List of Corrections

Referee 3

Approved without corrections

Referee 2

Minor errors and omissions were noted, and have been corrected in the thesis as follows:

- Appendix A has been removed as information is provided in Figure 1.1.
- Table 1.1 (page 2): unit for aperture provided (metres).
- GRoC: Regular font has been used throughout the thesis and the explanation has been provided at first occurrence (page 3).
- Page 13 – Equation (1.14): $\mathbb{N}$ has been specified (set of nonnegative integers).
- Regular font has been used throughout the thesis for abbreviations such as CTE_{steel}, VIF, SSE.
- Pages 32 (line -4) and 33 (line -1) – now pages 32 and 34: *that* has been changed to *than*.
- Section 3.2.5 (page 84): The appropriate parameters have been provided (smoothing factor and correction time for exponential moving average, and weighting factors for weighted moving average).
- Page 99 – now page 103: The operator $\mathbb{E}$ has been specified (mathematical expectation).
- Section 4.7 (pages 130–131) – now Section 4.6 (pages 129–131): More elaboration on conclusion has been provided.

Referee 1

Specific points made in this Report are minor corrections and have been included in the thesis as follows:

- Page 20: Equation (2.2) now (2.3) – x and y have been explained as follows:

Also note that, considering A as the matrix of a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ [28],

$$\|A\|_2 = \sigma_1 = \max_{\|x\|_2=1} \max_{\|y\|_2=1} \left| y^T A x \right| \tag{2.3}$$

where x varies in $\mathbb{R}^n$, y varies $\mathbb{R}^m$ and $\|\cdot\|_2$ is the standard Euclidean norm.

- Start of section 2.1.2 (pages 21–22) has been edited as follows, to emphasise the linear aspect of regression:

  Linear regression analysis is concerned with the estimation of the response variable as a linear combination of the explanatory variables. In this thesis, linear regression *will simply be referred to as* regression.

- Pages 22–23: s.e. from equation (2.8) has been explained (standard error).

- Page 94 – now page 98 (just before section 3.4 – Conclusion): Last sentence changed to:

  It also suggests that, in the context of the current analysis, humidity is not a serious concern and that the main aspect to explore is computation.

General comments and corrections required by the examiner

Minor corrections have been made as itemized and included in the thesis. Some significant responses are repeated here in detail (labelled by: pages 48–50, page 99, page 114, pages 128–129 [in the thesis]).

- Bibliography has been standardised.

- More elaboration has been provided to captions of figures throughout the thesis.

- Pages 48–50: In the second paragraph of his report, the examiner recalls that temperature and humidity are the main concerns regarding poor image quality, and that they are strongly correlated. Conclusion of chapter 2 has been extensively modified to emphasise the facts that data labelling was incorrect at start, and that besides the temperature of truss and humidity, time is the most significant explanation of the figure of merit and strongly suggests re-examination of the control algorithm.

- Page 99: In paragraph 3 of his report, the examiner recalls the well-known fact that SALT edge sensors are faulty. The following paragraph was inserted in the thesis before the last paragraph of the conclusion of chapter 3:

  We wish to make clear that perfect edge sensors (sensors providing zero measurement error) will not guarantee that SALT control system will work. This is because besides the temperature of truss and humidity, time is the most significant explanation of the figure of merit, which means computation is the most likely main cause of poor image quality. The numerics proposed in this chapter provide an improvement on the implementation of *the current control system. In particular, we found unacceptable accumulation of numerical errors unless SVD was implemented with exponential moving average (see Figure 3.23 page 96). With improved numerics, we found that RMS actuator precision must be stringently chosen at better than one micron (Table 3.2). SALT staff will be informed of this. In addition, we identified errors and omissions in SALT software (that is, deviations from specification and documentation of SALT). A trivial example was the inconsistency in the dimensions of tips, tilts and pistons. This chapter provides consistent documentation with our implementation of the re-designed SALT control system. Finally, we note that should the edge sensors be replaced with more accurate sensors, our software (or any other software used on SALT to implement the control algorithm), should be retested.*

- Pages 103–104: It has been clarified how problem formulations (4.1) and (4.2) are adapted from reference [3] as follows:

  Both formulations (4.1) and (4.2) above are adapted from [3] where in both cases, M, N, Q and R depend on the step k. The adjustment of the state z (in the SALT case actuator displacements) from step k to step $k+1$ involves a disturbance w_k. In formulation (4.2), the relationship between the state z (in the SALT case actuator displacements) and the output s (in the SALT case relative heights) at step k involves an observation noise vector v_k with a known probability distribution. The matrix A in this relationship depends on k and is known for each value of k.

- Page 115: It has been clarified that Lemma 4.12 is inspired by a similar result from [47].

- Page 117: It has been clarified that Theorem 4.16 is inspired by a similar result from [47].

- Pages 128–129: In the two figures 4.5 and 4.6, relative time is in seconds, not in hours. Captions have been modified accordingly. The examiner noted that a control response time of order hours would be too long to achieve control against environmental factors that might change more rapidly (for example, temperature changes on a scale of about half an hour). It is important that this constraint on the gradient flow method no longer holds, and in turn, that the method is more reliable in all aspects, compared to all methods of Chapter 3.

- Pages 123–129: Section 4.5 has been extensively modified accordingly.

- Pages 130–131: Section 4.7 – now Section 4.6 (pages 129–131) has also been modified accordingly.

- Pages 132–136: Conclusion (Chapter 5) has been modified, especially parts involving the gradient flow approach.

- Additional references have been inserted for the following results:

  - Section 2.1.3 (page 28)
  - Fourier transform (under spectral analysis, page 29)

- Remark 3.1 (under section 3.2.1 – pages 56–57)
- Lemma 3.2 (page 57)
- Definition 3.3 (page 57)
- Theorem 3.4 (page 57)
- Theorem 3.5 (page 57)
- Theorem 3.7 (page 57)
- Remark 3.8 (page 58)
- Normal equations approach (page 60)
- QR approach (page 62)
- Theorem 3.10 (page 62)
- SVD approach (page 63)
- Theorem 3.13 (page 63)
- Remark 3.15 (page 64)
- Proposition 3.16 (page 65)
- Proposition 3.17 (page 65)
- Proposition 3.18 (page 65)
- Z-transform (page 78)
- LTI digital filters (page 79)
- Lemma 4.14 (page 117)
- Lemma 4.20 (page 119)

• **The Gradient Flow Approach** (page 114): The first paragraph has been slightly edited to

The gradient flow technique is a relatively recent mathematical technique. This technique has been applied to a few classes of optimal control problems, including nonlinear quadratic optimal control problems in discrete time, linear quadratic optimal control problems in continuous time with stochastic jump parameters [5,44,47,48]. This technique can be adapted to the problems under study, provided we are dealing with infinite horizon problems, in discrete or in continuous time. The main idea behind the gradient flow approach is to transform an optimal control problem (in continuous time) into an ordinary differential equation problem whereby solving the ODE gives the solution to the original optimal control problem. A standard formulation of a linear output feedback optimal control problem has the form

$$\begin{array}{ll} \min\limits_{u} & J\left(t, x(t), u(t)\right) \\ \textit{subject to} & \begin{cases} \dot{x}(t) = Ax(t) + Bu(t); \quad x(0) = x_0 \\ y(t) = Cx(t) \\ u(t) = -Fy(t) \end{cases} \end{array} \tag{4.34}$$

In this formulation, x is the state variable; y is the output variable; u is the control variable; the function J to minimise is called the objective *function; C is the interaction matrix (relationship between the state and the output), and F is the linear output feedback gain matrix. The solution to the original optimal control problem is entirely determined by the computation of F. The gradient flow algorithm determines the F matrix by the addition of a differential equation for F, of the form*

$$\dot{F} = -\frac{\partial J}{\partial F} \tag{4.35}$$

called the gradient flow associated with the objective function J. This is done after J is made a function of F by a transformation

$$J\left(t, x(t), u(t)\right) \rightarrow J(F, P) \tag{4.36}$$

where $P = \mathbb{E}\left(x_0 x_0^T\right)$, in our case, as in equations (4.39) below, such that $\dot{F} \rightarrow 0$ as $t \rightarrow \infty$. Intuitively, we place the gain F of the standard control in a potential well defined on J. Clearly, equation (4.35) *finds the value of F that minimises J. The solution to the new problem gives us the solution to the original problem. Note that given our original optimal control problem, computation of F is executed, and then holds throughout the standard control process* (4.34). *Moreover, the transformation* (4.36) *can always be found. Furthermore, we can ensure controllability of* (4.34), *given F (see Theorem 4.18 below). Gradient flow is then no more expensive than other methods, and is robust.*

Additional changes

- The following has been added in Remark 4.22 (page 123)
  - *If the A matrix in Problem (4.4) is rank deficient, then so is the square matrix $\Sigma_r \lambda \Sigma_r^T$. Hence, the $\widetilde{F}$ matrix as given just above this remark, cannot be determined since the matrix $\Sigma_r \lambda \Sigma_r^T$ is singular. Therefore the optimality condition algorithm fails for all rank deficient systems.*
- The conclusion (Chapter 5 – pages 132–136) has been strengthen with specific recommendations for the future operations on SALT.