LIMIT EQUILIBRIUM AND NUMERICAL MODELLING APPROACHES IN SLOPE STABILITY ANALYSES WITH REGARD TO RISK ASSESSMENT FOR OPEN PIT MINING

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A research report submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

DECLARATION

I declare that this research report is my own unaided work. It is being submitted to the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

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The 5th of June 2015

ABSTRACT

The planning of open pit mines, and road and rail cuttings constitutes one of the activities usually undertaken by geotechnical engineers. However, this endeavour faces major challenges such as the correct design of rock slopes, the evaluation of their stability and the risk associated with them. Two main analytical methods are used in this process: the Limit Equilibrium (LE) analysis and the Numerical Modelling (NM) method; Slide and Phase2 programmes will be used respectively in this regard. Previous studies have shown some discrepancies between their results in assessing the probabilities of slope failure and the consequent economic risks. This research project aims to understand the reasons behind these divergences and possibly to find some ways of reducing them.

To attain these objectives, a homogeneous slope model was adopted. It required a detailed validation study depending on the analytical method, such that the settings would carefully be calibrated to avoid any further misinterpretation of the results. For Phase2, attention was given with regard to the number of mesh elements and their type, and for Slide, the number of slices. In addition, for both methods, attention was given to the adequate distribution of rock mass variables, the adequate failure criterion, etc. Deterministic and probabilistic assessments were performed to better interpret the differences to be found from these methods and results. The response surface methodology (RSM) facilitated the probabilistic studies, to avoid the constraints of long computer run times and to ease the study of the influence of the rock mass parameters on the slope stability.

For the considered model, 25 slices and 1500 mesh elements were found adequate to better assess the probability of failure (POF), while 1000 slices and 50 000 mesh elements provided results of estimated failure volumes. Well defined distributions of rock mass variables have proven indispensable to better assess the POF as well as the risk associated with the slope failure. Application of the Hoek Brown criterion resulted in the LE analysis predicting higher failure volumes than when Mohr Coulomb criterion was used. With the NM method, cares were taken not to under or overestimate the resultant failure volume when extracting the path of the failure surface. Deterministic assessments showed that the risk determined from NM analyses is not always greater than that from LE analyses. The addition of rock mass parameters not taken into account in LE analyses results does not sensibly influence the POF and the failure volume outcomes, but can in some conditions influence the behaviour of the outcome risk of slope failure. In essence, LE and NM methods can be relied on for probabilistic studies, or even for risk assessments conditioned by carefully setting the models, and in case of LE being adopted to assess the risk, it is recommended to introduce a multiplying factor for cases similar to those that have already been analysed.

Dedication

This research is dedicated to my children: "Ce travail est pour vous, mes tendres enfants que Dieu me donnera."

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NOMENCLATURE

2D: Two dimensional

3D: Three dimensional

C: Cohesion

D: Disturbance factor

FEM: Finite-element method

FOS: Factor of safety

GLE: Genera limit equilibrium

GSI: Geological strength index

kPa: Kilo Pascal

H-B: Hoek-Brown criterion

LE: Limit equilibrium

LIS: Locked-in stresses

M-C: Mohr-Coulomb criterion

MC: Monte Carlo

MPa: Mega Pascal

MR: Modulus Ratio

NM: Numerical modelling

NA: Not applicable

POF: Probability of failure

R: Response function

RFEM: Random finite-element method

RSM: Response surface methodology

SRF: Strength reduction factor

SSR: Shear strength reduction

UCS_i: Uniaxial compressive stress for intact rock

Chapter 1 INTRODUCTION

The design of rock slopes and the evaluation of their stability constitute one of the main issues geotechnical engineers have to face in the planning of open pit mines, and road and rail cuttings (Nicholas and Sims, 2001). The evaluation of slope stability requires the identification of potential modes of slope failure, sufficient knowledge of the geological characteristics of the slope, and of the shear strength, rock mass strength and rock mass deformation parameters which determine the slope behaviour and potential failure surfaces. Modes of failure of rock slopes are generally characterised as rotational, translational and toppling modes. In soil slopes and rock slopes in locally homogeneous well jointed rock masses, the rotational mode is common - its failure surface is generally circular or almost circular (Read and Stacey, 2009). Translational modes, however, occur when the geological structure dictates the failure behaviour of the rock mass. Here, analyses of kinematic stability involving structural planes are applied, and the resulting failures observed are planar or wedge failures. Finally, toppling modes are a special case of kinematic instability, determined by the presence of steeply dipping, persistent joints or structural planes.

Once the failure mode is identified and the shear strength parameters are sufficiently known, slope stability analyses can be carried out. Limit equilibrium (LE) analyses and numerical modelling (NM) methods are the two main approaches employed for slope stability analyses (Sjöberg, 2000 and Krahn, 2004). The traditional method of assessing slope stability has been the LE method. In this method, the stability of a sliding or rotating mass on a continuous surface is considered. For rotational modes, the analyses consider many potential locations of the circular surface and the "final" surface is the one that yields the minimum factor of safety. However, this method ignores the influence of the ratio of lateral to vertical normal effective stresses (K ratio) and the deformation properties of the rock mass. These parameters can be taken into account in numerical stress analysis approaches. In such analyses, the location of the failure surface is determined by the stress distribution in the slopes and not by "arbitrary" choice. It has been shown that there can be a significant difference between these two methods when it comes to the location of the failure surface and hence the volume of the unstable material (Chiwaye and Stacey, 2010).

1.1 Problem Statement, Research Question and Main Objective

It has been observed that, for the same rock slope, different values of risk are obtained depending on the fact that either the LE method or the NM analysis is employed. This has constituted the reason to undertake this research. Knowing that the same causes should generate the same effects, a thorough evaluation of the causes which lead to the differences in the results from the two methods of stability analysis needs to be undertaken to answer the following research question: which factors impact the discrepancy in the failure volumes obtained through the LE and NM methods in slope stability? The process of attempting to respond to this question constitutes the main focus of the research described in this research report. The aim is to improve understanding of the discrepancy and possibly to provide a solution to it.

1.2 Research Objectives

Based on the above discussion, the objectives of this research are:

- To optimally set up LE and NM parameter conditions for realistic slope analyses.
- To assess the impact of variations in slice and mesh properties in LE and NM rock slope analyses on consequences of failure.
- To identify reliable conditions for failure volume determination for both LE and NM methods.
- To assess the sensitivity of slip surface locations, and hence failure volumes, to variations in rock mass properties.
- To identify the impact of different failure criteria on slip surface locations, and hence on failure volumes, for an analytical method of slope stability.
- To efficiently compare the assessed risks related to slope stability when using LE and NM methods.

1.3 Research Methodology

Several parameters related to shear strength; rock mass strength and rock mass deformation will be varied to assess the sensitivity of the stability evaluation, and the failure volume, in order to understand the different results obtained from the two methods of analysis. Interest particularly arises regarding the impact of mesh density as well as the influence of the K ratio on greater volume of failure in NM analyses.

The adopted methodology will rely on the use of two programs called Slide and Phase2, corresponding respectively with LE analyses and NM methods. For the sake of avoiding the long running time required by a full NM analysis, the Response Surface Methodology (RSM) will be assessed and compared to the outputs of the probabilistic full Phase2 models. This is the validation of the RSM for its subsequent use as a replacement for a full NM approach. In other words, the adopted RSM/NM will be compared to the full LE, unless specified otherwise.

The process starts from the model settings, and goes all the way to the risk assessment.

1.3.1 Model settings

To simplify the problem, and due to the availability of only two dimensional (2D) continuum codes, the research will deal with homogeneous rock slopes, supposing that the same behavior of the volume discrepancy may be observed for a non-homogeneous rock mass. Moreover, this non-homogeneous case will be left open for further studies.

General material properties for an assumed rock slope will be considered, characterized by the Hoek-Brown criterion. And they will be obtained by resorting to case studies and RocLab data. The Hoek-Brown criterion will be the reference criterion. However, for the sake of assessing the impact of a chosen stability criterion, the equivalent Mohr Coulomb will be used for comparison. The RocLab program will be used to determine the equivalent Mohr Coulomb parameters.

1.3.2 Sensitivity of FOS to change of rock mass deterministic parameters and mesh settings

A series of models will be assessed to analyse the sensitivity of the Factor of Safety (FOS) while varying the Slope angles, Pit depth, K ratio (NM), Dilation angle (NM), number of Mesh elements and Mesh type (NM) and the number of Slices (LE).

1.3.3 Sensitivity of failure volume through deterministic analyses

As was shown in the previous section, variations of the same parameters will be undertaken, but this time for the purposes of assessing the sensitivity of the failure volume. The obtained graphical surface will be exported into a drawing eXchange format (dxf) extension program (AutoCAD) for an automatic volume calculation. Supposing that the out of plane extent of the failed block is equal to unity, the location of the failure surface will correspond with the failure volume.

1.3.4 Impact of failure criterion type on FOS

The Mohr Coulomb failure criterion has been most commonly used in slope stability analyses. However, some specialists prefer the use of the Hoek-Brown Criterion. Thus, this research will assess the impact of these two most used failure criteria on risk assessment using LE and NM methods for slope stability analyses.

1.3.5 Impact of failure criterion type on failure volume

Hoek-Brown versus Mohr-Coulomb results will be analysed for both LE analysis and NM methods. Afterwards, the resulting failure volumes will be analysed to understand the influence of a chosen failure criterion on the failure surface.

1.3.6 Probabilistic analyses

Any risk assessment requires knowledge of the probability of occurrence of the hazard. Thus, the research intends to evaluate such probabilities for the models, prior to any risk assessment. And for reasons of reducing the running times, the RSM will be introduced. However, prior to its use, and in accordance with Babu and Srivastava (2008) and Morgan and Henrion (1990), it will be applied after its validation. The Oracle Crystal Ball Excel add-in will be employed. Table 1-1 gives an overview of how the comparisons between the LE and NM methods can be made. However, in this study, only the homogeneous comparison will be used.

Table 1-1 Comparison of LE and NM probabilistic results

Comparison of LE and NM probabilistic results	
LE Analysis-Homogeneous Slope: Full LE run with Probabilistic analysis ↔	***NM method- Homogeneous Slope:*** Response Surface Methodology
LE Analysis-Non Homogeneous Slope : Response Surface Methodology ↔	***NM method- Non Homogeneous Slope:*** Response Surface Methodology

1.3.7 Risk Assessment

Having evaluated the consequences and the probabilities related to slope failure, the risk will then be calculated. Afterwards, the assessed risk will be compared from the LE and NM techniques. Here, only the adopted RSM will be used whenever required, as well as the adopted failure criterion for the optimum model conditions. Some patterns may also be established, which could confirm the reliability of the LE and /or NM methods.

1.4 Contents of the Research Report

In order to attain these objectives, the research is subdivided into six chapters: the first deals with the introduction, while the second contains a review of the literature that will contribute to putting the research problem into context. In the third section, the slope model settings are developed while the fourth chapter presents the use of the RSM. Results and discussions will be presented in chapter five, and the last chapter will deal with the conclusions and recommendations.

Chapter 2 LITERATURE REVIEW

Several researches regarding slope stability in surface mining have previously been carried out, especially looking at LE and NM approaches for slope stability. Hoek *et al.* (2000), Cheng *et al.* (2007), Chiwaye and Stacey (2010), etc. found that factor of safety (FOS) results obtained from LE analyses and NM techniques usually are in good agreement. However, these two methods diverge when it comes to showing the results of the slip surface locations, and hence the consequences of failure (Chiwaye and Stacey, 2010). To better understand and possibly address this divergence, a more thorough review of topics related to this issue is presented in this section.

2.1 Slope Stability in Surface Mining

2.1.1 Causes of instability

Mining activity generally contributes to the instability of rock mass slopes (Hustrulid *et al.*, 2000). On one hand, this issue is aggravated by a number of factors such as slope design, presence of groundwater, complex geology, geological information difficult to determine due to the vicinity of ore bodies and presence of significant alteration (Read and Stacey, 2009). Eberhardt (2003), on the other hand, stipulates that most of the rock slope stability problems are related to complexities of the geometries, in situ stresses, anisotropy and non-linear behaviour of the material. Additionally, many coupled conditions, such as pore pressures, seismic loading, etc constitute problem elements to be taken into account in the slope stability analyses.

Mining activity

The recovery of minerals implies a creation of openings in the ground. These openings may lead to underground or open pit mines. Due to the excavations, stress relaxation occurs and is associated with the initial movements in the slope, capable of reducing the confinement provided by the rock mass, and geological weaknesses inherent in the rock mass are exposed (Read and Stacey, 2009). Hence, mining activity generally causes diverse effects of rock mass deformation, leading to instability (Hustrulid *et al.*, 2000). An in-depth study of adequate slope design is therefore required for better conditions of stability.

Slope design

In the process of a slope design, the aim is to determine a slope height with a certain inclination that is economic and likely to remain stable over a reasonable life period (Nicholas and Sims, 2001).

According to Osasan (2013), excavating shallow ore bodies using open pit mining techniques results in the creation of a succession of benches commonly known as high-walls. For economic reasons, these benches should not have very flat slope angles, otherwise the cost of the removed waste material becomes very high, whereas for stability reasons, the opposite is required. The design process is therefore a trade-off between stability and economics, since steep cuts are often less expensive to effect than flat cuts due to a lower volume of excavated rock mass, lower acquisition of right-of-way and smaller cut face areas; however, they are likely to result in more unstable slopes than the flat cuts (Wyllie and Mah, 2004).

A large-scale failure can cause serious damage to the operation as well as to the personnel who work in the mine; therefore, it has become very important to design precise slope angles for open pit slopes, particularly because very small changes in the slope angle can translate into very important economic consequences (Nicholas and Sims, 2001).

Figure 2-1 illustrates the geometry of an open pit with its most relevant elements.

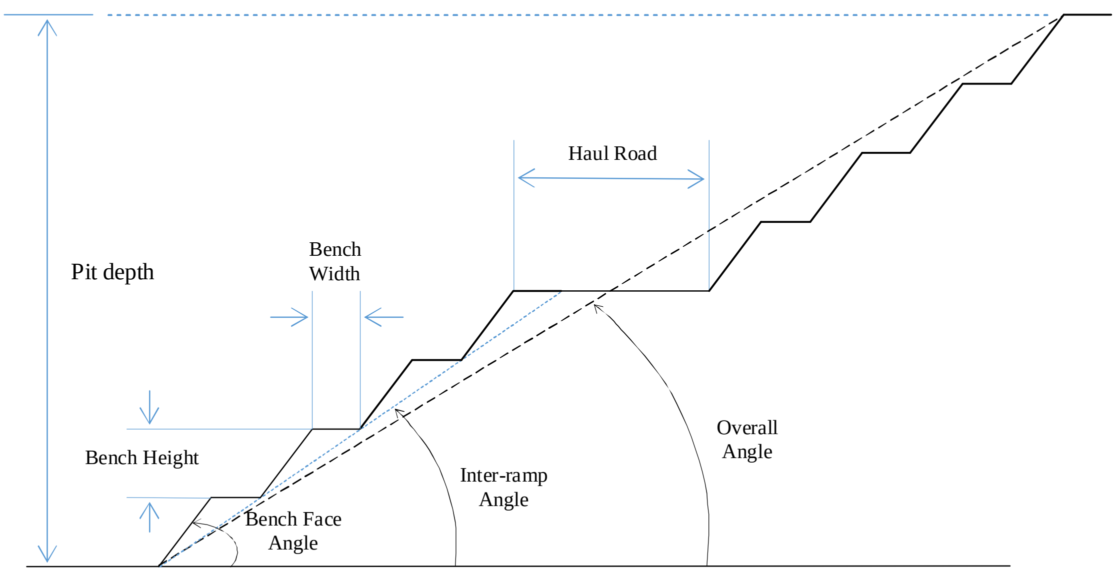


Figure 2-1 Typical open pit design, after Hustrulid *et al*., *eds*. (2000)

Once the geometry of the slope has been designed and the shear strength along the potential slip surface is known, computer programs may be resorted to in order to quickly determine its stability. However, it is recommended, rather than just using one or two values of strength parameters, that a considerable range of both the strength parameters and different slope geometries be assessed in slope stability analyses (Abramson, 2002).

The stability of the overall slope concerns only the failures that mostly incorporate the height of the pit slope (Nicholas and Sims, 2001). The increase in mining depths leads to an incremental risk of large-scale stability problems. Moreover, the risk is aggravated by the practice of mining with the steepest possible slopes in order to minimise the cost of stripping the waste, as mentioned previously (Wyllie and Mah, 2004). In the on-going research, the influences of either the height or the bench inclination will be investigated to determine their influence in the assessment of the risk using LE and NM methods.

Wyllie and Mah (2004) suggested that whenever the stability conditions of the slope design are not fulfilled, failure may occur. Generally, the geological conditions are one of the factors influencing the design, and therefore constitute the reason behind the cut (Abramson, 2002).

Geological constraints

When analysing the overall slope, all of the geological structure data-base should be incorporated. The rock texture and intermediate structure data used to characterise the rock mass allow one to predict any potential rock mass failure (Nicholas and Sims, 2001).

As discussed earlier, the factors influencing the design are the reason behind the cut. Alongside the geological conditions are the in situ material properties, seepage pressures, construction methods, and some natural phenomena likely to occur such as erosion, earthquake, freezing, flooding and abundant precipitation (Abramson, 2002). Dilation results from the rock mass loss of strength after failure (Crowder and Bawden, 2004), it represents the volume increase whenever the material is sheared. Parameters have been used to measure the amount of dilation. When Mohr-Coulomb material is concerned, the parameter corresponds to an angle, usually varying from zero (non-associative flow rule) to the friction angle value (associative flow rule).

And when the material obeys to Hoek-Brown criterion, dilation is identified by a dimensionless parameter, varying from zero to the Hoek-Brown parameter “m”. Low values of the dilation parameter are associated with soft rocks while high values are used for hard rock masses (“Phase2 FAQs: Theory,” n.d.).

Referring to the geological conditions, major-structure (intermediate and regional structure) data are more sensitive on the slope design than that of the rock fabric, which implies that the major structure data are likely to be the most important geologic structure data (Nicholas and Sims, 2001). Wyllie and Mah (2004) considered that the presence of discontinuities is often prejudicial to the stability of the slope, and this constitutes one of the reasons pushing designers not to completely rely on the advantage of the beneficial effects of concave slope curvature.

Influence of in situ stress on open pit design.

Prior to the use of an in situ stress measurement program for any particular site, it is advisable to carry out a parametric study using a three dimensional (3D) model. This allows for determining whether horizontal variations of in situ stresses have significant impact on the stresses induced in the rock mass encountered near the surface where slope failure may occur (Hoek *et al,* 2000). Not having at our disposal such a 3D program model, assumptions of plane strain conditions will be made.

Assessment of the behaviour of the horizontal stresses, usually determined at both civil and mining sites around the world, indicates a tendency towards high values of K ratio close to the surface, which decrease with an increase in depth (Hoek and Brown, 1980; Read and Stacey, 2009). The following figure 2-2 illustrates the trend of the K ratio with increasing depth below the surface:

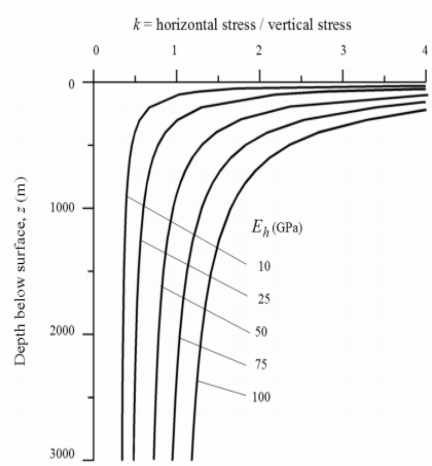


Figure 2-2 K ratio for different deformation moduli values (Sheory, 1994)

Particular focus should be directed towards the effect on slope stability of variation in depth. One of the analytical methods (NM) is able to take into consideration pre-mining initial stress conditions and evaluate their impact on the rock behaviour for simulated states of mining induced stress. These conditions are most appropriate in simulating shear failure modes, particularly brittle modes, which may be encountered at the toe of a steep slope in massive rock, characterised by a sudden increase of the k ratio over a short distance for deep pits (Read and Stacey, 2009).

Another in situ stress parameter is the locked in stress (LIS). Tan and Kang (1981) suggested that creep and locked-in stresses play an important role in the behaviour of rock masses. They affirmed that though the rock mass is subject to motions of rock formation, metamorphosis as well as tectonic motions, there are still stresses which persist within it. These are called residual stresses or LIS. Besides, they suggest that earthquakes' occurrence, localized in the rock mass, can be ascribed to the stress accumulation along definite seismic belts.

It is generally difficult to predict the effect of in situ stresses due to the fact that the rock mass behaviour depends on specific conditions such as the major structures orientations, rock mass strength and groundwater impacts; whereas one can say that the horizontal elastic displacement is directly proportional to the initial horizontal stress (Read and Stacey, 2009)

Seismicity and dynamic stability

Seismic and dynamic forces alter the shear strength profile, hence the slope stability. These forces are generally oscillatory, multidirectional and sporadic. And their effects, though not always resulting in a complete collapse of the slope, may lead to some unacceptable permanent deformation (Krahn, 2004).

Two coefficients Kh and Kv are proportional to the seismic forces depending on whether they are oriented in the horizontal or vertical directions. If the application of vertical seismic forces has little impact on the FOS, horizontal seismic forces have proven to play a significant part in the destabilisation process of the slope. Here, even a slight seismic coefficient can sensibly decrease the value of the FOS; but generally horizontal seismic coefficient increases are proportional to smooth decreases of the FOS (Bromhead, 1992 and Krahn, 2004).

Considering a condition of seismic shaking of a slope, the dynamic stresses oscillate dramatically such that the instantaneous FOS decrease and increase, respectively when the resultant oscillations translate into forward or backward movements of the slope as illustrated below:

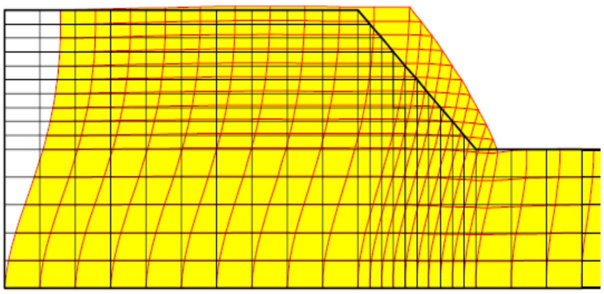


Figure 2-3 Snapshot of deformation during an earthquake, after Krahn (2004)

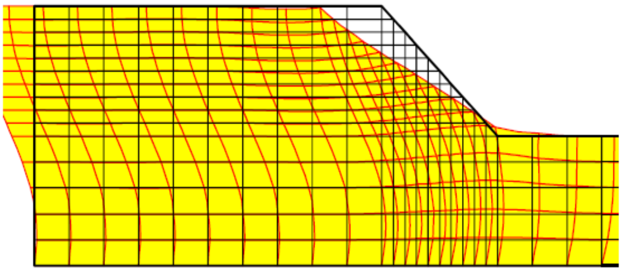


Figure 2-4 Snapshot of backward slope deformation during an earthquake, after Krahn (2004)

Such a wide range of FOS variation makes it difficult to advance any interpretation of the slope stability. The key issue is no longer the total collapse of the slope, but the number of shaking movements taking place during the earthquake. To each movement may be associated a corresponding average acceleration. And these average accelerations, plotted versus their respective FOS, may lead one to determine the acceleration equivalent to a FOS of unity; this is called the yield acceleration above which the slope is considered failed (Finn, 1988 and Krahn, 2004). Though this factor may have a considerable effect on the slope stability, its direct effect on the slope will be ignored in this study, supposing the pit to be deepened in a seismic-free zone.

Groundwater conditions

Another factor, capable of contributing to slope instability, is the groundwater impact. The most important problem related to groundwater conditions in a slope is the effect that water pressure can have on the stable angle of slopes. In the rock mass discontinuities, water pressure reduces the effective stresses on those discontinuities, resulting in a shear strength reduction (SSR). The problem is aggravated where there are critical features such as foliation, bedding, or a dipping wedge structure in a high-wall. The proposed solution may be the flattening of a wet slope; however, dewatering of slopes is a practical alternative which has been found to be both economic and desirable (Azrag *et al.*, 1998). This knowledge should be an incentive for opting for a completely drained slope; even though, the possibility of having groundwater conditions may need to be reviewed, if it is known before the slope is drained, that it was subject to the presence of water. Therefore, groundwater in a rock slope can have a negative effect upon its stability due to the following reasons (Wyllie and Mah, 2004):

- Water pressure reduces the slope stability by decreasing the shear strength of potential failure surfaces. And in tension cracks or similar near vertical fissures, water pressure reduces stability by increasing the forces that induce sliding.
- Freezing of ground water is likely to cause wedging in fissures filled with water due to temperature dependent volume changes in the ice. Additionally, freezing of the surface water on slopes can result in a build-up of water pressure in the slope, which may block drainage paths and consequently decrease the stability of the slope.
- The moisture content of some rocks can induce some changes, especially in shale, and can cause accelerated weathering which results in a decrease in shear strength.

Figure 2-5 presents the anatomy of groundwater flow in pit slope.

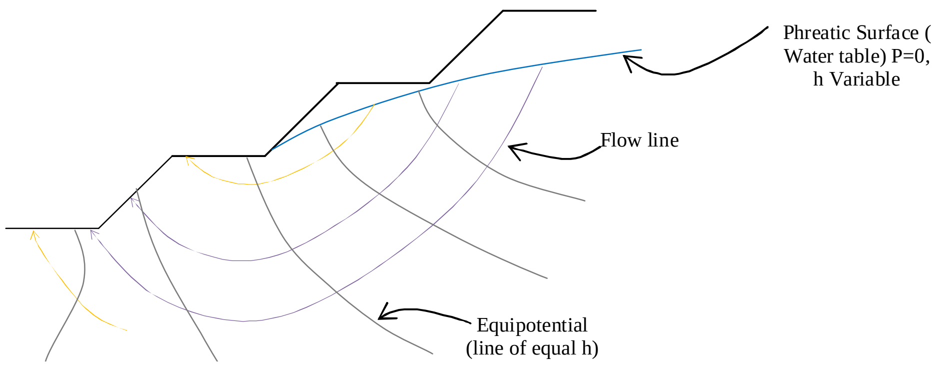


Figure 2-5 Groundwater Flow Anatomy System in Pit Slope (Hustrulid *et al.*, 2000)

2.1.2 Failure modes of slopes in surface mining

Considering a failure surface along which a mobilised strength exists, the magnitude of the latter is proved to be not uniform. And at any time that the shear stress is greater than the strength of some particular short sections of the surface, the equivalent excess loading will be redistributed to the neighbouring zones. However, if the rock mass is characterised as brittle, the redistributed stress may also induce the neighbouring zones to fail. Consequently, starting from some single points, the failure is then propagated to the entire rock mass until it fails (Abramson, 2002).

Three slope failure modes are commonly observed in a rock mass (Hustrulid *et al.*, 2000). These are:

- ✓ Circular (rock mass) shear failure, present in continuum slopes consisting of highly jointed or weak rock masses; this will be the failure mode of interest for the proposed research.
- ✓ Plane shear failure, which is likely to occur with pre-existing joints striking parallel to the slope angle, but dipping less than the slope angle. Moreover, failure of the rock mass can occur by sliding on the intersection between two planes of weakness - this is the two-plane wedge.
- ✓ Large scale toppling failures which are encountered in foliated slopes, and in persistently jointed or discontinuum slopes.

The Figure 2-6 illustrates the above mentioned failure modes.

In the case of closely fractured or highly weathered rock, a strongly defined structural pattern no longer exists, and the sliding surface is free to find the line of least resistance through the slope. Observations of slope failures in these materials suggest that this sliding surface generally takes the form of a circle, and most stability theories are based upon this observation (Hoek and Bray, 1974). These theories rely on the indicated analytical techniques for slope stability.

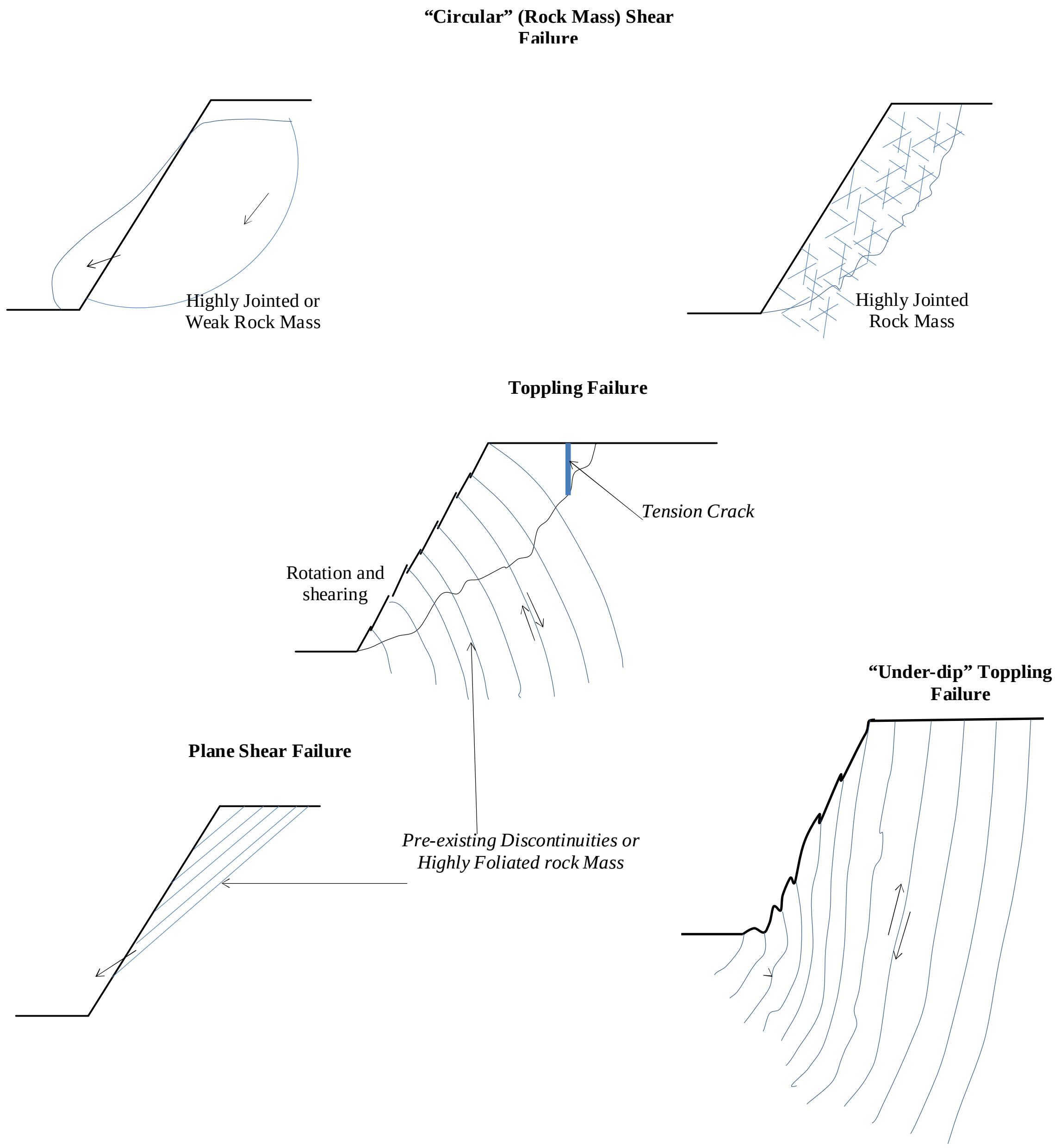


Figure 2-6 Analysed failure modes, after Hustrulid *et al*., (2000)

2.1.3 Stability Charts or Stability number

Slope stability charts are generally used for preliminary analyses. They help both to compare alternative solutions to be examined in detail at a later stage and to quickly check outcomes of detailed analyses (Abramson, 2002). Therefore, they are useful in determining an approximate FOS value, prior to any computing program; as such, they contribute to a quality control process and may lead to a comparison with results from a computer program.

Stability charts are more reliable for slopes characterised as ideal, such as homogeneous soil, and are divided into assumptions of 2D LE analysis, simple homogeneous slopes and circular slip surfaces shapes (Abramson, 2002). This is one of the reasons that should lead this study to adopt simple homogeneous slopes.

2.2 Analytical Techniques for Slope Stability

Slope stability analyses are generally undertaken by means of two well-known analytical tools: The LE and NM methods (Griffiths and Lane, 1999).

2.2.1 Limit equilibrium analysis

The LE method is the traditional method used for slope stability (Han and Leshchinsky, 2004). Narendranathan (2009) suggests that a potentially unstable block is at a condition of LE when the driving forces are exactly equal to the resisting forces. This condition involves the comparison of the available shear strength along the sliding surface with the force that is required to maintain the slope in equilibrium (Wyllie & Mah, 2004). LE stability analyses are usually 2D (Cala *et al.*, 2006). This fits well with the program “SLIDE”, which is a 2D LE program to be used in the current project.

LE methods are based on equations of statics to determine a certain constant called *Factor of Safety* (Krahn, 2004). These methods have seen improvement through time by relying on the success of some people’s studies such as Petterson (1955), who presented a stability analysis of the Stigberg Quay in Gothenburg, assuming a circular slip surface and a subdivision of the sliding mass into slices. Further, Fellenius (1936) introduced the so-called Swedish method of slices. Janbu (1954) and Bishop (1955), introduced this contribution to the method of analysis before it became much more practical through using computers, which allowed for the incorporation of even more rigorous formulations such as that devised by Morgenstern and Price (1965). The availability of commercial geotechnical software resources finally allows for the frequent use of slope stability analyses by means of LE approaches (Krahn, 2004).

LE Methods only provide data on the state or correspondence of forces and moment-balance conditions at the failure instant. Therefore, they are useful in providing simplified reasons for failure, whereas they are not detailed enough in terms of the time and progression of development from the condition of stability to instability (Balkema and Liden, 2004). Though the simplicity of the method is evident, it can still be very accurate (Baecher and Christian, 2002; quoted by Hammah and Yacoub, 2009).

LE approaches are frequently used nowadays, though the fundamentals of the approaches are, most of the time, not well assimilated and the expectations often exceed the boundaries of what they can really provide (Krahn, 2004). Cala *et al.* (2006) stipulate that the use of several 2D cross-sections may sometimes provide a reasonable understanding of the 3D condition. Nevertheless, applying LE analyses in the resolution of 3D problems is however limited due to several simplifying assumptions.

The LE analysis being one of the main tools to be employed in this study, a deep insight regarding its applicability and diversity is required. Therefore a brief review of principles circumscribing its ability is worthy of being undertaken.

Review of various principles

During the pioneering beginnings of slice methodology, LE analysis was mostly approached through the Fellenius method, also known as the ordinary or Swedish method. However, in this approach, no attention is given with regard to the inter-slice forces, the equilibrium equation only dealing with the moment concept. This eased the resolution process, since, at that time, no computer was available.

Here, the slice weight has two components which are parallel and perpendicular to the base of the slice. In the absence of pore water pressures, the ordinary approach is expressed in terms of the so called Factor of Safety (FOS).

The sense of the FOS parameter will be explained later in this project, within the section on probabilistic slope stability analysis. FOS can easily be computed using a spread-sheet or simply manually determined. Usually, the 2D slope is divided into a certain number of slices and properties and/or parameters are allocated to them such as cohesion, friction angle, slice width, mid-height, slice weight, inclination of the slice base and slice base length. From these data, manual calculations or spread-sheet techniques can be carried out to determine the FOS (Krahn, 2004).

Following the Swedish method, Janbu (1954) and Bishop (1955) included the normal forces in the inter-slices. This leads one to obtaining a non-linear equation for the FOS. Additionally, Bishop's simplified method only satisfies moment equilibrium (Krahn, 2004). Its non-linear equation is resolved by resorting to an iterative procedure. Still the shear forces were ignored.

The general limit equilibrium (GLE) is worth to be emphasised because it deals with two equations of FOS. The first equation presents the moment equilibrium based FOS, while the second equation expresses the FOS on the basis of the equilibrium of the horizontal forces. Morgenstern and Price (1965) proposed the following equation for the inter-slice GLE shear forces (Krahn, 2004):

$$X = E.\lambda.f(x) \qquad \textbf{(1)}$$

Where:

f(*x*): a function

λ :Percentage of *f(x)*, expressed in decimal form

E: Normal force in the inter-slice

X: Shear force in the inter-slice

Considering a range of Lambda (λ) values, the FOS computed through the GLE are represented by the following chart:

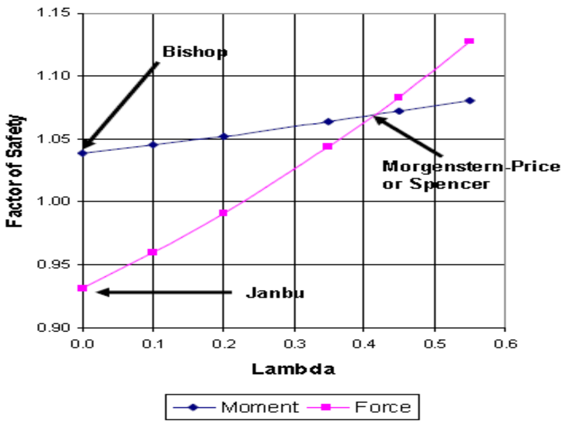


Figure 2-7 GLE Factors of Safety with dependence on Lambda (Krahn, 2004)

The charts of the dependence of the FOS with Lambda vary according to the shape of the slip surface (Krahn, 2004). As for the slip surface characteristics, a section of the current chapter will deal with this later on.

Consequently, it is known that an understanding of the method and knowledge of its limits and capabilities are required, prior to any use of an LE approach in order to avoid any misuse of the method. And as a guideline tool for the choice of an appropriate LE method, the Table 2-1 follows, indicating the abilities of different LE Methods to incorporate moment or force equilibrium.

Table 2-1 Equations of statics, after Krahn (2004)

LE Methods	Moment Equilibrium	Force Equilibrium
Ordinary or Fellenius	*Yes*	*No*
Bishop's Simplified	*Yes*	*No*
Janbu's Simplified	*No*	*Yes*
Spencer	*Yes*	*Yes*
Morgenstern-Price	*Yes*	*Yes*
Lowe-Karafiath	*No*	*Yes*
Janbu Generalised	*Yes (by slice)*	*Yes*
Sarma-Vertical slices	*Yes*	*Yes*

Steps prior to the choice of Limit Equilibrium Methods

The Essential first steps in the choice and use of Limit Equilibrium Methods are (Chowdhury and Rao, 2010):

a. The probable shape of the slip surface has to be visualised as soon as possible. For this, an understanding of the geological architecture of the site is of importance. Major discontinuities, existing slip surfaces, stratification, non-homogeneity, tension cracks and open joints have to be considered and examined with special attention.
b. A clear distinction must be established between first-time slides and possible repetitive movements along an assumed existing surface. In the case of old surfaces, rely only on the residual strength along them.
c. Decisions with regard to relative FOS and with respect to friction and cohesion have to be made.
d. Decisions have to be made whether to use effective stress analysis or total stress analysis. Exceptionally, the type of material has to be considered, the time, whether short or long term. Attention has to be oriented as to whether reliable estimates of pore pressure are feasible for field monitoring.

Though different techniques for resolution of the slice method have been developed, they all rely on the same principles: the differences emerging from the adequate equation of statics to be chosen, the forces between the slices to be incorporated and the relationship between the inter-slice shear and forces to be assumed (Krahn, 2004). LE analyses require the analysis of several slip surfaces, called trial slip surfaces, and from their FOS, one should find the surface with the lowest FOS value. Figure 2-8 and Figure 2-9 illustrate the slice discretization within a rock mass slope.

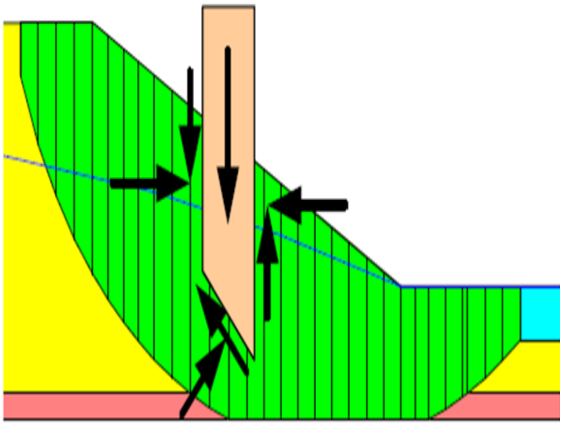


Figure 2-8 Slice discretization and slice forces in a sliding mass

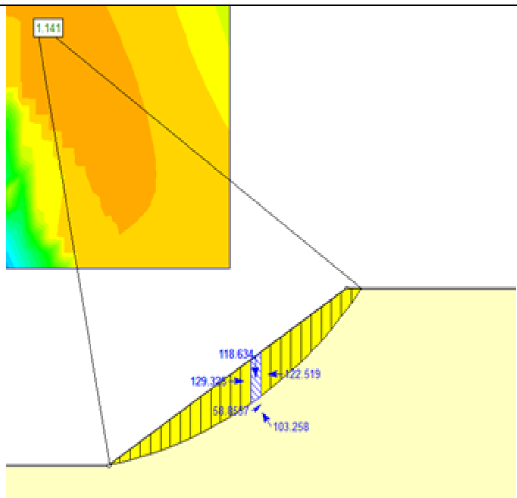


Figure 2-9 Slice discretization and slice forces in a sliding mass using SLIDE program with Bishop's method

Eberhardt (2003) suggested that there are coupled processes such as non-linear rock mass behaviour, pore pressure, seismic loading, etc. which constitute a problem in the analysis process of the slope stability. These conditions cannot be taken into account in LE analyses (Griffiths and Lane, 1999), but they can be incorporated into NM methods, though at the expense of running times.

2.2.2 Numerical modelling methods

NM methods are strain-stress based analytical techniques. They are mostly focused in analysing the rock mass deformation, unlike the LE analyses that deal with the condition of slope stability (Griffiths and Lane, 1999). Numerical techniques are used to evaluate slope stability particularly whenever there are complexities of parameters relating to geometry, material anisotropy, non-linear behavior and in situ stresses. These techniques are employed to interpret the rock mass physical behaviour and assess geological models, failure processes and eventual available slope designs (Read and Stacey, 2009). These methods are more recent than LE methods and are mostly used in open pit mining and landslide studies, where interest often focuses on slope displacements (Wyllie & Mah, 2004) and stress analyses. These techniques deal with the

behavior for different ratios of lateral to vertical normal effective stresses (K ratio), which is completely ignored by the traditional LE methods.
Generally, analyses of soil slope stability are identified through effective stress analyses. These methods assume the material to be fully consolidated as well as at equilibrium with the available stress system. Nevertheless, when additional stresses are suddenly applied and insufficient drainage is undertaken, failure usually takes place (Karzulovic and Read, eds. 2009).
In the numerical models, the rock mass is divided into elements, each one of them being assigned an optimal stress-strain behavior as well as properties to characterise the block behavior. Each element therefore is being assigned both material model and properties (Read and Stacey, 2009). In these models, the required effort of constructing a model is proportional to the number of stages simulated for the excavation. And knowing that the greater the number of stages, the more accurate are the solutions, there comes the matter of finding the optimal number of stages. This is sometimes resolved in considering two steps, an elastic model followed by simulated plastic behaviour (Read and Stacey, 2009).

Three approaches summarise the numerical methods of analysis when used for rock slope stability, i.e. continuum modelling, discontinuum modelling, and hybrid modelling. Only the first approach will concern this endeavor as it best suits the analysis of slopes which may involve massive or intact rocks, weak rocks, and soil-like or heavily jointed rock. The second approach, discontinuum modelling, is best suited for slopes controlled by discontinuity behaviour. The final approach, however, involves both the continuum and discontinuum approaches to maximise their advantages (Eberhardt, 2003). Included in this approach are different methods, but this study will limit its NM analyses to the use of the finite element method (FEM) due to the availability of a Roscience FEM program called “Phase2”. The FEM was first introduced into the geotechnical engineering sector by Clough and Woodward (1967). It represents a powerful tool for slope stability analysis as it is accurate, versatile and needs only a few initial assumptions, particularly with regard to the failure mechanism (Griffiths and Lane, 1999).

The principle of the FEM consists in modelling the rock mass as a set of discrete elements, so that the failure criterion condition results into a progressive, but not simultaneous failure behavior of elements (Abramson, 2002). Since the mesh quality, which is dictated by the number of elements and the type of element, has to be good to get reliable finite element outcomes, detailed studies have to be undertaken when generating a finite element mesh to optimise the tradeoff between a more refined mesh and the computing time (Thohura and Islam, 2013). Additionally, the quality of the FEM is dictated by the selected constitutive model, capable of simulating the real non-linear behavior of the slope (Abramson, 2002). LE methods are able to run thousands of safety calculations almost instantaneously, while numerical models require much longer ‘machine time’ to run only a single FOS calculation and are not easy to use (Hustrulid *et al.,* 2000).

These two most used analytical techniques are usually in agreement with regard to the outcomes of the stability parameters, as they almost converge. However, they deviate when it comes to the locations of their critical slip surfaces, which generally show a significant divergence (Chiwaye and Stacey, 2010). Investigation of the impact of the model settings on the failure surface location is therefore important.

2.3 Rock Mass Model

2.3.1 Introduction

The rock mass model is a component of the geotechnical model, apart from the geological, structural and hydrogeological elements. The aim of the rock mass model is to collect data in accordance with the engineering properties of the rock mass for further use in the stability analyses prior to the carrying out of the slope design. These properties include data, randomly selected and representative of the intact rock from the rock mass, of the structures, as well as of the proper rock mass (Karzulovic and Read, eds. 2009).

2.3.2 Rock Mass Strength

When the geotechnical engineering properties of the rock mass are set to be determined, the strength of the rock mass and the potential mechanism of failure have to be taken into account and included in the sampling and testing processes (Karzulovic and Read, 2009).

Mechanical properties are properties that determine the strength and the deformability of the rock. The well-known case is the uniaxial compressive strength (Karzulovic and Read, 2009). The strength of a rock mass is usually affected by an excavation such as a tunnel or a slope, and this translates into a relaxation of the confining stress. This causes the expansion of the remaining rock mass, which is also well known as the dilation. Thus, the strength of the rock mass will seriously be affected because it is highly dependent on the interlocking process between the pieces of intact rock that compose the whole rock mass (Hoek and Karzulovic, 2000).

Though it is not possible to correlate the rock mass strength and the dilation, one can still conclude that the loss of strength is not negligible for rock slopes. On the contrary, the strength loss of the rock mass due to blast damage is likely to be quantified (Hoek and Karzulovic, 2000).

As the study of the rock mass strength is directly linked to the evaluation of its condition of failure, some insight into the failure criteria is appropriate to define the conditions of the rock mass loss of strength.

2.3.3 Failure criteria

Abramson (2002) states that the study of failure criteria is useful to relate the available strength as a function of measurable material properties under imposed stress conditions.

Here, only two of the many criteria available will be referred to. These are the Mohr-Coulomb and Hoek-Brown failure criteria, which are the most frequently used.

Mohr-Coulomb Failure Criterion

The procedure of LE involves a comparison of the shear strength available along the sliding surface with the required force to maintain the slope in equilibrium. Generally, a rock mass with shear mode of failure can be assumed as a Mohr-Coulomb material. The corresponding shear strength will be expressed in terms of the cohesion and the friction angle (Wyllie & Mah, 2004).

Mohr-Coulomb shear parameters of weak planes or defects are the most used as the defect properties in the process of the stability evaluation of slopes. These shear strength parameters are the cohesion and the friction angle. When the analytical tool in use is the NM technique, it is advisable to incorporate into the analysis the stiffness of the defect (Barton and Choubey, 1977).

There exists a linear equation between the major and the minor stresses for the Mohr-Coulomb criterion, which is expressed as follows (Hoek and Karzulovic, 2000):

$$\sigma'_1 = \sigma_{cm} + k\sigma'_3 \quad \textbf{(2)}$$

Where:

σ'_1 and σ'_3 are respectively the major and the minor stresses

σ_{cm}: Rock mass uniaxial compressive strength

k : Gradient of the line joining σ'_1 to σ'_3

However, there is no straight correlation between the above equation (2) and the generalized Hoek-Brown equation, which makes the determination of the Mohr-Coulomb parameters, cohesion and friction angle, more difficult for a rock mass evaluated as a Hoek-Brown material (Hoek and Karzulovic, 2000).

Hoek-Brown Failure Criterion

Most rock mass models used in numerical continuum methods are linear or perfectly plastic stress-strain models. The Hoek-Brown failure criterion assumes a condition of isotropic rock and rock-mass behavior. Its use is only recommended for rock masses with a suitable number of closely spaced discontinuities and almost identical surface properties (Hoek and Brown, 1980).
These conditions are briefly illustrated as follows:

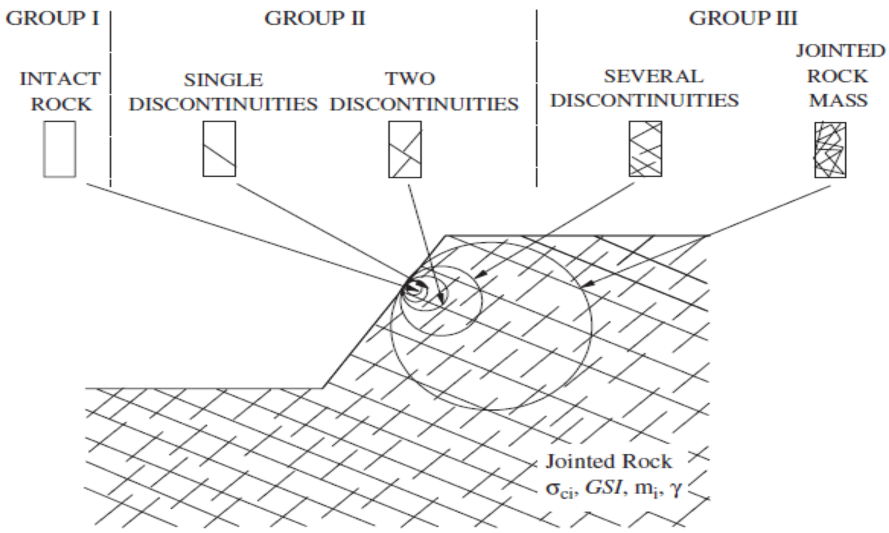


Figure 2-10 Ranges definition of Hoek-Brown application for slope stability problems

The above figure illustrates a rock mass slope, with sufficient number of closely spaced discontinuities. This case meets the applicability of the Hoek-Brown criterion, assuming an isotropic behaviour. (Hoek and Karzulovic, 2000).

The use of the Hoek-Brown criterion for estimating the strength of jointed rock masses as well as their deformability requires the determination of the following three properties (Hoek, 2007):

- ✓ Uniaxial Compressive strength of the intact rock mass fragments (σ_{ci})
- ✓ Geological Strength Index relative to the rock mass (GSI)
- ✓ Hoek-Brown constant relative to the intact rock (m_i)

Moreover, whenever feasible, it is recommended to determine the values of the above parameters via statistical assessment of the results of triaxial tests (Hoek, 2007).

The GSI is represented by a number, which may be associated with the properties of the intact rock to provide an estimation of the strength reduction of the rock mass considering a variety of geological conditions (Hoek *et al,* 1995). It is recommended that consideration of the undamaged rock face after blasting will offer adequate observational material to be used in the estimation of the GSI value, since the purpose is to assess the properties of the rock mass which did not suffer any disturbance (Hoek, 2007).

Mohr-Coulomb versus Hoek-Brown Criteria

In comparison to Mohr-Coulomb criterion, Hoek-Brown criterion has proved to be the practical failure standard (Lorig *et al.,* 2009).

Nevertheless, due to the importance of evaluating the impact of these two criteria on the stability of the slope, some other process may be undertaken as specified by Li *et al* (2008). These explain that, since most geotechnical engineering software are Mohr-Coulomb failure criterion based; equivalent friction angles and cohesive strengths have to be determined for each rock mass. These are found through the following expressions:

$$c' = \frac{\sigma_{ci}\left[(1+2\alpha)s + (1-\alpha)m_b\sigma'_{3n}\right](s+m_b\sigma'_{3n})^{\alpha-1}}{(1+\alpha)(2+\alpha)\sqrt{1+\left(6\alpha m_b\left(s+m_b\sigma'_{3n}\right)^{\alpha-1}\right)/(1+\alpha)(2+\alpha)}} \qquad \textbf{(3)}$$

$$\phi' = Sin^{-1}\left[\frac{6\alpha m_b (s + m_b \sigma'_{3n})^{\alpha-1}}{2(1+\alpha)(2+\alpha) + 6\alpha m_b (s + m_b \sigma'_{3n})^{\alpha-1}}\right] \quad \textbf{(4)}$$

Where $\sigma'_{3n} = \sigma'_{3\max} / \sigma_{ci}$

c': Cohesive Strength

ϕ' : Friction Angle

σ_{ci}: Uniaxial compressive strength of the intact rock

α, m_b, s: Hoek and Brown Constants

σ'_{3max} is determined for each particular problem. Li *et al.* (2008) suggested that instead of considering a unique way of determining the value of σ'_{3max}, this can be found through more realistic formulae, depending on the adopted slope angle. It allows a reduction in the errors of estimation of the Hoek-Brown curve to the Mohr-Coulomb line, as illustrated in the following figure:

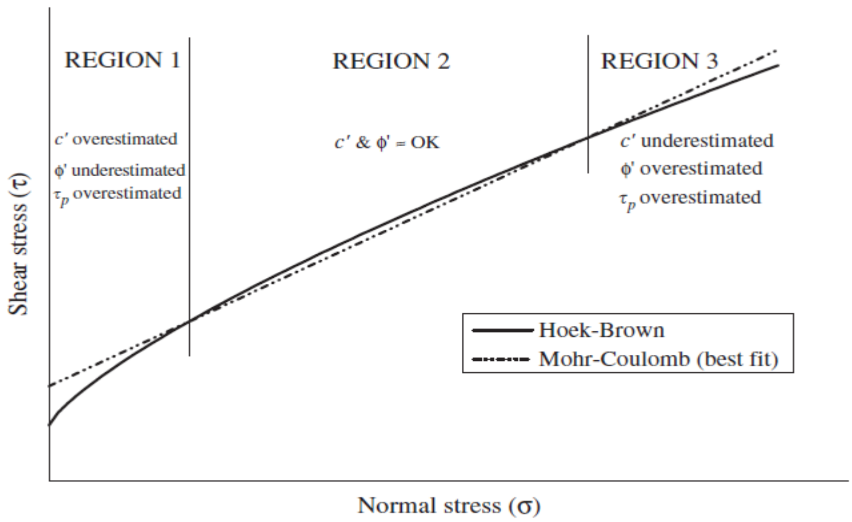


Figure 2-11 Hoek-Brown and equivalent Mohr-Coulomb Criteria, after Hoek *et al.*, (2002)

Explicitly, the way of reducing the error of estimation proposed by Hoek *et al* (2002), while considering the equivalent Mohr Coulomb from Hoek-Brown parameters, has been done by the equation: $\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.72\left[\frac{\sigma'_{cm}}{\gamma H}\right]^{-0.91}$. However, Li *et al.,* (2008) suggested the errors may be even more reduced while incorporating the impact of the slope angle in the determination of the equivalent Mohr-Coulomb. This led to considering two different equations depending on the fact that the slope angle in the study is either less or greater than 45 degrees. And the corresponding equations are illustrated below:

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.2\left[\frac{\sigma'_{cm}}{\gamma H}\right]^{-1.07} \quad \textbf{(5)}$$

For a steep slope, $\beta \geq 45^\circ$

$$\frac{\sigma'_{3\max}}{\sigma'_{cm}} = 0.41\left[\frac{\sigma'_{cm}}{\gamma H}\right]^{-1.23} \quad \textbf{(6)}$$

For a gentle slope, $\beta < 45^\circ$

Where:

σ'_{cm} : Rock mass strength

γ : Unit weight of the rock mass

H : Height of the slope

2.4 Slip Surfaces from LE and NM

Depending on the applied method of slope analysis, in two dimensions the failure is characterised by a certain line, which corresponds well to defined principles. The slip surface shape is mostly controlled by some parameters such as the geologic features, the stratigraphy, or combinations of linear and circular segments. However, the process of determining the critical slip surface, which has to correspond with the lowest FOS, constitutes one of the key issues in stability analysis (Krahn, 2004).

2.4.1 LE slip surface

LE stability analyses consist in determining critical slip surfaces of rigid rock mass volumes, so that FOSs of these volumes are assessed and that the overall slope slip surface is the one associated with the lowest FOS (Hustrulid *et al.,* 2000; Rocscience 2008). The critical slip surface is determined by resorting to a trial procedure, such that many possible slip surfaces are created and consequently their associated FOSs are computed. Finally, the trial slip surface with the lowest factor of safety is the adopted critical slip surface (Krahn, 2004).

A LE critical slip surface obtained from analysis does not necessarily represent the final failure surface. Actually, a new and more critical failure surface is likely to develop behind the first one in a case where the failing material slides away; it is also called a *retrogressive failure*. This phenomenon is frequently observed in flatter slopes and in cases when almost all, if not all, of the failed material is supposed to have slid away from the slope. The explanation of this statement relies on the fact that the resulting failure surface after the occurrence of the slope failure, creates a much steeper local slope than the original slope shape (near the crest). Consequently, the new slope has more instability than the original, with a flatter geometry (Sjöberg, 1999).
Depending on the type of sliding movement, the importance of the inter-slice force function may influence the FOS of the slope.

The slip surface may have different shapes such as circular, planar, composite slip surface, block slip surface and shoring wall (Krahn, 2004). However, as mentioned previously, only the circular surface will be of interest in this study, corresponding with the chosen material properties.

Circular failure surfaces are the most critical slip surfaces in homogeneous slope materials. The simplest analysis is based on the theory of assuming a rigid-cylindrical mass that will fail by rotation about its center. Here, the shear strength along the failure surface is equivalent to the undrained strength characterised by an internal friction angle equal to zero (Abramson, 2002).

2.4.2 NM slip surface

Unlike the LE analysis, the predicted failure development from NM methods is associated with the occurrence of deformations within the slope and important displacements will occur prior to the full development of the slip surface. Therefore, here the location of the slip surface is determined by the stress distribution in the slope (Sjöberg, 2000). It can be stipulated in this case that failure cannot be considered as movement of a rigid body as assumed in an LE analysis

(Hustrulid *et al,* 2000). And to better assess the slip surface location, it is recommended to select tabs with immediately higher shear strength reduction factor (SRF) values than the one displayed (Rocscience, 2009).

2.4.3 Comparison of LE and NM slip surfaces

Once the design choice has been made, and the model parameters related to the strength of the rock mass, as well as the failure criteria, specified, the locations of slip surfaces almost always follow different paths for LE and NM methods. Obviously, due to the variability of rock mass data, the slip surface generation should take variability into account, instead of considering only deterministic and limited values, hence the introduction of the probabilistic concept. Han and Leshchinsky (2004) suggest that there is a significant difference in the location of the critical slip surface predicted by LE and NM Methods. This is corroborated by Chiwaye and Stacey (2010) who found that the volumes above the critical slip surface from the NM analysis is almost always bigger than that from the LE analysis; except, as suggested by Cheng *et al* (2007), simple homogeneous soil slopes are likely to have their failure surfaces located almost at the same place for both analytical methods. However, they did not study the influence of the mesh properties on the slip surface location. Thus in this study, numerical models will be run by varying the mesh element densities and types to assess their effect on the failure surface location, though at the expenses of longer running times.

2.5 Probabilistic Slope Stability Analyses

Through the approach of the analytical techniques, the FOS is determined based on the circular mode of failure. Consequently, the POF may be calculated.

The context of slope stability and probabilistic studies requires an initial deterministic analysis of slope stability in the rock mass (Fenton and Griffiths, 2008). In geotechnical practice, the reliability is usually measured using indices such as the FOS, safety margin and reliability index. However, in this study, we will limit the determination to the FOS, as it is mostly straightforward as a result of a stability model.

2.5.1 Safety Factor

The FOS is known as the ratio of the load bearing capacity and the demand on that capacity (Harr, 1987). Several other definitions of safety factors have been suggested so far by researchers such as Morgenstern and Price (1965); Barton and Choubey (1977), Sjöberg (2000), Cheng (2003), Narendranathan (2009), etc. These definitions might be summarised in Read and Stacey's (2009) definition, which stipulates that the FOS is the ratio of the actual rock mass shear strength to the reduced shear strength at which the material failed.

The evaluation of the FOS depends on a number of factors and the evaluation itself also depends on the types of analysis to be used. Generally, gravity forces and the seepage forces influence the factors that affect the slope stability (Craig, 2004). Besides, the type of analysis depends on whether the FOS deals with a short or long term application. Though the short term application is considered simple and regardless of the seepage forces, both analysis types can be applied for slopes (Nash, 1987).

The FOS is logically computed by the means of reducing the shear strength until the rock mass fails, as in the Finite Element Method (FEM), a method to be applied in this research. Narendranathan (2009) indicates that a block is at LE condition when the FOS is equal to unity. As mentioned in the previous section, in LE analysis the FOS of the slope corresponds with one in which the rock mass failure volume has the most critical slip surface (the lowest FOS).
The methodology of determining the FOS through numerical modelling is summarised as follows (Hoek and Karakas, 2008):

a. Consider a material characterised by the Mohr-Coulomb criterion, with deterministic values of cohesion and friction angle, resulting in a certain value of the FOS.
b. Run simulations for a series of increasing trial FOS values (*f*). The shear strength properties (known as cohesion and friction angle) are reduced until collapse occurs. These properties are determined as follows:

$$C_{trial} = \left(\frac{1}{f}\right).C \tag{7}$$

$$\phi_{trial} = \arctan\left(\frac{1}{f}\right).\tan\phi \tag{8}$$

c. These new values will be entered as material properties while an assumed higher value of the strength reduction factor (SRF) will be considered

d. After repeating these previous steps, the different values of the increased SRF are plotted against the maximum total displacement

e. The value of the SRF at which the maximum total displacement starts showing a sudden increase is considered as the slope FOS.

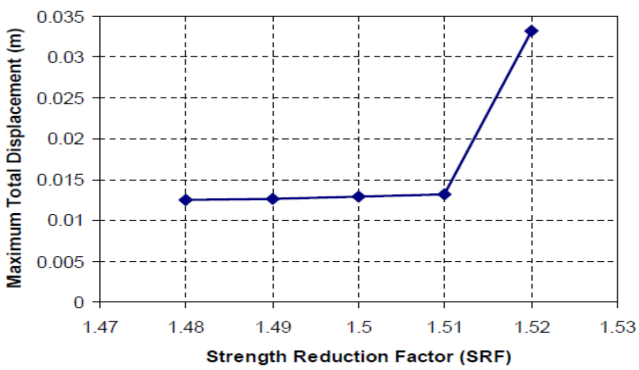


Figure 2-12 Strength reduction factor versus maximum total displacement

Factors of safety calculated with NM models coincide very closely with those determined by the LE analyses in cases where these analyses are known to give reliable results (Hoek *et al.*, 2000).

Any design of engineering systems basically requires that the engineer in charge conciliates both performance and safety under the constraint of economy. However, most of the rock mass data necessary to a successful design are affected by a certain level of uncertainty (Read and Stacey, 2009).

2.5.2 Data uncertainty and probability of failure

Because of the inherent characteristic of the data uncertainty in any engineering design, the process of assessing risk always confronts the varying degree of uncertainty of input data, resulting in a range of output risk estimates (Ang and Tang, 1984). Likewise, the choice of introducing the concept of the probability of failure (POF) in slope stability analyses is also due to the variability of the parameters involved in the rock mass behaviour (Abramson, 2002).

Data uncertainty for a block capable of sliding from a mine bench may always be associated with non-deterministic values such as the spacing, the orientations or the strength of the delimiting failure surface. Thus, levels of confidence should be allocated to all the analysis steps, i.e. from the geological model, through to the structural, rock mass, and hydrogeological models and finally to the geotechnical model (Read and Stacey, 2009).

When considering the two analytical methods, here in this study, Chiwaye and Stacey (2010) showed that the POFs obtained from these techniques also correspond well, but only when the dilation angle and the locked-in-stress are neglected in the NM method. Otherwise, the obtained probabilities of failure will diverge depending on the type of analytical method.

Some authors, among which are Griffiths and Fenton (2000), have introduced a more rigorous method of probabilistic analysis. This method, called Random finite-element method (RFEM), combines the non-linear FEM with the random field generation techniques. In the present study, the RFEM will resort to a random field generation technique known as the Monte Carlo (MC) Method.

The MC method is a technique in which the analyst creates a large number of sets of randomly generated values for uncertain parameters and computes the performance function for each set. The number of trials to be used in the MC simulation depends on the level of accuracy one would like to achieve; knowing that the accuracy is proportional to the increase in the trial number.

The Monte Carlo method shows clear advantages in the way that various probability distributions are able to be incorporated with fewer approximations, and that the correlations among variables are modelled with ease (Baecher and Christian, 2002). Information on the distribution of the response variables is then obtained from the MC simulations (Hammah & Yacoub, 2009), such as the probability of the FOSs less than unity corresponds with the POF. By means of the MC method, the LE analysis can make thousands of safety factor simulations almost instantaneously while the NM methods require longer periods of time to make just one safety factor calculation and are not easy to use (Valdivia and Lorig, 2000). These constraints may be avoided by resorting to a concept called “Response Surface Methodology”.

2.5.3 Response Surface Methodology

The Response Surface Methodology (RSM) is a collection of statistical and mathematical techniques which are useful for developing, improving and optimising processes (Box and Wilson, 1951). In the slope stability context, it is a safety factor regression function established on the basis of the more influential variables of slope stability, employed to avoid the time consuming constraints of NM methods (Read and Stacey, 2009).

The RSM is also defined as an assembly of statistical and mathematical methods capable of developing, improving and optimizing processes (Nicolai and Dekker, 2008). The response or output variable is influenced by numerous independent input variables. And multiple runs are assessed; each differently from the other, by changing the input variables so that the changes observed in the output may be studied. The decision of opting for an RSM in the process of design optimisation is aimed at reducing the time taken for the classical analysis techniques (Box and Wilson, 1951).

The steps adopted in the RSM formulation are mostly repeated in the following figure 2-13:

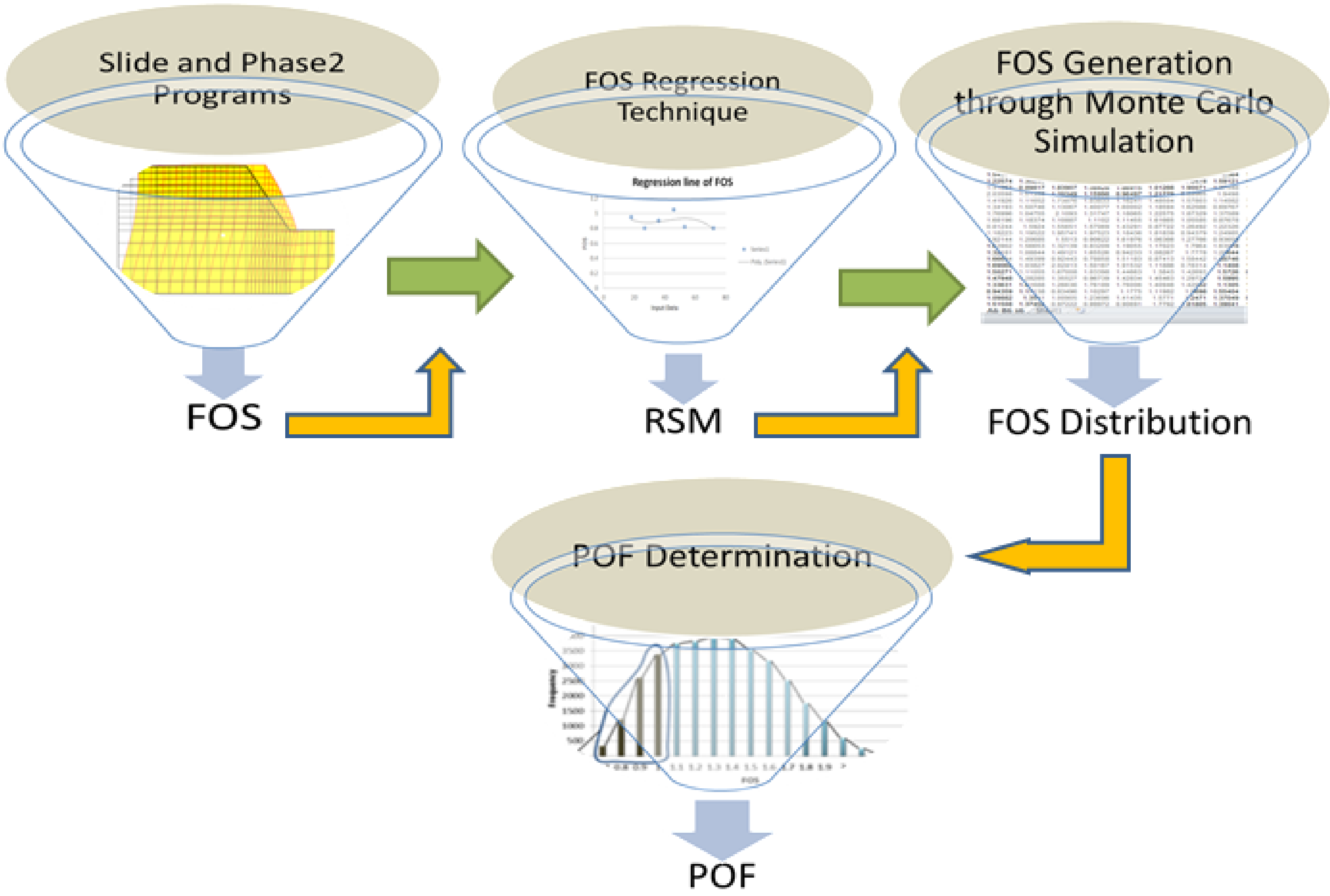


Figure 2-13 RSM Formulation

Three main questions arise in the implementation of the RSM (Morgan and Henrion, 1990, cited by Chiwaye and Stacey, 2010):

- How to get a small sample of the scenarios with which to run the large model?
- How to best select uncertain parameters and determine which particular variables should be modeled?
- How close are the response surface results in comparison to the traditional methods results?

In the study of regression functions, the second order polynomial response is mostly used as an approximation to the real response surface. This is due to the fact that it is a very flexible model and satisfies a range of various types of functions. The RSM technique only needs a small number of runs (only 2n+1 different runs) to assess the reliability process, in contrast with other more demanding simulation techniques (Myers *et al.,* 2009). Obviously, the linear form of the number of runs is reasonably limited, even when the number of variables is high (Morgan and Henrion, 1990).

The reliability analysis of geotechnical systems is associated to the application of the RSM. A sensitive parameter has therefore to be identified to help with quality control (Babu and Srivastava, 2008).

The general formula of the RSM is $y = f(x_1, x_2, x_3, ...) + e$ **(9)**

After coding the natural variables, x_i is expressed in a new variable ξ as following:

$$\xi = \frac{x_i - [\max(x_i) + \min(x_i)]/2}{[\max(x_i) - \min(x_i)]/2}$$ (Babu and Stivastava, 2008) **(10)**

Real and adjusted regression coefficients are then determined so that the difference between them must be small: $R^2 \approx R^2_{adj}$ **(11)**

Afterwards, a second moment reliability index β will be determined thanks to a matrix expression:

$$\beta_{HL} = \min_{x \in F} \sqrt{(X - M)^T C^{-1} (X - M)}$$ (Babu and Stivastava, 2008) **(12)**

To establish a performance function g(x) as a failure surface, the concept of the RSM will then be needed. Two ellipses (1-σ) and (1-β) will be plotted so that the one that will be tangent to the failure surface will be β times the size of the (1-σ) dispersion ellipse (Babu and Stivastava, 2008).

Below is an illustration of the RSM with reliability index:

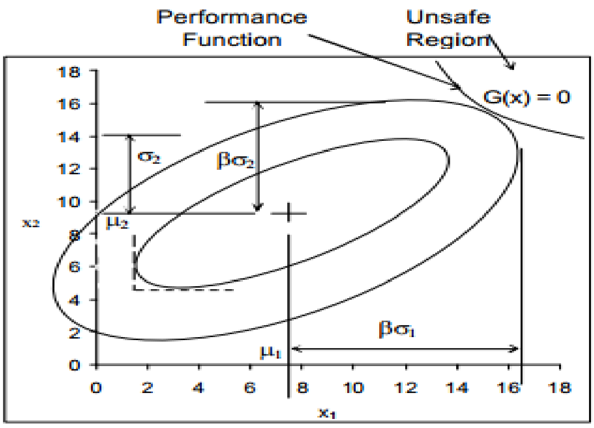


Figure 2-14 Mean Points and Reliability Index in a Plane, after Myers et al. (2009)

Issues relative to sampling data, screening uncertain inputs and finding the best fit of the RSM will be dealt with in detail in the methodological section where sensitive rock mass properties are to be defined (Morgan and Henrion, 2009). The validation of the RSM is done by comparing its results to the one obtained from the full NM probabilistic run without any use of RSM.

When the obtained probabilistic results are reliable, and having already determined the locations of failure surfaces, hence the consequences of failure, then risk assessment may be performed.

2.6 Geological Risk Management for Slope Stability

Due to the unavoidability of the risk existing in any engineering system, it is useful to adopt a reliability-based design to evaluate and monitor the “non-eliminable risk”. The reliability can therefore be considered as a probabilistic measure of certification of performance (Ang and

Tang, 1984), taking account of the specific consequences of the potential failures (Steffen *et al*, 2008). Risk is the probability of occurrence of an event combined with the consequence or the potential loss which is associated with the event (Tapia *et al*, 2007). Thus, risk can mathematically be expressed as:

$$\textbf{\textit{Risk= (Probability of the event)*(Consequence of the event)}} \qquad \textbf{(13)}$$

Read and Stacey (2009) suggested that the risk evaluations may be represented using a semi-quantitative risk matrix. In this case, results of their occurrence likelihoods, as well as their impacts should be determined prior to the use of the matrix, which has the following display:

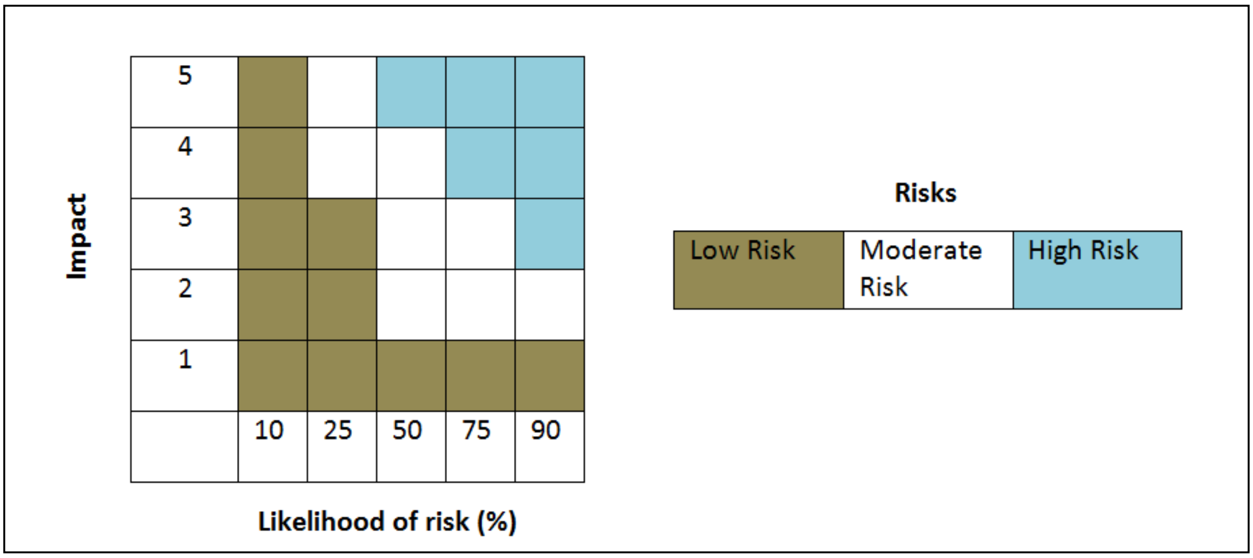


Figure 2-15 Semi-quantitative risk matrix

Duncan (1996); Griffiths and Lane (1999), Chiwaye and Stacey (2010), etc found that the risks determined from LE are usually less (but sometimes greater) than those determined from NM methods. Here, the difference of failure volumes obtained from the two analytical methods leads to different values of risk being calculated. Chiwaye and Stacey (2010) compared LE and NM approaches for risk analysis in open pit mining and concluded that the risk obtained from NM is generally greater than that from LE methods. They suggested that this is due to the fact that LE methods underestimate the failure volumes and hence the consequences of the slope failure. Therefore, they recommended that NM should be incorporated into risk analyses in addition to LE models, or to replace them when more confidence has been gained. Hence, the thrust of the current research is to investigate and to understand why there exist such differences in the usage of NM and LE analyses. To fulfil the series of research objectives, a methodological study designed to establish the rock mass model was undertaken.

Chapter 3 SLOPE MODEL SETTINGS

In order to reach the objectives stated in section 1.2, a series of model settings have been performed. A preliminary validation study allowed definition of the prototypes of Slide and Phase2 models to be employed throughout the research analyses. The validation of the Slide model is intended to adopt model parameters that are not biased or tendentious, but instead should generate the most representative results of the rock mass stability behaviour. FOSs, POFs and sliding volumes were then investigated with regard to variations in numbers of slices and in slope angles. The results of these preliminary analyses will dictate the number of slices to adopt for the subsequent analyses, investigating slope failure under Mohr-Coulomb or Hoek-Brown criteria. Distributions of rock mass parameters, whether normal or lognormal, were incorporated in this study. In the same way, the Phase2 model was validated before its use. The process consisted similar preliminary analyses, this time varying the number of mesh elements and their types for different cases of slope angles. A last validation process concerned the applicability of the RSM, aimed at easing the POF determination from the NM analyses. The results of the POFs determined using RSM were compared to the results obtained from a full probabilistic NM, with the aim of validating the use of the RSM.

Deterministic analyses were carried out to assess the sensitivity of the FOS while varying parameters such as slope angle, pit depth, k-ratio, locked in stress and dilation. This was done using the NM method as well as the LE analysis, depending on their respective ability to include one or the other model parameters. The impacts of the parameters variations could lead to further statements with regard to the performance of the two methods of analysis. Later on, the probabilistic analysis would also be carried out, for comparison purposes of the two methods in study and also to assess the impact of some parameters not included in the LE program.

Risk assessment was also undertaken to understand and assess the theory suggesting that the outcome risk from numerical models is often greater than the one found from LE analyses. Detailed studies, particularly associated with the inherent functionality of the programs, had to be undertaken to identify the reasons behind the low consequence values (specifically small volumes).

These analyses were not intended to simulate a particular case in detail, but rather to explore the general characteristics of slope failures; thus, some typical failures were simulated for ‘generic’ slopes geometries. Model parameters were verified through comparisons with reported laboratory model tests.

Prior to any detailed model setting, the geometry of the slope and external geometry of the model were chosen as follows. It is justified by the fact that this work, as it relies on the findings of Chiwaye and Stacey (2010), should almost consider the same data as theirs.

3.1 Slope Geometry

The slope external boundary was chosen, making sure that the height is 600 meters to ensure no divergence from Chiwaye and Stacey’s (2010) data. The generic slope geometry is configured in the following table:

Table 3-1 Slope boundary coordinates

Vertices	X	Y
1	0	0
2	3000	0
3	3000	1200
4	1000	1200
5	400	600
6	0	600

This geometry was adopted as a reference case, unless specified otherwise. This is in accordance with the criterion stipulated by Wyllie and Mah (2004), which proposed the FEM boundary conditions acceptance, and the corresponding dimensions are shown in the following Figure 3-1.

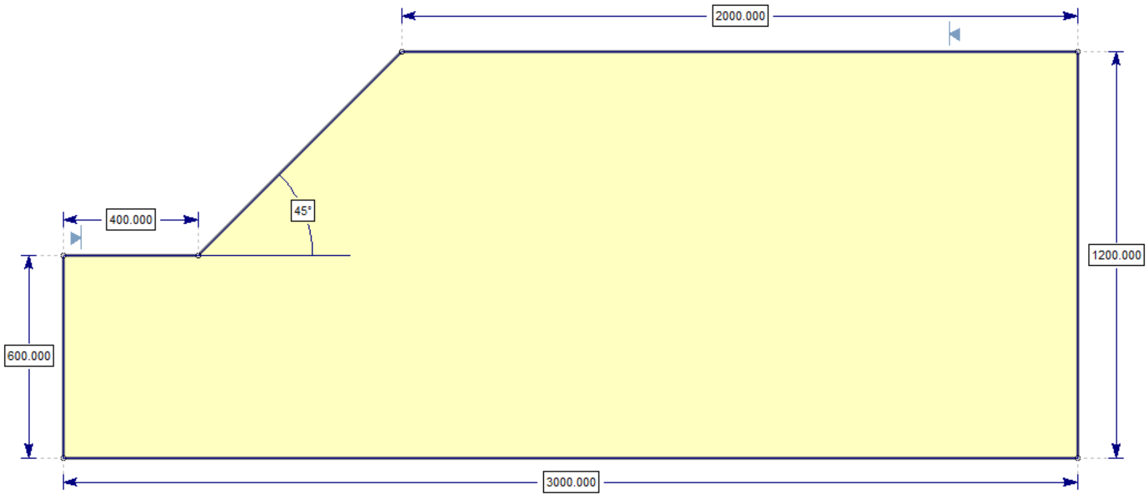


Figure 3-1 Generic slope elements dimensions

Though different geometries have been assessed, the above geometry was adopted for Numerical models as the reference model geometry in conformity with the earlier cited research; and, for the sake of uniformity, the same model geometry was used for LE analyses as well. The material properties were then allocated.

3.2 General Material Properties

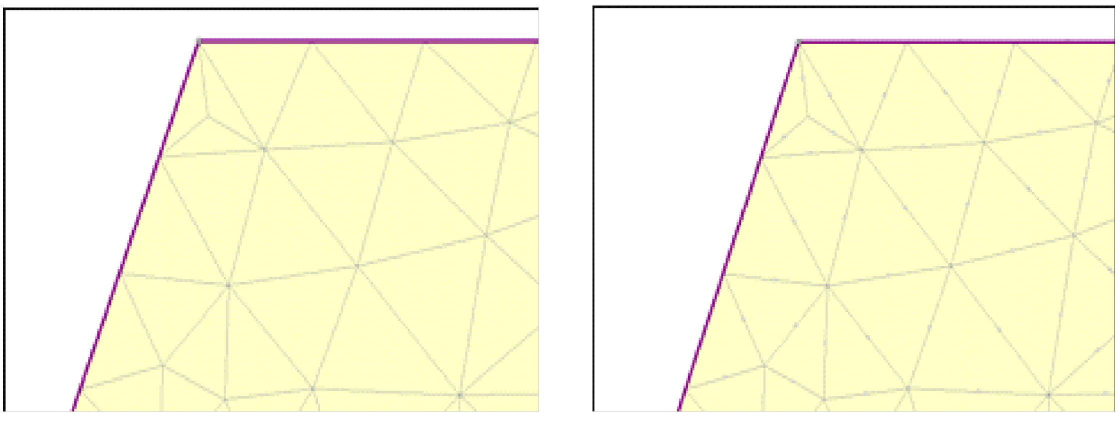
All the analyses were based on one type of material, which is homogeneous. However, for the sake of diversity and to compare some realistic non-homogeneous material, it will be proposed to carry out the same study for non-homogeneous cases. This could also be processed to determine and understand the behaviour of mass based rock on the two selected methods of analysis for further studies. The adopted failure criterion for all cases is Hoek-Brown, but, due to the fact that Mohr Coulomb is the most used failure criterion, some comparisons were undertaken to justify the choice of the criterion. In this case, the equivalent Mohr Coulomb method suggested by Hoek and Brown (2002) was referred to, after its validation.

There was no surface water and therefore no water Ru value had to be considered. The blasting was assumed to be “heavy”, leading one to consider the worst condition of disturbance factor (D) equal to one (1). This is justified by the fact that average conditions of rock mass were chosen and that the project was supposed to relate to some data from Chiwaye and Stacey (2010) in order to understand their outcomes and possibly minimise the discrepancy obtained in the comparison of the two methods of analysis.

The homogeneous material was assigned the following properties: unit weight of $27kN/m^3$, UCSi was assumed to be 80MPa, Geological Strength Index (GSI) equalling 50, and the intact rock yield parameter (m_i) equal to 10. This is a case of general rock mass properties. Much more specific model settings were selected based on the type of analytical method in use as explained in the next section.

3.3 Stability analysis methodology

Factors of safety were determined for various combinations of model properties. Variations in slopes angles and depths were made to study the stability sensitivities due to each parameter and their influence on the risk assessment. Four different angle values were chosen: 25, 45, 60 and 80 degrees. And before, for validation, a single Slide model and a Phase2 model were run for each angle, considering a homogeneous material subdivided respectively into 4; 10; 25, 500 slices (even 1000 slices for volume assessment) and 100, 1500, 50 000 mesh elements (and 100 000 elements for volume assessment). Four different Phase 2 element types were run: triangular (three and six nodded), and quadrilateral (four and eight nodded) as shown in figures 3-2 and 3-3, respectively. Only uniform meshes were applied. A validation of the adequate type of mesh element was the goal, which had obviously to corroborate the theory suggesting that accuracy of stability results generally improves for elements with mid-side nodes.



(a) (b)

Figure 3-2 Triangular mesh elements of (a) three nodes and (b) six nodes

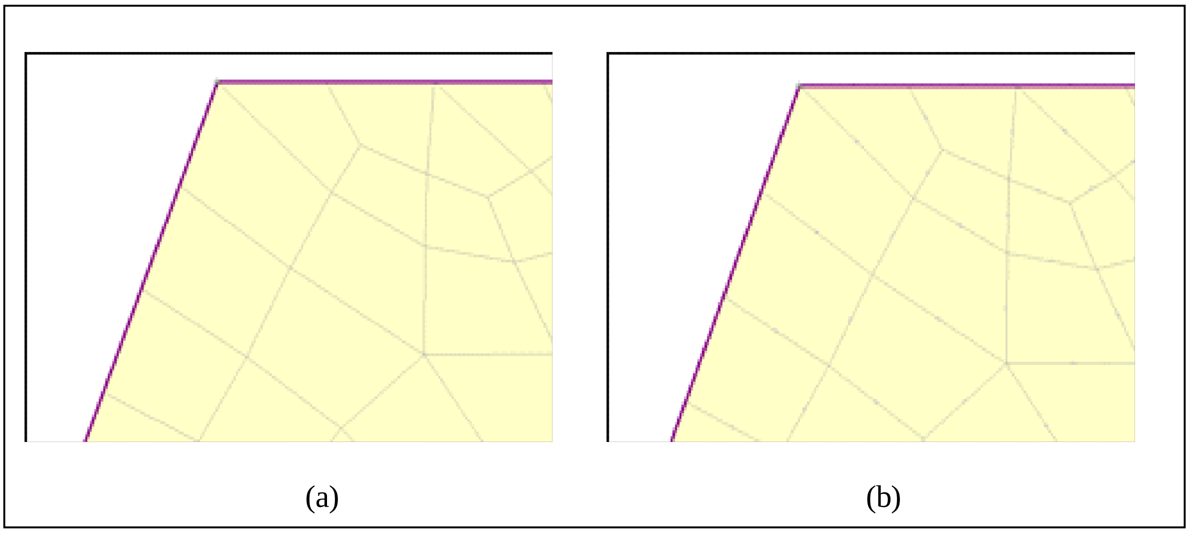


Figure 3-3 Quadrilateral mesh elements of (a) four nodes and (b) eight nodes

Similarly, a study of the stability sensitivity, and failure volume sensitivity was investigated. The same techniques employed by Chiwaye (2010) in the assessment of the surface were used. Slide indicates distinctly the sliding area with an incorporated failure surface, but Phase2 requires a careful estimation of the failure path found in drawing a polyline to clearly identify and therefore assess the failure zone. The likelihood of error should be taken into account with regard to the volume assessed from the NM. The modelling program being 2D based, an assumption was made to consider the extent of the failure dimension in the out of plane direction to be equal to unity for both Slide and Phase2. In other words, only the surface seen in 2D matters the most. The 3D extent of the failure was not considered in this research. Several models were run for both the LE analysis (Slide) and the NM method (Phase2). Both methods work in the same conditions as 2D models. 2D techniques have proved sufficient to analyse the failure surfaces differences, and furthermore the risk assessments differences when using LE and the stress analysis method.

3.4 LE Model Settings

As said earlier in this chapter's introduction, Slide was the chosen LE method due to its availability. Metric stress units (kPa) were adopted.

3.4.1 General settings

For uniformity reasons, failure directions were assumed from right to left in standard data output option, and a maximum of 25 material properties were considered. The “Global Analysis” was adopted as analysis type, and the analyses were based on the “Bishop simplified” method of slices, to conform to the recommendations suggested by Abramson (2002). This choice is justified by the fact that the project deals with a homogeneous slope. Each analysis will consider an optimum number of slices in the convergence options, which shall be proved reliable, except when the variation of number slices is required for sensitivity purposes. Therefore, the influence of the number of slices is worthy of a particular study. The default values of tolerance and maximum iterations have been kept, as in Chiwaye and Stacey’s (2010) work, and are respectively equal to 0,005 and 500.

3.4.2 Slip surfaces

A non-circular slip surface search has been assessed in preference to the circular slip surface search to better approximate a practical path of the failure surface. And due to the fact that this work relies on the findings of Chiwaye and Stacey (2010), the same search methodology called “path search” was kept, as it was proven to be much better than the circular one. Actually, in this method, no imposition of any slip surface geometry is to be made, however the program identifies the right path to be taken by the failure surface.

The default number of 5000 surfaces of failure was retained and the “optimize surface” option was always chosen. The pseudo-random surface option was enabled to ensure repeatability of the results for the same seeds, with its seeds automatically generated by resorting to Park and Miller v.3 number generator option. The segment length was chosen to be auto-defined. No grid requirements were needed, since the model searched for the optimum slip surfaces. The global minimum slip surface was always applied for the sake of getting the lowest factor of safety.

3.4.3 Deterministic analysis and sliding volume calculation

The main criterion used was Hoek and Brown, but due to the need of comparing its impact on the slope stability, an alternative criterion was chosen, the Mohr-Coulomb criterion, on which most stability programs are built. The original rock mass parameters are displayed in Table 3-2.

Table 3-2 Deterministic parameters for LE with Hoek Brown criterion

Material name	Property	Deterministic value
Material 1	UCSi	80MPa
	GSI	50
	mi	10
	D	1
	MR	400

And the equivalent Mohr-Coulomb parameters were determined resorting to the RockLab program. The same procedure as applied by Chiwaye and Stacey (2010), showing the minimum stress σ_{3max} had to be customised to 3 MPa. This ended in the values for the cohesion and friction angles respectively equal to 0,641MPa and 30,98degrees. Therefore, this research opted for equivalent Mohr-Coulomb parameters equal to 640MPa for cohesion and 30 degrees for the friction angle. The small changes to round the figures off will not invalidate the result as the rock masses have very variable parameters. Li *et al.* (2008) proposed an alternative to the equivalent Mohr-Coulomb criterion, which might be recommended in case the traditional equivalent Mohr-Coulomb shows some significant discrepancy when compared to the Hoek-Brown criterion. And once the deterministic parameters were set, the sensitivities of the FOS were assessed by varying some model parameters. The following demarche will be undertaken to evaluate the criteria of the FOS behaviour:

Sensitivity of FOS to change of rock mass parameters and slice number

A series of models will be assessed to analyse the sensitivity of the FOS while varying the Slope angles, pit depth and the Slice number.

Sensitivity of failure volume through deterministic analyses

As shown in the previous paragraph, variations of the same parameters will be undertaken, but this time, to assess the sensitivity of the failure volume. And the obtained graphical surface will be exported into the "dxf" extension program (AutoCAD) for an automatic surface calculation. Supposing that the out-of-plane length of the failed block is equal to unity, the value of the failed surface will coincide with the corresponding volume.

Impact of failure criterion type on FOS

Mostly, Mohr Coulomb failure criterion has been used so far. There are some specialists who prefer the use of Hoek and Brown Criterion. However, this research will assess the impact of the two most used failure criteria on slope stability while using LE and NM methods, which will eventuate in the most reliable failure criterion according to the current rock mass conditions.

Impact of failure criterion type on failure volume

Hoek-Brown versus Mohr-Coulomb results will be analysed for both LE analysis and NM methods. Afterwards, the outcome volumes will be analysed to reveal the influence of a chosen failure criterion on the failure surface.

Knowing that the rock mass is affected by many variations, it is therefore worthwhile to incorporate a probability study in the stability analysis.

3.4.4 Probabilistic analysis

The research intends to evaluate the probabilities of the models, prior to any risk assessment. The LE method being a simple and often quick stabilisation method, the RSM was not used for this case, only full probabilistic analyses were undertaken. However, in particular cases such as when the sensitivity analysis was needed, the RSM had to intervene even in the LE analysis.

The sampling method used was the Latin-Hypercube. It allows for better results using only a few samples when compared with the Monte Carlo method. The checkboxes of both the sensitivity analysis and the probabilistic analysis were ticked, with the considered number of samples adopted being equal to 10000, to get results close to the ones from the population.

Three parameters were considered as variables. These are the Uniaxial Compressive Strength of intact rock (UCSi), the Geological Strength Index (GSI) and the Hoek-Brown m_i constant. These variables were considered both as normally and log-normally distributed with the FOS to study the influence of a well-chosen distribution on the slope stability. To conform to the normal distribution, the standard deviation has to be equal to a third of the mean.

The following tables display the parameters on which these distributions were built.

Table 3-3 Hoek-Brown input variables for normal distribution

Material Name	Property	Distribution	Mean	Standard Deviation
Material 1	UCS intact	Normal	80000KPa	26660KPa
	GSI	Normal	50	16.6
	mi	Normal	10	3.33

Table 3-4 Equivalent Mohr-Coulomb parameters for normal distribution

Material Name	Property	Distribution	Mean	Standard Deviation
Material 1	C	Normal	640KPa	213KPa
	ϕ	Normal	30°	10°

Particular attention needed to be considered in the determination of the standard deviation of the lognormal distributions since it is no longer a simple matter of dividing the mean by a certain constant. Here it was made sure that when the means have increased with a step of a standard deviation, the strength parameters from the Hoek-Brown criterion will still almost correspond to the parameters from Mohr-Coulomb, as illustrated in tables 3-5 and 3-6.

Table 3-5 Hoek-Brown input variables for Log-normal distribution

Material Name	Property	Distribution	Mean	Standard Deviation
Material 1	UCS intact	Lognormal	80000KPa	16000KPa
	GSI	Lognormal	50	2
	mi	lognormal	10	3

Table 3-6 Equivalent Mohr-Coulomb input variables for Log-normal distribution

Material Name	Property	Distribution	Mean	Standard Deviation
Material 1	C	Lognormal	640KPa	146KPa
	ϕ	Lognormal	30°	6°

It is worth noting that the impact of a failure criterion choice was assessed with regards to the probability outcomes, likewise the sensitivity of the FOS and failure volumes to rock mass changes, prior to any risk assessment.

3.4.5 Risk assessment

Any risk assessment requires the knowledge of the probabilistic magnitude of the hazard. Thus, having evaluated the consequences and their corresponding probabilities, the resultant risk will then be calculated. Its value should be multiplied by a factor "t", which represents the cost related to each m^3 of failure volume. Since the main idea is to assess the slope risks in relation to their stability, the impact of the choice of failure criterion will also be evaluated in reference to its influence on the slope stability risk.

Similarly to the current section 3.4, NM settings are summarised in the next section.

3.5 NM Model Settings

The available 2D NM program for slope numerical analyses was Phase2. However, there was a possibility of using a demonstration version of the Flac2D program, limited unfortunately to only 600 zones. Since this would not allow the number of zones to be changed at will, it was believed that this could influence the stability analysis results in this comparative study. Therefore Phase2 was used in preference.

3.5.1 General settings

Plane strain conditions were assumed, and the "Gaussian Elimination Solver" method was chosen. The adopted units were "metric, stress as kPa". The number of iterations assumed was 500, with a tolerance of 0,001. And "Absolute Energy" was chosen as the convergence type in stress analysis. In this, the option of tensile failure reducing shear strength to residual was selected, while a factor of 0.01 was assumed for the option of joint tension reducing joint stiffness. Only a single stage was chosen for the analysis, particularly based on the fact that no groundwater condition had to be set, the rock slope being considered as completely drained.

Phase2 uses a method called "shear strength reduction" (SSR) to determine the Factor of Safety. The method consists of reducing the shear strength of the material until the model becomes unstable (Rocscience, 2008). The step size was set to "Automatic", with a SRF tolerance fixed at 0.01. The convergence parameters were determined automatically, with the option of accelerating the analysis by reducing the number of iterations after failure. The tensile strength was enabled.

An original un-deformed boundary area was established with the same geometry being used for the Slide program. Lateral boundaries were fixed only in the x direction, thus allowing displacements in y direction. The bottom boundary was restricted along both x and y directions; the upper boundaries forming the slope were free of restriction.

Apart from the general material properties mentioned in Chapter 3, Section 3.2, particular properties were required to be set for the NM. The material type was set to plastic for an isotropic elastic type. Though in Chapter 5, which presents the results and discussion, the Hoek-Brown results are compared with the Mohr Coulomb outcomes with regard to stability parameters as well as the failure volume, the first criterion was again taken as the main strength type, to maintain uniformity with the LE settings. The field stress type was chosen for gravity using the actual ground surface. Both the total stress ratios in-plane and out-of-plane were considered to be equal to unity, unless specified otherwise. The reference model applied values of locked-in stress and dilation parameters equal to zero kPa, unless specified otherwise. The following table 3-7 summarises the material properties for the homogeneous rock mass applied to the NM method:

Table 3-7 Hoek-Brown material properties applied to NM methods

Material Properties for Hoek-Brown Criterion	
Initial element loading	Field stress & body force
Unit weight	27kN/m^3
Elastic type	isotropic
Young's modulus	2.13GPa
Poisson's ratio	0.25
Failure criterion	Generalised Hoek Brown
Material Type	Plastic
Dilation parameter	0
Compressive strength	80 000kPa
m_b parameter	0.281157
s parameter	0.00024
a parameter	0.505734
Residual m_b parameter	0.281157
Residual s parameter	0.00024
Residual a parameter	0.505734
Piezo to use	None
Ru	0
D	1
MR	400

3.5.2 Mesh characteristics

One of the key parameters on which the analyses were focused was the type and number of mesh elements. This has been varied to check the sensitivity to the sliding volume. However, this could not be done in LE cases. Nevertheless, a fixed number of 3000 mesh elements were set for the reference model.

The basic mesh type was chosen to be uniform; the element types were 3 and 6 nodded triangles, 4 and 8 nodded quadrilaterals. The mesh generation was done automatically by the Phase2 program, resulting in good quality elements.

3.5.3 Slip surfaces

The NM slip surface was found by displaying the option of maximum shear strain. Sometimes, at the actual SRF no failure zone was formed. Therefore, it was decided to display the maximum shear strain corresponding to the SRF one step greater than the actual value. And even when this configuration was effected, the actual slip surface could not be found automatically. However, a polyline needed to be drawn beneath the failure zone. This shows a weakness of using Phase2 to determine the failure volume, because the estimation may be exaggerated if a cautious approach is not taken.

3.5.4 Deterministic analysis and sliding volume calculation

Sensitivity of FOS to change of rock mass parameters and mesh settings

As for the LE methods, a series of numerical models will be assessed to analyse the sensitivity of the FOS while varying the slope angle and pit depth, and other parameters not included in LE analyses such as K ratio, LIS, dilation parameter, mesh type and number of mesh elements. The considered FOS corresponds to the displayed value of SRF in the interpretation stage of the model. In this study, reliable ranges of the K ratio, LIS and the dilation parameter needed to be considered. Theory from Read and Stacey (2009) showed that K ratio lies in a range between zero and four. The LIS was considered from zero to an exaggerated value of 40MPa, while the dilation parameter varies in accordance with Rocscience recommendations ("Phase2 FAQs: Theory," n.d.), as elaborated in Chapter 2, Section 2.1.1, under the heading of geological constraints, or simply from zero to 2.81.

Sensitivity of failure volume through deterministic analyses

Similar to the proceedings of the previous paragraph, an assessment of the sensitivity of the failure volume to variations in the same rock mass parameters will be undertaken. The sliding area will be found using the AutoCAD Program. Assuming a constant sliding thickness of unity in the out of plane direction, the calculated value of the sliding area will automatically be equal to the value of the corresponding sliding volume. It is to be noted that this approach could not guarantee with 100% confidence the estimation of the area, since there was difficulty in identifying the exact slip surface location before exporting the failed area's shape to AutoCAD.

However, a cautious approach was taken to keep to a minimum the likely error of approximation, say 5%.

3.5.5 Probabilistic analysis

The current research intended to evaluate the probabilities of failure of models before dealing with the risk related to slope stability. As referred to in Table 3-3 to Table 3-6, three rock mass parameters (UCSi, GSI and mi) were considered as variables for the Hoek-Brown criterion, and two parameters (cohesion and friction angle) for the Mohr-Coulomb criterion. To reduce the running times, the RSM was introduced after its validation, to align this research with suggestions from Babu and Srivastava (2008), Morgan and Henrion (1990). The Oracle Crystal Ball Excel add-in was the chosen tool. Instead of running the probabilistic full model with Phase 2, only the deterministic models were run in Phase 2 and the results of the Factors of Safety were recorded.

3.5.6 Risk Assessment

Likewise LE risk assessment, NM evaluation of risk will revert to the results obtained from the probabilistic analyses and the volume calculations. Once the POF was determined and the corresponding failure volume calculated, and knowing the unit cost (t), which is the cost associated with each m^3 of failed volume, a study of risk assessment was undertaken to establish the LE and NM risk outcomes. The risk determination was found by multiplying the POF, the failure volume, and the unit cost (t) together. It is worth noting here that only the volume of failure was input as a consequence - other consequences might be relevant for both LE and NM methods.

3.6 LE versus NM Model Settings

The main purpose was to identify the reasons behind the greater predicted failure volumes from the NM methods than from LE techniques. Knowing that these latter techniques have limited data input possibilities; the NM method is then the more flexible analytical tool. It mostly helped to establish reasons behind the high failure volumes and could consequently offer more room, if possible, for reducing the volumes differences. The Hoek-Brown criterion having a wider range of input data than the Mohr Coulomb criterion, as mentioned earlier, will allow comparisons of the two analytical methods in study with regard to both Hoek-Brown and Mohr-Coulomb criteria.

Table 3-8 and Table 3-9 illustrate the LE and NM abilities regarding input parameters for Hoek-Brown and Mohr Coulomb, respectively.

Table 3-8 Model setting options for Generalised Hoek-Brown criterion

Model inputs	Limit Equilibrium	Numerical Modelling
Initial element loading	✓	✓
Analysis type	✓	✓
Elastic type	✓	✓
Young's modulus	✓	✓
Poisson's ratio	NA	✓
Peak tensile strength	✓	NA
Residual tensile strength	✓	NA
Material type	NA	✓
Dilation Angle (Parameter)	✓	✓
Piezo to use	✓	✓
Ru value	✓	✓
Total Stress Ratio (horiz/vertic in plane)	NA	✓
Total Stress Ratio (horiz/vertic in plane)	NA	✓
Locked-in horizontal stress (In plane)	NA	✓
Locked-in horizontal stresss (out-of-plane)	NA	✓
Field stress type	NA	✓
Strength Reduction settings	NA	✓
Probabilistic analysis type	Global Minimum	PEM*
Number of statistical variables	2	3
Distribution type	✓	NA
Mesh settings	NA	✓
Displacements settings	NA	✓
Slices settings	✓	NA
Predefined surface shape (settings)	✓	NA
Sensitivity analysis	✓	✓

*Point Estimate Method

Table 3-9 Model setting options for Mohr-Coulomb criterion

Material Properties	Limit Equilibrium	Numerical Modelling
Initial element loading	✓	✓
Analysis type	✓	✓
Elastic type	✓	✓
Young's modulus	✓	✓
Poisson's ratio	✓	✓
Failure criterion	✓	✓
Peak tensile strength	✓	✓
Residual tensile strength	✓	✓
Peak friction angle	✓	✓
Peak cohesion	✓	✓
Material type	NA	✓
Dilation angle (Parameter)	✓	✓
Residual friction angle	✓	✓
Residual cohesion	✓	✓
Piezo to use	✓	✓
Ru value	✓	✓
Total stress ratio (horiz/vertic in plane)	NA	✓
Total stress ratio (horiz/vertic in plane)	NA	✓
Locked-in horizontal stress (In plane)	NA	✓
Locked-in horizontal stress (out-of-plane)	NA	✓
Field stress type	NA	✓
Strength reduction settings	NA	✓
Probabilistic analysis type	Global Minimum	PEM*
Number of statistical variables	2	3
Variables correlation settings	✓	NA
Distribution type	✓	NA
Mesh settings	NA	✓
Displacements settings	NA	✓
Slices settings	✓	NA
Predefined surface shape	✓	NA
Sensitivity analysis	✓	✓

*Point Estimate Method

3.7. Probabilistic Rock Mass parameters Impact on Overall Slope

3.7.1 Case 1 and Case 2 definitions

A sensitivity study of various rock mass parameters was made. For such, the research resorted to Crystal Ball utility tools. Two cases of slope model settings were considered (see Table 3-10). Case 1 was considered the base case, referred to throughout the whole study, and mostly deals with the three main parameters defined in Table 3-3. Some changes to this case have been introduced, with particular attention being paid to three more parameters not applicable in the LE analyses, and this represents Case 2. This research intended to investigate the influence of each parameter on the overall slope stability.

The three parameters considered as main variables of the rock mass behaviour are the UCS, GSI, and mi, and the three other parameters investigated in Case 2 for the NM sensitivity analyses are the K ratio, Locked-in stresses (LIS) and the dilation parameter.

3.7.2 Case 1 and Case 2 distributions

The findings of the distributions for the three main parameters were made almost easy by referring to some case studies and their distributions were associated to normal distributions. However, when the distributions of the three additional parameters were concerned, the scenario was different. Almost thirty deterministic runs were processed for each of these parameters to fit the best distribution. And in the case of the best distribution not being normal, Crystal ball still gave an approximation of the normal distribution, providing also its mean and standard deviation. The ranges of K ratio and locked in stresses are from zero to four and from zero to forty, respectively. However, the dilation parameter only varied from zero to zero point zero six (or *m/3*). This is justified by the low value of the deformation modulus (2,1 GPa), corresponding to soft rock mass; and with this, only low values of dilation parameters should be considered (“Phase2 FAQs: Theory,” n.d.).

Besides, it was decided to use the RSM, instead of the full numerical modelling, to avoid the long running time usually required. There was no need to apply the RSM in an LE analysis due to its relatively short running time, except when the sensitivity analysis was required. The study needed more effort due to the variability of the input parameters and eventually their combinations while building the probabilistic model.

Table 3-10 Variability of rock mass parameters for case 1 and case 2

Input Parameters	Case 1				Case 2			
	LE_3 variables		NM_3 Variables		NM_ 3 variables		NM_6 variables	
	Mean	Standard dev.	Mean	Standard dev.	Mean	Standard dev.	Mean	Standard dev.
UCSi (Kpa)	80000	26660	80000	26660	80000	26660	80000	26660
GSI	50	16.6	50	16.6	50	16.6	50	16.6
mi	10	3.3	10	3.3	10	3.3	10	3.3
k ratio	NA	NA	1	0	1.5	0	1.5	1.34
LIS (KPa)	NA	NA	0	0	1330	0	1330	44
Dilation parameter	NA	NA	0	0	0.025	0	0.025	0.017

Chapter 4 RESPONSE SURFACE METHODOLOGY

As mentioned earlier in the literature review chapter, the RSM is identified as a collection of statistical and mathematical techniques which are useful for developing, improving and optimising processes (Box and Wilson, 1951). It has been used to limit the running times required to compute models, particularly for NM cases and when probabilistic analysis is involved.

Generally, there are many ways of finding a response surface. A completely manual technique, entirely based on a spread-sheet tool could be adopted. This may be employed in other research, particularly when the aim is to promote the use of manual techniques for generating the RSM for a large number of users, who might mostly be interested in its generation for academic purposes rather than in the perfection of the tool's results, or when a commercial package is not available. Additionally, this tool is almost completely manual, with no use of commercial software, apart from the Monte Carlo Excel simulation. However, based on the reasons cited above and due to the use of a commercial package, supposedly more reliable, the completely manual practice was ignored. Since the reliability of the results matters here, the commercial package was preferred. Among numerous types of commercial packages, the Oracle Crystal Ball package was chosen. This choice is justified by the fact that it had already been used in some previous research, including the Chiwaye and Stacey (2010) studies, and because it was available.

4.1 Response Surface Methodology Formulation

The RSM is an explicit or implicit function which establishes a relationship between input random variables and output values by means of regression techniques, as suggested by Babu and Srivastava (2008).

4.1.1 RSM based on "Oracle Crystal Ball"

The Crystal ball package is a well-furnished simulation program with most of the processes already incorporated and, in applying it; the work consists of carefully defining the input parameters as assumptions with a fitted distribution (Gentry *et al.*, 2008). The outcome, here identified as forecast, is obviously defined as a function of the assumptions capable of simulating the equivalent POF. Besides the formulation task of assumptions and forecast, it was simple to resort to this method, all the steps occurring almost automatically. However, this method is still fairly new and its validation needed to be assessed to make the work reliable. The RSM uses a process that will be well explained in the next sub-section.

4.1.2 RSM principles

The steps involved in the implementation of the RSM are described as follows (Read and Stacey, 2009):

- ✓ Use of the stability model (Phase 2 or Slide) to determine factors of safety with different combinations of input parameters
- ✓ Use of the regression techniques to determine the response surface
- ✓ Use of the response surface to generate a distribution of factors of safety, with the aid of Monte Carlo simulation
- ✓ Determination of the probability of failure

Determination of RSM safety factors

The input values needed in the running of the RSM are as follows for each of the variables (Read and Stacey, 2009):

i. Consider all the variables at their mean values; this is the first combination
ii. Repeat the first case but variable varied by minus one standard deviation of the mean value
iii. Repeat the second step, but this time with the mean plus one standard deviation. All other parameters stay at their mean values
iv. Steps two and three are repeated for the rest of the variables, and for each combination a deterministic model is run to provide a corresponding factor of safety

In contrast with the probabilistic NM run (also called as full NM), which requires a number of runs of 2^N using the PEM, the RSM only requires (2N+1) runs to generate the FOS distribution, N being the number of input variables. Afterwards, Monte Carlo simulations will be carried out to provide a distribution of the FOSs and consequently the POF (Myers *et al.*, 2009).

Deterministic numerical models were run and the parameters were manually varied to obtain the required number of FOSs for the regression process. Each combination of inputs corresponded with an FOS result, and the number of combinations is what was earlier referred to as the number of runs.

Use of regression techniques

This step consists in generating a regressive function from the output variables, before simulating a distribution of FOS. However, the response surface assumes independence between variables. This

statement is arguable in geotechnical engineering. Likely, the issue is solved by incorporating the reliability index β, determined with resort to the best estimate of the means as follows: $FOS_{bc}=R(X'_1, X'_2, X'_3, ..., X'_N)$ (Myers *et al.*, 2009).
Where:
FOS_{bc}: Base case of the FOS
$X'i$: Mean value of the input variable Xi.

Considering $FOS_i^{\alpha} = R(x'_{i-k}, x'_{i-k+1}, ..., x_i^{\alpha}, ..., x'_N)$ **(14)**

The sensitivity index of x_i^{α} was calculated as follows**:**

$$\beta(x_i^{\alpha}) = \frac{R(x'_{i-k}, x'_{i-k+1}, ..., x_i^{\alpha}, ..., x'_N)}{FOS_{bc}} \quad \textbf{(15)}$$

Where:
α: (-); (‘) or (+)
x_i^{α} : Variable *i* at the side α
N: Number of variables x_i
R: Response Surface
$\beta(x_i^{\alpha})$ may simply be written as β_i^{α}

The reliability index (β_i^{α}) has a high variability, thus it is to be plotted against a normalized variable z_i and regression techniques will be employed to fit a second order polynomial (this will be adjusted upfront). Any value of the factor of safety can be found randomly by combining the input variables (Myers *et al.*, 2009).
Therefore, considering N variables, defined by $X_1, X_2, X_3..., X_N$ the overall factor of safety for each parameter is expressed by:

$$FOS^{\alpha} = FOS_{bc} . \prod_{i=1}^{N} \beta_i^{\alpha} \quad \textbf{(16)}$$

Where:
$\prod_{i=1}^{N} \beta_i^{\alpha}$: Product of all the sensitivity indices at a specific side of the base case position (left, right or its own position).
β^-_i and β^+_i are the sensitivity indices respectively calculated at the left and the right sides of the mean variable $X'i$. This shows that the sensitivity coefficient was determined twice for each variable at a distance of sigma from the mean variable (Chiwaye and Stacey, 2010). Similarly, the FOS was

determined twice at each side of the base case, depending on the fact that the assessed sensitivity concerned the left or right side values of the base case.

Once these factors of safety were determined, regression techniques were used to best fit a second order polynomial. This is the Response Surface with the effect of sensitivity of the variables. However, it has been noticed that there were particular cases where the polynomial regression curve deviated from the reliability index trend. Sometimes when the best case is closer to the positive (or negative) reliability index, the resulting regression line's curvature has a long radius to adapt to the sudden change. This has been resolved by substituting the polynomial curve with two straight lines joining the best case to β^{-}_{i} and β^{+}_{i} as illustrated in figure 4-1.

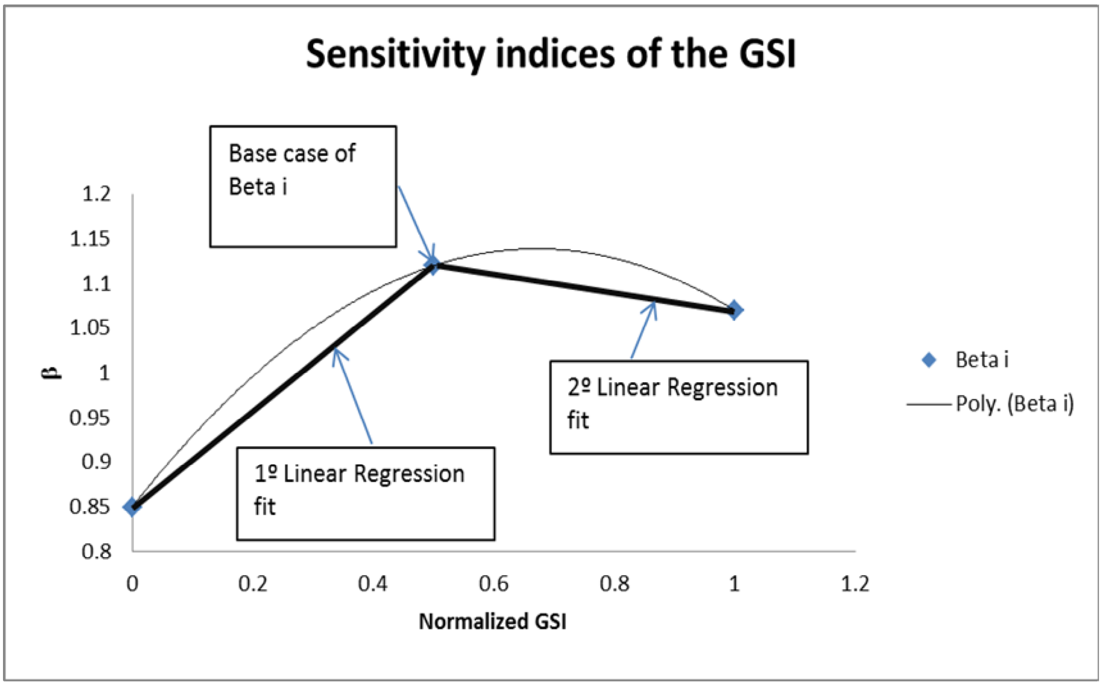


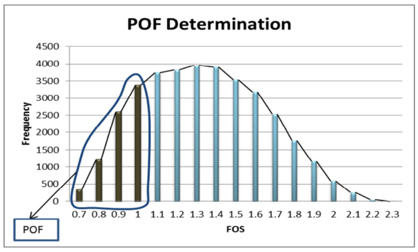
Figure 4-1 Conversion of RSM polynomial curve into two regression lines

Generation of FOS Distribution and Probability of failure Determination

On the basis of the obtained regression curve (or surface in 3D), a method of data generation called *Monte Carlo simulation*, was carried out to generate thousands of FOS values in a spread sheet package (Microsoft Excel) and which should follow a certain distribution. Therefore, the long runtime of the numerical modeling was sensibly reduced to very little time.

From the distribution of the FOS values encountered in the previous step, a statistical analysis was undertaken to provide the probability of failure, which actually is the ratio of the number of trials resulting in the FOS less than one and the total number of trials.

These steps are illustrated in figure 4-2 as follows:



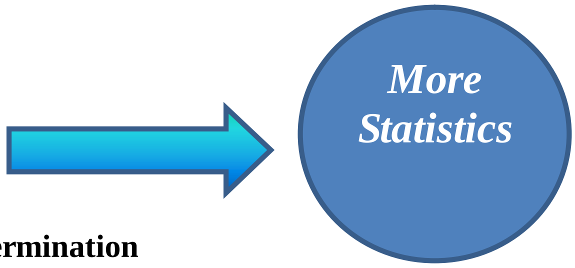


Figure 4-2 Steps of the RSM determination

4.2 RSM Validation Process

Systematically well-chosen models were run to validate the use of the RSM. As noted earlier in the current chapter, the well-known “Crystal Ball” simulation software (Oracle product) was the simulation tool employed, supplying results of 500 000 simulations for each run. But prior to its use, a critical process of validation was initiated which consisted of comparing POF results from the RSM with the results from a full NM run, before it could be used as a substitute of the full probabilistic NM and be compared with the full probabilistic LE analysis. Three different slope angles were chosen (25, 45 and 60 degrees). The adopted failure criterion was the Generalised Hoek-Brown and both normal and lognormal distributions were assessed, whose data were presented in section 3.4.4. In order to build the RSM, combinations of the main rock mass parameters were formed prior to the run of the different scenarios, considering both distributions as presented in the following tables 4-1 and 4-2.

Normal distribution

Table 4-1 RSM combinations for normal distribution

Runs	UCSi (KPa)	GSI	mi
Ugm	80000	50	10
$U^{-}gm$	53340	50	10
$U^{+}gm$	106660	50	10
$Ug^{-}m$	80000	33.4	10
$Ug^{+}m$	80000	66.6	10
Ugm^{-}	80000	50	6.7
Ugm^{+}	80000	50	13.3

Lognormal Distribution

Table 4-2 RSM combinations for lognormal distribution

Runs	UCSi (KPa)	GSI	mi
Ugm	80000	50	10
$U^{-}gm$	64000	50	10
$U^{+}gm$	96000	50	10
$Ug^{-}m$	80000	48	10
$Ug^{+}m$	80000	52	10
Ugm^{-}	80000	50	7
Ugm^{+}	80000	50	13

These simulations however had to be run only with the validated settings of the models in terms of mesh elements type and number, slices number, and with the chosen failure criterion. This study will be presented in section 5.1.

Once this study is done, the criterion of validation will depend on the similarity of the POF's determined using the RSM results and using the full probabilistic NM results. Afterwards, the degree of agreement between volume of failure results from the LE and NM should be assessed before assessing the risk. In cases of a lack of good agreement, the reasons behind the discrepancies in the results should be analysed in depth.

Chapter 5 RESULTS AND DISCUSSION

The results associated with the adopted methodology are presented in the current chapter. Some rock mass parameters and properties, as well as their variations, were studied to seek the reasons behind the high value of the sliding volume from NM in comparison to LE. But prior to the use of the analytical methods, models needed to be assessed and validated.

5.1 Models Validation

Particular attention was given to some parameters encountered in the LE and NM methods such as the influence of the number of slices, number of mesh elements, type of mesh, type of failure criterion, etc.

5.1.1 Slide model

The results obtained from the Slide model regarding the sensitivity of the FOS to variation in the slope angles are displayed in the appendices Table A- 1 and Table A- 2 respectively for generalized Hoek-Brown and Mohr Coulomb criteria; likewise the POF sensitivity results are displayed in

Table A- 3 and Table A- 4 from the Appendices. At the same time, different numbers of slices were run to determine the optimal number of slices to apply. Figure 5-1 gives the trends of the FOS's with the increase in the number of slices.

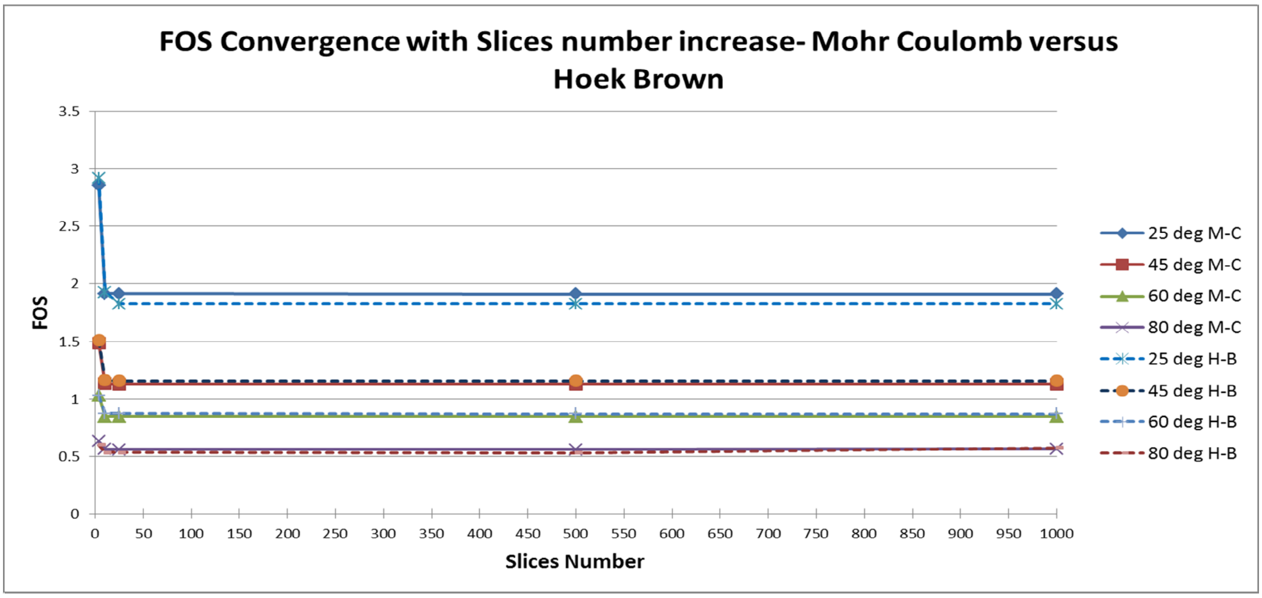


Figure 5-1 FOS Comparisons for increases in slope angle and number of slices - Mohr Coulomb versus Hoek Brown

The graphs show a decrease of the deterministic FOS with increase in the number of slices. However, from values close to 25 slices, the deterministic FOS's converge to constant values in respect of each slope angle. Moreover, a real increase in running time was observed for greater numbers of slices, such as up to 1000 slices, whereas the FOS results are almost the same as those obtained with only 25 slices. This is a strong finding that may lead to adopting slide models with only 25 slices as representative of all the models with regard to the FOS study.

When the Generalised Hoek-Brown criterion is applied, FOS's issued from the rock mass variables, which follow either the normal or the lognormal distributions, are almost equal. Therefore, the type of distribution does not have an impact on the slope stability prediction. As for the Hoek-Brown criterion, FOS results for the Mohr-Coulomb criterion give exactly the same values for both normal and lognormal distributions. However, FOS results from Mohr Coulomb and Hoek and Brown agree better for steeper slopes than is observed for the flatter ones. For flatter slopes, Hoek-Brown gives lower FOS values, hence indicating lower conditions of stability. Therefore, the suggestion may be made to use the Hoek and Brown criterion in evaluating flatter slopes, since it is the most realistic criterion. In addition, since it gives lower FOS values, its use will be conservative, which will contribute to ensuring greater stability of the slope.

Regarding the sensitivity of the FOS to slope angle variation, as expected, the FOS decreases as the slope angle increases; which is perfectly logical as the increase in slope angle generates instability of the slope. The same behaviour was observed regarding the sensitivity of POF to variation of slope angle. This time the condition of non-equilibrium creates an increase in the POF as the slope angle increases.

Considering the rock mass parameters normally or log-normally distributed, results from Mohr Coulomb and Hoek-Brown criteria can be compared by the means of the following Figure 5-2 and Figure 5-3, respectively for Hoek-Brown and Mohr-Coulomb criteria:

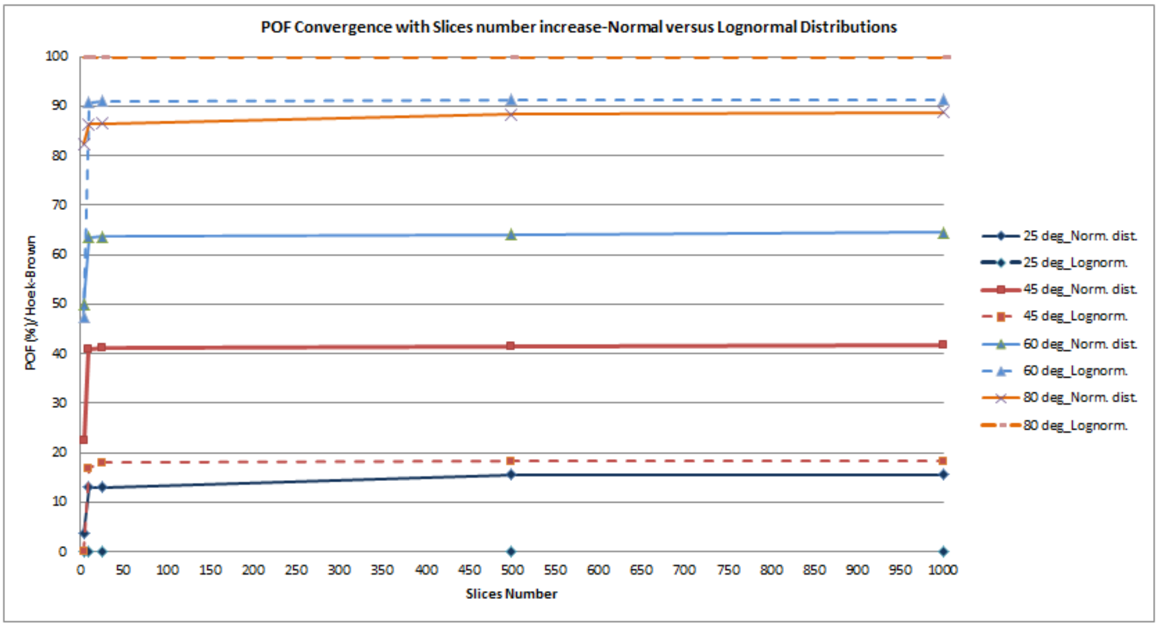


Figure 5-2 POF Comparison for increases in number of slices/Normal versus Lognormal distributions (Hoek-Brown)

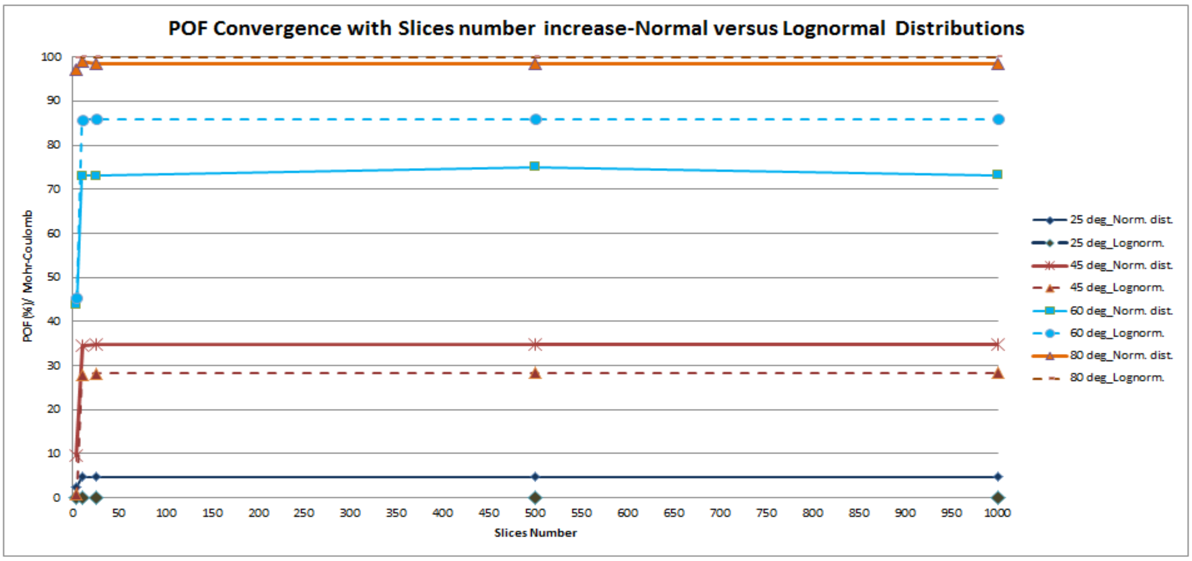


Figure 5-3 POF Comparison for increases in number of slices /Normal versus Lognormal distributions (Mohr Coulomb)

From the last two figures, it is generally observed that for numbers of slices less than 25 slices, the POF increases proportionally to the increase in the number of slices. However, from 25 slices, the POF remains almost constant, though slight variations were observed for slope angles of 60 degrees when the Mohr-Coulomb criterion was applied, and 25 and 80 degrees when the Hoek-Brown criterion was chosen. Nevertheless, Slide models with 25 slices can be suggested as representative when analysing the POF behaviour in slope stability. The two failure criteria show that, for lower slope angles, POF's from normal distributions are greater than from lognormal distributions. The POF outcomes from the two criteria do not permit conclusions to be drawn with regard to which of them could be underestimating or overestimating the realistic results, since they fluctuate (see Figure 5-4 and Figure 5-5). Nevertheless, this shows the importance of clearly defining the type of distributions of the rock mass parameters, prior to any probabilistic analysis.

Unlike the FOS analysis, POF results are not the same for normal and lognormal distributions. Because of this, it is recommended to analyse their trends, before suggesting any cautiousness in dealing with this case study. Figure 5-4 and Figure 5-5 display the comparisons of POF for increases in number of slices and slope angles, respectively, for normal and lognormal distributions.

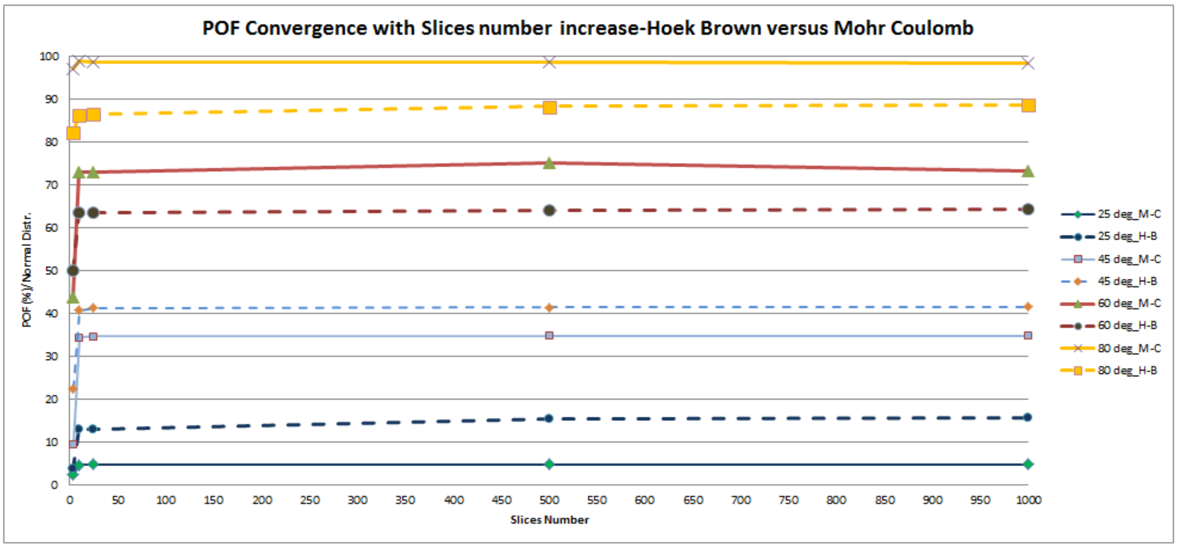


Figure 5-4 POF Comparison for increases in number of slices – Hoek-Brown versus Mohr-Coulomb criteria (Normal Distributions)

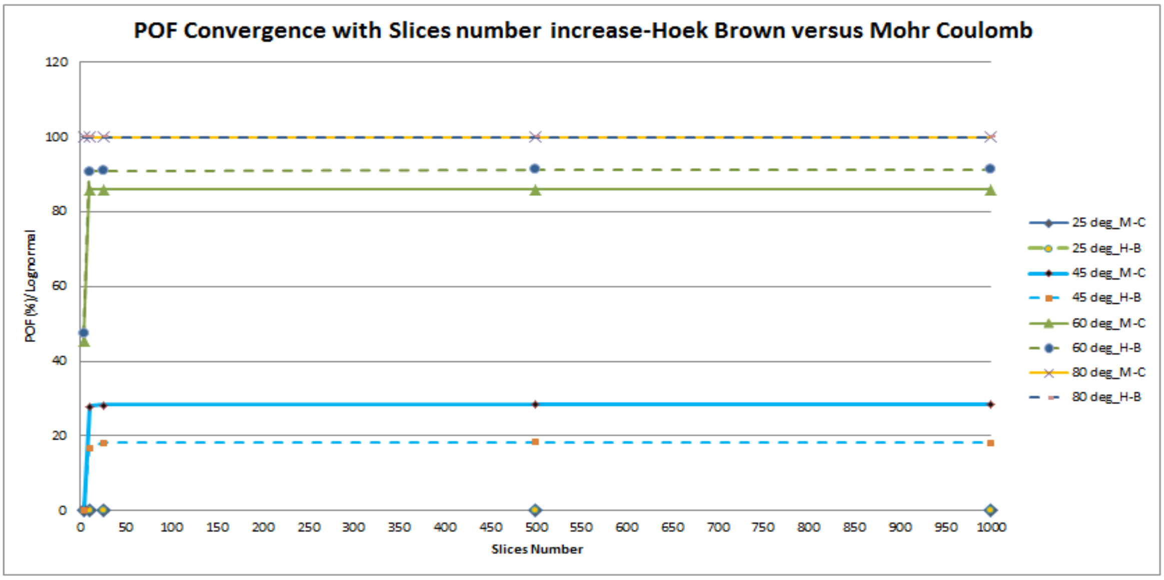


Figure 5-5 POF Comparison for increases in number of slices – Hoek-Brown versus Mohr-Coulomb criteria (Lognormal distributions)

In conclusion for this section, as the numbers of slices increase, the FOS decreases at fewer numbers of slices before remaining almost constant. However, for greater numbers of slices, say 500 slices, the running time increases significantly compared to 25 slices, for only a very slight change in the FOS. Thus, 25 slices are recommended as a reliable number of slices for determining the FOS using Slide. The Hoek-Brown criterion is also recommended in this case, independently of the type of distribution of parameters.

Similarly to FOS analyses, POF analyses show that 25 slices is a reliable figure for building a Slide model. However this time, a well-defined distribution of the rock mass parameters is imperative, likewise for the failure criterion. It was also verified that the FOS decreases as the slope angle increases.

In contrast with lognormal distributions, with rock mass parameters following normal distributions the Mohr-Coulomb criterion underestimates the POF for flatter slope angles, and overestimates it for steeper slope angles. Hoek-Brown can also be recommended in this case, to avoid the imprecision of the results when calculating the POF.

Graphs illustrating the predicted volumes of failure for different numbers of slices and failure criteria are presented in Figure 5.6. The graphs only deal with two different numbers of slices (25 and 1000), but detailed results are given in Table 5-1. Only one table is shown, due to the equality of failure volume values whether normal distribution or lognormal distribution is used. However, for the sake of verification, both tables are displayed in the appendices (Table B- 1 and Table B-2).

Table 5-1 Sensitivity of Slide failure volumes to number of slices and variation in slope angle

Slope angle (degrees)	Comparison of Volumes (m³)-Bishop Simplified									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B
25	25925	24860	218793	218284	218793	418706	218793	419424	218793	419424
45	26723	27186	122800	122860	122800	122800	122800	122800	122800	122800
60	29495	29495	67247	67247	67247	67247	67247	67247	67247	67247
80	38668	23753	53816	32672	62528	32672	62528	32672	70586	70586

An adequate number of slices were chosen following the observation made in the figures below:

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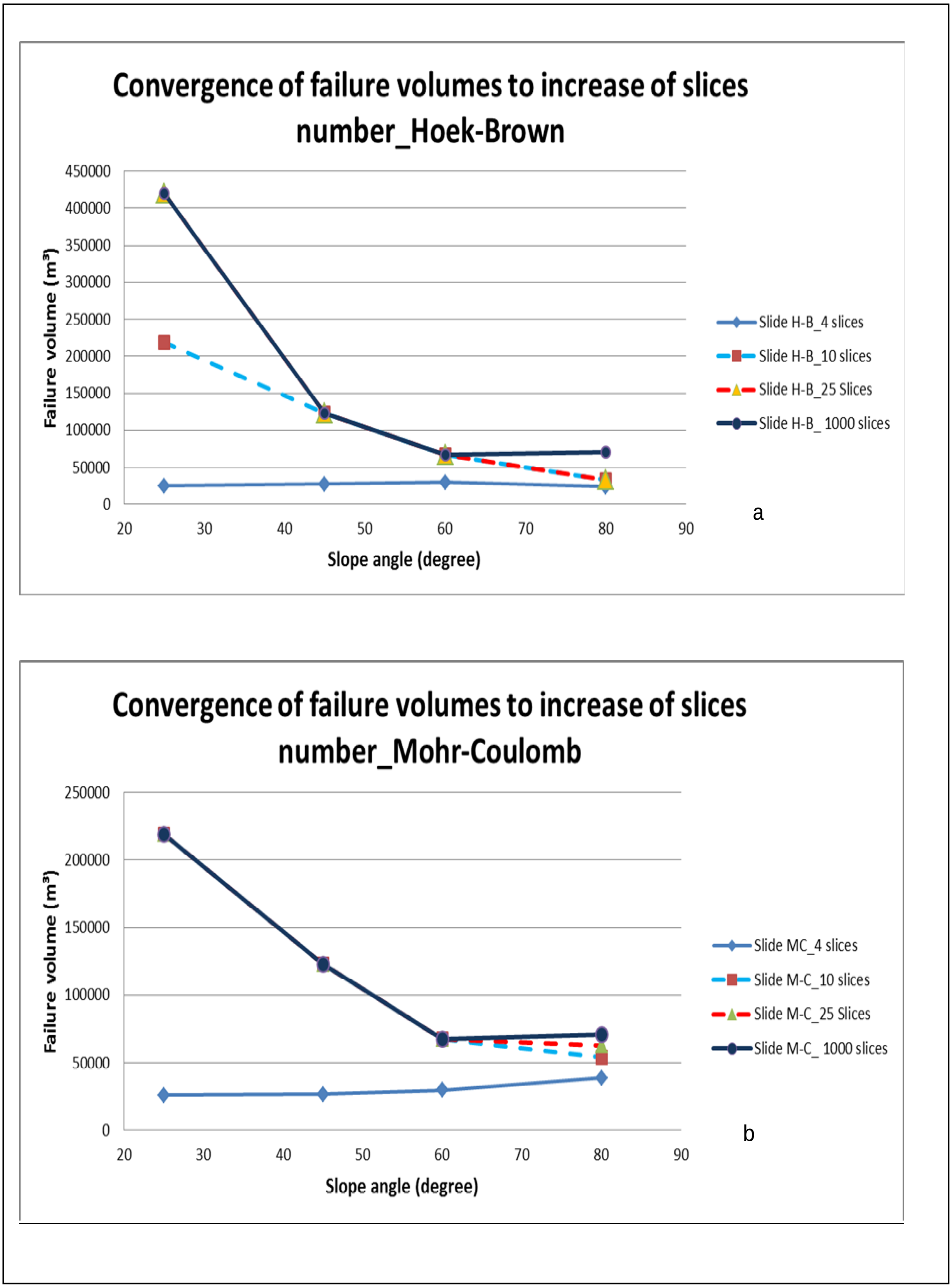


Figure 5-6 Comparisons of failure volumes with increase in number of slices

For both Mohr-Coulomb and Hoek-Brown criteria, the volumes converge for numbers of slices from 25 upwards. However, for the number of slices equal to 1000, which is the maximum possible number of slices, a sudden increase of the failure volume was observed. This could, at first sight, be assimilated to a program error for such a large number of slices. However, when looking at Mohr-Coulomb results and comparing them with NM results, these values approximate the failure volumes results obtained when using Hoek-Brown criterion with 1000 slices. And knowing that NM failure volumes are more reliable than volumes from LE, since they incorporate progressive deformations as well as the redistribution of stresses, realistic cases might well result from 1000 slices. It was therefore recommended to consider the number of 1000 slices where the assessment of volume is concerned.

Now that 25 slices or 1000 slices are recommended in the assessement of failure volume, with a preference towards 1000 slices, analyses were also undertaken to identify which failure criterion fits better in determining the sliding volume. Figure 5-7 illustrates the results obtained in this study.

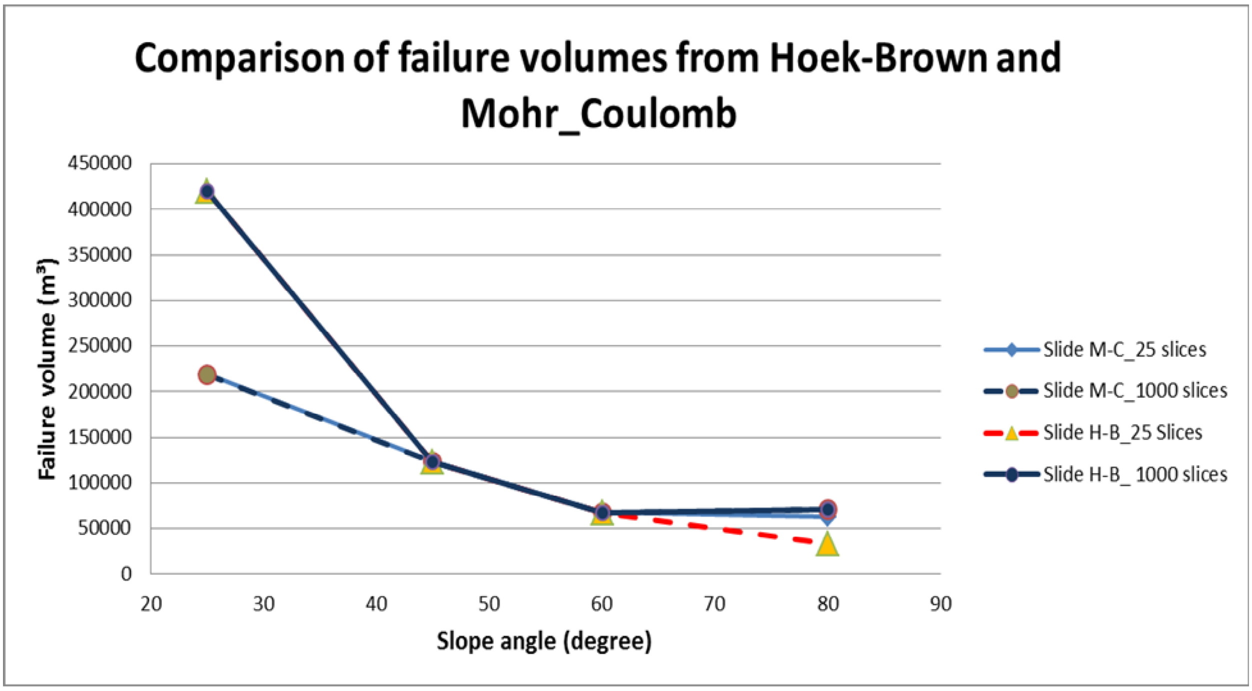


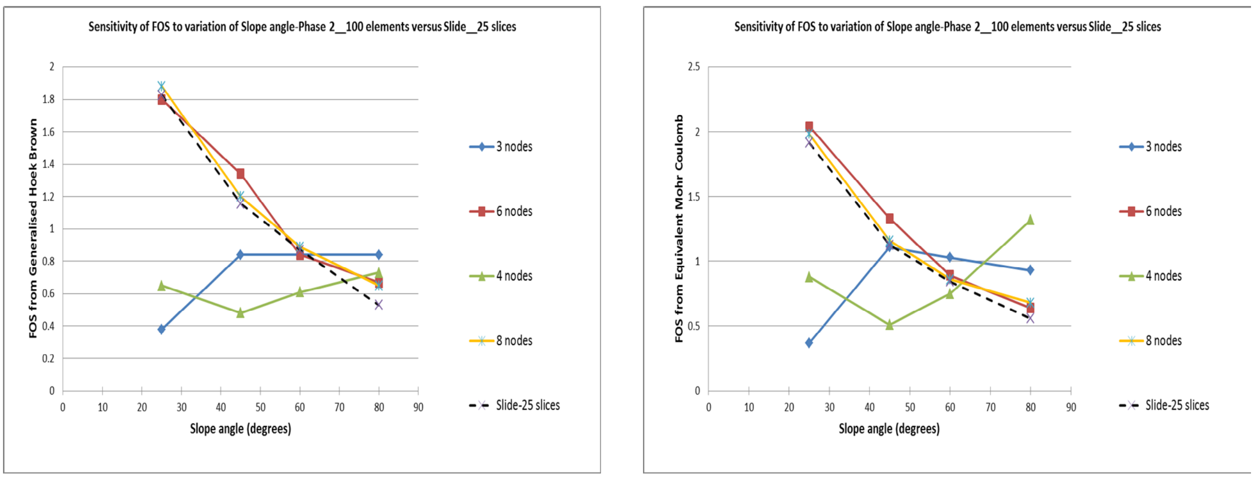
Figure 5-7 Results of sliding volumes _ Hoek-Brown versus Mohr-Coulomb criteria

As observed from Table 5-1, for numbers of slices equal to or greater than 25, Mohr-Coulomb and Hoek-Brown criteria result in the same failure volumes. However, the latter criterion shows greater volumes for slope angles less than 45 degrees or greater than 60 degrees. In this last case, the statement is valid only for 1000 slices. With this finding, it is seen that, using the Mohr-Coulomb criterion for the evaluation of failure volumes will lead to lower sliding volumes than when Hoek-Brown is employed. This may be a reason behind the finding that Slide underestimates the failure volume (Chiwaye and Stacey, 2010). The same is the case with the use of the Mohr-Coulomb criterion. Therefore Hoek-Brown can be recommended for the evaluation of failure volumes, particularly when the slope angle is flatter than 45 degrees or steeper than 60 degrees. However, in this latter interval, it would be advisable to consider a total of 1000 slices when running the Slide model. Thus, this study should apply the generalised Hoek-Brown to assess the volumes differences between Slide and Phase2 programs. This could be applied only after performing Phase2 analyses.

5.1.2 Phase2 model

Results of the validation of the Phase2 model to assess FOSs, POFs and failure volumes are presented in detail in Appendices C, D and F. Their tendencies are assessed assuming the rock mass parameters following both normal and lognormal distributions, respectively.

The following figures validate the use of elements with mid-point nodes and, at the same time, show the effect on FOS of an increase in the number of elements. Note that figures “a” refer to the generalised Hoek-Brown criterion while all the figures “b” refer to the equivalent Mohr-Coulomb criterion.



a b

Figure 5-8 FOS Sensitivity of element type to slope angle increase_ case of 100 mesh elements/ lognormal distribution

Before carrying on, it is obvious 3 and 4 nodded elements give unrealistic results for 100 mesh elements and should be discarded. And the increase of number of mesh elements results in the following graphs:

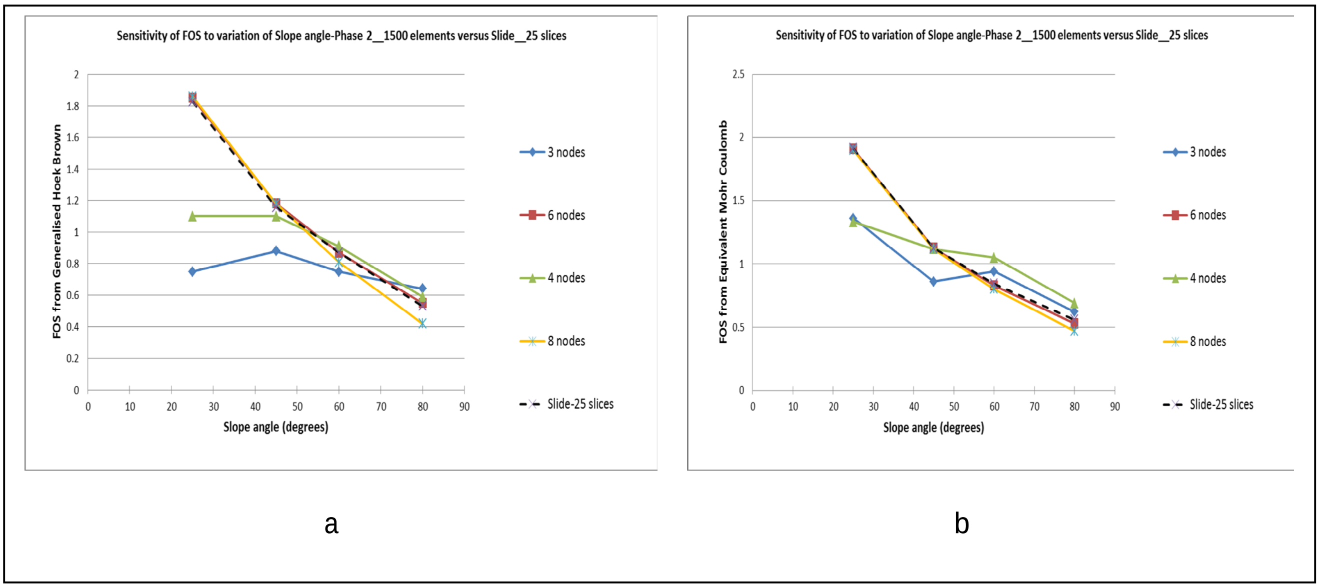


Figure 5-9 FOS Sensitivity of element type to slope angle increase_ case of 1500 mesh elements/ lognormal distribution

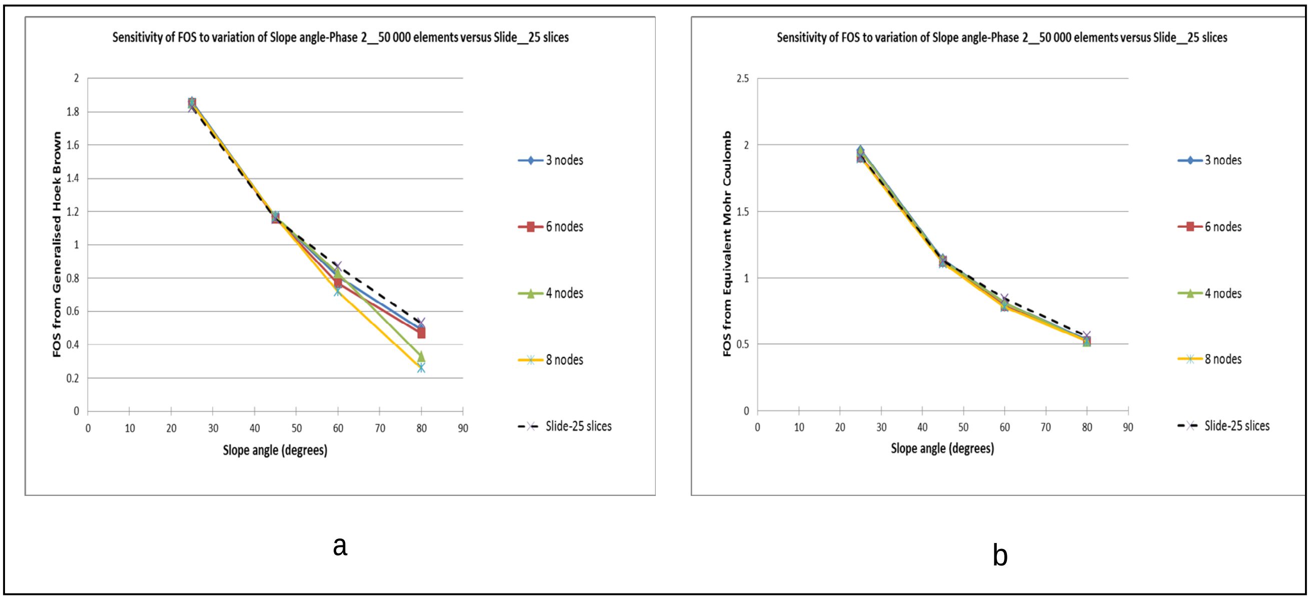


Figure 5-10 FOS Sensitivity of element type to slope angle increase_ case of 50 000 mesh elements/ lognormal distribution

Independently of the type of failure criterion, the results confirm that there is a convergence of FOS values as the numbers of elements increase, but at the expense of running time. The FOS results for 100 elements show “senseless” trends and are not reliable for any further interpretation. FOS trends for 1500 elements are just slightly different to those of 50 000 mesh elements. These results converge even more for mid-points mesh elements. Additionally, Slide FOS results with only 25 slices give almost the same values as 50 000 mesh elements. It is suggested that one can use 1500 six noded mesh elements in Phase2 models to assess the FOS values or just a Slide model with 25 slices.

Normal distributions have also been investigated, and based on previous results; only 6 nodded elements were used. Results are shown in Figure 5.11.

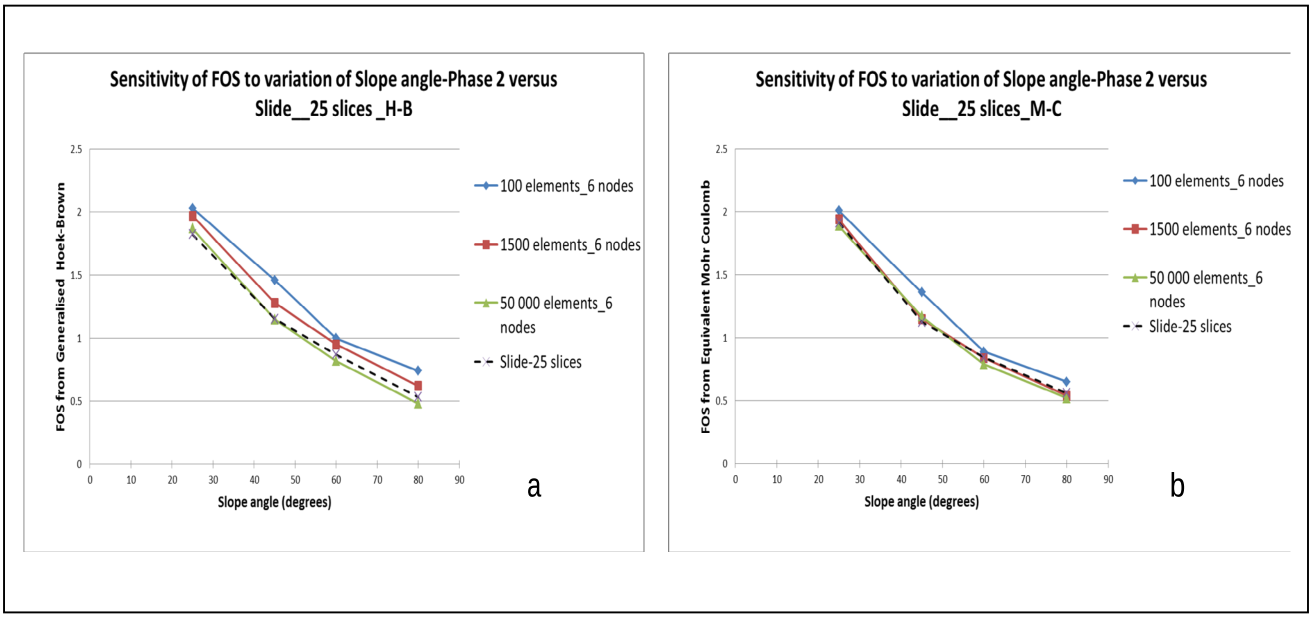


Figure 5-11 FOS Sensitivity of mesh element number to slope angle increase_ normal distribution

The same process was undertaken regarding the effect of the type of mesh element in assessing the POF, as well as for choosing an efficient number of mesh elements for probability studies. The proposed data of lognormal distribution were used here as well. The findings are illustrated by the figures 5-12, 5-13 and 5-14:

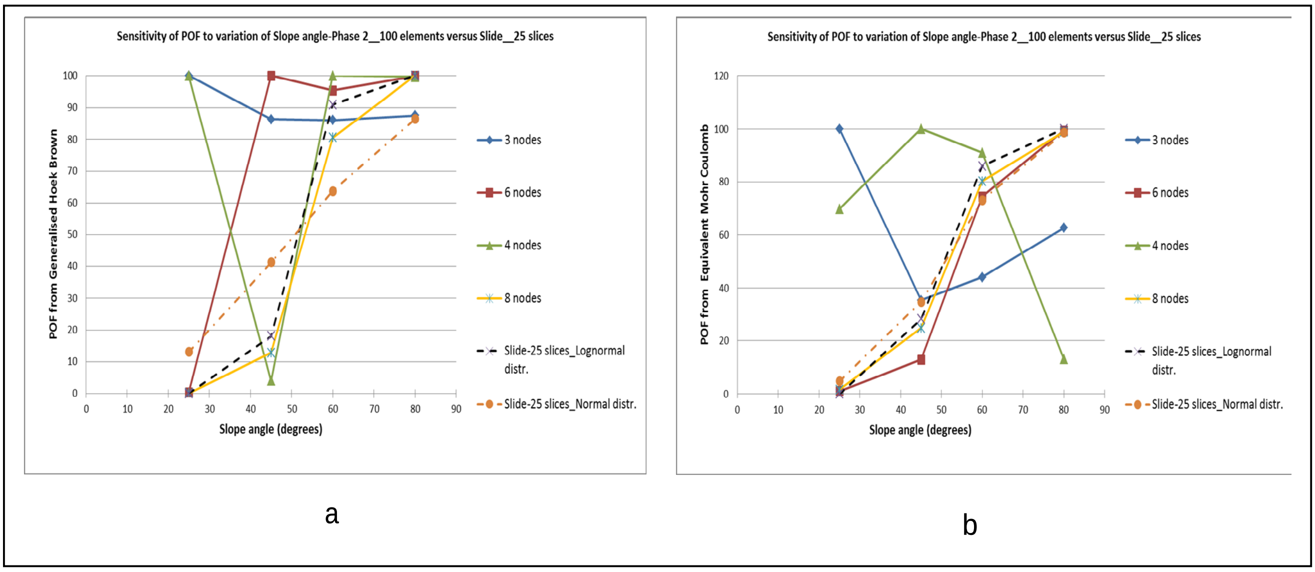


Figure 5-12 POF sensitivity of element type to slope angle increase_ case of 100 mesh elements

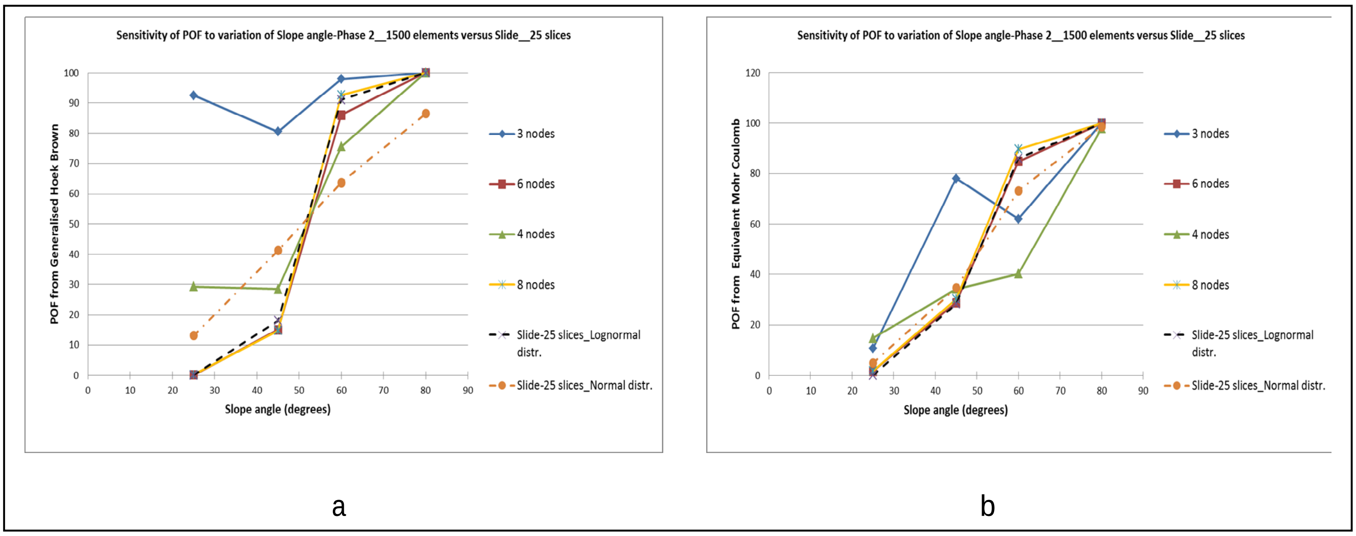


Figure 5-13 POF Sensitivity of element type to slope angle increase_ case of 1500 mesh elements

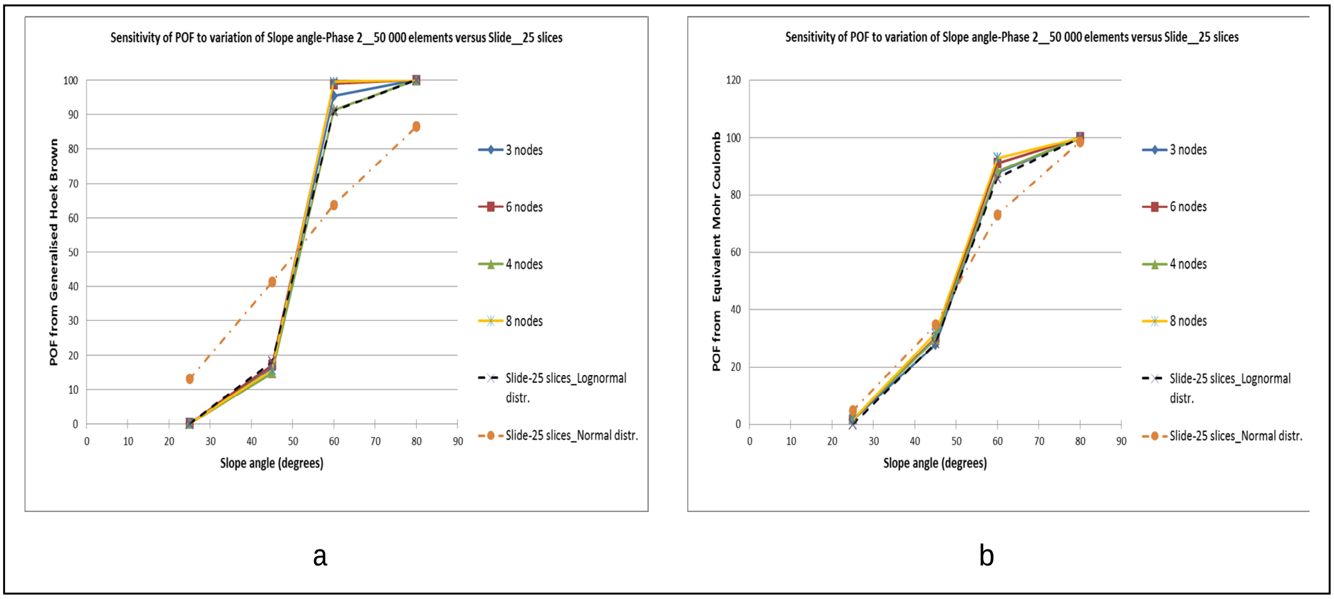


Figure 5-14 POF Sensitivity of element type to slope angle increase_ case of 50 000 mesh elements

The POF outcomes corroborate the findings from the FOS analysis. Low numbers of mesh elements should be discarded as illustrated in Figure 5.12. But increasing the number of mesh elements approximates the POF, for either Hoek-Brown or Mohr Coulomb criterion. POF results for six nodded elements are very close to the results for eight noded elements and also to the Slide model whose parameters follow a lognormal distribution and are composed of only 25 slices.

Similarly, normal distributions were also studied and similar results were found as follows in figure 5-15:

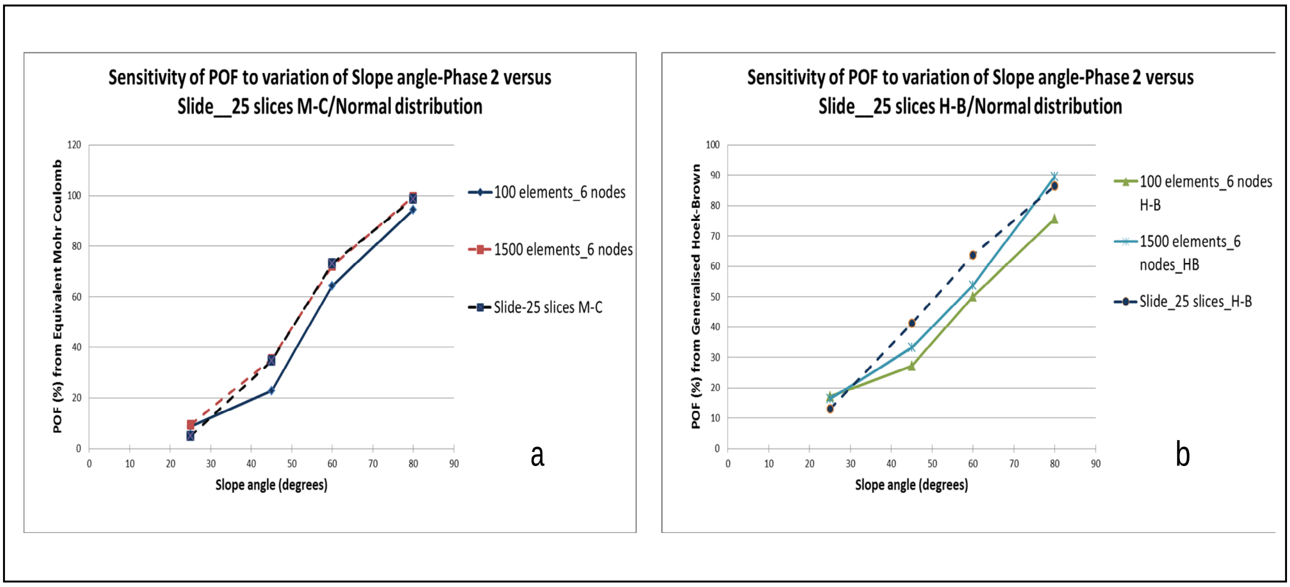


Figure 5-15 POF sensitivity of mesh element number to slope angle increase_ normal distribution

The figure 5-15 shows that, when a normal distribution is used, Mohr-Coulomb results converge rapidly and 1500 mesh elements are likely to give almost the same results of POF with Slide, unlike for Hoek-Brown criterion.

The last validation in Phase2, prior to recommending any prototype of a probabilistic model, is the sensitivity of the failure volume to changes in numbers of elements and slope angles. But this time, as Hoek-Brown Slide models with 25 slices have already been validated in the determination of the failure volume, this will be incorporated into the graphs to both validate the Phase2 model and then compare it with the Slide model. And due to the awareness of considering a slide model of 1000 slices for slope angles greater than 60 degrees, a 1000 slices Slide model will also be incorporated in the comparison process.

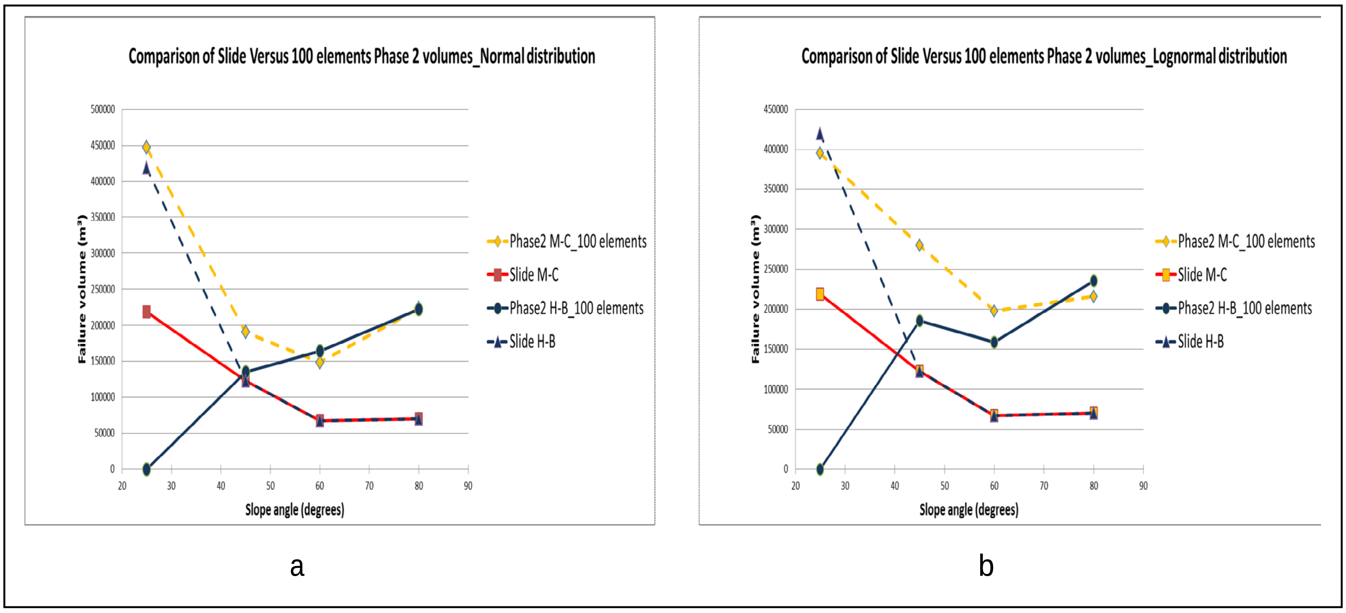


Figure 5-16 Comparison of Slide and Phase2 failure volumes with increase of slope angle_ case of 100 elements

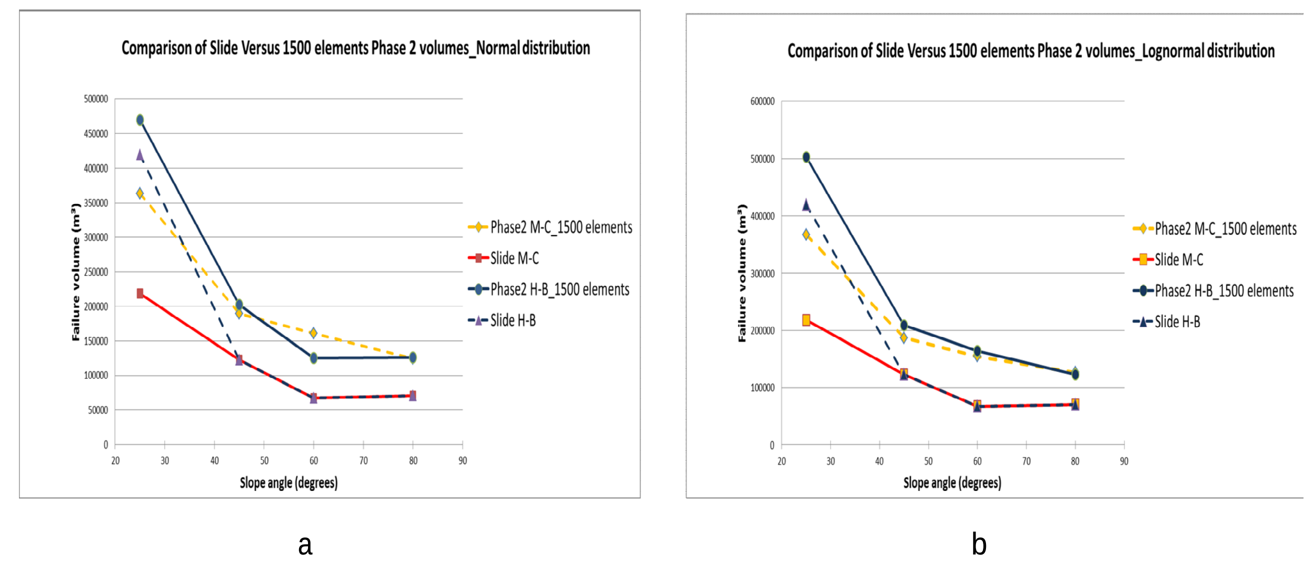


Figure 5-17 Comparison of Slide and Phase2 failure volumes with increase of slope angle_ case of 1500 elements

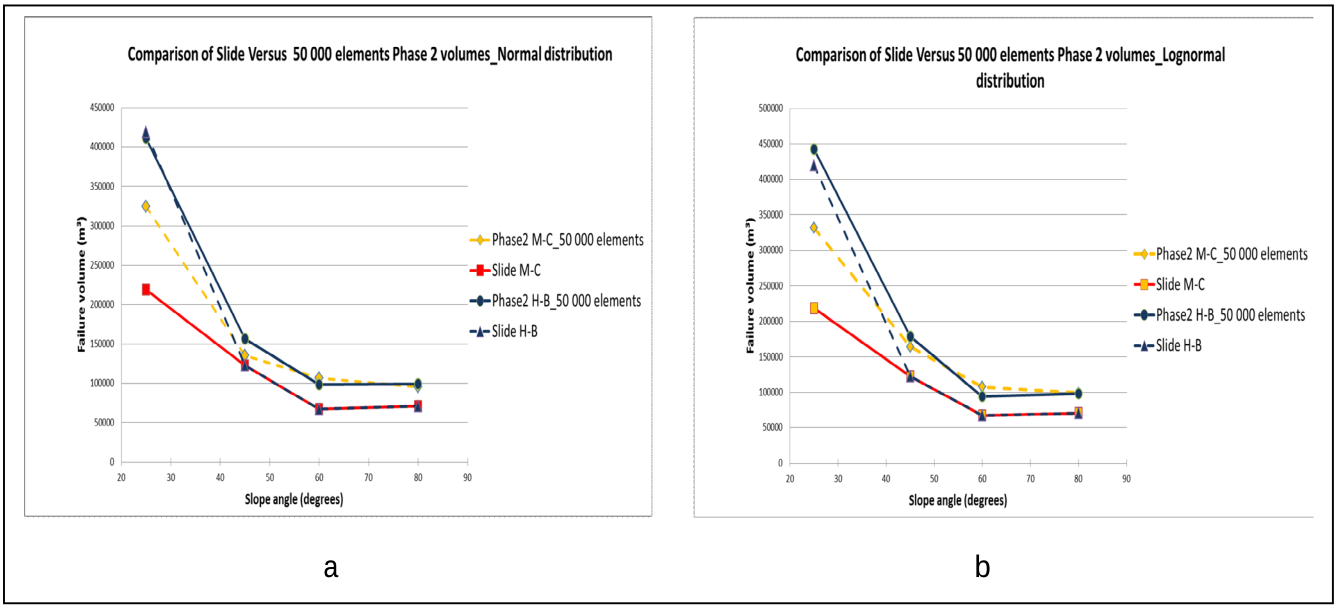


Figure 5-18 Comparison of Slide and Phase2 failure volumes with increase of slope angle_ case of 50 000 elements

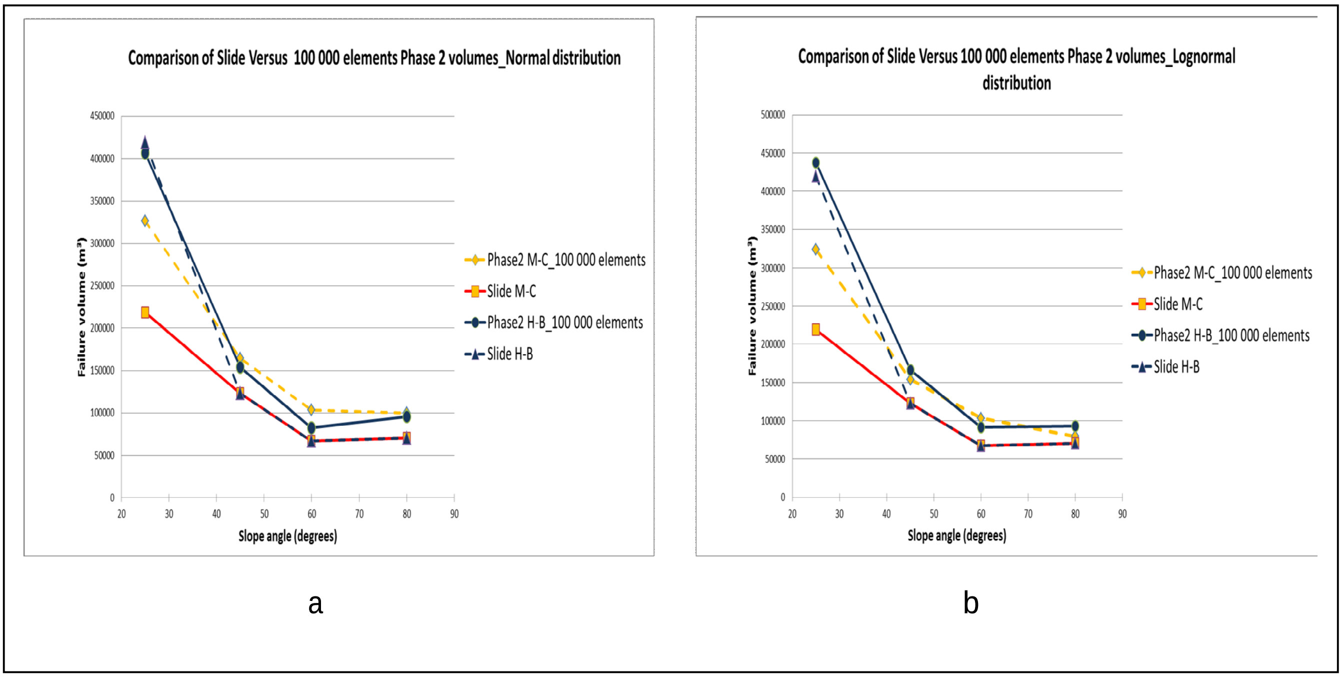


Figure 5-19 Comparison of Slide and Phase2 failure volumes with increase of slope angle_ case of 100 000 mesh elements

Prior to any analysis related to mesh impact on the sliding volume outcome, it is worth mentioning that when a flatter slope fails, the consequence is more disastrous than that of a steeper slope. Figure 5-16 to Figure 5-19 show that flatter slopes fail with greater volumes than steeper slopes, though the latter have a higher likelihood of failure. And, for better comparison of volumes, the results shown in Figure 5-20 will represent the base case of failure volume comparison as far as this study is concerned.

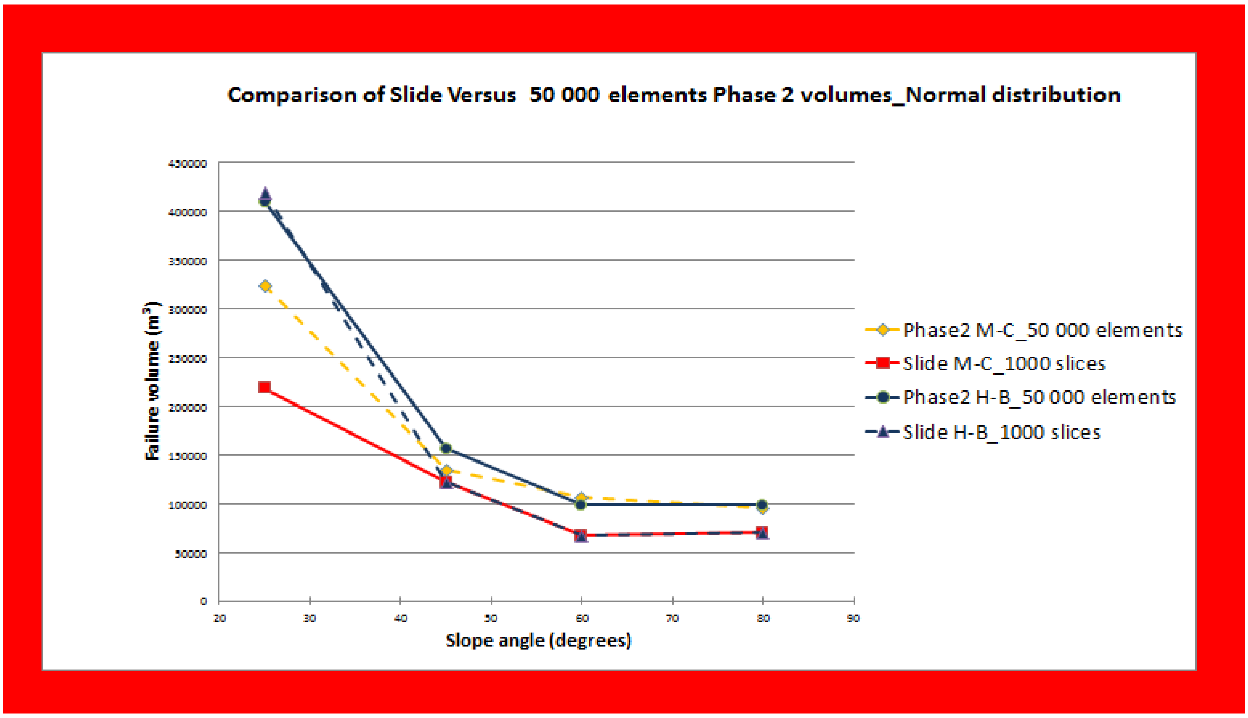


Figure 5-20 Base case model of comparison of LE and NM failure volumes

The number of mesh elements has a significant impact on the predicted failure volume. And because the accuracy of a FEM is proportional to this number of elements, the validation of Phase 2 models will also relate to their comparison. The results from Figures 5.16 to figure 5.19 show that using fewer elements can overestimate the failure volume. Therefore, caution should be used when evaluating the failure volume to ensure that one does not overestimate the failure volume simply by using a small number of mesh elements. The increase in number of elements shows an approximate agreement of failure volumes between Phase2 and Slide. Based on the results, it may be recommended the use of Hoek-Brown criterion, which gives a more accurate approximation than Mohr-Coulomb. And for this, preferably, deterministic Phase2 models with 50 000 mesh elements should be used to only determine the FOS and the failure volume and avoid time consuming constraints of the probabilistic analysis. The POF should be determined with resort to the RSM. Besides, in case of opting for a Slide model, the finding of section 5.1.1 should be applied, pointing out the use of the Hoek-Brown failure criterion with use of 25 slices for slope angles flatter than 60 degrees and 1000 slices when the angle is steeper than 60 degrees.

Apart from these validations, intrinsic to the type of analytical methods, another validation study was undertaken, this time dealing with the RSM.

5.2. Response Surface Validation

Various combinations of rock mass parameters were tested in accordance with the RSM principles and the results were obtained as presented in detail in Appendix C. However, in this section, a summary of the results is presented in Table 5-2. Though the validation should require a comparison of RSM to full NM methods, results of the full LE analysis are also presented in anticipation of the comparison and so that the divergences can be interpreted later.

Table 5-2 Comparative results from RSM and probabilistic stability methods

POF (%)						
Probabilistic Method	25 degrees		45 degrees		60 degrees	
	Normal	Lognorm.	Normal	Lognorm.	Normal	Lognorm.
RSM	14.2	0.1	37.8	16.2	59.9	85.7
Full NM	16.37	0.13	33.14	14.99	53.77	86.02
Full LE	13.04	0.01	41.29	18.04	63.69	91.02

The trends of these results are displayed in Figure 5-21.

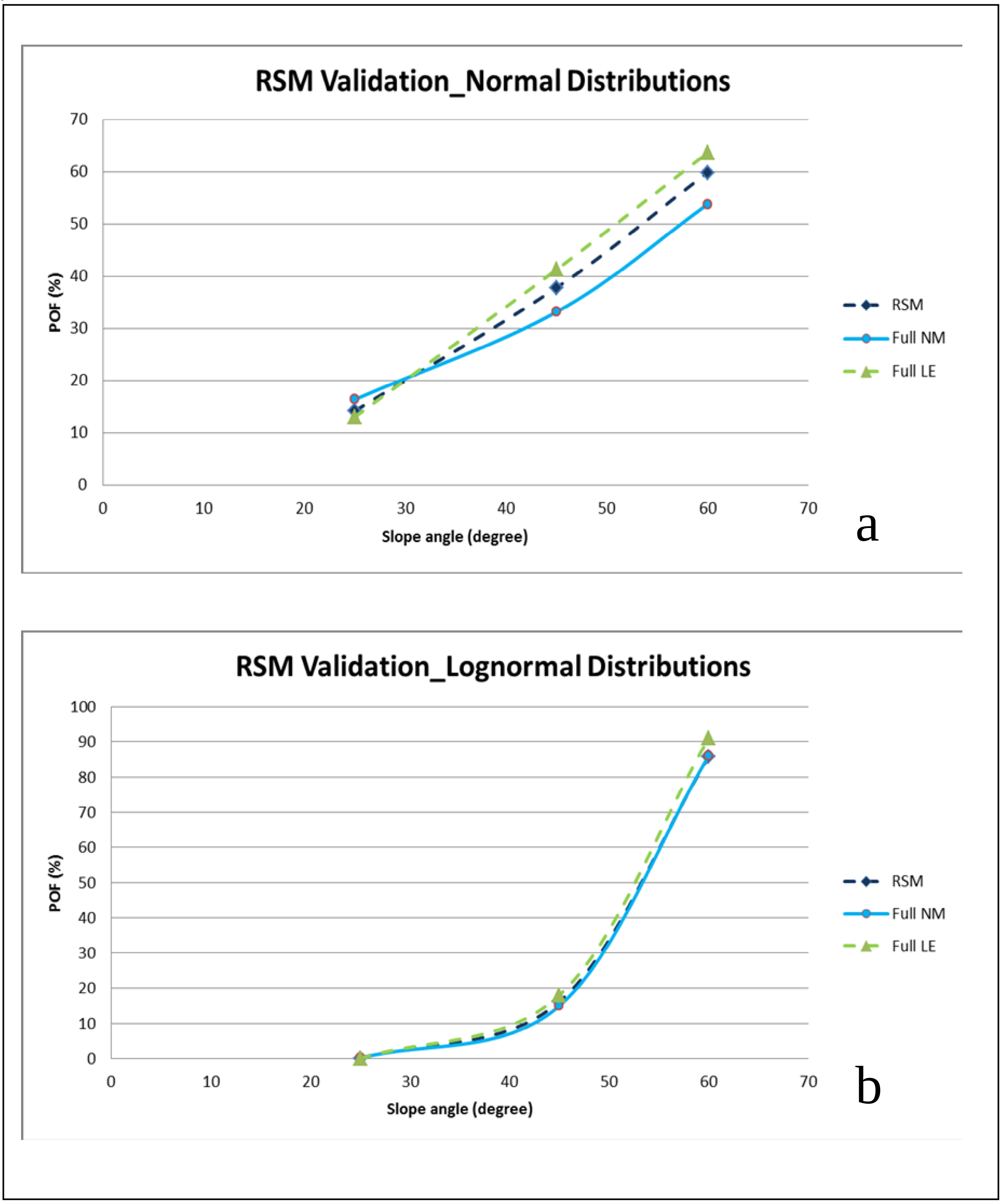


Figure 5-21 Comparison of RSM to NM results

There is good agreement in the results obtained from the RSM and full probabilistic NM method. Figure 5-21b shows almost no discrepancy in the results obtained from both methods, even in good agreement with the LE method for the lognormal distribution. Figure 5-21a however, shows some discrepancies. These, however, are of little concern since they are of the order of 6% out of almost 60% and can be considered as acceptable in highly varied material (standard deviation of the order of 33% to the mean).

5.3 LE versus NM Risk Assessments

Some deterministic parameters were examined as they may possibly influence the results from the analytical stability methods. This study aimed to find out the impacts of pit depth, K ratio, locked-in stresses and dilation angle on the slope risk of failure, the last three not accounted for in the LE analyses.

5.3.1 Effect of the pit depth variation

Since Slide and Phase2 show the same trend of a FOS proportional to the deepening of an open pit mine, with the latter method having slightly better results, and that the difference between the volumes increases proportionally to the pit depth, further analyses proceeded to confirm and probably complete the statement. For this, the effects of pit depth on the FOS, POF and failure volume were re-analysed, but this time with a validated number of mesh elements and after defining the appropriate failure criterion. Only Hoek-Brown was used, since it has proved more reliable for this research model. Table 5-3 displays results of FOS, POF and failure volumes in relation to the pit depth.

Table 5-3 Sensitivity of FOS, POF and Failure Volume to variation of pit depth

Pit Depth	FOS		POF (%)		Failure Volume (m^3)		
	Phase2_ 1500 elements_ 6 nodes	Slide_25 slices	Phase2_ 1500 elements_ 6 nodes	Slide _ 25 slices	Phase2_ 50 000 elements_ 6 nodes	Slide_ 1000 slices	Percentage (%) of volume increase from Slide to Phase2
100	3.17	2.22	9.84	8.94	6321	5915	6.9
300	1.79	1.49	20.44	24.74	51429	37910	35.7
600	1.28	1.16	34.14	41.74	178966	122800	45.7
1000	1.04	0.97	47.16	55.23	462430	393922	17.4

The comparative graphs are displayed in Figures 5.22 to 5.24.

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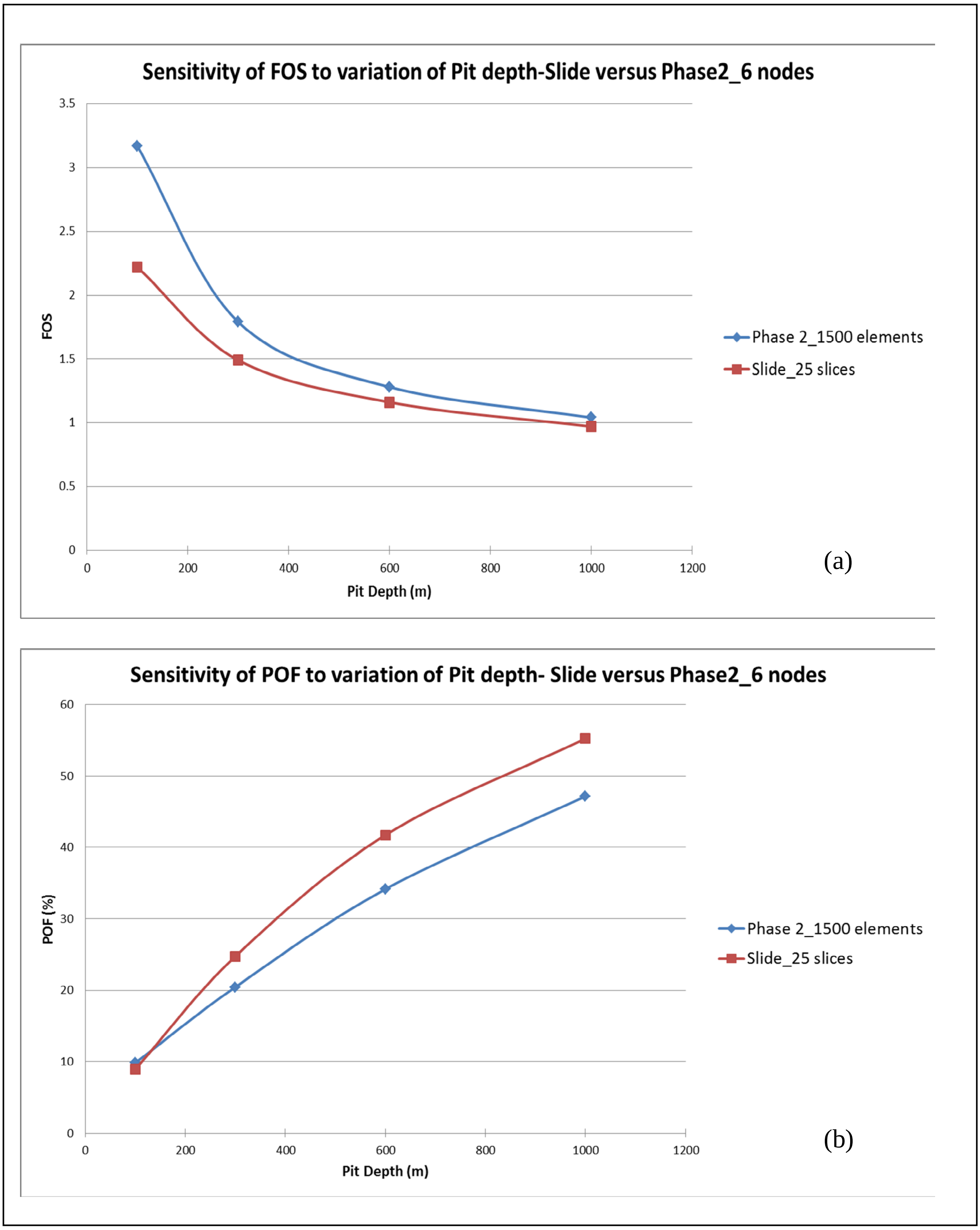

Figure 5-22 LE and NM comparisons of (a) FOS and (b) POF sensitivities to deepening of the pit

Both Slide and Phase2 show the same trends of FOS (see figure 5-22a) and of POF (see figure 5-22b) with depth increase. The differences between the reported FOS values are quite small, with Phase2 results slightly higher; unlike Phase2 results for the POF, which are obviously lower than the corresponding Slide results. However, the tendency of the failure volume results may dictate the potential comparative approach of the two analytical methods. Figure 5-23 reflects the results with reference to the comparison of failure results above.

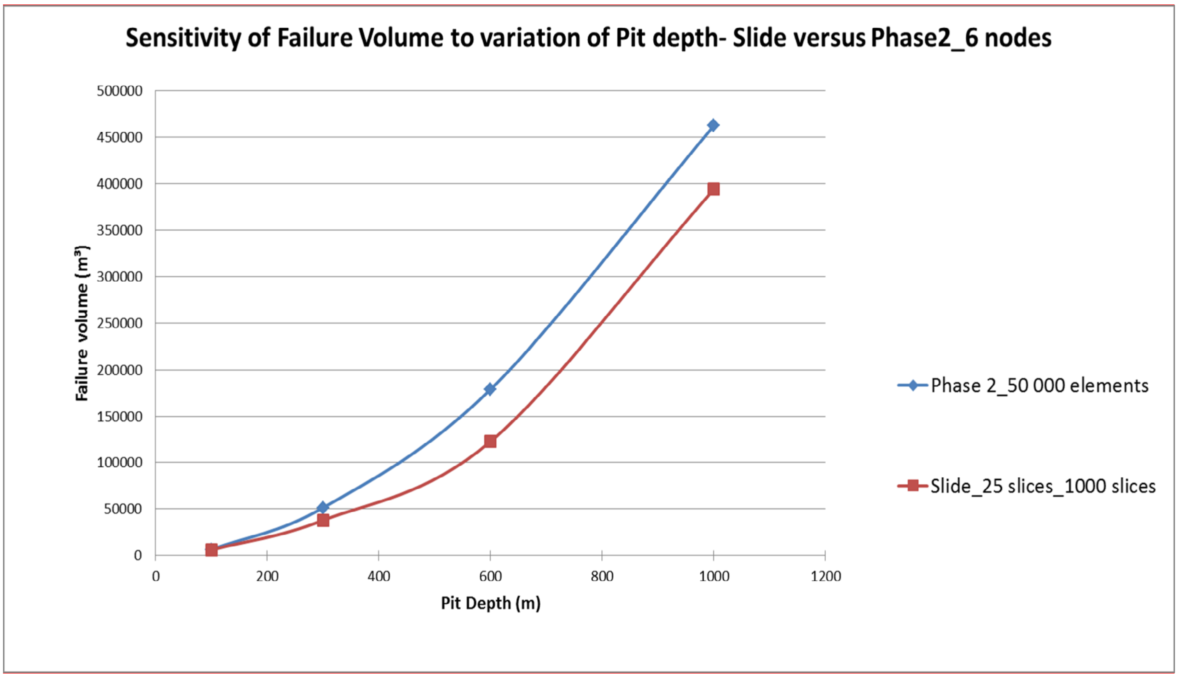

Figure 5-23 Comparison of LE and NM failure volume sensitivities to deepening of the pit

Figure 5-23 shows that SLIDE tends to be under-conservative in predicting lower failure volumes than Phase2. The difference between the volumes increases with increasing pit depth. A better illustration with regard to the percentage of volume increase is displayed in figure 5-24.

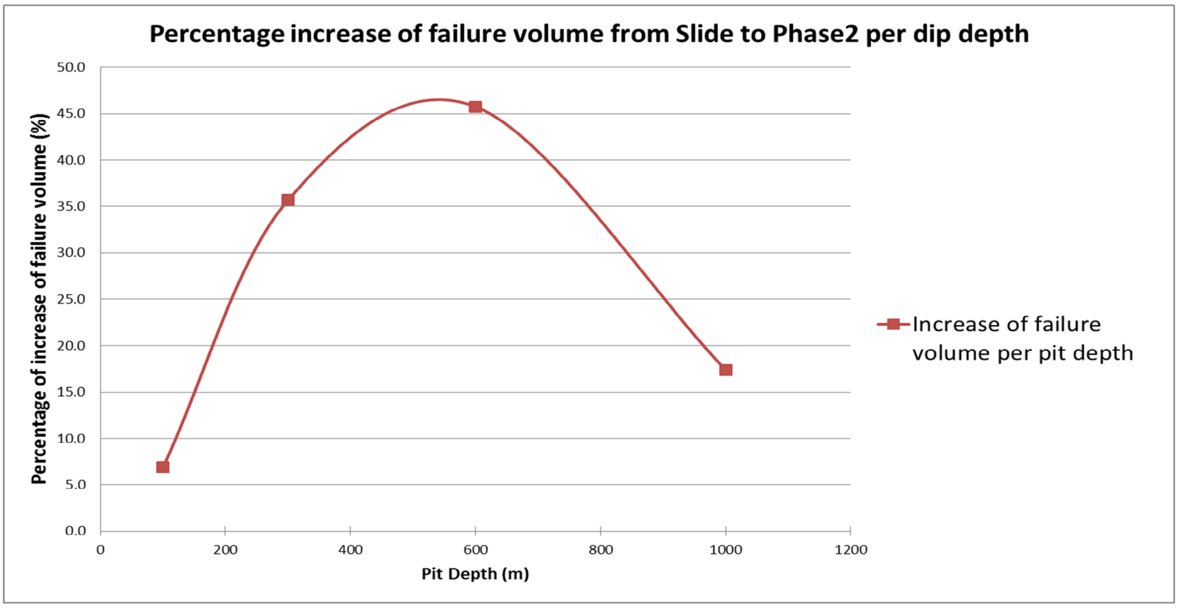

Figure 5-24 Evaluation of failure volume increase per pit depth from Slide to Phase2

From figure 5-24, the predicted failure volumes are different, with a higher NM failure volume increase on Slide volume of approximately 46%. This shows that with a better calibration of the models, though NM predicted failure volumes are greater than LE's, the earlier volumes are no more twice the LE outcomes, as suggested by Chiwaye and Stacey (2010). The comparison should be complete only after analysing the outcome risks from the LE and NM methods with regard to the pit depth, being the POF multiplied by the corresponding failure volume. These risks are determined for a unitary cost of each m^3 that fails. The total risks are then to be multiplied by this cost (t) associated with each m^3 of slid volume of slope. The risk results are displayed in Figure 5.25.

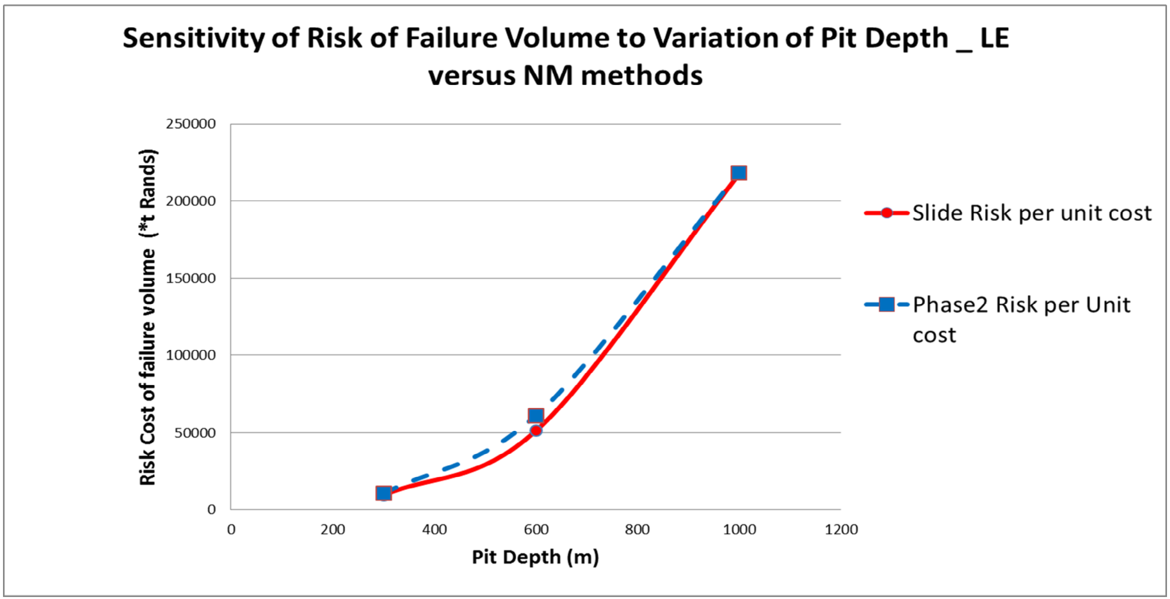

Figure 5-25 Comparison of LE and NM risks of failure volumes to variation of pit depth

As far as the deepening of the pit is concerned, pit risk sensibly increases proportionally to the depth. However the comparative study shows only a slight difference between the risk results from the LE and the NM methods; which is to say that the LE risk outcomes are almost equivalent to those of the NM approach and there is not any significant underestimation in the Slide assessments.

5.3.2 Effect of K ratio on the risk assessment

After a careful validation of the number of mesh elements, and resorting to the more adequate Hoek-Brown strength criterion, sensitivities of FOS, POF and sliding volumes to variations in K-ratio were analysed and the results are displayed in Table 5-4.

Table 5-4 Sensitivity of FOS, POF and Failure Volume to variation of K ratio

K ratio	FOS		POF (%)		Failure Volume (m^3)	
	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 50 000 elements_6 nodes	Slide_1000 slices
0.01	1.35	1.16	30.76	41.74	211695	122800
0.5	1.27	1.16	33.9	41.74	132051	122800
1	1.28	1.16	34.14	41.74	178966	122800
2	1.33	1.16	31.21	41.74	192659	122800
4	1.42	1.16	28.29	41.74	241243	122800

Figure 5-26 and Figure 5-27 show the trends of these results.

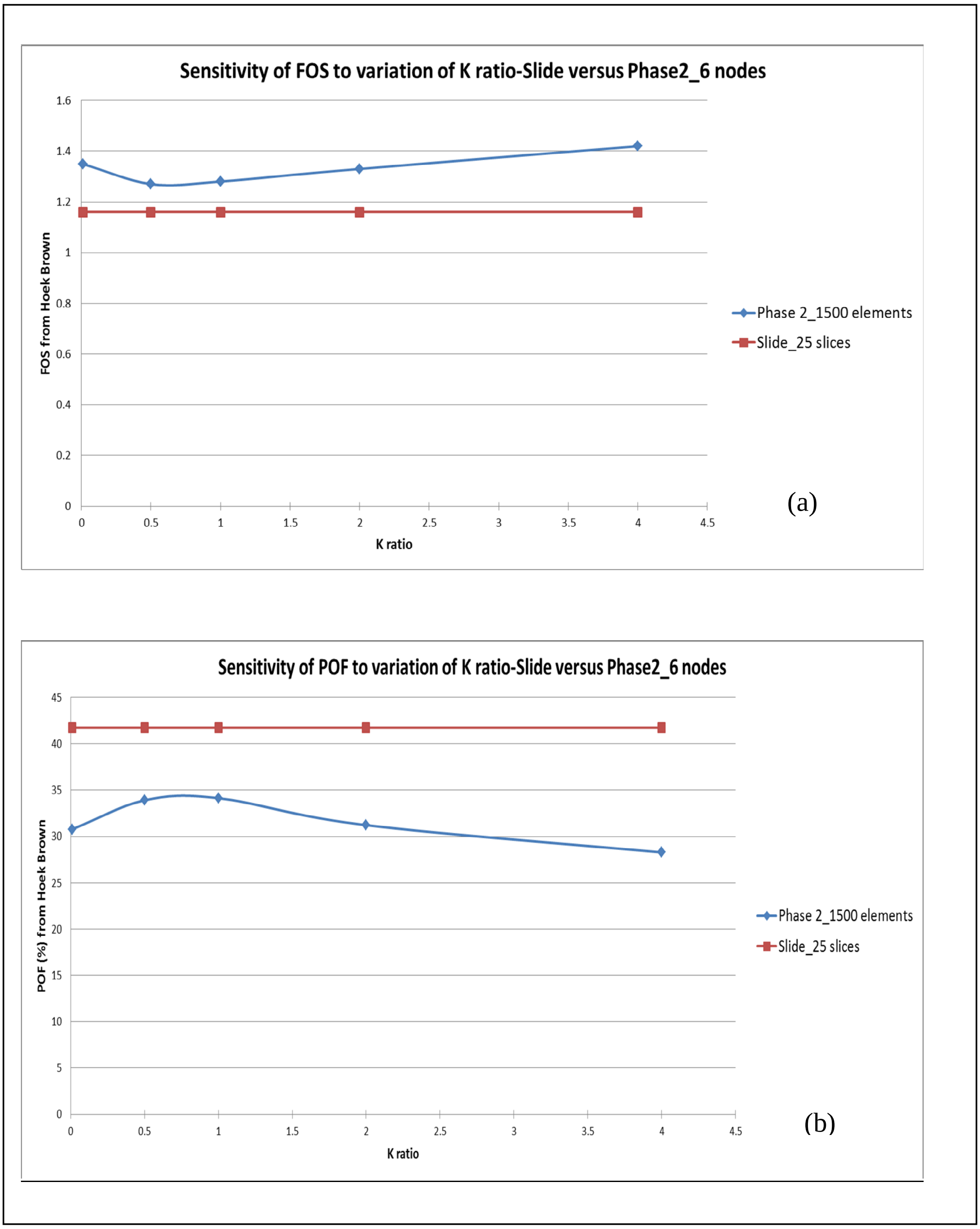

Figure 5-26 LE and NM comparison of (a) FOS and (b) POF sensitivities to variation of K ratio

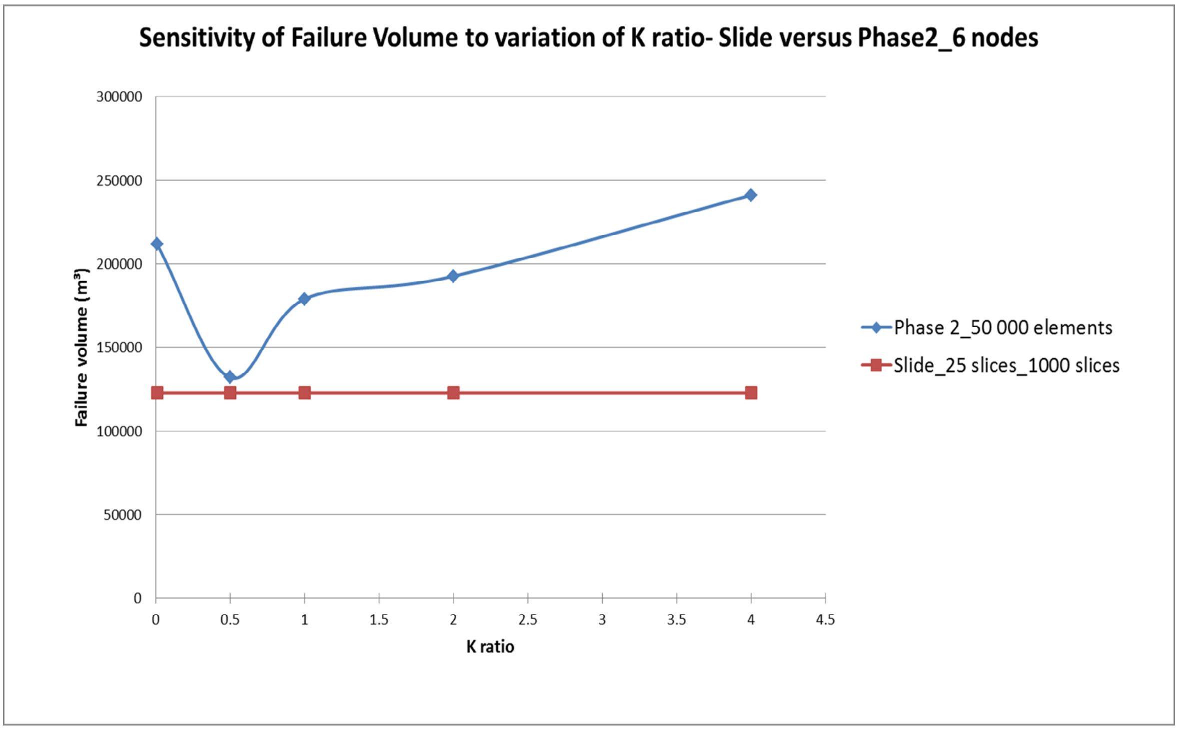

Figure 5-27 Comparison of LE and NM failure volume sensitivities to variation of K ratio

The change of K ratio does not significantly upset the stability of the slope, though its increase slightly enhances the FOS and reduces the POF. Phase2 always provides a better estimation of stability as well as a higher failure volume than the Slide program, with the volume differences having their minimum around a K value of 0.5. The "abnormal" behaviour of the FOS, POF and specially the failure volume around K value of 0.5 brings out the importance of taking more care of slope stability predictions, when the rock mass has a K ratio value near to 0.5. This might avoid overestimating the failure volume predicted through NM methods. The corresponding risk outcomes for Phase2 and Slide are displayed in Figure 5.28.

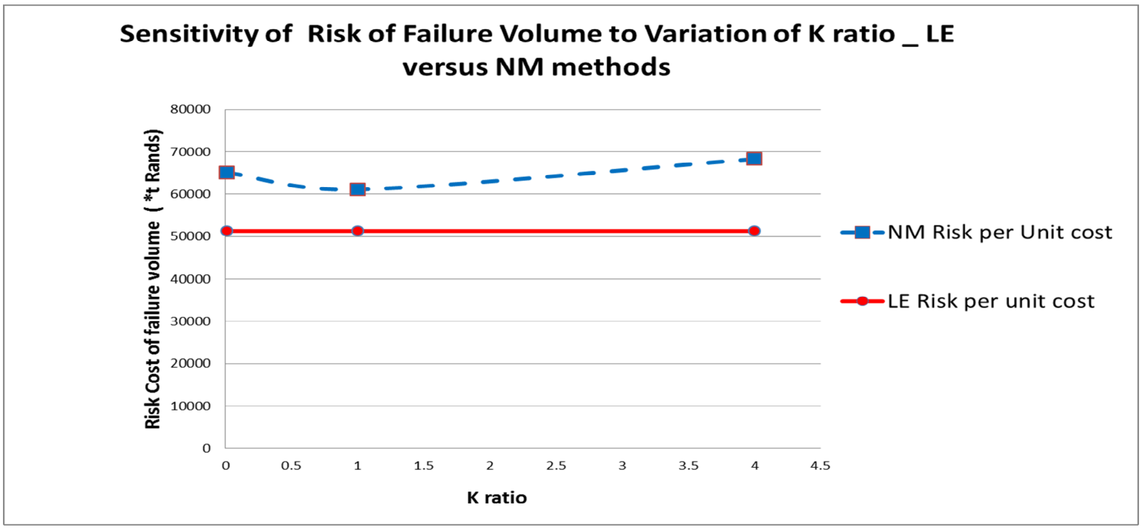

Figure 5-28 Comparison of LE and NM risks of failure volumes to variation of K ratio

Risk resulting from the NM is always greater than that from the LE method. The LE method underestimates the real risk of slope failure with regard to K ratio since it does not take this parameter into account.

Another rock mass parameter which is not included in the LE analyses is the LIS.

5.3.3 Effect of Locked-in horizontal stresses on risk assessment

The results of FOS, POF and failure volume sensitivities to variation of LIS are displayed in Table 5-5 and their trends illustrated from Figure 5-29 to Figure 5-31.

Table 5-5 Results of FOS, POF and Failure Volume based on the LIS variation

LIS (MPa)	FOS		POF (%)		Failure Volume (m^3)	
	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 50 000 elements_6 nodes	Slide_1000 slices
0	1.28	1.16	34.14	41.74	178966	122800
1	1.29	1.16	33.24	41.74	173048	122800
5	1.3	1.16	32.81	41.74	185613	122800
10	1.32	1.16	31.9	41.74	191467	122800
15	1.34	1.16	31.06	41.74	201228	122800
20	1.36	1.16	30.33	41.74	191105	122800
25	1.38	1.16	29.64	41.74	207145	122800
30	1.39	1.16	29.28	41.74	215643	122800
40	1.42	1.16	27.85	41.74	228359	122800

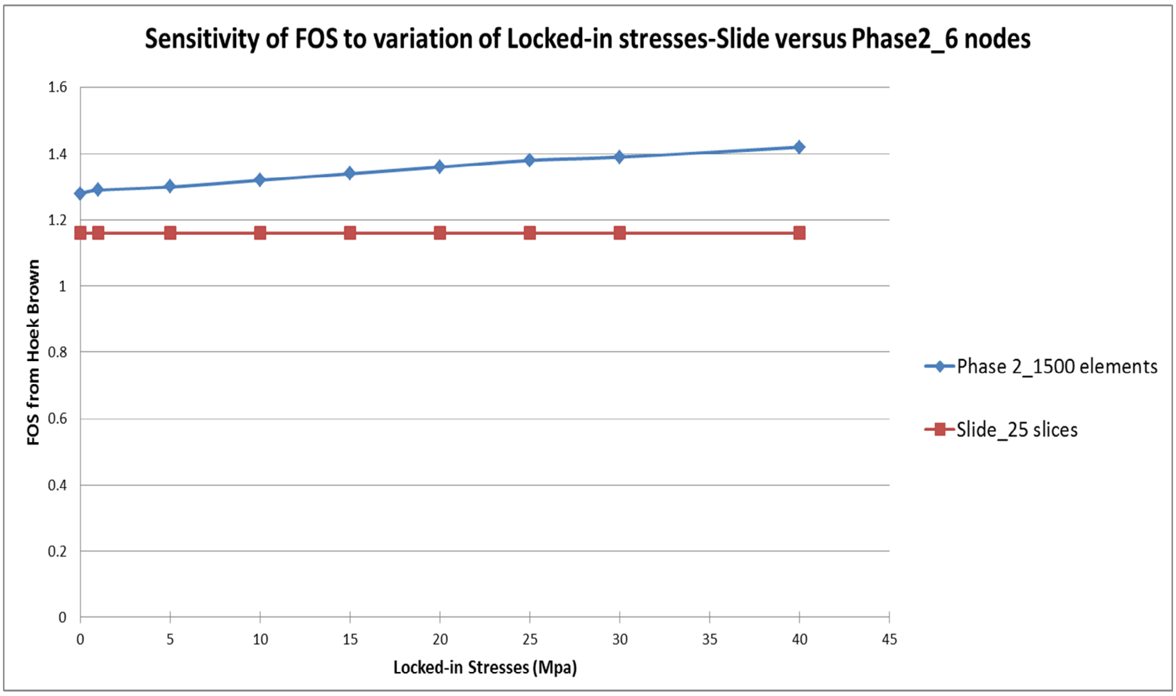

Figure 5-29 Comparison of LE and NM FOS sensitivities to variation of LIS

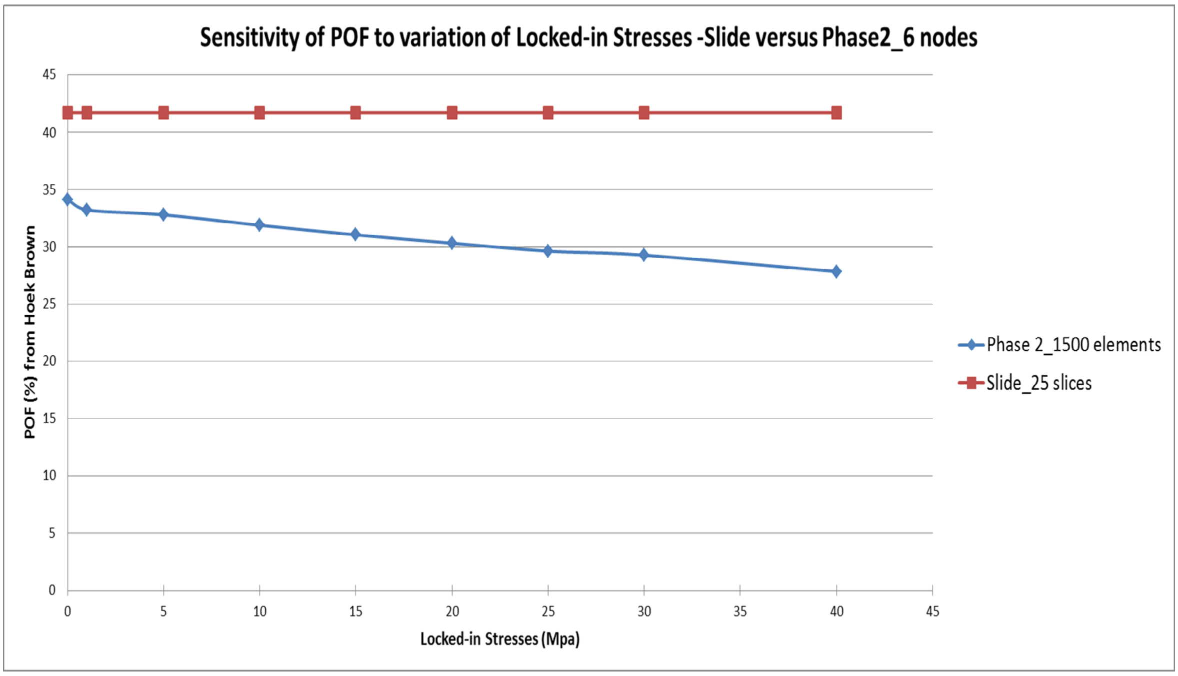

Figure 5-30 Comparison of LE and NM POF sensitivities to variation of LIS

The increase of LIS contributes to enhance the stability of slopes, with a decrease in the POF's (see Figure 5-29 and Figure 5-30). Not taking these stresses into account when analysing a slope may lead to under or overestimating the risk, by considering that the volume of failing material is independent to the LIS; which is not the case in reality. Therefore, no conclusion may be drawn on the risk behaviour without assessing the actual failure volume. This assessment has the results illustrated in Figure 5-31.

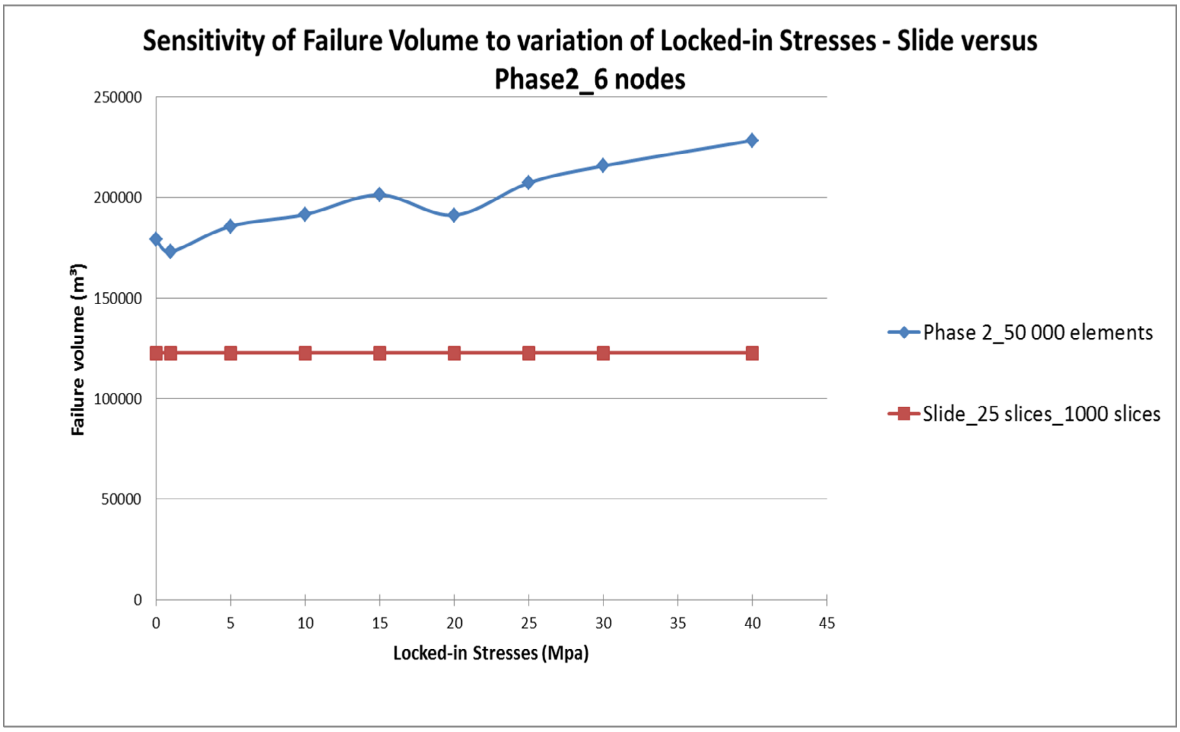

Figure 5-31 Comparison of LE and NM failure volume sensitivities to variation of LIS

This figure shows that failure volumes assessed from Phase2 are greater than those assessed from Slide. Since the POF and failure volumes are inversely proportional, the determination of the risk trend is important to provide a clear picture of the study. This trend is illustrated in Figure 5-32.

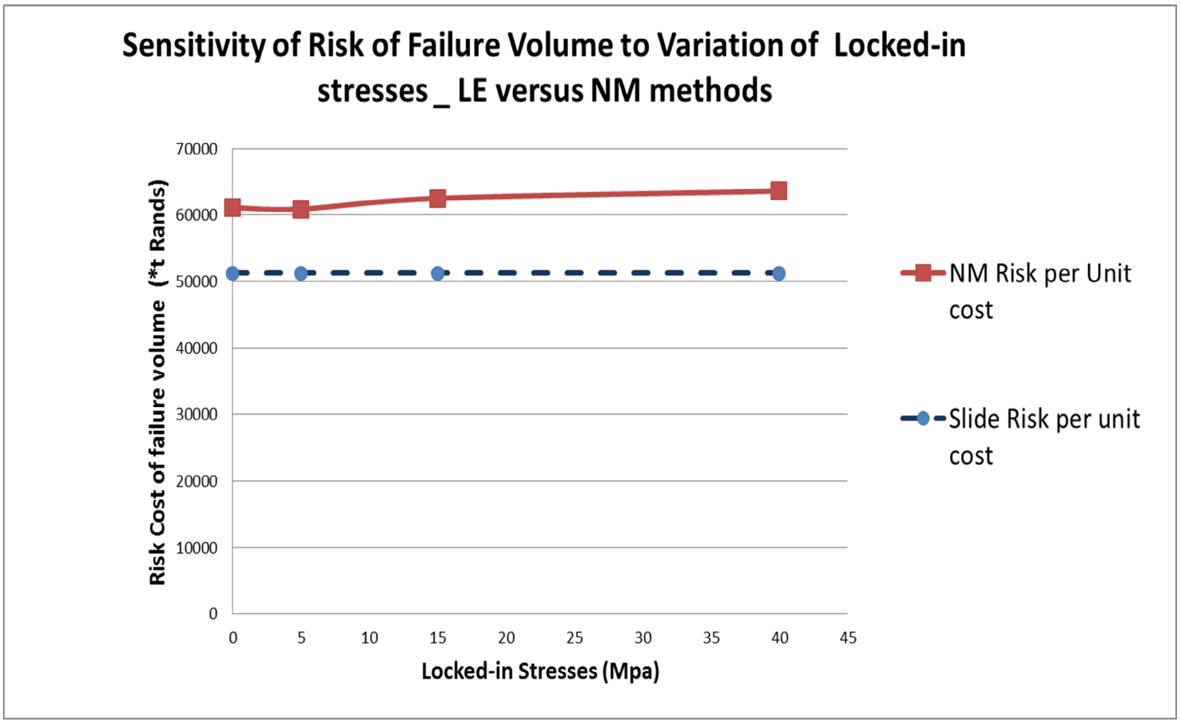

Figure 5-32 Comparison of LE and NM risks of failure volumes to variation of LIS

Risk assessed from the NM method is greater than that from the LE analysis. It is suggested that a multiplying factor should be incorporated when using LE analyses to rectify the underestimation of risk expected with regard to LIS.

In conclusion, in the case of a risk assessment solely based on failure volume, the risk calculated from Phase2 is almost steady, hence independent of the LIS and always greater than that from Slide, when the range of the LIS varies from zero to half the UCSi, as shown in the above figure.

5.3.4 Effect of the dilation parameter in slope stability study

The dilation parameter is also a factor that can affect the rock mass behaviour, and its impact on the slope stability is indicated by the results in Table 5-6.

Table 5-6 Results of FOS, POF and Failure Volume based on the dilation parameter variation

Dilation parameter	FOS		POF (%)		Failure Volume (m^3)	
	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 1500 elements_6 nodes	Slide_25 slices	Phase2_ 50 000 elements_6 nodes	Slide_1000 slices
0	1.28	1.16	34.14	41.74	178966	122800
0.01	1.29	1.16	32.92	41.74	170734	122800
0.02	1.29	1.16	32.73	41.74	161033	122800
0.03	1.3	1.16	32.46	41.74	153249	122800
0.04	1.3	1.16	32.47	41.74	144863	122800
0.05	1.3	1.16	32.44	41.74	135845	122800
0.06	1.3	1.16	32.56	41.74	132112	122800

The trends of these results are well displayed in the figures 5-33, 5-34 and 5-35:

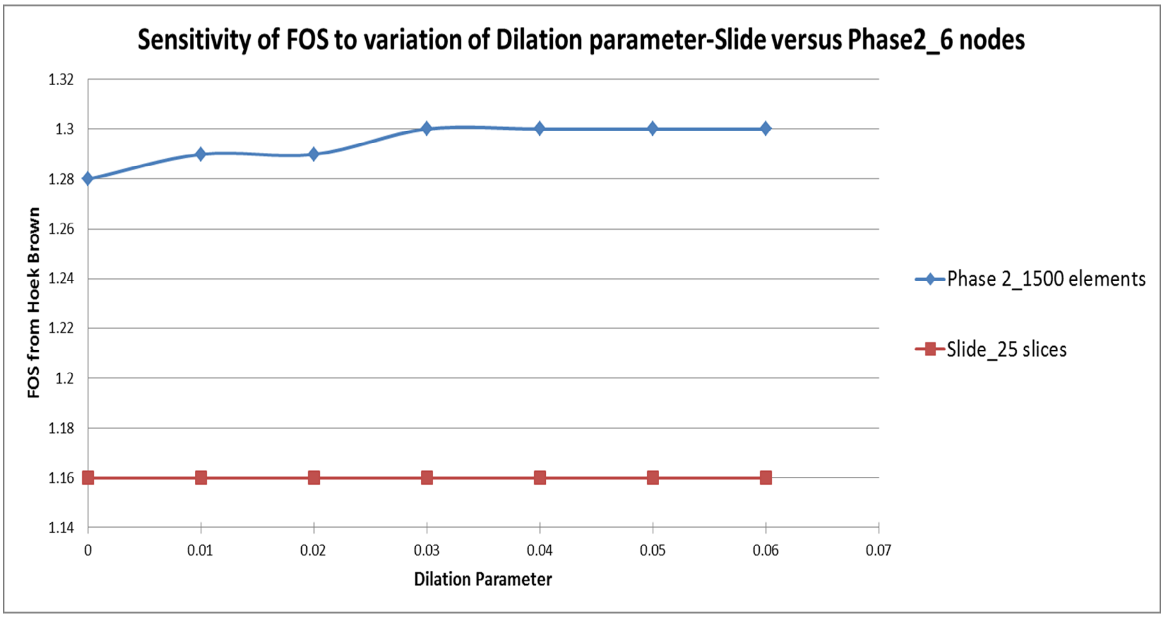

Figure 5-33 Comparison of LE and NM FOS sensitivities to variation of dilation parameter

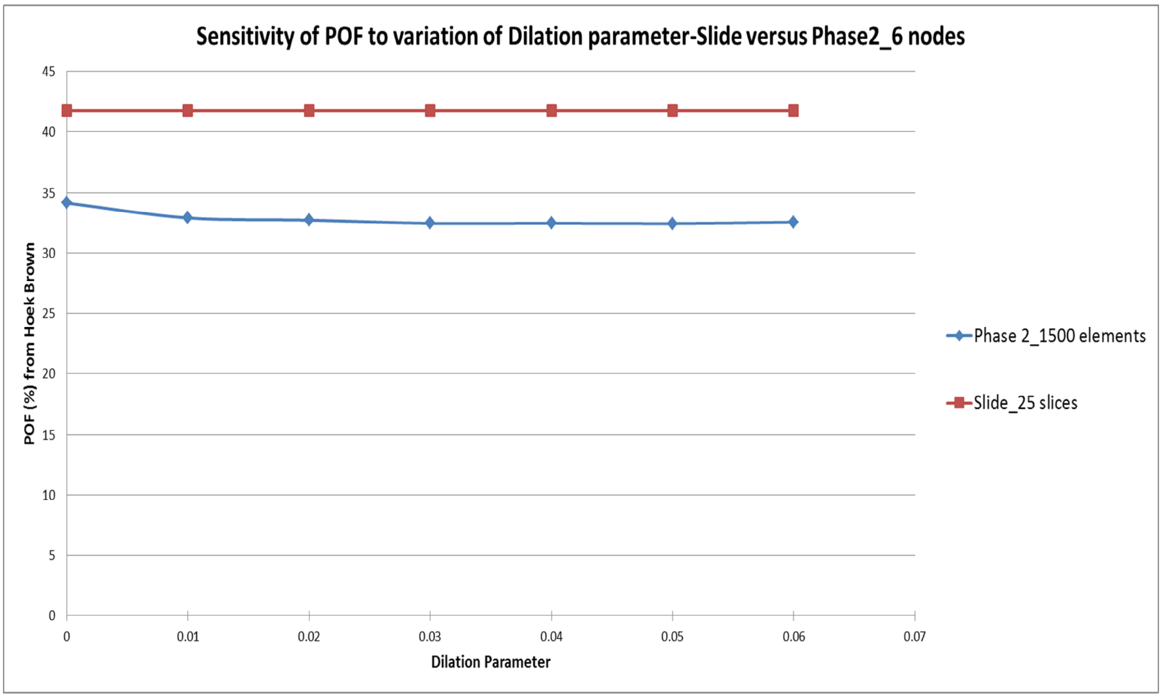

Figure 5-34 Comparison of LE and NM POF sensitivities to variation of dilation parameter

As a result of loss of strength, the dilation increase should have led to the reduction of the slope stability. This might be the case for values of dilation parameters greater than unity. However, this project relied on the range of values presented in Chapter 3 section 3.7.2 as proposed by "Phase2 FAQs: Theory" (n.d.). In this range (from zero to zero point zero six) of soft rock, the parameter increase appears to not really contribute to the slope instability; instead, it enhances its stability. The reason might that the values are very low and all of them almost equal to zero. But this parameter significantly influences the outcome failure volume as observed in Figure 5-35.

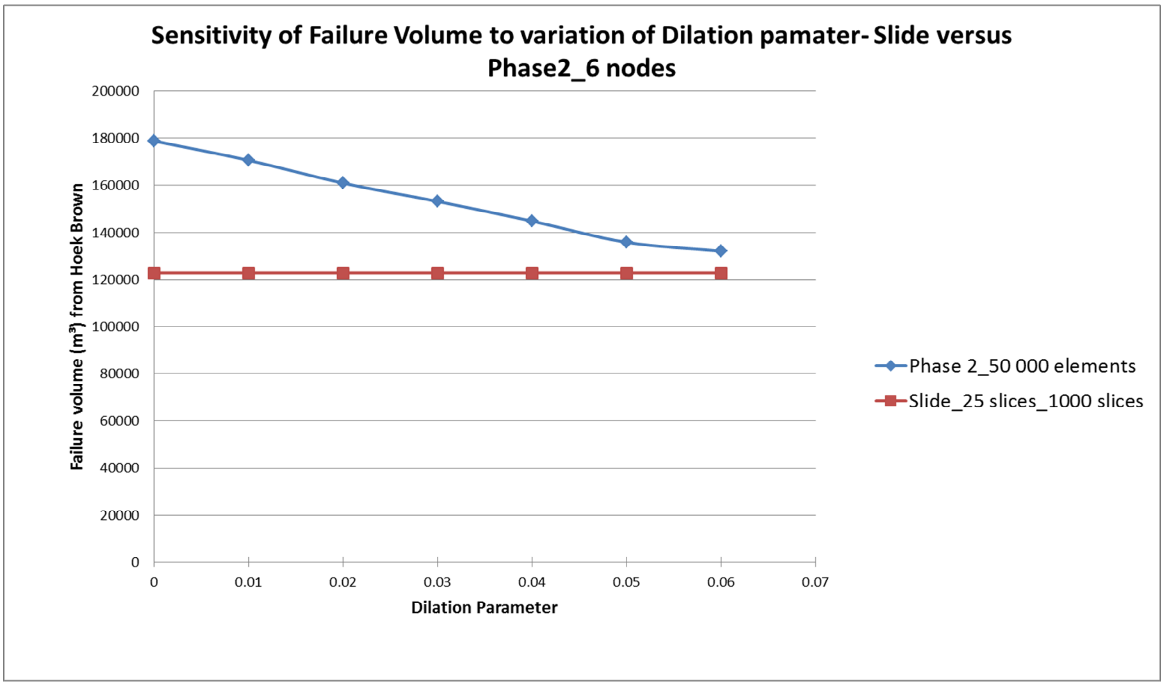

Figure 5-35 Comparison of LE and NM failure volume sensitivity to variation of dilation parameter

Considering a non-associative flow rule condition, the failure volume assessed from Phase2 is greater than that of Slide, though sensibly decreasing. This situation may drastically be inversed for greater values of dilation parameters, when it might be observed as lower failure volumes from Phase2 as compared to Slide results. But in order to conform with the practical behaviour, and because of the lower value of the deformation modulus (2.1 GPa), the study should only consider low values of dilation parameters, not greater than “m/3”, as mentioned in section 3.7.2.

The dilation influence on the risk of the slope stability is illustrated by the graphical results presented in Figure 5-36.

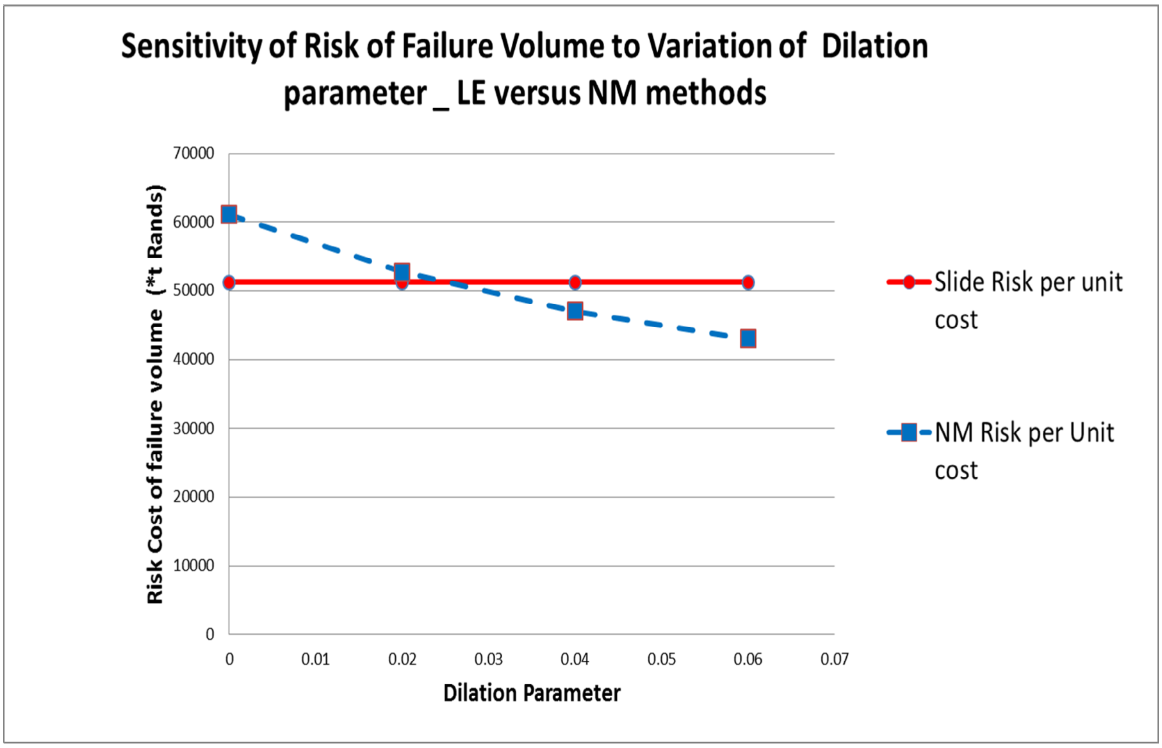

Figure 5-36 Comparison of LE and NM risks of failure volumes to variation of Dilation parameter

As expected, due to the decrease of the failure volumes, the assessed Phase2 risk also decreases, becoming lower than that of the Slide program with the dilation parameter increase. Here therefore, LE may overestimate the risk of slope failure when an associative flow rule is considered.

In conclusion, NM risks assessed with regard to the effect of each of the parameters studied in this Section 5.3 may in some cases be higher, equal to or lower than LE calculated risks. The sensitivity of these parameters on the overall risk was also investigated.

5.4 Impact of Probabilistic Rock Mass Parameters on Overall Slope Stability

The study considered two cases: in case 1, parameters are almost similar to Slide input properties, meaning that only the three main rock mass properties (UCSi, GSI and mi) are more influential in the NM method; and in case 2, three more parameters (K ratio, LIS and dilation parameter) with significant values have been added. This was only applicable to the NM method. All these parameters were considered as input variables, and the generic data of the normal distribution were considered. The aim was to assess the impact of parameters not accounted for in LE analyses on the overall stability of the slope while comparing the two analytical techniques. The obtained results are presented from figure 5-37 to figure 5-39:

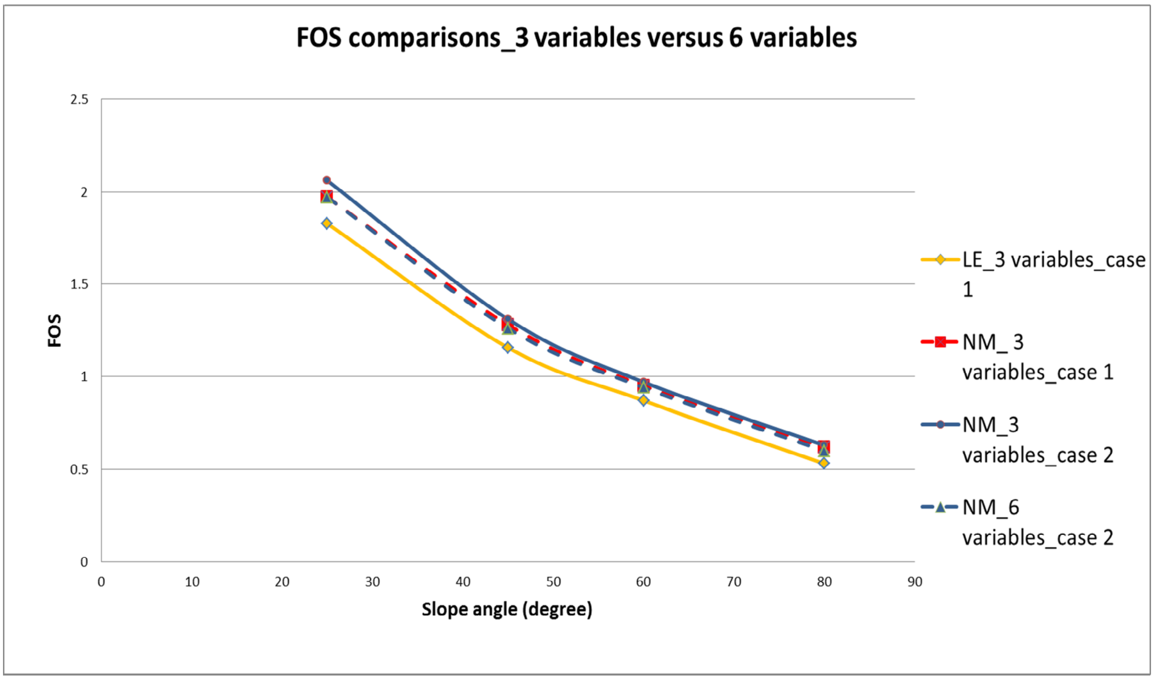

Figure 5-37 Impacts of 3 variables and 6 variables on slope stability

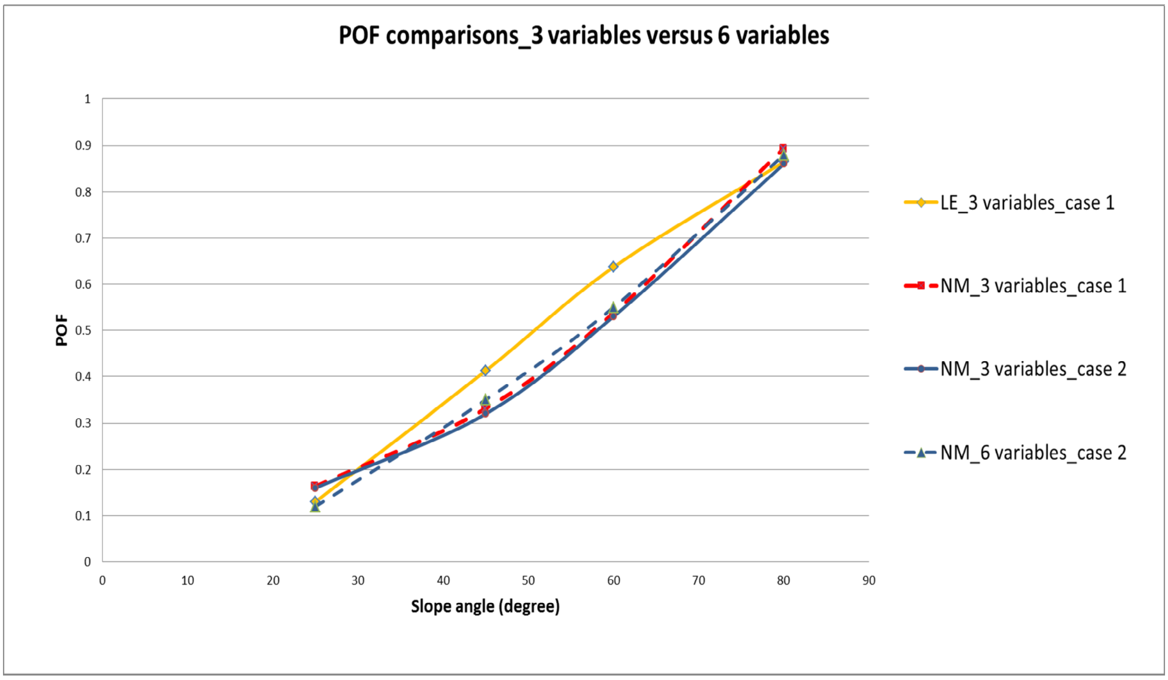

Figure 5-38 Impacts of 3 variables and 6 variables on slope POF

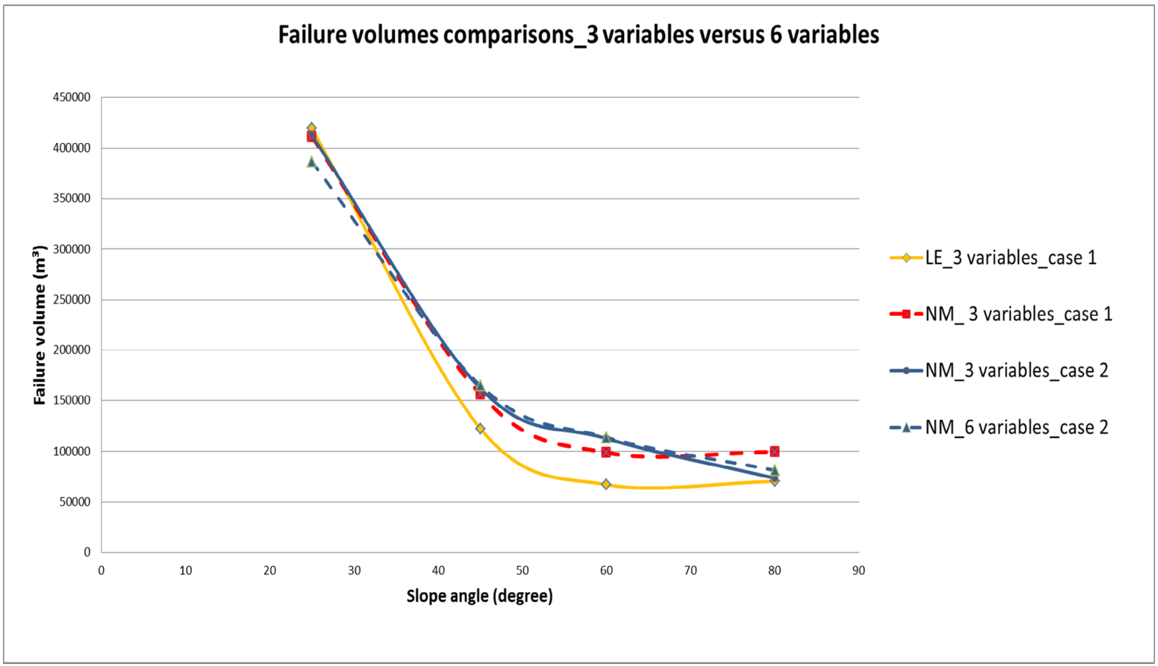

Figure 5-39 Impacts of 3 variables and 6 variables on failure volumes

Figure 5-37 to Figure 5-39 show that the increment of other parameters besides the UCSi, GSI and m_i does not influence the FOS, the POF and the failure volume results. However, Slide has shown a slight deviation in the range of slope angles between 35 degrees and 75 degrees, implying an underestimation of the failure volume.

Nevertheless, if the K ratio, LIS and dilation parameter do not have significant impact in the previous results, a careful analysis of the risk outcomes shows that these parameters have a slight impact on the assessed risk (See Figure 5-40). Particular attention should be paid when those three additional parameters are input variables. In this case, they show that, not taking them into account may result in overestimating the risk of failure (as compared with NM case 1), when the slope angle is between 40 and 70 degrees. Besides, the risk assessed by LE is almost always underestimated, except for slope angles less than 35 degrees; which range of angles is not practically resorted to.

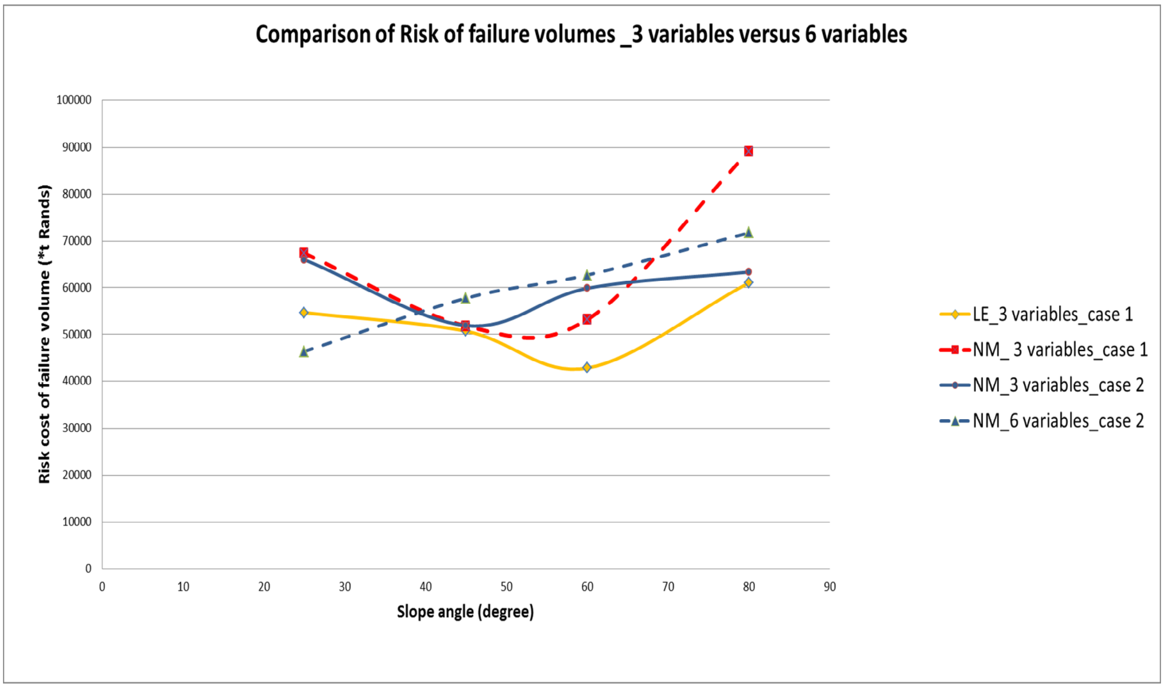

Figure 5-40 Impacts of 3 variables and 6 variables on risk assessment

In addition to the above figure 5-40, an illustration of the risk results is displayed in the semi-quantitative risk matrix represented by the following figure 5-41:

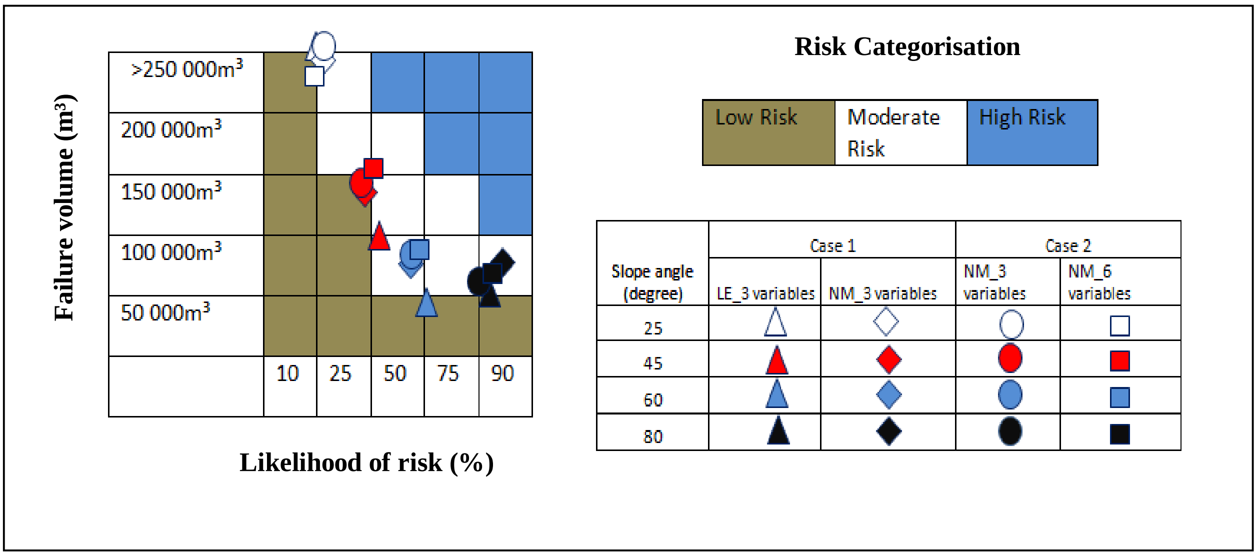

Figure 5-41 Semi-quantitative risk matrix_3 and 6 variables

Figure 5-41 presents results proving that there is no high risk associated with the failure volumes; though cares are to be taken for slopes of 80 degrees as they are much closer to the higher risk zone. And as expected from Figure 5-40, the semi-quantitative matrix presents lower risk results when additional rock mass parameters (from case 2) are used, except for slopes steeper than

60degrees, where the risk of slope failure becomes moderate, but not yet higher. This is due to the increasing tendency of the curve of 6 variables. Nevertheless, the risk assessed from the LE method is less than that assessed from the other three scenarios, except when the slope angle is less than 25 degrees. And, generally, the risk assessed with NM case 2 is almost the same as that assessed from NM case 1.

Phase2 contributed by displaying the contribution of each parameter included in case 1 and case 2 to variance of the slope POF. Based on the generic rock mass slope (45 degrees), Figure 5-42a, b, c, d and Figure 5-43a, b, c and d display the contribution of the rock mass parameters, respectively to variances and the POF results associated to case 1 and 2. And each group of these figures a, b, c and d respectively illustrates the results of LE 3 variables and NM 3 variables from case 1, NMs 3 and 6 variables from case 2.

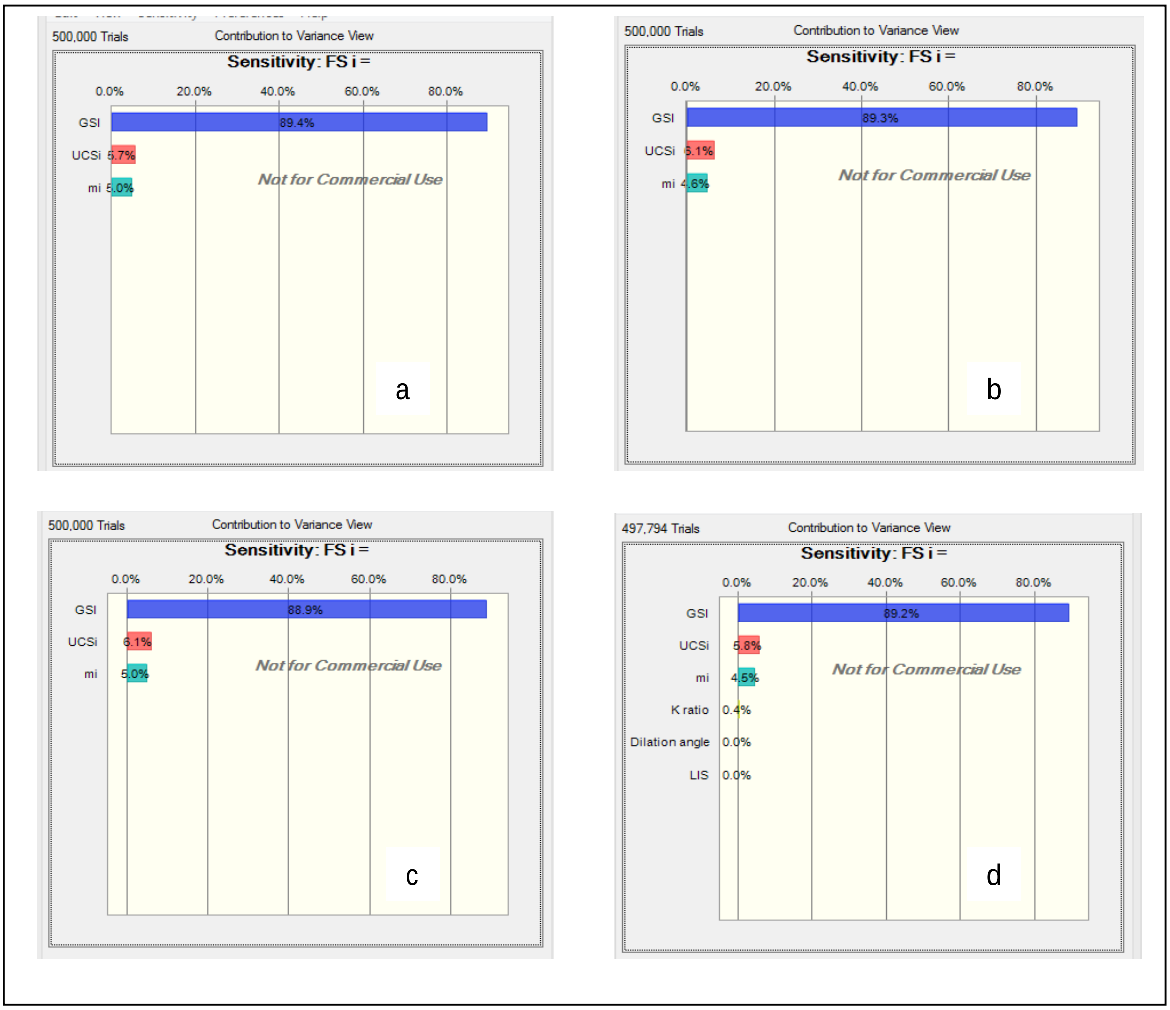

Figure 5-42 Sensitivity of FOS _Contribution to variance for 3 and 6 Variables

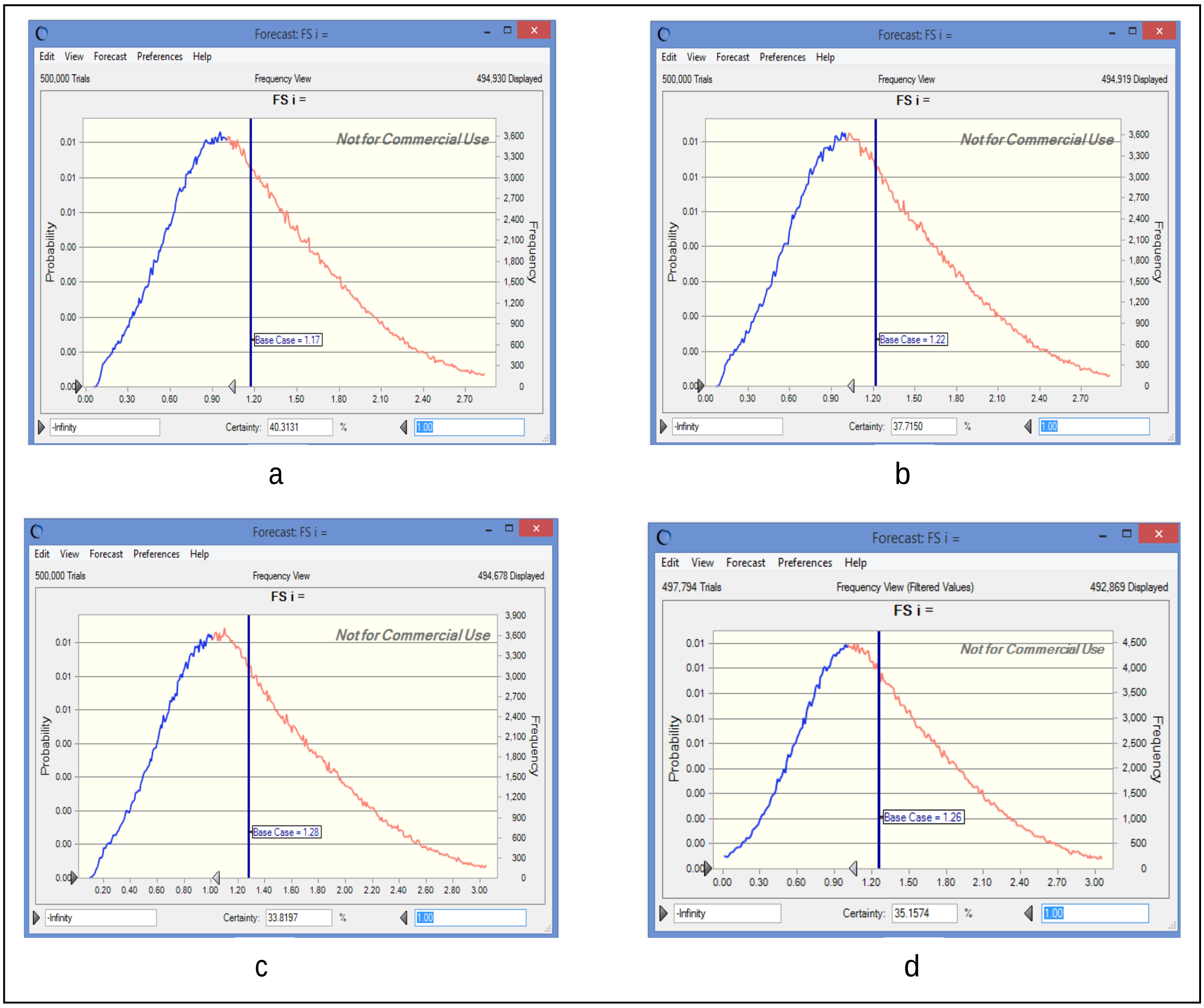

Figure 5-43 POF associated to cases 1 and 2

The displayed results show a huge impact of the GSI on the slope stability and at the same time corroborate the newly found results that show no significant impact of other parameters on the overall slope POF, besides the main ones cited upfront.

Chapter 6 CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary and Conclusions

The evaluation of slope stability and the associated risk has been one of the biggest challenges faced in open pit planning, particularly where the design of rock slopes is concerned. Two main analytical tools have been employed in the assessment of slope stability. Comparison of their outputs has shown some discrepencies, especially in the location of the failure surface, leading to the NM methods predicting higher failure volumes, and hence greater risk than the LE analyses. The research contained in this report investigated the reasons behind these discrepancies, with the following conclusions:

A. <u>LE validation</u>

1. FOS results from both Mohr-Coulomb and Hoek-Brown criteria agree more closely for steeper slopes than is observed for flatter slopes, the second criterion giving lower FOS values.
2. The Hoek-Brown criterion can be recommended for determination of FOS using Slide, independently of the type of distribution of input parameters.
3. The type of distribution (either normal or lognormal) does not have an impact on the stability prediction of the rock mass slope.
4. The type of distributions of the rock mass parameters should be well defined prior to any probabilistic analysis, since the predicted POF is affected by it, but not by the type of failure criterion.
5. The POF results from the two criteria do not permit definition with regard to which of them could be underestimating or overestimating the realistic results. Nevertheless, this confirms the importance of carefully defining the type of distributions of the rock mass parameters, prior to any probabilistic analysis.
6. For both FOS and POF analyses, 25 slices are reliable for building a Slide model. However for this research, a well-defined distribution of the rock mass parameters was imperative, likewise for the failure criteria.
7. With rock mass parameters following normal distributions, the Mohr-Coulomb criterion underestimates the POF at flatter slope angles and overestimates it for steeper slope angles.

Hoek-Brown can be recommended in this case, to avoid any imprecision of results when calculating the POF.

8. For numbers of slices equal or greater than 25, Mohr-Coulomb and Hoek-Brown criteria result in the same failure volumes. However, the latter criterion shows better (higher failure volume) results for slope angles less than 45 degrees or greater than 60 degrees. And in this last case, the statement is valid only for 1000 slices.

9. Using the Mohr-Coulomb criterion in the evaluation of failure volume leads to lower sliding volumes than when Hoek-Brown is employed. This may be a reason behind the finding expressed in the literature that Slide underestimates the failure volume.

B. <u>NM Validation</u>

1. Independently of the type of failure criterion, results confirm that there is FOS convergence as the number of elements increase, but at the expenses of running time.

2. FOS results for 1500 elements are just slightly different from those for 50 000 elements. These results are even more convergent for six and eight noded mesh elements.

3. There is convergence of FOS and POF results with the increase in numbers of elements, independently of the applied failure criterion. And these results are obtained with significantly fewer elements when using mid-point noded elements.

4. When a normal distribution of input parameters is used, for Mohr-Coulomb 1500 elements are likely to give almost the same POF results as Slide. This is not the case for the Hoek-Brown criterion.

5. A small number of mesh elements overestimates the failure volume. Therefore, a cautious approach should be taken, using a considerably larger number of elements.

C. RSM validation

1. There is good agreement in the POF results obtained from RSM and the full probabilistic NM method.
2. Rock mass parameters following a lognormal distribution, result in almost no discrepancy in POF results obtained from the two methods of analysis.

D. Volume determination

If the extraction of the LE failure surface is not affected by a margin of error, this is not the case for NM methods. In effect, caution should be applied when drawing the polyline representing the slip surface to avoid underestimating or overestimating the failure surface, hence its corresponding volume.

E. LE versus NM Risk assessments

1. Pit depth

   Pit risk sensibly increases proportionally to the depth. The comparative study shows only a slight difference in risk results between LE and NM methods - LE risk outcomes are almost equivalent to those of the NM approach.

2. K ratio

   Risk resulting from NM is always greater than that from the LE method. The LE method does not take the K ratio into account and consequently underestimates the real risk of a slope failure with regard this parameter.

3. LIS

   Risk assessed from the NM method is greater than that from the LE analysis in the range of LIS up to half of the UCSi (40MPa). Therefore a multiplying factor should be incorporated when using LE analyses to rectify the underestimation expected result of risk assessment with regard to LIS.

4. Dilation parameter

   - POF's resulting from NM methods are almost independent of the dilation parameter and always greater than those obtained from LE methods. Likewise, the failure volume predicted by NM methods is greater than that assessed by LE analyses. But this time, the NM assessed volume drastically decreases with the dilation parameter increase.

- For values of dilation parameter equal or very closer to zero, Phase2 assessed risk is always higher than the risk predicted with Slide. But a very slight increase of the dilation parameter shows a change of tendency of the Phase2 predicted risk, which becomes less than that obtained from LE techniques. Here therefore, LE methods overestimate the assessed risk of slope failure.
- Caution should be considered when choosing the range of the dilation parameter. This should depend on whether the rock mass to be analysed is a soft or hard rock.

F. Impact of Probabilistic Rock mass parameters on overall slope stability

1. The increment of other parameters besides the UCSi, GSI and m_i does not sensibly influence the FOS, POF and the failure volume results. However, Slide has shown a slight deviation in the range of slope angles between 35 degrees and 75 degrees, implying an underestimation of the failure volume. Thus, considering the K ratio, LIS and dilation parameter as variables in the rock mass, the assessed risk is slightly different compared to cases where these variables are ignored.

2. NM case 2 needs only to be resorted to for in-depth search of risk results; otherwise NM case 1 is sufficient for risk assessment.

3. LE risk assessment is generally underestimated when compared to NM with or without additional parameters (K ratio, LIS and dilation parameter). But this affirmation is maybe contradicted for slope angles less than 35 degrees and only when compared to the case of NM methods using all of the six parameters as variables. This however should be considered as a special case, since rock mass slopes are generally greater than 35 degrees.

4. LE and NM methods can be relied on for probabilistic studies, or even for risk assessments conditioned by carefully setting the models, and in case of LE being adopted to assess the risk, it is recommended to affect a multiplying factor based on case studies in order to level up the assessed risk and meet the one assessed from NM.

6.2 Recommendations for future Works

- The equivalent Mohr-Coulomb proposed by Li et al, (2008) should be investigated, since it has been proved to better approach the Hoek-Brown criterion than does the conventional equivalent Mohr-Coulomb.

- Charts of determination of more precise dilation parameters with regard to the rock mass characteristic should be investigated. The dilation parameter being a very sensitive input when assessing the failure volume, the study might help avoiding possible uncertainties in the current method of choosing the parameter in the range between zero and “m”.

- It is recommended for future studies to resort to screen dumps of typical analyses for both the LE and NM methods in order to evaluate depths of failure and shape of failure planes. This alternative method might help in validating the failure volume determination adopted in this report research.

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APPENDICES

APPENDIX A

SENSITIVITY OF SLIDE STABILITY PARAMETERS TO VARIATIONS OF SLOPE ANGLES AND SLICES NUMBERS BASED ON TYPES OF FAILURE CRITERIA

Table A- 1 Sensitivity of Slide FOS to variation of slope angles and slices number /Hoek-Brown criterion

Slope angle (degrees)	FOS-Bishop Simplified							
	4 slices		10 slices		25 slices		500 slices	
	Normal	Lognormal	Normal	Lognormal	Normal	Lognormal	Normal	Lognormal
25	2.92	2.92	1.927	1.927	1.827	1.825	1.825	1.825
45	1.508	1.508	1.162	1.162	1.156	1.156	1.155	1.155
60	1.028	1.028	0.872	0.872	0.87	0.87	0.869	0.869
80	0.592	0.598	0.532	0.532	0.531	0.531	0.530	0.530

Table A- 2 Sensitivity of Slide FOS to variation of slope angles and slices number / Mohr – Coulomb criterion

Slope angle (degrees)	FOS-Bishop Simplified									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	Norm.	Lognorm.	Norm.	Lognorm.	Norm.	Lognorm.	Norm.	Lognorm.	Norm.	Lognorm.
25	2.852	2.852	1.915	1.915	1.914	1.914	1.912	1.912	1.912	1.912
45	1.486	1.486	1.130	1.130	1.127	1.127	1.127	1.127	1.127	1.127
60	1.032	1.032	0.845	0.845	0.844	0.844	0.844	0.844	0.844	0.844
80	0.63	0.63	0.562	0.562	0.559	0.559	0.559	0.559	0.563	0.563

Table A- 3 Sensitivity of Slide POF to variation of slope angle and slices number /Mohr-Coulomb criterion

Slope angle (degrees)	POF (%)-Bishop Simplified/ Mohr Coulomb									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	Norm.	Lognorm	Norm.	Lognorm	Norm.	Lognorm	Norm	Lognorm	Norm	Lognorm.
25	0	0	4.77	0	4.79	0	4.79	0	4.79	0
45	9.63	0.8	34.46	27.91	34.74	28.25	34.82	28.33	34.82	28.33
60	43.79	36	73.07	85.89	73.16	85.97	75.11	86.02	73.21	86.02
80	97.1	99.9	99.12	100	98.62	100	98.62	100	98.53	100

Table A- 4 Sensitivity of Slide POF to variation of slope angles and slices number /Hoek-Brown criterion

Slope angle (degrees)	POF (%)-Bishop Simplified/ Hoek-Brown									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	Norm.	Lognorm	Norm.	Lognorm	Norm.	Lognorm	Norm	Lognorm	Norm.	Lognorm
25	3.72	0	12.99	0	13.04	0.01	15.58	0.01	15.615	0.01
45	22.53	0.1	40.8	16.82	41.29	18.04	41.45	18.32	41.607	18.216
60	49.97	47.44	63.466	90.77	63.686	91.02	64.04	91.34	64.456	91.263
80	82.328	100	86.322	100	86.482	100	88.34	100	88.775	100

APPENDIX B

SENSITIVITY OF SLIDE FAILURE VOLUMES TO VARIATIONS OF SLOPE ANGLES AND SLICES NUMBERS BASED ON TYPES OF PARAMETERS' DISTRIBUTIONS

Table B- 1 Sensitivity of Slide failure volumes to variation of slope angles and slices number/ lognormal distribution

Slope angle (degrees)	Comparison of failure volumes-Bishop Simplified/ Lognormal distributions									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B
25	25925	24860	218793	218284.5	218793	418706	218793	419424	218793	419424
45	26723	27187	122800	122860.7	122800	122800	122800	122800	122800	122800
60	29495	29495	67248	67247.9	67248	67248	67248	67248	67248	67248
80	38669	23753	53816	32673	62528	32673	62528	32673	70587	70587

Table B- 2 Sensitivity of Slide failure volumes to variation of slope angles and slices number/ Normal distribution

Slope angle (degrees)	Comparison of failure volumes_M-C versus H-B criteria/ Normal Distributions									
	4 slices		10 slices		25 slices		500 slices		1000 slices	
	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C	H-B
25	25925	24860	218793	218452	218793	419253	218793	419253	218793	419339
45	26724	27187	122800	122800	122800	122800	122800	122800	122800	122800
60	29495	29495	67248	67248	67248	67248	67248	67248	67248	67247
80	38669	23753	53816	32673	62528	32673	62528	32673	70587	70587

APPENDIX C

SENSITIVITIES OF PHASE2 FOS'S TO VARIATIONS OF SLOPE ANGLES AND MESH ELEMENTS CHARACTERISTICS BASED ON TYPES OF FAILURE CRITERIA

Table C- 1 Sensitivity of Phase2 FOS to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 100 elements

FOS_100 elements _Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0.38	0.37	1.8	2.04	0.65	0.88	1.88	1.98
45	0.84	1.11	1.34	1.33	0.48	0.51	1.2	1.16
60	0.84	1.03	0.84	0.89	0.61	0.75	0.89	0.86
80	0.84	0.93	0.67	0.64	0.73	1.32	0.65	0.68

Table C- 2 Sensitivity of Phase2 FOS to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 1500 elements

FOS_1500 elements _Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0.75	1.36	1.85	1.91	1.1	1.33	1.86	1.9
45	0.88	0.86	1.18	1.13	1.1	1.12	1.18	1.12
60	0.75	0.94	0.87	0.83	0.91	1.05	0.81	0.8
80	0.64	0.62	0.55	0.53	0.59	0.69	0.42	0.47

Table C- 3 Sensitivity of Phase2 FOS to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 50 000 elements

FOS_50 000 elements _Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	1.86	1.96	1.85	1.91	1.85	1.95	1.85	1.9
45	1.17	1.14	1.16	1.12	1.17	1.13	1.17	1.11
60	0.81	0.81	0.77	0.79	0.83	0.81	0.72	0.78
80	0.49	0.53	0.47	0.52	0.33	0.52	0.26	0.52

Table C- 4 Sensitivity of Phase2 FOS to variations of slope angles and mesh elements numbers for normal distributions

FOS_Normal distribution								
Slope angle (degree)	100 elements		1500 elements		50 000 elements		100 000 element	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	2.03	2.01	1.97	1.94	1.87	1.89	1.87	1.89
45	1.46	1.36	1.28	1.15	1.15	1.17	1.13	1.12
60	1	0.89	0.95	0.84	0.74	0.79	0.68	0.78
80	0.74	0.65	0.62	0.54	0.48	0.52	0.48	0.52

APPENDIX D

SENSITIVITIES OF PHASE2 POF TO VARIATIONS OF SLOPE ANGLES AND MESH ELEMENTS CHARACTERISTICS BASED ON TYPES OF FAILURE CRITERIA

Table D- 1 Sensitivity of Phase2 POF to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 100 elements

POF (%)_100 elements_ Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	100	100	0.44	1.1	99.98	69.78	0.11	1.59
45	86.43	35.38	100	12.9	4.03	100	12.75	24.81
60	86.13	43.97	95.41	74.49	100	91.03	80.6	80.13
80	87.59	62.59	99.99	99.42	99.69	13.11	100	98.99

Table D- 2 Sensitivity of Phase2 POF to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 1500 elements

POF (%)_1500 elements _Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	92.49	10.52	0.13	1.63	29.32	14.65	0.18	1.71
45	80.56	77.95	14.99	28.78	28.44	34.22	14.7	30.24
60	97.93	62	86.02	84.77	75.63	40.21	92.54	89.58
80	100	99.93	100	100	100	97.86	100	100

Table D- 3 Sensitivity of Phase2 POF to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 50 000 elements

POF (%)_50 000 elements_ Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0.13	1.58	0.13	1.7	0.13	1.83	0.13	1.73
45	16.09	27.82	17.07	30.31	14.72	29.75	15.61	31.73
60	95.43	87.74	98.94	91.1	91.32	88.15	99.45	92.95
80	100	100	100	100	100	100	100	100

Table D- 4 Sensitivity of Phase2 POF to variations of slope angles for normal distributions_cases of 100 and 1500 elements

POF (%)_Normal distribution				
Slope angle (degree)	100 elements		1500 elements	
	H-B	M-C	H-B	M-C
25	17.08	8.6	16.37	9.37
45	27.29	22.77	33.14	35.38
60	50.09	64.28	53.77	72.24
80	75.72	94.4	89.4	99.5

APPENDIX E

SENSITIVITIES OF PHASE2 FAILURE VOLUMES TO VARIATIONS OF SLOPE ANGLES AND MESH ELEMENTS CHARACTERISTICS

Table E- 1 Sensitivities of Phase2 failure volumes to variations of slope angles and mesh elements characteristics for lognormal distributions_case of 100 elements

Failure volume (m^3)_100 elements _Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0	0	0	394903	0	0	713781	469010
45	0	0	185622	279419	0	0	279755	200377
60	0	178465	158595	197672	0	0	164178	172356
80	0	301213	235697	215839	0	271736	161597	160637

Table E- 2 Sensitivities of Phase2 failure volumes to variations of slope angles and mesh elements characteristics for lognormal distributions_case of 1500 elements

Failure volume (m^3) _1500 elements_ Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0	0	503010	367566	0	0	524053	355460
45	0	0	209060	186442	0	0	198563	190354
60	0	73632	163826	154196	205830	175548	116097	150731
80	143077	140085	123211	126473	161806	129120	115189	113382

Table E- 3 Sensitivities of Phase2 failure volumes to variations of slope angles and mesh elements characteristics for lognormal distributions _case of 50 000 elements

Failure volume (m³) _50 000 elements_ Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	441064	386595	442299	331397	358831	344302	462720	325560
45	238263	183424	178966	164266	218201	182275	180568	154005
60	141154	125504	94091	107681	137047	119886	95182	101386
80	80877	92536	97783	98955	87497	92569	113906	115665

Table E- 4 Sensitivities of Phase2 failure volumes to variations of slope angles and mesh elements characteristics for lognormal distributions_ case of 100 000 elements

Failure volume (m³)_100 000 elements _ Lognormal distribution								
Slope angle (degree)	3 nodes		6 nodes		4 nodes		8 nodes	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	496731	373003	437614	323875	457235	333446	447404	317312
45	212196	173255	166423	153860	212898	176596	176772	152189
60	123338	103027	91790	103202	106019	108380	86961	114515
80	82791	84220	92977	79796	77207	88044	109788	115260

Table E- 5 Sensitivity of Phase2 failure volumes to variations of slope angles and mesh elements types for normal distributions

Failure volume (m³)_Normal distribution								
Slope angle (degree)	100 elements		1500 elements		50 000 elements		100 000 elements	
	H-B	M-C	H-B	M-C	H-B	M-C	H-B	M-C
25	0	446658	469702	363787	411347	324868	406299	326794
45	135533	190358	202450	190094	156337	135197	154267	164329
60	163941	148801	125536	160532	98930	107003	82675	103160
80	222512	223878	126073	124426	99703	95880	95409	99702

APPENDIX F

RSM COMBINATIONS AND RESULTS

Table F- 1 RSM combinations and results for slope of 25 degrees _Normal distributions

	25 deg_Normal Distribution			
	UCSi (KPa)	GSI	mi	FOS
Ugm	80000	50	10	1.88
$U^{-}gm$	53340	50	10	1.62
$U^{+}gm$	106660	50	10	2.07
$Ug^{-}m$	80000	33.4	10	1.11
$Ug^{+}m$	80000	66.6	10	2.95
Ugm^{-}	80000	50	6.7	1.62
Ugm^{+}	80000	50	13.3	2.07

RSM: POF=14.2%

Table F- 2 RSM combinations and results for slope of 25 degrees _Lognormal distributions

	25 deg_Lognormal Distribution			
	UCSi (KPa)	GSI	mi	FOS
Ugm	80000	50	10	1.88
$U^{-}gm$	64000	50	10	1.73
$U^{+}gm$	96000	50	10	2
$Ug^{-}m$	80000	48	10	1.77
$Ug^{+}m$	80000	52	10	1.98
Ugm^{-}	80000	50	7	1.65
Ugm^{+}	80000	50	13	2.06

RSM: POF=0.1%

Table F- 3 RSM combinations and results for slope of 45 degrees _Normal distributions

	45 deg_Normal Distribution		
UCSi (KPa)	GSI	mi	FOS
80000	50	10	1.2
53340	50	10	1.03
106660	50	10	1.32
80000	33.4	10	0.73
80000	66.6	10	1.89
80000	50	6.7	1.05
80000	50	13.3	1.3

RSM POF=37.8%

Table F- 4 RSM combinations and results for slope of 45 degrees _Lognormal distributions

	45 deg_Lognormal Distribution			
	UCSi (KPa)	GSI	mi	FOS
Ugm	80000	50	10	1.2
$U^{-}gm$	64000	50	10	1.1
$U^{+}gm$	96000	50	10	1.27
$Ug^{-}m$	80000	48	10	1.13
$Ug^{+}m$	80000	52	10	1.26
Ugm^{-}	80000	50	7	1.06
Ugm^{+}	80000	50	13	1.3

RSM: POF=16.2%

Table F- 5 RSM combinations and results for slope of 60 degrees _Normal distributions

	60 deg_Normal Distribution			
	UCSi (KPa)	GSI	mi	FOS
Ugm	80000	50	10	0.88
$U^{-}gm$	53340	50	10	0.75
$U^{+}gm$	106660	50	10	0.98
$Ug^{-}m$	80000	33.4	10	0.53
$Ug^{+}m$	80000	66.6	10	1.47
Ugm^{-}	80000	50	6.7	0.78
Ugm^{+}	80000	50	13.3	0.96

RSM : POF=59.9%

Table F- 6 RSM combinations and results for slope of 60 degrees _Lognormal distributions

	60 deg_Lognormal Distribution			
	UCSi (KPa)	GSI	mi	FOS
Ugm	80000	50	10	0.88
$U^{-}gm$	64000	50	10	0.8
$U^{+}gm$	96000	50	10	0.94
$Ug^{-}m$	80000	48	10	0.82
$Ug^{+}m$	80000	52	10	0.93
Ugm^{-}	80000	50	7	0.78
Ugm^{+}	80000	50	13	0.96

RSM: POF=85.7%