)

Algebra has contributed to the development of other areas. Kaput (in Fennema and Romberg, 1999) describes the importance of algebra by pointing out the role it has played in the development of civilisation. He says:

> *…algebraic reasoning in its many forms, and the use of algebraic representations such as graphs, tables, spreadsheets and traditional formulae, are among the most powerful intellectual tools that our civilization has developed. Without some form of symbolic algebra, there could be no higher mathematics and no quantitative science; hence no technology and modern life as we know them. Our challenge then is to find ways to make the power of algebra available to all students.* (Fennema and Romberg, 1999: 134).

Kaput (1999) here recognizes the role that algebra has played in the development of other areas, but notes the importance of algebraic reasoning as an intellectual tool that should be available to all students. It is the effect of algebra as an intellectual tool that I believe is very important to all citizens. It is a tool that can be used at many different levels. It can be used in the workplace, in business, in small enterprises and in academic fields. Every citizen must be provided opportunities to develop this intellectual tool and learn how to use it.

WHY EXPONENTS?

My interest in exploring teaching exponents in multilingual classrooms was stimulated by four observations about exponents. The first is the deceitful absence of language when an exponential expression is written. There is a difference between the simplicity of the written mathematical symbolic form of exponents and the wording or language that is needed to explain the simple mathematical form. If we take as an example a statement like, '*Find* $\sqrt[4]{16}$, *without using a calculator*'. There are no English words that would be a cause for concern to a learner who is still learning English. This could be the reason why some people believe that language is not necessary for understanding mathematics, as reported by Lee Wung (1994). However to understand this simple and short mathematical form one needs to read it either silently or read it out aloud. It would be read as '*Find the fourth root of 16*'. But for a learner to understand what is meant by the fourth root, the teacher will most likely say '*What number is multiplied by itself four times to give the answer 16*?' The simple and short mathematical form, translated into ordinary language, is no longer short; a lot of words are required to explain it. Attention must also be drawn to the fact that the words used in exponents like *base, power, exponent, squared, square root, cubed, cube root* have special mathematical meanings which are different from ordinary English. Consider the language that is required to read and understand the mathematical expression that also uses variables like

$$\frac{6\left(3^{n+1}\right)}{\left(3^{3}\right)^{n-1}} \div \frac{2\left(9^{n+1}\right)}{3^{n^{2}-1}}$$

While exponential expressions may look as though they do not involve the use of language, they are actually very dependent on good understanding of meanings in ordinary English as well as mathematical language.

The second reason for the focus on exponents is because of their appropriateness and effectiveness in developing algebraic thinking and understanding. Algebra has been described as generalized arithmetic by a few authors (Wu, 2001; Wheeler, 1996; Usiskin, 1988; Herscovics and Kieran, 1980). Elementary work on exponents usually starts with using exponents as a short method to represent the arithmetic operation of repeated multiplication of the same number. This provides the learner with numerical and realistic bases for recognizing patterns and therefore the learner can move from arithmetic to generalising the situations, thus leading to algebra. The similarities in $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ and $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^7$ can lead to the generalisation of the algebraic expression $a \times a \times a \times a = a^4$. The generalization of $2^4 \times 2^7$ to develop the equivalent form $2^{(4+7)}$ leads to the development of the laws of exponents $a^x \times a^y = a^{(x+y)}$.

The third point is the use of exponents in understanding events in real life. Everyone is affected by interest on loans or bond payments. Knowledge and understanding of how interest is calculated would help citizens make more informed decisions before getting into debts which they may not be able to repay because of the effect of compounding interest. Other occurrences in real life where exponents are important are the spread of diseases, growth of bacteria and exponential growth of world populations. Growth of many organisms exhibit exponential growth patterns. Often science deals with very large or very small values. Using the scientific notation makes it possible to represent very large and very small quantities in more appropriate ways. Thus exponents are used both at academic and non-academic levels.

The fourth is the occurrence of exponents in final national examinations. The importance of any topic in a subject can be reflected by how often questions involving the topic are examined in final examinations and the prevalence of the concepts that underlie the topic in everyday life. An analysis of the Matric Mathematics Higher Grade and Standard

Grade past examination question papers of the National Papers and the Independent Examinations Board for the years 2002 up to 2007 revealed the importance of exponents. The table below shows the percentage of marks for each paper that directly or indirectly depended on knowledge of exponents. The percentages do show that exponents have been considered important in both of the examining boards.

Year	2002	2003	2004	2005	2006	2007
National Higher Grade	29%	30%	32%	28%	26%	25,5%
National Standard Grade	29%	29%	26%	26,7%	30%	30%
IEB Higher Grade	30%	22,5%	22%	27%	28,5%	40%
IEB Standard Grade	26,7%	22%	21%	26,7%	23%	33%

Table 2.1: *Percentage of examination questions involving exponents (2002- 2007)*

An analysis of the 2008 examination papers from the new curriculum shows the percentage of marks in the examination as:

	November 2008
National Examination	30,5%
Independent Examination Board	27%

Table 2.2: *Percentage of exam questions involving exponents in year 2008*

The percentage of marks in the two past examination papers indicates that exponents are still an essential part of the curriculum. Exponents have been an important part of the mathematics curriculum in South Africa and continue to be an important part of the new mathematics curriculum. Examples of the examination questions in 2008 are included in Appendix A.

A further analysis of the new FET curriculum was done to find how much emphasis the new syllabus places on exponents. Exponents are examined in paper 1. Paper 1 consists of two learning areas, number and algebra. There are 2 sections under outcome 1 and 4 sections under outcome 2. Only the section on linear programming does not involve exponents.

Learning Outcome	Part of Syllabus Dependent on Exponents
LO 1: Patterns and sequences	Geometric sequence: nth term : $T_n = ar^{i-1}$ Sum : $S_n = \frac{a\left(r^n - 1\right)}{r-1}$
LO 1: Annuities and finance	Compound interest formulae: $A = P(1+i)^n ; F = x\left(\frac{\left(1+i\right)^n - 1}{i}\right); P = x\left(\frac{1-\left(1+i\right)^{-n}}{i}\right)$
LO 2: Functions, graphs and modeling	$y = ab^{x+p} + q$
LO 2: Algebra and equations	Exponential equations: $ka^{x+p} = m$
LO 2: Calculus	Finding $f'\left(x\right)$ if $f\left(x\right) = \frac{3\sqrt{x} + 4x^{-1}}{x^{3/4}}$
LO 2: Linear programming	No exponents

Table 2.3: *Distribution of exponents in learning outcomes of the new FET curriculum*

CONCLUSION

Chapter 2 has provided an explanation of the importance of algebra and exponents. The chapter has also discussed why some people may argue that there are no language demands in teaching and learning algebra and exponents. The importance of algebra as a stepping stone to further education and careers and as a tool for developing thinking and reasoning has also been highlighted in this chapter. Chapter 3 will discuss the theoretical framework that guided this study.

CHAPTER 3

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

THEORETICAL FRAMEWORK

In an effort to explore strategies that teachers use to promote mathematics learning in multilingual classrooms, several questions immediately arose. Questions like: how do children learn, how does multilingualism in classrooms enhance or hinder learning and teaching and what constitutes mathematics learning? This study was therefore guided by theories that address these questions. This chapter discusses the theoretical framework that guided the study. It starts by discussing constructivist theory of learning. Social constructivism is relevant to this study because it emphasises the role of language and the role of the social environment including the knowledgeable other in a child's cognitive development. The chapter further discusses language in the mathematics classroom. This will be followed by discussing what mathematical proficiency is and the chapter ends with a discussion of instructional strategies.

Constructivism

According to constructivist theory, learners construct their own knowledge by being active participants in learning rather than by being told and memorizing work. Olivier (1989) argues that in constructivist theory, knowledge cannot be transmitted from one person to another, but that it arises from the interaction between experience and one's current knowledge structures. The theory of constructivism can be traced back to the 19th century when the Neopolitan philosopher Giambattista Vico claimed that humans could only clearly understand what they themselves have constructed (Southwest Educational Development Laboratory, 1995). Dewey (1916) also has similar ideas when he argues that knowledge and ideas emerge from situations in which learners have to draw them out of experiences that have meaning and importance to them. Hatano (1996) in his discussion of how knowledge is acquired says that one of the characteristics of knowledge is that it is acquired by construction and by reconstruction and not by transmission. In describing constructivism, Ernest (1996) says:

> *Constructivism accounts for the individual idiosyncratic constructions of meaning, for systematic errors, misconceptions, and alternative conceptions in the learning of mathematics. Thus it facilitates diagnostic teaching, and the diagnosis and remediation of errors.* (Ernest, 1996:2)

Ernest here is emphasizing the personal nature of the constructivist theory while also pointing to the fact that in this process some errors and misconceptions can result although this is part of the learning process. Errors and misconceptions should be used as teachable moments for deeper understanding of the concepts. This also brings to light some concepts that could easily be mistaken for another. Thus it allows the teacher to find out if a learner has arrived at a certain construction. This process of clearing out errors and misconceptions is in itself a valuable learning process.

Ernest (1991) acknowledges that there are a variety of different forms of constructivism, but it is Piaget's radical constructivism and Vygotsky's social constructivism that have gained a lot of attention in education during the past several decades. Piaget's constructivism theory places the individual as the primary source of learning. According to Piaget's constructivist theory the individual himself learns by continuous interaction with the environment (Dahl, 2002). Thus Ernest (1996) describes Piaget's constructivism as knowledge that is embodied and is essentially a result of interaction in and with the world. Drawing on Piaget's short summary of his educational thoughts in the book "To understand is to invent", he argues that:

> *To understand is to discover, or reconstruct by discovery, and such conditions must be complied with if the future individuals are to be formed who are capable of production and creativity and not simply repetition.* (Piaget, 1973:20)

Here Piaget is emphasizing the importance of individual experiences and observations for learning to take place. According to Piaget only by personal experience do people learn and are able to apply this knowledge later on. Repetition does not necessarily result in acquiring knowledge that can be used later on. Applying Piaget's principle in a mathematics classroom would involve a teacher providing opportunities for the learners to discover patterns, similarities and being able to make conjectures. Let us say a teacher wants to teach the theorem that states, "*Sum of angles of a triangle is 180* ^{o}". The teacher can provide opportunities for the learners to measure the angles of each of several differently-shaped triangles and then look for patterns and formulate conjectures about the sum of the interior angles, rather than just informing them that the sum is 180^{o} and tell them to memorise the theorem. In this way learners can interact with different kinds of triangles and observe that the result of the sum of the angles is the same. Such exposure to interacting with the environment builds better understanding of knowledge that they learn. Such learners are also better capable of production in the future. They are more prepared for problem-solving than those who are simply told that the sum of the interior angles of a triangle is 180^{o}. Such learners develop their thinking skills during concept formation. Thus Piaget makes a distinction between learning for understanding and learning facts.

While Piaget's theory is a theory of development rather than a theory of learning, it is helpful in understanding learning. Piaget argues that a child who has not yet attained a certain level of cognitive development cannot be taught or learn particular concepts. For example, a child who has not developed to the formal operational stage cannot be taught calculus. Thus Piaget's theory involves the mind and cognitive development of an individual. As Ernest (1991) appropriately argues, the weakness in Piaget's theory is that he takes a child as an isolated cognizing subject.

Vygotsky's (1978) constructivist theory, on the other hand, places emphasis on the influence of social interactions in the construction of knowledge. He agreed with some of the assumptions that Piaget made on how children learn, however, he placed more emphasis on the social context of learning. Vygotsky (1978) argues that interaction with objects in the environment is only one element of activities that contribute to learning. He argues that social interactions are crucial to learning, thus his theory is referred to as social constructivist theory. Thus his social constructivism theory involves the mind and language. It includes such factors as linguistic factors, cultural factors, interpersonal interactions, peer interactions, teaching and the role of the teacher in learning. Vygotsky (1978) emphasises the use of language in cognitive development. He argues that a child cannot learn on his own without the interaction and activities of others. He further argues that a child learns in collaboration with adults or more knowledgeable peers who guide him and organise experiences for learning.

Vygotsky's emphasis on the importance of the social in the development of a child both culturally and cognitively is evident in his claim that:

> *Every function in the child's cultural development appears twice: first on the social level and later on, on the individual level; first between people (interpsychological) and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory and to the formation of concepts. All the higher (mental) functions originate as actual relationships between individuals*. (Vygotsky, 1978:57)

Thus according to Vygotsky a child first learns through his/her interaction with other people around him, through conversations, discussions or dialogue then later he internalizes what he has learnt in collaboration with the others as his knowledge. According to this theory, language therefore becomes a very crucial resource in the mathematics classroom. It is through language that a learner can share his understanding with other learners, discuss and compare his findings, and ask for help from the teacher. According to Njisane (1992), social constructivism encourages dialogue between learners as well as between learners and teachers, creating a two-directional flow of information between learners and teachers.

There are implications of this social constructivism theory for the teacher in the mathematics classroom. The teacher is the adult or the more knowledgeable other who guides and provides opportunities, activities and tasks that a learner can use in the construction of concepts and knowledge. In Mercer's (1995) terms, the teacher is a 'discourse guide' who conveniently acts to a considerable extent as an intermediary and mediator between the learners and mathematics, in part determining the patterns of communication in the classroom, but also serving as a role model of a 'native speaker' of mathematics (Pimm,1987). Thus the teacher is not a transmitter of knowledge, but facilitates cognitive growth by creating the appropriate context, encouraging collaborative learning among learners and facilitating learning. A teacher in a constructivist classroom encourages learners to discuss and share ideas. Language then becomes crucial as learners share ideas, discuss their findings and negotiate meanings of the new concepts that they construct. To the teacher, language becomes important as he/she directs and mediates for learners as they construct meanings. Language is the crucial mediational means in the meaning making process (Wertsch, 1991; Haywood et al, 1998).

> *As a discourse guide, the teacher is expected to help learners to develop ways of talking, writing, and thinking which will enable them (the learners) to travel on wider intellectual journeys, understanding and being understood by other members of the wider communities of educational discourse: but they have to start from where the learners are, to use what they already know, and help them to go back and forth across the bridge from everyday discourse into educated discourse*. (Mercer, 1995:83)

Zone of Proximal Development (ZPD)

Vygotsky also argues that there exists a 'zone of proximal development' (ZPD) in each child (Kozulin, 1998). He describes the ZPD as:

> *The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers*. (Vygotsky 1978:86)

The ZPD is the measure between what a child can do by himself/herself and what the child can do with the help of a more knowledgeable person, a teacher or a peer. Vygotsky argues that it is the ZPD which is important in the learning process and not what the child already knows. Learning occurs in the ZPD. Nicholl (1998) argues that it is in the ZPD that we learn, through social interactions, how to use the psychological tools that are available to us. Such tools include counting systems, using mnemonics, writing, diagrams, map signs and symbols and language. The importance of interaction, collaboration, discussion, comparisons, and help from peers or the more knowledgeable person affords learning in the ZPD and language becomes the most important of these tools in mediating the thinking process and guiding our thoughts. All these socially dependent activities require and depend on the use of language. Language therefore becomes central to learning and cognitive development of a child in the mathematics classroom.

A teacher in a multilingual mathematics classroom could use the ZPD in two aspects of learning mathematics: learning mathematical concepts and increasing language proficiency. It is possible that in a classroom each learner can be a more knowledgeable peer in just one aspect of a range of new knowledge. A learner can be more knowledgeable in language proficiency or in a mathematical concept or a mathematical skill. When each learner contributes what he knows, the result of the pooling together is new knowledge in different respects. Each learner has learned as a result of the social interaction. In the ZPD enough challenge can be provided for a learner without making the challenge too difficult for the learner to become despondent. Thus for learners to learn profitably in the ZPD, the teacher needs to be aware of each learner's ZPD so that appropriate grouping can be made where learners can be more knowledgeable peers in a certain area, thus creating an environment in which all learners feel worthy. The teacher must be aware of the upper level of the ZPD so that learners do not become frustrated. The teacher's responsibility as learners learn in the ZPD is to help them, by scaffolding for the learners by giving them hints, suggesting other ways of approaching the problem, prompting them by referring to past experiences, asking questions that can lead to jogging the learner's memory and thus enabling learners to find the solution through their own reasoning. The teacher must also create a positive classroom culture that will promote collaborate contribution and communication so that each learner benefits.

LITERATURE REVIEW

This part of the chapter discusses some of the literature on language, learning and teaching in multilingual mathematics classrooms.

Language as a thinking tool

From the time of Plato to the present, the relationship between language and thought has been of interest to people, especially linguists. Marlowe (2004) says that many people who are linguistically informed are of the opinion that language influences thought. He further says that one of Plato's chief concerns was to examine how words relate to concepts and to realities and to show how people will go astray in their thinking when they use words without adequate analysis of the concepts the words are supposed to express. Marlowe (2004) shows how the relationship between language and thought was treated in Germany by referring to the work of Johann Gottfried Von Herder. In answering the question "What exactly is the connection between language and thought?" Herder (1764) replied that whoever surveys the whole scope of language surveys a field of thoughts and whoever learns to express himself with exactness precisely thereby gathers for himself a treasure of determinate concepts. Recognition of the crucial relationship between development of concepts and language is evident in the above statements. Herder maintains that language is a form of cognition and that thoughts are shaped by language.

In France, the French philosopher Etienne Bonnot de Condillac shows the importance of language especially in relation to mathematics when he writes:

> *We think only through the medium of words… Languages are true analytical methods. Algebra, which is adapted to its purpose in every species of expression, in the most simple, most exact, and best manner possible, is at the same time a language and an analytical method… The art of reasoning is nothing more than a language well arranged.* (Marlowe, 2004:3)

De Condillac's statement helps to emphasise the importance of language in learning mathematics. Often people do not think language is necessary in mathematics classrooms and therefore do not see how lack of fluency in the language of learning and teaching can hinder the learning process in multilingual mathematics classrooms. This supposedly irrelevant view of language is explained by Lee and Jung who say, "Language needs tend to be ignored in the mathematics classroom because of the pervasive myth that students do not need proficiency in English to perform well in mathematics" (Lee and Jung, 2004:269).

Coming closer to our time, the Jewish-German anthropologist, Franz Boas maintains that when people think clearly, they think in words. He goes on further to point out that inaccuracy of vocabulary has been a stumbling block in the advancement of science (Marlowe, 2004). Personally I have noticed the stumbling block that results from learners' inaccurate understanding of the meaning and difference between the mathematical terms: *expression* and *equation*. Those learners who do not recognise the difference between an expression and an equation end up trying to solve for a variable such as x in an expression, making the question very difficult for themselves. I have learners who have previously tried to solve for x when asked to simplify the expression: $\sqrt[x]{\frac{6^x+3^{x+2}}{2^x+9}}$. Solving for x becomes impossible and thus students create their own stumbling block because of inaccurate vocabulary. Other inaccuracies occur when algebra is introduced and letters are referred to as representing objects, like the use of '*a*' to represent an apple and '*b*' to represent a banana. Such inaccuracies become stumbling blocks in the advancement of more complex mathematical concepts like a^4 or a^x. Whorf (cited in Brodie, 1989) argues that language is not just an instrument for voicing ideas but it is the shaper of ideas, it is a programme and a guide for the individual's mental activity, for the analysis of impressions and for the synthesis of the individual's mental stock.

The most influential authors on the relationship between language and thought are Piaget and Vygotsky. Piaget believes that cognitive development precedes linguistic abilities whilst Vygotsky believes that it is language that promotes thinking and development of thought (Kozulin, 1998). Vygotsky (1978) argues that language is necessary for thought, language is responsible for thought and language allows us to control our mental functions. For Vygotsky, language is a system of signs that help us to organize our world and that these signs enable us to express abstract aspects like feelings, attitudes and mathematics. He goes on further to argue that language is the most vital psychological tool for mental development and it is primarily responsible for the development of the mind and thought.

Levina, cited in Vygotsky (1978), gives an account of how a child faced with a difficult task of reaching candy, out of her reach in a cupboard, talked to herself as she reflected on possible ways to the solution. Vygotsky (1978) argues that the more complicated the task, the more talking through the action that the child does. In another experiment Vygotsky (1978) shows that if children are prevented from speaking (e.g. by making them keep a pencil between the teeth), they were worse at solving problems. All the arguments above support the idea of language as crucial to the thinking process of individuals.

In mathematics education Pimm (1991) has done seminal work on the relationship between language and mathematics. He argues that one of the functions of language is to externalize thought and that this externalization aids in the important process of reflection. Pimm says of mathematics and language:

> *In mathematics, language can be used to conjecture mental images as well as provide access to others. Language can be used to point---that is to focus attention—in situations where physical indication is not possible or prohibited for some reason. And despite the mathematician's deliberate (and often powerful) use of 'symbol is the object' metaphor, it is the mental images and acts of attention that are the stuff of mathematics, not marks on paper*. (Pimm, 1991: 23)

This quotation points to what happens when one learns mathematics. One recognizes for himself the patterns that exist and realises that a generalisation is possible. This recognition is in the mind, and that is what the mental images are. Barnett-Clarke and Ramirez (in Rubenstein, 2004) refer to the silent talk one does in an effort to find a solution to a problem as 'internal chatter'. They argue that internal chatter supports the thinking process which is necessary for solving problems. But what language do learners in multilingual classrooms think in? In what language is their internal chatter? Barnett-Clarke and Ramirez (2004) argue that students whose first language is not English will use their home language in their internal chatter or will use a combination of home language and English.

The emphasis in the new mathematics curriculum is that learners develop the ability to think for themselves and be able to use mathematical concepts they have learned in new and unfamiliar situations. Thinking however, as discussed above is through a language. It is therefore important to be aware of the language used in classrooms so that it can support the thinking process of each individual.

Language as a tool for communication

The importance of language as a communication tool is well expressed by Pirie (1998). She maintains that language is the mechanism by which teachers and pupils alike, attempt to express their mathematical understanding to each other. In her words she says "… we must remember that the only knowledge we can have of the pupils' understanding is gained from our interpretations of their communication to us through symbol or word" (Pirie, 1998:10). Pimm (1991) echoes the same idea when he says that the primary concern of mathematics teaching should be to improve and encourage the communication of mathematical meanings. He says that language in mathematics classrooms can be used to make accessible to others those mental images that one would have conjectured for oneself. As discussed earlier, mental images are formed individually and only by communicating what one holds in the mind to others does that particular knowledge become accessible to other people. This communication results in discussions and comparing with what others hold as their mental images. This results in mental images being modified, re-interpreted, constructed or reconstructed to match the true mathematical image. This communication with others is the social aspect of constructivism that Vygotsky (1978) refers to.

The new curriculum emphasizes that learners must discuss, justify their work, findings, and conjectures, describe, compare and negotiate meanings as they learn. All these activities require use of language in communicating. Language for communication in a mathematics multilingual classroom is not simple. It involves at least three forms of languages: Language of Learning and Teaching (LoLT), mathematical language (ML) and the individual's home language (HL). The majority of learners in South Africa learn in a language that is not their home language and in many cases their LoLT can be their third or even fourth language. The most common LoLT in South African schools is English. Thus at any time in a mathematics multilingual classroom, communication represents an interaction between the three forms of language. At every stage in learning mathematics, most learners in these classrooms must move between the three forms of languages if they are to be active participants in the construction of new concepts. This interaction can be represented by the diagram below:

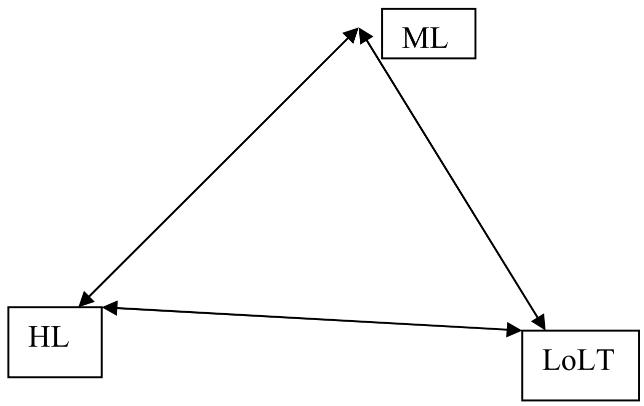


Figure 3.1: *Interaction of Language forms in Multilingual Classrooms*

At anytime during concept formation, a multilingual learner must not only construct new mathematical concepts, he/she must also manipulate the three forms of language to actually understand what is going on and then construct meaningful and accurate concepts.

Mathematical Language (ML)

According to Pimm (1981), mathematics is notorious for attaching specialized meanings to everyday English words. He uses the example of '*What is the difference between 30 and 7?'* In ordinary everyday English the difference could be that 30 is a two digit number and 7 is one digit, it could that 30 is an even number and 7 is an odd number, or it could be that 30 is a composite number and 7 is a prime number. In mathematical language the more expected meaning is 30 – 7 = 23. The term 'difference' in mathematics corresponds to the process of subtraction. Halliday (cited in Pimm, 1991) describes specialized meanings in mathematics to constitute a mathematics register. The mathematics register is not just a collection of words and their mathematical meanings; it includes phrases, symbols, abbreviations and the structures that express the meanings. Words like *root*, *power and table* have different meanings when used in the mathematics classroom. Even within the mathematics register itself, the same sign or symbol may have different meanings depending on the mathematical context or just the position relative to another mathematical term. The symbol '*-1*' has several different meanings depending on the mathematical context as well as depending on its position relative to other terms. In $sin^{-1}(x)$ it means *arcsin x* ; in $f^{-1}(x)$ it means the inverse function of *x*; in *120 km* h^{-1} it means 120 km per hour and 2^{-1} means ½ .

Not only is mathematical language different, often the way it is written is different from the way it spoken. When the statement like '*x* to the power 2 or *x* squared' is written, the conventional way of writing it is x^2 and yet the expression $(cos\ x)^2$ is often written as $cos^2 x$. After learning that x^2 is the same as $x \times x$, a learner must learn that $cos^2\ x$ is not the same as $cos \times cos\ \ x$, but that it is $cos\ x \times cos\ x$. For a learner to be able to communicate mathematically, understand explanations, be able to follow mathematical conversations, and discussions, be able to read mathematical texts and be able to write mathematically, it is important that the mathematics register becomes part of the mathematical language that the learner uses. Pimm (1991) suggests that there are two routes that learners could take in moving from speaking to writing mathematics as illustrated in the diagram below.

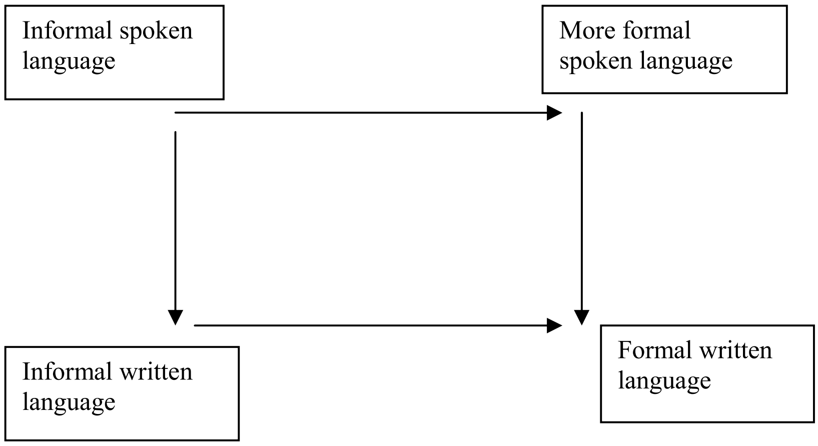


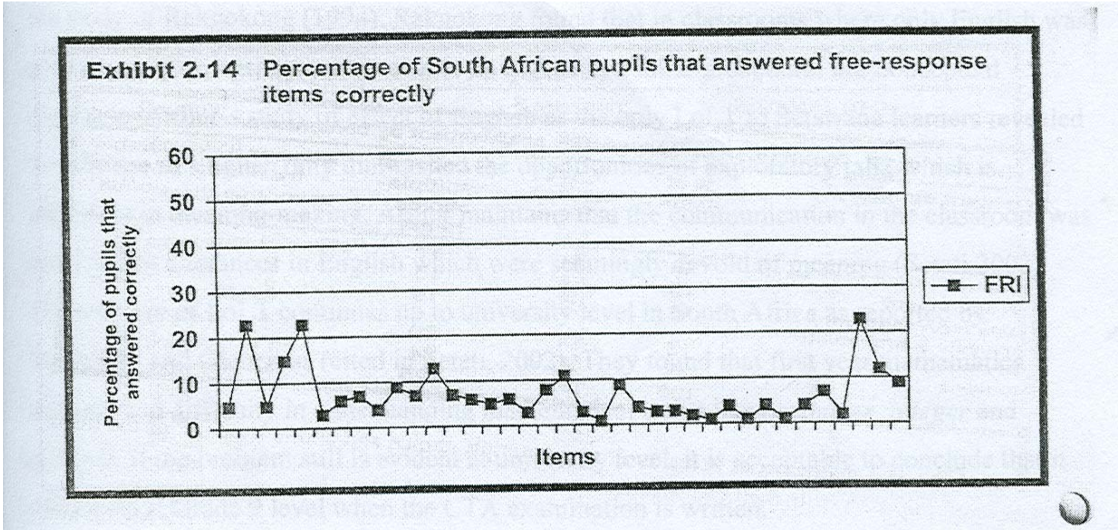
Figure 3.2: *Routes from speaking mathematics to writing mathematics*

An example here would be starting with 'the angle whose sine is equal to ½' (informal spoken language) to 'the angle whose sine is ½' (informal written/spoken language) to arcsin (½) (formal spoken language) to $\sin^{-1}(½)$, the formal written form. Thus besides manipulating the three language forms, learners must also manipulate movement within the mathematics register to become active participants in the mathematics community. However these routes do not cater for learners in multilingual classrooms. Setati, Adler, Reed and Bapoo (2002) argue that for multilingual learners these routes are more complex. Setati (2002) has reworked the routes that would be relevant for learners in multilingual classrooms. These routes for multilingual learners are discussed in more detail in the next section.

Language and learning (or communication) in mathematics multilingual classrooms

In the previous section, the importance of language as a tool for communication was discussed. The next section focuses on the literature that deals with the effects of language on learning in mathematics multilingual classrooms. As discussed earlier, communication in mathematics classrooms is a crucial activity in the learning of mathematics. It is worthwhile, at this point, to look at the present language policy in education in South Africa. In 1997, a new language–in-education policy was introduced in the South African education system. This policy recognizes each of the 11 official languages as a language of learning and teaching (LoLT). Governing bodies have the authority to choose their own LoLT. Whilst governing bodies can decide on an LoLT, according to the several studies done since the introduction of the new policy (PRAESA study; Brown 1998; Pile and Symthe; Setati, 2002; and Murray (cited in Vinjevold, 1999), not many governing bodies have adopted a language policy that uses mother tongue as the LoLT. Most of the governing bodies have opted for English as the LoLT. According to Brodie (1989), there are pragmatic reasons for the choice of English such as the lack of learning and teaching materials in the vernacular languages, a shortage of suitably qualified teachers who speak the languages fluently enough to teach in that language, grade 12 examinations still being written only in English and Afrikaans and the use of a language that is used for studies at tertiary level. Setati (2007) adds that it is because of access to power and social goods such as higher education that English is chosen as a favoured LoLT.

The choice of English as the LoLT by the majority of the schools in South Africa is accompanied by problems of learning in a language that learners are not fluent in. Learners have difficulties in understanding their teachers, understanding materials in textbooks and expressing themselves in English. Odendal's research findings (cited in Webb, 1999) clearly show these difficulties. For example, she claims that grade 5 learners in Kwazulu-Natal could not understand questions such as: '*Where do you live*?'; '*How far have you come*?'; '*What does your father do*?' and '*In what standard are you*?' Her research findings also revealed that 21% of the teachers confirmed the point that they felt their learners could not understand them when they used English for teaching and that 83% of these teachers thought their learners could not understand the textbooks which were used. Supporting this evidence is the TIMSS study which used English as the language of testing. The TIMSS study of 1999 ranked South Africa at the tail end in open-ended responses (Howie, 2001). The study claims that learners did not understand some of the questions and that language was a barrier to answering the questions and even understanding instructions. The diagram below shows the percentage of South Africans who answered open-ended questions correctly. Only three items were correctly answered by about 22% of the learners. The majority of the open-ended questions were correctly answered by less than 10% of the learners.



(Howie,2001)

Figure 3.3: *Percentage of South African pupils that answered free-response items correctly* **(**Howie, 2001)

Another complexity that is brought into the multilingual mathematics classroom is the varied levels of proficiency in the LoLT, English. Setati and Adler (2001) use and extend the definitions from linguists (Ringborn, 1987) to distinguish two groups of learners with regard to English as a LoLT. They talk about the learning environments in which learners learn in: 'additional language learning environment, ALLE' and 'foreign language learning environment, FLLE'. They describe ALLE learners as those whose contact with English extends to the environment outside the classroom. These learners see English writing around them like in the media and advertisements. English is accessible to them outside the classroom. These learners may be able to manage to some extent learning in English only. However, their English may not be well developed for academic learning. Cummins (1979) refers to this level of language competence as Basic Interpersonal Communicative Skills (BICS) and this takes 1 to 2 years to acquire. Cummins (1979, 1981, and 1984) suggests that there must be a minimum level of linguistic competence that a second language learner must achieve to be able to function effectively in cognitively challenging tasks. He calls this threshold 'cognitive academic language proficiency' (CALP). Cummins suggests that it takes a minimum of 5 to 7 years for a learner to develop this kind of proficiency in a second language. It must be noted here that for most learners in South African schools, English is not their second language but could be the third or even the fourth language. Using Cummins' CALP idea, it can be said that some of the additional language learners do not actually possess the linguistic competence to function effectively in learning mathematics in English. Setati and Adler (2001) describe the second group of learners, FLLE, as those whose contact with English, the LoLT, is confined to the classroom only. English to these learners is as foreign as any other language like French or Japanese.

The importance of language proficiency in the LoLT has not always been appreciated by many mathematics educators. Lee and Jung (2004) claim that language needs tend to be ignored in mathematics classrooms because of the pervasive myth that learners do not need language proficiency in English to perform well in mathematics. This myth stems from people's ideas that mathematics is all about numbers and symbols and therefore English is not necessary for dealing with numbers. However, there is a growing body of research that suggests a close relationship between language proficiency and success in mathematics. Mestre (1981) found a strong positive correlation between language proficiency and mathematics performance among college Hispanic engineering students. Cuevas (1984) showed that language is a factor both in the learning and assessment of mathematics achievement. Research studies by Brandy (1992); Secada (1992); Dawe (1983) have all independently found sufficient evidence to conclude that language proficiency in the LoLT is related to mathematics achievement. In her recent research in South Africa, Howie (2003) reports that pupils tended to achieve higher scores in mathematics when their language proficiency in English was higher and that they were more likely to attain low scores in mathematics when their scores on the English test were low. Thus both the additional language learners and the foreign language learners are likely to be negatively affected by their lack of language proficiency in English.

Success or achievement in mathematics is directly related to the success in developing faculties of thinking and reasoning in mathematics. The focus in mathematics education is thinking mathematically, construction of meaning and being able to transfer knowledge to unfamiliar situations. Aiken, in Brodie (1989) argues that linguistic abilities are important for mathematical thinking and problem solving and for the construction of meaning. Howie (1990) and White (1985) both maintain that lack of language abilities lead to inability of learners to deal with precise mathematical language in tasks, ending up with vague and ill-formed statements. As indicated earlier, this study was guided by social constructivist theory. In social constructivism, communicative skills are essential for discussions, negotiating meanings, being critical thinkers and describing findings in one's own words. Research findings by several authors (Collins, Brown and Newman 1989; Schoenfeld, 2002) all suggest that talking about mathematics enhances the understanding of mathematics. Cocking and Chipman (1988) maintain that students with stronger verbal abilities are better able to understand the mathematics being taught by working together in a group. Learners who are not proficient in the LoLT often do not talk because they are not confident enough to voice their thoughts. This lack of language proficiency will therefore rob these learners of the opportunities for developing their understanding. Such learners are more likely to resort to memorizing facts and formulae. Retention is more likely to be poorer when facts are memorized without understanding and no critical thinking will develop.

Moschkovich (1999) looked at how Latino students constructed mathematical meaning during bilingual conversations in mathematics. She found that these students had to move between English and Spanish to be able to understand what was going on and to get the meaning of certain terms. Not only did these students have to move between two languages, they also had to move between two mathematics registers. Locally, the work of Setati (2002) helps to understand what additional and foreign language learners have to do when they learn mathematics. She argues that code-switching in multilingual classrooms (by both teacher and learners) is beneficial in the formation of mathematical concepts. Setati et.al (2002) use the metaphor of complex journeys that these multilingual learners must take. Teachers who are not English first language might also have to take the same journeys. The diagram below taken from Setati (2002) shows the different routes that could be taken in an effort to understand what goes on in a multilingual mathematics classroom in the process of meaning-making.

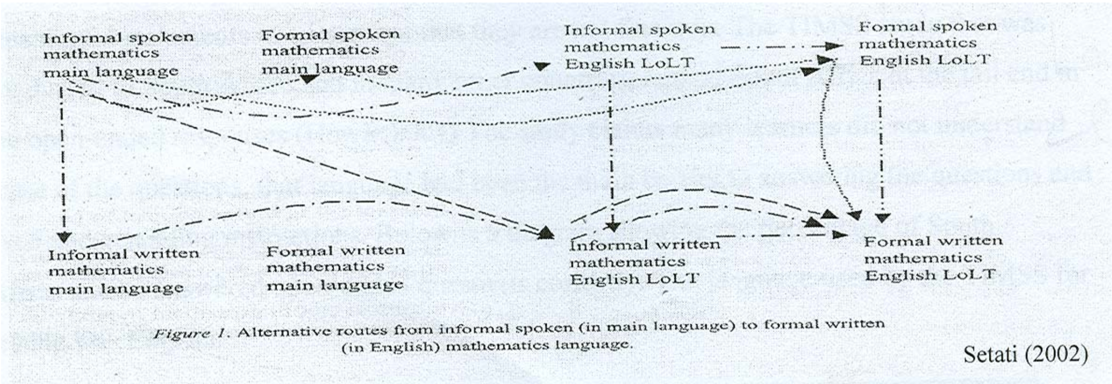


Figure 3.4: *Alternative routes from informal spoken to formal written mathematics language (Setati, 2002)*

Learners who are not fluent in English must switch codes several times as they make an effort to understand new concepts or try to solve problems. Both learners and teachers must pass through several stages to make the necessary transition. Setati (2002) maintains that the journeys that must be navigated from learners' informal exploratory talk in their main language to the formal written work in English appeared, for the most part, to be incomplete. If learners do not complete their journeys the quality of the new concepts they make and the knowledge that is constructed cannot be whole and accurate. If a teacher also needed to make such journeys then the guidance that he/she provides for the learners could be left wanting in some areas. The result of combining two incomplete journeys of the learner and the teacher cannot constitute a result that assures meaningful and accurate construction of knowledge. It is also important to mention the cumulative nature of mathematical knowledge. If a particular part is not well understood, it will influence the construction and acquisition of the next concepts.

This section has focused on the importance of language in learning mathematics and how proficiency in English, the LoLT, can affect learning in multilingual mathematics classrooms. It is also important to look at what mathematics learning is and what would constitute learning mathematics. The next section looks at what constitutes mathematics learning.

MATHEMATICAL PROFICIENCY

What society and the mathematics education community consider as successful mathematics learning has changed during the past century. One of the reasons for this change is the shift in the type of skills required by a dynamic economic world. Research findings in cognitive psychology have also contributed to what is considered as important mathematical attributes. The Rand Mathematics Study Panel (2002) approaches the debate by considering what it means to be competent with mathematics. The panel studied the activities in which mathematically proficient people engage as they structure and accomplish mathematical tasks. They looked at three core mathematical practices which include representation, justification and generalisation as the central practices to learning and using mathematics.

Kilpatrick, Swafford and Findell (2001) use the term mathematical proficiency to describe what they argue is a comprehensive way of looking at successful mathematics learning. They describe mathematical proficiency as comprising five strands that are intertwined, interdependent and support one another in their development. These strands are: Conceptual understanding, Procedural fluency, Strategic competence, Adaptive reasoning and Productive disposition. Mathematical proficiency is achieved by focusing on all the five strands at all levels of learning. Kilpatrick et al. (2001) argue that for children to achieve mathematical proficiency, it is important that instructional programmes that can address all the strands be organised.

The strands of mathematical proficiency have similarities with some of the descriptions of the mathematics learning outcomes in the National Curriculum Statement (NCS) which state that a learner should be able to identify, pose and solve problems; communicate appropriately using words, symbols, graphs or diagrams; collect, organise and interpret data and engage with problems relating to local, national and global issues. These learning outcomes are similar at all the levels in secondary education. They are developed to different levels during the schooling years. Thus the development of these outcomes is interdependent on each other and influences each other's development just as Kilpatrick's strands of mathematical proficiency.

Conceptual understanding

Kilpatrick et al (2001) describe conceptual understanding as an integrated and functional grasp of mathematical ideas. Knowledge is not just a collection of isolated facts. These facts are organised into functional wholes that can be called upon to help solve unfamiliar problems as well as to create new knowledge. Conceptual understanding enables a student to recognise connections among concepts and procedures, and aids in retention. Work that is understood conceptually is easier to remember than work that is learned by memorisation. Attainment of conceptual understanding can be recognised when a learner can represent mathematical situations in different ways. Kilpatrick et al (2001) argue further that when learners have conceptual understanding there is less to learn because they can recognise similarities, connections and generalisations. I had one learner who was busy memorising the fact that $log_2 2 = 1, log_3 3 = 1; log_{10} 10 = 1, log_5 5 = 1$. When I asked him why he was memorising all these statements his answer was that these were the bases where we had found that had a result of 1. It was evident to me that he had not understood the basic concepts of exponents and logarithms and how they relate to each other and generalise. He was busy trying to memorise all these facts. Of course he had a lot more to remember than the learners who understood the basis for exponents and realised or generalised that $log_a a = 1$.

Sometimes learners resort to memorizing because they do not understand. This, however, risks remembering the concepts and facts. The learner also ends up with many more facts and concepts to remember because these have not been organised into a functional whole. Imagine the difference in amount of work to be learned between a learner who has reached a generalised conjecture that $log_a a = 1$ and one who memorises the log statements mentioned in the previous paragraph. But this is not all; this strand is influenced and influences the acquisition of the other four strands.

The attainment of this strand can be problematic in multilingual classrooms if learners do not have fluency in the LoLT to help them understand the concepts. Kilpatrick et. al. (2001) argue that if learners do not understand they often resort to memorisation which, as mentioned earlier, makes remembering of concepts more difficult, but if learners learn with conceptual understanding forgotten facts can be reconstructed.

Procedural Fluency

This refers to the knowledge of procedures and knowledge of when and how to use them appropriately and efficiently. Learners who have mastered procedures as just order of operations often find it difficult to deal with algebraic manipulation when they begin work on algebra. Such lack of proficiency then constrains the development of further mathematical concepts. I have noticed that learners who still grapple with addition and subtraction of directed numbers get frustrated when working with simultaneous equations because their addition or subtraction is always wrong. Thus they take a lot more time to develop skills for solving simultaneous equations. They get bogged down on basic algorithms. When given two equations like $2x + y = 10$ and $2x - y = 2$, they do not realise that adding the equations will automatically eliminate the variable y leaving a very simple equation $4x = 12$.

Another example where procedural fluency costs learners a lot of time is with expressions written in exponential form. The term 2^3 is often simplified to 6 where the two numbers have been multiplied together. In multilingual classroom the statements multiplying 2 by itself three times and multiplying 2 by 3 seem to confuse many learners. Thus this lack of recognition of the differences will constrain the pace of achieving procedural fluency. Attainment of procedural fluency is beneficial to learners because it enables them to spend more time on more important relationships.

Strategic competence

This strand involves the ability to formulate mathematical problems, represent them in some form and then solve the problem as it appears in the real world. It requires the ability to apply mathematical knowledge, facts and concepts that have been learned in school to solve non-routine problems. In class often problems are neatly and appropriately laid out. However, in the real world, problems are not so neatly laid out. One must recognise what the problem is, recognise the features of the problem, recognise relationships within the problem, think of possible ways the problem can be solved, represent it in some mathematical form and then solve it. Kilpatrick et al (2001) argue that part of developing strategic competence is developing the ability to replace cumbersome procedures by concise and more efficient procedures. An example of this could be finding the cost of 5 pencils if each pencil costs 99 cents. One method would be to say 99 + 99 + 99 + 99 + 99. Another method would be to work out 5 x 99. Strategic competence would replace the long methods by saying the cost of each of the pencils is 1 cent less than R1, therefore the 5 pencils would be 5 cents less than R5. It could be argued that with the availability of calculators one could just do it on the calculator with great speed, but the issue at hand is developing thinking. It is the process rather than the end result that the focus is on.

Adaptive reasoning

Adaptive reasoning is the capacity to think logically about relationships among concepts and situations. This ability stems from careful consideration of alternative procedures and concepts. Kilpatrick et al (2001) argue that adaptive reasoning is the glue that holds everything together in mathematics. Examples of situations where adaptive reasoning can be demonstrated is the learner's ability to recognise the relationship between exponents and logarithms when dealing with logarithmic equations and the ability to recognise that one of the forms is usually more appropriate to use in particular solutions. Solving the equation $\log_2(x-1) = -3$ is easier to solve when the expression is represented in an alternative form like: $2^{-3} = (x-1)$ or expressing the equation in the form: $\log_2(x-1) = \log_2 2^{-3}$. Solving an equation like $\log_3 x + \log_3(x+6) = 3$ requires that the 3 on the right hand side of the equation be expressed in the form $3\log_3 3$ before attempting the solution. Thus talking about mathematical statements in their different forms should become a habit in the mathematics classroom.

In classroom situations adaptive reasoning is manifested when a learner can informally justify his/her work or can explain to others how he solved a problem or explain his ideas about a mathematical situation. Kilpatrick et al (2001) say that evidence of adaptive reasoning in children as young as 4 years old can be seen when they are allowed to talk about how they arrived at a certain solution to a problem. Kilpatrick et al (2001) say that research suggests that learners' adaptive reasoning develops when the following three conditions are met: when learners have a sufficient knowledge base; when a task is understandable and motivating and when the context is familiar and comfortable. Establishing a classroom norm where learners are encouraged to justify their work and feel comfortable to discuss their views is one way a multilingual classroom can help learners develop adaptive reasoning. At every stage of its development adaptive reasoning interacts with the other strands.

Productive disposition

Kilpatrick et al (2001) describe productive disposition as the tendency of seeing sense in mathematics and to perceive it as useful in one's life and worthy of learning. To develop this disposition, it is important that learners believe they can learn and understand mathematics and use it in their lives. They also need to realise that perseverance eventually pays. Many learners who struggle with mathematics lose patience too early in the learning process. It is therefore important that the teacher provides learners with opportunities to solve, successfully, mathematical problems that relate to the learner's situation.

Building learner's confidence and a positive attitude is another important aspect of developing productive disposition. This is possible if the classroom norms are positive. Learners must feel they are cared for, they are individually encouraged and can learn without fear of being made to feel inferior. Thus a classroom environment is critical for productive disposition. Developing productive disposition will motivate learners to spend longer sustained periods of time practicing skills and doing mathematics. This will in turn improve their procedural fluency which will also improve attainment of conceptual understanding and strategic competence. The importance of developing mathematical proficiency is summarised by Kilpatrick et al's words:

> *The currency of value in the job market today is more than computational competence. It is the ability to apply knowledge to solve problems. For students to be able to compete in today's world and tomorrow's economy, they need to be able to adapt the knowledge they are acquiring. They need to be able to learn new concepts and skills. They need to be able to apply mathematical reasoning to problems. They need to view mathematics as a useful tool that must be constantly sharpened. In short, they need to be mathematically proficient. Students who have learned only procedural skills and have little understanding of mathematics will have limited access to advanced schooling, better jobs and other opportunities. If any group of students is deprived of the opportunity to learn with understanding, they are condemned to second-class status in society, or worse.* (Kilpatrick et al, 2001:144)

The last section of this chapter discusses what strategies are and the strategies that can be used to support the development of mathematical proficiency.

INSTRUCTIONAL STRATEGIES

The Webster Encyclopedic dictionary (1975) describes a strategy as a method or plan to achieve some goal. Merell and Tennyson (1981) describe an instructional strategy as a specified sequence of presentation forms which include attribute isolation, attribute matching and a range of difficult levels. The State of Florida Department of Education (2004) describes instructional strategies as methods that are used in a lesson to ensure that the sequence and delivery of instruction helps students learn. Other educational definitions include 'a systematic plan, consciously adapted and monitored, to improve one's performance in learning'; a planned, deliberate goal-oriented procedure achieved with a sequence of steps that are subject to monitoring and modification (www.cpt.tsu.edu/ESE/in/strmain.html. Downloaded on 03/04/2009). From these definitions it is clear that one of the main characteristics of a strategy is that it is planned, it just does not happen. A strategy is well thought about before it is implemented. It can be argued that in planning a strategy prior experience is brought to the front so that the best possible plan is organized. A strategy is intentionally goal-oriented. It is planned with a specific goal in mind and achieved with a sequence of steps. While there can be a sequence of steps to be followed to carry out the strategy, keeping the strategy flexible to allow for monitoring and modification of the strategy can be a strength.

Jones, Bagford and Wallen (1979) argue that teachers need to have understanding of philosophies of education, learning theories and human development for them to choose and make use of appropriate instructional strategies. The District of Columbia State Improvement Grant, posits that, 'To raise student achievement and deliver what students need to know and be able to do, teachers need to be knowledgeable of effective instructional strategies, be mindful of how learning theory impacts instruction.' (www.dcsig.org/strategies.htm.Downloaded on 20/03/2009). Many instructional strategies are, therefore, based on educational theories. Constructivism has added a dimension to learning that depends on pupils working together and discussing ideas. Constructivism, therefore, backgrounds instructional strategies like interactive instruction and cooperative learning. Instructional strategies must address particular needs of students as well as the particular characteristics of the mathematics content being taught (mathVids, http://fcit.usf.edu. Downloaded on 03/05/2009). In most multilingual mathematics classrooms in South Africa English, the LoLT, is one of the challenges that learners face. Instructional strategies that address learners' language and communication challenges would be beneficial in multilingual classrooms.

The complexities that are brought into the mathematics multilingual classroom have been documented by several researchers (Adler 1997, 1998, 1999; Cuevas, 1990; Cummins 1984, 1986; Khisty, 1995; Mestre, 1981; Moschkovich, 1996, 1999; Ndiyakupfamiye, 1994; Olivares, 1996; Rubenstein, 1996; Setati, 1996, 2002, 2005; Spanos et al 1988). For any instruction in such situations to be as supportive as possible, it is crucial that educators plan ahead the strategies that they use for their teaching. What then in the multilingual classroom must a teacher plan strategies for? What could limited proficiency in the (LoLT) constrain learners from achieving readily? What could be the language pitfalls or the language pathways in the multilingual classroom? (Barnett-Clarke and Ramirez, 2004). As mentioned earlier, the aim of this study was to explore instructional strategies that teachers in multilingual mathematics classrooms are using. Knowledge of such strategies would help teachers in multilingual mathematics classrooms effectively support their learners.

Different ways of categorizing instructional strategies have been proposed, none of them being superior to the others, by several researchers. Jones et.al (1979) propose 30 categories; Price and Nelson (2007) propose seven categories. Florida State (2004) proposes six categories (Focusing on Essentials, Making Linkages Obvious and Explicit, Temporary Support for Learning, Use of Conspicuous Steps, and Reviewing Fluency and Generalization). Teaching Today (http://teachingtoday.glencoe.com.Downloaded on March 20,2009) propose five categories for English Language Learners in Math (Small groups, Varied Math Instruction, Teaching Math Vocabulary, Monitoring Interactions with Learners, Using Prior Learning as a Starting Point, and Valuing Learner's Background) and add three additional strategies for all learners: Multiple Representation, Real-life applications and Seating Arrangements). Cebrian M (www.mathusaelmaths-teachingstrategies. Downloaded on April 5, 2009) proposes four categories (Lecture-Discussion, Cooperative and Collaborative learning, Jigsaw method, and Think-Pair-Share. The Saskaton Public Schools (2004) propose five categories (Direct Instruction, Indirect Instruction, Interactive Instruction, Independent Study and Experiential Learning). However, these categories of strategies are not independent of one another, but it is possible to use several strategies at the same time and various strategies can fit into more than one category (Saskaton Public Schools, 2004).

In this study, instructional strategies have been divided into two broad groups: strategies that support a positive classroom environment and strategies that support the development of understanding of mathematical concepts.

STRATEGIES THAT SUPPORT AN ENABLING ENVIRONMENT

Chaplin and Eastman (1996) argue that the learning environment and classroom culture that a teacher establishes in his or her classroom has a tremendous influence on students' attitudes towards mathematics and the pursuit of knowledge and on their intellectual and social development. They say 'the learning environment can foster mathematical power' (Chapin and Eastman, 1996; 112). This statement is true for all classrooms. However, it is more of a challenge in multilingual classrooms where some learners may feel anxious about their lack of language fluency in the LoLT. Most researchers in multilingual mathematics classrooms have echoed the same observations (Setati, Adler, Reed and Bapoo, 2002; Setati, 2002; Rakgokong, 1994; Arthur, 1994). Chaplin et al (1996) argue further that there are two types of learning-environment characteristics that support students' developing mathematical power. These are the external characteristics and internal characteristics. The external characteristics relate to the physical environment (Lampert, 1998) in the classroom such as the arrangement of desks and whether or not such arrangements encourage sharing and discussing ideas. Chaplin et al (1996) report that some teachers found that just by arranging desks in a circle instead of rows helped to facilitate discussions in class. Other external characteristics reported by Chaplin et al (1996) are availability of resource materials (in South Africa, this would include textbooks, calculators, mathematical instruments) and the length of the mathematics lessons.

Chaplin et al (1996) say further that changing the external set up in and of itself however, cannot create an enabling learning environment. The internal characteristics of the teacher have the greatest potential to transform the classroom into an enabling learning environment that enhances mathematical abilities of all students. Noddings (cited in Hackenberg, 2005) refers to this internal characteristic as mathematical caring relations. Caring relations in multilingual classrooms could be expressed by the teacher listening to all learner contributions whether or not the language is correct or not. Moschkovich (1999) refers to this as focusing on the mathematical content and mathematical argumentation practices rather than focusing on the language. It must be remembered that learners who are not fluent in the LoLT (English in the case of most South African schools) need to know that their contributions are valued, accepted, respected and understood even though the language is not correct. Caring relations are manifested when a teacher's actions reflect his knowledge of his learners' needs in the classroom. Multilingual learners need more time to process questions and to formulate the solutions. They may need to travel through the several registers and languages before formulating a solution. They therefore need more time than usual to respond. Many learners in multilingual classrooms in South Africa are also from the poorer communities and many do not have all the mathematical resources needed in mathematics lessons such as calculators, mathematical instruments or dictionaries. Provision of these by the teacher, even if they have to share, would be evidence of a caring relationship and will make learners feel cared for. These learners will experience stimulation and this feeling helps them to sustain and increase their engagement in mathematical activity.

A very important strategy that fosters an enabling classroom environment in a multilingual classroom is to allow and encourage the use of the learner's main language in discussions. Learners can be split into same-language groups during discussions. Coelho (1998) recommends that grouping students in the same first-language groups can indicate to students that their first language is respected and promoted. They can also discuss in their own language as they learn English. Several studies have shown that learners are more involved in the learning process when they are allowed to use their main language for discussions (Adler, 1996, 1999, 2001; Arthur 1994; Khisty; Wong Fillmore and Valadez 1985(cited in Khisty, 1995); Moschkovich, 1996, 1999; Rakgokong, 1994; Setati, 1996, 2002; Setati and Adler, 2001).

The shift of emphasis in what is considered as learning mathematics globally has made talking in mathematics classrooms very important and thus the focus on the use of a learner's home language. Learners are expected to construct mathematical meanings as they interact with new learning situations. Mathematical learning should be an interactive and constructive activity where there are a lot of opportunities for creative discussions and each learner has a genuine voice. Presentation and discussion of conflicting points of view should be encouraged and the verbalization of mathematical ideas should be a commonplace (Lampert, 2001; Cobb, 1990). Another important aspect of using the learner's home language is that it facilitates exploratory talk in the mathematics classroom. Setati (2002) argues that exploratory talk is a necessary part of talking to learn and is likely to be most effective in the learner's main language. Setati et al (2002) say:

> *Exploratory talk is important for enabling learners to explore ideas and concepts in a comfortable environment. It is also important for enabling teachers to listen to learners' ideas and conceptions so that these can be worked with and built on. Code-switching, and through this the harnessing of learners' main language as a resource, becomes a means for exploratory talk in the multilingual classroom.* (Setati et al, 2002: 78)

Corwin, Storeygard, Price, Smith and Russell (1995) also argue for the importance of talking in mathematics classrooms. To the question 'Why is talk important in Mathematics?' they responded by giving 4 points:

- *Whether it is written, drawn, gestured or spoken, the medium of mathematical expression is human language. Mathematics is a specialized language developed to communicate about particular aspects of the world.*
- *Mathematical knowledge develops through interactions and conversations between individuals and their community. It is an intensely social activity.*
- *A major way of communicating in a mathematics community is through talk. Students use language to present their ideas to each other, build theories together, share solution strategies and generate definitions.*
- *By talking both to themselves and to each other, students form, speak, test and revise ideas.* (Corwin et al., 1995:1)

Corwin et al (1995) further argue that it is the student's own tentative language that should be used as a vehicle to enter into the mathematics community discussing ideas and then gradually adopting more refined, more precise and more theoretical expressions. They argue that early insistence on formal language may keep some of the students out of the mathematical community and worse, it may give the impression that mathematics is just a set of rules and definitions.

Creating an environment where learners have many opportunities to talk in whatever language is therefore a strategy that can be beneficial in multilingual classrooms. It is therefore, important to use of a language that a learner is comfortable in, as a resource, rather than as a hindrance. In most of these cases in South Africa it would the any of the nine local languages.

STRATEGIES THAT PROMOTE UNDERSTANDING AND COMPREHENSION

As mentioned earlier, mathematics education reform internationally has shifted the emphasis in mathematics learning from the traditional procedure-emphasis to meaning-construction, participation, discussion and explaining solutions. Previously, learners could easily resort to memorization of procedures and facts, but now it is crucial that learners understand and be able to communicate and apply the mathematics they learn to unfamiliar situations. Because of the emphasis on communication in learning mathematics, the use of language has increased and thus the dependency of language in learning has correspondingly increased. Instructional strategies, as mentioned earlier, are grounded on the basic needs of learners and should help learners learn. Therefore instructional strategies in multilingual classes should address challenges that learners would meet because of a lack of language proficiency in the LoLT. The instructional strategies used in this study are: Interactive Instruction, Code-Switching, Language Modes, Multiple Representations and Scaffolding Instruction.

Interactive Instruction

Interactive instruction strategy is a strategy that allows for a range of groupings and interactive methods (http://sasked.gov.sk.cs/docs/policy/approach/instrapp03.html downloaded on March 5, 2009). Interactive instruction may include such methods as discussion, cooperative learning, problem-solving and debates. Learners could work in small groups, large groups, and whole-class groups or even in pairs. Interactive instruction relies heavily on discussion and sharing among the participants. Seaman and Feellenz (cited in Saskatchewan Education, 1988) argue that by discussing and sharing, learners have the opportunity to react to the ideas, experiences, insights and knowledge of the teacher or of peers and to generate alternative ways of thinking and feeling. Interactive instruction helps learners to develop social skills and abilities, to organize their thoughts and to develop rational arguments. Saskatchewan Education further argues that interactive instruction often motivates students and that the opportunity to interact with others broadens the educational experience and this takes them beyond the limitations of the traditional classroom. In interactive instruction learners are not passive recipients of knowledge but are active participants (Price and Nelson, 2007). Learners listen to other peoples thinking, share their thoughts, compare and contrast conjectures and ideas and negotiate meanings of new concepts.

Use of interactive instruction has one challenge. This challenge is expressed by the Saskatchewan Learning as "the success of the interactive instruction strategy and its many methods is heavily dependent upon the expertise of the teacher in structuring and developing the dynamics of the group" (Saskatchewan Learning, http://www.sasked.gov.sk.ca/docs/policy/approach/instrappo3.html#strategies. Downloaded on April 5, 2009). Therefore the teacher needs to be very observant of the dynamics of the groups during discussion and sharing of ideas and apply methods that will help to promote talking, discussions and sharing ideas. In multilingual classrooms learners may not be able to say exactly what they want to say. The teacher needs to listen carefully to what the learners say. Davis (1997) argues that the way a teacher listens to learners can also promote talk in the multilingual classroom. He also argues that the quality of student articulation seems to be as closely related to the teacher's modes of attending as to their teaching styles. Coles (2000) who draws on the work of Davis (1997), describes listening in mathematics education as the act of listening which requires a full and conscious effort to tune into the how and the what of the student's idea.

Davis (1997) posits three types of listening that would promote discussion. These are interpretive, transformative and hermeneutic listening. He describes interpretive listening as when a teacher listens to the learner to try and make sense of what the learner is thinking. Interpretive listening reaches out to the student and may require that the learner clarifies a statement. This type leads to further discussions and promotes more mathematical talk. He describes transformative listening as the type of listening that is open to discussions and to altering points of view. It, therefore can lead to debate and or discussions. According to Davis hermeneutic listening is type of listening where the teacher also participates, interprets, transforms and interrogates. This would also lead to more participation by the learners together with the teacher and therefore keeps the discussion and conversation mathematical. Thus the type of listening that the teacher uses can aid in interactive instruction.

Another way the teacher can promote talk in multilingual classrooms is to ask questions that encourage learners to talk. Nystrand (1997) describes two types of questions: test questions and authentic questions. He describes test questions as those that aim to find out what students know or how close their solutions are to the teacher's solutions, whereas authentic questions create dialogue, increase engagement with concepts and increase critical thinking and achievement. Authentic questions require more conceptual answers and require learners to explore meaning and relationships. Authentic questions promote exploration, discussion, questioning and explanations by students (Zemelman, Daniels and Hyde, 1998). Nystrand (1997) found that authentic questions had a strong positive effect on achievement in Grade 8 classrooms, even though they were seldom used. Thus the use of authentic questions would be a positive method to use in a multilingual classroom. Boaler and Brodie (2004) identify nine categories of questions that teachers use. They categorise them according to their functions: gathering information; inserting terminology; exploring mathematical meanings and relationships; probing; generating discussion; linking and applying; extending thinking, orienting and focusing and establishing context. The categories that would promote mathematical talk would be the last seven categories. These would then be ways that could be used to promote mathematical talk and discussion in the mathematics multilingual classroom.

Another method that can be used to orchestrate and keep conversations mathematical is revoicing. The use of revoicing has been recommended by several researchers who have worked in multilingual mathematics classrooms (Moschkovich, 1999; Brodie, 2003; Enyedy, Rubel, Castellon, Mukhodhyay, Esmonde and Secada, 2008). Enyedy et al, (2008) argue that one of the primary purposes of revoicing is to promote a deeper conceptual understanding of mathematics by positioning students in relation to one another, thereby facilitating student debate and mathematical argumentation. They further argue that one significance of revoicing is its value in initiating and sustaining mathematical discussions. Forman, Larreamendy-Joerns, Stein and Brown (1998) say that revoicing occurs when one person re-utters another's contribution through the use of repetition, expansion or rephrasing. O'Connor and Michaels (1993) and Enyedy et al (2008) say that when a teacher revoices a student's contribution it amplifies the student's contribution. The teacher reformulates the contribution in a more precise language or into more technical terms but maintains the student as the owner of the contribution. In multilingual mathematics classrooms, learners often struggle to find the proper word or term to use because they are still learning English. Revoicing would be beneficial as learners' contributions are not neglected nor ignored for lack of accuracy, but are refined to the appropriate terminology.

Code-Switching

Though a teacher may listen in an interpretive, transformative or hermeneutic way and ask authentic questions, learners may still find it difficult to explain what they actually mean in English, the LoLT. The use of the learner's home language is a strategy that can help to promote talk and discussions and help learners to understand new concepts. The use of home language allows learners and/or the teacher to code-switch. Setati (2002) argues that when learners switch codes during discussions, explanations or justifications, they are better able to explore and develop mathematical ideas in a comfortable environment. Barnes, cited in Setati et al (2002), says that exploratory talk is a necessary part of talking to learn and that it is most effective in the learner's main language. It is this exploratory talk that helps them make meaning of mathematical ideas. Research by Setati, (2002); Setati et al., (2002); Adler, (2001); Rakgokong, (1994) and Moschkovich

(1996, 1999) has shown that when teachers encourage code-switching in their multilingual classrooms, learners become more engaged and contribute to discussions. Being active participants in discussions results in better understanding and learners are thus able to construct more meaningful mathematical concepts. Group discussions can also bring to light some misconceptions that would otherwise remain concealed in written work. Thus placing learners in same language groups and encouraging code-switching can be an effective way to promote interactive instruction in multilingual classrooms.

Language Modes

The Virginia Department of Education (2004) also recommends integrating the four language modes into the mathematics class for LEP students. These language modes are listening, speaking, reading and writing. "Discussing, writing, reading and listening to mathematical ideas deepen students' understanding of mathematics". (Zemelman et al 1998:91). Because most classrooms in South Africa are multilingual these language modes can be beneficial in South African classrooms. Listening is important so that the learner knows how the word must be pronounced, then he can hear and recognize the word later. Words like *sine ß* or its abbreviation *sin ß* and *sign of ß* can interfere with a learner's ability to follow a discussion if he or she does not recognize the sound or how it is written. The learner must also be able to communicate mathematics mathematically and verbally to others. The learner must therefore know how to say the word. It is also necessary to be able to read mathematics from mathematics textbooks. The final examinations are in written form. It is therefore crucial that learners are able to communicate the mathematics mathematically in writing. Another challenge in multilingual classrooms is the intonation of words. Words may sound different because of a different mother tongue. Writing the new words on the board, where learners can see the word written down, can avoid confusion of what the word is. A learner may not be able to pronounce the words correctly or explain his/her thoughts, but he or she may be able to write and communicate his thoughts, solutions or explanations and justifications. In a multilingual class, talking aloud to the multilingual learners would actually be multipurpose because the learner can hear the English words and learn the expected intonations. As the teacher talks aloud whilst thinking, he models the thought processes and learners can witness the process of mathematical thinking rather than just the end result.

Multiple Representations

Another strategy that would be important in multilingual mathematics classrooms is the use of different expressions (Moschkovich, 1999) and representations (McKendree et al., 2002; Seeger, 1998; Van Oers, 2000; Brenner, 1999) for the same concept. McKendree et al. describe a representation as: 'A structure that stands for something else, a word for an object, a sentence for a state-of-affairs, a diagram for an arrangement of things, or a picture for a scene'(McKendree et al, 2002:59). Seeger (1998) refers to representations as tools that are complimentary to other thinking tools and that they can be seen as exploratory artefacts that allow the production of multiple perspectives on mathematical content. Being able to see a concept in different forms or representations benefits multilingual learners because they can understand a concept from a representation that has reduced dependency on language. Seeger (1998) further argues that representations are so productive and generative that they are open for condensing a whole lot of information. This aspect makes representation appropriate for reducing the amount of work that learners have to remember. This is even more beneficial for a multilingual learner as he has to cope with a foreign language. However, Seeger quickly points out that 'in learning, representation without a constructive appeal is as empty as construction that does not represent anything' (Seeger, 1998: 329). This suggests a process through which learning must progress so that learners do not just memorise representations without understanding, like *b* represents bananas rather than that *b* represents the number of bananas at a particular time. To a multilingual learner the difference between the above statements needs to be emphasized and not taken for granted.

McKendree et al. (2002) argue that a good representation system must capture the features of a problem that are important in a particular situation rather than representing everything and learners must know which representation is appropriate for which problem. He further argues that being able to think about why a representation may or may not be good in a particular context is a big part of being a critical thinker. Many mathematical problems can easily be solved with one representation, but not so easy or even very hard with another representation. Below is an example of a problem that is easily solved graphically rather than algebraically:

> *Three friends Amo, Benny and Chung decide to meet on a Sunday afternoon. Chung lives 200 km. away from Amo. Amo starts at 09:00 and cycles at 30 km/h. Chung starts from his home at 10:00 travelling at 50 km/h. After one hour Amo has a puncture and spends 30 minutes trying to fix the puncture. He then walks for 30 minutes to get to Benny who lives 35 km from Amo. The two then drive at 100 km/h to meet Chung. Find the time that the friends will meet and how far they are from Amo's home.*

Dealing with the different traveling speeds for Amo would create more work in formulating an equation that could be used to find when the three friends meet. A graphical representation would be a much easier representation to use for solving this problem.

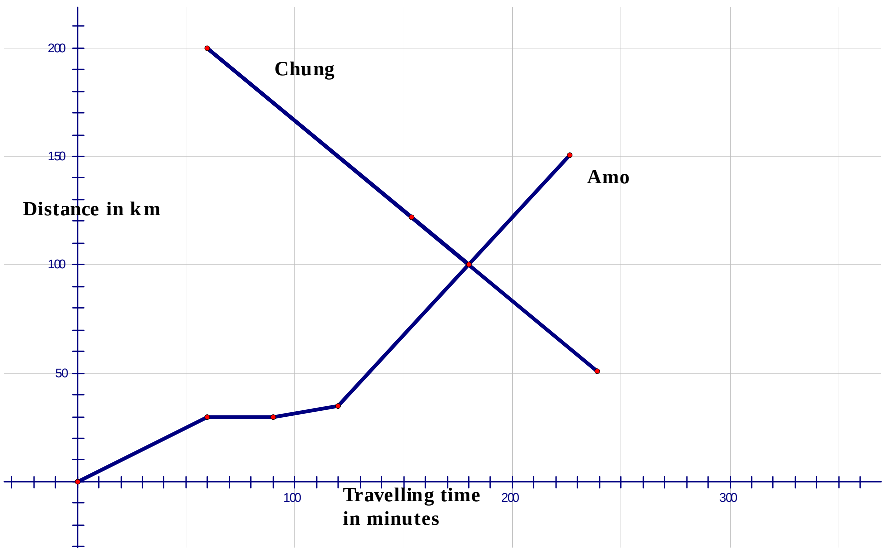


Figure 3.5: *Graphical Solution of when and where the friends meet*

The algebraic solution would involve setting up an equation where the total distance travelled by the two friends is equal to 200 km as:

Let x be the time in hours Amo and Benny drive before meeting Chung Distance travelled by Amo is Distance cycled + distance walked to Benny's house + distance travelled in Benny's car at 100 km/hr = 30 + 5 + 100x Since Chung starts at 10:00, he travels one hour less than Amo. Time Amo is on the road is 1 hr (cycling) + ½ hr (trying to fix the bicycle) + ½ hr (walking to Benny's house) + x hours (travelling in Benny's car) = (2 + x) hours. Therefore Chung travels for (2 + x -1) hours = (1 + x) hours Distance traveled by Chung = 50 (1 + x) km = 50 + 50x Total distance: 50 + 50x + 30 + 5 + 100x = 200 km. This gives x = 46 minutes. Therefore they meet at 11:46.

The graphical solution would take much less language to deal with the solution and would be faster. Learners would not have to deal with Chung traveling one hour less in their equations. Knowing the graphical method would be beneficial.

Flexibility in representation is therefore an important aspect of representation. Research results have shown that students who can readily translate between and manipulate representations score higher on measures of reasoning ability than those whose skill in representation is more limited (Stenning et al,1995; Monaghan, 1999 both cited in McKendree et.al, 2002; Brenner et al, 1999). Research studies by Mayer (1995) seem to suggest that even the use of textbooks that portray multiple representations of concepts or problems have a positive effect on conceptual development of students. Thus flexible representation is a good strategy that can be used in the multilingual classroom. McKendree et al. (2002) also recommends that 'modelling problem solving by talking aloud and drawing intermediate representations while working through a problem, and then reflecting on the process afterwards is an excellent way to show students how you would like them to think too'(McKendree, 2002:65). In a multilingual class, talking aloud to the multilingual learners would actually be multipurpose because the learner can hear the English words and learn the expected intonations. As the teacher talks aloud whilst thinking, he models the thought processes and learners can witness the process of mathematical thinking rather than just the end result.

Scaffolding Instruction

The last strategy is that of Scaffolding Instruction. Scaffolding Instruction is grounded on Vygotsky's (1978) concept of the Zone of Proximal Development, ZPD (Anghileri, 2006; Kiong and Yong; Van der Stuyf, 2002). Vygotsky (1978) describes the ZPD as "The distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers"(Vygotsky 1978:86). Goulding (1999) describes scaffolding as 'The process by which a more knowledgeable adult or peer can help a child move from her actual performance level to her potential level, giving just enough help to move the child from one to the other.'(Goulding in Johnston-Wilder et al., 1999: 44)

Vygotsky (1978) argues that it is the ZPD which is important in the learning process. It is in the ZPD that individuals learn through social interactions and with the help of a more knowledgeable person, a teacher or a peer. The term scaffolding was used for the first time by Wood, Bruner and Ross in 1976 in the article titled 'The Role of Tutoring in Problem Solving'. The concept of scaffolding instruction can be compared to scaffolding in building construction. Greenfield says:

> *The scaffold, as it is known in building construction, has five characteristics: it provides a support; it functions as a tool; it extends the range of a worker; it allows the worker to accomplish the task not otherwise possible; and it is used selectively to aid the worker where needed.* (Greenfield, 1984: 118)

Greenfield (1984) further posits that in the learning process the teacher can provide the support that a learner needs to complete a task successfully. 'At the beginning of learning students need a great deal of support and gradually this support is taken away to allow students to try their independence' (http://www.eduplace.com/rdg/res/literacy/litins4.html. Downloaded on March 4, 2009). Scaffolding is therefore a temporary support in the learning process that provides help for learners at specific points. It closes the gap between the task requirement and the skill level of the learner (Kiong and Yong, 2001).

Some of the characteristics of scaffolding that McKenzie (2000) posits are that scaffolding provides clear direction to learners and reduces confusion, helps keep students on the task, clarifies expectations and incorporates assessment and feedback, points students to worthy resources and reduces uncertainty, surprise and disappointment. McKenzie's descriptions of the characteristics of scaffolding match the needs of multilingual learners. If learners are not sure of the meaning of words they can be confused. Providing scaffold instruction would reduce possible confusion or uncertainty. Learners would benefit if they know that they are on the right track and what is expected of them. Giving them feedback at appropriate intervals would ensure correct direction and save on time as well. Scaffolding instruction can be delivered at different levels. It can be delivered in whole-class discussions, in small group work, individual work or during reporting back. Anghileri (2006) puts forward a three-tier hierarchy of scaffolding practices for learning mathematics. She describes level 1 as the environmental provisions that include peer collaboration and classroom organization. Level 2 includes explaining, reviewing and restructuring. Level 3 involves developing of conceptual thinking by making connections, developing representational tools and generating conceptual discourse (Anghileri, 2006:39). Other activities that have been suggested by researchers to scaffold learning include encouraging clarification and justification, modelling, cues, prompting and probing, providing hints or partial solutions, think-aloud modelling, simplifying tasks, increasing level of questioning and redirection (Kiong and Yong, 2001; Van Der Stuyf, 2002; Pennil, 2002; Henry and Stager, 2002)

The success of scaffolding instruction relies on the teacher's observation skills and decision skills to decide when and how much support is given and when to fade away the support. The teacher needs to observe all responses from the learners because some of the responses may be not be accurate and some may be non-verbal and give immediate feedback to the learners. Scaffolding must be flexible and dynamic where the teacher is responsive to individuals within the classroom setting (Anghileri, 2006). A teacher in a mathematics multilingual classroom must be aware of where a learner needs scaffolding. It could be a new English word; it could be the meaning or the pronunciation of the word or the mathematical concept. A learner may know what to do but does not possess the vocabulary to explain his/her work. As explained earlier, strategies should cater for the needs of individual learners, scaffolding instruction must attend to the needs of the individual learners within the multilingual classroom. Therefore, teacher's scaffolding must be dynamic. Anghileri's (2006) concluding remarks are worth noting for use in mathematics multilingual classrooms '…Teachers will be most effective if they are able to scaffold pupil's learning by employing a range of teaching approaches in an environment that encourages active involvement' (Anghileri, 2006:50).

CONCLUSION

This chapter has given an outline of the theoretical framework that guided this study. The chapter has explained why the theory of social constructivism is relevant in learning and teaching mathematics in South Africa and how it relates to the learning of mathematics in multilingual classrooms. The chapter has also given an account of the literature on language as it is used both as a tool in thinking and as a tool in communication. The chapter has also explained what constitutes mathematical learning and the strategies that can be used to support mathematical learning in multilingual classrooms. The next chapter discusses the methodology that was used for the study.

CHAPTER 4

RESEARCH DESIGN AND METHODOLOGY

The previous chapter discussed the theoretical framework and the literature on which the study was based. This chapter discusses the processes undertaken to obtain the information needed to answer the research questions as well as the rationale for the choices. Bassey (1999) describes and shows the importance of educational research by saying:

> *Educational research is a critical enquiry aimed at informing educational judgements and decisions in order to improve educational action. This is the kind of value-laden research that should have immediate relevance to teachers and policy-makers, and is itself educational because of its stated intention to "inform"* (Bassey, 1999: 59).

The nature of multilingualism in South African schools and poor mathematics results at matriculation level, served as an inspiration to undertake a qualitative research and explore what is going on in some of these multilingual classrooms.

Why Qualitative Study?

Leedy (1997) describes a qualitative study as an inquiry process of understanding a social or human problem, based on building a complex and holistic picture from words that detail the views of informants in their natural setting. He goes on further to say that qualitative researchers usually collect an extensive amount of verbal data from a small number of participants and present their findings in descriptions that are intended to accurately reflect the situation under study. Qualitative researchers tend to adopt an attitude of exploration and discovery. McMillan and Schumacher (2006) describe it as an inquiry in which researchers collect data in face-to-face situations by interacting with selected persons in their natural settings and interpreting the data. They further say that purposes of qualitative research include being exploratory, examining new or little understood phenomena and possibly contributing to educational practice by building rich descriptions from complex but real situations. Opie (2004) describes qualitative research as research that seeks to obtain insights into how individuals modify and interpret the world about them.

These descriptions resonate with the focus of this study, whose purpose was to observe teachers in the classroom in teaching situations as they interacted with their learners. The main focus was to explore what teachers in different multilingual classrooms are doing to facilitate learners' development of mathematical proficiency in algebra. Findings from the study could help inform other teachers in similar situations of possible strategies that could be used. The interest was not only in the strategies they used in their classrooms, but also on understanding the reasons for their actions and choices. Thus it was important to see the teachers teaching and also be able to discuss their lessons with them. Thus a qualitative study which accommodates interactive processes between the researcher and the teacher was found suitable.

Two cases were identified for the study. Opie (2004) describes a case study as an in-depth study of interactions of a single instance in an enclosed system. Opie further argues that:

> *Crucially the focus of a case study is on a real situation, with real people in an environment often familiar to the researcher. Its aim then is to provide a picture of a certain feature of social behaviour or activity in a particular setting and the factors influencing this situation. In this way the interactions of events, human relationships and other factors are studied in a unique location* (Opie, 2004:74).

McMillan and Schumacher (2006) say that a case study examines a bounded system over time in detail, employing multiple sources of data found in the setting. They further explain how case studies can contribute to theory when they say:

> *Case study design is appropriate for exploratory and discovery-oriented research. An exploratory study examines a topic about which there has been little prior research, is designed to lead to further inquiry. The purpose is to elaborate*

a concept, develop a model with its subcomponents or suggest propositions (McMillan and Schumacher, 2006:318).

In this study, the issue at hand is the multilingual nature of most of South African schools combined with the poor performance in algebra for most of these learners. It was important to have a methodology that allows for the exploration of what goes on in the real classroom situation. Case studies investigate and report on the complex dynamic and unfolding interactions of events, human relationships and other factors in unique instances (Cohen, Manion and Morrison 2002). Every classroom situation is always dynamic and the interactions within each classroom contribute to the learning process. Being able to catch the close-up reality and thick descriptions of participants' lived experiences (Gertz, 1973, in Cohen et al 2002) makes case study an ideal methodology for this study. Cohen et al (2002) say that case studies are strong in reality and recognize the complexity and the embededness of social truths. There is no doubt that the multilingual mathematics classroom in South Africa is a complex social classroom with learners speaking any number of languages from two up to six.

The Sample

Purposive sampling was used in this study. In a purposive sample, the subjects are hand-picked by the researcher on the basis of an estimation of their typicality (Opie, 2004). The subjects are chosen for a specific purpose. Patton, cited in McMillan and Schumacher (2006), says purposive sampling is selecting information-rich cases for in-depth study. It is done to increase the utility of information obtained from small samples. The sample is chosen because it is likely to be knowledgeable and informative about the phenomena under investigation. The power of purposive sampling is that a few cases studied in depth yield many insights about the topic.

The purpose of this study was to explore strategies that teachers use to support the development of mathematical proficiency in multilingual classrooms. For purposes of this study it was important to work with teachers who understood the complex nature and effects of language of learning and teaching in multilingual classrooms. It was also important to select those teachers who were interested in addressing the language complexities and who were also familiar with what constitutes mathematical proficiency. These would be teachers who have attended the Mathematics and Science Education courses offered by the University of the Witwatersrand. Chapin et al (1996) argue that teachers' beliefs and attitudes, and familiarity with knowledge of mathematics content as well as methodology affect learning. They further report that:

> *Research indicates that teachers' beliefs and knowledge affect the decisions they make about instruction. Since teachers' decision making is the core process that creates and shapes the classroom environment, teachers' beliefs and knowledge affect their willingness and ability to change environment.* (Chapin et al 1996:113).

The teachers chosen for the study, Jean and David (not their real names) have attended the Mathematics and Science Education course at University of Witwatersrand and they have completed courses in 'language and communication in mathematics education'. Both teachers were also keen on implementing this knowledge in their mathematics classrooms.

Jean, who has been teaching for 21 years, shares a common language with all of the learners and thus she is able to communicate and code-switch for all of the learners in her class. David does not share a common language with most of the learners. David has been teaching for 20 years and teaches in English. English is not David's main language but is his second language. His first language is Afrikaans and he has always taught in Afrikaans until six years ago. He is, however, fluent in English. Choosing these two teachers provided me with an opportunity to consider two different language attributes in mathematics multilingual classrooms and not to compare the two teachers in any way at all.

Data Collection

Data was collected in two stages. For Jean, the first stage of data collection was observing the video of Jean teaching exponents that had been recorded previously. The video tape was analysed and notes and transcripts of the lessons were made. This was followed by a semi-structured interview with Jean. In a semi-structured interview a schedule is planned but it is sufficiently open-ended so that contents can be reordered, digressed and expansions made. New avenues can be included and further probing can be undertaken to get in-depth understanding of participant's response, actions and/or knowledge (Cohen et al., 2002). Silverman, cited in Cohen et al (2002), says that interviews are useful for gathering facts, accessing beliefs about facts, identifying motives and feelings and eliciting reasons and explanations.

The interview was recorded on tape. The recording of the interview helps to ensure accuracy of the conversation during the interview. McMillan and Schumacher (2006) argue that the primary data of qualitative interviews are verbatim accounts of what transpires in the interview session and tape recording ensures completeness of the verbal interaction and provides material for reliability checks. The interview schedule was designed after analysing the video of the lessons. This was so that focus during the interview would be on exploring, in depth, the strategies and language issues that were apparent in the lessons.

In the case of David, I spent four days at the school during which five lessons were observed and video recorded. Field notes were also taken during the lessons. The field notes included gestures, social interactions and observed behaviour of the subjects (Opie, 2004) during the lessons. At the end of each lesson I discussed the lesson with David using my field notes. This discussion also helped to check the accuracy of my data. Data and non-verbal behaviour can make observational research superior to experimental research (Cohen et al, 2002). Recording of ongoing behaviour adds to information on the nature of the classroom environment. The first two lessons were used to minimize the effect of observers and presence of instruments in the classroom. The lessons were also used to check if the instruments were working properly. Data analysis was done on the last three lessons. The video recordings of the last three lessons were analysed and transcribed. Analysis of the video tape helped in designing the questions that were used in the semi-structured interview that followed. The interview, which was semi-structured, was tape recorded.

RIGOUR IN THE RESEARCH

Lincoln and Cuba (1985) posit that for any research study to be worthy, one must provide information that will persuade the readers that the research findings are worthy paying attention to. It is therefore important to establish rigour in any research. They argue that the trustworthiness of any research report depends on issues described as validity and reliability. But, according to Golafshani (2003) reliability and validity are concepts that are rooted in positivist perspectives which result in quantitative research studies. In quantitative research experimental methods and quantitative measures are used to test an hypothesis. Information in the form of numbers is quantified and summarized. On the other hand, qualitative research uses a naturalist approach to understand phenomena in real world settings. Golafshani (2003) adds that qualitative research seeks to illuminate, understand and extrapolate to similar situations. The purposes of quantitative and qualitative studies are also different. Quantitative studies aim to establish the relationship between measured variables whereas the purpose in qualitative studies is to understand a social situation from the participants' perspectives. Because of these differences some qualitative researchers (Lincoln and Cuba, 1985; Bassey, 2003; Elliot, Eisner and Peshkin, 1990) argue that the terms reliability and validity as they are described in quantitative research are problematic in qualitative research. They have, therefore, proposed alternative descriptions to the terms reliability and validity. Alternative terms such as credibility, trustworthiness, rigour, transferability, dependability, conformity, consistency and applicability have been proposed by such researchers as Bassey (2003); Davies and Dodd (2002); Lincoln and Cuba(1985) and McMillan and Schumacher (2006); Mishler (2000); Seale (1999); and Stenbacka (2001).

In this study the terms validity and reliability are used as they are defined in qualitative research to address issues about rigour in the research.

VALIDITY

Denzin and Lincoln (2000) argue that the cornerstone of qualitative research has been the descriptions of persons, places and events and argues that validity in qualitative research has to do with descriptions and explanations and whether or not the explanations fit the descriptions. McMillan and Schumacher (2006) argue that validity refers to the degree of congruence between explanations of the phenomena and the realities of the world. Validity addresses the questions on whether researchers observe what they think they see and whether they hear the meanings that they think they hear. Validity is the degree to which the interpretations have mutual meaning between the participants and the researcher (McMillan and Schumacher, 2006). Opie (2004) argues that the issue of validity concerns the relationship between the claim and the accompanying process of data gathering. Maxwell (1992) uses a realist approach to the concept of validity which sees the validity of an account as inherent in its relationship to those things that it is intended to be an account of and not in the procedures used to produce and validate it. He argues that validity is not an inherent property of a particular method, but it pertains to the data, the accounts or conclusions reached by using a method in a particular context for a particular purpose. Maxwell (1992) describes five types of validity that are commonly used in qualitative research: descriptive validity, interpretive validity, theoretical validity, generalisability and evaluative validity. This study used the first four types of Maxwell's concept of validity.

Descriptive Validity

Descriptive validity deals with the factual accuracy of the participants' utterances. This refers to the actual statements or accurate descriptions of utterances made by the participants or informants rather than the interpretations of the researcher. This refers "to what Kaplan calls 'acts' rather than 'actions'… activities seen as physical and behavioral events rather than in terms of the meanings that these have for the actor or others involved in the activity" (Maxwell, 1992: 286). Descriptive validity can be enhanced by making use of verbatim accounts of participants as well as including features of the participant's speech and/or expressions. In this study to enhance descriptive validity, literal accounts and quotations of the exact words and phrases used by the participants were recorded without any changes. A video recorder was used to capture accurate utterances, actions and gestures of the teacher and the learners. A tape recorder was also used during the interviews with the teachers. This made it possible to capture the exact statements made by the teachers, including any features of the teacher's speech like different pitches of voice or emphasis on particular phrases. Verbatim transcripts of the lessons and interviews are attached as appendices.

Interpretive Validity

Interpretive validity refers to the meaning of events, words, actions or behaviours. The term 'meaning' also includes the intentions and beliefs of the people engaged in the events. This meaning is according to the people who are engaged in events, words or actions and not meaning as determined or interpreted by the researcher. According to Maxwell (1992) interpretive research 'seeks to comprehend phenomena not on the basis of the researcher's perspective and categories, but from those of the participants in the situation studied—that is from an emic rather than an etic perspective' (Maxwell, 1992: 289). He adds that 'accounts of meaning must be based initially on the conceptual framework of the people whose meaning is in question.'(Maxwell,1992:289). Interpretive validity does not apply only to the conscious concepts of participants; it can also pertain to the unconscious intentions, beliefs, concepts and values of the participants. To enhance interpretive validity in this study, the classroom events and my observations of the lesson were discussed with the teacher after the lessons. In the case of David, we discussed after each lesson and in the case of Jean we discussed after I had watched the video. This was to make sure that the correct interpretation of the teacher's words, actions and his or her intentions were made. An example in this study was that David always asked the learners to read aloud mathematical statements either from their textbooks or from learners' work on the board. During one of our informal discussions after the lesson, I asked why he emphasized reading the statements aloud. This enabled me to correctly interpret the meaning/intentions of the teacher's insistence on learners reading of statements aloud. Interviews were also used to check on the accuracy of the participant's meanings of words and actions that were captured on the video recording. Interpretive validity was enhanced by showing and discussing excerpts of my interpretations of the classroom observations and interviews with the participants. By doing this, the participants were able to corroborate, correct and/or disapprove of my interpretations.

Theoretical Validity

Theoretical validity addresses the theoretical constructions that a researcher brings to the study or develops during the study. Maxwell (1992) describes theoretical validity as that which is concerned with the legitimacy of the application of a given theory to establish facts. He argues that theoretical validity depends on whether there is consensus within the community concerned with the research about the terms used to characterize the phenomena. In this study the theoretical constructs used are Kilpatrick's (2001) strands of mathematics proficiency and Vygotsky's (1978) social constructivism. As data was analysed, instances of where and how the concepts of these theories were applied were noted. Notes were also made of where and how these theories supported each other in the multilingual classroom.

Generalisability

Generalisability concerns the usefulness of the research to other similar situations. Maxwell (1992) argues that generalization refers to the extent to which one can extend the account of a particular situation or population to other persons, times or settings than those directly studied. He further posits that generalisability is normally based on the assumption that the theory may be useful in making sense of similar situations. McMillan and Schumacher say:

> *Qualitative researchers provide for logical extension of findings, which enable others to understand similar situations and apply the findings in subsequent research or practical situations. Knowledge is produced by the preponderance of evidence found in separate case studies over time.* (McMillan and Schumacher, 2006: 330)

It is generally accepted, and as mentioned earlier, that the majority of classrooms in South Africa are multilingual. While the findings cannot necessarily be generalised to all multilingual mathematics classrooms in South Africa, they will certainly provide more understanding on the kind of instructional strategies that mathematics teachers in different multilingual classrooms use or can use to support their learners. To enable extension of findings of this study, adequate descriptions of how the participants were chosen, the physical and social scenes, language composition of the learners in each classroom, main language of the teachers and sites from which the study was done are included. The rationale for the study and theoretical framework that informed the study have also been included.

RELIABILITY

Reliability is, as mentioned earlier, another concept that has its roots in quantitative research and whose definition and meaning causes concern in qualitative research. Joppe defines reliability as:

> *The extent to which results are consistent over time and an accurate representation of the total population under study is referred to as reliability and if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable*. (Joppe, 2000:1)

Embodied in this definition is the idea of replicability or repeatability of results or observations (Golafshani, 2003). According to Wellington (2000) reliability can be understood as the extent to which a test, a method or tool gives consistent results across a range of settings, and if used by a range of researchers. This idea of reliability is the reliability of the research instrument. The problem in qualitative research is that the researcher is the instrument and replicability is impossible in this case. The environment in which observations are made can never be replicated because the classroom environment is dynamic.

In this study the concept of reliability used is in the same sense as defined by Opie (2004). He uses 'reliability to describe the extent to which a data-gathering process produces similar results in similar conditions' (Opie, 2004: 68). He regards reliability as a property of the whole process of data gathering, rather than a property solely of the results. He uses reliability to judge the process of data-gathering and not to judge the product. Here reliability deals with the accurateness of methods and techniques that are used to produce data. One way of enhancing reliability is the use of multiple methods of data collection so that data sources can be corroborated. In this study multiple methods of data collection were used. Data was collected by mechanically recording the lessons, collecting field notes and mechanically recording interviews with the participants.

Triangulation

The idea behind triangulation is that the more agreement there is of different data sources, the more reliable is the interpretation of the data. Paton (2001) states that triangulation strengthens a study by combining methods. Mathison (1988) says that triangulation is a strategy that is used to improve the validity and reliability of research. Triangulation assists in correcting bias that might occur during observations (McMillan and Schumacher, 2006). In this study two types of triangulation were used: data triangulation and investigator triangulation (Denzin & Lincoln, 2000). The multi-methods (video-recordings, field notes and interviews) that were used to collect data allowed for triangulation of data. Peer debriefing, member checking and participant reviews (McMillan and Schumacher, 2006) were used for investigator triangulation. Peer debriefing was achieved by formal corroboration of the interpretation of data with other members of the research team. Member checking was achieved by checking with the participants the accuracy of the data collected. I always had informal discussions with the teacher after each classroom observation.

Ethical Considerations

Since this study involved persons in their natural teaching and learning environment, it was important that permission from these individuals as well as permission from institutions that were involved was sought. The consent of the two teachers involved in the study was secured first. Thereafter permission from the Gauteng Department of Education to do the study in schools in their region was applied for. This permission was granted and was followed by application for permission to the school where the study was going to be undertaken with data collected through lesson observation as well as video recording of the lessons. A meeting with the head of the school together with the teacher concerned was held to discuss how the study would be carried out. Through this meeting, permission was granted. Lastly I met with the learners and explained to them what the study was about and how the study is expected to contribute to the teaching and learning of mathematics in South African multilingual classrooms. A letter, explaining the study, was sent to parents of learners and consent forms were received back from all the parents.

As this study involved researcher's interpretations of the activities that were going on in the classrooms, I promised the participants that I would share with them my findings and check with them about the accuracy of my interpretations of the observations. Anonymity of all participants in the study was assured and the use of the video recorded sections would only be done with participants' prior consent, if this was to be used at all.

Data Analysis

Data analysis is a systematic search for meaning. It requires organizing and interrogating data in ways that allow researchers to see patterns, identify themes, discover relationships, develop explanations and make interpretations (Hatch, 2002). Analysing data therefore requires some organized way of seeing what is in the data, putting data into categories or coding data and interpreting what is in the data. The process of data analysis is guided by some working theories as well as personal experiences.

This research focused on exploring the strategies that teachers in multilingual classrooms use to support development of mathematical proficiency in algebra. Data analysis was therefore guided by concepts and theories on mathematical proficiency. Hatch (2002) posits five models (typological, inductive, interpretive, political and polyvocal) of analysing qualitative data. I used the inductive model to analyse the data. Hatch says that to argue inductively is to begin with particular pieces of evidence, then pull them together into meaningful wholes, looking for patterns across individual observations, then arguing for those patterns as having the status of general explanatory statements (Hatch, 2002). The data in this study was transcribed and then coded. Coding of the data was guided by the instructional strategies and the strands of mathematical proficiency. The categories and their corresponding codes, in brackets, used for analyzing data are described below.

CATEGORIES USED FOR ANALYSING DATA

Interactive Instruction (TII): Teacher using and/or encouraging learners interactive participation.

Language Modes (TLM): Teacher using and/or encouraging learners to use different language modes.

Scaffolding (TSc): Teacher scaffolding work for learners.

Code-Switching (TCS): Teacher code-switching.

Code-Switching (LCS): Learners code-switching.

Informal Mathematical language (TIM): Teacher using and accepting use of informal mathematical language.

Multiple Representations (TMR): Teacher using and/or encouraging learners to use multiple representations.

Data was also analysed in search of where the development of a particular strand of mathematical proficiency was promoted. The table below explains how teacher's actions were coded as promoting a particular strand of mathematical proficiency.

<u>Strand of Mathematical Proficiency</u>	<u>Indicator</u>: Evidence seen when teacher requires, asks or encourages learners to:
Conceptual Understanding (**CU**)	• Represent mathematical ideas in different ways. • Make connections • Know how different representations can be used for different purposes

Procedural Fluency **(PF)**	• Know procedures • Know how and when to use procedures appropriately • Develop skill in performing procedures flexibly and accurately
Strategic Competency **(SC)**	• Be flexible in approaches to solving problems • Choose effective procedures • Replace long methods with concise and efficient procedures • Form mathematical representations from words
Adaptive Reasoning **(AR)**	• Think logically about relations between situations and concepts • Explain ideas • Justify solutions and procedures • Explain procedures
Productive Disposition **(PD)**	• Persevere and exert more effort • Develop confidence

Table 4.1: *Mathematics Proficiency Strands and corresponding Indicators*

CONCLUSION

This chapter outlined the research design and methodology used in the study. The chapter also gave explanations for the choice of the type of study, the rationale for choosing case study as a research approach, the sample chosen and how data was collected. The chapter also discussed how concerns over validity and reliability were dealt with, as these are not approached in the same manner when doing a qualitative research as one undertaking quantitative research. The next chapter draws on the data collected to describe the strategies that each of the teachers used during their lessons.

CHAPTER 5

ANALYSIS: DESCRIPTIONS OF STRATEGIES IN LESSONS

Chapter 4 discussed how data was collected and the measures that were taken to ensure rigour in the research project. The purpose of this study was to explore the instructional strategies that mathematics teachers in different multilingual classrooms use. The study was, therefore, both descriptive and exploratory. It is descriptive because it describes the strategies that the teachers are using, and goes further to explore which of the strategies could be beneficial in the multilingual classroom. The analysis is therefore presented in two parts. This chapter describes the strategies that each of the two teachers used in their classrooms. Chapter 6 focuses on exploring strategies that could be appropriate for multilingual classrooms.

STRATEGIES USED BY JEAN

Data in Jean's classroom was collected through pre-recorded videoing of her teaching exponents to Grade 10 learners. Jean and the learners all speak IsiZulu fluently; therefore they all understood each other when switching between languages during discussions. The lessons that were analysed all focused on exponents at Grade 10 level. The rectangular desks were arranged in groups where between four and six learners could sit. With this arrangement, learners in each group were in close proximity to each other.

Lesson 1

This lesson focused on rational exponents using the radical form, i.e. using the terms 'square root', 'cube root', 'fourth root' and 'fifth root'. At this stage learners had already done some basic work on exponents with natural numbers. Jean started the lesson by discussing with the learners how one feels when one does not understand a language. Jean put a message on the overhead and asked the learners to read the message. None of the learners could read the message. The learners said they did not know what the language was. Then she asked the learners how they feel when they do not understand a language. The learners told the teacher that when they do not understand a language they get bored, lose interest or they just do not care. By doing this, Jean started by creating an environment in which learners knew that the teacher was aware of how lack of knowledge of a language can affect the process of learning. Jean therefore started by using strategies that create a positive learning environment for the learners.

Jean then distributed work to be done in groups. She divided the class into nine groups. There were three different activities, so three groups worked on the same activity but at different positions in the classroom. The three activities involved finding the value of a house, the value of a car and the value of an investment. The value of each was a six digit number. Each digit was given in a radical form, like $\sqrt{4};\sqrt[5]{32}$ and learners had to discuss in their groups what the value of each digit was. Then each group had to report their answer to the whole class and explain how they had obtained their answer. There was discussion in small groups and then the discussion moved to whole-class discussion. Jean here was using interactive instruction as well as varied instructional settings in the classrooms.

During the discussion of the meaning of the exponent ½ one learner approached the teacher and showed her something on his calculator. Jean first discussed with the learner and then brought this discussion to the attention of the whole class. In this case the teacher moved the setting from an individual discussion to whole class discussion.

There was a lot of interaction between the teacher and learners as Jean used questioning to develop the lesson. Jean used different types of questions. She asked questions that required the learners to explain how they had arrived at a solution. She asked questions like '*Can you explain what it means?'; 'How did you get that 2?'; 'Why are you using the square root?'; 'What must I do?'* The teacher asked for meaning of terms: '*What does cube root mean?', 'What does ½ mean?', 'What is the answer for 81* x *0,5?*'. Often she rephrased the questions when learners did not respond or when the responses were wrong.

In explaining square roots, cube roots, fourth and fifth roots Jean used different forms of representations. In explaining the cube root she said, "*cube root of 8 means*" and she wrote $\sqrt[3]{\square\times\square\times\square}$ on the board. Then she expanded on this by saying '*We want a number to place in the boxes. It is the number that you multiply three times to give us 8*'. Then she inserted 2's in the spaces. Jean then used another example to explain the cube root. She used $\sqrt[3]{512}$ and wrote $\sqrt[3]{512} = 8 = \sqrt[3]{8\times 8\times 8} = 8$. She went on to explain the fourth root by writing ($\sqrt[4]{\square\times\square\times\square\times\square}$) and saying '*It means there must be four numbers inside'*. The fifth root was explained in the same manner. The teacher said, '*We need the same number 5 times',* and wrote $\sqrt[5]{\square\times\square\times\square\times\square\times\square}$. '*We need a number that can fit in there'*. In explaining these mathematical concepts the strategies that the teacher was using were multiple representations and use of informal mathematical language.

Jean then proceeded to explain the relationship between fractional exponents and root signs. At first learners were confused by how to read the root signs. When $a^{\frac{1}{3}}$ was converted to the root form, $\sqrt[3]{a}$, learners read this as '3 to the square root of a'. Jean explained how to convert from fractional exponents to root signs and how to read it mathematically. She used only unity fractions where the numerator is one, like ½; $1/3$; ¼; and 1/5 . Then she gave a generalisation for converting from a fractional exponent to root signs. She said, '*If it starts with a 3 then it is the cube root*' and wrote $a^{\frac{1}{3}} = \sqrt[3]{a}$. The teacher wrote, read and then asked learners to read the mathematical expression aloud. In this part the teacher was using different modes of language which are listening, speaking, reading and writing.

Lesson 2

In this lesson Jean developed the meaning of a negative exponent. The discussion during this lesson was a whole-class discussion with learners writing their explanations and simplifications on the board. Each time a learner had a different idea or opinion, Jean encouraged the learner to share the idea with the rest of the class and to write it on the board. Jean asked all questions in English and did all her explanations in English but the learners explained their work in their home language. Often as learners wrote on the board they would speak softly to the teacher, in their home language, as though asking for approval. The teacher would always respond using English. From the manner in which learners switched codes and the frequency with which it happened, it can be assumed that code-switching was an acceptable strategy in Jean's classroom.

The meaning of negative exponents was explained through the use of different procedures in the simplification of proper fractions. Jean used different representations and different methods to simplify $3^2 \div 3^3$. She led the learners by using a lot of scaffolding questions, leading learners to change 3^2 to 9 and 3^3 to 27 and then simplifying to the simplest fraction, 1/3. She then used a different form of the expression by writing $3^2 \div 3^3 = \dfrac{3\times 3}{3\times 3\times 3}$ and then used the cancellation method to simplify the expression to 1/3. In the last method, Jean led the learners, again by using many scaffolding questions, to the use of exponential laws in simplifying $3^2 \div 3^3$. This time learners were reminded of the law of exponents when dividing expressions that have the same base. This led to the result 3^{-1}. Jean then explained that they had just shown that $3^2 \div 3^3$ can be written as 1/3 or it can be written as 3^{-1}. She went on to explain that the numbers 3^{-1} and $\dfrac{1}{3}$ must be the same but written in different representations. The different procedures that were used are shown below.

$$\frac{3^2}{3^3} = \frac{\cancel{9}}{\cancel{27}} = \frac{1}{3} \qquad \text{OR} \qquad \frac{3^2}{3^3} = \frac{\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}\times 3} = \frac{1}{3} \qquad \text{OR} \qquad \frac{3^2}{3^3} = 3^{2-3} = 3^{-1}$$

Jean, in this case, was using the strategy of multiple representations and scaffolding.

Some of the scaffolding questions that Jean used were low order questions where learners were required to remember some skills or facts. These were questions like *what is* 3^3 *?* , *'How much is this? Is this simplified?', 'What is left at the top?', 'What is left below?'*

Other questions were of higher order where learners were required to explain or demonstrate their work. Jean asked questions like '*How is this different from this one?', 'What does this mean?', 'Can you explain that?' 'How do you explain the one that is positive and the one that is negative?' 'What is the problem here?' 'Why are these not giving the same answer?*

Some learners thought they could combine the methods to simplify the expression. To clarify this, Jean allowed two learners to come to the board and show what they thought were correct procedures for simplifying the expression. One leaner wrote

$\boxed{\begin{array}{l} \cancel{3}^2 \div \cancel{3}^3 = \\ 2-3=-\frac{1}{3} \end{array}}$ and then he changed it to $\boxed{\begin{array}{l} \cancel{3}^2 \div \cancel{3}^3 = \\ \frac{2}{3}-\frac{3}{3}=-\frac{1}{3} \end{array}}$, still cancelling the base 3's. As he

wrote this other learners mumbled, indicating disagreement, but the teacher said, '*Let us not make him uncomfortable. He is trying to deal with this.*' Another learner came to the board and wrote his simplification as $\boxed{\frac{2-3}{3}=-\frac{1}{3}}$.

Jean then allowed time for whole-class discussion on what these learners had written and concluded by emphasizing how the laws must be used. By allowing these learners to write up their simplification even after the correct answer had been obtained, the teacher was respecting each learner's contribution. She created a positive learning environment by saying, '*Let us not make him uncomfortable. He is trying to deal this.'*

Jean then used another example, $5^2 \div 5^4$ and let learners simplify this. One learner wrote:

$\boxed{5^2 \div 5^4 = \frac{2}{5} \quad ; \quad \frac{4}{5}-\frac{2}{5}=-2}$ leaving both 2/5 and -2 as solutions.

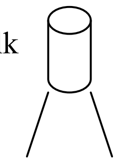
Jean then directed the learners to refer to previous work where the meaning of 3^2 was understood to be 3×3 and encouraged the learner to use this on the new expression $5^2 \div 5^4$. By scaffolding the process, Jean led the learners to express the expression as $\frac{5\times5}{5\times5\times5\times5}$. The learner then wrote the answers as 5^{-2} and 5^{2} and the learner still thought the two answers were the same value. Jean then guided the learners to convert 5^{-2} and 5^{2} to other forms so that they could see the difference in the two values. Learners correctly converted the 5^{2} to 25 and 5^{-2} to $\frac{1}{25}$ but still said these represented the same number. Learners kept on saying '25 and $\frac{1}{25}$ were the same thing and that they are just different representations'. At this point it was clear that learners had a misconception of the meaning of the term 'different representations'.

Jean then used an example from learners' real world. Jean said, '*Let's go to a smaller number. Now this is about lunch ok. This is my loaf of bread. I am giving you 2 loaves wena [you]. I am giving rona [us] ½ loaf. Are these the same?'* At that point the learners said, emphatically, '*No*'. It was at this point that the learners realised that 25 and $\frac{1}{25}$ were not the same. At this point Jean emphasized that when a different representation is used the value of the expression should not be changed. It was the use of real-life examples that Jean used here to make the learners understand the concept of different representations.

<u>Lesson 3</u>

Jean started the lesson by recapping the relationship between rational exponents and their radical equivalent forms. The fractional exponents were all unity fractions (fractions with one in the numerator). Jean demonstrated a different way of explaining root signs. In lines 6-17 Jean explained the other way of looking at root signs as:

6. *Jean:* *My way of understanding the root sign is if you have a maize crop, the maize crop will come out as one stalk but underneath you have many roots. Right? I have made a diagram here to say that if that is your maize crop (Jean draws a diagram), square root means there are two roots underneath but we are taking one of them. Cube root, there would be how many roots?*



7. *Ls:* *Three*

8. *Jean:* *But we are taking how many roots?*

9. *Ls:* *One.*

Jean then used the strategy of scaffolding to find the cube root of 27 and fifth root of 32. She proceeded as:

6. *Jean:* *----- How many roots?*

7. *Ls:* *Three*

8. *Jean:* *But how many roots are you taking?*

9. *Ls:* *One*

10. *Jean:* *So the cube root of 27 is?*

11 *Ls:* *3*

12. *Jean:* *In the case of* $\sqrt[5]{32}$ *you are saying there are how many roots?*

13. *Ls:* *5*

14. *Jean:* *But you are taking how many of them?*

15. *Ls:* *One*

16. *Jean:* *But you are taking one of them. Which is what?*

17. *Ls:* *2.*

In the discussion quoted above, Jean and the learners were using informal mathematical language to develop the meaning of the radical form of exponents. Jean and learners were using their own agreed form of mathematical language in the development of the concept of root signs.

Jean also used a series of questions. Some of the questions she used required one word answers but more of the questions required learners to explain or demonstrate their work or solutions. Jean asked questions like, '*Now what does that mean? Who can give the meaning? What does this one require us to find? What do you think it means when you have square root of a square root? How do you work it? How do you work out bracket of a bracket?* The lesson was a whole-class discussion that was guided by the teacher's use of different types of questions.

Jean also used a lot of different representations in this lesson. She used different representation for exponents, root signs and fractional exponents. She also used different representations for numbers. The number 8 was represented as 2^3; 64 was represented as 8^2 or 4^3 or 2^6. The number 2^8 was represented in four different ways as shown below: $\left((2)^2\right)^4 = (2\times 2)^4 = (2\times 2)(2\times 2)(2\times 2)(2\times 2) = 2^8$. So Jean used the strategy of multiple representations in this lesson.

Jean used gestures like using hands to show terms that must be in a bracket. When dealing with root signs, she often pointed to the number in the arm of the root sign to emphasise where the number of roots is represented in the root sign. When talking about cube root of 8, she would point at the 3 in the arm of the root sign.

In this lesson, the strategies that Jean used were interactive instruction, multiple representations, and scaffolding.

Lesson 4

The aim of the lesson was to develop the meaning of a zero exponent. The lesson started with learners working in groups on simplifying the expression $2^3 \div 2^3$. The learners were told to use laws of exponents. While the learners were working in groups, Jean moved around explaining to each group. Jean explained in English. Then each group gave its final result. The teacher encouraged all answers to be given. There were actually six different answers from the groups. The answers given were $2^1; 1^3;\ 1^0;\ 5^1;\ 8^1;\ 2^0$. Jean did not immediately accept the correct answers nor did she reject the wrong ones but she wrote down all the answers from the groups. Her expressions did not give away which answers were correct or which ones were incorrect. She accepted all learners' contributions and then followed with a whole-class discussion on how each answer had been obtained.

Jean asked each group to explain how they had obtained their answers. As learners explained their solutions, Jean insisted that they identify the laws that they had used. One group explained how they got the answer 2. This group explained that they said 2 to the power 3 minus 3, giving them 2 to the power zero which is equal to 2 to the power 1. The group that got 1^0 explained that they had said '2 divided by 2 is 1 and 3 minus 3 is zero'. This group had divided the bases and then subtracted the exponents. As the groups explained their solutions, the teacher used questioning techniques that required learners to explain the process of their simplification. The teacher used questions like: *What does the exponent mean? You are repeating the base so many times, so when it is zero are we repeating the base? When it is 1 are we repeating the base? Would this then mean the same thing? I want you to explain how you moved from* 2^0 *to* 2^1.

During the explanations there was a heated discussion and disagreement on whether 2^0 and 2^1 were the same number. Some learners thought that since $2^0 = 1$, the meaning of the zero exponent could be transferred to mean zero can be 1. One learner shouted at the top of her voice saying '*Zero can represent 1, it's the law*'. At this stage learners took turns to come to the front to explain their points of view and go back to sit down. Some would erase what had been written by previous learners and write what they believed was the correct answer. One learner cancelled 2^1 and replaced it with 1. There was a lot of discussion and arguments in home language about whether zero can be equal to 1. One learner used a calculator as empirical verification. This learner even moved around the classroom showing other learners what was on the calculator. She then wrote on the board: $2^3 \div 2^3 = 2^{3-3} = 2^0$ √, putting the tick herself. It was evident from the discussions that the meaning of power zero had not been well understood and was causing confusion as shown by how learners completed the statements as shown: $\begin{pmatrix} 1^0 = 1 = 1 \\ 2^0 = 2 = 2 \\ 3^0 = 3^1 = 3 \end{pmatrix}$

Jean then used a different example to help learners understand. She led the learners to see the pattern in the terms: $2^3; 2^2; 2^1; 2^0; 2^{-1}$. She asked learners to find the value of each term. The learners did this correctly except for 2^0. One learner insisted that 2^0 is 2 to the power 1, which is equal to 2. The teacher then explained the pattern in the value of the terms by saying that as the index decreased by 1 the value of the number decreased as well. Eventually the teacher said, '*The effect that 1 has on a number is it the same as the effect that 0 has on a number? If the effect is the same then we can argue 1 is the same as zero'*. This discussion went on for longer than expected and thus there was not enough time to discuss the other solutions. The discussion in small groups, the whole-class discussions, the use of home language and the use of different types of questions supported interactive instruction in this lesson.

From the four lessons analysed the strategies that Jean used in her classroom were multiple representations, scaffolding, code-switching by learners, interactive instruction and language modes.

STRATEGIES USED BY DAVID

David was teaching a class in which there were six different home languages, namely IsiZulu, Setswana, Xhosa, Sotho, Afrikaans and English. David's first language is Afrikaans. All his schooling and college studies were done in Afrikaans. David is however fluent in English and uses both languages at home. He has always taught in Afrikaans. Only in the past six years has he used English as the LoLT. At this school Afrikaans, English and Zulu were offered as subjects. There were only two learners in his class whose home language was English. In contrast with Jean, David did not share a home language with most of the learners. David was therefore chosen so that the strategies that a teacher who does not share a common home language with most of the learners uses could be explored and analysed.

All the lessons observed and video-recorded focused on exponents. In the rest of this chapter the strategies that David used in the lessons are described.

Lesson 1

The learners had been given homework in the previous lesson. The first part of the homework was for learners to translate the mathematical statement $a^0 = 1$ into ordinary language. Learners had been instructed to use their own words to translate the statement. The second part of the homework was to simplify the following exponential expressions:

$x^{-3}; \quad 2y^{-9}; \quad 3t^{-6}; \quad \frac{2}{r^{-2}}; \quad \left(2p^2\right)^4 \times \left(3p^2q\right)^{-2}$

The lesson started with David asking for four learners to write their English translations to the mathematical statement $a^0 = 1$ on the board. The following translations were written on the board by four learners:

1. *When the exponent of a base is zero the answer is 1*
2. *Anything to the power of 0 is equal to 1*
3. *If a power is divisible by 0 it automatically equals to 1*
4. *Any number to the power 0 is equal to 1.*

David asked learners to read out their translations. Then he read the translations one at a time and discussed with the learners if the translation could be accepted. As the teacher read the translations he would point to the base and exponent as he read each word in the translation. The first two translations were readily accepted. Then David read the third translation and the learner responsible for it changed his mind and said that what he had written did not make sense. The learner then changed the translation to 'If the exponent of a power is zero the answer equals 1'. The strategies that David was using here were interactive instruction and different modes of language (writing, speaking, hearing and verbalizing). David also used gestures to match mathematical terms to the written and spoken word. He pointed to the exponent when he said *exponent* and pointed to the base when the word *base* was spoken.

David then raised a question on the fourth translation. He asked the learners if the base could be any number. Most learners said yes, only one learner said 'no'. David asked learners to check, using their calculators, the value of 0^0. Learners' calculations resulted in two different results. Some calculators showed the result as a 'maths error' and other calculators showed the result as 'undefined'. David asked the learners why some calculators say 'maths error'. This question led to the following explanations from learners about what they thought were the properties of the number zero (lines 35, 37, 55, 57, 58,and 60):

35. L4: Because zero is not classified as a number
36. L4: not classified as a whole number
55. L1: It does not have a value.
57. L1: Naught is not a whole number because it does not have a
58. [L]: It is a place holder.
60. L1: It is alone

David used probing questions and acknowledged contributions from all learners. This helped to create an environment in which learners felt their contributions were considered and valued. This discussion made it possible for the teacher to pick out and deal with misconceptions that some learners had about zero and whole numbers. David then told the learners that the issue about zero would be dealt with later on and he proceeded with the exponential expression a^0 and the effect of a zero base. Then David, by using a series of scaffolding questions, led the learners to the formulation of the mathematical representation of the exception to the rule 'any number to the power 0 is equal to 1' i.e. $a^0 = 1,\ a \neq 0$.

The lesson then progressed to marking the rest of the homework. The teacher asked for volunteers to come up to the board and write down their solutions on the board. Each learner had to explain how he or she obtained the answer. The teacher instructed the learners to identify and read aloud the exponential law that they had used on each simplification. The simplifications of $2y^{-9}$ and $3t^{-6}$ were both incorrect. David emphasized the importance of identifying the exponent and its corresponding base. He used informal mathematical language to match exponent with the appropriate base. He used statements like '*The law must apply where it belongs'; 'This negative belongs to?' 'Minus 6 belongs to the t'* As the learners tried to explain their work their English and grammar were not correct, but David accepted the use of informal language by the learners. Learners used explanations like '*It lets us the way to the law*'; '*Sir I think it is to explain how the law is done'; 'We are not allowed to multiply powers of the same base*'. This indicated that David was focusing on the development of mathematical concepts rather than on the correct grammar. Thus the teacher used strategies of interactive instruction, informal mathematical language and language modes by letting individuals write and explain their work and then turning to whole-class discussion.

Lesson 2

The lesson started by David giving two learners a dictionary each. There had been different misconceptions of what a whole number is from lesson 1. David asked the learners to look up the meaning of whole number from the dictionaries and read the definitions to the whole class. The two learners read the definitions as:

1. *A whole number is a number with no fractions attached.*
2. *Whole numbers are natural numbers or integers.*

David repeated the two definitions as read by the learners. He then moved on to recap where the lesson had ended the previous day. One learner had simplified $2^4 \times 3^{-2}$ as 6^2. The learner had multiplied the bases (2 and 3) and then added the exponents (4 and -2). David used the scaffolding strategy to guide the learner into correcting his mistake. He used questions like: *Which law did you use? Can you use that law? Why not? What did* *we say about multiplying bases and exponents?* This then cleared the misconceptions from the previous lesson.

David also used the strategy of multiple representations to correct the simplification of $2^4 \times 3^{-2}$. He asked learners to use their calculators, read out their solutions and apply the 'SD' button that converts fractions into different forms. The three forms for the answer given by learners were all written down i.e $\frac{16}{9}$; 1,7777 and $1\frac{7}{9}$.

The lesson then progressed onto simplifying exponential expressions by converting bases into prime bases. The lesson was based on whole-class participation with the teacher using many questions to develop the lesson. David scaffolded the work by first discussing the meaning of the terms *converting* and *prime* separately. David started by making sure that learners knew the meaning of the term '*convert*'. He asked questions '*What does it mean?' 'What do you understand about converting to prime numbers?' 'What do we do?' 'What does converting mean to you?' 'Do you agree with him?'* As he questioned the learners, David also rephrased his questions.

The discussion then moved to the meaning of prime numbers and what the first prime number is. Again David used questioning (lines 31-44) to scaffold and develop this section of the work.

31. David: Now what is prime?
32. L5: An odd number
33. David: So prime numbers are odd numbers? What are prime numbers?
34. L6: No factors except itself and one.
35. David: Prime numbers have two factors; itself and one. What is the first prime number?

39.	*David:*	*Listen to the definition of prime numbers. The definition is 'It is a number with two factors; one and itself. Now I am asking what is the first prime number?*
40&41.	*Ls:*	*1, 2*
42.	*David:*	*If you say 2 tell me why.*
43.	*L8:*	*Because it can be divided by one and itself.*
44.	*David:*	*What about 3? (David shows the factors of 1, 2 and 3 and circles the prime factors. David then emphasizes that 1 is not a prime number).*

David then demonstrated converting 44 into the product of prime numbers, using questioning to scaffold the work. A question from one of the learners led David to show two other methods of breaking down 44 into its prime factors. The number 44 was broken down in the following three different ways.

44	44	2|44
2 22	11 4	2|22
11 2	2 2	11|1

David's use of different ways of breaking down 44 showed how he used the strategy of multiple representations.

David then led learners to simplify the expression $\dfrac{12^{n+1} \times 9^{2n-1}}{36^{n} \times 8^{1-n}}$ by scaffolding the questions. He started by breaking down the bases 12; 9; 36 and 8 into prime bases circling the prime factors.

12	9	36	8
4 3	3 3	6 6	4 2
2 2		2 3 2 3	2 2

As he did this, he repeatedly asked why they were converting to prime bases. David used gestures to emphasise the base and the corresponding exponents, making sure learners could see that the value of the bases had not been changed. David said, '*Did I change the value (pointing to* $2^2.3$*)? The exponent belongs to the 12 therefore the bracket (gestures bracket with his hands)*'.

Then he guided the learners, by asking a series of questions, which terms could be combined and what law was used in each case. He then scaffolded the algebra involved, in adding and subtracting like terms, so that exponential laws could be used. David did not accept learners' answers of just 'law 1' or 'law 2'. He always said 'Read the law, translate the law'. For each stage in the simplification process the teacher asked for the appropriate law to be used and probed learners to explain why it was the appropriate law to be used. He kept the learners engaged by the frequent use of questions of different levels of cognitive demands. When the expression had simplified to $2^{3n-1}.\ 3^{3n-1}$, David asked learners to look for a law that could be used to simplify the expression to one base. The learners did not seem to remember this law and the teacher wrote $a^m \times b^m = a^m b^m$ and asked if he could write it as $(ab)^m$. David explained this law and proceeded on to complete the simplification and wrote $2^{3n-1}.\ 3^{3n-1} = 6^{3n-1}$.

The solution to the simplification was:

$$\frac{\left(2^2.3\right)^{n+1} \times \left(3^2\right)^{2n-1}}{\left(2^2.3^2\right)^n \times \left(2^3\right)^{1-n}}$$

$$= \frac{2^{2n+2}.3^{n+1} \times 3^{4n-2}}{2^{2n}.3^{2n} \times 2^{3-3n}}$$

$$= \frac{2^{2n+2}.3^{5n-1}}{2^{-n+3}.3^{2n}}$$

$$= 2^{3n-1}.3^{3n-1}$$
$$= (2.3)^{3n-1}$$
$$= 6^{3n-1}$$

David then put up another expression to be simplified: $\frac{10^n \times 25^{n-1} \times 2}{50^{n+1}}$. This time learners took turns to come to the board and apply one law to either numerator or denominator, starting by breaking down the bases to prime bases. There was only enough time for the simplification to go as far as $\frac{5^{3n-2}}{5^{2n+2}}$.

Lesson 3

The lesson started with David recapping the point at which the previous lesson had ended. A learner completed the simplification of the previous problem on the board. Learner wrote:

$$= 5^{3n-2-(2n+2)}$$
$$= 5^{n-4}$$
$$= \frac{5^n}{5^4}$$

This was followed by a whole-class discussion of each step, guided by David asking questions of the laws used and why each law was used, asking learners to read out the law and translate the laws into their own words. David kept the learners engaged in the lesson by the number of questions that he used during the lesson. The questions asked included questions like *'From this point to this point which law?' 'What does the law say?' 'How did she get the minus 4?' 'Okay what did she do here? Explain the removal of brackets.'*

The lesson then moved to marking of homework from the previous day. This started with one learner putting her proof of the law that says $\frac{1}{a^{-m}} = a^m$. She wrote

$$\boxed{\begin{aligned}\frac{1}{a^{-m}} &= \frac{1}{a^{-m}} \times \frac{a^m}{a^m} \\ &= \frac{a^m}{a^0} \\ &= \frac{a^m}{1} \\ &= a^m\end{aligned}}$$

David then asked the other learners if they agreed with the proof, but there was no response from the other learners. He then asked how many of them did not know what was going on. David then explained each step of the proof, starting with why the expression was multiplied by $\frac{a^m}{a^m}$. The teacher used the example of equivalent fractions: $\frac{1}{3} = \frac{1}{3} \times \frac{5}{5} = \frac{5}{15}$, emphasizing that multiplying numerator and denominator by the same number had no effect on the value of the original fraction. He also reminded learners that $a^0 = 1$. Then marking of the rest of the homework continued. Learners wrote solutions on the board; followed by whole-class discussions of the solutions. The expressions $\left(2t^{-4}\right)^2$ and $\frac{4}{m^{-7}}$ were correctly done and learners understood these. There were problems with the expression with $\frac{2}{s^{-3}}$ which was surprising because learners seemed to have understood the previous work. The expression $\frac{2}{s^{-3}}$ was simplified first as:

$$\boxed{\begin{aligned}\frac{2}{s^{-3}} &= \frac{2}{s^{1/3}} \\ &= \frac{2}{s^3}\end{aligned}} \quad \text{and then changed to} \quad \boxed{\begin{aligned}\frac{2}{s^{-3}} &= \frac{2}{s^{-3}} \times \frac{s^3}{1} \\ &= \frac{2}{s^0} \quad s \neq 0,\ s = 1 \\ &= \frac{2}{s}\end{aligned}}$$

Then David used another example to show how to simplify without changing the value of the expression. He used the example 2/3 and multiplying it by 3, doing cross cancellation and ending up with 2 and compared it to taking 2/3 and multiplying it by 3/3 where cross cancellation still ends up with the value 2/3.

$$\frac{2}{3} = \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{1} = 2 \quad \text{versus} \quad \frac{2}{3} = \frac{2}{\cancel{3}} \times \frac{\cancel{3}}{3} = \frac{2}{3}$$

Here David used numerical verification of equivalent fractions to develop the appropriate procedure for exponential expressions.

Three more solutions were written on the board by learners, followed by whole-class discussion of the solutions. The teacher kept the learners engaged by asking questions about which law was appropriate, why the law was appropriate and probing for alternate solutions. The learners by then were offering and asking about alternate solutions without the teacher prompting them. One learner asked if he could simplify the bracket of the expression $\left(\frac{a^2b^3}{b^2}\right)^2$ before squaring the bracket. Another learner came to show the teacher the method he had used to simplify the expression. Throughout the lesson the teacher asked questions like, '*Why are we changing bases?'; 'So we can do what?', 'Why do we want the bases to be the same?', 'Do the laws work when the bases are not the same?', 'Why did she multiply by* a^m *over* a^m *?*', '*My questions is did she change the value?' "Is* $\frac{1}{3}$ *the same value as* $\frac{5}{15}$ *?', 'Why do we do this?, in exponents why do we do this?', 'Why do we want the bases to be the same, why must the bases be the same?' 'Do the laws work when the bases are not the same?'*

The strategies that David used in his lessons were interactive instruction, multiple representations, scaffolding, informal mathematical language and different language modes.

CONCLUSION

This chapter has presented descriptions of the strategies that Jean and David used in their classrooms. The next chapter explores these strategies and puts forward strategies that could be considered as beneficial in multilingual mathematics classrooms.

CHAPTER 6

ANALYSIS: EXPLORING STRATEGIES

Chapter 5 focused on the descriptions of the strategies that Jean and David used to promote learners' development in mathematical proficiency as they learned new concepts and skills on exponents. As mentioned earlier, the focus of the study was to explore the instructional strategies that mathematics teachers in different multilingual classrooms use. The study was guided by the following research questions:

- What instructional strategies do teachers in multilingual classrooms use in order to support learners' development of mathematical proficiency in algebra?
- How do they use these instructional strategies to support mathematical proficiency in algebra?

This chapter explores these strategies. By definition, to explore is to look into something carefully, to examine carefully, to analyse or to traverse a region for the purpose of discovery (Webster Encyclopedic dictionary, Merian Web, 1976). The term 'explore' is also often linked to travel where people travel to a region previously unknown or where little is known in order to learn more about the area. Multilingual mathematics classrooms in South Africa are very common, yet little is known about how teachers can best support the development of mathematical proficiency in algebra. It is with this analogy that I use the term explore in this study. By exploring these strategies, some indications can be gained as to **why**, **how** and **when** the strategies are beneficial in the multilingual mathematics classroom.

The strategies described in chapter 5 are Multiple Representation; Interactive Instruction; Code-Switching; Language Modes; and Scaffolding. This chapter looks in detail how, why and when these strategies were used by the teachers and compares this with what some researchers have argued or have posited as effective instructional strategies.

MULTIPLE REPRESENTATIONS

One of the strategies that both Jean and David used was the use of multiple representations. McKendree et al (2002) describe a representation as a structure that stands for something else, a word for an object, a diagram for an arrangement or a picture for a scene. He further argues that a representation can support or hinder problem solving. In what follows I look at how Jean and David used multiple representations to support learning.

Jean used multiple representations to develop the meaning of exponents. She encouraged learners to start by using the representations that they were used to rather than getting bogged down with representations they were not yet confident with. In an excerpt below from lesson 2 (Appendix D2, lines 1 -14) Jean explains to the learners the importance of understanding the meaning of new mathematical representations.

1. Jean: *…. Sometimes maths can look like Spanish or Italian.*
3. *Jean But we really want to understand the symbols that we are using. Take the number* 3^2. *I have seen people express it in two ways. Some are saying* $3^2 = 9$, *some are saying* $3^2 = 6$. *The person who introduced exponents wanted to use a symbol way of expressing the numbers that we are used to. So,* $a^n = a \times a \times a \times ....n^{th}$. *So it is* a *n times. For* a^3 *we are used to:* $a \times a \times a$. *They are representing* $a \times a \times a$ *in a different?*
5 *Ls: way.*
6. Jean*: What we know is now represented in another way*

12. Jean*: Remove the hidden message—the hidden message is this n means so many of this multiplied together. So when you see* a^2 *go back to what you know. So* $3^2 = 3 \times 3$ *which is equal to what?*
13. *Ls: 9*
14. Jean*: Because this is very common to you.*

Here Jean was emphasizing the use of a representation that has meaning to the learners and a form that learners can manipulate. Jean used different representations to help develop an understanding of the meaning of exponent. Paul Cobb (1994) says that symbols used in mathematics are also instruments for attributing meaning in new contents. Jean used different representations to connect the already familiar meaning (line 4) of $3^2 = 3 \times 3$ to the new meaning in $a^n = a \times a \times a \ldots . n \text{ times}$. It can be argued that by encouraging learners to use representations that they are used to, Jean is aware of how some representations can hinder learning when learners are not confident of new representations.

Jean also used multiple representations to develop the meaning of terms like square root, cube root, fourth root as shown in the excerpt below from lesson 1 (Appendix D1, lines 36, 40, 41, 111 and 112):

36. Jean : … Cube root of 8, can you say what it means?

40. Jean : The meaning here is we are all agreeing that we must have the same number multiplied three times (Jean writes $\sqrt{\square \times \square \times \square}$ and then inserts $\sqrt[3]{2 \times 2 \times 2} = 2$ and $\sqrt[3]{512} = \sqrt[3]{8 \times 8 \times 8} = 8$)

41. Jean : What will be fourth root? (Jean writes $\sqrt[4]{\square \times \square \times \square \times \square}$)

111. Jean: And if I write (Jean writes $a^{\frac{1}{5}}$) then it would be?

112. Ls: 5^{th} root of a (Jean writes $a^{\frac{1}{5}} = \sqrt[5]{a}$).

The use of three spaces (line 40) for cube root inside the root sign and four spaces (line 41) for the fourth root emphasised the fact that a number had to be written three times and four times respectively. McKendree et al (2002) argue that a good representation must capture exactly the features that are important, whilst Bell (1996) refers to this as manipulating it into a form which displays the desired property. In multilingual classrooms, the expression: ‘the number that multiplies itself three times to get 8’, as a way of explaining $\sqrt[3]{8}$, can be confusing. Learners can have difficulty understanding what the statement actually means. But by writing it down in mathematical symbols and showing the 3 or 4 spaces that must be filled can help learners understand. Here, Jean emphasized that in finding the cube root, the important feature is that the same number must be written three times inside the root sign (line 40) and for the fourth root the important feature is that the same number must be written four times inside the root sign. The use of the symbols as an alternative form of representation also reduces the dependency on language for understanding concepts (Vorster, 2008). This therefore, would be helpful in multilingual classrooms where learners are still learning English, the LoLT. Jean used multiple representations as a tool to regulate learners' thinking process and as a complementary way of learning and explaining (Cobb, 1993; Seeger 1998).

Jean extended this in lesson 3, line 78, where she wrote a number in different forms: Jean wrote

$$\left((2)^2\right)^4 = (2\times 2)^4 = (2\times 2)(2\times 2)(2\times 2)(2\times 2) = 2^8$$

Here Jean showed the same expression in four different ways, each showing an important aspect of the original expression. In the expression on the left hand side, the exponent 2 in the inner bracket emphasizes that 2^2 is 2×2 and then the exponent 4 on the outer bracket emphasizing that the (2×2) must be multiplied 4 times and the exponent 8 on the final expression emphasised that the 2 is actually multiplied 8 times.

Research results have shown the effectiveness of multiple representations. Brenner (1999), reports research results that indicated that diverse problem representations contributed to greater conceptual knowledge and enhanced problem solving, whilst Yerushalmy (1997) reported that multiple representation enhanced students' understanding of functions. Brenner (1999) further reports that good problem solvers tend to be sufficiently flexible in their use of relevant representational systems that they instinctively switch to the most convenient representation.

Now looking at how David used multiple representations in his lessons, David used different representations to develop procedural fluency. Procedural fluency is important in the simplification of expressions. In lesson 2, David probed learners to give him different forms for the value of $2^4 \times 3^{-2}$. Learners give the answers as $1,\dot{7}$; 1,777; $\frac{16}{9}$; $1\frac{7}{9}$ and 1,77. Though all the answers are correct, $\frac{16}{9}$ makes combination with the rest of the terms easier, rather than a result like $1\frac{7p^4}{9q^2}$. David wrote the final answer as: $\frac{16p^4}{9q^2}$. It is procedural fluency that helps give answers in most simplified forms. David also used multiple representations to achieve procedural fluency that was required to simplify exponential expressions. To simplify the expression: $\frac{12^{n+1} \times 9^{2n-1}}{36^n \times 8^{1-n}}$, the bases 12 ; 9 ; 36 and 8 must first be expressed in the form of prime factors. It is only when the numbers 12, 9, 36 and 8 have been written as products of prime numbers that the expression can be simplified. David emphasised this as shown in the excerpt from lesson 2 (Appendix D6, lines 65 -68)

65. *David:*	*In order to apply the law. If I want to apply the law the bases must be the same. Now in this case can I apply the law? (David reads the expression* $\frac{12^{(n+1)} \times 9^{(2n-1)}}{36^n \times 8^{(1-n)}}$ *) Can I apply the law?*
66. *L1:*	*No*
67. *David:*	*Because the bases are not the same. I am going to convert the bases to prime and see what happens. (David now breaks the bases and writes*

12 9 36 4

4 3 3 3 6 6 2 2

2 2 2 3 2 3

68. David: *(David now writes* $\dfrac{\left(2^2.3\right)^{n+1}\times\left(3^2\right)^{2n-1}}{\left(2^2.3^2\right)^n\times\left(2^3\right)^{1-n}}$*). The exponent belongs to the 12 and therefore a bracket (David gestures brackets with hands). Now can I apply the law? Read the first law.*

David also emphasized the importance of flexibility in representation by demonstrating the reverse process where numbers can be multiplied together instead of breaking them down. He showed this when he simplified $2^{3n-1}\times 3^{3n-1}$ to 6^{3n-1} as follows:(Appendix D6, lines 100 – 105).

100. David: *Now I want to write a base. I have two bases 2 and 3. Look for a law where I can write as one base. (David pointing to* 2^{3n-1} and 3^{3n-1}*)*

101. L11: a to the power m times b to the power m is equal to a to the power m b to the power m

102. David :(Writes: $a^m\times b^m = a^m b^m$ *). Can I do this? (Writes:* $\left(ab\right)^m$ *). The only time I can do this is when the exponents are the same.*

103. David :(Writes: $\left(2\cdot 3\right)^{3n-1}$ *). Can we go further? Now I can multiply 2 times 3*

104. L10: Yes sir

105. David: (Writes: 6^{3n-1} *). Laws must be read from left to right and right to left.*

In line 100 David instructed learners to find a law they could use to represent the expression in another form. In line 102 David showed the three different representations of the expression. The first emphasized the multiplication sign, the second emphasized the normal algebraic form and the third emphasised a law of exponents. In line 105 David emphasized the importance of reading laws from left to right and right to left. Developing this flexibility is what Brenner (1999) says good problem solvers do.

Another situation where Jean used multiple representations was in the development of the meaning of negative exponents and zero exponent. These two concepts can be confusing to understand, especially coming from the representation that a^3 means $a \times a \times a$, multiplying *a* three times. This representation then fails to explain a^{-3} and a^0. A different approach must be used to explain the meaning of these two concepts. Jean used different representations to develop the meaning of negative exponents. Jean started by leading learners to simplify $3^2 \div 3^3$ using the exact value of numerator and denominator as 9/27, then simplifying to 1/3. She then guided learners to use laws of exponents as follows (Appendix D2, lines 49 – 56):

49. *Jean: Now we can use the law* $a^n \div a^m$. *The law says* $a^n \div a^m$ *is the same as what? Can someone finish it for us?*
50. *L3:* a^{n-m}
51. *Jean: Now I am coming back here.* $3^2 \div 3^3$ *is equal to what according to the law?*
52. *Ls:* 3^{2-3}
53. *Jean: And what is 2-3?*
54. *Ls: -1*
55. *[L]:* 3^{-1}
56. *Jean: Can you see that we have just shown that* $3^2 \div 3^3$ *can be written as 1/3 or it can be written as* 3^{-1}.

In lines 49-55 Jean led the learners, by scaffolding the process, to simplify the expression ending with 3^{-1}. She then concluded in line 56, explaining that $3^2 \div 3^3$ can be written in two different ways.

David also used multiple representations to develop the meaning of zero exponents. David had assigned learners, as homework, to translate the exponential law: $a^0 = 1$ into words. The learners were required to explain what the symbolic representation meant. The translations (different representations) of the expression $a^0 = 1$ that the learners wrote were:

1. *When the exponent of a base is 0 the answer is 1.*
2. *Anything to the power 0 is 1.*
3. *If a power is divisible by 0 it automatically equals 1.*
4. *Any number to the power 0 is equal to 1.*

The four translations that were written on the board helped to expose wrong conceptions of the symbolic expression. Akkus, Hand and Seymour (2008) argue that by requiring students to explain we may promote and reveal the students' understanding of the concept. This assignment did expose a wrong conception of the symbolic expression as shown by the third translation. It also paved the way for promoting the learners' understanding because the discussion of point 4 lead to the inclusion of the exception to statement a^0=1, namely that $a \neq 0$.

Both Jean and David used different representations to promote learning in their classrooms. They used multiple representations to develop meaning and understanding of new concepts and to develop procedural fluency. Jean used different representations more in developing meaning and understanding, whilst David used different representations more in developing procedural fluency. It can be argued that multiple representations can be used for meaning construction and for developing procedural fluency as shown in Jean's and David's lessons. The strategy of Multiple Representation was not used in isolation but also incorporated the other strategies like scaffolding and language modes.

INTERACTIVE INSTRUCTION

A second instructional strategy that was used by Jean and David was interactive instruction. Interactive instruction is a strategy that provides opportunities for learners to interact with peers and with their teachers. Learners can learn from peers and teachers and develop abilities to organize their thoughts and rational arguments in mathematics. Interactive instruction is a strategy that relies heavily on discussion and sharing of ideas among the participants. The Saskatchewan Education argues that interactive instruction has advantages in that the interaction is often highly motivating for learners and that the opportunity to interact with others broadens the educational experience of the learners beyond the limitations of the traditional classroom and the knowledge and abilities of the individual teacher (SaskatchewanLearning, downloaded on March 5, 2009). http://www.sasked.gov.sk.ca/docs/policy/approach/instrappo3.html#strategies. Interactive instruction allows for a range of groupings and interactive methods such as discussion, cooperative learning groups, role playing, problem solving, debates and think-pair-share. However, according to Saskatchewan Learning (2001), there are challenges faced when using interactive instruction. One of these challenges is that the "success of interactive instruction strategy and its many methods is heavily dependent upon the expertise of the teacher in structuring and developing the dynamics of the group" (http://www.sasked.gov.sk.ca/docs/policy/approach/instrappo3.html#strategies.downloaded March 5, 2009). So the teacher has to be proactive and creative in maintaining group dynamics that keep discussions going and productive. The teacher must know how to control the groups, allow for arguments yet maintain the focus of the discussions, ask questions that encourage learners to explain, know how to encourage a learner who is struggling with language continue to explain, know how much time is spent in small group discussions before reporting to the whole class.

The interactive instruction methods used by both Jean and David were whole class discussion and cooperative learning groups. The excerpts that follow show how each of the teachers structured the dynamics of the learning groups and the instructional skills that they used to maintain and prolong discussions within the groups. The discussion below (taken from Appendix D5, lines 33-61), followed after David had asked learners to use their calculators to find the value of 0^0.

33. *L5: Maths error.*

34. *David: Maths error. Why does it say maths error?*

35. *L4: Because zero is not classified as a number.*

36. *David: Because zero is not classified as a number?*

37. *L4: Because zero is not classified as a whole number.*

38. *David: So it is not classified as a whole number?*

39. *L5: We should change that to a whole number.*

40. *David: Any whole number? I want to come back to your zero is not a number.*

43. *L5: To any whole number to the power zero.*

44. David*: Any whole number?*

45. *L1: Yes. Can't we say anything to the power naught is equal to 1 except naught?*

48. David*: Anybody else?*

51. *L2: Anything to the power of a whole number is 1.*

54. David*: It's the whole number issue again. Why is zero not a whole number?*

55. *L1: It does not have a value?*

56. David*: So if something does not have a value it is not a whole number?*

58. *L1: It is place holder.*

59. David*: Naught is place holder?*

60. *L2: It is alone.*

61. David*: Alone where?*

Here David's question of why the calculator read 'maths error' (line 34) created what Brodie (2005) referred to as 'dialogic' classroom. This led to the discussion of what type of a number zero is. The learner first said zero was not a number (line 35). When David questioned this by repeating the learner's statement, the learner changed to 'zero is not a whole number' (line 37). David's questions showed that the teacher was interested in what the learners thought about zero as evidenced by David's '*I want to come back to* *your zero is not a number*' (line 40). What David was doing here resonates with what Davis (1997) calls transformative listening. Transformative listening is the type of listening that is open to discussions and open to altering points of view. The learners had several different concepts of zero as evidenced by their responses: *zero is not a number, it is not a whole number, it does not have a value, it is a place holder, it is alone* (lines 35, 37, 55, 58, 60). David did not tell the learners what the correct statement should have been but kept using probing questions. David's use of listening skills and authentic questions led to a discussion which not only helped in explaining limitations on a^0, but also corrected previous misconceptions of zero.

David structured the discussion of the meaning of 0^0. He did not just say 0^0 was undefined but allowed learners to discuss why some calculators had 'maths error'. David structured the discussion by the use of several probing questions. Researchers who have looked at questioning in classrooms say authentic questioning has positive effects on learning mathematics (Nystrand et al., 1997; Brodie and Boaler, 2004). Brodie describes authentic questions as those questions that "convey teacher's interest in what students think and which serve to validate student ideas and bring them into the lesson" (Brodie, 2005: 45). Brodie (2005) also argues that authentic questions create dialogic classrooms and will increase learner engagement. Nystrand (1997) and his colleagues did a study on Grade 8 classrooms and found that authentic questions had a strong positive effect on achievement even though such questions occurred seldom.

In the excerpt above, David also used revoicing to sustain the discussion. In multilingual mathematics classroom situations a teacher can use revoicing to rephrase a learner's statement using the appropriate English term, or rephrase for clearer understanding, or rephrase to make the statement more mathematically precise, or to bring the discussion into the broader audience of the classroom (Enyedy , Rubel, Castellon , Mukhopadhyay , Esmonde and Secada , 2008). O'Connor and Michaels (1993) describe revoicing as a move by the teacher where the teacher repeats or rephrases a student's comment. They argue that revoicing amplifies the student's contribution and sometimes reformulates it in a more precise language, more technical terms but maintaining the student as the owner of the contribution. Moschkovich (1999) argues that revoicing can be used to support multilingual students' participation in mathematical arguments. She adds that a teacher can move past a learner's unclear utterances and use the learner's utterances to uncover the mathematical content in what the learner is saying. Brodie (2004) says that one function of revoicing is to make a student's idea the focus of the discussion which then facilitates other students' responses to it thereby sustaining the interaction in the discussion. If learners' ideas are kept as the focus of discussion, learners will feel their contributions are valued and they are thus motivated to contribute more. By doing this, the teacher creates an environment that supports learning.

In line 34, David revoiced the learner's contribution but also extended by asking for a reason for the statement. David's revoicing of the learners' statements in lines 36, 38, and 44 and rephrasing of learners' statements in lines 55 and 56 encouraged other learners to contribute as seen by the number of other learners who contributed to the discussion. Learners contributed to the discussion by offering their own understandings of the number zero.

In lesson 1 Jean encouraged and sustained the discussion of the meaning of the power ½ by using questioning technique as shown below : (Appendix D1, lines 42-77)

42.	Jean*:*	*Let us look at* $81^{\frac{1}{2}} = 9$. *You got 9? How did you get that?*
43.	*L5:*	*9 times 9 i 81. Lokusa i square root* (*It is the square root*)
44.	Jean*:*	*Why are you using square root?*
47.	*L3:*	*½ of 81.*
48	Jean*:*	*½ of 81? So we are saying 81 divided by 2? What does ½ mean?*
49.	*Ls:*	*½ is 0,5*
50.	Jean*:*	*What should I do? Multiply by ½ ?*
51.	*Ls:*	*Yes.*

52. Jean: *So this and this would be the same thing?* (Jean *pointing to* $81\times 0,5$ and $81^{\frac{1}{2}}$*)*

54. Jean: *If you say no why not?*

66. Jean: *Is it 9? 81 divided by 2 is 9?(writes 81/2 =?)*

67. Ls: *40,5*

70. Jean: *81 to the power ½ ? I am saying is it the same as 81* x *0,5?*

71. Ls: *Yes.*

72. Jean: *It's the same?*

73. Ls: *Yes.*

74. Jean: *Ok what is the answer for 81* x *0,5?*

75.` Ls: *40,5.*

76. Jean: *40,5? So is it the same as 9?*

77. Ls: *Yes , no.*

Jean used a lot of probing questions to get the learners to understand the meaning of the power ½. Her rephrasing of the question, "What does ½ mean?" (line 48) to "What should I do, multiply by ½ ?" (line 50) helped learners to think about the operations that were needed for dealing with the exponent ½. Instead of learners trying to explain the meaning of the exponent ½ they could say what the operation should be. This eventually helped to explain the wrong concept that you multiply the bases and the exponent. Jean did not just readily give learners the correct answers but she used a series of questions which generated a dialogic classroom as many learners got involved in the discussion. One learner even got a calculator and showed the teacher the result from the calculator.

During another lesson in Jean's classroom, the meaning of the power zero became confusing when one group of learners simplified the expression 2^{3-3} as $2^{3-3} = 2^0 = 2^1$. The excerpt below, taken from lesson 4, shows how Jean used questioning to promote and sustain the discussion of whether zero can be equal to 1.(Appendix D4, lines 24-47)

24. Jean: *Why are you saying zero is 1? Can you explain to us? Someone else in the group?*

25. L8: *If x to the power zero is equal to x to the power 1.*

26. Jean: *Give us the law that says that. You have got the paper, right?* (*Jean encouraging learners to refer to where the laws are written). We have law 1, law 2, law 3. Is it law 1 or 2 or 3? Which one is that? Or did you get it from a different paper? Here you know what an exponent means?*

29. Ls: *Yes*

30. Jean: *What does exponent mean? You are repeating the base so many times. So when it is zero are we repeating the base?*

31. Ls: *No.*

32. Jean: *When it is 1 are we repeating the base?*

33. Ls: *No*

34. L4: *Yes we are repeating the base one time.*

37. Jean: *Would this mean the same thing? (pointing to* 2^0 *and* 2^1*)*

38. Ls: *Yes, no.*

39. Jean: *I would like other groups to respond because you are saying u zero can be 1. Can you respond to that?*

40. L7: *No, never. We can't say zero is 1. You can't say zero is equal to 1. Never mam.*

43. Jean: *I want you to explain how you moved from* 2^0 to 2^1

47. L9: *u zero u representa 1* (zero represents 1). *It's the law. If there is zero it means there is one of them.*

The use of probing questions to establish meaning also resulted in creating a dialogic classroom. There was a lot of discussion, some with raised voices, on whether zero was the same as 1. Some learners used calculators to verify the arguments to their points of view and other learners used code-switching to let their points of view be known (line 47). One learner came to the board and erased 2^1 which he thought was incorrect and wrote 1 which he thought was correct. Some learners had misunderstood the law $a^0 = 1$ to mean the zero exponent could be simplified to 1. One learner even tried to explain this by saying 'If there is a zero it means there is one of them' (line 47). The teacher's questioning encouraged discussions, improved understanding of concepts as well as revealing misconceptions that some learners had. Some of these misconceptions could be because of the learners' lack of English language proficiency. For example, the meaning of learner's statement: ' …repeating the base one time' (line 34). I have had experience with learners who think repeating once is the same as writing once. While the teacher did not probe these statements further, their presence could be indications of poor language understanding.

Jean also encouraged learners' participation by allowing them to demonstrate their understanding in other ways other than explaining as shown below (Appendix D1, lines 36-40):

36. Jean: *What is the cube root of 8? Can you say what it means?*
(no response, one learner is hesitant) How do we find it? (still no response)

37. L4: (Explains something very softly to teacher)

38. Jean: Come and show us. How did you get the 2?

39. L4: (Writes on the board: 2x2=4
4x2=8.)
Must multiply 2 three times

40. Jean: Oh! We must multiply 2 three times. The meaning here is we are all agreeing that we must have the same number multiplied three times.

So the cube root of 8 is 2. The number that you multiply three times to give 8 is 2.

In line 36, one learner wanted to respond but was hesitant. Jean rephrased the question from ‘What is the cube root of 8?’ to ‘How do we find it?’ and on to ‘Come show us’ (lines 36 and 38). By rephrasing the question, Jean offered the learner a choice of how to demonstrate her knowledge. The focus shifted from explaining what the cube root is to explaining how to find it and on to demonstrating how to find it. The learner did demonstrate that she knew how to find the cube root (line 39). She was hesitant to explain but was comfortable enough to demonstrate. By offering the choice, Jean was able to get a learner to participate by her actions rather than by her explanations. Moschkovich (2002) argues that “Even a student who is missing vocabulary may be proficient in using mathematical constructions or presenting clear arguments” (Moschkovich, 2002:207). The learner could not explain but could demonstrate. It can, therefore, be argued that this is one way that opportunities for participation in multilingual classrooms can be encouraged, especially where learners’ fluency in English is still developing. In the excerpt above the learner did not have to rely on her English language proficiency to explain the meaning of cube root but demonstrated how to find it.

David also used questioning to help learners recognize their own mistakes and correct them. A learner had simplified $3t^{-6}$ as $3t^{\frac{1}{6}}$ and when she was asked what law she had used she was not sure which part was the law. The extract below (Appendix D5, lines 126-155) shows how the David used questioning to help learners see the difference between the law and the proof of the law:

126. *David: Can you explain to us which law did you use?*
127. L12: I used this one.
128. David: Read the law.
129. L12: (Learner reads the derivation of the law)

132. David: Which one are you reading? Read the law.

134. David: *Is that the law she read? How is that different from the one we have on the board? What did she read? Did she read the law? The things that come in front of everything else. What do those things do?*

145. L10: *It let us the way to the law.*

150. David: *So what did she read before we got to that? What comes in front of the law?*

155. L14: *Sir I think it is to explain how the law is done.*

David used the questions in different forms, rephrasing the questions and even using informal spoken Mathematics (Setati, 2002) in English language (lines 134 and 150) so that learners could understand and see the difference between the proof of the law and the law. The learner also responded using informal spoken Mathematics (line 145 and 155). The learner did not have the vocabulary or the correct grammar to explain, but her explanations indicated that she understood the difference between the proof of a law and the law itself.

The discussion above shows how both Jean and David created and sustained Interactive Instruction in their classrooms. They used questioning techniques; they allowed learners to contribute in different ways like showing and demonstrating and not just by explaining. Proficiency in the LoLT was not an issue, learners' lack of correct vocabulary and grammar was not a hindrance to their contributions. In Jean's classroom, use of code-switching was a norm, whilst in David's classroom grammatically wrong sentences were condoned. Both teachers showed interest in what their learners thought as evidenced by them not giving away the correct responses automatically, but encouraging learners to explain their solutions. In Jean's classroom one learner went to the board to write what he believed was correct, another used a calculator as basis for her argument. This can be evidence that learners were engaged in the discussion in several different ways.

LANGUAGE MODES

According to Thompson and Rubenstein (2000) students build their mathematical understanding as they process new ideas through language. Learners need to learn new mathematical vocabulary as well as know how to say or pronounce the word and be able to hear and recognize the word when it is used by the teacher or by other learners. The Virginia Department of Education (2004) recommends that instructional strategies for Limited English Proficient students must include integration of four language modes in the mathematics classroom. These four language modes are listening, speaking, reading and writing. Bell (1996) argues that the development of algebra requires the development of algebraic abilities in writing, reading, representing and manipulating and interpretation of symbolic expressions.

My study focused on strategies that teachers use in teaching exponents. Most of the words and terms that are used when dealing with exponents are specific to mathematics. The symbolic form of these terms is short and precise whilst the meaning in ordinary language often results in lengthy English expressions. An example is the mathematical notation $\sqrt[4]{16}$ is read the fourth root of 16 yet the meaning is 'the number that is multiplied by itself 4 times to give 16'. The written mode is much shorter than the reading mode. The four language modes are therefore important when dealing with work on exponents. Below I highlight the modes that Jean and David used in their classrooms.

Throughout all the lessons, when dealing with exponents, Jean always wrote down the expression as she said the word or soon after saying it as seen in the excerpt below (Appendix D1 lines 40, 41 and Appendix D3 line :133):

40. *Jean:* *Cube root of 8 is 2 (teacher writes* $\sqrt[3]{2\times2\times2} = \sqrt[3]{8}$ *. The number that you multiply three times to give 8 is 2*

40. Jean: The cube root of 512 . So in this case it will be (teacher writes $\sqrt[3]{512} = 8 = \sqrt[3]{8\times8\times8}$ *)*

41. Jean: *What will the fourth root mean? (teacher writes* $\sqrt[4]{\square\times\square\times\square\times\square}$ *)*

133. Jean: *Eight to the power minus 1 = one-eighth* (*writes* $8^{-1} = \frac{1}{8}$)

The 4 lines above show that Jean always said what the root was and immediately wrote it down, in different ways. In line 40 she even proceeded to expand on the meaning of cube root by using ordinary language to explain the meaning. By doing this, learners could hear how the expression must be read and they could see how it must be written. Jean's use of the expanded explanation helped in consolidating the meaning of cube root.

Jean proceeded to use different language modes when she dealt with the meaning of fractional exponents as shown in the excerpt below (Appendix D1 lines 94-127):

94. Jean: *For example a to the power half would be the square root of a (writes* $a^{\frac{1}{2}} = \sqrt{a}$ *)*

95. Jean: *a to the power one third will be what? (writes* $a^{\frac{1}{3}}$ *)*

96. L8: *Becomes* $a \times a \times a$

97. Ls: *square root of a*

102. Jean: *What root is this? (pointing to* $a^{\frac{1}{3}}$ *)*

103. Ls: *Square root of a. 3 to the square root of a.*

104. Jean: *If it starts with a 3 then it is cube root (writes* $a^{\frac{1}{3}} = \sqrt[3]{a}$ *)*

119. Jean: *81 to the power ½ . Let us write it with a root sign.*

120. L11: *Second root of 81*

121. Jean: *Second root ah!! Have you ever seen the 2?*

122. Ls: *No*

123. Jean: *Do we write the 2?*

124. Ls: *No*

127. Jean: So when we have not written the 2 it is square root

Lines 96, 97, 103 and 120 show that learners had not yet mastered how to read fractional exponents. Jean demonstrated how the fractional exponents must be read and the radical form of writing them (lines 94, 104 and 127). She also emphasized that for square root, the 2 is not written down. The importance of language modes was demonstrated in the line 103 where a learner reads '3 to the square root'. By asking the learners to read the expression, Jean was able to find if learners knew how to read the expression and then tell learners the correct way of reading the expression. Incorrect reading could cause confusion later on. Usiskin (1996) says that if a student does not know how to read mathematics aloud, it is difficult to register the mathematics. Thompson et al., (2000) report on the discovery by a teacher of mistakes her students were making only when she asked them to read the expression $\log_2 8$. Some students were reading it as log of 2 to the eighth. In this case the correct mathematics was not being registered by these students. They further argue that only by listening to their oral communication and by reading their mathematical writings are we able to diagnose and assess students' understanding

The importance of using different language modes in the learning process was also observed in David's class. Learners could say, in words, the restriction on the base for exponential statement $a^0 = 1$ but could not write down the mathematical form of this restriction. The excerpt below, from Appendix D5 lines 62-103, shows how David used language modes to help develop understanding of the new concepts.

62. L1: Anything to the power naught is 1 except naught.

63. David: The part with 'except naught', give it to me in a mathematical symbol. Just the part: 'exclude naught'.

64. L1: (learner comes to the board and writes $\begin{Bmatrix} x^0 = 1 \\ 0^0 \neq 0 \end{Bmatrix}$

65. David: (inaudible)

66. L1: *x stands for any number and naught to the power naught is not equal to 1 (learner corrects what he had written and changes the second statement to* $0^0 \neq 1$*)*

87. David: *… And there is the translation. There is an exception rule that says that what cannot be equal to naught?*

88. Ls: *The base.*

89. David: *The base cannot be equal to? Naught. So, from that let's translate to symbol. The base is not equal to naught. Translate that into mathematical symbol. Let's start with the base. What can we place in place of the base?*

90. L4: *x*

91. David: *In place of the base we put x, right? (David writes x on the board) is not equal to, what shall we write?*

92. L4: *The equal sign with a line.*

93. David: *Is not equal to?*

94. Ls: *Zero (David completes writing* $x \neq 0$ *and high lights it by drawing a rectangle around it .*

95. David: *In algebra what does x, a represent?*

96. Ls: *Anything; variable.*

97. David: *What does the x represent?*

98. Ls: *The base.*

99. David: *What does x represent?*

100. L4: *Variable.*

101. David: *Also a variable. Can I replace the x with a?*

102. Ls: *Yes.*

103. David: *(David replaces the x with a so that it is now* $a \neq 0$*) Now we have a to the power naught is equal to 1 but a cannot be equal to naught. Then we can come to your translation anything to the power naught is equal to 1 except naught.*

Though the learners could say the restriction in words as shown in line 88, it took about 15 rounds of conversation (lines 89-103) between the teacher and learners before the learners understood how to write the restriction. David had to scaffold several times (lines 89-103) to help learners understand how the restriction must be written mathematically, i.e. $a \neq 0$. For learners to communicate mathematics, they should know how to explain as well as write the mathematics mathematically.

David also used language modes to help learners correct their mistakes in algebra. Learners had made an error in subtracting negative exponents in lesson 3. To help the learners see the error, David continued as follows (Appendix D7 line 26):

> 26. *David: For r^8 divided by r^{-3} you got r^8. I would like to see how you got that (No response).Now look at this. When dividing powers with same base we subtract. r to the power 8 minus, from the law, minus 3 (writes 8 –(-3).*

In line 26, David reminded learners of the law to be used, then he used the numbers in the question saying 8 minus negative 3 and then wrote '8 – (-3)'. From my personal experience in teaching, when the statement '8 minus minus 3' is written down as 8 – (-3), learners are more likely to realize that this changes to '8 +3'. By writing down what he had said, learners could recognize where the error was in their simplification. David used the 2 modes of language (speaking and writing) to help learners correct their mistakes.

Not only did David write all his explanations, he encouraged learners to write their work down. It was normal practice in David's class to have solutions of homework written on the board by learners and then the learners would explain how they had obtained the solution. The excerpt below shows this. (Appendix D5, lines 177-184)

177.	*David:*	*Ok finish it. Write down all the laws used.*
	L10:	*(Writes the solution)*
	David:	*Which laws?*
178.	*L10:*	*Law 5.*

179. David: According to law 5, it is law number 5? Read law number 5.

180. L15: In brackets a times b to the power m equals a to the power m times b to the power m is equal to a to the power m, b to the power m.

David: (Writes $(ab)^m = a^m \times b^m = a^m b^m$) Ok is that the law you used?

181. L10: Yes sir.

182. David: Let's see. Did he apply the law exactly? How did you get 16?

183. L10: I used two laws, law number 3.

184. David: Ok read law number 3.

Here, not only did David ask learners to identify the laws (lines 177,179 and 180), but he asked them to write the laws down (line 177), and then asked them to read the laws aloud (lines 179 and 184). Line 180 shows the difference between reading out the law and its short written algebraic form. David wrote the law that the learner had read out aloud and asked for confirmation from the learners. David was emphasising the relationship between the law as read out by the learner and its written form. Thompson et al., (2000) argue that fluent use of mathematical terminology is necessary though not a sufficient condition for overall mathematics achievement. What David was doing here resonates with the importance placed on reading aloud by Usiskin (1996) and Thomson et al., (2000)

Both Jean and David paid attention to the way learners spoke the mathematics. The excerpts below highlight the speaking language mode as encouraged by the teachers (Appendix D1, lines 104-110).

104. Jean: So if it is $a^{\frac{1}{4}}$ then it would be what?

105. L9: 4 to the square root of a.

106. Jean: How do we say this?

107. L9: 4 to the square root of a

109. L3: Square root to the square root

110. Jean: Fourth root of a.

Here Jean was making sure learners knew how to read the rational exponents (lines 104 and 106). She corrected the learner's inaccurate answers (lines 105, 107 and 109) by rephrasing to the appropriate mathematical form in line 110, '*fourth* root of *a*'. Thompson et al., (2000) argue that immersion in mathematical language usage is necessary for developing fluency in the subject. It is important that learners know how to read mathematical expressions if they are going to succeed in communicating their ideas accurately.

Appendix D3, lines 24-41, shows how Jean corrected the reading of the square root sign.

24. *Jean: For this one, how many roots?(pointing to* $\sqrt{49}$ *)*
25. *Ls: 1*
26. *Jean: There is one?*
27. *Ls: Yes*
28. *Jean: Where is the 1? The number that is not written there (teacher pointing to* $\sqrt{\ }$ *)?*
29. *Ls: Yes*
30. *Jean: How do you say this sign?*
31. *Ls: Square root*
32. *Jean: Square root?*
33. *Ls: Yes*
34. *Jean: And we say that square root means?*
35. *Ls: 2*
36. *[L]: It is just a root because there is no number in the arm of the root sign.*
37. *Jean: How do we say this that (Jean writes* $\sqrt{25}$ *on the board)?*
38. *Ls: Square root of 25*
39. *Jean: You say the square root of 25?*
40. *Ls: Yes*
41. *Jean: We say square root but we don't write the 2. When there is a 1, we don't have 1 square root. We start from square root.*

Lines 30-35 indicate that learners were already familiar with the square root notation, ($\sqrt{\ }$), and how it is read. However, the introduction of other roots seemed to confuse this prior knowledge. Lines 25-29 indicate some learners thought there must always be a number in the arm of the root sign. In lines 37-41 Jean explained and re-emphasised the meaning of a root sign when there is no number in the arm of the root sign.

David always asked the learners to read out aloud the particular exponential law that was used in the solution and to read out their solutions. Every time a solution was completed on the board or a learner answered verbally, David would ask the learner which law she/he had used and then ask the learner to read out aloud the law (Appendix D5, lines 124-130).

124. *David:* *Who is responsible for c? Read your answer.*
125. *L12:* *It's 3t to the power 1 over 6*
126. *David:* *3t to the power of? 1 over 6? Which Law did you use?*
127. *L12:* *I used this one.*
128. *David:* *Read the law.*
129. *L12:* *(Inaudible)*
130. *David:* *No, read the law.*

Sometimes learners were reluctant to read but David still encouraged them to read, as shown from lesson 1, (Appendix D5, lines 110-112):

110. *David:* *Anyone else give me a law.*
111. *L10:* *(Writes on the board* $a^{-m} = \frac{1}{a^m}$ *)*
112. *David:* *Beautiful. Now read that in your own words. How do you read that?*

In line 111 the learner answered by writing down the law. David positively commented on the answer but insisted on the learner reading out the law (line 112).

According to David, he sometimes asked learners to read so he could check if they had written the correct initial expressions (Appendix D6, lines 10-15):

10.	*David:*	*Everyone on your calculator. 2 to the power of 4 times 3 to the power of minus 3 equals? Read your calculator screen.*
11.	*L2:*	*1,777*
12.	*David:*	*No read the full screen*
13.	*L2:*	*2 to the power 4 times 3 to the power negative 2*
14.	*David:*	*Yes that equals?*
16.	*L2:*	*1,7777*

The learner had given the answer as it appeared on the calculator (line 11), but David insisted that the whole calculator screen must be read (line 12).

The excerpts above have shown that both Jean and David used different language modes in their teaching. Both Jean and David used language to develop new concepts, for correcting errors in learners' work and learners' misconceptions about mathematical ideas and for developing fluency in reading mathematics. The language modes were not used in isolation to other strategies, but incorporated with scaffolding and interactive strategies.

CODE-SWITCHING

The last strategy used was that of the use of home languages. Only Jean shared a home language with her learners, so it was only in Jean's class that code-switching was used during the lessons. The use of home languages in multilingual mathematics classrooms has been well researched and recommended by several researchers (Adler, 1996, 1998, 2001; Arthur, 1994; Barton et al., 1995, 1998; Khirsty, 1995; Gorgorio et el., 2001; Moschkovich, 1996, 1998, 2001; Rakgokong, 1994; Setati, 1996, 1998, 2005; Setati &

Adler, 2001; Setati et al., 2002; Setati and Barwell, 2008; Setati, Molefe and Langa, 2008). The new mathematics curriculum emphasizes learning mathematics for understanding. To achieve this, learners are required to be actively involved in the learning process. This means that learners must, amongst other activities, be able to discuss, share, compare, contrast and explain their ideas to other learners and to the teacher. Since learners are not yet fluent in English, the LoLT, the use of home language can help learners in communicating their ideas and increase their participation in the class. This will in turn help them in developing conceptual understanding of new concepts. Jean hardly ever used home language herself in her teaching, but her learners readily used home language when they discussed in small groups and in whole class discussions. They used their home language when they explained their ideas to others or discussed ideas and concepts. The discussion below shows how home language was used in the classroom.

During the lesson Jean had asked learners to find $81^{\frac{1}{2}}$. She then asked learners to explain how they had got the answer. The excerpt below (Appendix D1: lines 42, 45 and 81) shows how home language was used in Jean's class to explain the process of finding $81^{\frac{1}{2}}$

42. *Jean:* *Let us look at* $81^{\frac{1}{2}}$. *You got 9? How did you get that?*

45. *L1:* *Saloku la isquare root sasikhipha u-9, kusho ukuthi Mam ngithe ulo. ..Ulo.. u-9 ungena kangaki ku-81 uhalf ka -81*(For this the square root gives us 9 which means that mam I said that how many times does 9 go into 81, it is half of 81)

81. L3: *Thina Mam ngoba sithe u-9 OK loya ½ lo -1 Eish sivele samshaya ngesi thende sathi u-81 uyi- square root sa -81*^{2} *sase sithi isquare root sika -81 kwaba ngu-9.* (We say that 9 ok that 9 is half, and this one we just throw it away as though it doesn't exist. The square root of 81 squared and then we said the square root of 81 is 9).

Though the explanations offered by both learners were not accurate, the learners were engaged in the discussion, trying to explain what they had done to get the 9. From the explanations of L1 and L3, it is clear they were both not sure of the difference between $\sqrt{81}$ and ½ of 81. By explaining what they did, their thoughts and reasoning were then visible (Mercer 1995) and Jean was able to attend to the confusion. The idea of what the learners had done comes through when they each mention 'square root'. It is the talking through and the participation that aids the learners as they construct meaning of new concepts.

Code-Switching was also used for explaining the solution $2^3 \div 2^3$. Learners had worked in groups on the simplification of the expression: $2^3 \div 2^3$. The different solutions that the groups had ended up with were: $2^3 \div 2^3 = 1^0$ and $2^{3-3} = 2^0 = 2^1$. One learner explained the group's solution as shown in the excerpt below (Appendix D4, line 16 and 19 and 20):

16. Jean: Those who got 1 to the power zero, how did you do it?

19. L7: Sithole u-1^0 [We got 1^0]

20. L7: Ngithe 2/2 =1 then sathi 3-3 u 0.[I said 2/2 = 1 and then we said 3-3 =0.]

Using home language the learner could explain how the group had arrived at the answer, 1^0. The process was wrong, but the explanation helped to uncover wrong misconceptions about dividing exponents that the learners in that particular group had used. This discussion led to learners using their calculators to check what the correct solution was. However, one of the learners had a calculator with a different display and she interjected to get an explanation of how her calculator operated (Appendix D4: lines 58-65)

58. *L11: Ngicela ukubuza [Can I ask a question?]*

62. *L11: What is the difference between exponent and base? Calculator 2 exponent 3 divided 2 exponent 3 (writes 2E3 ÷ 2E3 =1)*

63. L12: Bona into abayenzile bathe kucalculator …[This is what they did they ---- showing calculator]). Bathathe as if ibase bayenze i-expnent [what they did they took the base and made it an exponent.]

64. *L15: Nina into eniyenzile nithathe u-2 namenza i-exponent ka -3[It means that what you did you took 2 and made it an exponent of 3]*

The discussion above shows how two other learners, L12 and L15, tried to explain the different notation 2E3 $\div$ 2E3. While the explanations were not accurate the learners were actively participating and letting their thoughts be heard. Only by letting their thoughts known could they be corrected if they were wrong. The use of home language benefited learners because it promoted their participation in classroom discussion. The importance of use of home language for communication and participation is supported by several authors (Arthur,1994; Khirsty, 1995; Moschkovich, 1996, 1999, 2002; Setati et al., 2002; Usiskin, 1996).

> *One of the goals of mathematics instruction for bilingual students should be to support all students, regardless of their proficiency in English, in participating in discussions about mathematical ideas. … Classroom conversations that include the use of gestures, concrete objects and student's first language as legitimate resources can support students in learning to communicate mathematically* (Moschkovich, 2002:208).

Then Jean proceeded to find out how the other group had obtained the solution: $2^3 \div 2^3 = 2^0 = 2^1$. The excerpt below from lesson 4 shows the disagreement about zero being equal to1. (Appendix D4 lines 39, 40, 41,44 and 47)

*39. Jean: I would like other groups to respond to their argument because they are saying u zero can be 1. Can you respond to that?**(TII; AR)***

40. L7: Ngeke uthi ngingekho eskoleni uthi ngikhona, ngeke uthi u-0 ngu-1 into engekho leyo, never Mam [you can't say when that Iam not at school I am at school. You can't say zero is 1 there is no such, Never mam] ***(LCS)***

41. L8: It means ukuthi u- x^0 uthi there are no numbers kanti x this side uyi-one, u-x urepresentor u-1 [it means that x^0 it means there no numbers and this side there is one x, x represents 1] ***(LCS)***

43. Jean: (To another learner) I want you to explain how you moved from 2^0 to 2^1*. How are you moving there?* ***(TII; AR)***

44. L6: thina sithe i-exponent laphaya ngoba sazi ukuthi u-0 urepresenter u-1 makahamba---([we say it is the exponent over there because we know that zero represents one] ***(LCS)***

47. L9: u zero u representor 1 [zero represents 1] (other learners argue). It's the law. (Learner goes to the board and crosses out the 2^1 *and writes in its place 1). It's the law . If there is zero it means there is one of them. (Another learner raises the text book) There is a lot of discussions and arguments about this.* ***(LCS)***

The discussion between L7 and L6 is what Mercer (1995) termed disputational talk. There was a lot of disagreement about the meaning of zero when it was the exponent. There was no negotiating of meaning. Each was adamant that their interpretation was the correct one and each was trying to convince the other that their interpretation was the correct one. One other learner went to the point of standing up and raising his book saying that 'It is a law written in the book'. There were assertions and counter-assertions and individualized decision-making (Webb and Webb, 2008). The use of home language enabled the learners to be engaged in the disputational talk.

The use of home language did not only promote disputational talk, it also promoted exploratory talk. Mercer (1995) describes exploratory talk as when learners engage critically but constructively, with other people's ideas, alternative explanations are given and justifications are offered. The excerpt below (Appendix D4 lines: 48-52) shows how learners' talk later on in the lesson displayed exploratory characteristics.

48. L7: U-2^0 *is* 2^1 *uyazi ukuthini ngicela uthathe u* 2^0 *ubuyisele u-2 then uthathe u-* 2^1 *umbuyisele back u-*2^0 *ukuthi uzokunika i-answer eyi-one [*2^0 is 2^1 *you know what remove the* 2^1 *and bring back* 2^0 *it's the same answer].*

49. L13: Mam ngivumelana nabo L7 ibase ine-exponent u-1 noma i-exponent ingabhalwanga ukhona u-1 [I agree with L7, base has 1 as its *exponent and then you can write it as it without the exponent which is 1].*

50. L7: *Ibase yakho u-1 hhayi i-exponent iwu- 0 ngeke ichange isala injalo [if a base is 1 and the exponent is 0 it doesn't change, it remains the same]*

51. L7: The law says akumelenga uyichange so it doesn't make sense [The law says we musn't change, ..]

52. L7: Akumelanga nithi u-0 akuna -2 [You mustn't say it's zero there is no 2].

The learners here critically put forward their thoughts and understanding about the exponents 1 and zero. L7 agreed with L13 and tried to explain in a different way why 2^0 and 2^1 were the same. Her reason was that 2^1 was the same as 2 without an exponent written, which she claimed was the same as 2^0. It can be argued here that in algebra $1x$ is written just as x. The coefficient 1 is not written. Yet the meaning of zero can be understood as meaning nothing, thus 2^0 can be incorrectly taken at though there is no power on the base 2. This would result in 2^0 and 2^1 being written as 2. Then the explanation of L13 of 'If base is 1 and the exponent is zero it doesn't change' tries to explain that $1^0 = 1$ means it remains the same. Though the explanations were not accurate, the learners were trying to collaborately come to an understanding of the meaning of the zero exponent. This discussion made it possible for learners' reasoning to become visible (Mercer, 1995).

The discussions, disagreements and exploratory talk in home language all helped towards promoting understanding and construction of new mathematical ideas and concepts. By allowing the use of home language in her classroom, Jean facilitated the development of the construction of new mathematical concepts and also facilitated the journey that multilingual learners must make as they move from a stage of informally talking about mathematics in their main language to a stage where they use the formal language of mathematics in English (Setati, 2002). Arthur's (1994) study reveals that the absence of learners' main language in mathematics classrooms (where learners were not yet fluent in

English, the LoLT) diminishes the opportunities for exploratory talk and thus diminished opportunities for meaning-making. In Jean's classroom learners readily switched to home language and they did this very often. There were, therefore plenty of opportunities for exploratory talk and therefore opportunities for developing mathematical understanding of new concepts.

In lesson 2 there was disagreement on the simplification $5^2 \div 5^4$. Solutions that were put forward by learners were: 5^{-2}; 5^2; 25 and 1/25. The excerpt below shows how learners participated in the discussion of the procedure for simplifying the expression: $5^2 \div 5^4$. (Appendix D2, lines 72-110)

72. *Jean: Can someone come and write the answer?*

75. L2: Kusho ukuthi mina Mam…… abawu – 2 sibakhanselile (5s). ngiqale kuma negative [What I did mam is the two fives cancel. I started with the negatives and there are no negative]

77. L2: Mina kule ephezulu kumele sithathe unegative 4 u-2 upositive [What I did from the one at the top we have to take negative 4 and positive 2].

78. Jean: Yes I can see that. Are you happy with this one? Now explain this one.

81. L2: Angithi manje lena u-5, abo-5 basuke bakhanselana ngoba bekuyi division kwasala u-1 phezulu [Now these 5's cancel each other because it is division and then we are left with negative one at the to].

82. Jean: So how do you explain the one that is positive and the one that is negative?

85. L1: Uyazi mam ukuthi ngiyithole kanjanileyo answer? kusale u-1/25 [Do you know mam how I got that answer? What is left is 1/25].

86. Jean: Kusale u1/25 uma wenza kanjani?[How did you end up with 1/25?]

87. L1: Mina ngithe kanjenga leya oyenzile laphaya. [I did like the one you did over there]

100. Jean: Is this the same as that? How is this different from this one? (pointing to 25 and 1/25).

101. L6: Le ephezulu sifumene u lent , u-5. Kule esenzansi sifumene u- ..u [the one at the top we got 5 and then bottom …]

106. L7 : Ama…abo -2 no-4 laba abaphezulu bazokunika i-minus u-2 no-4 mawuzi uthola u-2minuser la uma uyazi multiply angeke uzenze njengaphezulu [Eh ! the 2 and 4 at the top they will give you minus 2. But if you multiply you can't do as you did at the top]

110. L3 : The question engiqale ngayi-raiser mina ngokuthi uma ngiqale…. Mina bengenza amanegative signs, so ngicela ukubuza ukuthi if ngifaka ama negative signs. So ngicela ukubuza ukuthi if ngifaka amasigns ..is the problem iproblem yethu amasigns [The question you raised is that if I start – if use negative signs so I want to ask do I put the negative sign? Our problem is the signs].

The excerpt shows that 5 learners were actively involved in the discussion. They all tried to explain the procedures that they had used, with learner 2 contributing to the discussion three times, but all using their home language. Helme and Clarke (2001) posit that some of the indicators of cognitive engagement in a classroom are when learners ask and answer questions, when they explain their procedures and their reasoning and when they contribute ideas. The excerpt above shows that the learners were cognitively engaged in the classroom discussion. L7 even tried to explain that the process would be different when you multiply. L3 pointed out that their problem was with the signs.

It was apparent in Jean's classroom that the level of discussion, the number of learners who got involved, the number of questions asked and the explanations given all increased when learners used their home language. There was a lot of exploratory talk and disagreements when learners used their home language. Misconceptions that would have remained hidden were exposed and corrected. By letting learners use their home language, Jean facilitated the journey that they must make as they move from a stage of informally talking about mathematics in their main language to a stage where they use the formal language of mathematics in English (Setati, 2002). The use of home language promoted exploratory talk (Mercer 1995), made their thoughts and reasoning visible (Mercer 1995) and increased cognitive engagement in the classroom (Helme and Clarke, 2001).

SCAFFOLDING INSTRUCTION

The last strategy that both Jean and David used in their classrooms was that of scaffolding. Scaffolding instruction is grounded on Vygotsky's (1978) concept of the ZPD. He argues that it is in the ZPD that learning actually takes place. According to Goulding scaffolding is 'The process by which a more knowledgeable adult or peer can help a child move from her actual performance to her potential level, giving just enough help to move the child from one to the other (Goulding in Johnston-Wilder et.al., 1999:44). Anghileri (2006) argues that scaffolding practices in learning mathematic can be divided into three levels. She posits that level 1 provides for favourable environmental conditions, level 2 includes explaining, reviewing and structuring. The last level, level 3, involves development of conceptual thinking by making connections, developing representational tools and generates conceptual discourse.

Both Jean and David used scaffolding instruction to develop understanding of new concepts, to explain and correct mistakes and to demonstrate procedures for solutions to work. Jean used scaffolding instruction to develop the meaning of negative exponents. The concept of a negative exponent is a concept that can be very confusing to learners . Jean started by simplifying the expression $3^2 \div 3^3$ using two different but familiar procedures, both resulting in the answer 1/3. She then used lawso exponents to simplify the same expression, but scaffolding the process as shown in the excerpt below (Appendix D2 lines 49-56)

49. Jean: *Now we can use the law* a^n *divided by* a^m. *The law that says* a^n *divided by* a^m *is the same as what? Can someone finish it for us?*

50. L3: a^{n-m}

51. Jean: *Now I am coming back here.* 3^2 *divided by* 3^3 *is equal to what according to this law?*

52. Ls: $3^2 \div 3^3 = 3^{2-3}$

53. Jean: *and what is 2-3?*

54. Ls: *-1*

55. [L]: 3^{-1}

56. Jean: *Which is that one there? Can you see that we have just shown that* 3^2 *divided by* 3^3 *can be written as 1/3 or it can be written as* 3^{-1}. *(Jean bracketing 1/3 and* 3^{-1} *and pointing to the two forms).*

In line 49, Jean makes sure learners remember the law that they need to use and asks someone to complete the law. Then in line 51, she focuses on the use of the law on the given expression. In line 53, she focuses on just doing the subtraction. Then by making the connections in line 56 she develops the meaning of the negative exponent.

David used scaffolding instruction to develop knowledge of the algebraic representation of the exception to the rule: $a^0 = 1$. Though learners could verbally say the exception, they could not write the algebraic representation. In lesson 1 lines 63, 64, 87-103, David applied the scaffolding process as shown below: (Appendix D5)

63. David: *The part with except naught. Give it to me in a mathematical symbol. Just the part: exclude naught.* ***(TMR; SC)***

64. L1: *(learner comes to the board and writes* $\begin{matrix} x^0 = 1 \\ 0^0 \neq 0 \end{matrix}$

87. David: *….There is an exception rule that says that what cannot be equal to naught?*

88. Ls: *The base*

89. David: *The base cannot be equal to? Naught so from that let's translate to symbol. The base is not equal to naught. Translate that into mathematical symbol. Let's start with the base. What can we place in place of the base?*

90. L4: *x*

91. David: *In place of the base we put x, right? (David writes x on the board) is not equal to, what shall we write?*

92. L4: *the equal sign with a line*
93. David: *Is not equal to?*
94. Ls: *zero (David complete writing* $x \neq 0$ *and high lights it by drawing a rectangle around it) .*
95. David: *In algebra what does x / a represent*
96. Ls: *anything; variable*
97. David: *What does the x represent?*
98. Ls: *the base*
99. David: *What does x represent?*
100. L4: *variable*
101. David: *Also a variable. Can I replace the x with a?*
102. Ls: *Yes*
103. David: *(David replaces the x with a so that it now reads* $a \neq 0$*) now we have a to the power naught is equal to 1 but a cannot be equal to naught. Then we can come to your translation anything to the power naught is equal to 1 except naught.*

The response given by the learner in line 64 above shows there was a lot of confusion on how to represent the exception to the rule algebraically. David then split the conversation of the statement to algebraic notation into smaller bits. First he focused on what could not be equal to naught (line 87). He then focused on how this could be represented algebraically (line 89 and 91). Then he focuses on the sign for not equal to. Learners knew the sign (line 92). In line 101 David finally brings in the base a. Learners were able to follow through the process and at each point learners knew the correct representation. Their ability to represent the exception to the rule algebraically resonates with what McKenzie refers to as one of the characteristics of scaffolding. He argues that scaffolding provides clear direction and reduces confusion.

Both Jean and David used scaffolding instruction to model solutions to newly learned concepts. In lesson 3, lines 78-85, when Jean wanted to simplify the expression $\sqrt[3]{\sqrt{64}}$, she scaffolded the work as follows:

78. Jean: *……So what will be square root of 64?*
79. Ls: *8*
80. Jean: *So we are actually looking for the cube root of 8 (writes* $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} =$*) which is how much?*

81. Ls: 8

82. Jean: What is the cube root of 8? (writes $\sqrt[3]{8}$) What does it mean?

83. L1: It means $2\times2\times2$

84. Jean: Which is now how much?

85. L1: 2

Jean split the solution by working from the inner most bracket (line 78). She then shows how it simplifies to just the cube root of 8 (line 80). She then completes the solution in lines 82 and 84 by focusing on the cube root of 8.

In lesson 2 David also used scaffolding to model the solution to the simplification of the expression $\dfrac{12^{(n+1)}\times 9^{(2n-1)}}{36^{n}\times 8^{(1-n)}}$.(Appendix D6, lines 65-73)

*65. David: In order to apply the law, if I want to apply the law the bases must be the same. Now in this case can I apply the law? (David reads the expression: $\dfrac{12^{(n+1)}\times 9^{(2n-1)}}{36^{n}\times 8^{(1-n)}}$).Can I apply the law?**(TSc; TLM; AR)***

66. L1: No

67. David: Because the bases are not the same. I am going to convert the bases to prime and see what happens (David now breaks the bases and writes

12 9 36 4
4 3 3 3 6 6 2 2
2 2 2 3 2 3

68. David: (writes $\dfrac{\left(2^{2}.3\right)^{n+1}\times\left(3^{2}\right)^{2n-1}}{\left(2^{2}.3^{2}\right)^{n}\times\left(2^{3}\right)^{1-n}}$) The exponent belongs to the 12 and therefore a bracket (David gestures brackets with hands). Now can I apply the law? Read the first law.

69. L10: (reads) a to the power m to the power n is equal to a to the power mn

70. David: What does mn mean?

71. L10: Means multiplying

72. David: So it means m times n. Now we use the law. (David writes):

$$\boxed{\begin{array}{l}\dfrac{\left(2^2.3\right)^{n+1}\times\left(3^2\right)^{2n-1}}{\left(2^2.3^2\right)^n\times\left(2^3\right)^{1-n}}\\[2em] =\dfrac{2^{2n+2}.3^{n+1}\times 3^{4n-2}}{2^{2n}.3^{2n}\times 2^{3-3n}}\end{array}}$$

73. David: WE have applied the same law over and over, now what? Simplify numerator and denominator if I can apply the law (David writes :

$$\frac{3^{5n-1}.2^{2n+2}}{2^{-n+3}.3^{2n}}$$

I applied one law here and a second law to here and the same law here (pointing to the relevant parts). What's next? Which law can we apply? Read the law.

In the scaffolding process, David uses a lot of questions. This resonates with what Kiong and Yong (2001) argue as one of the activities that can be to scaffold work. In line 65, he asks learners if he can use the law, then explains in line 67 that because the bases are not the same, the law cannot be used. After converting to prime bases he asks if the law can then be used (line 71). He then focuses on the meaning of the law that can be used (lines 69-71).

David also used scaffolding to explain to learners the mistakes that they had made. In lesson 2, one learner had written $2^4 \times 3^{-2} = 6^2$ and David helped correct this as shown below (Appendix D6, lines 4-10)

4. *David : (starts by recapping where lesson ended the previous day). L5 said $2^4 \times 3^{-2} = 6^2$. He took the bases 2 x 3 = 6 and the powers 4 + -2 = 2. Therefore answer = 6^2. The question now is which law have you used?*
5. *[L]: I used law 1*
6. *David: You can't use that law. Why?*
7. *L1: Bases are not the same*
8. *David: If the bases are the same then I can use the law. Can I say 2 times 3? This is 2 to the power 4 times 3 to the power minus 2. We must get this*

in order (pointing to 16 and 3^{-2} *). So this must be cancelled (David erases the part* $2^4 \times 3^{-2} = 6$. We have cancelled so we can continue. What did we say about multiplying a base and exponent?

9. L1: *We can't do that.*

10. David: *The 16 we got by saying 2 times 2 times 2 times 2 four times. Now we want 2 to the power 4 times 3 to the power minus 2. Everyone on your calculator. (as learners work on calculator the teacher repeats 2 to the power of 4 times 3 to the power of minus 3 equals.) L5 read your calculator screen.*

In the above excerpt, David again uses a lot of questions to scaffold the work (lines 4, 6 and 8). The questions encourage interactive participation as well as thinking about when the laws can be applied. In line 10, David directs learners to use their calculators. He restructures the questions, but still with the aim of scaffolding the work. This is equivalent to the level 2 that Anghileri (2006) suggested ad the level at which explaining, reviewing and restructuring can be used in scaffolding.

In the excerpts above scaffolding was not used in isolation of the other strategies. Both teachers used scaffolding instruction with different combinations of the other strategies. Bothe teachers used scaffolding instruction at different levels as required by their learners.

CONCLUSION

The analysis of data in chapter 5 and 6 shows that the teachers in the two multilingual classrooms used similar strategies. The strategies used were not limited to language strategies but also included strategies that can be used to support for mathematical development in any other classroom. The strategies used were:

- Interactive instruction
- Multiple representation
- Code-Switching
- Scaffolding

- Language Modes

The next chapter concludes the study by discussing recommendations for strategies in multilingual mathematics classrooms, limitations of the study and possible future research.

CHAPTER 7

FINDINGS OF THE STUDY

This study focused on exploring instructional strategies that teachers in multilingual mathematics classrooms use to promote development of mathematical proficiency in algebra. Two case studies were used for this study. One case focused on a teacher who shared a home language with all the learners and the other case focused on a teacher who did not share a home language with most of the learners. This study was guided by two research questions:

- What instructional strategies do teachers in two different multilingual classrooms use in order to support learners' development of mathematical proficiency in algebra?
- How do they use these instructional strategies to support their learners' mathematical proficiency in algebra?

Chapter 6 explored these instructional strategies in some detail by looking at and analysing when and how the strategies were used. This chapter concludes the study by discussing the findings of the study, suggesting recommendations from the study and discussing limitations of the study.

ISTRUCTIONAL STRATEGIES USED IN MULTILINGUAL CLASSROOMS

One of the research questions the study set to answer was "**What instructional strategies do teachers in different multilingual classrooms use in order to support learners' development of mathematical proficiency in algebra?** The findings of the study reveal that both Jean and David used the strategy of **interactive instruction**. Interactive instruction was encouraged in Jean's class by the learners first working in small groups followed by whole class discussions. Jean even told her class that mathematicians work together as a community and that was how she wanted her class to work, discussing ideas and questioning when they did not understand. The findings also show that Jean encouraged and provided opportunities for learners to participate during the lesson as evidenced by her saying '*It is important to give your responses. It does not matter whether it is wrong or right. If you are not satisfied you are allowed to say hmmm I don't understand'.*

The findings from Jean's classroom further reveal the application of Vygotsky's (1978) social constructivist theory of learning. During the interview Jean said '*I also encourage them to speak English so that they learn from each other. They may not necessarily be aware that they are learning from each other as the others are expressing themselves in English. But those who are not fluent, as time goes on, they will catch up.'*

Findings from David's class also reveal the strategy of interactive instruction. Interactive instruction was shown in the way homework was marked. All homework was marked collectively on the chalk board. Learners always presented their solutions on the chalk board and then explained to the rest of the class how they had obtained their solutions. The rest of the class and the teacher would then challenge their methods, procedures and accuracy of the solutions and the learner then had to explain his or her procedure and/or correct the solution accordingly. The use of many different questions by David promoted participation by learners during the lessons.

The use of interactive instruction promotes general learner engagement and discussions of concepts. Learner engagement and discussions are some of the indicators of the process of developing conceptual understanding and adaptive reasoning.

The findings further reveal that both teachers used the strategy of **multiple representations.** Explaining the concepts and meanings of fractional, negative and zero exponents especially to learners, who are still learning the LoLT, can be tricky because of the need to use wordy and long phrases in English. Seeger (1998) says that being able to see a concept in different forms or representations benefits multilingual learners because they can understand a concept from a representation that has reduced dependency on language. Jean used multiple representations and different methods and procedures to develop the understanding of these concepts. The concept of a negative exponent was developed by simplifying the expression $3^2 \div 3^3$ in different ways. The concept of a zero exponent was developed in a similar way using $2^3 \div 2^3$. She even used an analogy of roots of a maize plant under the ground and the number of stalks that appear above the ground.

The findings also show the use of multiple representations in David's classroom. David used multiple representations to develop the understanding of the exponential law that says $a^0 = 1$. He asked learners to translate and write the law into their own words. One of the translations given by the learners was '*If a power is divisible by 0, it automatically equal to 1.'* From this it was clear that the learner did not understand the meaning of the law. It was the use of multiple representations that uncovered this misunderstanding. Findings further reveal the importance that David placed on conceptual understanding and multiple representations. During the interview David said, 'I *always want to see the interpretations at 3 levels. First the symbolic, second the mathematical and third the English form'*.

Access to multiple representations provide learners with a choice of forms for expressing a mathematical term and this can lead to the development of procedural competence, strategic competence and adaptive reasoning. Therefore multiple representations supported the development of 4 of the strands of mathematical proficiency, i.e. conceptual understanding, procedural fluency, strategy competence and adaptive reasoning (Kilpatrick et al., 2001)

Findings further show that the strategy of multiple representations was not used in isolation, but that it was used together with the other strategies by both Jean and David.

The findings also show that both Jean and David used the strategy of **scaffolding.** As mentioned earlier in chapter 3, explaining the concepts and meaning of fractional, negative and zero exponents to learners who are still learning the LoLT can be tricky because of the need to use wordy phrases or long sentences in English. Findings show that not only did the teachers use the strategies of multiple representations and interactive instruction but they also scaffolded the new concepts they were dealing with. The findings reveal that both teachers broke the work into smaller doable bits. They gave verbal cues and modelled parts of solutions. The teachers acted as the more knowledgeable others providing temporary support to learners so that the learners could successfully complete a task that they could not complete without assistance. This showed the teachers' use of the Zone of Proximal Development (Vygotsky, 1978). The scaffolding strategy also promoted interactive instruction as it kept learners engaged in the work. When work was scaffolded learners could follow and understand how concepts had been developed. Learners were able to understand the meaning of 3^{-1} and why $a^0 = 1$. By scaffolding the work, learners were able to understand that negative exponents could be represented with positive exponents. Scaffolding therefore supported conceptual development, strategic competency and adaptive reasoning.

Findings further reveal that both Jean and David used the strategy of **language modes.** Language modes include listening, speaking, reading and writing. Zemelman et al., (1998) argue that discussing, writing, reading and listening to mathematical ideas deepen students' understanding of mathematics. These four language modes are very important in multilingual classrooms because learners must learn how to communicate mathematics mathematically both orally and in written form. Both Jean and David used language modes when they asked learners to write and explain their procedures on the chalk board. The use of language modes resonates with what Thompson et al., (2000) refer to as diagnosing and assessing students' understanding by listening to their oral communication and their mathematical writings. Findings also show that David used language modes and multiple representations together very often. David always said '*Which law? Read the law. Interpret the law.*' By asking learners to find a law, David was promoting adaptive reasoning. By asking the learners to read the law and interpret it, he was promoting conceptual understanding.

Findings of the study further reveal that there was a lot of **code-switching** in Jean's class. Jean is the teacher who shared a home language with all the learners. The findings in this study show that Jean, herself, hardly ever used any of the learners' home languages. She always used English. The findings show that the learners are the ones who used code-switching. They switched codes when they needed to explain their work or discuss among themselves or with the teacher. The findings indicate that the learners used their home language as a resource that facilitated exploratory talk (Setati, 2000) in the mathematics classroom. During the interview Jean said:

Jean: I encourage them to use their own mother tongue especially if I am interested in getting out their understanding. But I also encourage them to speak English so that they teach each other. They may not necessarily be aware that they are learning from each other as the others are expressing themselves in English. But those who are not fluent as time goes on; they will catch up because it is still important for them that they have access to English.

Researcher: So you think it is important for them to have access to English?

Jean: Yes I do think so 'cause they are using it in examinations. We do not have options. But I don't want them to be conscious and say the teacher wants me to say it in English. It should be something that happens freely. I want them to just do it maybe something like incidental learning.

The findings here echo with what Adler (1998) referred to as dilemmas in multilingual classrooms. Jean, herself, hardly ever switched codes. As she said in the interview, she encouraged her learners to code-switch so that learners could develop the necessary understanding of the concepts, but also provided opportunities for them to develop their English, which is necessary for examinations. This resonates with what Setati et al., (2006) refer to as gaining epistemological access without loosing access to English.

In David's class, code-switching did not take place in the same manner. David is the teacher who did not share a home language with most of the learners. Findings from David's lessons indicate a lot of use of informal spoken mathematical language. Setati (2002) describes and shows the different routes that can be taken by learners who are not fluent in English in an effort to understand what is going on in a mathematics classroom. David and the learners used statements like '*Whose exponent is this? The -9 belongs to* *the y; The things that come before everything else, what do those things do? It lets us the way to the law'*. It is the use of such statements that I have categorized David's strategy as part of code-switching starting at the informal spoken mathematics in English stage and completing the journey in the possible routes as shown on the diagram.

The diagram below shows part of the diagram suggested by Setati (2002). (The part that is not included shows routes used in a home language). The findings from David's class show that he used the informal spoken mathematics stage as the starting point of the routes taken to develop understanding and completed the journey using the possible routes shown in the diagram.

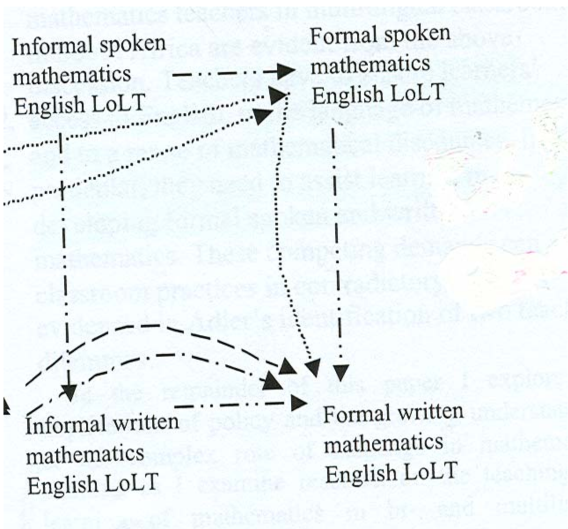


Figure 7.1: *Part of the diagram of 'Alternative routes from informal spoken (in main language) to formal written (in English) mathematics language' taken from Setati (2002)*

HOW INSTRUCTIONAL STRATEGIES ARE USED IN MULTILINGUAL CLASSROOMS

The second research question that guided the study was "**How do they use these instructional strategies to support mathematical proficiency in algebra?"**Findings of the study reveal that both teachers used a lot different types of questioning. Some of the questions helped to sustain learner participation in interactive instruction, other questions promoted the strategy of multiple representations or scaffolding or language modes. Findings further reveal that each of the strategies was supported by the use of appropriate questions and that the questions promoted the development of conceptual understanding, procedural fluency and adaptive reasoning Findings also indicate that the technique of revoicing encouraged learner participation and therefore promoted interactive instruction. Revoicing helps the learners by providing them with the correct mathematical term, correct wording, and keeping the discussion mathematical. This, in turn, promoted conceptual understanding and adaptive reasoning. Kilpatrick et al., (2001) argue that the strands of mathematical proficiency are intertwined and interdependent and support one another in their development. Therefore the development of conceptual understanding, procedural fluency, strategic competence and adaptive reasoning supports the development of productive disposition.

Findings further reveal that both teachers did not use each of the strategies in isolation, but the strategies were used in different combinations as needed by the learners or required by the nature of the mathematics to make the new concept accessible to learners. In one situation, scaffolding strategy was used alternatively with multiple representations and in another situation it was used together with code-switching, yet in another situation scaffolding was used with language modes and multiple representations and with code-switching. Findings show that each of the strategies was used in different combinations with all or some of the other strategies. Findings also reveal that a strategy or a combination of strategies could be used to develop any of the 5 strands of mathematical proficiency i.e. interactive instruction could be combined with language modes to develop conceptual understanding or to develop strategic competence. Findings further revealed that the combinations of strategies were dynamic, always changing according to the needs of the learners at the particular time as well the demands placed on the learning process by the concept at hand. These findings resonate with Fairhall, Trinick and Meaney (2007) who argue that combining a range of strategies seems to be part of what makes effective support for students who are operating at different stages. The different combinations of the strategies that were used to support development of mathematical proficiency can be represented by the diagram below.

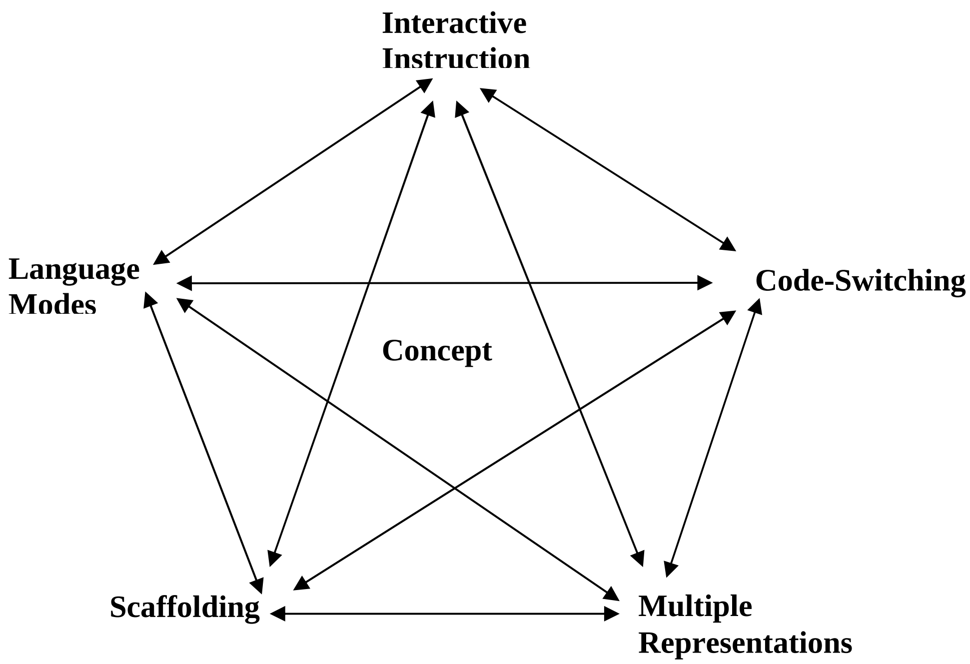


Figure 7.2: *Combinations of Instructional Strategies*

In the diagram, each of the strategies is connected to all of the other 4 strategies by a double-headed arrow. The double arrows indicate that the connections between the strategies operate in both directions i.e. using scaffolding and then multiple representations or using multiple representations and then scaffolding to promote the development of the algebra concept. The different arrows show the different ways that the five strategies were combined to support the development of mathematical proficiency in algebra. The position of the concept, in the middle of the diagram, indicates the resultant concept being developed through the use of a range of different combinations of the strategies.

RECOMMENDATIONS

The cries, for better matriculation results in mathematics, continue to be headlines in the media every time the matriculation results are published. There is a shortage of qualified professionals in professions that require mathematics as a prerequisite. As mentioned in chapter 1, the majority of classrooms in South Africa are multilingual. The aim of the study was to explore strategies that teachers in multilingual classrooms use to support development of mathematical proficiency in algebra. Purposive sampling was used to make sure that the participants were knowledgeable of the phenomena under investigation. Exploring these strategies can highlight strategies that can be used to support mathematics learning in other multilingual classrooms and thereby improve the matriculation results in mathematics.

Strategies must address the needs of the learners as well as the characteristics of the mathematics content at hand. Knowing the needs of the learners as well as mathematical content becomes very important in selecting the strategies. Knowing the needs of each learner in a particular multilingual classroom is then important; whether it is need of appropriate mathematical vocabulary, opportunities for exploratory talk or opportunities for communication. Use of strategies that reduce dependency on language while still developing meaning of a concept can be helpful. This could be combining code-switching and multiple representations, or code-switching and language mode, or scaffolding and multiple representations. The teacher must have knowledge of the nature of the mathematical content to be taught. It could be the process of conceptual development or the language necessary for the particular mathematical content. Strategies that cater for these needs can then be selected. Learning can be maximized by using the best combination of strategies. Teachers can be creative in trying different combinations of strategies and keeping the strategy combinations dynamic.

From the findings of the study, I am arguing that language strategies are not the only strategies that would support the development of mathematical proficiency in multilingual mathematics classrooms, but that they are only part of strategies that can be considered in multilingual classrooms. Most of the strategies explored would be seen in any other mathematics classrooms, but the language strategies would be unique in multilingual classrooms. I therefore propose the use of the language strategies, together with the appropriate combination/s of the other strategies as the most effective strategies for multilingual mathematics classrooms.

LIMITATIONS OF THE STUDY

While the findings of the study reveal the strategies and how they were used, there are some limitations to the findings being generalised. In this study one limitation is the proportion of the topic that was explored to all of the algebra in the curriculum. The study focused only on exponents. Though exponents are an important part in algebra the field of algebra is wide and there are many other areas that could have been considered. Would these strategies be the same for these other sections of algebra? Would the combinations of these strategies be the same for other sections of algebra, or would they be different?

Another limitation was the sample size. The sample size of only two teachers is very small. A larger sample can be used. Also the sample that was used for the study was specific. The teachers were hand-picked because of their knowledge of the relationship between the language of learning and teaching (LoLT) and learning mathematics. What about those teachers who are not aware of this relationship? What do they do to support their learners' mathematical development in their classrooms?

Another limitation to the study was the extent of multilingualism in the classroom. In the sample that was used in the study, Jean shared a home language with all her learners, whilst in David's class only two learners had English as their home language. English is however David's second language. The two case studies, therefore, stand at the two extreme ends of a continuum on the percentage of learners who share a common language with the teacher in multilingual classrooms.

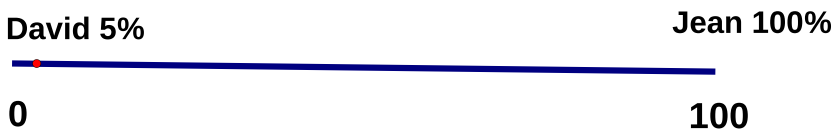


Figure 7.3: *Percentage of learners who share a home language with teacher*

Along the continuum, there are many other possible multilingual situations. What would be the implications of the strategies used or the combinations of the strategies in different positions along the continuum?

CONCLUSION

This chapter has given a summary of what can be done in multilingual classrooms to support learners' development of mathematics proficiency. The chapter has not only given a summary of the strategies that can be used in multilingual classrooms, it has also shown and described how the strategies can be combined to provide the best support for learners. The chapter has also explained that the same combination of strategies can be used to support different strands of mathematical proficiency. The success of using these strategies depends on the creative nature of the teacher in selecting the appropriate combinations. It is crucial to recognize that combinations of strategies are not static but dynamic.

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APPENDIX A

MATRICULATION EXAMINATION QUESTIONS INVOLVING EXPONENTS, FOR THE YEAR 2008.

( ) Indicates number of points assigned for the question.

NATIONAL SENIOR CERTIFICATE:

Question 3:

Given the geometric series $8x^2 + 4x^3 + 2x^4 + ---$

a. Determine the n^{th} term of the series. (1)

b. For what value(s) of x will the series converge? (3)

c. Calculate the sum of the series to infinity if $x = \frac{3}{2}$. (3)

Question 5:

Given $h(x) = 4^x$ and $f(x) = 2(x-1)^2 - 8$

5.1 Sketch the graphs of h and f. Indicate all intercepts with the axes and any turning points. (8)

5.2 Without any further calculations, sketch the graph of $y = \log_4 x = g(x)$ on the same system of axes. (2)

5.3 The graph of f is shifted 2 units to the LEFT. Write down the equation of the new graph. (2)

5.4 Show, algebraically, that $h\left(x + \frac{1}{2}\right) = 2h(x)$ (3)

Question 7:

7.1 R1 570 is invested at 12% p.a. compound interest. After how many years will the investment be worth R23 000? (4)

7.2 A farmer has just bought a new tractor of R800 000. He has decided to replace the tractor in 5 years' time, when its trade-in value will be R200 000. The replacement cost of the tractor is expected to increase by 8% per annum.

7.2.1 The farmer wants to replace his present tractor with a new one in 5 years' time. The farmer wants to pay cash for the new tractor, after trading in his present tractor for R200 000. How much will he need to pay? (3)

7.2.2 One month after purchasing his present tractor, the farmer deposited ***x*** rands into an account that pays interest at a rate of 12% p.a., compounded monthly.

He continued to deposit the same amount at the end of each month for a total of 60 months.
At the end of 60 months he has exactly the amount that is needed to purchase a new tractor, after he trades in his present tractor.
Calculate the value of ***x*.** (6)

7.2.3 Suppose that 12 months after the purchase of the present tractor and every 12 months thereafter, he withdraws R5 000 from his account, to pay for maintenance of the tractor. If he makes 5 such withdrawals, what will the new monthly deposit be? (4)

Question 8;

8.2 Determine , using the rules of differentiation;

$\frac{dy}{dx}$ if $y = \frac{\sqrt{x}}{2} - \frac{1}{6x^3}$. Show ALL calculations. (3)

Question 10:

A drinking glass, in the shape of a cylinder, must hold 200 ml of liquid when full. The radius of the cylinder is *r* and the height is *h*

10.1 Show that the height of the glass, *h*, can be expressed as $h = \frac{200}{\pi r^2}$ (2)

10.2 Show that the total surface area of the glass can be expressed as

$S(r) = \pi r^2 + \frac{400}{r}$. (2)

10.3 Hence determine the value of *r* for which the total surface area of the glass is a minimum. (5)

INDEPENDENT EXAMINATION BOARD (IEB)

Question 2:

2(c). Evaluate (1) $\sum_{x=1}^{4}\left(x^2 - x + 1\right)$ (3)

(2) The sum of the geometric series :

$\frac{45}{4} + \frac{135}{16} + \frac{405}{64} + \frac{1215}{256} +$ ---- to 15 terms. (4)

Question 3:

(a) The Vorsters buy a new car for R255 000. It depreciates in value (on a reducing balance) by 12,5% per year.

(1) Determine the value of the car after 3 years. (2)

(2) Determine how long (to the nearest year) it will take for the Vorsters' car to be worth R100 000. (5)

(b) Calculate the depreciation rate (using reducing balance) for another brand of vehicle that will depreciate to be half of its original value in exactly 6 years. (4)

(c) Nosizwe's parents are considering taking a bank loan of R40 000 to cover the costs of her first year at university. They are offered an interest rate of 14,75%p.a. compounded monthly and wish to pay back the loan in 12 equal monthly instalments starting one month after receiving the loan. Calculate what these monthly payments will be. (7)

Question 4:

(a) Find $\frac{dy}{dx}$ if $y = 3x^3 + x$ (2)

(b) Find $f'(4)$ if $f(x) = \frac{1}{2\sqrt{x}}$ (4)

Question 7:

(a) Given $a^x = 1$, state the values of a if

(1) $x = 0$ (1)

(2) $x \neq 0$ (1)

(b) Given: $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \text{---}}}}}$

(1) Write x^2 in terms of x (2)

(2) Hence determine the value of x (3)

APPENDIX B--- CONSENT FORMS

UNIVERSITY OF THE WITWATERSRAND
MATHEMATICS RESEARCH PROJECT

Dear Learner

My name is Sophie Mparutsa. I am currently doing my MSc degree in Mathematics Education. As part of my course, I am doing a research exploring what methods and approaches teachers in multilingual classes use to teach algebra. The focus of the research is to find out what teachers are doing to help learners whose home language is not English to learn algebra.

Your mathematics teacher and headmaster have given me permission to send you this letter to invite you to participate in this research project. If you and the other learners agree to participate in this project, I will come to observe your mathematics teacher teaching algebra to your class. During the lesson, I will video record the lesson. This is important so that I can record the exact things that your teacher says and does during the lesson.

I will protect your anonymity and confidentiality. Your real name will not be used in the final report. I will remove any information that might give away your identity. If some of information on the video is needed for conferences, your face will be hidden from public viewing. In such a case, permission from you and your parents will be asked before the video is used for conferences.

Your participation in this project is voluntary. You will not suffer any consequences if you not agree to participate. If you require more information, you can contact me. My phone number is given below.

If you agree to participate in this research project please complete the consent forms that are attached and sign in the space provided.

Mrs Sophie Mparutsa

(011) 6453219

LEARNER CONSENT FORM: Video-recording.

I --(please print your name in full) a mathematics learner at --School , am aware of all the data collection process in the research project as listed in the information sheet attached.

I give consent to the following:

- Being videotaped during the mathematics lesson

  Yes ☐ or No ☐

Signed --- Date-------------------------

TEACHER CONSENT FORM: Video-recording.

I --(please print your name in full) a mathematics learner at --School , am aware of all the data collection process in the research project as listed in the information sheet attached.

I give consent to the following:

- Being videotaped during the mathematics lesson

  Yes ☐ or No ☐

  Signed --- Date------------------------

- The possible use of the videotext for conference purposes

  Yes ☐ or No ☐

  Signed -- Date -----------------------

TEACHER CONSENT FORM: Tape recording.

I ---(please print your name in full) a mathematics learner at --School , am aware of all the data collection process in the research project as listed in the information sheet attached.

I give consent to the following

- Being interviewed at some point during the study.

  Yes ☐ or No ☐

  Signed --- Date------------------------

- The tape recording of my interview with the researcher

  Yes ☐ or No ☐

  Signed -- Date -----------------------

APPENDIX C :CATEGORIES USED FOR ANALYSING DATA

Interactive Instruction (TII): Teacher using and/or encouraging learners interactive participation.

Language Modes (TLM): Teacher using and/or encouraging learners to use different language modes.

Scaffolding (TSc): Teacher scaffolding work for learners.

Code-Switching (TCS): Teacher code-switching.

Code-Switching (LCS): Learners code-switching.

Informal Mathematical language (TIM): Teacher using and accepting use of informal mathematical language.

Multiple Representations (TMR): Teacher using and/or encouraging learners to use multiple representations.

Strand of Mathematical Proficiency	Indicator: Evidence seen when teacher requires, asks or encourages learners to:
Conceptual Understanding (**CU**)	• Represent mathematical ideas in different ways. • Make connections • Know how different representations can be used for different purposes
Procedural Fluency **(PF)**	• Know procedures • Know how and when to use procedures appropriately • Develop skill in performing procedures

	flexibly and accurately
Strategic Competency **(SC)**	• Be flexibility in approaches to solving problems • Choose effective procedures • Replace long methods with concise and efficient procedures • Form mathematical representations from words
Adaptive Reasoning **(AR)**	• Think logically about relations between situations and concepts • Explain ideas • Justify solutions and procedures • Explain procedures
Productive Disposition **(PD)**	• Persevere and relate more effort • Develop confidence

Table 4.1: *Mathematics Proficiency Strands and corresponding Indicators*

APPENDIX D--- TRANSCRIPTS OF LESSONS

Names in the transcripts are not the real names.

[L]: Indicates a particular learner who could not be identified from the video.

APPENDIX D1: JEAN: LESSON 1

1. Jean: We are doing exponents, but before we go there I want to read this message. After that I want you to tell me whether you understand what it says. (Jean puts the message on overhead projector and gives learners time to read). What language do you think it is? Tipicheperepe opo hapinapa kara/da lipibunukapa tsapa niropa **(TII; TCE)**

2 L1: *Yini yona leyo*? [What is this?]
L2: Isi French [it is French]

3. Ls: Setwsana, Sotho, French. (Learners argue about language it is)
4. Jean: You don't understand what it says, right? How do you feel when you don't know what it means? **(TCE)**
5. Ls: Bored; don't care; loose interest.
L1: *Asinamsebenzi nayo*[We don't care]
6. Jean: You just loose interest. Does maths sometimes make you feel like that? **(TCE)**
7. Ls: Yes.
8. Jean: Like Greek, like Spanish?
9. Ls: Yes.
10. Jean: Ok now for us as learners of maths we want to unravel this language that we don't understand. We want to know what it means. Right?
Read task #1 as a group (Jean divides learners into groups) Do the task in your group for 10 minutes. Every group after that will give us their answer on the board, alright! Some of you might need calculators. (Jean distributes calculators, one per group and moves around the groups) **(TII; TCE; CU; AR; SC)**
11. Ls: (There is a lot of discussion in groups)
12. Jean: There are nine groups I hope one person from each group to give us responses. Can we have the responses for the house, the value of the house There will be three responses for the house, for the car the same, for the investments. (Jean makes a table with place values) **(TII)**
13. Jean: If you did not finish then we will do it together as a group. What I want us to realize, it will be very powerful if we learn together as a group. This is why I am giving you these problems to start in small groups then discuss together as a class. Then we will get to understand better. That's how mathematicians work. They work as a community. It is important to give your responses. It doesn't matter whether it is wrong or right. If you are not satisfied you are allowed to say – hm hm I don't understand Ok!! Now for those who did the house what values did you find? I've got millions. If you did not finish it is

fine. There are six digits there. What is the first value that you got?**(TII; TCE; CU; SC)**

14. L3 2
15. Jean: Let's concentrate on one group. Your group
16. L3: *Bekuyi cube root ka -8*.[It was cube root of 8] **(LCS)**
17. Jean: The cube root of 8 is--- (Jean repeating learner's response in English) **(TII)**
18. L3: 2
19. Jean: next one
20. L3: *kuma hundred kwaba ngu-9* [For the hundreds, it is 9](**LCS)**

21. L3: 7
22. Jean: u 7 and then?
23. L3: *Kuma thousand kwaba ngu-8* [For the thousand it is 8] (**LCS**)

24. Jean: ten thousand
25. L3: *uma hundred u 9* [for hundreds 9] **(LCS)**
26. L3: four square root of 2
27. Jean: on this space?
28. L2: *Iphinde into eyi –one kayi two* [you repeated one thing twice] **(LCS)**
29. Jean: *Ngiphindile* sorry [did I repeat? Sorry] **(TCS)**
30. Ls: *akayiphindanga* [she didn't repeat] **(LCS)**
31. Ls: *uyiphindile,uphinde le kuma thousand* [she did repeat, she wrote the thousands twice]

32 L3: *kuma tens ngu-4 ne square root sika -2, kuma hundreds ngu-… nguma tens ngu- 4 nesquare root sika 4 i- answer ngu -2. Ngithole I square root sika- 4. O i-answer* ngu-2 [for tens it is 4 and square root of –2. for the tens it is 4 and square root of 4, the answer is 2. I got the square root of 4, so the answer is 2] **(LCS)**

L2: Yiyo lento esixakayo ekhipha i-answer engu-4 [this is what confuses us, which gives an answer 4] **(LCS)**

33. Jean: thirty two to the power one-fifth.
35. L3: thirty two to the power 1 over 5.
36. Jean: Now what we should do, what is important to us is to discuss. How can we have that, because in maths one thing cannot mean two different answers. So we must agree, ok? We must make sure we must agree why the value here must be 2. If we don't agree we must use the law whatever. The first one was cube root of 8. Do you understand what this means, the cube root of 8? Can you say what it means? How do we find it? **(TII; CU; AR)**
37. L4: (explains something very softly to the teacher)
38 Jean: Come and show us. How can you get that 2? We want to know to get the 2. We want everybody to agree that the 2 is correct.**(TII; AR; CU)**
39. L4: Writes on board 2x2 = 4
    4x2 = 8
    Must multiply 2 three times.
40. Jean: Oh! We must multiply 2 three times. So if I want to say there was another (Jean refers to worksheet) which had the cube root, the cube root of 512 and

they are saying the answer is 8, why (Jean shrugs her shoulders) are we agreeing that is right?

The meaning here is we are all agreeing that we must have the same number multiplied three times. (At this stage Jean writes $\sqrt{\square\times\square\times\square}$ =. Then Jean inserts the 2's in the rectangles and writes):

Cube root of 8 = 2, $\sqrt[3]{2\times2\times2}=\sqrt[3]{8}$. So the cube root of 8 is 2. The number that you multiply three times to give 8 is 2.

So in this case it will be $\sqrt[3]{512}=\sqrt[3]{8\times8\times8}=8$, to give us $\sqrt[3]{512}$. So we all understand the meaning of cube root. **(TMR; TLM; CU; SC;)**

41. Jean: What will the fourth root, the meaning be? (Jean writes $\sqrt[4]{-\times-\times-\times-}$). It means that there must be 4 numbers inside. From now on when we are dealing with cube root, fourth root we must remember the meaning ok! So we are not expecting people to come up with different meanings.**(TMR; TLM)**
42. Jean: Let us look at $81^{\frac{1}{2}}=9$. You got 9? How did you get that? Bongani **(TII; AR)**
43. L5: 9 x 9 i81. *Lokuza isquare root* [what we call the square root] **(LCS)**
44. Jean: Why are you using the square root?**(TII; AR; CU;SC)**
45. L3: *saloku la isquare root sasikhipha u-9, kusho ukuthi Mam ngithe ulo. ..Ulo.. u-9ungena kangaki ku-81 uhalf ka -81*[for this the square root gives us 9 which means that mam I said that how many times does 9 go into 81] **(LCS)**
46. Jean: Remember we did not have this 9. We want to understand what the power ½ means. **(TII; AR; CU SC)**
47. L3: ½ of 81
48. Jean: ½ of 81? So we are saying 81 divided by 2? What does ½ mean? **(TII; CU; AR)**
49. Ls: ½ is 0,5
50. Jean: What should I do? Multiply by ½ ?
51. Ls: Yes
52. Jean: So this and this would be the same thing? (Jean writes: $81\times0,5$). Are we saying $81^{\frac{1}{2}}=9$ and $81\times0,5$ mean the same thing? (Jean points to the two statements)
(TII; SC; AR; CU)
53. Ls: Yes
54. Jean: If you say no why not? (One learner uses calculator) **(TII; AR)**
55. [L]: 81,2 (there is a lot of talking now)
56. Jean: Remember we need to listen to each other. Sibusiso you were saying something.**(TCE)**
57. L6: *Ayisafani i-answer ayisafani izoba ngu-81*[the answer is not the same, its going to be 81] **(LCS)**
58. Jean: You are saying this is not the same? **(TII; SC; AR; CU)**
59. L6: *iyafana,* [it is the same] **(LCS)**
60. Jean: So what would be the answer here? This is the same as 81 divided by 2? **(TII; SC; AR)**
61. L1: *ayishintshi le nto* [this thing doesn't change]. **(LCS)**

The answers are the same.
62. Jean: You are saying this is the same answer as this one?
63. Ls: Yes
64. Jean: And what is the answer?
65. [L]: (Inaudible)
66. Jean: Is it 9? 81 divided by 2 is 9? (Jean writes 81/2 =)**(TII; TLM; AR; SC)**
67. Ls: No. 40, 5
68. Jean: Remember one person at a time. Who is talking here? You were saying something right
69. L6: It does not
70. Jean: 81 to the power ½? I am saying is it the same as $81 \times 0,5$? **(TII; AR; SC)**
71. Ls: Yes
72. Jean: It's the same? **(TII; AR; SC)**
73. Ls: Yes
74. Jean: Ok what is the answer for 81 x 0, 5?
75. Ls: 40, 5
76. Jean: 40, 5? So is it the same as 9? **(TII; AR)**
77. Ls: Yes, No
78. Jean: Now let's go back to their answer. Your group is saying the answer is 9
79. Ls: Yes
80. Jean: We want to understand where did you get the 9? **(TII; AR; CU)**
81. L3: *thina Mam ngoba sithe u-9 OK loya ½ lo -1 Eish sivele samshaya ngesi thende sathi u-81 uyi- square root sa -81^2 sase sithi isquare root sika -81 kwaba ngu-9* [We say that 9 ok that 9 is half , and this one we just throw it away as though it doesn't exist. The square root of 81 squared and then we said the square root of 81 is 9] **(LCS)**
82. Jean: Now you are saying 81^2. (There is a lot of talking now)
83. Jean: But where did the 9 come from? Shhh I want to get this very clear. Eighty-one squared and eighty-one to the power ½ (Jean writes 81^2 and $81^{\frac{1}{2}}$) are not the same thing, right?**(TII; TLM; AR; CU)**
84. Ls: Yes
85. Jean: You are saying $81^{\frac{1}{2}}$ is what? You are saying it is 9?
86. Ls: Yes, no
87. L7: *isquare root sakhe ngu*-[the square root only is 9] **(LCS)**
88. Jean: Yes. The square root of 81 is 9. So what is 81 to the power ½ equal to? **(TII; TSc; AR)**
89. Ls: 9
90. L1: *Isquare root siphumaphi*? [Where is the square root coming from?] **(LCS)**

91. Jean: Themba is asking where does the square root come from. (At this point there are lots of small group discussions going on)
92. Jean: Bheki wants to show us something. (Learner shows teacher something on the calculator. Another learner moves to join to see what Bheki is showing the teacher)

Alright there is something interesting here. Bheki is showing me something. Bheki is saying 81 (pause) she is doing it on the calculator, 81 to the power ½ = 9. What is important here is we need to know what this ½ means (Jean points to the exponent).

So there is a definition of exponents, a strange one that explains that. I don't know did you do this before or you are meeting this for the first time? You did it in grade 9? **(TII; TCE; CU; AR; SC)**

93. Ls: Yes

94. Jean: When you have any number to the power of a rational number, a fraction (here Jean writes $a^{\frac{1}{n}} = \sqrt[n]{a}$) the nth root of that number. For example $a^{\frac{1}{2}}$ it would be the square root of a (Jean writes $a^{\frac{1}{2}} = \sqrt{a}$) **(TLM; TMR; SC)**

95. Jean: $a^{\frac{1}{3}}$ will be what?

96. L8: Becomes *a* x *a* x *a*

97. Ls: square root of a

98. Jean: We want to see where this fraction comes from. **(TII; CU; AR)**

99. L1: a squared

100. Jean: You see it becomes a root. So what root in that case? **(TII; AR; SC)**

101 Ls: Square root

102. Jean: What root is this?

103. Ls: Square root of a. 3 to the square root of a

104. Jean: If it starts with 3 then it is the cube root of *a* (Jean writes $a^{\frac{1}{3}} = \sqrt[3]{a}$).

So if it is $a^{\frac{1}{4}}$ then it would be what? **(TII; SC, CU)**

105. L9: 4 to the square root of a

106. Jean: How do we say this? **(TII; TLM)**

107. L9: 4 to the square root of a

108. Jean: Ellah says 4 to the square root of *a*.

109. L3: Square root to the square root.

110. Jean: The fourth root of *a*.

111. Jean: And if I write (writes $a^{\frac{1}{5}}$) then it would be **? (TII; TLM; SC)**

112. Ls: 5th root of a (teacher writes $a^{\frac{1}{5}} = \sqrt[5]{a}$)

113. Jean: Now we have $32^{\frac{1}{5}}$ what does it mean? **(TII; CU; AR)**

114. Ls: 5 square root of 32

115. Jean: Mala you were trying. What were you saying?

116. L5: 5 square root of 32

117. Jean: If you were to write it, it would be 5th root of 32 **(TLM; TMR)**

118. L10: *iquestion yami ok bengifuna ukwazi ku5th root ka -32 loya -5 uvelaphi*?[my question is where does this fifth root of 32 come from] **(LCS)**

119. Jean: Yes what does the fifth root mean? Remember with the ½ (writes $81^{\frac{1}{2}} =$) 81 to the power ½, what does it mean? Let us write it with a square root. **(TII; TMR; CU; AR; SC)**

120. L11: Second root of 81, $81^{\frac{1}{2}} = \sqrt[2]{81}$
121. Jean: Second root ah! Have you ever seen the 2? **(TII; AR; SC)**
122. Ls: No
123. Jean: Do we write the 2? (Jean pointing to the 2)**(TLM; TMR)**
124. L5: No, second root.
125. Jean: Have you ever seen a 2 here? **(TII; TMR; AR)**
126. Ls: No.
127. Jean: So when we have not written the 2 it is the square root. This one is (writing 81^2 and $\sqrt{81}$; comparing the two). Square stands for 2's **(TLM; TMR; SC; AR)**
128. [L]: $10^{\frac{1}{2}}$ is equal to what, what does it mean?
129. L3: the answer must be 5.
130. Jean: What does it mean? **(TII; AR; CU)**
131. L3: Square root of 10 (Jean writes $10^{\frac{1}{2}} = \sqrt{10}$)
132. L3: Manje –ke uma kuwu10 ^{½} ? why singathi u-5 yi-answer? [So now if it's 10 ^{½} ,why don't we say 5 is an answer?] **(LCS)**
133. Jean: The answer here must be 5?
134. Ls: Yes
135. Jean: Are you saying the square root of 10 is 5? (**TII; AR; CU)**
136. L11: Yes. Why singathi 10 ^{½} =5, why singathi ngu1/2 ka -5 kuphele kanjalo?[why don't we just say 10 ^{½} =5 why not just say half of 10 is 5 instead of making things complicated] **(LCS)**
137. Jean: (Explains in home language but inaudible) So is this correct? **(TCS)**
138. Ls: No
139. Jean: According to the definition how should we write 10 to the power ½ (writes $10^{\frac{1}{2}} = 5$). And is it equal to 5? **(TII; TLM; TMR; CU; AR; SC)**
140. L1: No. *akanaso isquare root u-10* [10 does not have a square root] **(LCS)**
141. Jean: Remember this – if there is a 3 outside there must be 3 same numbers okay.
142. [L]: (Asks question in home language, inaudible)
143. Jean: This number outside there indicates how many numbers you multiply together to give you that number (Jean pointing to the numbers). In this case (pointing to $32^{\frac{1}{5}}$) how many of the numbers do you need? **(TII; TIM; CU; AR; SC)**
144. Ls: 5
145. Jean: So which number multiplies by itself 5 times? (there is no response)
146. Jean: We are having the fifth root of 32. We want a number that you multiply by itself 5 times and that number is what? **(TII; PF)**
147. [L]: 4
148. Jean: Okay let's go (teacher using hands/ fingers) **(TII; TMR;TSc; CU; SC)**
149. Ls: 4, 8
150. Jean: No! no! no!. Multiply by itself.
151. Ls: 4, 8

152. Jean: We are saying only 1 number. You started with 4 so it must be 4 all the way. **(TSc; PF, CU)**

153. Ls: 4

154. Jean: same number 5 times. Maybe I should write this. We need something like this (Jean writes $\sqrt[5]{-\times-\times-\times-\times-}$). We need a number that can fit in there.**(TMR; TLM; CU; SC;AR)**

155. Ls: 2

156. Jean: You all find 2

157. Ls: Yes

158. Jean: So it means 5th root of 32 is equal to (Jean writes $\sqrt[5]{32} =$) **(TLM; TMR; PF; CU; AR)**

159. Ls: 2

160. Jean: Ok. What then would be cube root of 27 (writes $\sqrt[3]{27}$ =) **(TII; TLM; TMR; CU; PF; SC)**

161. L1: The cube root of 27

162. Jean: (Completes $\sqrt[3]{27} = 3$). How else can I write it using a fraction, a rational power? How can I write that? Remember we are saying this law explains how we can write a rational **(TII; TMR; PF; SC; CU)**

163. L1: $\sqrt[3]{27} = 3$

164. Jean: I am going to take a different answer.

165. L2: 27^3

166. L3: 3^3

167. Jean: I am looking for a different way of writing this (pointing to $\sqrt[3]{27} = 3$) No I want to use a fraction. **(TII; TMR; CU; SC)**

168. L12: $27^{\frac{1}{3}}$

169. Jean: Anybody with a different form of writing this? Which of the three now is correct? **(TII; TMR; AR; CU; SC)**

170. Ls: $27^{\frac{1}{3}}$, second one.

171. Jean: What is 3 to the power 3? **(TSc;)**

172. Ls: 9, yes it is 27.

173. Jean: You are giving me 3 to the power 3, but I want the answer to be 3.

174. Jean: When we say we are writing same thing in just a different way we are not changing the value. Which is the correct one? **(TII; TMR; CU; SC)**

APPENDIX D2 : JEAN: LESSON 2

1. Jean: Yesterday I gave you an expression here, do you still remember? The one you could not say. The reason why I did that is sometimes maths can look like Spanish or?
2. [L]: Italian
69. Jean: But we want it to be real to us. We want to be familiar. For us to do that we really want to understand the meaning of the symbols that we are using. Meaning of symbols we talk about. Take the number 3^2. I have seen people express it in 2 different ways. Some are saying 3^2 is equal = 9 . Some are saying 3^2 is equal to 6. I want to see why can't people see it as one thing, because it should be one, the same way. (Jean writes 3^2 --→ 9 and -→ 6. Therefore that symbol 3^2 is saying different things to different people. Right? The person who introduced exponents wanted to use a symbol way of expressing the numbers we are used to Ok? And according to definitions they are saying $a^n = a \times a \times a \times a \times ----n$th. So it is a n times alright. So this number means something (pointing to the exponent n). They introduced a way of writing what we are used to. This is what we are used to (Jean points to $a \times a \times a$). $a \times a \times a$ is not a problem. But what is a problem to us is how this being represented now. **(TMR; TLM; CU; SC)**
70. Jean: They are representing $a \times a \times a$ in a different?**(TII; CU)**
71. Ls: way
72. Jean: What we know is now represented in another way (Jean bracketing $a \times a \times a$ and explains how the message was hidden in the sentence)
73. Jean: Remove the hidden message. What does this *n* mean (Jean pointing to the exponent *n*. When 3^2, write it as 3 x 3. Write it in the way that is familiar. Don't be afraid. When you see new things coming in mathematics do not be afraid, go back to what you understand because you need that information to understand. Nothing is wrong with going back. Do you understand?**(TCE; TMR; TLM; CU; SC; PL)**
74. Ls: Yes
75. Jean: I would like us to go to what we were doing yesterday cos we are going to another level. We are not only finding exponents of positive numbers, exponents but we also have to say now what if is $3^{\frac{1}{2}}$ or what if it is 3^{-1}. Bringing in now other kind of numbers that we haven't met, fractions and negative numbers. What is that you don't know? What is new to you? It is this foreign thing that you don't know. Ok? The two things mean the same. It is just this is hidden. It is hidden because it is not a usual it is a common way of writing. It is just written in a different way. It is a different representation of the same thing. **(TCE; TMR; CU; SC;)**
76. Jean: *Nyiyakutola manje*) [I understand you now]
77. Ls: Yes
78. Jean: So what do you do as a human being- your mind will be able to understand this if you can only go back to what you know (pointing to $a^n = a \times a \times a \times a$)

Remove the hidden message and the hidden message is what does this *n* mean (pointing to the *n* in a^n). It means so many of this thing multiplied together. Alright ? So when you see a^2 write it, go back to what you know. Write 3 x 3 so that you can remove the hidden message. Ok?. So 3^2 = 3 x 3 which will give you what? **(TLM; TMR; CU; SC)**

79. Ls: 9
80. Jean: Because this is very common to you (Jean pointing to 3 x 3) Do you understand? When you see $\sqrt{3}$ what does it mean? **(TII; CU; AR)**
81. L1: Square root of 3 .
82. Jean: Does anyone (pause) is there anybody who can remember the meaning of 3^{-1}? **(TII; CU, SC)**
83. L1: Square root of 3
84. Jean: Also square root
85. L2: 1/3
86. Jean: For us to remember lets go back to what we know. OK?
87. Jean: Let me say 3^2 divided by 3^3. You know this number (Jean pointing to 3^2). This is a different representation. It is the same number. **(TMR; TCE;)**
88. Ls: Yes
89. Jean: How much is it? **(TII; PF)**
90. Ls: 9
91. Jean: I am not changing the number. Still means 9 divided by ?**(TII; TMR; PF; CU; SC)**
92. Ls: 9
93. Jean: 9?
94. Ls: 27
95. Jean: What is 3^3 ?
96. L3: 27
97. Jean: What is 9 divided by 27? Remember you can write it as ?**(TII; TMR; CU; SC; PF)**
98. Ls: 9/27
99. L3: 0,3
100. Jean: Can we write it as a simpler fraction? Let us just simplify. How can we write it as a simple fraction? Simplify. 3 over ?**(TSc; TMR; CU; PF; SC)**
101. L3: 8, 3/8. *ngiye ku-table ngithe 3,6,9 ngathola 3,6,9 ngathola u-* [using tables 3,6,9 and got 8] **(LCS)**
102. Jean: 3/8 ? Do you agree?**(TII; AR; SC)**
103. [L]: 3/9
104. Jean: Is this simplified? **(TII; SC; AR)**
105. Ls: No
106. Jean: Get another simpler **(TII; TMR; SC; AR)**
107. Ls: 1/3
108. Jean: So this will be 1/3 . I am going back to 3^2 divided by 3^3 = 1/3. Nothing is hidden here. It is clear. Now this statement I can write it in a different representation.

109. Jean: I can write in a different form (Jean writes $\frac{3\times3}{3\times3\times3}$) using the definition 3^2 = 3×3 and 3^3 = $3\times3\times3$. We all agree? **(TMR; TLM; CU; SC; AR)**
110. Ls: Yes
111. Jean: Using canceling method, what is left at top? **(TII; TSc; SC; CU)**
112. Ls: 1
113. Jean: What is left below?**(TSc; CU)**
114. Ls: 3 (Jean demonstrates canceling and writes = 1/3)
115. Jean: Now we can use the law a^n divided by a^m. The law that says a^n divided by a^m is the same as what? Can someone finish it for us? **(TII; CU)**
116. L3: a^{n-m}
117. Jean: Now I am coming back here. 3^2 divided by 3^3 is equal to what according to this law? **(TII; TMR; SC; AR; CU)**
118. Ls: $3^2 \div 3^3 = 3^{2-3}$
119. Jean: and what is 2-3? **(TII; PF)**
120. Ls: -1
121. [L]: 3^{-1}
122. Jean: Which is that one there? Can you see that we have just shown that 3^2 divided by 3^3 can be written as 1/3 or it can be written as 3^{-1}. (Jean bracketing 1/3 and 3^{-1} and pointing to the two forms).**(TII; TMR; CU; AR; SC)**
123. L4: (demonstrates what he thinks can be a way to simplify) 3^2 divided by 3^3 . (learner cancels the bases, then writes 2-3 = -1/3
124. Jean: (clears the board to give space for learner to write down clearly what he is asking) Do it for us here. (Other learners begin to mumble) **(TII; TCE; CU)**
125. Jean: Let us not make him uncomfortable. He is trying to deal with this. **(TCE)**
126. L4: (Learner talks in home language as he continues to write on the board. Writes 3^2 divided by 3^3 $\frac{2}{3}-\frac{3}{3}=-\frac{1}{3}$
127. L5: (Another learner writes : $\frac{2-3}{3}=-\frac{1}{3}$ and talks very softly in home language as he writes. At this point there is a lot of discussion in different home languages)
128. Jean: (Jean allows learners to try the laws using their understanding) Remember whatever we agree here that it can be done like that must work for everything. **(TCE;TMR; CU**
129. Jean: (Jean writes on board $5^2 \div 5^4$) Moses try your law and see if it will work here.**(TCE; TLM; CU;)**
130. L4: (Writes on board as he explains to teacher) $5^2 \div 5^4 = \frac{4}{5}-\frac{2}{5}=\frac{2}{5}$ and $\frac{2}{5}-\frac{4}{5}=-2$
131. Jean: Ok just give us a complete final answer. What is your answer?
132. L4: It is negative 2 (Jean writes -2)

133. Jean: Before we go far, remember where we are coming from. We are coming from the fact that 3^2 means what? **(TII; TMR; CU)**

134. Ls: 3x3

135. Jean: Now if we are saying 5^2 divided by 5^4 what does it mean? **(TII; CU; AR)**

136. Ls: $5 \times 5 \div 5 \times 5 \times 5 \times 5 = ?$

137. Jean: We start there. Now what would be the answer according to the normal way you would do your division? (At this time there is a lot of discussions in home languages)**(TII)**

138. Jean: 25 over? Can someone come and write the answer?**(TII; TLM; TSc; SC; CU; AR)**

139. L2: (Talks to teacher in home language, inaudible)

140. Jean: Yes, yes you can.**(TCE)**

141. L2: (Learner works out the solution) $5^{2-4} = 5^{-2} = 5^2$. *kusho ukuthi mina Mam…… abawu – 2 sibakhanselile (5s). ngiqale kuma negative* [what I did mam is the two fives cancel . I started with the negatives and there are no negatives] **(LCS)**

142. Jean: So what is the answer? Ok now Lebo you, one person have written 2 different answers. Can you explain that? Which one do you go with? Do you take both of them? Lebo has 5^{-2} and 5^2. Lebo says for her this 5^{-2} and 5^2 work. Does this work? Is this the same as this?(teacher pointing to 5^{-2} and 5^2) Let him explain himself. **(TCE; TII; TMR; CU; SC; AR)**

143. L2: *mina kule ephezulu kumele sithathe unegative 4 u-2 upositive* –[what I did from the one at the top we have to take negative 4 and positive 2---] **(LCS)**

144. Jean: Yes I can see that. Yes that's right. Are you happy with this one? (method where the exponents are subtracted) **(TCE; SC)**

145. Ls: Yes

146. Jean: Ok fine. Explain this one now. **(TII; AR)**

147. L2: *angithi manje lena u-5, labo-5 basuke bakhanselana ngoba bekuyi division kwasala u-1 phezulu* [Now these 5's cancel each other because it is division and then we are left with negative one at the top] **(LCS)**

148. Jean: So how do you explain the one that is positive and the one that is negative? What is the problem here? **(TII; AR)**

149. L6: Sign.

150. Jean: Why? Why is it a problem? Why are these not giving the same answer? **(TII; CU; SC;AR)**

151. L1: *uyazi mam ukuthi ngiyithole kanjanileyo answer ? kusale u-1/25* [Do you know mam how I got that answer? What is left is 1/25] **(LCS)**

152. Jean: How are you doing that? How do you get 1/25? **(TII; AR)**

153. L1: *mina ngithe kanjenga leya oyenzile laphaya*[I did like the one you did over there] **(LCS)**

154. Jean: Can you come and show us? *Kusale u1/25 uma wenza kanjani*? *Woza* [how did you end up with 1/25? Come]. **(TCS; TII; TCE; CU; AR)**

155. L1: Learner writes $\frac{\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times 5\times 5}=\frac{1}{25}=0,04$

156. Jean: Why does it give us positive? What is the problem or was there any problem?**(TII; CU; AR)**

157. Ls: No problem

158. Jean: No problem so we can accept this?

159. Ls: Yes

160. Jean: As positive? So are you saying 25 is the same as 1/25?**(TII; TMR; CU;SC; AR)**

161. Ls: No

162. Jean: But why are you saying this and that are the same (Jean pointing to the 25 and 1/25 and calls on one learner). **(TII; TMR; CU; SC;AR)**

163. L3: (Learner writes on the board: $\frac{\cancel{5}\times\cancel{5}}{\cancel{5}\times\cancel{5}\times 5\times 5}=5\times 5=5^2$, cancels two pairs of 5', and writes = 5x5 = 5^2

164. Jean: $5\times 5\times 5$ comes from? Let us come to a conclusion. What are we expecting here between this and this? Are these the same thing? **(TII; TMR; SC; CU; AR)**

165. Ls: Yes , no

166. Jean: Is this the same as this? How is this different from this one? (Jean pointing) **(TII; TMR; SC; CU; AR)**

167. L6: *le ephezulu sifumene u lento ,u-5. Kule esenzansi sifumene u- ..u* [the one at the top we got 5 and then bottom--] **(LCS)**

168. Jean: Let us forget what answer we got. I am just saying the representation--- according to the definition, how we can represent 5^2?. According to the definition this (Jean pointing to 5^2) is the same as ?**(TII; TMR; SC; CU; AR; PF)**

169. Ls: 5×5

170. Jean: So whatever we get here should be the same.

171. Ls: Yes/ no

116. L7: *ama…abo -2 no-4 laba abaphezulu bazokunika i-minus u-2 no-4 mawuzi uthola u-2minuser .la uma uyazi multiply angeke uzenze njengaphezulu*[eh ! the 2 and 4 at the top they will give you minus 2. But if you multiply you can t do as you did at the top] **(LCS)**

117. Jean: (Writes and repeats learner's statement in English) Are you having a problem with this type of representation?

108. Ls: No

109. Jean: Do the two types of representation give you a problem? For those who say no (pause) it is okay to say no. You have the right to say no. Lyn are you one of those people saying no? **(TCE)**

110. L3: *The question engiqale ngayi-raiser mina ngokuthi uma ngiqale…. Mina bengenza amanegative signs, so ngicela ukubuza ukuthi if ngifaka ama*

negative signs. So ngicela ukubuza ukuthi if ngifaka amasigns ..is the problem iproblem yethu amasigns [the question you raised is that if I start – if use negative signs so I want to ask do I put the negative sign. Our problem is the signs] **(LCS)**

111. Jean: You can't have the same thing giving different answers. Is it a problem if the two representations give us different things? Between these two which one is right? Remember this sign is what? (Jean pointing to division sign). When canceling you are getting 1 and 1. Numerator becomes what?**(TII; TSc;AR; SC)**
112. Ls: 1
113. Jean: Denominator becomes what?**(TSc; CU; SC)**
114. Ls: 25
115. Jean: So it means this must be the correct answer. Don't you think so? Does this represent the same? **(TII; TMR; SC; AR)**
116. Ls: Yes
117. Jean: But you just said 5^2 is how much?**(TII; TSc; PF)**
118. Ls: 25
119. Jean: So is 25 the same as 1/25 **(TII; TMR; SC; AR)**
120. Ls: Yes / no. (At this point two learners check using a calculator)
121. Jean: What about 1/25
122. L5: 0,04. (Learner comes to the board to show his answer)
123. Jean: Let us go to a smaller number. Now this is about lunch ok? This is my loaf of bread. I am giving you 2 loaves wean[you]. I am giving '*rona'* [you]½ loaf. Are these the same?**(TII; TMR; CU; SC; AR)**
124. Ls: No
125. Jean: So is 2 and ½ the same?**(TII; SC)**
126. Ls: No.
127. Jean: Now is 25 and 1/25 the same?**(TII; SC; CU)**
128. Ls: No
129. Jean: So why are we getting different answers here? Are we happy? Confused? Let us come to a conclusion. We should understand that there is something wrong we are doing that is giving us different answers. We must be protected by our laws. Is it okay to write the answer as 1/25? **(TCE; TMR; CU; PD;**
130. Ls: Yes, '*yebo'*
131. Jean: Therefore 5^{-2} is the same as 1/25. Go to the calculator and find (pause) use calculator to find 1 divide 2. Does it give you 2?**(TCE; TMR; SC; CU; PD)**
132. Ls: No
133. Jean: What do you conclude? Whenever you see ½ it is the same as --- you can also read it as 1 divided by 2. Most of the time we don't see it that way. This is the problem with representation. Do you understand? These things are being represented in different ways and we fail to see they mean the same. So we must remember that in maths this is a very symbolic language Ok? Putting things in different ways doesn't have to confuse us. Let us go back and remember the basic. ½ comes from 1 divided by 2. 1 over 2 comes from 1 divided by 2. When you are getting 1 over 25 we should not write it as 25.

Remember 5^2 mean 25 and divide by 5^4 = 625. If you say 25/625 what do you get?**(TCE; TMR; CU; SC; AR; PD)**

134. [L]: 25

135. Ls: 1/25

136. Jean: As long as you move from one representation to another, you are on the right track. Use this to check yourself. When you are using negative exponents check them. Do not force it to go to what you want. Check yourself with a different representation. Go to the calculator, use the real number**(TMR)**

137. L1: When the bases are not the same?

138. Jean: We multiply. The thing is can we use the law? Can we use the law? In the case when the law does not apply, find other means. You cannot force a law where it does not apply. The laws are very strict. They control you. This is not the same condition. We only use this law if the bases are the same. In the case when bases are not the same, what would you do?

139. L3: (Learner writes 5^{-2} = 1/25)

140. Jean: This is working beautifully for us. $5^{-2} = \frac{1}{5^2}$. Then we can move on. When we have a negative exponent we can write 5^{-2} = 1/25.**(TCE; TMR; PF; PD)**

APPENDIX D3: JEAN: LESSON 3

1. Jean: I want us to look at those problems as different groupings of how we can represent numbers. Before we revise I want us to look at those groups. First group: we are just looking at the exponent. You can use an index of a natural number. Do you understand what I mean by natural numbers? Alright? Numbers that --- 1, 2, 3. The definition we got about exponents is the base has been repeated so many times. You mustn't be afraid to move on to those forms that you understand. You must be able to switch. It is just a different form of the same thing. You want it to go to your own reality, ok? You must be able to switch between these different forms when you have a natural number. **(TMR; CU)**
2. Jean: We can move on to the next representation and I have called this the rational exponent. You know rational numbers okay?
3. Ls: Yes
4. Jean: Although some of the natural numbers are rational numbers. We have seen that from definition that we can translate rational exponent into a root. (Jean puts on overhead and points to the rational exponent and the root. On the over head are the following: $4^{\frac{1}{2}} = \sqrt{4} = \sqrt{2\times 2}$

   $27^{\frac{1}{3}} = \sqrt[3]{27} = \sqrt[3]{3\times 3\times 3} = 3$

   $32^{\frac{1}{5}} = \sqrt[5]{32} = \sqrt[5]{2\times 2\times 2\times 2\times 2} = 2$

   When it is a 2 there, it means you have a square root. In maths we know that a square means 2.(**TLM; TMR; CU; SC)**
5. Jean: The 2 represents two numbers multiplied, the same number multiplied twice (Jean points to the 2 in the root sign). 3 represents three numbers multiplied (Jean pointing to the 3 in the cube root sign and the 3 threes under the root sign), the same number multiplied 3 times. 5 represents 5 numbers that are multiplied five times.**(TMR; CU; SC)**
6. Jean: My way of understanding the root is if you have a maize crop, the maize crop will come out as one stalk but underneath you have many roots. Right? I have made a diagram here to say that if that is your maize crop (Jean draws diagram) Square root means there are two roots underneath. Cube root, there would be how many roots?**(TMR; TSc; CU; AR)**
7. Ls: 3
8. Jean: But we are taking how many roots?**(TSc; CU)**
9. Ls: 1
10. Jean: So cube root of 27 is?
11. Ls: 3
12. Jean: In the case of $\sqrt[5]{32}$ you are saying there are how many roots?**(TII; TSc; CU; AR)**

13. Ls: 5
14. Jean: But you are taking how many of them?
15. Ls: one
16. Jean: But you are only taking one of them(Jean agreeing) which is what?**(TII)**
17. Ls: 2 (At this stage Jean writes the following on the board $\sqrt[3]{8}$; $\sqrt{49}$; $\sqrt[3]{512}$; $\sqrt{25}$; $\sqrt[4]{16}\ \times 10^6$)
18. Jean: I want us to interpret the way I have been showing. Remember the number that says how many roots you have. (Jean pointing to position in arm of the root sign). Remember the maize plant. Let us just stick to the number of roots. How many roots here (pointing to $\sqrt[3]{8}$) **(TII; TSc; CU; SC;)**
19. Ls: 3 roots
20. Jean: So which number will give us 8?
21. Ls: 2
22. Jean; (Jean writes $\sqrt[3]{2\times2\times2}=2$.) So the cube root of 8 is?**(TII; TLM; CU' SC)**
23. Ls: 2
24. Jean: Therefore the number is 2. For this one, how many roots?(pointing to $\sqrt{49}$)**(TII; TSc; CU; SC)**
25. Ls: 1
26. Jean: There is one?
27. Ls: Yes
28. Jean: Where is the 1? The number that is not written there (Jean pointing to $\sqrt{\ }$)?**(TII; CU; AR)**
29. Ls: Yes
30. Jean: How do you say this sign?**(TLM; CU, SC)**
31. Ls: Square root
32. Jean: Square root?
33. Ls: Yes
34. Jean: And we say that square root means?**(TII; CU; AR)**
35. Ls: 2
36. [L]: It is just a root because there is no number in the arm of the root sign
37. Jean: How do we say this that (writes $\sqrt{25}$ on the board)?**(TII; TLM; CU; SC;)**
38. Ls: Square root of 25
39. Jean: You say the square root of 25?
40. Ls: Yes
41. Jean: We say square root but we don't write the 2. When there is a 1, we don't have 1 square root. Therefore in this case I will have a number times another number. (Jean writes $\sqrt{49}=\sqrt{(\ )(\ )}\ =$) So that number will be what? (writes in the 7 in the empty brackets before learners answer the question)**(TII; TMR; TLM; CU; SC)**
42. Ls: 7
43. Jean: We don't have 1 root. We start from the square root. Let's go to this one, cube root. How many numbers am I multiplying together?**(TSc; CU)**
44. Ls: 3

45. Jean: Am multiplying 3 of them. (writes $\sqrt[3]{(\ )(\ )(\ )}=3$) So the number must be what? I want to get 512. It will be $8\times8\times8$ (Jean inserts the 8's in the brackets under the root sign). So what is the cube root of 512?**(TSc; TLM; CU; SC)**
46. Ls: 8
47. Jean: Are you getting it now?
48. Ls: Yes
49. Jean: Now let's go to this one $\sqrt{25}$. How many numbers do I have inside?**(TSc; CU)**
50. Ls: 2. (teacher writes $\sqrt{(\ )(\ )}$ **(TLM)**
51. Jean: So now we know if there is no number **(TMR; CU; SC)**
52. Ls: 5
53. Jean: So the answer is?
54. Ls: 5
55. Jean: This one, one of the numbers is expressed as a root and the other number an exponent. We have got two groups. So work them separately. Let's start with this one (pointing to $\sqrt[4]{16}$). How many roots? **(TSc; PF; CU)**
56. Ls: 4
57. Jean: How many? (writes $\sqrt[4]{(\ )(\ )(\ )(\ )}$) and that number must be what?**(TII; TSc; TLM; CU; SC)**
58. Ls: 2
59. Jean: 2 because I want that number to give me 16. But there is another one next to it**(TSc; CU; PF)**
60. Ls: Yes

61. Jean: 10^6 . Now what does that mean? Who can give me the meaning?**(TII; CU; AR**
62. L1: It means we multiply that 10 six times
63. Jean: Can you imagine if we did not have exponents. We would be writing like this (writes $\sqrt{(\ )(\ )(\ )(\ )}\times10\times10\times10\times10\times10\times10$) the maths book would be a biiig book. Exponents are useful. They are compressing. What would be the answer to for this?(pointing to $\sqrt[4]{16}$) **(TLM; TMR; CU; SC; PD)**
64. Ls: 2 (Jean writes 2 $\times$what)
65. Jean: What is the answer here (pointing to the part with 10x10 10)
66. Ls: 60; 10; $\times10$
67. Jean: Lyn is saying 1 million. So what is 2 x I million?
68. Ls: 2 million
69. Jean: So the answer is what?
70. Ls: 2 million
71. L1: (Inaudible :asks a question)
72. Jean: Is this the same as this (writes the two expressions: $\sqrt[4]{16}\times10^6$ and $\sqrt[4]{16\times10^6}$) **(TII; TLM; AR; CU)**
73. L1: No

74. Jean: What does this one require you to find? If it is the fourth root of 16 million then it would be (writes: $\sqrt[4]{16000000} = \sqrt[4]{(\ )(\ )(\ )(\ )}$ and the number that we multiply 4 times to get 16 million would be what? If we could divide these zeros into four then it would be a perfect number. First lets find the square root (writes $\sqrt{16000000}$)**(TLM; TMR;TSc; CU; SC)**

75. Ls: 4000

76. Jean: Yes (writes $\sqrt[2]{16000000} = 4000$). Then we want another square root. It is not a perfect number. It is not a whole number. It is not a rational number. So you see the difference? Do you see why we cannot do that? You must be careful. You must know where the root is going up to.**(TLM; CU; PF)**

77. Jean: Let's do the last one. What do you think it means when you have square root of a square root? This comes to something like a bracket of a bracket. How do you work it? Vee how do you work out bracket of a bracket? (There is no response from Vee)**(TII; AR)**

78. Jean: Let's say we have (teacher writes $\left((2)^2\right)^4 = (2\times 2)^4 =$ $(2\times 2)(2\times 2)(2\times 2)(2\times 2) = 2^8$. So how many 2's altogether? So if you have a power to a power you actually mean 2×4 which gives us 8. Which agrees with the law which says when you have got a power to a power is the same as 2 to the power 2×4. So what will be square root of 64?**(TMR; CU; SC; PF)**

79. Ls: 8

80. Jean: So we are actually looking for the cube root of 8 (writes $\sqrt[3]{\sqrt{64}} = \sqrt[3]{8} =$) which is how much?**(TSc; CU; PFSC)**

81. Ls: 8

82. Jean: What is the cube root of 8? (writes $\sqrt[3]{8}$) What does it mean?**(TII; PF; AR)**

83. L1: It means $2\times 2\times 2$

84. Jean: Which is now how much?

85. L1: 2

86. Jean: Now there is another way we could do this. We can express the root as fractions. How can I write as a rational exponent?**(TMR; CU; SC)**

87. [L]: Rational exponent?

88. Jean: Yes. I want to write it as (writes $\sqrt{64}$ =? and moves back to overhead and reads): square root of 4 is the same as $4^{\frac{1}{2}}$. So we can express this one as?**(TMR; TLM; CU; SC)**

89. Ls: 64 to the power 1 over 2

90. Jean: Now you are saying the cube root of that, I am putting a bracket (puts a bracket around the $64^{\frac{1}{2}}$) what does cube root mean as a rational exponent?**(TII; CU; AR)**

91. L2: $8\times 8\times 8$

92. Jean: No I just want to represent this

93. L2: power 1/3

94. Jean: To the power 1/3. So ($64^{\frac{1}{2}}$) you can also express your 64 as an exponent. Which number can you write as an exponent to give you 64?**(TII; TMR; CU; SCS; PF)**

95. Ls: 8

96. Jean: (writes $\left(\left((8)^2\right)^{\frac{1}{2}}\right)^{\frac{1}{3}}$. Alright? 64 is the same as 8 to the power?**(TMR; TLM; CU; SC)**

97. Ls: 2

98. Jean: But we still have ½ and 1/3 . (Blocks 64 and draws attention to ½ and 1/3). All we have done is to put 8^2 in place of 64. So then what is 2 x ½ (writes this on the side) **(TSc; TMR; CU; SC)**

99. [L]: It gives 1

100. Jean: So it means now I have now 8 to the power?

101. Ls: 1 over 3 (teacher writes $8^{\frac{1}{3}}$)

102. Jean: But I can also write 8 as an exponent. We can write a certain number as something to the power 3 (writes $(\ )^3$). Which number goes there?**(TSc; TLM; CU; SC)**

103. L2: 2

104. Jean: Ah so now $\left(2^3\right)^{\frac{1}{3}}$ is 2 to the power what; 1/3 will be the same as what?**(TII; CU; SC)**

105. L3: 8 to the power 1/3

106. Jean: We are here (pointing to $\left(2^3\right)^{\frac{1}{3}}$). Remember what we did to the powers. No we don't want to go to the 8; we just want to deal with the powers. So we finally get what?**(TSc; CU; PF)**

107. L4: 2

108. Jean: *Kanjan*i, which *kanjani*? Exponents are so diverse. They can be written in many ways. What we are doing is, we want to say exponents are—they can be written in so many ways. Okay? We don't have to be stuck on just one way. You can go this route of saying $\sqrt{64} = 8$ **(TMR; CU; SC; PD)**

109. L3: (Asks question in home language, inaudible)

110. Jean: This you can multiply your fractions. Remember how your multiplication of fractions. So here it is just fractions there. It is multiplication. We are multiplying

111. L1: (Asks another question in home language, inaudible)

112. Jean: Now let's go to *kanjani*. If you are not comfortable with this one you always have another place to go to. You are not happy with fractions. We are at a place where we have a power to a power; we can just multiply the two. You can just multiply the two powers. On mine this is what I have done.**(TCE; PF)**

113. Jean: Just high light to remind you because you are going to use this al the time. So to remind yourself. It is much easier when you work with the law because it has been simplified for you, you don't always have to go this route (pointing) But because we are still exploring, we really want to understand what it means, we can go back. We mustn't be afraid to explore. The law generalizes. I can use it when the power is a rational number, a big number. If I know the law it makes it much easier. If you understand the longer you don't have to use the other longer routes.**(TCE; PD)**

114. Jean: I want to finish the other two. We have looked at natural numbers, it doesn't matter how big the number is, if you see it you know it means this. If it is a rational number, you go to roots. If it is negative number what does it really mean now? What does 8 to the power -*3* mean? How am I going to interpret this? I know what 8 to the power 3 means

115. Ls: $8 \times 8 \times 8$

116. Jean: So what does 8 to the power negative 3 mean?

117. L1: 1/3 or -1/3

118. Jean: Where does it come from? Anybody?**(TII; CU; AR)**

119. [L]: Means 3/8

120. Jean: Someone else

121. L4: $\frac{1}{-3}$

122. Jean: Anybody else? Let us try and explore. You all have your--- whatever you are thinking. Lets go and explore now and it's your exploration that will clarify if this is okay or not.

123. Jean: Let's say I have $8^2 \div 8^3$. I am giving you two choices. You can do this using the laws of exponents or you can do it just dividing in the normal way. Ok? $\left(\text{writes } \frac{8^2}{8^3} = \right)$ What is the answer here? Let's explore both ways.**(TMR; TLM; CU; SC)**

124. [L]: The answer is 8

125. Jean: 8 to the power (2-3) .We are using the law here. So what is 2 – 3?—there comes-- 8 to the power -1. Now a negative exponent is coming in $\left(\text{Jean writes } 8^{-1}\right)$ Is this the same as this? (Jean pointing to the two different forms). Remember we did this yesterday. We can do it this way $\left(\text{Jean writes } \frac{8^2}{8^3} = \frac{8 \times 8}{8 \times 8 \times 8}\right)$ and we just do what?**(TMR; TLM; TSc; CU; SC)**

126. Ls: Cancel. (Jean shows the canceling on board)

127. Jean: What is remaining on top?**(TSc; CU)**

128. Ls: 1

129. Jean: What is remaining at bottom?**(TSc; CU)**

130. Ls: 8 (teacher completes writing = 1/8)

131. Jean: So it means this is the same thing but written in? $\left(\text{connects } 8^{-1} \text{ and } \frac{1}{8} \text{ on board by an arrow}\right)$ **(TII; TMR; CU; SC)**

132. Ls: different ways.

133. Jean: So it actually means 8 to the power minus 1 is equal to one eighth $\left(\text{writes } 8^{-1} = \frac{1}{8}\right)$ **(TMR; CU; SC)**

134. L4: (Asks question -- inaudible)

135. Jean: *sithole le problem,irepresentation yiyo eneproblem*[we had this problem, the representation is the problem] Are you happy with what (inaudible) is telling you? Are you happy?**(TCS; TCE)**

136. Ls: No she is not happy.

137. Jean: Can we deal with the problem, or is it way out of our understanding? This issue of representation. We are saying we are representing 3^2 in a different way. When we are getting higher-----When we are getting bigger--- Lets do it the way we are used to so that we are dividing here using the law--- we just say two minus three. Are you happy with that law?**(TMR; TIM; CU)**

138. Ls: Yes

139. Jean: Does it give us?

140. Ls: Yes

141. Jean: Can we divide according to grade 4?—we can just cancel. What is remaining?

142. Ls: 1/8

143. Jean: These are the same but giving me two answers. It means they are the?**(TII; CU)**

144. Ls: Same

145. Jean: Can I accept that this and that must be the same although they are written in different ways?

146. Ls: Yes

147. Jean: This and this are the same it means they are giving me the same thing

148. L1: *Ngeke Mam Mam, Mam, Wo Mam! Wo Mam!Ngicela ukubuza. Angithi Mam usifundisa iMaths njengothisha wethu wemaths uthi kuma quadratic equations maybe u-x wethu abe yi -2 bese i-exponent epositive. Why -ke laphaya ngoba le ipositive le inegative*[no ways mam wait. can I ask a question As our maths teacher you say quadratic equations if you have x is negative 2 and the exponent is positive why then there one positive the other negative?] **(LCS)**

149. Jean: What is negative here?**(TSc; CU, PF)**

150. Ls: 1

151. Jean: What is positive here? **(TSc; CU; PF)**

152. Ls: 8

153. Jean: May be I can use a different example. Let's say $8^2 \div 8^5$. What will be the answer?

154. Ls: 8 to the negative 3

155. Jean: Lets go to the $\left(\text{writes on board: } \frac{8\times 8}{8\times 8\times 8\times 8\times 8} \text{ and cancels}\right)$. The answer is?**(TMR; TLM; CU; SC; PF)**

156. Ls: 1 over 8 to the power 3

157. Jean: Now what is this? When the number is left as 8 my exponent is?**(TII; CU; PF)**

158. Ls: Negative

159. Jean: When we write as a fraction and take the 8 to the denominator then my exponent becomes what?**(TII; CU; PF)**

160. Ls: Positive.

161. Jean: It means these are still the same things. When my exponent goes to the numerator the exponent is negative. When the exponent goes to the denominator it is?**(TMR; CU; PF; SC)**

162. Ls: positive.

163. Jean: It is just two things. It is written in a different form. Two things that are exactly the same. So we are not saying sometimes a negative number is positive. This is in the denominator. This one is in the numerator.

164. Jean: If I have 2^{-5}, how will I represent it now in this form?**(TII; TMR; CU; SC; PF)**

165. L4: $-2\times-2\times-2\times-2\times-2$

166. Jean: I want to write it as fraction. It would be 1 over what?**(TSc; CU; SC)**

167. Ls: 1 over 2 to the power 5

168. Jean: This means there are five 2's in the denominator. But because they are in the denominator I write as a positive. When they are in the numerator?**(TII; TMR; CU; SC; PF)**

169. L4: Can we say 1 over 10?

170. Jean: You have to identify the exponent. What is the exponent here?**(TII; CU; SC)**

171. L4: There is no exponent

172. Jean: There is no exponent? What is the exponent?

173. Ls: 1

174. Jean: So in the numerator we write 10^{-1}.

175. Jean: We still have to deal with zero exponent which will be last grouping.

APPENDIX D4: JEAN : LESSON 4

The lesson started with learners working in groups simplifying $2^3 \div 2^3$

1. Jean: Is this equal to that? $\left(\text{Jean writes Is } 8^{-1} = \frac{1}{8^3}?\right)$. This was from yesterday. How does this compare with the problems. (Jean moves around the groups and explains in English in these groups). Let's finish up. Those who have not finished will catch up in the discussions.
2. L1: (Asks a question in English)
3. Jean: Good excellent. What we are going to do is we will ask one group to give us feed back. If your answers are not the same as ---(inaudible). Remember here the answer you got it by using your exponents. So which group is going to give us their first?**(TCE; TII)**
4. L2: 2^2
5. Jean: And for this one? If you have a different answer, give us your answer. (learner gives another answer). Any other difference?
6. L3: 1 to the power 3
7. Jean: Any other?
8. L4: 1 to the power zero, 5 to the power one; 8 to the power one
9. Jean: Now let us look at why we are having differences. For those who said they got 2 to the power zero, explain. It is not only them, there are many others.**(TII; AR)**
10. L5: When we were dividing bases we minus the exponents.
11. Jean: So you said 2 to the power 3
12. L5: minus 3
13. Jean: Which gives you 2 to the power?
14. Ls: zero. sithe 2^0 [we got 2^0] **(LCS)**
15. Jean: nithe 2^0 uthole u2 [You said you got 2^0 the answer is 2] **(TCS)**
16. Jean: Those who got 1 to the power zero, how did you do it?**(TII; AR)**
17. L6: mina ngithole la ngi OK ngapha ngithole u-1/1 [I got 1/1] **(LCS)**
18. Jean: idifference ila, [the difference is here] **(TCS; TII)**
19. L7: sithole u-1^0 [We got 1^0] **(LCS)**
20. L7: *ngithe 2/2 =1 then sathi 3-3=0* ….[I said 2/2 = 1 and then we said 3-3 =0.] **(LCS)**
21. Jean: You said 2 divided by 2 is 1 and 3 minus 3 is zero. Okay. And those who got—**(TII)**
22. [L]: Indlela eziningi [there are many ways of doing the sum] **(LCS)**
23. Jean: The house is divided. We have to come to an agreement. Okay? (On the board is written ; $2^3 \div 2^3 = 1^0$ and $2^{3-3} = 2^0 = 2^1$
24. Jean: Why are you saying zero is 1? Can you explain to us? Someone else in the group.**(TII; CU; AR)**
25. L8: If *x* to the power zero is the same as *x* to the power 1.
26. Jean: Give us the law that says that. You have got the paper, right? (Jean encouraging learners to refer to where the laws are written). We have got law

1, law 2, law 3. Is it law 1 or 2 or 3? Which one is that? Or did you get it from a different paper?**(TII; AR)**

27. Ls: (Learners try to find the law)
28. Jean: Here you know what an exponent means? Yes?
29. Ls: Yes
30. Jean: What does the exponent mean? You are repeating the base so many times. So when it is zero are we repeating the base?**(TII; AR)**
31. Ls: No
32. Jean: When it is 1, are we repeating the base?**(TII; AR)**
33. Ls: No
34. L4: Yes we are repeating the base one time (other inaudible discussions in home language in groups)
35. L1: kurepresentor mam ukthi ibase asiyimultiply iyi-one injalo --- [it represents mam --- that we don't multiply the base, the base is the same] **(LCS)**
36. L2: Sithe……., sathola ukuthi inebase…… [We say that--- the bases are the same] **(LCS)**
37. Jean: Would this then mean the same thing?**(TII; AR)**
38. Ls: Yes / no
38. L3: Mam le injengaleya Mam le ephezulu [Mam this one is the same as the one at the top] **(LCS)**
39. Jean: I would like other groups to respond to their argument because they are saying *u zero* can be 1. Can you respond to that?**(TII; AR)**
40. L7: Ngeke uthi ngingekho eskoleni uthi ngikhona, ngeke uthi u-0 ngu-1 into engekho leyo, never Mam [you can't say when that Iam not at school I am at school. You can't say zero is 1 there is no such, Never mam] **(LCS)**
41. L8: It means ukuthi u- x^0 uthi there are no numbers kanti *x* this side uyi-one, u-*x* urepresentor u-1 [it means that x^0 it means there no numbers and this side there is one *x*, *x* represents 1] **(LCS)**
42. L6: uma ngizothi maybe x ne,bese ngithi 1/x angithi uyazi ukuthi lona omunye u-x awubhalanga u-1….. sesiyazi ukuthi u-x wethu uyi-one. [if I say x and then I say 1/x and you know that the other x I don't write 1----We know that x is one] **(LCS)**
43. Jean: (To another learner) I want you to explain how you moved from 2^0 to 2^1. How are you moving there? **(TII; AR)**
44. L6: thina sithe i-exponent laphaya ngoba sazi ukuthi u-0 urepresenter u-1 makahamba---([we say it is the exponent over there because we know that zero represents one] **(LCS)**
45. Jean: Are you saying this is the same as this? (pointing to 2^0 and 2^1.) Even those learners who had been quiet have joined in the argument) Can you come and write it?(cleans the board for the learner to write) **(TII; TLM; TCE; CU; AR)**
46. L5: $\begin{pmatrix} \text{Learner writes:} \\ 2^3 \div 2^2 = \\ 2^3 \div 2^3 = \end{pmatrix}$ and goes to sit down

47. L9: u zero u representor 1 [zero represents 1] (other learners argue). It's the law. (Learner goes to the board and crosses out the 2^1 and writes in its place 1). It's the law . If there is zero it means there is one of them. (Another learner raises the text book) There is a lot of discussions and arguments about this. **(LCS)**

48. L7: u-2^0 is 2^1 ? wo wo wo uyazi ukuthini ngicela uthathe u 2^0 ubuyisele u-2^3 then uthathe u- 2^1 umbuyisele back u-2^0 ukuthi uzokunika i-answer eyi-one [2^0 is 2^1 wait wait wait you know what remove the 2^1 and bring back 2^3 and take 2^1 and bring back 2^0 its the same answer] **(LCS)**
(Learners now argue about the following statements which are already written on the board)

$$2^3 \div 2^3 = 1^0$$
$$2^{3-3} = 2^0 = 2^1$$
$$x^0 = x^1$$

49. L13: Mam ngivumelana nabo L6 ibase ine-exponent u-1 noma i-exponent ingabhalwanga ukhona u-1 [I agree with L6 base has 1 as its exponent even though the exponent is not written] **(LCS)**
50. L7: Ibase yakho u-1 hhayi i-exponent ewu zero izoremainer iwu- 0 ngeke ichange isala injalo [if your base is 1 and the exponent is not zero it doesn't change, it remains the same). **(LCS)**
51. L7: The law says akumelenga uyichange so it doesn't make sense [the law says we musn't change, ---] **(LCS)**
52. L7: Akumelanga nithi u-0 akuna -2 [you mustn't say its zero there is no 2] **(LCS)**

53. L11: Ngicela ukubuza [Can I ask a question?] If you have 2 to the power 3 divided by 2 to the power 2, what will be your answer here? **(LCS)**
54. Ls: 2 to the power 1
55. Jean: She is saying it does not make sense.
56. L10: (Comes to the board and cancels 2^1 and replaces it with 1)
57. L4: (another learner comes to the board and explains but it is inaudible, there is too much noise, but it appears the rest do not agree with him and he goes back to his seat)
58. L11: Ngicela ukubuza [Can I ask a question?] what the difference is between exponent and base?
59. Jean: You are asking that?
60. L11: Yes because bona into abayenzile bathe kucalculator …[this is what they did they ---- showing calculator] **(LCS)**
61. Jean: (Gestures to the learner to write on the board) **(TLM; TII; CU; SC)**
62. L11: (writes on the board 'what is the difference between B & E?). Calculator says 2 exponent 3 $\div$ 2 exponent 3 (writes 2E3 $\div$2E3 =1).
(There is still a lot of discussions and argument. The learner moves around showing other learners what is on her calculator Then she completes the writing as $2^3 \div 2^3 = 2^{3-3} = 2^0$ √ ; putting in the tick herself)

63. L12: Bona into abayenzile bathe kucalculator ---[This is what they did --- showing calculator]. Bathathe as if ibase bayenze i-exponent [what they did they took the base and made it an exponent] **(LCS)**
64. L14: Nina into eniyenzile nithathe u-2 namenza i-exponent ka -3 [it means that what you did you took 2 and made it an exponent of 3] **(LCS)**
65. L9: Mamela amabase uma …. Amabase they remain the same bese uminusa ama –exponent u-0 urepresenter u-1 [Listen if the bases are the same and you divide the bases they remain the same you subtract the exponents—zero represents 1] **(LCS)**
66. Jean: I am going to give you a chance, but before I give you a chance Lyn is this what you are saying is your answer?
67. L9: Yes
68. Ls: No (More arguments and discussions)

69. Jean: If I have this $\begin{Bmatrix} x=1 \\ 2x= \\ 3x= \end{Bmatrix}$ **(TMR; TII; CU; SC;**
70. L11: (says something to the teacher, inaudible)
71. Jean: Tell them, tell them
72. L11: (reads something form her book, inaudible)
73. Jean: If you have 1 to the power zero is equal to? (writes $1^0 =$) **(TII; CU; SC; PF)]**
74. [L]: 1
75. Jean: 2 to the power zero is equal to? **(TII; CU; SC; PF)**
76. Ls: 1, 2, 0 (arguments start again)
77. Jean: I am closing on this note. I don't have an answer, you have the answer. 3 to the power zero would be what?
78. Ls: 3
79. Jean: I am changing, now I really understand you. I really now understand you. She is saying: $\begin{pmatrix} 1^0 = 1^1 = 1 \\ 2^0 = 2^1 = 2 \\ 3^0 = 3^1 = 3 \end{pmatrix}$. Now I am going to ask you. What is 1 to the power 1, 2 to the power 1 and 3 to the power 1? **(TII; TMR; TLM; CU; SC)**
80. L9: It is the same as 1, 2 and 3. That's all.
81. Jean: If you are saying that then you are saying 1 to the power zero is 1 and 2 to the power 1 is 1.
82. Lsa: Yes
83. Jean: Now let's come to this (writes: $2^3 \quad 2^2 \quad 2^1 \quad 2^0 \quad 2^{-1}$) and so on. You see I am decreasing my exponents by 1. So give me the answers. **(TII; TLM; CU; SC; PF)**
84. [L]: is equal to $2 \times 2 \times 2$ which is equal to 9.
85. Ls: 8.
86. Jean: Okay. This one? **(TII; CU; SC; PF)**

87. Ls: 4
88. Jean: 2 to the power 1? **(TII; CU; SC; PF)**
89. Ls: its 2
90. Jean: 2 to the power zero? **(TII; CU; SC; PF)**
91. L9: its 2 to the power 1, which is equal to 2
92. Jean: And this one: **(TII; CU; SC; PF)**
93. L7: It will be 1 over 2
94. Jean: When I am decreasing my indices, it looks like the number to be decreasing as well here, but when we get here (pointing to 2^1 and 2^0) we get stuck.
95. L9: Which means, mam, that these two things are the same?
96. Jean: So it means 2^1 and 2^0 are the same? There is no difference? **(TII; CU; SC; PF)**
97. L9: Yes mam
98. L9: So it means zero is the same as 1
99. Jean: So 1 represents zero? **(TII; CU; AR)**
100. Ls: Yes
101. Jean: So instead of 1 I can just put zero? (Jean writes 1 = 0 and frowns). One day if you find yourself in court you must stand your ground. I am not saying you are right or wrong. This is what is important in mathematics when we are making a mathematical argument. You must look at things like this. The effect that 1 has on the number is it the same as the effect that zero has on the number. If the effect is the same can we argue that 1 is the same as zero? If I do the same operation with 1 and with zero and I get the same answer am I implying that 1 is the same as zero? If not then you must start revising. What we are all agreeing is the stage with 2 to the power zero. Can we safely say 2 to the power zero is equal to 1?(Jean writes: $2^0 = 1$) **(TLM; TMR; SC; AR)**
102. Ls: Yes
103. Jean: This is what I want us to compare with what we got yesterday. Can we say the representations are just the same? So we can say $2^0 = 1$

APPENDIX D5: DAVID : LESSON 1

1. David: I want four people to come to the board and write their interpretation of $a^0 = 1$ in words on the board.**(TII; CU)**
2. Ls: (Four learners write their interpretations on the board)
3. David: Reads what the learners have written on the board: When the exponent of a base is 0 the answer is 1; anything to the power 0 is equal to 1; If a power is divisible by 0 it automatically equals to 1; Any number to the power 0 **(TLM)**
4. David: Ok let's see. Anything to the power 0 is equal to 1
5. David: Any response to what is written on the board and what we had to translate was (teacher writes a^0 at the top of the board). That's the translation we had. Which of them describes it?**(TII; SC; AR)**
6. L1: When the exponent of a base is naught the answer r is 1
7. David: Let's check what he has done. When exponent (David points to the exponent 0) of a base (David now points to the base *a*)is equal to? Let's read it when the exponent of a base is naught the answer is 1. So that one is ok? **(TLM)**
8. Ls: Yes
9. David: Why are you saying this is better
10. L1: Because *a* is the base
11. David: Did you use that a as a base? Are you referring to that a as the base or (David inaudible) What are you using that *a* for?**(TII; CU; SC)**
12. L1: I am not using it as base
13. David: If I had x to the power naught (David writes $x^0 = 1$) Who is responsible for that? Would this *a* be *x*?**(TII; SC; CU)**
14. L1: No
15. David :Okay the next one if the exponent of a base is naught the answer is 1
16. L2: (reads the next translation) any number to the power of naught is equal to 1.
17. David: So this a becomes any number now?**(TII; CU; SC)**
18. Ls: No; never
19. David :We will come back to the letter number issue later.-------
20. David :Who is responsible for this one(David pointing to the last translation)
21. L3: (acknowledges he wrote it)
22. David: Okay explain it. **(TII; AR)**
23. L3: I think it is wrong. I realize it now. It should be if the exponent of a power is zero the number equals to 1
24. David: So you are going to abandon this one(pointing to the original translation)
25. L3: Yes
26. David: Why?**(TII; AR)**
27. L3: That doesn't make sense to me
28. David: It doesn't make sense to you, you abandon it and change it to what?**(TII; AR**

29. L3: If the exponent of a power is zero the number is equal to 1
30. David: Anything to the power of naught is equal to 1 are we accepting that one?**(TII; AR**
31. [L]: No(only one learner says no) (L3 comes to the board and changes his translation)
32. David: Okay, take out your calculators. If you say anything to the power naught is equal to 1. Okay put on you calculator 0^0 is equal to? Read your screen.**(TII; AR; SC)**
33. L5: maths error
34. David: Maths error. Why is it saying maths error?**(TII; AR)**
35. L4: Because zero is not classified as a number.
36. David: Because zero is not classified as a number?**(TII)**
37. L4: Not classified as a whole number
38. David: So it is not classified as a whole number?**(TII; AR)**
39. L5: We should change that to any whole number
40. David: Any whole number? I want to come back to your zero if it is a number or not. **(TII; CU)**
41. L6: The base is not equal to naught.
42. David: Why? The translation says anything to the power naught is equal to 1. Anything so I can choose naught but it is failing to give us an answer. Now the two of them are saying naught is not a number, so it must change to what?**(TII; AR)**
43. L5: To any whole number to the power zero
44. David: (repeating) any whole number (David putting emphasis on whole). Abel are you going to respond to what we are saying not any number but any whole number? **(TII; AR)**
45. L1: Yes can't we say anything to the power naught is equal to 1 except naught?
46. David: Except naught?**(TII)**
47. L1: Yes
48. David: Anybody else? He says anything to the power naught is equal to 1 except naught… and what about the whole number issue? **(TII; CU, AR)**
49. L6: (inaudible)
50. David :(inaudible)
51. L2: Anything to the power of a whole number is 1
52. David: (repeating) anything to the power of a whole number is 1? So it comes back to the whole number issue? (There is discussion among several groups.David tells them to be quiet) **(TII; AR)**
53. L2: (Explains but inaudible)
54. David: it's the whole number issue again, why is it not a whole number?**(TII, AR)**
55. L1: it does not have a value.
56. David: So if something does not have a value it is not a whole number?**(TII; CU; AR)**
57. L1: naught is not a whole number because it does not have a value
58. L8: it is a place holder
59. David: Naught is a place holder?**(TII; CU; AR)**
60. L2: It is alone

61. David: Alone where? (learners laugh)
62. L1: (Repeating his previous statement) Anything to the power naught is 1 except naught
63. David: The part with except naught. Give it to me in a mathematical symbol. Just the part exclude naught. **(TMR; SC)**
64. L1: (learner comes to the board and writes $x^0 = 1$
$0^0 \neq 0$
65. David: (inaudible)
66. L1: x stands for any number and naught to the power naught is not equal to 1(learner corrects what he had written so that the second statement is $0^0 \neq 1$
67. David: Someone said no (pointing to a learner who had said no)
68. L2: Calculator said undefined and they wrote something?
69. David: Undefined?
70. L2: Yes
71. David: You said naught to the power of naught (teacher writes on the board $0^0 = \infty$) Is that what you are talking of? Hmmm? I am using the symbol for undefined(David pointing to the symbol) **(TMR; SC)**
72. L7: Sir what if the base was a negative number?
73. David: What if the base was a negative number?**(TII; SC; AR)**
74. L2: Yes
75. David: For example…? Anyone? What if the base was a negative number?**(TII; SC; AR)**
76. L8: If the base is a negative number it is equal is negative 1.
77. David: (repeating the statement) If the base is a negative number it is equal to negative 1
78. Ls: Yes
79. David: We are looking at the base and the exponent(David makes gestures for position of base and exponent as he says the words) **(TII; CU; AR)**
80. L2: As long as the base is naught it is still undefined
81. David: (repeating) as long as the base is naught it is still undefined? We are excluding the naught only so to me we include everything else. I have asked you to translate the last part, excluding naught in mathematical symbol. He translated excluding naught to that (David circling the translation written by $0^0 \neq 1$) We need to look at is the translation correct but her argument is naught to the power naught is undefined from the calculator am I right?(teacher pointing to the learner who gave this answer). We are looking at is the translation correct? Does that mean exclude naught? Can the base be equal to naught? Can the base be equal to naught? **(TII; CU; AR)**
82. Ls: No.
83. David: So translate that.
84. L1: If the exponent is not equal to naught.
85. David: (Inaudible)
86. L1: Yes
87. David: Okay the exponent should not be equal to naught, then we change this(teacher reading from first translation) when the exponent of a base is

naught the answer is 1. She is not contesting the base, you are contesting the base as well. I want to translate this --- you said it means $a^0 = 1$ right? And there is the translation. There is an exception rule that says that what cannot be equal to naught?**(TII; TMR; TScCU; SC)**

88. Ls: The base

89. David: The base cannot be equal to? Naught so from that let's translate to symbol. The base is not equal to naught. Translate that into mathematical symbol. Let's start with the base. What can we place in place of the base?**(TSc; TMR; SC; CU)**

90. L4: *x*

91. David: In place of the base we put *x*, right? (David writes *x* on the board) is not equal to, what shall we write?**(TMR; TLM; TSc; CU; SC)**

92. L4: The equal sign with a line.

93. David: Is not equal to?**(TSc; TLM; CU)**

94. Ls: zero (David complete writing $x \neq 0$ and high lights it by drawing a rectangle around it) .

95. David: In algebra what does x / a represent**(TSc; CU; SC)**

96. Ls: anything; variable

97. David: What does the *x* represent? **(TMT; TSc; CU; AR)**

98. Ls: the base

99. David: What does *x* represent?

100. L4: variable

101. David: Also a variable. Can I replace the *x* with *a?* **(TMR; SC; CU)**

102. Ls: Yes

103. David: (David replaces the x with a so that it now reads $a \neq 0$) now we have *a* to the power naught is equal to 1 but *a* cannot be equal to naught. Then we can come to your translation anything to the power naught is equal to 1 except naught.

104. David: Now lets us mark the homework. We mark exercise 2 number 1 a-g (David asks one to come to the board and asks for volunteers for the rest).**(TII; TLM)**

105. Ls: (learners write their solutions on the board): *a*) $x^{-3} = \frac{1}{x^3}$ b) $2y^{-9} = \frac{1}{2y^9}$ c) $3t^{-6} = 3t^{\frac{1}{6}}$ d) $\frac{2}{r^{-2}} = \frac{2r^2}{1}$

106. David: What is the law? (David asks a learner to write the law on the board)**(TII; TLM; AR)**

107. L9: I don't know sir

108. David: If I had a 5 there, what can I use a number that represents anything?**(TMR; SC; AR)**

109. Ls: *a*

110. David: Anyone else give me a law?

111. L10: (writes on the board $a^{-m} = \frac{1}{a^m}$)

112. David: beautiful. Read that in your own words. How do you read that?**(TLM; CU)**

113. L11: *a* to the power of negative *m* equals 1 over *a* to the power *m*.

114. David: (David repeats while pointing to the law on the board). So the first one is correct and that is the law we used.. This one(teacher pointing to the b))
115. L10: The 2 is supposed to have--- (inaudible)
116. David: Why?**(TII; AR)**
117. L10: It is in the study guide (other learners laugh)
118. David: Okay it is in the study guide but I need to know why. That negative 9 whose exponent is it?**(TIL; CU; SC)**
119. L10: y
120. David: What is the 2 exponent?**(TIL; CU; SC)**
121. L5: It is a positive 1
122. David: It is a positive 1. Do you see that? So what happens is the, this negative belongs to the? base y (David pointing to the y and the -9) so it is y to the power of negative 9 and 2 to the positive 1. So that 9 is correct. The 2 must go to the top. The study guide doesn't show this (teacher writes $\left(\begin{array}{l} \frac{2}{1}\times y^{-9} \\ \frac{2}{1}\times\frac{1}{y^{9}} \\ =\frac{2}{y^{9}} \end{array}\right)$

The law must apply where it belongs. Okay let's look at c. Is c the same as b?**(TIL;TMR; TLM; CU; SC)**
123. Ls: Yes
124. David: Who is responsible for c? Read your answer. **(TLM)**
125. L12: Its *3 t* to the exponent of 1 over 6
126. David: 3 t to the power of? 1 over 6. (David re-writes the answer).Can you explain to us which law did you use?**(TII; TLM; SC; AR)**
127. L12: I used this one
128. David: Read the law.**(TII; TLM; CU)**
129. L12: (Inaudible)
130. David: No read the law.**(TII; TLM; CU)**
131. L12: a to the power of divided by (inaudible)
132. David: Which one are you reading? Read the law. Which law are you reading? Law number? (David moves to look at where the learner is reading from) Which one? No there is two which one? She is reading on page 2. She is reading the following law. It says negative indices: consider the second index law *a* to the power *m* divided by *a* to the power *n* and it says so . Do you see that law? She is reading that. Read it; let's hear.**(TII; TLM; AR; CU)**
133. L12: *a* to the power of *m* divided by *a* to the power *n* is equal to *a* to the power *m* minus *n* so *a* to the power naught divided by *a* is equal to 1 divided by *a* is equal to 1 over *a*
134. David: Is that the law she read? How is that law different from the one we have on the board? What did she read there? Did she read the law? Read it. **(TLM; AR)**
135. L12: Reads if *a* to the power naught divided by *a* equals 1 divided by a equals 1 over *a*. Therefore *a* to the power negative 1 equals 1 over *a*

136. David: What she has read there, is it the law? If she read the law how is it different from the law there $a^{-m} = \frac{1}{a^m}$?(Silence for a while) She says she read the law but how is it different from a to the power negative m equals to 1 over a to the power m? or is it the same?**(TII; AR; CU)**
137. L10: It is the same
138. David: Just read the last part. **(TLM; AR;CU)**
139. L10: This one?
140. David: Yes
141. L10: (learner reads)
142. David: Just the last part. This leads us to—
143. L10: This leads us to a to the power negative m equals 1 over a to the power m
144. David: The things that come in front, all the things that, what do those things do? See what I am getting to? What she read was it a law? What did she read?**(TII; TIL; AR; CU)**
145. L10: It lets us the way to the law.
146. David: I am trying to get to what she read before she got to the law. What was that she read before? She read a lot of symbols. **(TII; AR; CU)**
147. L13: I don't understand the question.
148. David: You don't understand the question? Did you follow what she read
149. L13: Yes sir
150. David: Now I am asking what did she read? And we ended with: a to the power negative m equals 1 over a to the power m. So what did she read before we got to that? Is that the law she read? I want you to read that section four; everything up to where it says this leads us to: Anyone who volunteers?**(TII; CU; AR)**
151. L4: (learner volunteers and reads)
152. David: Why does it say this leads us to? We have lots of symbols at the beginning and then we come this way.**(TII; AR; CU)**
153. L1: Where it says this leads us to a to the negative m equals to 1 over a to the power m, that's the law
154. David: That's the law. We have that as the law. So what comes in front of the law? What was that all about? Why do we have that in the notes?**(TII; AR; CU)**
155. L14: Sir I think it is to explain how the law is done
156. David: To explain how the law is done? Anyone else?
157. L1: Sir it is a conclusion, basically
158. David: Yes we draw conclusions from what? That is what she said. What did you say? **(TII; CU; AR)**
159. L14: It is to show how the law is done
160. David: What do you call this in geometry? If I make a statement in geometry how I use it, how I get it? **(TII; AR; CU)**
161. L6: Proof
162. David: Yes. So that is the proof to show that that law works. You see that. You can see how we derive. The proof is not just for geometry (David reads the derivation of the law again)

Start with something that we know, what we have now is $a^0 \div a$. How do we get from $a^0 \div a = a^{0-1}$? Apply the law. Where does -1 come from?

163. David: (writes $\boxed{\begin{aligned} a^m \div a^n &= a^{m-n} \\ a^0 \div a^1 &= a^{0-1} \\ &= a^{-1} \end{aligned}}$ But a to the power naught is equal to 1. So a to the power naught divided by a to the power 1 but $a^0 \div a^1 = 1 \div a$ and I can write 1 divided by a in another form as 1 over a (David completes writing: $= \frac{1}{a}$) Therefore a to the power negative 1 is equal to 1 over a (David pointing to the two forms). And that is the proof. The proof is not only restricted to geometry. We can use the proof here. We use what we have. Now back to c. I am not going to accept that. **(TLM; CU; PF; SC)**

164. L2: Sir I was taking the negative 6 to the positive side so I put 1 over 6

165. David: Look at the law. What does the law say? Does it put it as m over 1? Look at this that is the base that is the exponent that is the power (David pointing to each). What happens when I apply it? So now I use that law I am just going to apply it to the t. So 3 to the power 1 times t to the power of negative 6 (David writes $3^1 \times t^{-6}$) We split the numbers . -6 belongs to the t. Now we apply the law and it becomes 3 over 1 times 1 over t to the power of 6(David writes $\frac{3}{1} \times \frac{1}{t^6}$) I multiply and get 3 over t to the power of 6 (writes $\frac{3}{t^6}$). Now d. To me it looks like you just moved the r to the top. Is that what it is? Why did he move it up? Can I do that?**(TLM; TSc; TIL; TMR;TII; PF; CU; AR)**

166. L1: Yes sir

167. David: Can I do that?

168. L10: No sir.

169. L10: Sir obviously the *r* is negative and then the over 1---we don't need to put over 1 because anything over 1 is equal to itself

170. David: We can write the answer as $2r^2$ because anything divided by 1 stays the same.

171. David: Which law did we use? According to Sizwe, this is what he is saying (David writes $\frac{1}{a^{-m}} = a^m$ Is it true? **(TII; AR)**

172. L10: Yes sir

173. David: Why is it true? **(AR)**

174. L10: Obviously if it is negative in the denominator then it must go up to the numerator, is positive in the numerator.

175. David: Can we prove that mathematically? How did they prove this one? Where did they start? Can we come up with a proof for this? That will be your homework. (learners copy the homework) Now I need a volunteer for 3c.

We are marking a, b, c and d. Who did not do 3c? Who did not do b? L4 did you do b? **(TII; TSc; CU; AR)**

176. L4: Yes but I did not finish it

177. David: Ok you finish it while L3 writes on board. Write down all the laws used.**). (**Learner writes the solution below :) **(TLM; CU; AR; SC)**

$$\left(2p^2\right)^4 \times \left(3p^2q\right)^{-2}$$

$$= 16p^8 \times -9p$$

$$= 144p$$

178. L10: Law 5

179. David: According to L5, it is law number 5? Read law number 5.**(TLM)**

180. L15: In brackets *a* times *b* to the power of *m* equals *a* to the power *m* times *b* to the power *m* is equal to *a* to the power *m* , *b* to the power *m* {from the text the law is shown as: $\left(a \times b\right)^m = a^m \times b^m = a^m b^m$ }

181. David: Ok is that the law you used?

182. L10 Yes sir

183. David: Let's see did he apply the law exactly. How did you get the 16?**(TII; PF)**

184. L10: I used two laws—law number 3

185. David: Ok read law number 3. You used law number 3 and law number 5? **(TLM; AR)**

186. L10: Yes sir

187. David: What is the same, what is different between law number 5 and law number 3?**(TII; CU; AR)**

188. L15: Law 5 two variables

189. David: Law 5 two variables—and number 3?

190. L10: One variable

191. David: the 16 If I apply the law is 2 to the power 4. How did you get 16?

192. L10: I said 2 times 2 times 2 times 2

193. David: Ok, the 2 times 4 is 8(David pointing to the power of p). Then negative 2 times 3 equals 9(David pointing and circling the -9) How did you get that?**(TII; PF)**

194. L10: How did I get negative 9

195. David: Uhmmm

196. L10: Because you are multiplying with a negative

197. David: Let me just get this, you said a negative times a positive is a negative? **(TII; PF)**

198. L10: Yes. I said 3 times 3 is equal to 9

199. David: You said you multiply the sign of the exponent and the sign of the base. Can I do that? Can I multiply the sign of the exponent to the sign of the base? Are we allowed to multiply bases and exponents? That is the exponent, that is the base. He is multiplying the sign of the exponent there and the base. Are we allowed to do that? If so give me the law. (No response from learners. David repeats the question) He is multiplying the signs and not the numbers. If that is true can 6^2 be equal to 12?, if that is true (David pointing to base 6 and exponent 2). **(TII; TIL; CU; AR; PF)**

200. Ls: No

201. David: Six squared is equal to?---- 36. One important thing you need to know and I want you to write it if I haven't given it to you yet. We never multiply bases and exponents --not at this level. Maybe at another level I don't know, but not at grade 10 level. We are not allowed to multiply bases and exponents. (David writes on the board $2^2 = 2 \times 2 = 4$). Are we multiplying base and exponent here?**(TII; TIL; PF; TLM; AR; CU)**

202. Ls: No

203. David: This one may look like we are multiplying base and exponent, but we are not multiplying base and exponent. 2 is special. So according to this I cannot accept that answer. Consistency is the key. (David rubs off the -9). Three to the negative 2.**(CU; AR)**

204. David: The law says when we multiply powers of the same base we add, but you are subtracting

205. L2: If we are not allowed to multiply powers of the same base then how come on the other side we multiplied them?

206. David: (Inaudible)

207. L10: If $2^4 = 2 \times 2 \times 2 \times 2$ why can't we do the same with $3^{-2} = 3 \times 3$?

208. David: Must make the powers positive first before we can do that.

209. David: You said 2 to the power of 4 and 3 to the power negative 2 is 6 to the power of negative 2. Is that what you are saying?**(TII)**

210. L10: Yes sir.

211. David: Which law did you use?**(TII; AR)**

212. L4 The bases are the same.

213. David: Repeating—the bases are the same? Is 2 the same as 3?**(TII; AR; CU)**

APPENDIX D6: DAVID: LESSON 2

1. David: (Gives dictionary to one learner) Look up whole number and come back to me. We are marking exercise 1 and 3 and we can come back to yesterday's homework. Do you need more time with your homework?
2. Ls: Yes
3. David: Then Monday first double I want to see it
4. David : (starts by recapping where lesson ended the previous day). L5 said $2^4 \times 3^{-2} = 6^2$. He took the bases *2 x 3 = 6* and the powers 4 + -2 = 2. Therefore answer = 6^2. The question now is which law have you used?**(TII; AR)**
5. [L]: I used law 1
6. David: You can't use that law. Why?**(TII; AR)**
7. L1: Bases are not the same
8. David: If the bases are the same then I can use the law. Can I say 2 times 3? This is 2 to the power 4 times 3 to the power minus 2. We must get this in order (pointing to 16 and 3^{-2}). So this must be cancelled (David erases the part $2^4 \times 3^{-2} = 6$. We have cancelled so we can continue. What did we say about multiplying a base and exponent? **(TII; AR; SC)**
9. L1: We can't do that.
10. David: The 16 we got by saying *2 times 2 times 2 times 2* four times. Now we want 2 to the power 4 times 3 to the power minus 2. Everyone on your calculator. (as learners work on calculator the teacher repeats 2 to the power of 4 times 3 to the power of minus 3 equals.) L5 read your calculator screen.**(TII; TLM; PF;**
11. L2: 1,677
12. David :No read the full screen
13. L2: 2 to the power 4 times 3 to the power negative 2
14. David: Yes that equals?
15. L2: 1,7777
16. David Press the SD. What do you see?**(TMR; TII; SC; PF;)**
17. L2: sixteen over 9
18. David:I assume there is another calculator—read me yours (David moving to another learner) Read me your screen. I have 16 over 9; yours is? **(TMR; TII; TLM; SC; PF;)**
19. L3: one and seven ninths
20. David :And yours (pointing to a third learner)
21. L4: 1,77

22 David :Once we have this rule (pointing to $a^{-m} = \frac{1}{a^m}$, we can change this part (pointing to q^{-2}) to q^2 in the denominator. We can leave the answer as (David writes on board: $\frac{16p^4}{9q^2}$.) **(TMR; PF; SC)**

The idea of a whole number. Carl your idea was naught was not a whole number. Now read the definition of a whole umber.(**TLM; CU)**

23 L1: A whole number is a number which has no fractions attached
24. David: (Repeats the definition as read). Read the other definition
25. L1: Reads the second definition.
26. David: Compares the two definition**(AR; CU)**
27. David: Now today's lesson is converting to prime bases. Converting to prime bases. What does it mean? What do you understand about converting to prime numbers. What I need you to look at is converting to prime bases, just that statement. What do you understand about it? Converting to prime numbers. What do we do? What does converting mean to you?**(TSc;TII; CU; AR)**
28. Ls: Change
29 David: Do you agree with him?**(TII; AR)**
30. Ls: Yes
31. David: Converting means in simple terms change. Ok therefore change to prime bases. Now what is prime?**(TSc; CU)**
32. L5: An odd number
33. David: So prime numbers are odd numbers? What are prime numbers?**(TII; CU; AR)**
34. L6: No factors, except itself and one.
35. David: Prime numbers have two factors; itself and one.
36. L7: What is the first prime number? (Repeating the statement)
37. David: What is the first prime number?
38. Ls: one
39. David: (Writes on board 1; 2; 3) Listen to the definition of prime number. The definition is it is a number with two factors one and itself. Now I am asking you what is the first prime number?**(TLM; AR; CU)**
40. Ls: one
41. L8: two
42. David: If you say 2 tell me why.**(TII; AR)**
43. L8: because it can be divided by one and itself
44. David: (Repeats the learner's statement as he writes 1, 2, 3).Now what about three? **(TII; AR)**
45. L8: Can be divided by one and itself
46. David: It can be divided by one and itself. What about one?**(TII; AR;CU)**
47. L8: It can be divided by one and itself.
48. David: Now what is the first prime number? What is the first prime number (pointing to the statement one and itself)
49. L8: It is the same number
50. David: Let's go back to the definition. Prime number is a number with two factors, the number itself and one. But what are the factors? What do they look like? The factors are different. Prime number has two distinct factors. Note that 1 is not a prime number. The number and one are the same. We are going to convert, change bases. What are the bases itself?**(TSc; AR; CU)**
51. L4: (reads definition of prime numbers from dictionary)

52. David: Go back to page one where it shows the –(inaudible0. We are going to convert bases to products of prime numbers. If I have a number, for example 44. What times what is equal to 44? Any two numbers. **(TSc; AR; CU)**
53. Ls: 2 x 22
54. David: (writes 2 x 22). I will circle the prime number. Which one is the prime number?**(TLM; AR)**
55. L9: two
56. David: What times what is equal to 22? **(PF)**
57. Ls: 11 x 2
58. David: Is two a prime number?**(CU)**
59. Ls: Yes
60. David: Is eleven a prime number **(CU)**
61. Ls: Yes
62. David: Now we are taking 44 and writing it as a product of its prime. Now we write (teacher write on the board : 2x2x11 = 44
$2^2 \times 11$.) . Now you said something else.**(TMR; SC; CU)**

63. L9: What about 11 x 4 ?**(TMR)**
64. David: (shows decomposition of 44 into prime on board)

11 4
2 2. Another way is (David writes : 2 44
2 22
11 11
1

Use calculator to find $2^2 \times 11 = ?$.**(TMR; TLM; PF;CU)**
65. David: In order to apply the law, if I want to apply the law the bases must be the same. Now in this case can I apply the law? (David reads the expression $\frac{12^{(n+1)} \times 9^{(2n-1)}}{36^n \times 8^{(1-n)}}$) Can I apply the law?**(TSc; TLM; AR)**
66. L1: No
67. David: Because the bases are not the same. I am going to convert the bases to prime and see what happens (David now breaks the bases and writes

12 9 36 4
4 3 3 3 6 6 2 2
2 2 2 3 2 3 **(TSc; TLM; TMR; PF; CU)**

68. David: (writes $\frac{\left(2^2.3\right)^{n+1} \times \left(3^2\right)^{2n-1}}{\left(2^2.3^2\right)^n \times \left(2^3\right)^{1-n}}$) The exponent belongs to the 12 and therefore a bracket (David gestures brackets with hands). Now can I apply the law? Read the first law.**(TMR; TIL; SC; PF; AR)**
69. L10: (reads) a to the power m to the power n is equal to a to the power mn
70. David: What does mn mean?**(TII; CU)**
71. L10: Means multiplying

72. David: So it means m times n. Now we use the law. (David writes):

$$\boxed{\begin{array}{l}\dfrac{\left(2^2.3\right)^{n+1}\times\left(3^2\right)^{2n-1}}{\left(2^2.3^2\right)^{n}\times\left(2^3\right)^{1-n}}\\[2ex] =\dfrac{2^{2n+2}.3^{n+1}\times 3^{4n-2}}{2^{2n}.3^{2n}\times 2^{3-3n}}\end{array}}$$

73. David: WE have applied the same law over and over, now what? Simplify numerator and denominator if I can apply the law (David writes :

$$\frac{3^{5n-1}.2^{2n+2}}{2^{-n+3}.3^{2n}}$$

I applied one law here and a second law to here and the same law here (pointing to the relevant parts). What's next? Which law can we apply? Read the law.**(TSc; TII; TMR, TLM; AR; CU; SC)**

74. L6: (reads the law)
75. David: Ok, the translation. Law number 2 translate **(TMR; CU; SC)**
76. L11: When dividing same base subtract the bases.
77. David: When dividing powers
78. L11: When dividing the same base subtract the exponents
79. David: Lets break it up into pieces. Let's break into 4 (David demonstrates breaking into 4 groups and points to the 4 groups). Can I apply the law on those two? (pointing to base 2 and 3). **(TSc; CU; AR; PF)**
80. Ls: No
81. David: Can I apply the law on these two (teacher pointing to base 3 and 3)**(TSc; AR)**
82. Ls: Yes
83. David: 3 to the power (David pointing and underlining the powers). Which law are we using?**(TII; AR)**
84. Ls: Law number one
85. David: Read law number one (learner reads) Give me the translation for that law**(TLM; CU)**
86. L11: When multiplying and bases are the same add the bases.
87. David: (Correcting the statement) When multiplying powers of same base we add the exponents. The law says add exponents but I am not doing so. In some cases I am adding in other cases I am not. Can I add 5n – 1?**(TII; CU; PF; AR)**
88. Ls: No
89. David: Why not?**(TII; AR)**
90. L4: Are unlike terms
91. David: Algebraic rules. I cannot add unlike terms. Even if law says add, algebra laws say you can only add like terms. Which law? Read law number 2**(TII; TLM)**

92. L1: (reads)
93. David: Right, the translation **(TMR; CU)**
94. L11: When dividing numbers by same base then you have to minus the exponent.
95. David: When dividing powers (pointing to powers) of same base (points to base) we subtract the exponents. That (David pointing to (-) sign) comes from the law. (writes the law , $a^n \div a^m = a^{n-m}$ on the side. **(TLM; PF)**
96. David: (continues to write; $3^{5n-1-(2n)}$. Where does the minus sign come from?**(TLM; AR; PF)**
97. L10: 5n-1-2n doesn't that become 3?
98. David: I have 5 groups of n and I take away 2 groups of n, how many groups now? (David using fingers: raising 5 fingers and dropping 2 fingers) **(TMR; CU; AR)**
99. Ls: (Inaudible)
100. David: Now I want to write a base. I have two bases 2 and 3. Look for a law where I can write as one base. (David pointing to 2^{3n-1} and 3^{3n-1}). **(TMR; TSc; AR)**
101. L11: a to the power m times b to the power m is equal to a to the power m b to the power m
102. David: (writes $a^m \times b^m = a^m b^m$) Can I do this? (writes $(ab)^m$). The only time I can do this is when the exponents are the same. **(TSc; PF; SC**
103. David: {Writes $(2.3)^{3n-1}$}Can we go further? Now can I multiply 2 times 3?**(TII; AR; PF)**
104. L10: Yes sir
105. David: (writes 6^{3n-1}) Laws must be read left to right and right to left. Now do number 9. Start by converting to prime bases. (writes $\dfrac{10^n \times 25^{n-1} \times 2}{50^{n+1}}$)
106. L12: Writes $\dfrac{(5.2)^n \times (5^2)^{n-1} \times 2}{(5^2.2)^{n+1}}$
107. L13: If I write 5 x 5 instead of 5^2
108. L10: Why not leave 10^1 instead of 5.2?
109. L11: cos 10 is a prime number.
110. David: Can I write this as 2 x 5?**(TII; AR)**
111. L4: Yes it is the same thing
112. David: Another come and apply one law.**(TII; AR; PF)**
113. L4: (writes $\dfrac{5^n.2^n \times 5^{2n-2} \times 2}{5^{2n+2}.2^{n+1}}$)
114. David: The position of the 2? (David emphasizing the accurate position of terms) Next person to come and apply one law for numerator and denominator?**(TII; TSc; PF, SC)**
115. L14: (writes $\dfrac{5^{3n-2}.2^{n+1}}{5^{2n+2}.2^{n+1}}$)
116. L12: Where did something go?

117. David: You mean where did the 2 go? (David circles the 2^n and 2 and joins them with a line). Can I cancel the 2^{n+1} divided by 2^{n+1} ?**(TSc; PF; SC)**

118. L13: I do not understand

119. David: Where exactly (pointing to the additions)

APPENDIX D7: DAVID: LESSON 3

1. David: (Asks learners to complete putting solutions on the board from previous lesson).
Which law did she use? From this part to this point which law? **(TSc; TII; AR)**
2. L1: Law 2
3. L1: When dividing powers you subtract the exponents
4. David: (Repeating) When dividing powers of the same base we subtract. (Explains the final answer by splitting it into smaller units of the same base.

$$\boxed{\begin{aligned} &5^{3n-2-(n+2)} \\ &= 5^{2n-4} \\ &= \frac{5^{2n}}{5^4} \end{aligned}}$$

5. L2: Writes proof of the homework : $a^0 = 1$;

$$\boxed{\begin{aligned} \frac{1}{a^{-m}} &= \frac{1}{a^{-m}} \times \frac{a^m}{a^m} \\ &= \frac{a^m}{a^0} \\ &= a^m \end{aligned}}$$

6. David: Do you agree with this proof? She has multiplied by $\frac{a^m}{a^m}$. My question is did she change the value of the fraction? For example if I take 1/3 and multiply by 5/5, does the value change? 1/3 x 5/15; is the value the same as 1/3?**(TII; TMR; AR; SC; PF)**
7. L2: (inaudible answer)
8. David: Yes thank you for that. Now we are marking exercise 1. Can I have volunteers to put solutions on the board? (4 volunteers put their solutions on the board) **(TII; TLM; PF)**
9. L2: {Writes $\left(2t^{-4}\right)^2 = 2^2 t^{-8} = 4 \times \frac{1}{t^8} = \frac{4}{t^8}$
10. David: What does the law say?
11. [L]: (Inaudible)
12. L3: (inaudible)
13. David: What do I call that law? Which law?**(TII; AR)**
14. L1: Law 6
15. David: Read the law. Which law? She is writing a negative exponent. How did she get the 4? 2^2 (David points to the base and exponent) or 2 times 2 **(TLM; TII; PF)**
16. L4: base times base
17. L5: (wants to change her solution)

18. L5: Writes solution as $\boxed{\begin{aligned}\frac{2}{s^{-3}} &= \frac{2}{s^{\frac{1}{3}}}\\ &= \frac{2}{s^{3}}\end{aligned}}$ then changes to $\boxed{\begin{aligned}\frac{2}{s^{-3}} &= \frac{2}{s^{-3}}\times\frac{s^{3}}{1}\\ &= \frac{2}{s^{0}} \quad s\neq 0; s=1\\ &= \frac{2}{s}\end{aligned}}$

19. L2: Next solution : $\frac{4}{1}\times\frac{1}{m^{-7}}$ or $\frac{4}{m^{-7}}\times\frac{m^{7}}{m^{7}}$

20. David: 4 divided by m^{-7} is… another translation is $\boxed{\begin{aligned}&\frac{4}{1}\div\frac{m^{-7}}{1}\\ &=\frac{4}{1}\times\frac{1}{m^{-7}}\\ &=\frac{4}{1}\times\frac{m^{7}}{1}\\ &=4m^{7}\end{aligned}}$ (David accepts both solutions). Now e (meaning solution to number e). If I have 2/3 x 3/1 = 2 . Cross cancellation; this would give 2. My question is: is 2 equal to 2/3? This means you are changing the value. If I take 2/3 and multiply by 3/3 gives 2/3. It does not change the value. Or you could have said $\boxed{\begin{aligned}&\frac{2}{1}\times\frac{1}{s^{-3}}\\ &=\frac{2}{1}\times\frac{s^{3}}{1}=2s^{3}\end{aligned}}$. Now number 3. **(TMR; AR; SC; PF)**

21. L6: (Writes his solution: $\boxed{\begin{aligned}&\left(a^{3}b^{2}\right)^{3}\times\left(a^{2}b^{4}\right)^{-1}\\ &=a^{9}b^{6}\times a^{-2}b^{-4}\\ &=a^{7}b^{2}\end{aligned}}$

22. L7: Writes his solution: $\boxed{\begin{aligned}&\frac{\left(3rs^{2}\right)^{4}}{r^{-3}s^{4}}\times\frac{\left(2r^{2}s\right)^{2}}{s^{7}}\\ &\frac{3r^{4}s^{8}\times 4r^{4}s^{2}}{r^{-3}s^{11}}\\ &\frac{12r^{8}s^{10}}{r^{-3}s^{11}}=12r^{5}s^{-1}\end{aligned}}$

23. L8: Writes his solution:

$$\boxed{\begin{array}{l}\left(\dfrac{a^2b^3}{b^{-2}}\right)^2 \div \left(\dfrac{ab^4}{a^2}\right)^{-2} \\ \dfrac{a^4b^8}{b^{-4}} \div \dfrac{a^{-2}b^{-8}}{a^{-4}} \\ \dfrac{a^4b^8}{b^{-4}} \times \dfrac{a^{-4}}{a^{-2}b^{-8}} \\ \dfrac{b^6}{a^{-2}b^{-12}}\end{array}}$$

24. David: $\left(\frac{2}{3}\right)^{-1}$? (Use your calculator. What does the negative 1 do to the fraction? What is the word we use when the numerator and denominator change? **(TSc; PF)**

25. [L]: Reciprocal

26. David: For r^8 divided by r^{-3} you got r^8. I would like to see how you got that (No response).Now look at this. When dividing powers with same base we subtract. r to the power 8 minus, from the law, minus 3 (writes 8 –(-3). (David corrects solutions of L7 to $\frac{324r^{11}}{s}$ and rewrites solutions of L8) I write that as a^2b^{18}. Can I do that and why?**(TII; AR)**

27. L4: Making the powers positive

28. L1: Can I just divide the a's and the b's??

29. David: Is that what you did? Read. What did you get?**(TLM)**

30. L1: (Reads out the working and it is correct)

31. L9: Sir what if you had done the cancellation one?

32. David: If you cancel first?

33. L9: Yes

34. David: Right. What if I apply the law here first (pointing) then I work outside. (David works out solution). We are simplifying first **(TSc; AR; PF)**

35. L10: Can we cross cancel?

36. David: We cannot cross cancel cos there is a division sign. We can cancel this way (pointing to numerator and denominator. demonstrates the working using division first). **(TSc; PF)**

37. L2: (inaudible. Brings book to teacher and shows teacher)

38. David: (Stops and looks at learner's book and explains simplifying the brackets)

39. David: What does it mean to convert to prime bases?**(TII; CU)**

40. L3: You square root the number until you get we get prime 4

41. David: give me an example **(TII; CU; SC)**

42. L3: Eg 25 take square root of 25 = 5 and 5 is a prime number

43. David: Anybody else, is it always square roots? What about 6? Shall I find the square root of 6?**(TII; TSc; AR)**

44. L4: 2 times 3

45. David: Converting to prime bases, what does it mean? What are we doing? The law says breaking the number up. Now breaking the number up to what?**(TII; CU; AR)**
46. Ls: Into factors
47. David: Yes breaking into factors.
48. L3: Lowest factors that can be possibly broken into
49. David: (Repeats learner's statement)
50. L3: Lowest whole number pattern
51. David: Why do we do this in exponents? **(TII; CU; PF; AR)**
52. L11: So the bases can be the same.
53. David: So the bases can be the same.
54. David: Why must the bases be the same? So that we can do what? **(TII; CU; AR;PF**
55. L5: So we can add the same number again and again
56. David: Why do we want bases the same?**(TII; CU; AR)**
57. L4: It is easier
58. David: Yes but why? Go back to the page. **(TII; AR, CU**
59. L5: So we can get to a real number.
60. David: Someone else. Why? (**TII; AR; CU)**
61. L1: So we can apply the law
62. L4: Sir so that the laws work when the bases are not the same?
63. David: Do the laws work when the bases are not the same? (**TII; CU; PF; AR)**
64. Ls: No
65. David: Law says if bases are the same and we are multiplying we add the exponents and to divide we subtract. These are the conditions of the law. That is why we break up composite numbers to prime bases. Homework Exercise 3 numbers 7, 10, 11 and 12.**(PF; AR)**

APPENDIX E--- TEACHER INTERVIEWS

R- represents the researcher.

APPENDIX E1: INTERVIEW WITH JEAN

R: Good afternoon Jean. Thank you for making time for me. I would like to ask you a few questions about your lesson you encourage them to speak in their own language?

Jean: I encourage them to speak in their own mother tongue, especially if I am interested in getting out their understanding. But I also encourage them to speak in English so that they teach each other. They may not necessarily be aware that they are learning from each other as the other are expressing themselves in English—But those who are not fluent as time goes on they will catch up—because it is still important for them that they have access to English.

R: So you think it is important for them to have access to English?

Jean: Yes I do think so cos they are using it in examinations. We do not have options

R: So you still want them to have access to English?

Jean: Yes I do, but I don't want them to be conscientious and say that the teacher wants me say it in English. It should be something that happens freely. I want them to just do it, maybe something like incidental learning.

R: Yes incidental learning! That is interesting. They learn from each other. They are helping each other without knowing. But the ones that are contributing in English, are these the ones who are more able to communicate in English? The ones who are contributing their ideas in English.

Jean: In my experience, I have realized that there are some learners who are just not going to speak in their mother tongue. I don't know whether they feel free or they think this is school so you must speak in English. But I do not know the reason. But I have ealized particular kids in most classes who give answers in English only.

R: Then your lesson on representations. I ealized that sometimes they seem to get confused. Is it the language or the topic or is it algebra especially with representation of exponents. I know that at one point when you were talking about 1/25 and 25, they did not see the difference. They kept on saying they were the same. What was the problem? Is it representation or the English language? What do you think?

Jean: That one I think it is the mathematics basic concept. The class I was teaching, most of them struggle. It was a struggle with the concepts.

R: So it is mostly mathematical concepts?

Jean: Yes

R: By allowing them to use own language would help with what?

Jean: I think allowing them to use their mother tongue is something I have developed over time. Something I just do all the time. I just believe that the mathematics itself is a problem to them. That is why you find that most of the time I also want them to get the concepts in this language that they use often. But when they communicate in mother tongue, I allow it too so I can understand their thinking. I want to understand how they are ealized lizing the mathematics. I am interested in their knowledge more than how thy put it across. Because sometimes you may find that they say it in English and they are not very fluent or they are not really getting through to what they want to say. But when you say okay say it in your language, now you can understand a little better what they were trying to say. So I have found that it just assists me to understand them more than me making meaning to them.

R: I was interested in after them not understanding 25 and 1/25, you used another example. You said I am going to give you ½ a loaf and I am going to give you 2. That made such a big difference to their understanding. I wonder why? They could not see 25 and 1/25. But the moment you said ½ a loaf and 2 loaves, you actually wrote it ½ and 2 loaves, they said oh! They are different.

Jean: I have found fractions are difficult. When one is teaching grade 10 do not be afraid to unpack the concepts. Sometimes you will find they don't understand what ½ means. Now I started asking what is 3/9; 3/3. I ask 1/3 is this 3? For them they will say 1/3 is 3. It shows me how much these learners do not have the basic concepts. Concepts of fractions are a big problem. You can imagine now moving from negative exponents converting them into fractions. Now you are moving into another world altogether. I was researching my own teaching. I just assumed they are in grade 10 they know, but I ealized they don't know. I really understood how much injustice I was doing.

R: When you prepare for these lessons do you look at a lesson and say there are going to be language issues and mathematical issues within a lesson and do you prepare for them differently and plan for them differently?

Jean: Yes now I am planning very carefully. Now I am aware of problems. My planning changes altogether cos I think for me, it is no longer about the concept at hand. I must incorporate things that are prior to this in some ways. The nature of mathematics does not allow them to grasp if there is no prior knowledge. So I have also become more flexible in my teaching. But in grade 12 you just rush. But in other classes I try very hard not to rush.

R: What actually started you looking into this issue, especially the language issues. When was the turning point?

Jean: It was this course I did , expressing mathematics. It was totally new to me. I never even conceptualized that this language can have an impact in my mathematics class. I still allowed learners to express themselves in their own language. I have never forced them to really speak in English in my class. I encouraged it but I was not strict about it; if learners could not express themselves in English. After the course then I was more aware and I made conscientious decision to be more aware I am going to take care of this issue. I am going to take it into consideration--- look at the concept I am dealing with, look at the terminology and find means of explaining things so that they can relate more to what is accessible to them in everyday context.

R: And another thing I noticed, often you revoiced. You repeated. You said the same idea in different ways. Is it because of language or mathematics?

Jean: I think also with the course, it was also the course. I knew that you could express one issue, use one representation. But reading those articles and also going into the curriculum, I then noticed it is even stated in the curriculum. I became more conscientious, more aware.

R: One thing I noticed was that you always spoke in English; most of the times. Once in a while, in small groups, you spoke in Sesotho or IsiZulu when explaining to learners.

Jean: Yes I think it is important, personally, that our learners need to understand these concepts. They need to have access through their language, but I also feel that mathematics grows in certain communities. Now also something I also learned about communities of mathematics. I take my class as a community and for them to also become members that are growing, express themselves, having mathematical communications. If they go and meet other people with different languages they should be able to express mathematical knowledge in English because this is the language that will be used. They will be going to university. I feel that it is important that they learn how to communicate mathematically to everybody. I still believe that I am the resource that they have in this mathematics community. They have a chance to hear how we speak mathematically in English. We have not developed like other countries like Germany who are using totally their language. That is my argument.

R: Thank you very much Jean. It has been very informative.

APPENDIX E2: INTERVIEW WITH DAVID

R: Good morning David. I would like to thank you very much for allowing me to observe your lessons and interview with you. As you know I am doing research on what teachers in multilingual classrooms are doing specially to help teach topics like algebra. All the questions I am going to ask come from what I observed. I have gained a lot of knowledge. I would like to probe you further into some of the approaches that you use and why. Once again I really want to say thank you very much

David: Thank you Sophie.

R: The first thing that I noticed in your class was you tell your learners to always read aloud. You asked your learners to read exponential laws to the whole class. Why did you insist on the learners reading out aloud?

David: Yah The reason being you hear yourself say the law. I want them to get confidence in using the symbols and the way in which they read the law—you would say that it is for symbolism and for meaning. So in that case I believe the more they use the law and communicate mathematically they will understand the law and how to use it appropriately.

R: Ok! Ok! They achieve something in reading out aloud? as far as mathematics is concerned?

David: One thing I looked at—the why is from my experience with one child in geometry. I spoke about adjacent angles on a straight line and in the test the learner wrote adjacent, adjacent, adjacent. That's when I realized the learner heard it from me once and interpreted adjacent in that lesson. But by reading out aloud they pick up the pronunciation and the law itself and apply it correctly. That's why I do that.

R: Ah! Do you sometimes ask your learners to read silently?

David: Yes sometimes it works silently first, then you underline what you don't understand and then you come out publicly. But the strategy basically is you are on your own, alone, in a pair, in fours, then whole class. That is the approach I am using now.

R: Do your learners take advantage of doing this? Do they realize what the advantages are of reading out aloud or silently?

David: I explain to them I need to know. The only way for me to see their understanding is one of two ways, communicating their thinking verbally or displaying on the chalk board so that I can see. When they write on paper it is difficult to see; but when they display publicly most of them have similar problems. Then that's when they see you are not alone, there are others. The board is basically used not just as information, but as a field to rectify, display, to argue and then change. My reason is with wrong answers on the board you can erase them

R: How do your learners feel about reading out aloud? Are they all comfortable? Do they all just read out aloud? Are there some that with-hold?

David: No not all of them. But I look at them. Our English department is quite good. They teach all about public speaking and so forth and how to read so by the time I get them in grade 10, they have had experiences from the English department. When someone is not comfortable to read, then I ask someone to help. More than one accession the learner then says ‘no I can do it. I want to read it’

R: Would you say there is a relationship between their fluency in English and their freedom to read out aloud or their not shyness to reading out aloud?

David: No they are not shy. The majority are not shy. Again coming back to the system of the school, they get a lot of debate and drama

R: I notice one time when you had to verify the value of 2^4 x 3^-2, you insisted that the readers read out the full calculator screen, rather than just reading out the answer.

David: Yah the idea is that I look at the calculator screen as well because the calculator when it says negative or a minus it generates different answers. In order to validate to see if they are using the calculator correctly, we need to see the calculator screen, if they are punching the correct keys. Therefore we read the whole screen.

R: There was one time the discussion about whole numbers. You gave the dictionary to one learner and said read aloud. You are still emphasizing the child reading out aloud. Would it make any difference if you had read it yourself?

David: To me, it comes to classroom discussion. I like to see myself as an equal participant and not the one with all the answers. So by giving dictionary is another we bring in another source in order to see it is not Mr. David’s answer but others say the same. Sometimes we bring another dictionary and compare. They need to know how to use a dictionary and use it within an argument. That’s the idea. One child who reads—that’s the participation, others have to listen and follow through, because sometimes I do remove myself from the conversation and leave them only afterwards I come back. A dictionary is an important tool. In 2000 at Amesa conference I saw a maths dictionary.

R: Oh I actually was going to ask you later on whether you aware of the dictionaries that are available?

David: Yes at Amesa conference, I saw the first one in South Africa and bought one and that was the one I started using, others came into the market now. Depending on what is in the department that’s what I use. I always bring a dictionary to class.

R: Do you find there is one favourite dictionary among these that are coming out now?

David: The one I got in 2000. I forgot what it is called, that was the one I started using others have come into the market now but I lost it.

R: Ah.

David: Some are not user friendly. In my maths classrooms we don’t--- especially when it comes to definitions we use the dictionaries quite a lot and textbooks glossary at the back . Sometimes if we cannot find it in the dictionary we send one to the English department and use one of those.

R: These dictionaries, now that you have brought in dictionaries, so I am going to go on with dictionaries. Do you find that aahhh—what are the different languages that the dictionaries are in?

David: The ones that I currently use are mainly just English because it English medium. I had an Afrikaans one but I don't use it because I don't teach in Afrikaans. It is just different authors, mainly just in English. The only other one I had similar to a dictionary was a chart where I had symbols and all official languages like the equal sign, where the language does not have it there would be a blank, but I think I think I have lost the chart.

R: So for the dictionaries it is just English or Afrikaans

David: Just English

R: What about the multilingual dictionary. Have you heard of the multilingual dictionary?

David: I have seen one for grade 1. It was for IsiZulu, but I haven't looked at it I only saw it on display.

R: Do your learners find or accept the explanations in the English and Afrikaans dictionary satisfactory?

David: Some of them. I will revoice certain things and show them what things are out there to help them when I am not there.

R: Ahh. Thank you. Now I want to take you to interpretations, how learners interpret or interpreted some of the meanings in there. I noticed that when you—after you read out the law aloud you asked for the meaning of the law in your class. You asked them to read out aloud and then you asked them to explain.

David: Ahh . The reason for this is the symbolism.

R: Ahh

David: Because what happens is they can read the symbols a to the power m and so on, but they don't know how to use it. In order to say when multiplying powers of the same base we add the exponents. So that idea I push it left right and in. So when you use the law you can see it is multiplication the bases are the same we add the exponents. My experience is when they know how to interpret the law and understand the law they can apply it much better, they don't forget.

R: From your experience are there some of these laws that learners find difficult to interpret?

David: The one law is when multiplying numbers with the same base, we add the exponents versus raising a power to another power. There is always confusion, they don't know when to multiply and when to add. Then there one that is not a law, we call it a rule we never multiply base to an exponent. The one that confuses is 2^2. I never that as an example because it looks like you are multiplying the base to an exponent. If I use that I use colour and show that the base is multiplied with another base not exponent. The other law that difficult and I rely on the calculator is 0^ something.

R: Oh yes

David: That's the one. We start by saying 0^1, 0^2 trying on the calculator it will give you different type of error. Then this brings to when we use error and when is it mathematically undefined. Understanding of why base cannot be 0 is limited knowledge. Nobody knows. We accept it and use it and continue. The same as multiplying two negatives, we accept it and move on.

R: Can I come to your approach of learners writing on the board. I notice that is one activity that is very prominent in your classroom.

David: The writing on the board is something I have experienced as a child at school. Our homework was checked on the board. Some of the maths teachers never check homework in books. It was a matter of you display your answers on the board and then take it from there. One thing is if you go with your book that's only for—go with your book when you have done it in class. If it is homework you go without your book. That way I can see the difference of learners copying the homework and learners doing their homework. They know that they will go to the board and that gives them a sense of responsibility to sit and do the work. The chalk board is not just for information, but they have a chance to display their answers properly. The board is not reserved for just for correct answers. You write answers on the board and then you can change and at that point you look at the setting. There are two things on the board one that you understand how to do the work and two that you represent it in such a way that shows you know what is going on . So I am dealing with two issues on the board; the setting out versus interpretation.

R: I noticed there was one law a^0 =1. You asked learners to write their interpretations on the board and not just read them. You actually had each one of those students writing out their interpretations on the board.

David: It is a law they struggle with and that brings conversation to my class. I would like to see their interpretations .Once the interpretations are on the board the whole class can see these are the interpretations. You can pick up the wrong interpretation and deal with them immediately. In most classes I have noticed they say a^0 =1 and there not the extra addition but a =/ 0. They assume it is true for all cases. So I always want to see the interpretations, especially in exponential laws. I feel are three levels. First the symbolic form , a to the power of m and so on, second is the mathematical form when multiplying powers of the same base we add the exponents and then we have, if I can call it, the English form, the normal way. So these are the interpretations.

R: Would you say most of your learners will volunteer to go and put work on the board?

David: Yes , especially when they grasp the concept . Then they want to show off. It is a place where the confident comes through. If someone did not go to the board the learners can tell . They will ask when is he going to the board. So the board becomes a place where we all work on.

R: Oh Ahh. I notice on coming to mathematical representation; we have looked at one of them already but I notice that you do a lot of pointing on the board. You say something and you go and point. How do you expect this to help with the understanding?

David: Because the answers are on the board and there is lot on the boeard. So when I refer to something on the board I need to point where I am because I can talk about one thing and they can focus on something else. If I am teaching and using a certain sequence from left to right. I need to point to and give focus on a specific setting out or representation. For example if I say coefficient, I take my fingers and close the rest and say this is what it basically means; the body parts get used. You use your hands and everything can be used for communication in the class.

R: Okay. Then I am coming to your revoicing and repeating. I notice that you often took a statement and repeated it. Why do you do it so often?

David: That way I can interpret and learner can see if my interpretation is correct. In case my interpretation is wrong the learner can correct me. The other reason is for the rest of the class to hear what you are saying and in order to keep them all going because if one talks someone can say this conversation is too high for me I am going to switch off. By revoicing and working with them you can keep the conversation going, making sure most of them are participating. Some of them will break down when I start revoicing, they will start scribbling and so on

R: How do you determine which statements you are going to revoice more often?

David: It entirely depends. Not necessarily correct answers. Who ever responds. The idea is to give different points in the class and give opportunity and use different learners in the class. Revoicing is with anyone to keep the conversation going.

R: Thank you very much. This has been very informative. I am sure we will keep on talking as we meet at the university during the year.