TEACHERS' USE OF EXEMPLIFICATION AND EXPLANATIONS IN MEDIATING THE OBJECT OF LEARNING

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DECLARATION

I declare that this Research Report is my own, unaided work, except as indicated in the acknowledgements, the text and the references. It is being submitted for the Degree of Master of Science at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

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ABSTRACT

This study examined teachers' use of exemplification and explanations in the teaching of algebraic expressions. In particular the focus was on the selection and sequencing of examples as well as what a teacher does with these examples in terms of their explanations to maintain the focus or object of learning. Adler and Ronda's (2015) Mathematical Discourse in Instruction (MDI) framework was the foundation of this study's conceptual and analytical framework and was complemented with the work of Stein (2000) and Moschovich (1999, 2015). Data was collected from two Grade 8 teachers through the use of video-recordings and transcripts. The data was then analysed based on the themes that emerged from the conceptual framework. The findings revealed that the examples themselves had the potential to restrict the object of learning and together with the teacher's corresponding explanatory talk could reduce or shift the object of learning from translating algebraic expressions to focusing on procedures. The findings show how each component of MDI worked separately and then together to mediate the of the object of learning, but this study has additionally highlighted how the components themselves, namely exemplification and explanatory talk, have a direct effect on each other.

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CHAPTER 1

AN INTRODUCTION TO THE STUDY AND RESEARCH QUESTIONS

1.1 Introduction and background to the study

It is well known that learning takes place through the use of exemplification, and that examples are used not only in the learning of mathematics but also in the development of mathematics itself (Bills, Dreyfus, Mason, Tsamir, Watson and Zaslavsky, 2006). In every textbook, worksheet, discussion on mathematics or classroom lesson an example can be found because some mathematical content is being explained through exemplification. From the work of Zodik and Zavlasky (2008) an example is defined as "a particular case of a larger class, from which one can reason and generalize" (p. 8). In the present study, my use of the term 'example' resonates with this definition as it can be extended to examples of concepts, techniques, visual representations vocabulary/ mathematical terminology, methods of answering questions and so on. The term exemplification describes "any situation in which something specific is being offered to represent a general class" (Bills et al., 2006, p.2).

It would be impossible to even begin describing mathematical content without having a corresponding set of examples to demonstrate its purpose or to answer what it is, when do we use it or how do we use it? So then the question becomes not if we need examples but rather what is the role of examples in the teaching and learning of mathematics. Watson and Mason (2006) discuss that examples are used by teachers to demonstrate, communicate and introduce the abstract ideas of mathematical concepts, techniques and procedures. Examples are therefore a central part of any mathematics teacher's resources and teaching practices, and are a necessary tool used to introduce and discuss abstract mathematical concepts and procedures. However, in a classroom environment, one has to consider that the process of exemplification requires specific examples to be chosen and used. The challenge lies in choosing specific examples because there are a range of possibilities to choose from yet only a handful of examples are actually used. When I was a novice teacher this challenge presented itself to me and although I had access to many different resources, I questioned which of my examples I would use? Choosing good examples is a difficult task and at times the choice I made hindered the learning opportunities afforded to the learners.

Below I recall a lesson from my first year of teaching in which one of my learners was experiencing difficulty with determining the lowest common denominator whilst working with the addition of algebraic fractions. The example on the board was from the prescribed textbook and was as follows:

Simplify the following (assume all denominators are nonzero)

$$\frac{1}{x}+\frac{1}{x+2}$$

When discussing the lowest common denominator (LCD) I had written the expression $x(x+2)$ and was stopped by the learner and asked why we did not just add two to the first fraction's denominator. In my attempt to explain to the learner that x and $(x+2)$ are factors that multiply together to give the LCD, I introduced a spontaneous example to aid in my explanation. This example is presented below:

$$\frac{1}{x}+\frac{1}{x^2+5x+6}$$

My intention was to highlight that adding a value to x would not present with a common denominator as in the above example you could not add x $+5x$ $+6$ to x and result in x^2+5x+6. Instead the learners would need to find the factors of x^2+5x+6 by factorising and then multiply the denominators together to find the LCD. In hindsight my choice of example obscured the learner's difficulty as the introduction of x^2+5x+6 meant that I had increased the task demand of the example, as he was now expected to factorise and then find the LCD, when he did not know how the LCD was found with variables to begin with.

In my attempt to use a specific example with an increased task demand, I had potentially lost the opportunity to engage with the learner's confusion. A more beneficial approach would have been to build up an example set in which the denominators started off as constants and progressively varied to expressions with variables. Poor example choices have been documented by Rowland, Thwaites & Huckstep (2003) and they acknowledge that there are difficulties that novice teachers have when selecting examples. They caution that a teacher's choice of example may facilitate or hinder the learners' opportunity to learn and generalise a mathematical concept. Given my own background in experiencing this difficulty, I was motivated to learn more about sequencing examples so that I could create an example set that encouraged learners to discover patterns and make generalisations from the few scattered procedures and facts given to them (Michalski, 1983).

1.2 Defining the problem

The use of examples in the mathematics class in an essential yet difficult aspect of a teacher's job. The sequencing and choice of examples is a difficult task and one that requires many considerations (Zavlasky and Zodik, 2007). Choosing, generating and using examples requires careful consideration of relevant and irrelevant features to create patterns of variance that lead to generalisations.

In addition to the difficulties experienced with sequencing examples are the mathematical tasks embedded within the examples themselves. Mathematical tasks for the present study are defined as the instruction given to the learners in the examples. For example:

$$x^2 + 5x + 6 = 0$$

The above equation is an example of a quadratic equation. The associated tasks for this example may be to *factorise* or to *solve for x*. These tasks are different and are set at differing levels of complexity or cognitive demand. Cognitive demand is defined by Stein (2000) as "the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 11). This means that the level of thinking generated from doing the different tasks within the example will contribute to problem solving skills required to successfully complete the example. Together the example and its related tasks serve a purpose, which is to limit or broaden a learner’s ability to generalise the abstract mathematical concept. The variation aspect of the examples engage with the generalisation of the concept, while the cognitive demand engages with the connections between the concept and the procedures or tasks required to complete the sum. Together the example and its tasks mediate the abstract mathematical concept.

Every lesson has a purpose or a goal in terms of the content that it aims to cover. Marton and Tsui (2004) refer to the purpose or goal of a lesson as the object of learning. Adler and Ronda (2015) argue that this goal is achieved through the use of examples and their associated tasks. In other words the object of learning is mediated through the use of examples and tasks. It is through the sequence and build up of the examples themselves that determine the manner in which the object of learning comes to focus. Through the use of correct sequencing and variation of examples, the object of learning can be encapsulated. However, examples alone are a starting point for achieving the object of the lesson as they are dependent on the nature of a teacher's explanation.

The accompanying explanatory talk is an additional factor to consider when determining the successfulness in achieving the object of learning. It is of the utmost importance that the explanatory talk maintains the integrity of the example so that there is no shift in the object of learning. This however is not a simple feat and Stein (2000) cautions that during the implementation of the example a teacher may willingly or mistakenly change the demand of the example's associated tasks (as was the case with me in the example cited above), which in turn may shift the domain of the example and object of learning. The problem is thus twofold. First, the examples and tasks selected and sequenced by a teacher is of utmost importance for achieving the object of learning. Second, the accompanying explanatory talk must engage with the examples in such a way as not to shift the domain of the object of learning or undermine the integrity of the examples and tasks themselves.

What is then expected from a teacher to be able to select and sequence appropriate examples and tasks to successfully achieve the object of the lesson? Additionally what factors during the explanatory talk contribute to maintaining the integrity of the tasks and in turn the object of learning? Both of these questions refer to the growing interest in determining the mathematical knowledge needed for teaching (Bill et al. 2006). From the discussion it shows that the choices that teachers make when it comes to examples and the accompanying explanatory talk may hinder or support student's learning, and thus the design, sequence, and use of examples is a central issue in instructional use (Watson and Mason, 2006).

1.3 Purpose of the study

This study uses Adler and Ronda's (2015) Mathematical Discourse in Instruction (MDI) framework to explore the selection and sequencing of examples, as well as the explanatory talk in the context of maintaining the integrity of the object of learning. This study also focuses on how teachers deal with the complexity of set examples in terms of presentation and implementation. Hence the purpose of the study is twofold. The first purpose is to identify the example set and the opportunities that the examples afford in attaining generalisation of the concept through variation. The second is to examine and illustrate the ways in which the explanatory talk shapes the predetermined cognitive demand of the presented examples and their associated tasks. Using MDI as a framework provides a useful theoretical stance to analyse the choices and actions of the participating teachers in order to provide answers to the two research questions that underpin this study:

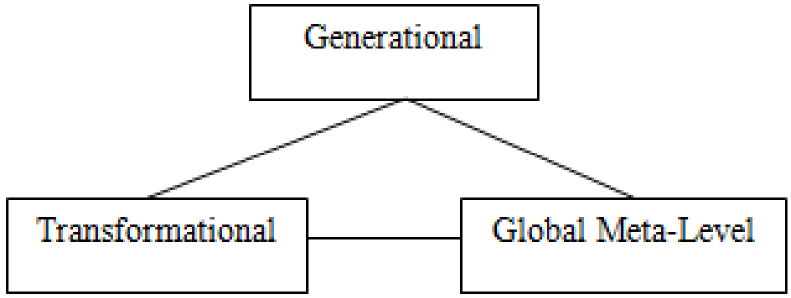
- What type of examples are presented during the teaching of algebraic expressions?
- How does a teacher's discourse impact the intended cognitive level of the presented examples?

By applying the notion of variation, as described by Watson and Mason (2006) and used in exemplification in MDI, I was able to assess a teacher's succession of examples and could determine the example sets' success in highlighting and generalising the critical features of the demonstrated concept or procedure. Additionally by focusing on the explanatory talk and learner responses from MDI that accompanies the lesson, I was able to explore what a teacher did with the examples and explanations to maintain the intended cognitive demand or, alternatively, the factors that resulted in a decline or increase of cognitive demand. To engage with the research questions I situated the study within the domain of algebraic expressions.

1.4 Why Algebraic Expressions?

There are a vast number of authors who have defined algebra in numerous ways, but three major themes emerge. First, that algebra provides a means of generalising arithmetic, second, that it requires some sort of solving (unknowns or problems), and lastly, that algebra is an activity (Kieran, 2007). School algebra, and the many challenges associated with the teaching and learning of school algebra, derive from the difficulties faced in choosing and presenting appropriate activities that exemplify the concepts surrounding algebra and algebraic thinking (Kilpatrick 2001, and Kieran 2007). During the early stages of algebraic development these activities involve working with variables and unknown values to form expressions and equations, and hence focus on generalising arithmetic (Kieran, 2007).

Consequently, one can imagine that there would be certain activities that promote generalisations, processes and problem solving. A model developed by Kieran (2007) synthesised the school activities that promote the above activities, namely *generational, transformational and global/meta-level* (GTG model).



<u>Figure 1.1: GTG model for conceptualizing algebraic activity</u>

Generational activities - These activities involve the formation of expressions and equations typically related to algebraic objects. An example would be forming an expression that modeled or represented a problem situation. I relate generational activities to concept formation as the meaning of algebraic objects stems from these types of activities (Kieran, 2007).

Transformational activities - These activities involve what one can think of as the rules of algebra such as collecting like terms, factoring, distribution, expanding, substitution, adding and multiplying polynomials, solving equations, simplifying expressions and so forth. Transformational activities can be thought of as the procedural methods and processes needed to complete a sum.

Global Meta-Level activities - These activities create a sense of purpose for the previously established transformational and generational activities by engaging in problem solving, modeling, justifying and generalisable patterns that draw on the concepts and procedures learnt in the transformational and generational activities. These activities are the thought-provoking sums that engage with general mathematical activities as well as algebraic activities.

Kieran (2007) argues that the meaning of algebraic objects are built up from generational activities and therefore assist learners in developing their fundamental algebraic understanding. In South Africa, the Curriculum Assessment Policy Statement (CAPS) states that learners are theoretically introduced to algebra in Grade 8. This is done by an introduction to both algebraic expressions and algebraic equations. Algebraic expressions are introduced first and according to the CAPS document one of the first activities that learners are expected to master is the ability to "recognise and identify conventions for writing algebraic expressions" (2011, p.23). The influence of both Kieran (2007) and the CAPS document motivated me to situate this present study in the domain of algebraic expressions because of its function in developing meaning of algebraic objects and introducing the accepted conventions of algebra. This means that the object of learning for this study would be the formation of algebraic expressions, and, in particular, I have decided to focus on translating worded statements into algebraic expressions. This decision is motivated by the idea that word statements contained within the introduction of algebraic expressions mimic the word statements learners have dealt within their fourth to seventh grades. For example, a Grade 6 learner might be presented with the question: *What is the sum of 5 and 12*, while a learner in Grade 8, while being exposed to algebra, they might be presented with the question: *What is the sum of a number and 12?*. Watson (2009) argues that a good understanding of the connections between numbers, relations, and their operations assist in the successful learning and usage of algebra. Watson further states that learners who have developed mental strategies for arithmetic operations are more susceptible to expressing arithmetical generalities through algebra. Thus by investigating an algebraic section that is closely linked to arithmetic operations I was able to focus on how the accumulation and variation of the example set highlighted the concept and promoted movement towards generality.

1.5 Significance of the study

There is a great demand for researchers to address the concerns raised in Bills et al. (2006) and the Zodik and Zaslavsky (2008) articles regarding teachers lack of awareness in their selection and sequencing of examples. This is a crucial or key component of examples as the sequence of examples limit the manner in which the object of learning is focused on. Therefore the findings of this study will contribute to the growing interest of examples and their role in teaching mathematics by exploring what examples teachers use, but additionally what a teacher does with these examples in terms of their explanations to maintain the focus or object of learning. The hope is that the present study will uncover critical areas in the selection and sequencing of examples as well as uncover the relationship between the presented examples and the teacher explanations of these presented examples.

1.6 Conclusion

In this chapter, I have described the background to the study and presented the problem that the study sets out to explore. I have also engaged with the purpose of the study and introduced the research question the study seeks to answer. Lastly I situated the study to pay particular attention to the examples that aid in the formation of algebraic expressions.

To foreground the underlying assumptions and theories for the study, Chapter Two reviews literature on the use of examples, variation, mathematical discourse and instruction as well as difficulties and challenges in the teaching and learning of algebraic expressions. Chapter Two also describes the conceptual framework that underpins the study and draws on variation theory, MDI as well as the adaptations and additions made to MDI using Stein's (2000) literature on cognitive demand.

Chapter Three provides a discussion and justifies the choices for methodology, sample choice, data collection and instruments. It further states the efforts that went into the study to ensure reliability and validity of the findings and this includes the ethical considerations concerning the study. Lastly chapter Three informs the reader on the techniques used in the data analysis.

Chapter Four provides analysis and findings from the exemplification component of the study and highlights the relationship between examples and tasks.

Chapter Five presents the findings (Teacher A and Teacher B) for the explanatory talk component of the study and discusses the strengths, weaknesses, similarities and differences of both teachers.

The study is concluded in chapter Six by discussing the summative findings and reflecting on the success of the study in its aim to determine teachers' use of exemplification and explanations in mediated the object of learning. Lastly chapter Six offers insight into the limitations of the study as well as any implications that it had for the mathematics education community.

CHAPTER 2

REVIEW OF LITERATUR RELEVANT TO THE STUDY AND THE CONCEPTUAL FRAMEWORK

2.1 Introduction

In this chapter, I review the related literature and the theories that underpin the study. The study focuses on examples and mathematical discourse, and I begin by defining the term example and then subsequently I discuss the pedagogical importance and role of examples in the mathematics classroom, as well as the different types of examples used. In presenting examples the teacher will use mathematical discourses as the medium to discuss and develop the concept, hence the next section describes mathematical discourse.The current concerns surrounding learner misconceptions associated with algebraic expressions and their relevance to this study is brought to light and discussed. This section further explores the recent studies that focus on errors made by learners when translating from English to mathematical symbolism. I then move on to discuss the conceptual framework which draws its theoretical resources from a previously established framework called Mathematical Discourse in Instruction (MDI). This section pays particular attention to the theories that form the basis of the critical components that make up MDI. Following some limitations of MDI, this chapter addresses the extensions made to MDI for the relevance and need of the study.

2.2 Teachers choice of examples

As indicated in Chapter One, this study defines examples in the same way that Zodik and Zaslavsky (2008) did. This means that any question presented to the learners that in some way aims to generalise a concept can be considered an example. These questions can be teacher generated, student generated, found in textbooks, worksheets or exams and tests. Leinhardt (2001) suggests that examples are used as a communication device during teachers' mathematical explanations and classroom discussions. In particular, teachers use examples to assist learners in "conceptualization, generalization, abstraction, argumentation, and analogical thinking" (Zodik and Zavlaskay, 2008, p. 165). This means that teachers require knowledge of examples, specifically as Zaslavsky and Zodik (2007) argue that it is the knowledge of choosing examples in terms of its content and the way in which a teacher presents it to the learners. This is particularly important as a poor choice of example may lead to an inappropriate level of mathematical correctness. Zodik and Zaslavsky (2008) determines mathematical correctness with respect to the claims by made the example itself. For example they suggested that a teacher may maintain that 0,3333 is irrational, however this is incorrect. The teacher may have meant that $0,\dot{3}$ is irrational but the distinction of the non-terminating decimal needs to be made explicit in the example and explanation. Furthermore, examples that are intended to demonstrate a particular concept but are visually incorrect may hinder the concept altogether as the example in itself is non-existent. This was demonstrated by Zodik and Zaslavsky (2008) by looking at an isosceles triangle with sides of 6, 6 and 12 units. Although the two lengths (6 units) are the same, the triangle does not follow the inequality rule, and therefore the triangle itself does not exist. Therefore the need to study exemplification is strengthened by the difficulties that previous teachers have faced regarding examples and example choice. Arising from these difficulties, is the question on what guides a teacher to generate or chose an example and in particular chose an example that promotes a good level of mathematical correctness. Bills and Bills (2005) suggest that a teacher's choice of examples may be guided by the example's ability to illustrate and exemplify the particular or intended feature of the concept formation. For instance, examples that aid in the abstraction of a concept will differ from examples that illustrate the core idea/s of the concept. Therefore, the knowledge involved in the teacher's choice of examples forms part of the specialised knowledge that teachers require.

This specialised knowledge is pedagogical content knowledge (PCK) classified by Shulman (1987) as being able to incorporate knowing the content, and the best methods of representing and adapting the content, so that it aligns with the learners' capabilities. In general, PCK is a teacher's knowledge of what to teach, how to teach and how learners learn (Ball & Bass, 2000). In terms of examples, PCK is elucidated through the choices that a teacher makes when choosing which examples to use in the classroom. The content knowledge incorporates the knowledge that surrounds the subject content and what needs to be taught. The pedagogical knowledge and role of the teacher is knowing how to take the content, choose particulars, and exemplify the content or concept in a way that will help learners to generalise (Bills & Bills, 2005). The two components work together so that a teacher has a well rounded PCK. In addition to what has been mentioned about PCK, it is also the knowledge that teachers develop as they experience classroom teaching. This includes being able to offer alternative solutions to a solving a worked problem, or the alternative methods of answering a learner's question during teaching instruction (Zazkis & Leikin, 2008). PCK, as specialised knowledge supports teachers in choosing relevant examples that align with the curriculum, object of the lesson and with how learners begin to understand the content. Thus the examples that teachers choose to use are an extension of their own understanding. A study done by Rowland et al. (2003) identified key characteristics of chosen examples that could hinder or obscure the object of learning. They are as follows (Rowland et al., 2003, p. 245):

- examples that obscure the role of the variables within it
- examples intended to illustrate a particular procedure, for which another procedure would be more sensible
- examples for instruction (as opposed to exercise examples) being randomly generated, typically by dice, at a point when it would be preferable for the teacher to be making careful choices.

A teacher's PCK provides them with the knowledge of which examples to use, when to use them, how to use them and what choices could hinder or obscure the object of learning. The next section looks specifically at the role and use of examples in the mathematics classroom.

2.3 Role and use of examples in the mathematics classroom

Rowland et al. (2003) distinguish two ways in which teachers use examples in the classroom. The first use is an inductive use of examples, which is to demonstrate and provide examples of something. Inductive examples are particular instances of the generality which show a general procedure or idea through a performance of that procedure or idea. Therefore, teachers using inductive examples must provide a finite set of well-chosen and thought out examples that will demonstrate the object of learning (Freivalds, Kinber & Wiehagen, 1993). Using examples inductively corresponds to the definition of examples as discussed above, whereby teachers use examples to assist in abstraction or generalisation of concepts. In this regard, the focus is not necessarily on the result or answer but rather these examples encourage learners to process the important information and become self-sufficient (Freivalds et al., 1993). Ling (1991) stresses that in order to successfully implement inductive examples the initial set of examples should contain well-balanced or 'good' examples. The reason for this can be justified by Michalski (1983) who claimed that inductive examples must be able to make accurate generalisations from a few facts. Thus when choosing inductive examples, one must keep in mind that the underlying structures and choice of numbers are important (Freivalds et al., 1993). Good examples should be able to distinguish features from previous examples given and most importantly provide the opportunity to give a general description of concepts from specific instances.

Exercise or practice-orientated examples on the other hand, are primarily for illustrative purposes. Once the procedure or concept has been demonstrated exercise examples are used to rehearse and practice what has been taught. These exercise examples are still considered a use of examples as the teacher is choosing an exercise for the learners to complete (often from a larger set). The intention is to "assist retention of the procedure by repetition, then to develop fluency with it" (Rowland et al., 2003, p. 243). Teachers give exercises for the purpose of class work, homework, projects and investigations for the purpose of testing a general principle through practice. For this study, practice-orientated examples are important because from my experience as a mathematics teacher, I have witnessed teachers focus on class work and exercises as part of their teaching practices. The exercises given, however, should match the object or goal of the lesson, whether it is to understand a concept, use a procedure or both.

Inductive and practice-orientated examples are a deliberate and informed choice from a selection of possible examples (Rowland et al, 2003). These choices are either pre-planned (chosen before the lesson) or spontaneous (created during the lesson). Pre-planned examples can be determined from looking at a teacher's lesson preparation and are generally influenced by their own content knowledge, teaching resources such as textbooks, past papers and worksheets and lastly by the curriculum itself. Spontaneous examples, however, would need to be observed during the teaching of the lesson as they stem from the interactions between the teacher and the learners. These examples almost derive from the spur of the moment, and are largely dependent on the teachers' content knowledge. Spontaneous examples rely heavily on the teachers' abilities to reflect on their lesson (Zodik & Zavslasky, 2008). Both the role and use of examples are important in determining the choices that teachers make regarding the examples that they present in the classroom. In addition to this, examples can be examined and discussed in terms of their sequence and variation.

2.4 Variation of examples

Every question posed to learners, whether pre-planned or spontaneous can act as an example, as every question demonstrates a particular case of a larger class. For instance an exercise from a textbook can be broken down and every question looked at as a different example. The example set can then be looked at in terms of the sequencing of examples and the use of variance. To develop a clear understanding of examples and variation, I turned to the work done by Watson and Mason (2006), Marton and Tsui (2004) and Lo (2012).

To begin to understand the importance of variation and its use in choosing and using examples, one must highlight a critical aspect of learning, that is according to Marton and Tsui (2004) learning can only happen if there is something to discern. This means that in order to learn something a learner must see something, whether it is a change in pattern or affirmation of an idea. According to Marton and Tsui (2004) learners pay attention to particular features that stand out from the given information, and in doing so begin to associate patterns or aspects of that particular feature. This means that teachers can use examples to explain a new concept by taking into account the features that the examples highlight, and use this to create a point of reference. From this point of reference teachers can then provide more examples to discern other characteristics or concepts that they deem important.

Effectively a good set of examples provide opportunities for learners to experience variation that discern features that were important in the past but also to use these past features to discern new and critical features (features that are necessary for defining the object/concept presently being taught). Marton and Tsui (2004) caution that using more variation in an example set does not necessarily lead to better opportunities for learning but rather it is the use of variation in discerning critical features that lead to development of the concept. Therefore critical features must be varied in different dimensions. Marton and Tsui (2004) refer to this as dimensions of variations and Watson and Mason (2006) dimensions of possible variations.

2.4.1 Dimensions of Variations

Watson and Mason's (2006) claim resonates with that of Marton and Tsui (2004), in that tasks that have been carefully constructed in terms of their variation generally produce a better outcome than randomly varied tasks. The prerequisite for this though is that these tasks are completed in a controlled and supporting environment, where the teacher is aware of the variation and critical features. Observing a teacher's choice of examples and the dimensions that are being varied in the example set may provide insight into the teacher's purpose or goal and subsequently answer if their example set aligns with their goal. Alternatively if the example set is used randomly and the environment is not supportive then the examples may lead to just practice and consolidation of the concept.

Careful choice and consideration of examples and their variance draw learners attention to critical features, patterns, expectations and mathematical meaning with regards to the concept or goal of the lesson. I know turn my attention to the different ways that teachers can vary these examples and tasks. Using Lo (2012) and Marton and Tsui (2004) I discuss three patterns of variation. These are generalizations, contrast and fusion.

2.4.2 Generalization

Marton and Tsui (2004) suggest that to fully understand a concept, one must be exposed to varying examples of it. Controlling the dimension of variation allows the learners to engage with a particular feature of the sum and through a supportive environment, such as a classroom discussion, the feature can be highlighted. Example sets that promote learners to identify regularities in terms of techniques, images, representations, terminologies or context may assist learners in constructing generalizations of the concept. Variation plays a large role in this process as in order to generalize learners must be exposed to examples that highlight the critical features (they remain invariant) while out of focus or non-critical features vary. This is what Lo (2012) refers to as generalisation and argues that this process allows the invariant feature to become generalised.

Lo (2012) cautions that there is a large emphasis on providing similar examples and although providing examples of a similar structure and promoting the same concept are the building blocks of the concept and assist in developing firm foundations, they are not sufficient in long term concept formation. This is because a complete sense of the concept and "meaning derives from difference; not sameness" (Lo, 2012, p. 97). Thus certain criteria must still be made explicit to a learner so that they can differentiate between what the concept is and what it is not. This is made possible by contrasting examples which is discussed in the next section.

2.4.3 Contrast

Learning from sameness or similar examples can lead to generalisation, however this process is made clearer when comparing that quality with another. Effectively sameness becomes more apparent after it is compared with something else and hence the importance and significance of using contrasting examples. Entrenching the idea of a concept is much easier once a learner knows what to compare it against. Through experiencing differences, learners can begin to discern similarities (Lo, 2012). Thus similar and contrasting examples work together to generalise a concept. With controlled variation an example set can be created and used to demonstrate examples with sameness and differences.

In summary contrasting examples provide learners with the means of comparing two or more values to each other. Learners are then able to discern different features based on the variance of the example set chosen. There are however considerations that need to be made when there are several critical features that could be varied and possibly need to be varied to complete the concept formation. These types of examples illustrate simultaneous aspects that could be varied and thus leads me to discuss fusion examples.

2.4.4 Fusion

Lo (2012) provides an example of fusion with regards to fractions that demonstrates the significance of varying more than one feature of a concept. The example provided by Lo (2012) reveals how focusing on one aspect of a concept is limiting in terms of forming complete and full generalisations and conjectures. At the same time, the example provides a method of how to approach fusion examples by first separating each critical feature and then bringing them together at the end. In a lesson on fractions a teacher may decide to keep the numerator invariant and vary the denominator, for instance an example set like $\frac{1}{2};\frac{1}{3};\frac{1}{4};\frac{1}{5}$ could be used to discuss and compare the size of each fraction where the critical feature is the denominator. A contrasting set of examples could then be given where the denominator remains invariant but the numerator changes, such as in the example set $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$. Together the two example sets provide opportunities for learners to generalise that when the numerator remains constant, the larger the denominator the smaller the fraction, and when the denominator is kept constant a larger numerator results in a bigger fraction. Thus learners are then able to quite confidently compare fractions such as $\frac{4}{7}$; $\frac{6}{7}$ or $\frac{1}{3}$; $\frac{1}{8}$ because they have developed generalisations, however when tasked with comparing fractions where both the numerator and denominator have been varied, learners struggle to make correct conjectures. For example if learners are asked to compare the fractions $\frac{5}{8}$ *and* $\frac{3}{5}$ they might only focus on the numerator or the denominator without considering the effect that both have on each other. Examples that vary simultaneous features offer opportunities for learners to become aware of each feature, how they work independently and together. If a learner is not exposed to fusion examples, they may never have the chance to develop a full understanding of the concept as they would not have been exposed to the many features that define and affect it. The challenge however is being able to show aspects of the different features in their own regard (separately) and then bringing them together in a way that shows learners that the features can work simultaneously. Marton and Tsui (2004) suggest that teachers should first demonstrate the features separately and then fuse them together as the example with fractions above does. The smaller parts of the concept should be distinct and then pieced together to help form the bigger picture or overarching concept.

From the discussion on the variation theory and in particular the patterns of variation we see that variation refers to the use of examples in a classroom setting with the purpose of obtaining a particular goal or concept. The examples are varied in terms of generalisation, contrast or fusion so that certain features of the concept are kept constant while other features are varied to draw attention to these features and help develop meaning. Careful consideration should go into planning and using examples in a lesson as variation theory provides a means for evaluating the successes and challenges of the presented example set. That is the successes and challenges associated with achieving the object of the lesson.

2.5 Mathematical discourses most commonly used by teachers in expressing and teaching mathematical concepts.

The term discourse varies in its definition and many authors have taken a stand as to what mathematical discourse entails. For the purpose of the present study, the use of the term mathematical discourse is used in the way that Moschkovich (2003) suggests, in that "mathematical discourse includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view" (p. 2). The study looks at a teacher's use of mathematical discourse (i.e. the words, and other semiosis) in maintaining the cognitive demand of the examples used in their teaching. Therefore, the interactions that occur between the teacher, learners and the examples used promote teaching mathematics as a social process, centralised around learning the mathematical discourse.

Vygotsky (1978) argued that the human consciousness had the capacity to take voluntary control over environmental factors through the use of higher level cultural tools. These tools act as a bridge between the individual and the environment in which they aim to expand themselves. That is to say, if one wishes to learn mathematics successfully, they require the social mediations provided through interactions. Thorne and Lantolf (2006) suggest that children need to learn the words but not as specific objects, rather as a concept through the interactions of the community. Initially learners rely on the input of adults, or in this case their mathematics teacher, and eventually they are able to develop the capacity to regulate their work themselves. This study investigates how teachers present examples and maintain cognitive demand through the use of different mathematical discourses that encourage concept development. It is first necessary to discuss the concept of scaffolding and its use in guiding learners in a particular goal. Moschkovich (2015) describes the many purposes of scaffolding and what it aims to achieve. For instance a teacher could use scaffolding to develop procedural skills, conceptual understanding or mathematical strategies and common practices. Learning and understanding how to carry out mathematical procedures fluently, flexibly, efficiently. Being able to choose the right procedure describes procedural fluency, while conceptual fluency describes a learner's ability to make sense of the concepts, operations and inherent relationships. Some typical or common mathematical practices that learners need to become familiar with are problem solving, mathematical modelling, generalising through patterns and justification skills. In order to use scaffolding to engage learners with these scenarios, I need to first define it.

For the present study scaffolding is defined as the methods used to mediate learning between a novice and more experienced other. Stone (1998) as cited in Moschkovich (2015) highlights some critical features of scaffolding. I will briefly discuss these features with the adult (expert) in mind being the mathematics teacher.

i. The mathematics teacher has the responsibility of engaging with the learner/s so that they become involved and participate in a lesson that encourages learning beyond their current means.
ii. The mathematics teacher needs to identify the current level of understanding of the learner/s and then decide on the level of the learner with their assistance.
iii. The mathematics teacher applies a range of supporting teaching strategies that guide the learner/s in achieving a higher level of understanding in comparison to their current state.
iv. As the learner/s are exposed to the teaching strategies, they expand their knowledge and level of understanding. Therefore after some time the support is no longer needed for that particular concept.

Careful consideration needs to go into scaffolding and teachers need to be aware of the difference between providing learners with opportunities to participate and explore the mathematical task, and with telling them the answers. Scaffolding should occur when a learner is unable to work through the task on his/her own, and a teacher through particular assistance enables the learner to complete the task. Henningsen and Stein (1997) emphasise that scaffolding should not reduce the complexity or cognitive demand of the task. To investigate the different types of teaching strategies that can be used to assist learners in completing tasks I turn to Moschkovich (1999, 2015) who discusses some supportive teaching strategies that are associated with successful scaffolding.

Interpreting mathematical content- This is the process of listening to the learners ideas and isolating the mathematical content. At times, learners struggle to formulate their ideas with the correct mathematical language, but their argument is mathematically sound. It is at these times that the teacher needs to focus on the content behind the words and not the vocabulary.

Repeating and Re-voicing- I have grouped these two teaching strategies together since they often lead from each other, however they differ in their execution. 'Repeating' occurs when the teacher clarifies the learner's idea by using the language that the learner used, so that the whole class can hear what was said. 'Re-voicing' is then taking the learner's contribution and rephrasing it with more technical or mathematical terms. This technique aims to introduce the mathematical discourse (specifically the words) in a meaningful and relatable way.

Building on a response- Both Moschkovich (1999) and Anthony and Walshaw (2009) discuss the benefits of using a learner's response. It creates an initial starting point. The learner feels involved since their answer is accepted and then developed further into a full argument. The teacher uses a learner's idea to form a basis of the concept and then expands on the content.

Research done by Walshaw and Anthony (2009) shows that not all teaching strategies support learning in the way that it is intended. Teaching techniques that appear to assist learners in answering the task but however lead to a decline in cognitive demand are not successful implementations of scaffolding. This was called path smoothing and it is discussed below.

Path Smoothing -. It was found that teachers, who support learning through giving chunks of information that fixed the direction of the lesson, actually hindered mathematical thinking (Walshaw and Anthony,2009). Effectively, path smoothing is simplifying the mathematics into a step by step rehearsed answer so that a pre-planned particular solution is achieved. Although this technique is not favoured, it needs to be considered since there is evidence in the research that this takes place in the classroom and will affect the development of conceptual understanding.

The above teaching strategies are employed by teachers during their classroom teaching, and it is through the use of their mathematical discourse that I intend to examine the impact that it has on the cognitive level of the examples that they present in the classroom. In the next section I discuss the importance of algebraic expressions during the introduction of algebra to grade 8 learners and provide supporting studies for choosing to focus on translating worded statements into algebraic expressions.

2.6 The importance of Algebraic Expressions in Mathematics

The significance of using examples of algebraic expressions stemmed from the South African grade 8 curriculum. This section is new in the South African senior phase (grades 7-9) and thus the introduction to algebra and formal algebraic language is done during these years. In particular if one looks at the curriculum and assessment policy (CAPS) for the grade 8 year, it describes that learners are expected to represent and describe situations in algebraic language and to recognize and identify conventions for writing algebraic expressions. This topic works extensively with language (both ordinary and mathematical) as well as the notion of a variable. I have found from my own experience that teaching algebraic expressions is difficult and that learners develop misconceptions from the examples and teaching aids used.

Stacey and MacGregor's (1997) research showed that learners with little experience in algebraic notation developed ideas about variables that were incorrect. Some of the common misconceptions are that variables must represent a specific numerical value, or perhaps, a value dependent on its position in the alphabet. This research was based on Küchemann 's (1978) six levels that letters can be used by learners:

i. *Letter evaluated* - in this instance, learners are able to answer the question and determine the variable through trial and error. There is no need to conceptualize the variable as they can look purely at the numerical values i.e. $x + 5 = 10$. Learners are familiar with this set up from their arithmetic work and are merely filling in the blank: $\blacksquare + 5 = 10$

ii. *Letter ignored*- when learners are moving through the stages of conceptualizing a variable they may initially ignore the variable and its contribution to the sum. Their focus is still on the arithmetic values and although they may arrive at the correct answer, as with the example in Küchemann's (1978) article (a+ b + 2 = 43), they have not grasped the effect of the variables.

iii. *Letter as an object* - Learners assign the variable to a particular object that is often linked to its association with the letter of the alphabet, i.e. a for apple or b for banana. This is frequently seen when learners are required to translate a word problem into mathematical symbols and therefore, this plays an important role in this research study.

iv. *Letter as a specific unknown*- As the learners progress through the idea of a variable, they realise that a variable represents a number. However, they associate the variable to a particular value and, in some instances, join the variable to a constant. This is because the variable is just another number that can be operated on. The distinction of the variable being able to represent an infinite amount of numbers is not yet conceptualized.

v. *Letter as a generalised number* - Learners begin to feel comfortable and confident with the idea that a variable can be used to represent a series of values. They may still, however, choose only one value when answering the questions.

vi. *Letter as a variable* - Learners are now aware of the relationship between a variable and all the values that it can represent. Their understanding extends to variables representing amounts, specific values (as in equations) and how variables can be used to represent a generalised idea.

Küchemann 's (1978) research paved the way for many researchers who have gone on to defining specific misconceptions and errors that learners have when working with algebra, particularly the concept of variable, and how to use variables in translating mathematically worded statements to mathematical symbols, in the form of algebraic expressions or equations. Learners in the Stacey and MacGregor (1997) study attempted to write expressions involving variables by ignoring the letter and giving a numerical answer, or perhaps choosing the specific value as mentioned above. These types of misconceptions develop as a result of learners relying on their previous knowledge of arithmetic to guide them in understanding algebraic notation. I would expect to see many of these difficulties present in a South African grade 8 mathematics classroom as these learners are using their previous arithmetic skills to build new algebraic skills.

In addition to learner driven misconceptions, Stacey and MacGregor (1997) found that learners in the higher grades developed misconceptions from a variety of other sources. In particular, learners had misunderstood the new learning material, and had begun to make analogies based on their previous experiences with letters in mathematics. Learners in older grades confused concepts that had been taught in class or had interpreted the idea different to how the teacher had intended it to mean. For example, learners developed a belief that any letter can just stand for the number 1. This misconception arises from the lessons whereby the teacher explains that when the letter stands alone such as x or y then the coefficient is 1. One can begin to question what examples were the learners presented with, and did these examples demonstrate the intended purpose of the lesson.

In order to find out how a teacher can assist learners in the translation of words to algebraic expressions, it is important to look at the struggles that are present when trying to write algebraic expressions from a given phrase. MacGregor and Stacey (1994) suggest that there are words used in the statement that have both an everyday context and mathematical context. The idea of 'and' and 'plus' are similar when used in everyday language, but mathematically, they could not be further apart. If we compare the two phrases: 'Add 7 and 5 together' versus '7 and 5', operationally the first instruction results in an addition sum, while the second answer requires the product 7×5. These instructional words can be used incorrectly at an algebraic level as well. Learners may confuse 'a' and 'b' with $a + b$ or 'a' plus 'b' to mean $a \times b$. Instructional words create opportunities for error, especially if not addressed or acknowledged by the teacher. When looking specifically at algebraic expressions, errors emerge from the structure and nature of the language and the skill required to translate the words to symbol form. Learners need to be shown examples of translated word statements and to discuss the difficulties associated with the mathematical language. This section of algebraic expressions has been a cause of concern and a study done by Rodríguez-Domingo, Molina, Cañadas and Castro (2012) focused on the errors that learners make when translating algebraic statements in the symbolic and verbal representation systems. Their study looked at learners abilities in translating from the symbolic form to the verbal representation, and found that there was a concern for movement from the verbal to symbolic representation, as learners had great difficulty in doing this.

MacGregor and Stacey (1997), along with McNeil, Weinberg, Hattikudur, Stephens. Asquith and Knuth (2010), suggested that some of the misconceptions have been introduced through inappropriate teaching materials or even from teachers explanations. The present study focused on the examples presented to the learners and the discourse practices used by a teacher in maintaining the intended purpose of the example. Earlier I discussed the purpose of scaffolding and will now apply this concept to algebraic expressions, focusing on worded statements.

The purpose of working with and scaffolding worded statements is to support learners in making sense of the question through simultaneously reading the information and extracting the relevant mathematical information. If the intention of the lesson is to purely translate the statement into an algebraic expression Moschkovich (2015) cautions that teachers may simplify the sum into a procedure of underlining key words and phrases. From my own experience as a teacher, the technique of emphasizing key words and translating the sum step by step leads to many challenges and incorrect algebraic language. For example if the statement is *4 less than x* and learners translate this in the order that the words were given they would get the expression $4 - x$. However, to correctly translate this statement into an algebraic expressions, learners have to be familiar with the language and more importantly they need to understand the concept of subtraction, and the idea behind what is being subtracted so that they come to the expression $x - 4$. When implementing examples and tasks of this nature and especially to a whole class it is important to assess the learner's current understanding and then apply the appropriate instruction.

It is important to note that the structure of mathematical texts are different to what learners have encountered in their English classes or everyday use of language. Learners are expected to read the given information and instead of interpreting the story, they need to make a mathematical argument and effectively create a mathematical model. The examples chosen to demonstrate these mathematical models contain multiple pieces of information and test the learners' ability to recognise how the quantities are related. While learners engage with the example and model the situation they also need to make strategic decisions, such as deciding on the correct procedure to use, and determining if their model fits the mathematical practices that they have encountered previously. I was drawn to using worded statements as it required the learners to read the information, make sense of what was given to them and then apply the mathematical practices and conventions taught to them. Therefore there are many opportunities for teachers to use real world scenarios to make sense of the mathematics and introduce algebra in a meaningful way. Additionally the CAPS document supported this decision as worded statements and representing algebraic expressions was a main focus of the grade 8 mathematics curriculum. The next section presents the conceptual framework of the study which is the lens upon which I will use to answer the two research questions of this study.

2.7 Conceptual Framework

In this chapter thus far, I have discussed and examined a selection of literature surrounding the role and use of examples, variation in examples, mathematical discourse and the challenges associated with algebraic expressions. I now present and describe the study's conceptual framework which is based on Adler and Ronda's (2015) framework: Mathematical Discourse in Instruction.

The study draws its theoretical and analytical resources from the framework developed by Adler and Ronda (2015) during their studies on mathematical discourse. Mathematical Discourse in Instruction (MDI) was built on the premises that teaching mathematics requires a network of interrelated concepts that could be analysed over a lesson to determine the mathematics made available to learn in a lesson. In particular MDI has the ability to focus on the sequence of examples and the accompanying explanatory talk that legitimises a concept (Adler & Ronda, 2017). The theoretical resources and analytical tools associated with MDI are the four components that define it, namely: object of learning, exemplification, explanatory talk and learner participation. In the MDI framework, the object of learning is subdivided into exemplification, explanatory talk and learner participation. The meaning of the object of learning is then mediated (Adler and Ronda, 2015) through the use of examples, tasks, naming and legitimations. These four components are said to be interacting, in that the abstract mathematical ideas are exemplified and explained within a classroom community (teacher and learner discourse and interaction). The overarching aim of the present research study is to investigate "*Teachers' use of exemplification and explanations in mediating the object of learning".*

Thus the emphasis is on the teacher and his/her choices and presentation of examples and the impact that their mathematical discourse has on determining the extent of the mathematics made available to the learners. In order to understand the nature of the examples, MDI was chosen for this research project as it provided a means of analysing a lesson in terms of the sequence of examples and their variation, hence assisting me in answering the first research problem:

- What type of examples are presented during the teaching of algebraic expressions?

MDI also describes the explanatory talk and learner participation, which provides insight into the interactions of the classroom community so that the lesson moves towards a particular goal or the object of the lesson. These key components will assist me in answering the second research problem:

- How does the teachers' discourse impact the intended cognitive level of the presented examples?

The MDI framework draws from the Vygotskian perspective that scientific concepts are built up by the teacher working with the learners in a social environment. This means that a key component of the MDI framework is the mediation between the object (abstracted mathematical concept) and the subjects (learners) through the use of physical and psychological tools such as examples, mathematical language and general social interactions (Adler and Ronda, 2015). Consequently the authors (Adler & Ronda) developed the MDI framework as a means of documenting and interpreting differences in mathematics teaching, particularly during the instruction of mathematics. The MDI framework is presented below in figure 2

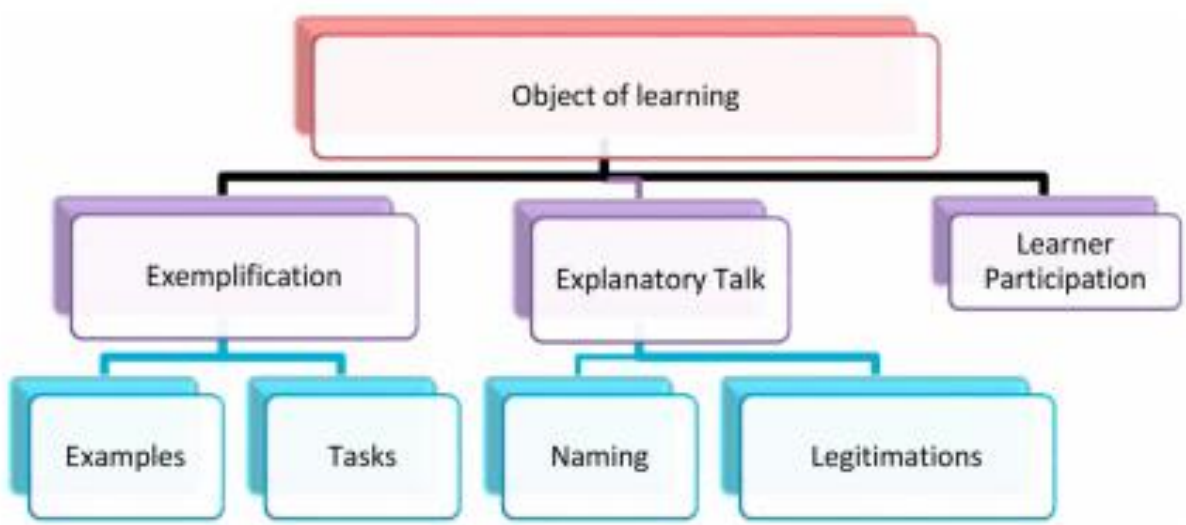


Figure 2.1: MDI framework (Adler & Ronda, 2015, p. 239)

The second research question looks primarily at the second level of the MDI framework and investigates how the explanatory talk with its own subdivisions, and learner participation influence and effect the presentation of the examples and tasks found under exemplification. Below I will discuss the theoretical underpinnings of the components of MDI used in this study.

2.7.1 Object of learning

Adler and Ronda (2015) state that learning begins with knowing what it is you are meant to do or know. The content itself and what it is that learners are supposed to do with the content is the object of learning (defined in Chapter One). Adler and Ronda (2015) further state the starting point of a lesson is achieved by a teacher bringing the content, whether it is a concept, procedure or algorithm into focus. The lessons particular goal or purpose must be made explicit so that a learner participating in the lesson i.e. reading the textbook or participating in a classroom discussion, is able to answer what the lesson was about in terms of its content and related capabilities. The capability of the lesson is what is expected of the learners in order to learn the content and together the content and its related capabilities form the *object of learning*. In order to establish the object of learning Adler and Ronda (2015) suggest that a teacher will typically announce the topic or goal of the lesson, which I will then take as the object of learning.

The object of learning draws attention to the content that will be learned, but these are merely titles or names of the mathematics and a starting point for the lesson. The object of learning will then influence the types of meditational tools that a teacher will choose to use and implement in the lesson. The meditational tools in MDI are found in the three subdivision, exemplification, explanatory talk and learner participation. These components are discussed next.

2.7.2 Exemplification

Exemplification is a subdivision of the object of the lesson and is implemented and influenced by two components, namely examples and tasks.

2.7.2.1 Examples

The focus is on how examples accumulate during a lesson and whether this movement assists the learners in moving towards generalisation of a concept. Thus the sequencing of examples and the use of variance is of the upmost importance. Examples in MDI draws resources from the overarching theory of variation based on the work done by Marton and Tsui (2004) and Watson and Mason (2006) which was discussed in section 2.4. To look specifically at how variation theory is used in MDI, I turn to Adler and Ronda (2015) to describe examples in terms of this framework. The three categories were developed based on Watson and Mason's (2006) ideas on variance, similarity (S), contrast (C) and fusion (F).

- Similarity - "Focusing on what something is through a set of similar examples brings possibilities for generalizing that which is invariant" (Adler & Ronda, 2015, p. 240).
- Contrast - Examples that look at the differences, which then work in conjunction with similarity examples to create opportunities to generalise.
- Fusion - These examples exhibit variance of more than one aspect.

MDI looks at how these three categories work separately and then how they work together over the full lesson to achieve generality. In addition to considering the patterns of variation, one must also consider the tasks associated with the examples to gain a full and clear picture of the presented examples.

2.7.2.2 Tasks

Tasks are the procedures and methods required to solve or complete the given example. Examples exemplify the content of the mathematics while tasks are associated with the object of learning's capabilities (Adler & Ronda, 2016). The second aspect of exemplification in MDI is the task that is associated with doing the example itself. With regards to MDI, the task level is limited to high and low cognitively demanding tasks with emphasis on three categories, known operation or procedure (K), application (A) and problem solving (C/PS). These three categories are based on the idea that cognitive demand increases as the connections between the concepts and procedures becomes more complex and intertwined. Using MDI, the three categories are discussed below:

<u>Known operation or procedure (K)</u> - These are tasks that require learners to solve for *x*, multiply and factorise for example. Effectively these are tasks that require a low cognitive demand to carry out as they focus on work that learners have routinely mastered. This means that these tasks have little connection with the procedure and the concept.

For example in algebraic expressions a learner could be asked to label the following diagram:

$$5x^{3} + 2\underline{x}^{2} - 3x - 10$$

In this instance learners are expected to label the exponent, variable, coefficient and constant which they can do from memorisation or studying their terminology.

Application (A) - These tasks require learners decide on the best procedure to use for solving the question. Thus there is a connection between the procedure and the concept and learners must apply their known skills.

To provide an example of A, I have used a word statement typically seen in the introduction of algebraic expressions. *Write an expression that represents 5 less than a number*. In conjunction with the learner having to make the connection of using a variable as well as a subtraction sum, I also considered that during the introduction of algebraic expressions (particularly at a grade 8 level) this would be new work and they would have to apply their known skills as well as their new skills.

Multiple connections and problem solving (C/PS) - These are tasks that could be solved in different ways or with different representations. Learners may be asked to prove or justify their answers and thus they need to make connections between the different procedures and the concept.

The three levels of tasks are set for the example at the time it is given to the learners, i.e. its level before implementation or teaching has begun. This is important, as Adler and Ronda (2015) caution that task levels can change during instruction. To accommodate for this MDI shows movement between the given task and implementation during instruction as a movement from application (A) to known operation or procedure (K) or from complex/problem solving (C/PS) to application (A) etc.

I however found that MDI briefly discussed tasks and their cognitive demand and thus there are opportunities but also limitations to these definitions and specifically limitations of the movement during the task implementation. This limitation is addressed in section 2.8 and adaptations are made to the task movement to account for the teacher's actions and mathematical discourse that may influence these movements in task level.

Exemplification in MDI and subsequently in this study looked at the examples and its associated tasks used in the classroom, however there is still a need to look at the mathematical language used in presenting and discussing these examples during the instruction of the lesson. Examples alone are a starting point to learning a concept because it is dependent on the nature of the teacher's explanations and discussions. Crucial to this study is the role of a teacher's talk, classified as the sub-division explanatory talk, which will be discussed in the next session.

2.7.3 Explanatory talk

The discourse in a mathematics lesson is used to transmit messages that communicate what is valued in terms of the object of the lesson. For this study mathematical discourse " includes not only ways of talking, acting, interacting, thinking, believing, reading, writing but also mathematical values, beliefs, and points of view" (Moschkovich, 2003, p.2). The mathematical discourses used to communicate these messages is exposed to learners during the explanatory talk. In MDI this is defined as "the function of which is to name and legitimate what is focused on and talked about, that is, related examples and tasks" (Adler & Ronda, 2015, p. 241). By looking at how the objects from the examples are named and legitimated in the lessons I will be able to describe the learning opportunities that are created and how the examples are incorporated and implemented in the lesson.

2.7.3.1 Naming

Naming refers to using words to give a mathematical object such as the mathematical words, symbols, images or relationships used in mathematics a particular reference or name. This connects the informal way that learners talk about the mathematics to the formal language accepted by the mathematical community. Naming the mathematical objects occurs in three ways.

- Colloquial (NM) which is using the non-mathematical (as seen by the community of practice) words to talk about the object of the lesson.
- Mathematical language used as a string of words (Ms) which refers to words that have mathematical meaning but used in a way that only identifies the object and not necessarily with meaning.

- Mathematics using formally (Ma), this is when the words are used with meaning to describe the object of the lesson.

The aim of a lesson is to name the object with meaning, so throughout the episode there needs to be movement between the informal and formal mathematical language.

2.7.3.2 Legitimating

This category focuses on the "criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning geared towards scientific concepts" (Adler & Ronda, 2015, p. 243). Effectively the legitimations are the substantiations made when describing the mathematical procedures and concepts. In a classroom lesson these substantiations would be made by the teacher and hence assist the learner in identifying what counts as an important mathematical feature or an explanation on why the procedure was done.

In terms of legitimations, I will consider the methods, techniques and hints that the teachers provide in the lesson as these often indicate the important mathematics to the learners and highlights the significance of the concept.

Studying the explanatory talk during the instruction of the lesson will help focus on the talk happening in the mathematics classroom and assist in discovering themes of what is intended and compare this to what is actually happening during instruction.

2.7.4 Learner participation

This particular component of MDI looks at what opportunities are given to the learners in the class to participate in lesson. During the presentation of the concept, what were the learners invited to say and answer and what were they expected to do. Was the teacher dominating the lesson in terms of talk and actions or were the learners expected to be active participants in the lesson. The interactions viewed here are between the learners and the teacher as well as the learner interacting with the examples given. No interaction is discussed between the learners themselves, both in the MDI framework and in the structure of this report.

- Level 1 – Short (one word) answers or yes and no answers.
- Level 2 – Learners answer with phrases or short sentences.

- Level 3 – Learners are encouraged to participate and discuss their answers or inputs to the lesson.

2.8 Using, complementing and extended the MDI framework

Adler and Ronda's (2015) MDI framework provides the foundation of how a lesson can be documented and interpreted through its mathematical instruction. The exemplification component provides a means on how to describe examples as they are presented in terms of their variation. The fist research question 'What types of examples are presented in a lesson during the teaching of algebraic expressions?' therefore draws on the component of exemplification. Primarily the focus is on the sequence of examples given, specifically on their ability to generalise through similarity, contrast and fusion and their capability in terms of their cognitive demand. Figure 2.2 below represents the approach that will be used to answer the first research question.

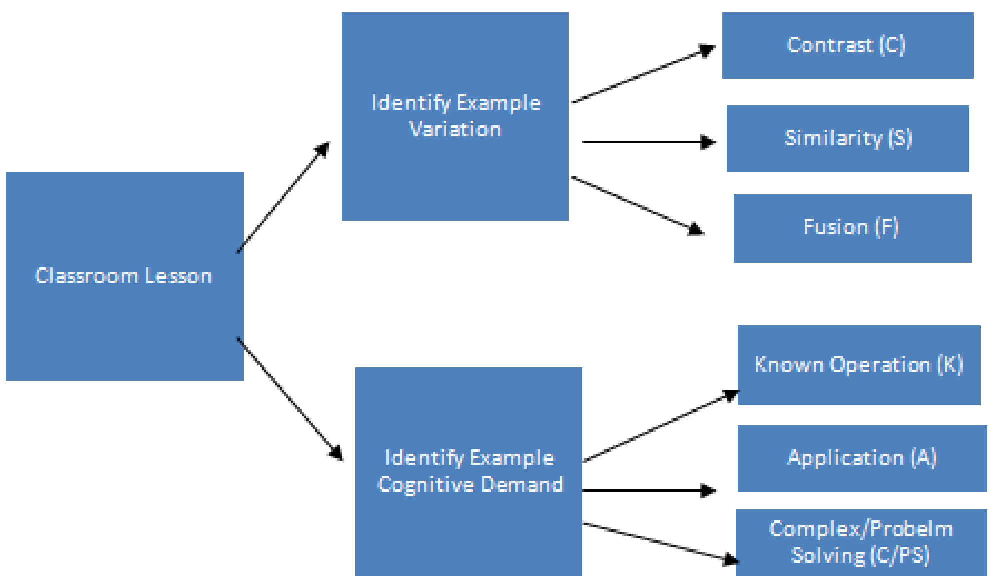


Figure 2.2: Taking a classroom lesson and classifying the extent of the examples generalisation opportunities through S, C and F and the tasks' capabilities in terms of K,A and C/PS..

Tasks in the MDI framework are classified as known operation or procedure, application or complex/problem solving. These three categories are based on the idea that cognitive demand increases as the connections between the concepts and procedures becomes more complex and intertwined. Despite the insight of low and high level tasks, the MDI framework lacks a coherent analytical tool for describing how to identify and classify tasks into these three categories. Limitations in the tools and methods for identifying which tasks can be classified as knowledge, application and complex/problem solving can lead to discrepancies in the data analysis (discussed in chapter 3) and thus I drew on the literature of cognitive demand and common patterns of task set up by Stein (2000).

Cognitive demand is defined by Stein (2000) as "the kind and level of thinking required of students in order to successfully engage with and solve the task" (p. 11). This means that the level of thinking generated from doing the different tasks within the example will contribute to the kind and level of thinking required to successfully complete the example. Stein (2000) argues that tasks of varying cognitive demand will engage learners in different ways. For instance, tasks that require applying a procedure will provide a different learning opportunities than tasks which require learners to actively engage with the concept. Consequently, the task analysis guide was created with the intention of describing and categorising four levels of cognitive demand. These four categories are memorisation, procedures without connections, procedures with connections and doing mathematics. Memorisation and procedures without connections, can be further classified as low level tasks, while procedures with connections and doing mathematics as seen as higher level tasks. The tasks are rated based on the kind of thinking required of the students to complete or solve the task. In order to establish the examples cognitive level before the implementation of the examples, I will draw on the levels Known operation or procedure, Application and Complex/Problem solving from MDI but I will use the cognitive task analysis created by Stein (2000) to assist me in determining the level of cognitive demand, as it provides a more thorough description of the task levels and will maintain consistency in determining the level of the tasks.

Known operation or procedure being a low-level cognitive task in MDI resonates with the memorisation and procedures without connection tasks and will be identified through the following features:

- Reproducing previously learned facts, formula, rules or definitions.
- Procedure is specifically stated or called for based on previous experience with similar tasks.

- Little ambiguity on what needs to be done, or how it needs to be done.
- No connection to the concept and can be solved by following a routine procedure without understanding needed.
- Focus is on the correct answer and no justifications are needed.

For this present study, I decided to name the low -level cognitive demand tasks as *Knowledge* and this incorporates the combination of known operation or procedure with the memorisation features from the task analysis guide.

Application in MDI resonates with procedures with connection tasks and the following features will assist me in determining these examples.

- The question suggest pathways to solving or completing the question.
- Can be represented in multiple ways.
- Procedures can be followed but with intention and understanding.

Complex/Problem solving in MDI resonates with doing mathematics tasks and can be identified as tasks that:

- Require complex thinking as no clear cut method is suggested.
- Require multiple concepts, processes, procedures and relationships to understand and complete.
- Draw on previous experiences so that learners think about the best approach to use.
- May make learners feel uneasy and challenged as the outcome is not obvious.

The task analysis guide from Stein (200) addresses the concerns I had with maintaining consistency and identifying the cognitive demand of each task. A second concern regarding the MDI framework was the movement of the tasks cognitive demand as they were implemented in the lesson. Although the MDI framework acknowledges that there is the potential for tasks to decline from application to knowledge or problem solving to application, there is no indication or explanation as to why this happens and what factors contribute to this movement. Therefore the challenge was to identify the factors that contributed to this movement and how to identify these factors.

Stein (2000) discusses that when a task is enacted in the classroom, it is influenced by the goal of the lesson, intentions, actions and interactions between the teacher and the learners. Both MDI and the task analysis guide framework acknowledge that during the implementation of the examples and tasks movement in terms of the level can and often does occur, thus I found the work by Stein (2000) compatible and comparable with the MDI framework. Stein (2000) accounts the movement of task level to the interactions between the teacher and the learner, and so through modifying the explanatory talk and learner participation components of MDI, I can begin to identify the factors that contribute to task movement. In order to maintain a high level of cognitive demand (in MDI this would be tasks at an Application or Complex/Problem solving level), learners need to reason with the mathematics in a meaningful way. Therefore, I have introduced a list of factors that have been considered from the works of Stein (2000) and Moschkovich (1999, 2015) that I will consider when determining if the teacher maintains the level of cognitive demand. These factors complement the explanatory talk component of MDI and will be used in conjunction with naming and legitimations.

Factors involved with maintaining examples and tasks at an Application or Complex/Problem solving level are deduced from the scaffolding techniques established in section 2.5. This consideration was not evident in the MDI framework and thus complements the MDI framework.

- Repeating and Re-voicing
- Prompting justifications and explanations
- Building on from a learner's response
- Interpreting mathematical language and content

Subsequently, there are also factors that will be associated with a decline in the cognitive demand. A decline in cognitive demand is most often associated with teacher dominant lessons, where the work is answered and explained by the teacher, these factors have been considered based on the work of Stein (2000) and Walshaw and Anthony (2009).

Factors involved with seeing a decline in examples and tasks at an Application or Complex/Problem solving level

- Path Smoothing – This is when the teacher reduces the complexity of the task by giving away the procedures and reasoning skills needed to solve it.

- Routine answer – the emphasis is on the correct answer only
- Not giving enough time
- Inappropriate task for the learners at their current knowledge level.

Lastly, there is a third category that although rare, may still occur in class. This is when a lower level task like knowledge is explored and discussed in such a way that actually raises the cognitive demand of the example. This movement was not discussed or acknowledged in the MDI framework and is therefore an extension and adaptation to the MDI framework. Thus the movement will be from $Knowledge \rightarrow Application$ or from $Application \rightarrow Complex$.

The adaptations were necessary for this project as the MDI framework lacked in providing a full and detailed description on how to code each task as knowledge, application or complex/problem solving. By complementing the framework with that of Stein's (2000) task analysis guide I can now classify each example and task into their respective cognitive demand. The second research question of the report aimed to investigate the effect of the mathematical discourse from the explanatory talk subdivision on the cognitive demand of the examples during the implementation stage of a lesson. As the researcher, the additional factors provide a means for exploring the data and identifying the mathematical discourses that maintain, decline or increase the examples task level. I have indicated the extension to the explanatory talk as a subdivision called scaffolding. Below is an overall view of the conceptual framework.

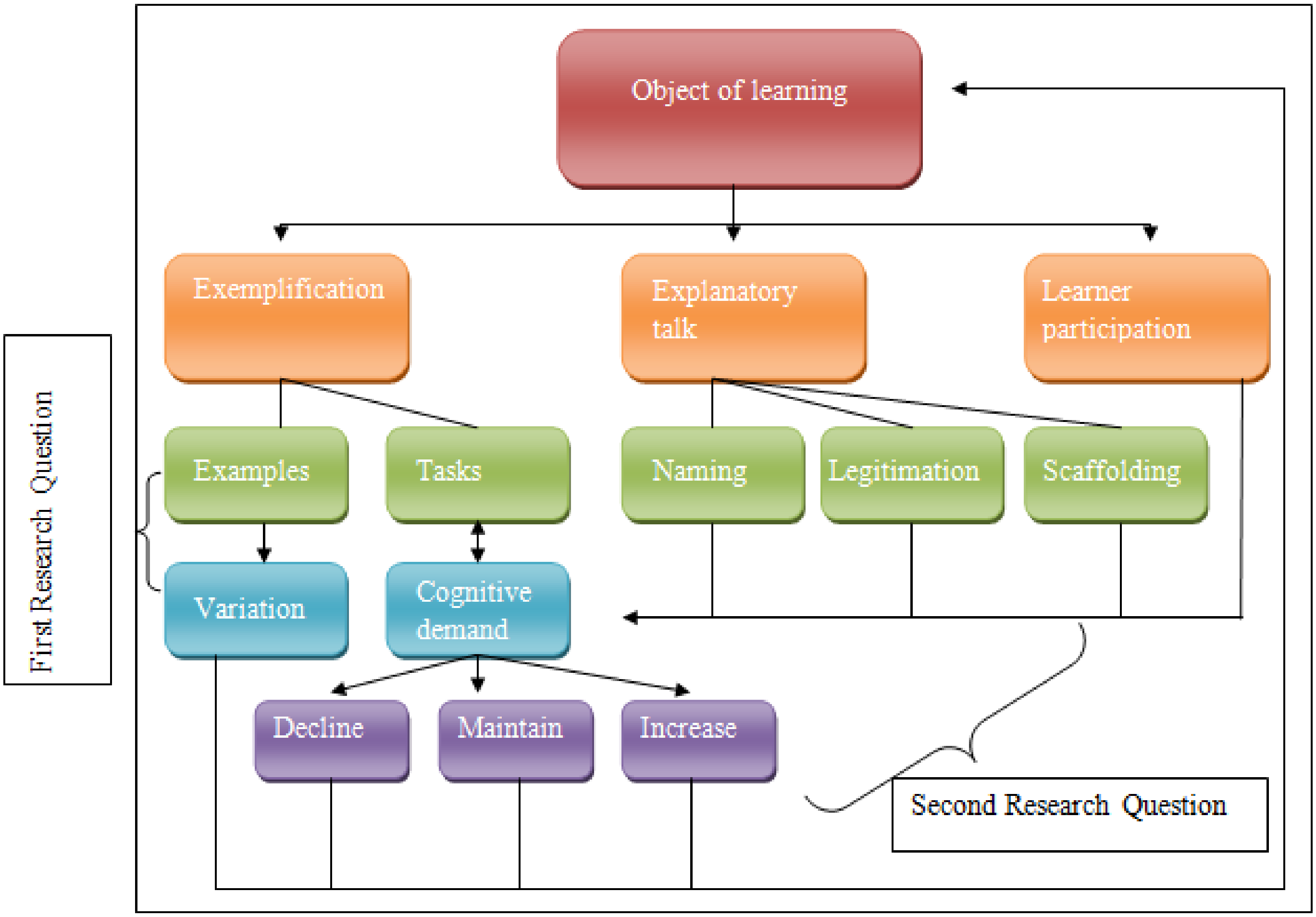


Figure 2.3 Conceptual framework of this Study

The diagram above shows the global conceptual framework of the study with the complementary addition of cognitive demand (task analysis guide) under tasks, as well as the addition of mathematical discourses presented as scaffolding under explanatory talk. I have also indicated the direction of the components by introducing connections between the different components through the use of lines and arrows.

The object of learning is mediated through the use of exemplification, explanatory talk and learner participation. Exemplification is further divided into examples and tasks, which assisted me in investigating and exploring the variation and cognitive demand of the examples presented to the learners in a lesson. Through variation and cognitive demand developed insight into the set of examples used to encapsulate the object of learning and explored the way that the examples themselves constrained the manner in which the object of learning was brought into focus.

The examples alone are a starting point in achieving the object of the lesson because mediating the object of the lesson was in turn dependent on the learners' responses as well as the nature of a teachers' explanation and discussion of the content. The explanatory talk and learner participation therefore provided cognizance of what a teacher does with the examples to maintain the focus of the lesson. Consequently there was a connection between the explanatory talk subdivisions, learner participation and the cognitive demand. This connection was the basis for the second research question as the explanatory talk and learner participation worked together to deliver and present the example/s. It was this connection that I explored and analysed to determine what factors in terms of the explanatory talk effected the cognitive demand of the example. The outcome of this connection will maintain, decline or increase the examples' capabilities in terms of the task cognitive demand.

Lastly the object of learning was the focus of the lesson and it is important to refer back and determine if this was achieved. That is why I have introduced one last connection between the variation and cognitive demand of the examples back to the object of learning. Variation engages with the generalisation of the content while the cognitive demand engages with the connections of the content and procedures, however both aspects mediate the object of learning. Thus the outcome of the example set in terms of achieving generality and maintaining (or not) the cognitive demand will determine if the object of learning has been achieved or if there has been a shift in the domain of the object of learning.

In summary, the conceptual framework of this study has been identified as an adapted version of the Adler and Ronda (2015) MDI framework. The adapted MDI frames the conceptual aspects of the project so that the two research questions can be addressed.

2.9 Conclusion

This chapter discussed the nature of examples by looking at the pedagogical role and use they have in the classroom as well as the different types of examples. Teaching strategies surrounding mathematical discourses were discussed and the concept of scaffolding was defined and explained. Challenges with maintaining purposeful scaffolding and its use in maintaining the cognitive demand of the chosen examples was brought to the reader's attention. In addition to challenges associated with discourse practices, this chapter discussed the challenges learners encounter in their early learning of algebra. In particular the errors that learners make when translating from ordinary English statements to algebraic expressions. Lastly this section presented the conceptual framework for this study with a detailed account of the object of learning, exemplification, explanatory talk and learner participation. The limitations of MDI were addressed and the MDI framework was complemented with the work of Stein (2000) and Moschovich (1999,2015) for the present study. The next section discusses the research design and methodology of the present study. It also engages with the limitations of the study and measures that were put in place to ensure that the study was ethically conducted.

CHAPTER 3

RESEARCH DESIGN AND METHODOLOGY

3.1 Introduction

This chapter discusses the methodology and methods of data collection used for this study and justifies these decisions. This section provides an outline of the research setting, sample and how the data is analysed and presented, including how the data was organised. In addition to this, I have also discussed the concepts of validity and reliability in terms of my study, as well as ethical considerations related to the study and its participants. Lastly this chapter briefly discusses the limitations with regards to the data collection that I encountered, and the methods used to address these limitations.

3.2 Methodology

This research project studied two qualified secondary teachers' choice and presentation of examples, and the impact that their discourse had in maintaining (or not) the intended cognitive demand. The research paradigm of this study is an interpretive one, and uses qualitative methods to answer the research questions. Thus this project does not aim to predict what the teachers will do, but rather it aims to describe and understand their actions and subsequently the consequences of their actions. The interpretive approach to this study is fitting as I recognise that the results of the study are created through the interpretation of the collected data (Bertram & Christiansen, 2014), and thus lends itself to interpretations informed by my conceptual framework (discussed in Chapter Two). An interpretive approach assumes that there may be multiple reconstructions or representations of the data (Bertram & Christiansen, 2014), and therefore these perspectives must be taken into consideration when drawing on conclusions. The notion of multiple perspectives can also be linked to the teachers' example set and choice when presenting examples of algebraic expressions to the learners, as well as the conscious and subconscious use of discourse when delivering these examples to the learners. The interpretive approach assisted me in describing the teachers' presentation of examples and their discourse practices used in delivering the lesson to the learners.

According to Halliday and Hasan (1985) learning takes place through a social process. In the school system the social context that promotes learning exists between teacher and learner as well as between the learners themselves. The nature of the research problem is exploratory, and seeks to understand how the participants in the study engage in a social context of learning, with regards to the examples presented and the discourses used in communicating the targeted mathematical areas. This ties in with the qualitative and interpretive research approach as it is "research that attempts to understand the meaning or nature of experiences" (Strauss & Corbin, 1990, p. 11). Therefore two case studies were used to describe and understand these factors and the decisions made by a teacher during the teaching of algebraic expressions. The decision to use case studies was motivated by its alignment with the interpretive paradigm as case studies focus on a real situation with the intention of providing a detailed account of the social setting (Opie, 2004).

3.3 Research Setting

The study was conducted at a high school in the Gauteng Province (RSA). The participating school has enrolled just under 1000 learners ranging from Grade 8 to Grade 12. The school has a good infrastructure with classrooms, computer rooms and a library. Learners enrolled in this school are expected to pay school fees and reside within the feeder areas of the school. There are however a number of learners who travel distances of up to 30km to attend this school. The learners come from a variety of home and culture backgrounds resulting in a rich and diverse school environment.

3.4 Sample Discussion

The sample consisted of two GET (Grades 7-9) mathematics teachers from the participating secondary school. The sample is small but purposeful, and from a qualitative and interpretive perspective the sample size is satisfactory to provide insight into the specific case of the targeted concept of interest. The sample size was able to generate quality data that aided in answering the research questions by providing opportunities for describing the teachers' presented examples and corresponding mathematical discourse.

I had established a good rapport with the school and the two teachers who participated in the study, as this is the school I currently work for. The two teachers were selected out of convenience sampling which is described by Bertram and Christiansen (2014) as choosing a sample which is practical and easy for the researcher to reach. At the time of the study I was also teaching, and therefore had difficulty in using random sampling or approaching teachers at other schools, purely based on a time constraint, and thus opted to use the resources available to me at my own work place. I did however, make specific choices in regards to which teachers I approached and invited to participate in the study. The two teachers that were selected from the participating school had the following criteria:

i) The mathematics teacher had to have a completed tertiary mathematics qualification (BEd or PGCE with mathematics as a major) to ensure that he or she was qualified for the profession, and had subsequently taught mathematics at a secondary level for at least three years.

ii) The mathematics teacher had to currently be teaching a GET mathematics class.

iii) The mathematics teacher was viewed as enthusiastic, willing to participate and had been recommended from the head of department.

iv) The mathematics teacher was viewed as competent as they had achieved a class average of +60% (and a pass average of > 90%) for the last two years.

To maintain consistency and to collect data that provided an opportunity for one particular aspect to be studied, described and understood (as case studies strive to do), each teacher chose one of their grade 8 classes for the study. All the grade 8 classes are of mixed mathematical ability, gender, race and age. In total two teachers and and their grade 8 classes participated in the study.

3.5 Methods of Data Collection

This study was dependent on the interactions that happened naturally between the teacher and the learners in the classroom environment. These interactions were in the form of classroom discussions and lesson presentation which were observed, video recorded, transcribed and analyzed so that the research questions may be addressed. The focus of this study was on the examples presented to the class and the classroom discussions, specifically the teacher's discourse used to explain and present the example with the intention of introducing algebraic expressions. The interpretive paradigm and case study approach is centred around the authenticity of the interactions and genuine viewpoint of the participants, and thus the methods used to collect the data must reflect these ideologies.

Cohen, Manion and Morrison (2005) recommend using a method to gather data that aligns with the research methodology and research questions. To maintain the integrity of the natural setting of the study and to ensure that detailed descriptions could be made, this study was dependent on audio-visual sources and data collection methods that focused on the presentation of examples and discourse of the mathematics teachers.

The method used was in the form of classroom observations with the assistance of audio-visual recordings of the classroom.

3.5.1 Observation and Video-Recordings

Cohen et al. (2005) discuss the traditional aspects of an observation, whereby the researcher has no involvement in the observed situation. They further mention that in a qualitative study the researcher seeks a dynamic view of the situation and to make connections from what is seen. It is for this purpose that I required information that needed to be seen and heard from the actual classroom lessons.

I however had one constraint, in that I was not able to be physically present at all of the lessons as, at times I was teaching my own classes. To tackle this challenge I looked for a method that would hold true to the traditional aspects of observation, in that the method had to in no way influence the lesson and decided on video recording the lessons. The video-camera was in essence filming and observing the lessons, and subsequently when I watched the recordings I was able to observe the lessons. The appeal of video-recorded data is that it gives the researcher access to "live data from live situations" (Cohen et al., 2005, p. 306). The participants in the study can be observed in their daily activities, allowing for a chance to uncover meaning and interpretation of the classroom setting while they are taking place. In particular the video-recordings were used to identify the build up of the examples that happened, thus assisting in describing the presentation and sequence of the examples. In addition to this, the video recordings were used to describe and understand the teacher's mathematical discourse.

The observations and audio-video recordings took place in two Grade 8 mathematics classes over a span of up to seven lessons, lasting thirty-five minutes each, during the first two weeks of August 2016. There were other considerations that I made when choosing to use observations and video-recordings as my data collection methods. As Bertram and Christiansen (2014) caution, the presence of an observer or video camera can be unsettling for the participants and may even have an influence their behaviour. To try and eliminate this disturbance and to put the teacher and the learners at ease, I introduced myself to the class and explained the procedure for the observed and recorded lessons. It was still of the utmost importance that my presence and mainly the presence of the video camera was minimal, so I made sure to set up the video camera at the back of the class. This was to make sure the video camera was away from all the learners and so that it would not interfere with their view of the board. After all considerations, I found that observations and video-recordings provided a full account of the situation and was valuable in making sense of and understanding non-verbal discourses such as gestures, facial expressions and representations.

3.5.2 Limitations experienced in data collection

During the lessons where I was unable to be present I set up the video camera prior to the lesson and set it to record until I was able to return and end the recording. Some challenges were met as a consequence of this. For instance, if the teacher moved around to check an individual's work, the camera was unable to move and focus on the teacher's movement around the class. Regardless of this limitation the recordings were of good quality, easily audible and the teacher and the board are visible throughout the lesson. Since the main focus of the study was on the examples and the teacher's discourse, this limitation did not present with major implications to the study.

I encountered one other limitation in conducting this study. This was not being able to collect the teachers' lesson plans. Initially I had planned on gaining access to the lesson preparation notes to isolate the examples that were going to be presented in the lesson. This would have been planned examples and I would have used these notes to gather data for my first research question. However, I found that both of the participating teachers relied on the prescribed textbook to source their examples and plan their lessons. I describe this as a limitation in that I could not prepare myself for the observation lessons with regards to the examples being used as the teacher would pick and choose examples out of the textbook during the lesson. Thus leading again to the importance of the video recordings as these provided me with the opportunity to play back the lesson and establish which examples had been used.

3.6 Transcriptions

The video-recordings of the lessons were made into transcriptions which are seen as convenient forms of representation and were used to check against bias or misinterpretation of information (Opie, 2004). The transcriptions provided me with access to a range of examples, board work, interactions and classroom discussions. To identify the two different teachers I refer to the first teacher, as Teacher A and the second as Teacher B. To preserve the anonymity of the teachers, I used the pronoun 'she' to refer to both teachers in the study. With all these considerations in place I found that transcriptions are not without constraints.

The structure of the video recordings was that the focus was on the teacher and not the learners thus the learners could not be identified by face. This presented a challenge when transcribing the video recordings as when the learners contributed to the discussion it was difficult to differentiate between them and to allocate pseudonyms. To overcome this challenge when transcribing each video recording of a lesson it was broken down into smaller pieces. Every new question was transcribed as one section so that I would be able to listen to the video recordings and identify the different voices of the learners who contributed to that particular question. To make this process easier, I had asked the teachers to use the name of the learner when engaging in a discussion or calling upon the learner to answer a question. This process assisted me in learning who the different learners were, and although the teachers did not always use the learners names, I was able to recognise the voices and when necessary approached the teacher for confirmation. The transcriptions were checked against the video recordings and any discrepancies were adjusted. The transcriptions from the video recordings allowed me to familiarise myself with the data which assisted me in selecting particular instances from a lessons that aligned with conceptual framework and analyse these particular situations.

3.7 Data Analysis

In Chapter Two, I described the conceptual framework that had been developed from Adler and Ronda's (2015) MDI framework and complemented by Stein's (2000) and Moschovich's (1999,2015) task analysis guide and mathematical discourse work respectively. In this section, I describe the analytical framework developed for analysis of data and the concepts that informed the framework. To begin the analysis, I look at the overall object of learning that spans over the entire analysis, which is translating word statements into algebraic expressions. Each of the lessons analyzed, however is informed by their own object of learning, which I will refer to as the lesson object of learning. Recognizing the object of learning is further discussed in Table 3.1.

As shown in Chapter Two (2.7), the object of learning is then mediated through three categories:

Exemplification - This category pays attention to both examples and tasks. With regards to analysing examples, this will take place in the form of variation and with regards to analysing tasks, this will be done from a cognitive demand perspective. Both examples and tasks need to be analysed over an example set as the accumulation of the examples and tasks are pertinent to the analysis.

Explanatory Talk- The sub-divisions naming, legitimations and scaffolding are analysed under the category of explanatory talk. The analysis focused on the way that examples and tasks are talked about and in turn described the mathematics made available to learn.

Learner Participation- The analysis of learner participation is focused on the opportunities provided for the learners to engage with the mathematical language. Both in terms of what they are invited to say by their teacher and their ability to reason mathematically. Learner to learner discussions are not included in the study and the focus remains on the teacher and the interactions that she has with the learners. At times the video camera recorded interactions with the teacher and an individual learner or small group of learners as she inspected their work.

These three categories of MDI and their associated sub-divisions provided the foundation of the analytical framework. As indicated in Chapter Two (2.8), the MDI framework had limitations in determining the level of tasks (cognitive demand), and limitations in determining the factors that contribute to the movement of the task level (from Application to Knowledge for example). As such, the MDI framework had limitations in terms of providing a clear and descriptive analytical tool for analysing the task level and limitations in describing the movement of tasks from one level to another. Hence the need for complementary frameworks and literature to fill the identified gaps. Stein's (2000) task analysis guide was incorporated into the exemplification category, as a subdivision called cognitive demand. While Moschovich (1999,2015) and Walshaw and Anthony (2009) was used to supplement the explanatory talk by providing an additional subdivision called scaffolding techniques. Thus the adapted and modified framework satisfied my purposes for analysing the collected data to investigate:

Teachers' use of exemplification and explanations in mediating the object of learning.

The analytical framework divided each category of MDI into subdivisions that represented the processes within each category. Codes are used for each category and the corresponding recognition rules are provided for each code. Additionally each category was coded into levels, which are again provided with a set of recognition rules. The categories and subdivisions were developed *a priori* by using the MDI framework and complementary literature mentioned above, however the connection between the exemplification and explanatory category with regards to the object of learning was developed *a posteriori* from working with the data itself. Below I present the analytical framework in terms of exemplification.

3.7.1 Coding for exemplification data

I began the analysis of exemplification by identifying and creating a list of examples used in each lesson. To do this, I watched the video recordings and identified each example that the teacher presented to the learners. I used the pre-planned examples and by pre-planned I mean the examples that the teacher sourced from the prescribed textbook. I did not consider the spontaneous arithmetic examples used by the teachers when they justified or explained a concept as this falls under the explanatory talk and scaffolding techniques. As a reminder the guiding question for exemplification was based on the first research question posed in the present study:

What types of examples are presented in a lesson during the teaching of algebraic expressions?

In the table below, I present the category of exemplification and its subdivisions from MDI, as well as the codes and recognition rules that informed the analysis. Each recognition rule has an example provided in *italics*.

<table>
<tr><th colspan="3">Division</th><th>Recognition rule</th></tr>
<tr><td colspan="3">Object of learning in the lesson</td><td>This is the content in terms of a procedure, concept, algorithm or activity that is the focus of the lesson. This was determined through looking at:
• What heading did the teacher give to the learners?
• What was the critical feature or content featured or discussed in the lesson?</td></tr>
<tr><th>Sub-division</th><th>Categories</th><th>Code</th><th>Recognition rule</th></tr>
<tr><td rowspan="6">Exemplification</td><td rowspan="3">Examples</td><td>Similarity (S)</td><td>Examples that demonstrate similarity have similar structure and promote the same concept.
'The product of a number and three' and 'three times a number' are both translated to x +3.</td></tr>
<tr><td>Contrast (C)</td><td>Contrasting examples compare one quality with another, thus highlighting a critical feature.
'The difference between 12 and a number' and '12 less than a number' are translated to 12 -x and x-12 respectively and highlight the significance of the order of terms.</td></tr>
<tr><td>Fusion (F)</td><td>Fusion examples vary more than one feature of the concept simultaneously
The sum 'Three times the amount of 12 less than a number' will be 3(x-12)</td></tr>
<tr><td rowspan="3">Tasks-before implementation</td><td>Knowledge (K)</td><td>Knowledge tasks resonate with memorisation and procedures without connection and can be recognized through the following:
• Reproduce previously learned facts, formula, rules or definitions.
• No ambiguity as the pathway is specifically given.
• Following a routine procedure
• The Focus on the correct answer only
The sum of a number and 5 is represented as x + 5</td></tr>
<tr><td>Application (A)</td><td>Application tasks resonate with procedures with connection and can be recognized through:
• Multiple representations
• Procedures that can be followed but with intention and understanding as the pathways are only suggested.
In the sum 'Three less than a number' the order of the terms are important and requires the learners to understand the difference between x-3 and 3-x.</td></tr>
<tr><td>Complex/ Problem Solving (P/CS)</td><td>Complex/Problem solving tasks resonate with doing mathematics and are recognized based on the following:
• There is no suggested or straightforward pathway, learners must make their own decisions.
• The sum is non-routine and requires complex thinking.
• Requires multiple concepts, procedures and relationships to understand and complete.
John is x years old. His brother Peter is double his age. Determine the sum of their combined ages in y years time.
This sum results in an algebraic expression of (x+2x) +2y or the simplest form 3x +2y</td></tr>
</table>

Table 3.1: Coding scheme and recognition rules for exemplification.

Sub-division	Categories	Coded levels	Recognition rule
Exemplification	Examples	Level 0	Simultaneous variation in the example set without a prior build up of similar and contrasting examples. • Level 0 indicates an example set that does not show awareness of variation, as sequencing of examples may be random and use fusion examples without considering similar and contrasting features first.
		Level 1	There is only one form of variation in the example set. • Examples that promote similarity OR contrast.
		Level 2	In the example set there is variation of more than one feature of the concept • An example set that demonstrates both similarity and contrasting questions/features.
		Level 3	The example set leads to simultaneous variation of more than one feature. • Level 3 differs from level 0 in that the fusion examples used were first built up from similarity and contrast examples, thus indicating that sequencing is important.
	Tasks	Level 1	An example set with tasks that rely on knowledge tasks only.
		Level 2	An example set that incorporates knowledge and some application tasks.
		Level 3	An example set that includes knowledge, application and complex/problem solving tasks.

Table 3.2: Coded Levels for exemplification

Below I provide an example of coding for exemplification in terms of both categories: examples and tasks.

Sum	Expression	Variation	Cognitive demand
The sum of a number and 5.	$x+5$	S	K
3 less than a number	$x-3$	C	A

In terms of variation, the above recognition rules determine that using an example of addition, as well as an example of subtraction demonstrates both similarity (S) and contrast (C). Similarity in the structure of having two terms operated on and contrast in terms of comparing one quality to another, in this instance addition and subtraction.

Using the above recognition rules for cognitive demand *the sum of x +5* is a known procedure or operation (K) because:

- The procedure of addition required in the sum is specifically stated and is based on similar tasks from arithmetic examples.
- The tasks within the examples are straight forward and explicitly stated, therefore minimizing possible ambiguities.
- The answer can be found by following a routine procedure.

While on the other hand the example of 3 less than a number results in x -3 and is coded as an application (A) example because instead of following a routine procedure, the learners must engage with the question and its suggested pathway to complete the question. In particular the order of the terms becomes important and cannot be completed as the sum was given.

In terms of coding examples and tasks as a level, I determined the accumulated examples and tasks throughout a lesson and made a decision based on the full example set used. This is because the sequence of the presented examples is important in determining the level of variation and the overall example set's cognitive demand must be accounted for to determine the level of tasks.

3.7.2 Coding for explanatory talk and learner participation data.

The second level of analysis explored the teachers' mathematical discourse through the categories of explanatory talk and learner participation. The guiding question for this level of the analysis was the second research question of the study:

How does the teachers' discourse impact the intended cognitive level of the presented examples?

The subdivisions naming, legitimations and scaffolding techniques, as well as the learner participation was used to investigate the relationship between the exemplification (first level analysis) and the explanatory talk. In particular the goal was to investigate the impact that the explanatory talk and learner participation had on the exemplification component. The intention was to identify relationships between the mathematical discourse emerging from the explanatory talk and the predetermined cognitive demand of the examples used (from exemplification). A key step at this stage of the analysis is described by Hatch (2002) as finding pieces of the data that both support and challenge the findings or relationships that emerged from the interpretation. Therefore extracts where an example and its corresponding explanatory talk created opportunities for maintaining, declining or improving the examples' cognitive demand were chosen**.** These opportunities were determined by the scaffolding techniques and corresponding naming and legitimations which can be viewed in Table 3.3.

Below I have included a detailed characterization of the category explanatory talk from MDI that I used to analyse these extracts.

		Coded levels	Recognition rule
Explanatory Talk	Naming	Colloquial (Non-Mathematical)	The teacher and learners engage with non-mathematical, everyday words and phrases to describe the mathematical concept. *A teacher may refer to the algebraic expression as an ambiguous pronoun such as this, thing or it. For instance we add this and that together when discussing an example such as x+10.*
		String of words (Ms)	Formal mathematical words and phrases are used to identify an object but no meaning is created or connected to the mathematical concept. *In asking the class why 12 less than a number cannot be written as x-12, a learner may respond by saying because the question says 12 less than. Although the language is formal mathematics it is read as a string of words and not as a means of explaining the concept.*
		Formal mathematics (Ma)	Formal language is used to create meaning and understanding of the object or mathematical concept. *The teacher discusses that the number 5 and a variable added together make an algebraic expression.*
	Legitimations	Non-mathematical	The statement made about the mathematics is mnemonic/ visual and does not infer mathematical reasoning. *A teacher may refer to the rainbow method when talking about distribution.*
		Positional (P)	This is when a teacher uses their authority and makes a statement to be taken as true without justification or mathematical reasoning. *A teacher may instruct learners to always follow the order of the sum as it is given to them.*
		Local (L)	The statement made is specific to one example or task. *Replace the word variable with the letter x. This may work in a particular case, where at other times the question might ask for another letter to be used.*
		Generality	The statement about the concept or procedure can be applied across numerous or all similar examples and was reasoned mathematically. This can be done partially or fully depending on the scope of the example. *A teacher might say that adding a constant and a variable, will result in an the expression of two separate terms.*
	Scaffolding	Decline	Techniques/mathematical discourses that result in lowering task demand demonstrate the following: *Path Smoothing, routine answer not giving enough time or the choice of an inappropriate task. Teacher dominant lesson and completes the example for the learners. Initiate-response-evaluate lesson.*
		Maintain	Techniques/mathematical discourses that have the potential to maintain task demand demonstrate: *Repeating and Re-voicing, prompting justifications and explanations or building on from a learner's response Initiate question, response from learner, prompt learner for more information, response and then finally feedback from the teacher.*
		Increase	Techniques/mathematical discourses that have the potential to increase task demand demonstrate: *Discussions that are thought provoking, such as linking the commutative property to the addition of an algebraic expression but not to the subtraction of an algebraic expression.*

Table 3.3: Coding scheme and recognition rules for Explanatory Talk

Sub-division	Categories	Coded levels	Recognition question
Explanatory Talk	Naming	Level 1	The lesson is coded as having dominantly colloquial and non-mathematical talk.
		Level 2	In a lesson there is movement between non-mathematical everyday talk and formal mathematics talk (as a string of labels). From the lesson, an observer can see that there is a progression in the mathematical talk.
		Level 3	There is movement between colloquial talk and formal mathematics talk that was used appropriately. *For instance a teacher may refer to a sum initially as diving the letter by 2 and then progressing to saying the algebraic expression represents division by halving the variable* ($\frac{x}{2}$).
	Legitimations	Level 0	The criteria stated to the learners in a lesson is predominantly non-mathematical and provided by the authority as a given statement. *A teacher might constantly refer back and rely on a non-mathematical rule given to the learners. For instance, always do the sum in the order given to you.*
		Level 1	Criteria stated to the learners gives specific and localised examples and justifications. *A teacher may constantly refer to each sum separately and provide justification based on what happened in that sum.*
		Level 2	There is an attempt at achieving generality but it is partial generality *In a sum given as a 'two more than a number', a teacher may offer two solutions,* $x+2$ or $2+x$ *but have a limited discussion on why this is possible.*
		Level 3	Full generality is achieved as the mathematical concept or procedure is derived or proved to the learners. *A teacher may discuss that 'any natural even number' can be represented by the algebraic expression of 2n.*

Table 3.4: Coding levels for Naming and Legitimations

Learner Participation	Code and levels	Recognition rule
	Y/N Level 1	The learners are invited or only given opportunities to respond questions in a yes/no format or fill in the blank left by a teacher. *The teacher says the 5 and x are...*and learners respond with *added. Or perhaps a teacher states "the answer is x+5, do you all agree?"* and the learners in unison say *yes*.
	Phrases (P) Level 2	Learners provide sentences and give their own idea. *The learner presents their answer: "Maam, the two numbers added together are 5+x".*
	Discussion (D) Level 3	Learners are engaged in a discussion with the class/ teacher and are prompted for more information. *Learner "The sum of a number and 5 is x+5"* *Teacher "Why did you decide to write the answer as x+5 and not 5x.* *Learner 'They do not join together"* *Teacher " What the learner means is because the terms are unlike they remain as two separate terms"*

Table 3.5: Codes and levels for Learner Participation

Together the components that make up the conceptual framework (discussed in chapter 2) and the data analysis procedure creates a whole coherent picture of the processes involved in investigating teachers' use of exemplification and explanations in mediating the object of learning. This process is explained below and visually represented as the global analytical framework in figure 3.1.

In total three lessons of Teacher A were transcribed and analysed and four lessons of Teacher B. Each lesson spanned a total of 35 minutes, and lesson 2 and 3 of Teacher B was a double lesson thus 70 minutes long. The overall object of learning was to translate word statements to algebraic expressions, however each lesson has its own object of learning that is first identified through the recognition rules stated above. Once the object of the lesson has been established the first level of analysis (depicted in the diagram below in orange) begins through the identification of exemplification. Each lesson of Teacher A and Teacher B was analysed in terms of the examples and their associated tasks through variation and cognitive demand. A summative judgment was made on the overall set of examples and tasks used throughout the lessons. Once this had been completed, the analysis moved onto the second level.

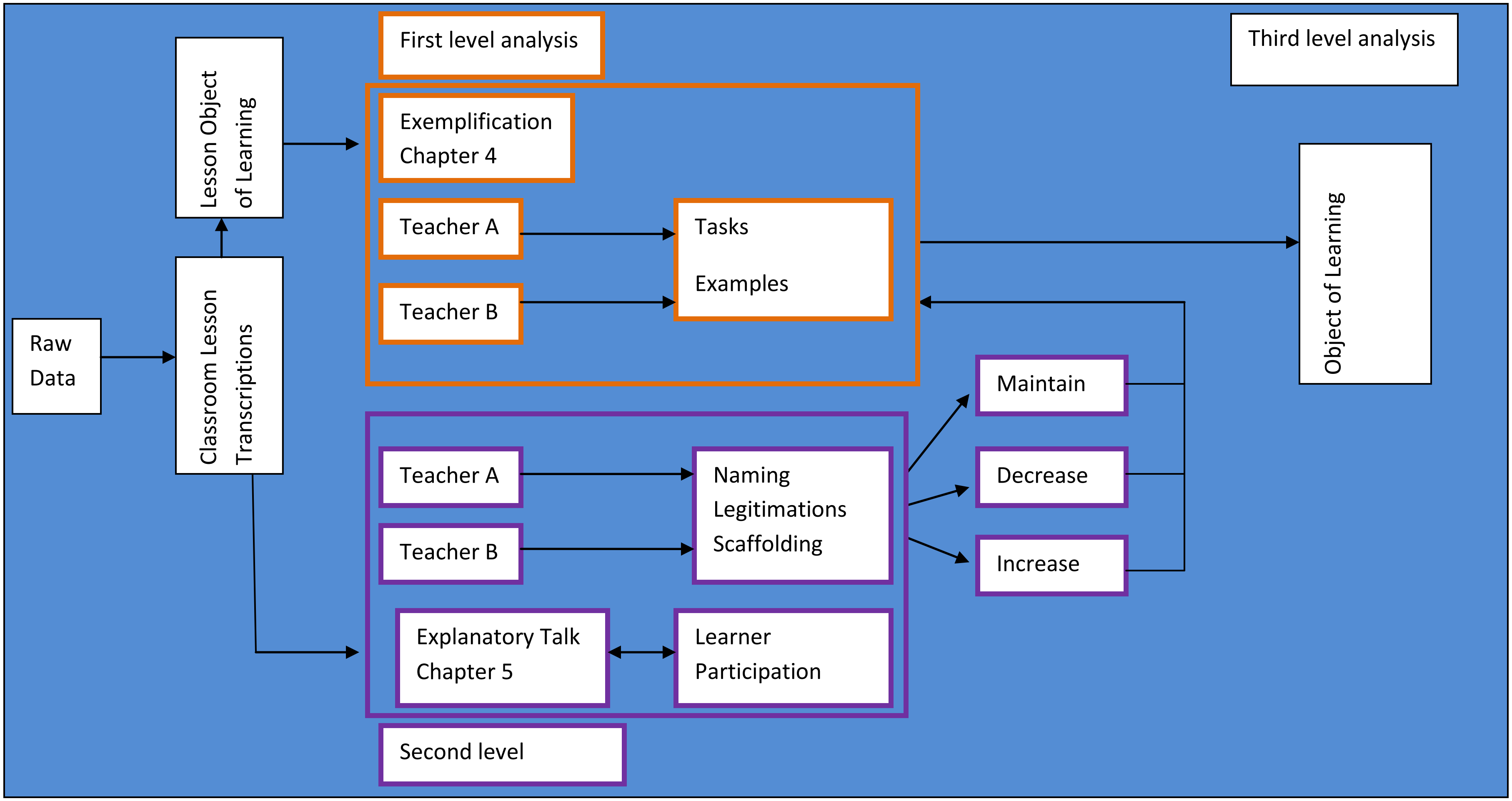


Figure 3.1: The global/overall analytical framework of the study

The second level of the analysis (depicted in purple) again used the transcribed lessons but focused on specific transcripts. The chosen transcripts were motivated by their ability to demonstrate explanatory talk that potentially maintained, decreased or increased the task demand, and therefore it became essential to analyse the teacher's use of naming, legitimations and scaffolding. In conjunction, the learners participation was used to explore the adjustments (if any) that Teacher A or Teacher B made on their explanations. Extracts were used from both teachers to provide clear arguments for factors that in any way impacted task demand. The interplay between exemplification and explanatory talk manifests in the connections between the naming, legitimations and scaffolding techniques and the examples and tasks. The relationship between the exemplification category and explanatory talk category create the final connection back to the object of learning. The process in its entirety analysed teachers' use of exemplification and explanations in mediating the object of learning.

Thus the study was guided by a systematic method for finding typologies (drawn from literature) and generating interpretations (drawn from emerging patterns). Due to the interpretation factor of the analysis it is of the upmost importance that the study is credible and that maximum effort went into maintaining valid and reliable results.

3.8 Validity and Reliability

Throughout this research study, the concepts of validity and reliability have been considered to ensure that the associated risks are reduced. For qualitative research it is suggested by Cohen, Manion and Morrison (2000) that validity centers around the depth and richness of the data and the objectivity of the person conducting the research. The emphasis of the study was to conduct the research in a natural setting to develop an understanding in a given context. The results were therefore created from the interpretation of the data and thus trustworthiness was of the utmost importance. Bertram and Christiansen (2014) suggest that trustworthiness is a better indicator of validity for qualitative data as there are no measurements. Therefore the credibility of qualitative research is strengthened by detailed descriptions of the data and by the conformability of the findings. Although I will incorporate trustworthiness in the discussion below, I still find it necessary to talk about validity and turn to Maxwell (1992) for assistance.

3.8.1 Validity in data collection

This type of validity focuses on the factual accuracy of the data collected and used (Maxwell, 1992). The recording of the data was done with a video camera that recorded visual and audio recordings. This meant that the transcripts made from the video recordings were done verbatim and were more accurate than notes taken purely from the observations. I felt that my own field notes and observations could be viewed against the recordings and checked for prejudice, ambiguity and any potential bias that I brought with. This precaution was spurred on by Ten-Have (1990) who cautioned that data observed by the researcher may be seen to produce a product influenced by the researcher's preconceived ideas of what is important and suitable for the study. In this way, having observational notes and video recordings allowed for a more trustworthy data collection.

The video recordings and transcripts create a trustworthy data collection, but to maintain validity in the transcript itself requires a discussion on descriptive validity. Descriptive validity ensures that a researcher maintains the integrity of what they heard or saw and that the information is not distorted, exaggerated or biased. Therefore to maintain descriptive validity I had to ensure that I transcribed as accurately as possible, using the exact words and structure of the participants.

Once the transcriptions had been accurately completed, the findings of the study were based on the interpretations of the data and the patterns that emerged. Interpretive validity is concerned with the events that take place and the meanings derived from these events. Maxwell (1992) stresses that interpretive validity is constructed from the perspective of the participants in the study and must accurately describe the individuals actions as they were presented. To do this I engaged with the transcriptions with the intention to interpret the meanings of the interactions and discourses that took place. My efforts to maintain interpretive validity in the data analysis is described in the next section.

3.8.2 Validity in data analysis

Interpretations rely heavily on the observations made by the researcher in response to the actions of the participant/s. To maintain validity in my interpretations I had to ensure that the meanings I established from the transcripts were based on the conceptual and analytical frameworks and were easily identified from the recognition rules and not solely on my perspective (Maxwell, 1992). This was done to avoid judgmental, subjective or prejudice from the researcher. The literature review, MDI and the adaptations made aided my decisions in the data analysis and analytical framework.

From the above discussion, it is apparent that descriptive validity was maintained in the data collection and that interpretive validity was strived for by referring to the literature and conceptual framework in the data analysis and developing clear and coherent recognition rules.

3.8.3 Reliability

Traditionally reliability refers to the possibility of replication of the study and findings, and is often thought of with quantitative studies. However Cohen et al. (2000) suggest that qualitative studies should strive to achieve the same level of replication. To achieve this I considered the data that I used and the processes that were involved in collecting the data. The video recordings enabled accurate transcriptions to be made, and if the study were to be replicated the transcriptions would remain the same. Therefore I have maintained stable transcripts and achieved reliability in terms of collecting data. With regards to maintaining reliability in the analysis of the data, one must consider that the natural setting creates a unique setting, where the intention is not to generalize the findings but rather to develop an in-depth description and understanding of the unique context. Thus in terms of the analysis itself Bertram and Christiansen (2014) state that the interpretive approach does not aim to replicate findings. This study achieves reliability in terms of the data collected, the transcripts made and through the description of how the analysis will be performed.

3.9 Ethical considerations

The first step in this process was the application for permission to undertake the research by the University of the Witwatersrand as well as the Gauteng Department of Education (GDE). Permission was granted from both institutions and I was able to approach the school principal and teachers to invite them to take part in the study. This process took place before the data collection could begin, and careful consideration went into the discussion with the participants to ensure all information was clearly explained. The purpose and aim of the study was discussed with the participants (including the learners of the two grade 8 mathematics classes) and all information was included in the written information letter that was handed out to the participants. Permission was sought from the principal of the school, the mathematics teachers and learners. In addition to this, letters of consent were sent to the learners' parents as the learners are all minors. In brief the information letter discussed:

i. An introduction to the study as well as an introduction to the researcher.
ii. The purpose of the research and the role that each participant would play in the study.
iii. What data would be collected and how the data would be collected. This included the time frame in which the data would be collected.
iv. The participants choice in participation and their right to be excused from the study at any point.

For the full information and consent letters see Appendix D.

3.10 Conclusion

This chapter outlined the research paradigm, data collection methods and conditions for the study. The processes involved in conducting the study were explained and described. I provided justifications for the choice of methods and the limitations involved with these methods were addressed. Issues surrounding validity and reliability of data collection and data analysis were brought to the reader's attention and a plan was discussed to minimize these risks. Lastly all ethical considerations were discussed and accounted for. The next chapter presents the analysis and findings of the exemplification component of the study.

CHAPTER 4

ANALYSIS AND FINDINGS FROM THE EXEMPLIFICATION COMPONENT OF MDI

4.1 Introduction

This study set out to investigate teachers' use of exemplification and explanations in mediating the object of learning. In this chapter, which primarily focuses on the exemplification component, I engage with the examples and tasks used in Teacher A and Teacher B's lessons. To do this I first describe the examples and tasks used by Teacher A across all three of her lessons and then move onto doing the same thing for Teacher B. Although this approach is a lesson by lesson analysis, the three lessons for Teacher A are progressions of each other and act as one continuous lesson in the overall spectrum of achieving the object of learning, the same can be said for Teacher B's lessons. The benefit of doing this separate analysis though, was that I was able to determine what each teacher made available to learn in terms of the selection and sequencing of examples. I then merged the findings from both teachers to form an overall collaborative finding on what examples the teachers used in teaching algebraic expressions.

4.2 Teacher A's use of exemplification

Teacher A used the prescribed textbook as her primary source of examples. Together with the textbook exercises and board work, I was able to determine each lesson's object of learning.

Lesson 1 - Teacher A used textbook exercise 6.3 (see Table 4.1 below) as her starting point for introducing word statements. The key focus of exercise 6.3 is to provide examples of word statements with only one mathematical operation. Thus the object of learning in Lesson 1 was to translate word statements with one mathematical operation and one variable into an algebraic expression.

Lesson 2- This lesson was a continuation of the first lesson, where the focus was on more complex examples from another but different textbook exercise 6.6. Thus the object of learning was to translate word problems with various mathematical operations and numerous variables.

Lesson 3 - This lesson focused predominantly on marking questions 5-10 from exercise 6.6 and thus establishing the object of the lesson as the content and examples from the remainder of exercise 6.6. This lesson concluded the work done on the overall object of learning: translating word statements into algebraic expressions.

Below I present the analysis of Teacher A's three lessons. The analysis for each lesson starts with a summary table of the examples used, as well as the codes as discussed in section 3.7.

Categories and codes	Examples	Tasks	Comments
Lesson 1: Textbook Exercise 6.3 **Examples:** Similarity and Contrast **Example level:** Level 2 **Tasks:** Knowledge and Application **Task level:** Level 2	1) The sum of a number and 5.	K	Teacher A presented each example (1-18) one at a time and asked the learners to present their answers for each question. The task (instruction given) for each question is to write an algebraic expression.
	2) The difference between 12 and a number.	A - procedure must be connected to the concept	
	3) Multiply a number by 6.	K	
	4) The product of 3 and a number.	K	
	5) Three times as many as a number.	K	
	6) Double a number.	K	
	7) Halve a number	K	
	8) Triple a number.	K	
	9) One more than a number.	K	
	10) 3 less than a number.	A	
	11) Multiply a number by 5.	K	
	12) Divide a number by 2.	K	
	13) Square a number.	K	
	14) Square root of a number.	K	
	15) Subtract a number from 8.	A	
	16) Add a number and 4.	K	
	17) 3 greater than a number.	K	
	18) 7 less than a number.	A	

Table 4.1: Examples and Tasks for Teacher A's first lesson

With respect to Lesson 1, exemplification was consistent in terms of the structure of the examples. In each of the 18 examples there was one mathematical operation acting on a variable at a time, and thus this approach observes generality of structure through similarity (S). The 18 examples, did however, differ in terms of the instructional language and the mathematical operation used in the sum itself. The mathematical operations are thus contrasted against each other, and consequently contrast (C) was used to differentiate between the different instructional languages and mathematical operations. Accordingly the use of similarity and contrast in the example set led me to judge the set of examples used in lesson one as a potential level 2, that is the simultaneous use of two variations. This means with respect to the object of learning - translating word statements into algebraic expressions- the set of examples has the potential to create generality from using both similarity and contrast.

The set of examples, however did not gradually introduce the varying features (instructional language). Although it is important for learners to be exposed to all four mathematical operations, Watson and Mason (2015) caution that if the examples are used randomly, then instead of experiencing generalisation learners may end up in a routine of drill and practice. The following table shows the scattered order of the sums in terms of their mathematical operation:

Question	Operation
1 ; 9 ; 16 ; 17	Addition
2 ; 10 ; 15 ; 18	Subtraction
3 ; 4 ; 5 ; 6 ; 8 ; 11	Multiplication
7 ; 12	Division
13 ; 14	Squaring and square rooting

Table 4.2: Grouping the sums into their mathematical operation

The ordering of the examples should have been informed by a particular purpose, or goal of the lesson. In this lesson, it would have been beneficial to base the purpose of the lesson on the different mathematical operations, or perhaps the position of the variable. In this way, a teacher would first show examples of addition but they would vary the instructional language. This would assist the learners in building up to generality of algebraic expressions involved with addition. Then this process can be repeated for subtraction, multiplication, division and squaring.

Having a thorough grasp of algebraic expressions with addition and then contrasting these examples with other mathematical operations provides a better means for generality.

The use of the textbook exercise as Teacher A's source of examples demonstrates reliance on the textbook and the examples and exercises within it. Ultimately in the first lesson the types of examples used was determined by the exercise itself, and even though the examples themselves promoted generality (being a potential level 2) the unsystematic order draws away from achieving this generality. Therefore to conclude the first lesson's analysis of examples, even when using a textbook, Teacher A needs to select and sequence examples according to a set of principles or in alignment with the purpose of the lesson.

In terms of tasks, the set of examples in lesson one involved the same task, which was to translate a word statement into an algebraic expression. Task demand however, is dependent on the connections between the procedure/s and the concept. The questions involved with addition, multiplication, division and squaring follow a routine procedure as the question itself implies the pathway for the learners to follow, and were coded as Knowledge (K). However, in terms of subtraction, learners cannot follow a routine procedure, they must engage with the question and its suggested pathway to complete the question. The order of the variable and constant is significant and the correct order must be established so that BODMAS is maintained. In this instance, ' *the difference between 12 and a number'* and *'3 less than a number*' demonstrate different task demand. *3 less than a number* cannot be written in the order presented, but rather as $x - 3$. Both MacGregor and Stacey (1997) and McNeil et al. (2010) suggest that some of the learners misconception is introduced through inappropriate teacher explanations. The above mentioned difficulty often stems from teachers explaining to learners that they must translate the sum word by word and this would result in an answer of 3 - *x*. This is further discussed in section 5.3.1 where I show Teacher A demonstrate and explain example 10 which was *'3 less than a number*'.

These tasks provided learners with different ways of engaging with the algebraic expressions. The combination of knowledge and some application tasks, results in the tasks being judged as a level 2. The potential at level 2 is for the tasks to engage with opportunities for application, this however is determined by the enactment and implementation of the tasks during the lesson and in conjunction with the explanatory talk.

Overall the exercise used (6.3) was an introduction to algebraic expressions by exposing learners to word statements that extended their previously known and established arithmetic skills to a generalised state. This example set before implementation was coded as a level 2 in terms of examples and level 2 in terms of task capability. The next section looks at the examples and tasks presented in the second lesson.

Categories and codes	Examples	Tasks	Comments
Lesson 2: Textbook Exercise 6.6 **Examples:** Fusion **Example level:** Level 3 **Tasks:** Knowledge and Application. **Task level:** Level 2	1) Lerato is *p* years old. Her sister is twice her age. a) How old is her sister? b) How old will Lerato be in 10 years? c) How old was her sister three years ago? d) What will their combined ages be in *q* years time? 2) What is the next natural number after the natural number t? 3) What is the sum of three consecutive natural numbers, with t being the smallest?	 K K K A- pathway suggested but connections between the procedures is needed. A - No pathway is suggested and previous knowledge required. A	Teacher A presented each example (1-4) one at a time and asked the learners to present their answers for each question. The task (instruction given) for each question is to write an algebraic expression based on the given information. Task demand differed for each sum as the connections between the procedures and concepts increased.

Table 4.3: Examples and tasks of Teacher A's lesson 2

Lesson 2 was a progression from Lesson 1, in that the examples used in Lesson 2 were a build up of the similar and contrasting examples used previously. The examples in Lesson 2 demonstrated more than one aspect varying together and thus presented the learners with an opportunity to experience fusion examples. The summative judgement of the two lessons in terms of the examples thus far was a level 3 and was coded as such due to its connection with the similar and contrasting examples from the first lesson. Although each example in the second lesson showed simultaneous variation, the examples themselves were random and once again the questions were from the learners prescribed textbook. There does however seem to be a link between the 2nd and 3rd example, both in the language (natural number) and mathematical operation (addition). Yet this connection was made purely by Teacher A's decision to follow the textbook and not by her own awareness of the similarity these two examples expressed.

Although the main task of each example was to write an algebraic example, each subsection of the examples involved different tasks in order to reach that goal. The examples in the second lesson became more complex in terms of determining the algebraic expression that represented the corresponding word statement. Additionally recognising and applying the mathematical operation became more abstract and demanding. Example 1 had knowledge and application tasks within it. 1a, 1b and 1c are coded as knowledge as it resonated with the examples from the first lesson. 1d however relied on more connections with the procedure and the concept and was thus coded accordingly as A. Examples 2 and 3 focused on prior knowledge; natural numbers and consecutive numbers, and incorporated these important ideas with the newly introduced algebraic expressions. These tasks engage the learners with a high cognitive demand and because of the example set's high task demand it was coded as level 2.

Questions 5-10 from the same exercise (6.6) were given to the learners as a homework activity and marked in the third lesson. The process was simple, Teacher A read out the question and called on numerous learners to give their answers. Very little discussion happened in this part of the lesson, until no answers are provided for questions 8, 9 and 10 and Teacher A decided to do the corrections for these on the board. For a full description of exercise 6.6 and its list of examples, see Appendix A. Question 8,9 and 10 were set at a high cognitive demand, due to their complexity and reliance on procedures with understanding. Therefore the task level for the third lesson was coded as a level 3. The examples variation remained at a level 3 as examples 8-10 were a build up from the first two lessons.

Looking at the examples and tasks from all three lessons, we see that Teacher A's sequencing in Lesson 1 was unstructured and followed the order as found in the textbook. This again happens in Lesson 2 and 3 where the textbook exercise was followed without careful consideration of the sequencing of examples. The cautionary warning from following a set of textbook exercises is the chance of variation happening very quickly and almost randomly. Thus teachers using a textbook must first select and sequence examples based on a particular goal. Although Teacher A did not do this, and demonstrated a lack of awareness of variation and its benefit in achieving generality, her choice of presenting exercise 6.3 first and then exercise 6.6 showed awareness of increasing complexity. Complexity particularly in terms of task demand, and subsequently cognitive demand. Overall possibilities for generalisation were constrained due to the random sequencing of examples but learners were presented with opportunities to engage with thought provoking and challenging questions in exercise 6.6, which required their previous knowledge from exercise 6.3. Overall examples and tasks were coded as a potential level 3.

4.3 Teacher B's use of exemplification

Teacher B incorporated some of her own examples and activities into her introduction of algebraic expressions but her primary source of examples was the prescribed textbook. Together with the textbook exercises and board work, I was able to determine each lesson's object of learning.

Lesson 1 - Teacher B used exercise 6.3 and exercise 6.4 as a self discovery for her learners. The objective was to use the two exercises, specifically their instructional words to create a table that incorporated the mathematical language for each operation. Thus the object of lesson 1 was to create a table that summarised the four mathematical operations with their corresponding instructional terms.

Lesson 2 and 3- This double lesson was a continuation of the first lesson, where the table of operations was put into practice and examples from exercise 6.6 were completed. This is the same exercise that Teacher A used. Thus the object of learning was to translate word problems with various mathematical operations and numerous variables.

Lesson 4 - This lesson focused predominantly on marking questions 6-10 from exercise 6.6 and thus establishing the object of the lesson as the remaining content from this exercise. This lesson concluded the work done on the overall object of learning: translating word statements into algebraic expressions.

Categories and codes	Examples	Tasks	Comments
Lesson 1: Table of operations **Examples:** Similarity and Contrast **Example level:** Level 2 **Tasks:** Knowledge **Task level:** Level 1	On the board Teacher B wrote the following: Plus - sum - increase - add - more This was her example of how the learners must set out and complete the table.	K- recall facts from previous knowledge. Pathway was suggested directly by Teacher B	The instruction from Teacher B at the start of this activity to the learners was: "I want you to write a summary of operations. After you've done this exercise. Operations means all the signs, plus, minus , divide etc. For example, for plus. You are going to do this in table form, please. For plus you are going to write, it is also known as sum. It is also known as increase. It is also known as add, as well as more and addition. Ok then you do that for plus and then you go onto the next operation, which is minus."

Table 4.4: Lesson 1-Teacher B's summary table

Teacher B's first lesson required the learners to create a summary table of the different mathematical operations, made up with the mathematical words that imply the operation. The example provided to the learners by Teacher B, was addition. In this way, the example set promoted the learners to identify the regularities of addition in terms of mathematical language and this might have assisted the learners in constructing generalizations (Marton and Tsui, 2004). Generalizations occur through noticing the similar aspects which was achieved by discussing each operation separately. The activity can only be fully grasped when the language that suggests addition is compared to the language that implies subtraction, multiplication and division. Thus by Teacher B introducing an activity in which the mathematical operations were compared to each other, she actually introduced contrast. It was for this reason that the examples were coded at a level 2 in terms of the potential to achieve generality of the mathematical language and operations. In terms of the task demand (cognitive demand), representing addition to its full extent required learners to call on their previous knowledge and find the numerous words that corresponded to addition. This was the same for all the mathematical operations. This activity relied on the learners reproducing previously learned facts, definitions and words. For example, a learner may be familiar with the term quotient and they may have memorised that quotient implies division has taken place. Knowing that quotient and division is linked does not however imply that a learner is competent in division itself. As a result of the low cognitive demand, the task level was coded at a level 1. Overall Teacher B's first lesson was coded as having the potential to present examples at level 2 and cognitive demand at a level 1. Below I begin the analysis and discussion for Teacher B's second and third lesson (double lesson).

Categories and codes	Examples	Tasks	Comments
Lesson 2 and 3: Exercise 6.6 **Examples:** Fusion without a build up from Similarity and Contrast **Example level:** Level 0 **Tasks:** Knowledge, application and complex/problem solving. **Task level:** Level 3	1) Lerato is *p* years old. Her sister is twice her age. a) How old is her sister? b) How old will Lerato be in 10 years? c) How old was her sister three years ago? d) What will their combined ages be in *q* years time? 2) Nigella sells a cupcake that cost her *x* cents at a profit for *y* cents. What is her profit? 3) What is the next natural number after the natural number t? 4) What is the sum of three consecutive natural numbers, t being the smallest? 5) Write down an expression for: a) Any real number. b) Any odd number.	 K K K A A A A C/PS C/PS	This was essentially a double lesson (two 35 minute lessons in a row) and I have coded the two lessons together as it provides a summative judgement of the examples and tasks used. This was the same exercise that Teacher A used, however Teacher B continued up until question 5 with her learners.

<u>Table 4.5: Teacher B's lesson 2 and 3 -exercise 6.6</u>

Although Teacher B used the same exercise (6.6) as Teacher A, the build up to the example set was different. The rapid progression from Lesson 1 in which the mathematical operations and their instructional language was generalised, and the examples relatively simple (low cognitive demand) to the complex examples and tasks seen in Lesson 2. The examples in lesson Two showed multiple aspects varying simultaneously and learners were expected to translate the words into an algebraic expression. A task in which they had limited exposure and practice in. As a result of Teacher B's sequencing of examples, the learners had a limited or constrained opportunity to develop generality with respect to translating word statements into algebraic expressions. Consequently I have coded the set of examples as a level 0, in which there are opportunities of seeing fusion examples but without developing the learners' abilities through similar and contrasting examples first.

The tasks involved in exercise 6.6 were discussed above in the discussion on Teacher A's second lesson and coded as a potential level 3, however I would like to draw attention to the difficulties learners may have with a task set at level 3. Tasks with a high cognitive demand engage with learners in thought provoking and challenging ways. Often with the intention of creating opportunities for learners to work independently and develop their own reasoning skills (Stein, 2000). My concern with the set of examples and tasks used in Teacher B's second lesson is the quick progression from simple tasks (1a and b) and the tasks that require more connections with the procedures (1c,d etc). My concern centres around the idea that examples that demonstrate simultaneous variation without building up from previous example sets, result in a quick fixes and strategies to help learners attain the correct answer. It will therefore be interesting to see the development of the examples and tasks during the implementation stage of the lesson, especially with the combination of level 0 in terms of example variation and sequencing and level 3 in task demand, as Teacher B demonstrates examples of Knowledge, Application and Complex/Problem Solving abilities.

At the end of the third lesson, Teacher B requested that the learners complete the exercise (6.6) for homework and it is important to note that Teacher B spent the next lesson (lesson 4) discussing the answers to questions 6-10. For a full description of exercise 6.6 and its corresponding set of examples, see Appendix A. Since the fourth lesson was a continuation of the previous lesson and no new work was covered, the variation remained at level 0 and task demand remained at level 3.

4.4 Findings and summary of Exemplification

Teacher A and Teacher B predominantly used the prescribed textbook to source their examples. The selecting and sequencing of the examples from the textbook differed depending on the teacher. Teacher A began her lesson on algebraic expressions by using exercise 6.3, while Teacher B incorporated her own activity, one in which learners created a summary table of the mathematical instructions for the different operations. Both teachers progressed to their second lesson with the intention of using exercise 6.6 as their source of examples. To conclude the topic of translating word statements to algebraic expressions, both teachers marked the outstanding questions from exercise 6.6.

As a result of both teachers relying on the textbook as their source of their examples, the types of examples presented to the learners was dependent on what the textbook exercises had to offer. Although both teachers used the same textbook, the exemplification components, examples and tasks were selected and sequenced differently in the two teachers' classes. A summary of this selection and sequencing can be seen in table 4.6.

	Lesson 1		Lesson 2		Lesson 3		Lesson 4	
	Examples	Tasks	Examples	Tasks	Examples	Tasks	Examples	Tasks
TA	S, C Level 2	K, A Level 2	S,C,F Level 3	K,A Level 2	S,C,F Level 3	C/PS Level 3	NA	NA
TB	S, C Level 2	K Level 1	F Level 0	K,A,C Level 3	F Level 0	K,A,C Level 0	F Level 0	K,A,C Level 3

Table 4.6: Summary of Exemplification in Teacher A and Teacher B's lessons

4.5 Conclusion

The analysis of the examples and tasks provided a means of engaging with the work that each teacher made available to learn. What is apparent, regardless of exercise choice, is that the awareness of sequencing is a weakness for both teachers as the common trend was to follow the textbook exercise as it was presented. This is not to say that textbooks cannot or should not be used, but rather that they should be used in a way that pays significant attention to the selection and sequencing of examples. Teacher A and Teacher B should have selected examples from the textbook exercises based on a particular goal or aim of the lesson. The object of learning can be used to focus the selection and sequencing of examples.

For example, the overall object of learning examined in the present study was the translation of word statements into algebraic expressions, but in achieving this goal, each lesson that built up to this goal had its own object of learning. The techniques and skills required to complete the overall object of learning must have developed in the lessons that precede it. These necessary skills, whether they are procedural or conceptual should be considered when selecting and sequencing examples, so that the chosen examples exemplify these attributes. This concludes the analysis and findings from the Exemplification component of MDI. Chapter Five presents the analysis and findings from the Explanatory Talk and Learner Participation components of MDI.

CHAPTER 5

ANALYSIS AND FINDINGS FROM THE EXPLANATORY TALK AND LEARNER PARTICIPATION

5.1 Introduction

In chapter Four I focused on the exemplification component of MDI, however selecting and sequencing examples are only the first stage of presenting the examples. Implementing the examples is dependent on the nature of a teacher's explanation and discussion of the content as well as the learners' responses. Therefore in this chapter I engage with the explanatory talk and learner participation, with the intention of providing answers to the question:

- How does the teachers' discourse impact the intended cognitive level of the presented examples?

The implementation phase of an example begins as soon as the learners and teachers work on the example. Teachers begin their explanations and learners provide their thoughts and ideas, thus creating a network of interrelated actions. These actions, whether they stem from the teacher explaining the example in terms of what to do or how to do it, or even the teacher's response to assisting a learner in completing the task, has the potential to alter (or not) the intended cognitive demand of the task.

In my attempt to answer the guiding research question mentioned above, I grouped the data into three categories. The first category was based on extracts that demonstrate the potential to decrease cognitive demand, the second category had the potential to maintain cognitive demand and lastly, the third category presented with extracts that had the potential to increase cognitive demand. In each presented extract, the mathematical talk is underlined and the legitimating criteria is bolded.

5.2 Examples resulting in a low level of cognitive demand.

As a reminder from Table 3.3, techniques and mathematical discourses that result in lowering task demand demonstrate the following:

- Path Smoothing .
- Focusing on a routine answer.

- Providing an inadequate amount of time (too much or too little).
- The choice of an inappropriate task.

5.2.1 Developing a quick strategy and rule to find an algebraic expression

One of the prominent themes emerging from the data of Teacher A's lessons was the reliance of using a procedure to complete the sum. Teacher A introduced a rule to follow at the start of her first lesson, and at the start of her second lesson. By exploring the explanatory talk used by Teacher A, I can gain insight into the effect that this rule has on the cognitive demand of the example. The first extract is taken from Teacher A's first lesson, where she is discussing the first example of exercise 6.3. Already at the very first example, Teacher A introduces a 'rule' to follow.

20	TA	No, it is an exercise, I will do the exercise with you. Alright question 1. <u>The sum of a number and 5.</u> Hands up, who can tell me how you are going to write this down? The <u>sum of</u>...Yes
21	L9	<u>5 plus *x*</u>
22	TA	<u>A number add five</u>, **so you first have to start with a number**. *x*+5. Ok you **have to do it in the order they give it to you**. Ok, so they say <u>the sum of</u>, so it means **you have** <u>to add</u>, ok so be very careful Grade 8's, you really need to read carefully.

<u>Extract 5.1: Producing a rule in Lesson One</u>

Teacher A then continues with the rest of the lesson in a similar fashion, first reading out the sum, calling on a learner for their suggestion and then lastly concluding the sum by repeating the learner's correct answer or providing the correction herself. The next extract presents how Teacher A introduces another, yet similar rule to the first extract in lesson One. The extract below is taken from Teacher A's second lesson, where she is discussing the first example from exercise 6.6.

68	TA	2p - 3 and then question d. <u>What will their combined ages be in q years time?</u> So how are you going to do that. If you need to combine their ages, **you always take it step by step**. You **first do the first part of the sentence and then you do the second part**. What will their combined ages be?
69	L5	2p + p
70	TA	Aha, yeah Writes (p+2p) And then they say <u>in q years time</u>. So let's say in 20 years time, what will you do then?
71	L6	You plus uh 20.
72	TA	Plus 20, ok but now they didn't give you the 20 so...
73	L5	<u>You add q</u>
74	TA	So it can be <u>plus q</u> Writes (p+2p) + q

<u>Extract 5.2: Introducing and using a rule in Teacher A's Lesson Two</u>

The focus of both extracts is the order that the sum must be done in. Statements such as "**you have to do it in the order they give it you"** and " **you always take it step by step**. **You first do the first part of the sentence and then you do the second part**." assert Teacher A's authority. The solution of 5 +*x* provided by L9 in extract 5.1, although correct was modified by Teacher A to *x* +5 so that it would represent the sum in the order of the given words. The key focus for Teacher A was on the order of the words given in the original sum and not in terms of the mathematics at hand.

Although in extract 5.1, the rule was able to produce the correct algebraic expression, the reliance on a rule was limiting in extract 5.2 as following the order of the given sum resulted in an incorrect solution. Breaking down the sum into two separate parts caused Teacher A and the learners to treat the variables as separate objects, and then later try to recombine them to give meaning. In doing this, both Teacher A and the learners missed the relationship between the ages represented by *p* and the future time in year given by *q*. The error that develops by relying on a rule resonates with the misconceptions that McNeil et al.(2010) mentioned, in the learners take the teacher's explanation quite literally and without question and end up following a procedure that they do not understand.

It was only later on in the lesson when using arithmetic to check their answers did the error unfold. This can be seen in the extract below.

80	TA	OK, I'm just thinking ok. Ummm *Writes on the board 15+7 and 30 + 7*
81	L5	Won't it be <u>plus *q* times 2</u>?
82	TA	Yeah, I'm picking that up now. Right if you look at this, ok good. So now if they say in 7 years time, right so it means this one (points to (p+2p) + q, **is going to be 2q**.

<u>Extract 5.3: Error unfolding in Teacher A's second lesson</u>

Focusing on a rule takes away the mathematical reasoning and logic that one needs to connect the procedures with the concept. In particular a rule without mathematical substantiations results in a non-mathematical legitimation and due to the rule being determined by the authority of the lesson, i.e. Teacher A, the legitimation is positional. With regards to naming, in both extracts the mathematics was read out as a string of labels. These labels were then used to identify the different segments of the sum, and subsequently acted upon in terms of the order that they appeared in. For example, combined ages was mentioned first in extract 5.2 and it was for that reason that Teacher A focused on adding p and $2p$ first. In terms of naming and legitimations, Teacher A exposed learners to a level 2 and level 1 respectively, with the mathematical language read out as a string of labels and the focus on the answers and not necessarily on the mathematics of the underlying concept.

What comes out clearly in both extracts is the reduction of the question from translating word statements to following a procedure. The emphasis is on finding a routine procedure to complete the sum. There is a shift in the object of learning, where instead of translating the word statements the learners are involved in following a procedure. The learners were exposed to a strategy that helps them to develop an answer instead of developing a strategy to deal with the complexity of understanding and translating word statements into algebraic expressions. The emphasis on following a procedure does not align well with the aim of achieving generality. In terms of cognitive demand, following a procedure without connections to the mathematics maintains a low cognitive demand in Lesson 1, but ultimately becomes a factor that shifts the cognitive demand in Lesson 2 from application to knowledge.

5.2.2 Low-level of cognitive demand when the emphasis is on finding the correct answer

The focus in the extract below was on the correct answer and although Teacher A presents some opportunities for learners to engage with the mathematics, her quick responses and search for the right answer counteracts these opportunities. This results in a missed opportunity for developing deep and meaningful connections between the concept and procedure.

33	TA	Alright question 5. Three times as many as a number?
34	L6	x to the power of 3.
35	TA	*Thinks about the answer* **No**, three times as many as a number. Yes Lauren
36	L7	$3x$
37	TA	Yes Lauren. Three times as many as two...so then it means it is going to be three times two
38	L1	Ah wait
39	TA	Right, three times two. So three times as many as a number, so that will also be three times x. (*writes on the board* $3 \times x = 3x$). Right number 6. Double a number.
40	L2	x squared
41	TA	**No**
42	L5	x times 2
43	TA	x times 2, if you double it you two times x. Um double a number, when you double a number you times it by two. Right, if you have to double 60, if you have to double 60 whats the answer.

Extract 5.4: Focusing on the correct answer only

In the short exchange between L6, L7 and Teacher A in turns 33-37, a common misconception is brought to light, however it is not addressed. L6 has incorrectly connected the instruction three times as many to the power of 3 instead of multiplying by 3. Teacher A rejects this answer without any explanation and moves onto the next learner who does get it correct. This extract demonstrates that Teacher A is focused only on the correct answer. This resonates with a strategy deemed as routine answer from Moschovich (2015) whereby the teacher only listens for the right answer and neglects to discuss the incorrect answers. The same misconception is brought up again in turn 40 because the underlying concept of multiplication has not been addressed. L2 thinks of x^2 when he hears the instruction double a number and again instead of a discussion or explanation, Teacher A rejects the answer. Moschovich (2015) argues that it is the mathematics teacher's responsibility to engage with the learners so that they are able to learn something beyond their current means. If Teacher A continually ignores the misconception or the confusion between multiplication and raising to a power, then these learners will remain indifferent to the two operations, or rather to the instructional words that imply these operations. This section looked at potential factors that contribute to cognitive decline and I argue that Teacher A's actions of searching for a learner with the right answer and avoiding discussions on the wrong answer does indeed contribute to cognitive decline. This is because the learners will copy down the correct answer without any consolidation on why their answers were incorrect. Teacher A's actions do not directly decline the cognitive demand of the examples 'three times as many' or 'double a number' per say, but she does however limit or restrict the learners potential to work with these examples and grapple with their misconceptions and poor understanding.

If I examine the naming in extract 5.4, Teacher A predominantly uses mathematical language as a string of labels. For instance in turn 35, she repeats the question in hope that another learner would say the correct label. Teacher A's mathematical talk lacks in creating meaning as her explanations constantly refer back to basic arithmetic examples, thus limiting the learners in using mathematical language formally. The legitimations involved in this extract are localised as Teacher A provides specific arithmetic examples without explanations to justify the answers. Naming is therefore coded as a level 2 and the legitimating criteria as a level 1. What I have done with this extract is look at the combined factors from naming, legitimations and scaffolding techniques to build up evidence of factors that can be associated with low levels of task demand, particularly the demand expected from the learners themselves.

5.2.3 A decline of application and complex tasks due to the use of inappropriate tasks

The intention of scaffolding is to assist learners in thinking about the question by asking thought-provoking questions that maintain the complexity of the question (Stein et al., 2000). There are however times when a teacher has the intention of assisting learners with thinking questions, but due to the pressures of the class environment, i.e. keeping learners engaged and completing the sum that they end up taking over and over assisting the learners in completing the sum. Extract 5.5, demonstrates how Teacher B decreased an example's cognitive demand by eventually over scaffolding and providing the answer herself. This example was the second question from textbook exercise 6.6.

48	TB	Nigella sells a cupcake that costs her *x* cent. So she went to the store, pick n pay and bought a cupcake and it was *x* cents. Cents back in the days, back in the days it was cents. Now she takes that cupcake and goes sells it for a certain amount of money. At a profit of y cents, that is what that means. What is her profit, write your answer down, raise your hand, I want to see how you use your brain. She buys the cupcake x, she sells it y. What is her profit? She buys x, sells y. Sell it for y, what is her profit.
49	TB	*Teacher B walks around the class and mutters* "Nope, nope, well done you smartie pants, you must keep trying, don't look at her answer, well done, no".
50	TB	Ok next question, listen next question, which one is going to have a higher amount. The one that she bought it for, or the one that she is selling it for?
51	Class	Selling
52	TB	How much she is selling it for, **that's the first thing**. So what are you doing now? *Continues to walk around the class checking answers.*
53	TB	Ok for those other people who are still struggling with it, I am going to give you real prices now. Ok I am giving you real prices. I went to china town yes china town, they sell cupcakes, they sell everything. I went to china town, I bought cupcakes for my birthday, for myself. Ok focus, I went to china town and bought cupcakes. 50cents for one cupcake, I come to School and because I am in charge of the c-team I sell for fundraiser, I am selling those cupcakes that I bought, only for 50cents, for R1,50. What is my profit?
54	Class	R1.00
55	TB	Write it down write out the sum that you would write out to find out the profit. **Then substitute that sum with the variable** (pointing to the x and y). Do you understand it now? No multiply, I don't know why. Must I tell my story again, once you are done raise your hand. *Walks around examining answers.*
56	TB	Ok so guys **I am going to do this one**. So I gave you a nice story about me going to china town and buying cupcakes for 50c, now listen everyone pay

		attention. That means it cost me this amount, which means I bought it at 50cents, so what is the variable going to take?
57	Class	*X*
58	TB	*x*, and then come to school and then raise funds and sell the cupcakes for R1,50 and I want to work out my profit. Profit meaning how much money do I get after selling these cupcakes after spending all this money (underlines 50centes). R1,50 minus the cost price of 50 cents that will give me R1 which I keep. Which is not y, this is my sell amount (underlines 1,50), this is y. At the end of the day in terms of variables, we have the sell amount minus my cost price. That will be my profit, writes y-x on the board. **You leave it as y-x.**

Extract 5.5: Teacher B's use of an inappropriate task

Throughout the extract, there was frequent non-mathematical talk from Teacher B who attempted to situate the example for the learners in a realistic scenario. Teacher B used descriptive pronouns as she named the expression for cost (x) and selling price (y), while simultaneously pointing at the board and underlining each expression. She repeated the phrase "she buys x, sells y" to highlight the key features of the sum.

The lesson progressed as Teacher B walked around the classroom and commented on the individual's work. Although we hear very little in terms of learner participation in the transcripts, Teacher B encouraged her learners to discuss the mathematics and their ideas with her as she passed their desks. Teacher B evaluated the learners' answers and in turn 50 she decided to introduce a new question to the class, highlighting the relationship between the cost price and selling price of the cupcake. Teacher B maintained colloquial or everyday language, and focused on the action of selling the cupcake. Continuing with the evaluation of answers she decided to bring in specific arithmetic examples in turn 53.

With each new question that Teacher B posed to the class (turns 50, 53, and 55), she was asserting the everyday notion of buying and selling products in an attempt to engage with the learners at a level that they might understand. She broke down the question in terms of the words that it used and she provided an arithmetic example in attempt to link the arithmetic sum to the algebraic expression. This can be seen in turn 55 when Teacher B tells the learners to write out the arithmetic sum and then to substitute the values with the variables. This substantiation is positional as Teacher B is directing the learners to use a method without providing justification. After providing the learners with more time to work on the sum, Teacher B decided to do the sum with them. In turn 58 she points out that cost price is x and selling price is y, thus resulting in the expression $y - x$. Teacher B did the task by herself, without any input from the learners and in the end decreased the example's cognitive demand.

Across the example (turns 48-58), the naming and legitimating in the MDI of Teacher B consisted of mainly non-mathematical and colloquial terms, as well as non-mathematical (everyday scenarios) and positional (authority of Teacher B) criteria for confirming the algebraic expression derived from the sum. Therefore with respect to explanatory talk, both naming and legitimating were coded as level 1. This level of analysis however is limited in determining why Teacher B provided the learners with the answer and subsequently decreased the cognitive demand of the example. As a result of this limitation, I turn to the scaffolding techniques that complement the explanatory talk.

Appropriate teacher scaffolding occurs when a teacher asks thought provoking questions that maintain the complexity of the task. What happened in Teacher B's lesson was a slow reduction of the example's cognitive demand. Teacher B attempted to find a level of cognitive demand whereby the learners were able to work independently. At first she maintained the complexity of the task by situating the learners in a scenario that would be familiar to them. When this did not work, she introduced 'real' prices and tried to incorporate the learners arithmetic knowledge into the question. It was at this point at turn 55, when Teacher B reduced the complexity by telling the learners to write down the sum and then replace it with the variables. This still did not work and many learners were unable to complete the task. Thus the analysis shows that reducing the complexity and creating a link back to the arithmetic model was not enough and Teacher B felt that she had to do the sum for the learners. This demonstrates that the example with its tasks was inappropriate for the learners as they lacked the prior knowledge needed to perform and engage with the high-level cognitive activity. Although the overuse of scaffolding declined the task demand of this example, the underlying cause for concern was the example itself. The learners in Teacher B's class are unable to engage with the examples and work independently to solve them. This is further evident in each example presented to the learners in Teacher B's lesson Two and Three. Below I present an extract that demonstrates the difficulty that the learners had in translating an algebraic expression concerned with addition.

8	TB	How old will Lerato be <u>in 10 years' time</u>?
9	L2	10p
10	TB	Nope
11	L3	20p
12	TB	Ok let me ask it like this. How old are you now?
13	Class	Murmurs of 12, 13,14
14	TB	Mkay, in 10 years time, how old will you be?
15	Class	Murmurs 23, 24
16	TB	23, 24, did you multiply. Listen, **did you multiply your age with 10 or did you add 10 years with your age**?
17	Class	Add
18	TB	**You added**. Ok, so how old will Lerato be in 10 years?
19	L4	12p
20	L5	p+10

<u>Extract 5.6: Learners in Teacher B's class struggling with a Knowledge task</u>

Two learners attempted to answer the question by applying multiplication techniques to the variable *p*. It was only after Teacher B related the scenario to their own ages and she questioned if they added or multiplied that the learners associated this task with addition. Teacher B led the students to the operation of addition in two ways, first, by relating the work to a scenario they understand and second, by telling the learners they had a choice of multiplying or adding. This is a common occurrence in lesson Two and Three of Teacher B's class, and with every example that she goes through at some point she gives the learners the answer. Below is an extract that demonstrates Teacher B discussing a learner's suggestion and then deciding to give the learners the answer instead.

110	TB	Three, is that an even number, no. If I substitute it with two, what would that give me. Then it would give me four. **What about instead of adding say 2t, or 2x, 2** any variable. Ok, so now if we replace that with any number, let's replace it with five, what would that give me?

Extract 5.7: Teacher B providing learners with the correct answer

The trend that emerges from Teacher B's lesson (Two and Three) is that the use of exercise 6.6 at the time was inappropriate as her learners were unable to work independently. Although the learners took the work seriously and were hard at work coming up with solutions and conjectures, they failed to progress to understanding how to translate word statements into algebraic expressions. When the problematic aspects of the tasks become routine procedures, or even as seen above the procedures themselves are given by Teacher B, Stein (2000) argues that this will decline high-level cognitive demand.

5.2.4 General discussion

The data shows that when explanatory talk and learner participation levels are kept low, as seen in the above extracts where naming and legitimation were at a level 2 and level 1 respectively, cognitive demand is also low. In the extracts shown above, task demand remained low or declined from application to knowledge (extract 5.2, 5.4 and 5.5) and explanatory talk, particularly the legitimating criteria remained non-mathematical and positional and learner participation in terms of mathematical talk remained restricted to short one word answers or phrases. The scaffolding techniques associated with the low level task demand were focusing on a routine answer by developing a procedure or rule, focusing and searching for a correct answer, the use of path smoothing and using inappropriate tasks. To conclude, examples that were implemented with the following characteristics had the potential to decline in cognitive demand:

- Scaffolding techniques such as path smoothing, finding a routine answer and inappropriate tasks.
- Colloquial talk that dominated the discussion, or mathematical words were only read out as a label.
    - Naming coded at level 1 or 2.

- Non- mathematical or positional legitimations made, showing the authority of the teacher.
    - Legitimation coded at a level 1.

5.3 Examples that have the potential to maintain their cognitive demand.

Maintaining high-levels (application and complex/problem solving) of cognitive demand requires adequate teacher support in terms of providing learners with thought provoking questions (Moschovich, 1999). Both Teacher A and Teacher B ran teacher dominant classrooms, where the majority of the talk was done by the teacher. It was only on the rare occasion that either teacher asked a learner to explain themselves, justify their answer or to expand on their solution.

5.3.1 Examples where learners are requested to provide additional information and justifications.

I have presented two extracts from Teacher A's lessons in which there was an opportunity for the learners to provide their own thinking and reasoning. The first extract is from Teacher A's first lesson where the discussion centres around the example 'three less than a number', and the second extract from Teacher A's second lesson where the focus is on consecutive numbers. Both examples were coded at an Application level and thus had the potential to be maintained (or not).

53	TA	Right, number 10. <u>Three less than a number</u>. Yes
54	L9	x - 3
55	TA	x - 3. Well done, right why can't we say <u>3 minus x</u>?
56	L7	Cause, **because it says three less than a number**.
57	TA	So if you do 3 less than 10, what is the answer going to be?
58	L7	7
59	TA	7, so it is 10 minus 3 so that is why in this case x - 3

<u>Extract 5.8: Justifying the order of terms in a subtraction expression</u>

Naming in extract 5.8, moved between reading the mathematics as a string of labels in turn 53, to formal mathematics in turn 55. The use of mathematical language was used appropriately and drew the learners' attention to the difference between two expressions, x - 3 and 3 - x. As a result of formal mathematical language I coded naming at a potential level 3. When Teacher A addressed the order of subtraction she questioned why the expression could not be 3 - *x*. Encouraging learners to justify their solution is a technique used to maintain the cognitive demand as this question can lead the learners to organise their thoughts, make conjectures and make further connections with the concept (Stein, 2000). Additionally providing learners with the expression 3- *x* contrasted the two expressions together and highlighted the critical feature (order of terms), resulting in Teacher A providing an opportunity for the learners to engage with the mathematics. The justification for why the expression must be *x* -3 and the criteria used to legitimate this expression was provided by L7. However this justification was a repeat of the question itself, and learner 7 used formal mathematics but only as a string of labels. Consequently, with the low level of legitimation, i.e. being localised to one specific sum, and with the learner basically just repeating the words of the example, I conclude that this is a weak justification. What I mean by this, is there is no new content or conjecture to the learner's justification, she does not connect the concept of subtraction to the important order of the terms. Teacher A's response in turn 57 did not re-voice or explain this justification but rather her response was localised and once again drew on a spontaneous arithmetic example to explain the expression. The scaffolding technique in turn 55 did not reach its potential and the two expressions were not connected to the commutative property thus losing the connection between the concept and procedure.

I propose that there was certainly good intention behind Teacher A's question in turn 55 and that she was aware of the importance of drawing attention to the correct order of the terms. The response from the learner was however limited in explaining the critical aspect of why the two expressions *x*-3 and 3 -*x* are different. Stein (2000) and Moschovich (1999) suggest that there are scaffolding techniques that assist teachers in eliciting information. For instance Teacher could have pressed learner 7 for more information, or asked her to try explain herself in a different manner. Alternatively Teacher A herself could have revoiced learner 7's response or even built on from it and produced a formal mathematical explanation for the difference in the order of the expressions. Although the discussion that appeared in extract 5.8 had the potential to engage with formal mathematics through the naming being coded at a level 3 and through Teacher A's support in the thought provoking question in turn 55, the result of a weak justification and localised legitimation cut the discussion short.

The next extract shows an example that has all of the right components to maintain a high-level task demand. First, the example itself incorporates learners' prior knowledge on natural numbers as well as the term consecutive, and second, Teacher A builds on the knowledge and response of learner 2.

105	TA	Question 4 they talk about, <u>consecutive natural numbers</u>. What is the meaning of the word consecutive? Yes.
106	L2	They go one after each other
107	TA	Ok, **give me an example**?
108	L2	1,2,3,4
109	TA	Ok, so for example 1 2 3, 4 5 6, 7 8 9 (*in unison with class*). Ok 21 22 23 24. Those numbers are consecutive. So whenever you see the word consecutive you must know that it **means something that is following.** Right, <u>what is the sum of three consecutive natural numbers</u>, <u>with t being the smallest</u>. So t is your first one. So, we are going to work with three numbers. Ok so first number. *Writes 1st number on the board.* Then you will have your second number and then you have your third number. *Writes 2nd number and 3rd number on the board.* Ok, so what is your first number going to be?

<u>Extract 5.9</u>

Teacher A began the example by asking the learners what the word consecutive meant. She did not opt to provide the definition herself but was searching for a learner to provide it. What was strikingly different in this extract was that Teacher A probed L2 for additional information and did not use her definition immediately. Teacher A wanted an example first. In addition to calling for an example, Teacher A then used L2's example and built up more consecutive numbers from it. These are all techniques that Stein (2000) argues for maintaining high cognitive demand. From an MDI perspective the naming in extract 5.9 uses both colloquial language and partially generalised legitimating criteria when L2 says "they go one after each other" and when Teacher A re-voices this and says "something that is following". Both definitions are informal and lack a full mathematical description of the term consecutive and are accordingly coded as level 2 for naming and legitimating criteria.

Teacher A then went on and provided the learners with a procedural approach to solving the example presented. In her approach of creating a table where she writes out the three consecutive numbers, Teacher A provided the learners with a starting point and method for solving the problem. This diminished the learners' opportunity to think about the discussion on the meaning of consecutive terms and haltered their chances of applying their own ideas. It appears that Teacher A's learners are able to work independently but we seldom see their full capabilities as Teacher A takes over the example and often completes the work for them. Therefore we are only exposed to their limited and constricted inputs, usually at the start of an example. Having all the right components in terms of naming, legitimating and scaffolding are a starting point in achieving high levels of task demand, however these levels need to be maintained throughout the example and explanation for it to yield positive results.

5.3.2 The influence of high levels of coding for naming and legitimating in maintaining cognitive demand.

The next extract was chosen based on its high coded levels of naming and legitimating criteria. The theme that emerged from the data was that teacher explanations that provided opportunities for formal mathematical talk, and that provided criteria for generality such as making distinctions between properties of mathematical objects promoted high levels of cognitive demand, or at least increased the chances to maintain the intended cognitive demand.

24	TA	There's a difference between this one and that one. *Points to the 6x and the x+5* Ok, so that number 1 and number 3. Number 1 I cannot go and write it as **5*x* because then it means five times *x***. are you with me?
25	Class	Mmmm
26	TA	ok, but when I multiply you put them together. Are you with me?
27	Class	yes ma'am

Extract 5.10: Introducing accepted conventions of algebraic expressions

In extract 5.10 there was movement between the colloquial pronouns and the formal mathematical talk in turn 24 when Teacher A said "5*x* because then it means five times *x*". She used appropriate mathematical talk to discuss the object 5*x* and linked it to the mathematical procedure of multiplication. The distinction made between the property of expressions where terms are multiplied, compared to expressions where terms are added was partially generalised as the convention for addition was not specifically mentioned. The emphasis was on the convention for multiplying terms.

The significance of this extract is to show that creating opportunities for learners to hear formal mathematical talk and to be informed of the accepted conventions provides them with the opportunity to reproduce what they have seen. Therefore this particular extract was not used to show one particular example's cognitive demand being maintained (these expressions developed from Knowledge examples), but rather how learners can be exposed to conventions and particular ways of thinking so that in future examples and tasks they may be able to apply this knowledge and work independently.

5.3.3 Discussion

In order to maintain high levels of cognitive demand, learners need to be supported in terms of the following:

- Scaffolding techniques
  - Student self-thinking and reasoning.
  - Being pressed for explanations, examples and justifications.
  - Using learners' prior knowledge to build up more elaborate answers.
- Naming- mathematical language that encourages learners to improve their colloquial talk into formal mathematical talk.
  - Coded level 2 or 3.
- Learners must be introduced to what counts as accepted mathematics through accepted conventions, comparison of mathematical properties, derived procedures or proofs.
  - Legitimating criteria must therefore include generality and accordingly coded at a level 2 or 3.

The findings from the extracts show that in order to maintain high task demand, the above techniques must be implemented, however they need to be implemented throughout the example. If the scaffolding techniques change and a teacher gives away the procedure then it does not matter how formal the language was or how general the case was as the learners will not need to think about the connections between the concept and procedure as the teacher did the question. My argument is that all three components of explanatory talk need to work in unison and all at high levels to maintain cognitive demand. This argument is further supported in the next section which discusses the methods used to potentially increase cognitive demand.

5.4 Examples that had the potential to increase their task demand.

Increasing cognitive demand is related to the connections made between concepts and procedures (Adler and Ronda, 2015). The different levels of task demand yield different learning opportunities and correspond to a particular goal or outcome. I set out to investigate in this section if it were possible to increase a task from a low-level cognitive demand to a higher level. To do this I found two extracts (one from Teacher A and one from Teacher B) in which low-level tasks (Knowledge) were in some way connected to multiple procedures or concepts.

5.4.1 Exploring an example with two correct solutions

This particular extract was used to demonstrate an opportunity in which two solutions are viewed as correct and compared against each other. Teacher A created an opportunity to discuss the two solutions, in terms of their similarity and differences.

67	TA	Now it's getting complicated. "One more than a number."
68	L4	x+1
69	TA	Well done, easy ok. One more than a number, ok so, one more. So it's 1 + x ok if I say <u>one more than two</u>, what would it be?
70	Class	Three
71	TA	So <u>one</u> (*points to the 1*) <u>more</u> (*points to the* +) <u>than a number</u> (*points to the x*). Ok so it's one plus x. Now please be careful now, **the answer is not going to be *x*** but the answer is 1 + x. Ok, or even if you wrote x + 1 you will get away with it because it is a positive so it doesn't matter, ok. Right, if **you do one plus two or two plus one, you get the same answer.**

<u>Extract 5.11: Comparing the solutions of *x* +1 and 1+ *x*</u>

In terms of MDI, the extract showed criteria that extended beyond localised and non-mathematical substantiations and was thus coded at a level 2 in terms of legitimations. Naming remains mathematical and was coded as a level 3 due to the links made between the term 'more than' and the operation of addition. I now look at the specific connections made in the extract to determine if there was a possibility to increase the cognitive demand.

In turn 71, Teacher A discussed the solution to be $1 + x$ even though L4 had said $x + 1$. This is repeating L4's answer while rewording the order of the terms, however Teacher A mentioned that both solutions, $1 + x$ or $x + 1$ would be "ok" as the numbers were positive, and she then provided an arithmetic example to justify this statement. Teacher A used partial generality to legitimate that $1+x$ and $x + 1$ are the same answer, just as two plus one or one plus two would give the same answer. Teacher A made an attempt at relating the properties of the addition of algebraic expression and the commutative property of addition. Her reference was however colloquial and only achieved partial generality as she used one basic arithmetic example. The arithmetic example shows qualities of the commutative property, however Teacher A does not directly link the algebraic expression to it. A discussion linked to previous knowledge has the ability to introduce links to full generality as well as to create more connections with the procedures of addition. This is further motivated by Stein (2000) who contends that tasks that build on prior knowledge have the capability of maintaining cognitive demand. Effectively if learners are exposed to the commutative property in algebraic expressions, they can develop the understanding that $1 + x$ or $x + 1$ would produce the same result but $1 - x$ and $x - 1$ do not. In developing or extending the learners knowledge with the commutative property, Teacher A would have made conceptual connections between prior arithmetic work and the new algebraic work. Consequently with the example ' One more than a number' being set at a Knowledge level, this connection to the commutative property would have ultimately increased its cognitive demand. This however, was not achieved and although learners may be able to reproduce these answers in a similar task, they might not have made the connection between their prior knowledge of the commutative property and their new knowledge of algebraic expressions.

In Teacher B's lesson on summarising the instructional languages for each mathematical operation she created a link between using division to prime factorise a number which is a technique that assists learners in square rooting a number. By drawing attention to this connection, Teacher B demonstrated multiple conceptual connections. This is what the next extract shows.

5.4.2 Creating links between the operations square root and division

TB	Ah ok, this one, square root and square. Square root 25 and square. Can everyone see that. When you are working out the square root using the ladder method, you are dividing all the prime numbers, into this number to get find the product, right. Because you are dividing that is why we put square root under divide. For this side the square, when you have 5 squared, or 5 cubed or 5 to the power of four, it means you are repeating the five how many times.
Class	Twice
TB	Twice, five times five, because you are multiplying, we put it under multiplication. That's it.

Extract 5.12: Connecting the procedures of square root and squaring

Teacher B used a non-mathematical term, the "ladder method" when she referred to a method used in prime factorisation, however this term was well understood by the learners and appeared to be a valid term in her mathematics classroom. Overall, the language used showed movement between colloquial and formal mathematics used appropriately and was thus coded at a naming level 3. Teacher B linked the procedural processes of finding a square root with the division operation and squaring a number to multiplication. Teacher B's explanation drew on the learners prior knowledge of prime factorising and showed generality as her explanation is valid for any real number (perfect) square rooted or squared. Therefore the summative judgement for extract 5.12 in terms of naming and legitimation is that they were both at a level 3. Subsequently as with extract 5.11, this extract shows how a teacher makes use of prior arithmetic work to develop connections to the work that will be used in algebra. Once again the activity presented above was at a Knowledge level and thus this extract only shows how learners can develop connections between different sections of work, with the intention of providing the learners with new ways of thinking about their work. Drawing on frequent conceptual connections is another factor that one can consider in the maintenance of high-level cognitive demand tasks (Stein, 2000) but since this task was already set a low task demand, what this extract shows is a teacher creating opportunities for her learners to think about low-level tasks in ways that create links to previous sections of work.

5.4.3 Discussion

The findings from the two extracts show that the components of explanatory talk need to be coded at high levels to create opportunities for the learners to engage with the mathematical concepts and procedures in a meaningful and thought provoking way. Creating more connections between the concepts and the procedures or required tasks is difficult and an area that requires more research. In this study, the findings for increasing cognitive demand were limited, and the main achievement in both extracts leaned towards assisting learners in working independently and being able to reformulate what they learned or heard. This was due to the level 3 naming and legitimating criteria.

5.4 Summary of Findings of Explanatory Talk and Learner Participation

The pattern that emerged from each category of cognitive demand, was that the level of naming and legitimating criteria provided insight into how Teacher A or Teacher B was able to provide opportunities for learners to engage with the mathematics and cognitive demand of the examples.

When Teacher A and Teacher B used their authority to make an assertion, it was in an attempt to legitimate a procedure (extract 5.2) or to make meaning for her learners during the lesson (extract 5.5). However, when these teacher assertions are made with the absence of mathematical criteria, they only partially supported learners in achieving generality and therefore reduced the possibilities for learners to succeed in translating algebraic expressions independently. When Teacher A and teacher B engaged in low levels of explanatory talk, their presented examples reflected this, and as a result of this task demand remained low or in the case of extracts 5.2,5.4 and 5.5 task demand declined. This decline was attributed to the scaffolding techniques associated with low levels of explanatory talk, which were focusing on developing a procedure, path smoothing and the inappropriate use of tasks. When engaged in low levels of naming, legitimations and scaffolding, learners were introduced to methods used to cope with achieving a routine answer and not with the complexity of translating word statements to algebraic expressions without the assistance of their teacher.

When Teacher A and Teacher B engaged with the examples in such a way that brought on higher levels of naming and legitimations, this too emerged in their presented examples. Providing formal mathematical talk and localised or general legitimations increases the chances for learners to work independently and for them to be self-sufficient in translating algebraic expressions. A main theme that emerged from the data was that in order to maintain task demand all of the components of explanatory talk had to be maintained at a high level and integrated in a way that complemented each other. In this study, Teacher A and Teacher B dominated the mathematical talk and demonstrated most of the examples themselves. This is why the analysis focused on the decline of tasks as it was the prominent theme in the two teachers' lessons. Another finding in itself is the lack of learner participation. Learners in both teachers' classes had restricted opportunities in developing or at least practicing the formal mathematics themselves. The point I am trying to make, is to successfully implement high level tasks, not only does the components of the explanatory talk have to be considered and rated highly, but learners must also be encouraged and motivated to participate in their learning.

5.5 Conclusion

Using the adapted version of the MDI explanatory talk has allowed me to see the mathematics made available to learn from both Teacher A and B in terms of their presented examples. Engaging with the Explanatory Talk and Learner Participation I was able to highlight the connections between the subcomponents of naming, legitimating and scaffolding. This concludes chapter Five. In chapter Six, I will present the overall findings of the present study with the intention of describing the meditational properties of both the exemplification and explanatory components from chapter Four and chapter Five respectively, how they worked separately but quite importantly how they work together to achieve the object of learning.

CHAPTER 6

SUMMARY OF FINDINGS AND CONCLUSION

In undertaking this study, I aimed to analyse teachers' use of exemplifications and explanations in mediating the object of learning. The motivation to do this research stemmed from my own difficulties that I experienced as a novice teacher with the selection and sequencing of examples. Thus in doing this research I hoped to uncover critical areas in the selection and sequencing of examples, as well as uncover the relationship between the presented examples and the teacher explanations of these presented examples. In this concluding chapter, I provide the summary of my findings from Chapter Four and Five, and further discuss the connection between exemplification and explanations as they work together to mediate the object of learning. I also consider implications from these findings and make recommendations based on the findings. Lastly I reflect on the study and the implications it has had on my own personal teaching.

6.1 Summary of the Findings

The object of learning is mediated through the use of exemplification, explanatory talk and learner participation (Adler & Ronda, 2015). The first component, exemplification, was explored and examined in Chapter Four by analysing the two subdivisions: examples and tasks. In doing this I was able to determine the degree of variation and cognitive demand of the examples presented to the learners over the course of their lessons on algebraic expressions. The second and third components of MDI, explanatory talk and learner participation, were examined with the purpose of finding out what a teacher does with the examples in terms of their explanations, and how effective their explanations were in maintaining (or not) the set cognitive demand. The findings from Chapter Four and Five are summarised and concluded in sections 6.1.1 and 6.1.2.

In Chapter Two, I indicated that the object of learning brought focus to the start of a lesson. I contend that not only is the object of learning the starting point of a lesson, but also the end result of a completed lesson. Therefore it is important to engage with the components of the MDI framework and determine to what extent the object of learning had occurred. To do this I describe a range of meditational elements brought on by the three components working together and conclude to what extent the object of learning has been achieved.

6.1.1 What type of examples are presented during the teaching of algebraic expressions?

This section focuses on using the findings from Chapter Four as a summary to answer the research question:

- What type of examples are presented during the teaching of algebraic expressions?

Although the activities differed, when both teachers began teaching translating word statements into algebraic expressions in their very first lesson, the examples used demonstrated similarity and contrast and were set at a low task demand. Teacher A focused on translating word statements into algebraic expressions when there was only one mathematical operation in the sum, while Teacher B focused on summarising the four mathematical operations with their corresponding instructional terms. To maximise the opportunities to present generality, I suggest that the examples should work together to achieve a particular goal or purpose. Teacher A in particular used the examples from the textbook as they were sequenced, when she would have increased the potential to promote generality by building up example sets that focused on one mathematical operation at a time.

Both teachers then proceeded to use the textbook exercise 6.6. In this instance the two teachers now used largely similar activities as the examples were from the same textbook exercise. For Teacher A this was a progression of translating word statements into algebraic expressions when there were multiple operations and tasks, and for Teacher B this exercise was her starting point for demonstrating the translation process. The choice to use exercise 6.6 shows a high degree of commonality in both teachers presentation of algebraic expressions. The focal point of this section of algebra was an exercise that demonstrated fusion examples that had a high cognitive demand. Lo (2012) cautions that if a learner is not exposed to fusion examples, they may never have the chance to develop a full understanding of the concept as they would not have been exposed to the many features that define and affect it. Therefore using exercise 6.6 exposed learners to varying features of algebraic expressions and provided the learners with examples that had the potential to engage with them in complex ways of thinking and reasoning. Although both teachers chose to present examples that incorporated multiple operations, concepts and procedures which created the potential for the learners to acquire or experience different levels of complexity, this was achieved differently in the two classes. Teacher A's approach attempted to show the aspects of the different features of translating word statements into algebraic expressions separately by first showing the learners algebraic expressions with only one mathematical operation and then progressively increasing variation and task demand. In this regard Teacher A demonstrated how the different features work separately and then also how the features can work together simultaneously. This is not to say that Teacher A did not encounter any difficulties and in section 4.2 I highlighted that Teacher A should have selected examples from the exercise based on a particular purpose, and I provided the suggestion of building up to the generality of algebraic expressions in each mathematical operation and then contrasting the expressions against themselves.

Teacher B, while still using the same exercise 6.6 to achieve fusion examples, did so without using simpler algebraic expressions first. She focused on the mathematical language and the instructional words that imply each mathematical operation. Although this activity is useful in its own regard, it did not prepare her learners for working with word statements that had multiple operations and complex tasks. Marton and Tsui (2004) recommend focusing on the smaller or simpler parts of the concept first, make them distinct and then piece them together to help form the bigger picture or overarching concept. I argue that the advancement to fusion examples without prior knowledge and experience of purely similar and contrasting examples, caused many of the difficulties that Teacher B had in presenting and implementing exercise 6.6 with her learners. Therefore Teacher A's selection and sequencing of examples (although not without its own issues) set a better foundation for preparing the learners to cope and experience the fusion examples than Teacher B's introduction lesson.

6.1.2 How does a teacher's discourse impact the intended cognitive level of the presented examples?

As discussed above, the presented examples ranged from a low cognitive demand when Teacher A and Teacher B introduced the topic of algebraic expressions to their learners, and then ranged to presented examples that had a high cognitive demand when the teachers used exercise 6.6. In Chapter Five, I explored the effect of Teacher A and Teacher B's discourse as they explained the presented examples to their learners. The findings from Chapter Five show that the explanatory talk aids the presented examples in ensuring the object of learning is brought into focus. The main theme that emerged from the data was that the three subcomponents of explanatory talk-naming, legitimations and scaffolding - all work together to deliver the presented examples. For example, when informal mathematical language dominated the lesson, classroom discussions were limited in their exposure of mathematical words used appropriately and effectively, which often resulted in discussions that were not very interesting or enlightening. Additionally when the teachers provided non-mathematical or positional legitimations they effectively told the learners what to do. Giving away the answer, or taking away challenging aspects of the questions are poor scaffolding techniques, and are associated with a decline in cognitive demand (Stein, 2000). Therefore the combination of discourses that focused on correct answers and described the procedures and methods on how to solve the question resulted in the task demand declining and leaving the learners with limited opportunities to think about, reason or justify their work.

On the other hand, when the naming and legitimating criteria where coded at higher levels, there became a greater potential for the teacher to engage with their learners in meaningful and complex forms of thinking. I emphasis that the use of naming, legitimating criteria and scaffolding are potential factors of mathematical discourse that impact the cognitive demand of examples. There may be additional factors that could be considered that this study did not focus on. For example, there was limited learner participation and therefore I was not able to comment effectively on the impact that the learners had beyond the struggles that Teacher B faced in her class (see section 5.2.3). What this study did highlight was that the three components of explanatory talk are in some way interlinked and need to be considered when explaining the examples, particularly if the goal is maintain high levels of cognitive demand. As a final finding of the study, I present the combined effect of exemplification and explanatory talk on the object of learning.

6.1.3 The overall effect of exemplification and explanations in mediating the Object of Learning.

In Chapters Four and Five, I analysed the exemplification and explanatory talk components (respectively) separately. This allowed me to work with exemplification and explanatory talk within the lessons and illustrate the differences in what each teacher made available to learn. The framework that I have used has however allowed me to look at how examples and explanatory talk work together to either enable or limit the object of learning. Additionally the framework has exposed the connections between the exemplification and explanatory talk components, something that would have otherwise been impossible to see.

In Teacher A's lessons the sequence and selection of examples provided an example set that incorporated all three levels of variation: similarity, contrast and fusion. Thus from the perspective of exemplification, Teacher A selected two exercises from the textbook that provided a set of examples that had the potential to encapsulate the object of learning and both examples and tasks and were subsequently coded as a level 3. However once Teacher A implemented and presented the examples to the learners, the task level (particularly of exercise 6.6) declined. This was due to the explanatory talk, particularly the legitimations as they were predominantly non-mathematical and positional, and by Teacher A removing the challenging aspects of the examples by providing learners with the answer or method to complete the sum. The use of routine procedures and methods can result in a limited understanding of translating word problems to algebraic expressions and learners may develop limited resources in working independently. Merging the strengths and weaknesses in exemplification and explanatory talk for Teacher A's lesson, I came up with a model that showed the interplay between the two components.

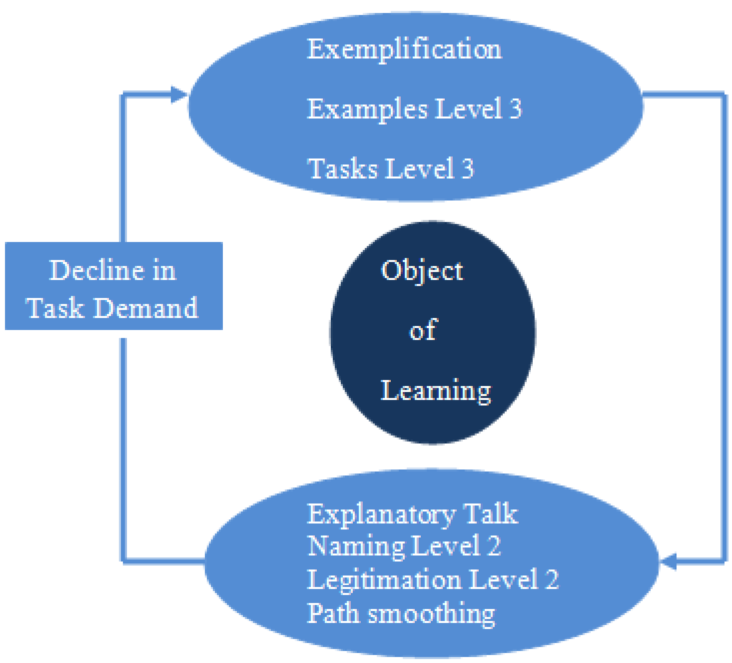


Figure 6.1: Model of the interplay between Exemplification and Explanatory Talk in Teacher A's lessons.

In figure 6.1, the object of learning is placed in the centre as both components play a role in mediating the object of learning. Starting with the exemplification component, the example set's variation and cognitive demand was set at a high level which shows that there is the potential to engage learners in complex forms of thinking. This however is not implemented in the lessons and the second connection from Explanatory talk back to Exemplification decreases the task demand and lowers the cognitive demand of the examples. It is the result of the examples declining in task demand that actually shifts the object of learning. In summary the set of examples created the potential to embody the object of learning but Teacher A's assertions made with the absence of mathematical criteria only partially supported her learners in achieving the object of learning, and possibly reduced the learners ability to work on this section independently. Indeed, learners in Teacher A's class are able to provide answers and mimic or imitate their teacher's technique but we rarely get to see the learners attempt a sum on their own or justify their work.

Given this result, I can conclude that from Teacher A's lessons that the object of learning was achieved partially as the mathematics made available to learn was a strategy of asserting a procedure, and this resulted in a coping mechanism for learners to translate words into algebraic expressions. Next I turn to Teacher B and her lessons to establish how she used exemplification and explanations to mediate the object of learning.

Exploring Teacher B's use of examples and tasks led me to conclude in Chapter Four that there was a weakness in her sequencing of examples. Variation in Teacher B's examples was random and fusion examples were introduced to the learners without a prior build up of similar and contrasting examples, and I subsequently coded Teacher B's use of examples at a level 0. The tasks however were coded at a level 3, as the tasks themselves demanded a high level of cognitive demand. Examining the explanatory talk and learner participation in Chapter Five provided insight into how the conflicting components of exemplification affected the explanations. Teacher B was unable to successfully scaffold the questions and eventually did the examples for the learners on the board. This was apparent in every example that Teacher B presented to the learners (from exercise 6.6). Considering the contrasting levels of exemplification and the difficulties that Teacher B had with the explanatory talk, I present the following model:

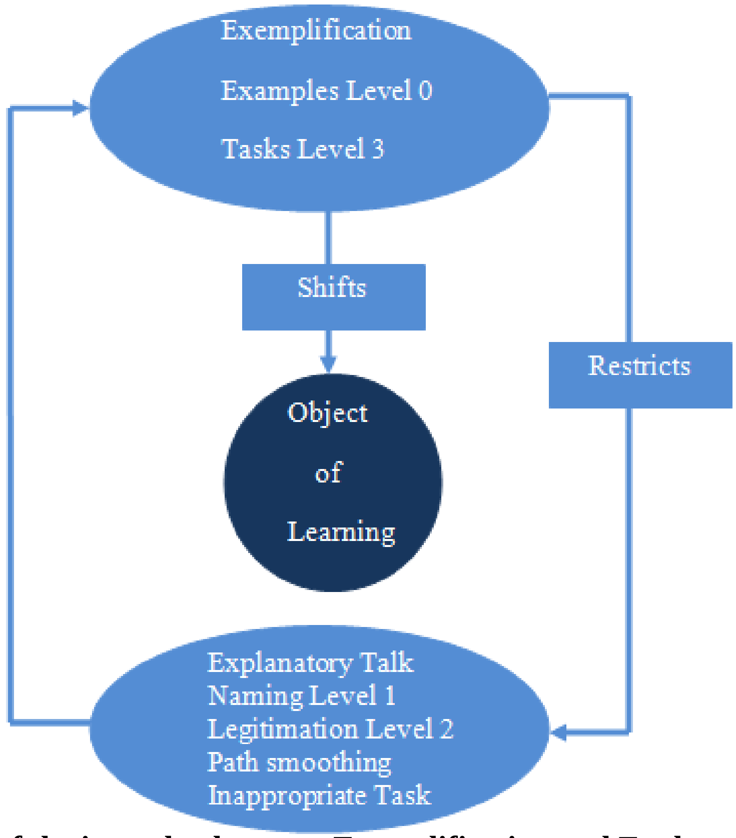


Figure 6.2: Model of the interplay between Exemplification and Explanatory Talk in Teacher B's lessons

The striking difference between the model for Teacher B in comparison to Teacher A, is the immediate shift of the object of learning from the Exemplification component itself. The model that I present shows that the examples themselves shifted the object of learning before Teacher B implemented the examples. This is because if the example set is used randomly (variation is not structured correctly) then the examples may lead to just practice and consolidation of the concept or as Watson and Mason (2015) suggest an exercise of drill and practice. I agree with Watson and Mason but further contribute to this statement by explaining the additionally difficulties Teacher B had in terms of the Explanatory Talk, and I attempt to answer why the exercise reduced to drill and practice. From analysing the Explanatory Talk in Chapter Five I found that Teacher B reduced the mathematical talk to mainly colloquial or informal language in an attempt to engage with the learners at a level that they understood the question. I argue that Teacher B reduced the complexity of the examples because she was unable to find a balance between scaffolding and task demand. The tasks themselves were inappropriate for the learners because they had not experienced the critical features of algebraic expressions separately as Marton and Tsui (2004) suggest. The smaller parts of the concept, which in this case are the individual operations, should have first been presented and then later on pieced together to help mediate the object of learning. This could have been achieved by using an exercise like 6.3 as Teacher did, or by Teacher B presenting her own examples that focused on the critical features separately.

The lack of support from the examples variation created a cycle in which Teacher B would introduce numerous arithmetic examples and real life situations in an attempt to explain the question to the learners, but with every new example and explanation the cognitive demand declined as the learners had less opportunities to make the connections with the concept and the procedures. This led me to conclude that the learners were unable to complete the examples independently. Consequently the object of learning was limited and the task demand declined which resulted in the learners depending on their teacher to conclude each sum.

From the work of Adler and Ronda (2015,2016), I had expected to see the components of MDI work separately and then together to mediate the of the object of learning. What this study has highlighted is how the components themselves, namely exemplification and explanatory talk, have an effect on each other.

6.2 Limitations of the Study

The framework used in this study did not take into account the learner participation from groups of learners or even individual learners working by themselves. This was most apparent in Teacher B's lessons where learners were first given the opportunity to work by themselves in an attempt to answer the sum. When the explanations occurred for Teacher B's lessons, the transcripts show that Teacher B spoke most of the time and she seldom asked the learners for their opinion or their answers, but I was unable to account for the private discussions had by the learners or by the learners and Teacher B. Although this presented as a limitation in this study, it did not have major implications as the explanations from Teacher B were the main focus of the study.

6.3 Potential Implications of the Study

In Adler and Ronda's (2016) article they suggested that resources that proposed examples and step by step instructions on how to deliver lessons (such as a lesson plan), could not guarantee a successful lesson. They proposed that this was due to the critical reliance on the teacher and their corresponding explanations. They argue that one should consider both components of exemplification and explanatory talk to inform instruction and particularly strategies to improve these instructions. The findings from the present study concur with this statement and there are two possible implications from these findings that I would like to elaborate on.

The categories and coded levels within and across the two teachers' lessons allowed me to provide a summative judgement on each lesson but also on the overall effect of the object of learning. Additionally I was able to make comparisons between the two teachers in terms of their selection and sequencing of examples and present a model of their exemplification and explanatory talk working together to mediate the object of learning. This provided insight into the different patterns and themes that emerged from each teacher, as well as the common patterns and themes that emerged from the factors that contributed to maintaining (or not) the cognitive demands of tasks. These insights can be used in teacher development, particularly for the participating teachers. The intention or hope would be that informing teachers of their approaches to the selection and sequencing of examples, as well as their habits in explanations may enable them to shift their practices.

The second implication from the findings, although beyond the scope of this project, provides potential future studies into the use of variation in textbook exercises or even teacher resources. This study has shown that textbooks and particularly their exercises influence the examples used in a classroom lesson. Some teachers, like as with seen with Teacher A, use the textbook as their primary source of examples and will therefore rely on the textbook to provide varying forms of concepts, procedures and representations of the content. Watson and Mason (2006), Marton and Tsui (2004) and Lo (2012) argue that when introducing a new concept, the examples used should provide opportunities for learners to experience variation that discern features that were important in the past but also to use these past features to discern new and critical features. If these examples are coming from the textbook exercises then it becomes imperative that variation is used appropriately and effectively. This paves the way for future studies as one can investigate multiple textbooks and their use of variation or perhaps investigate how different teachers use these exercises to enhance the different aspects of variation. If however a teacher still depends on the textbook and the exercises it provides, I recommend a strategy below that holds the best interest in terms of demonstrating variation.

6.4 Recommendations from the Findings

The results of this study indicate that when teachers use the textbook as their primary example source, they take the examples and exercises at face value. This is to say, that the teachers used the exercises as they are set in the textbook. This is not to say that textbooks cannot or should not be used, but rather that they should be used in a way that pays significant attention to the selection and sequencing of examples. Stein (2000) contends that the examples used in a lesson should match a specific purpose or goal. Furthermore Watson and Mason (2006) argue that variation amidst invariance afford learners the opportunity to discern critical features of the object of learning. Therefore I believe that these findings highlight the importance of teachers first examining the examples from the exercises and then selecting the examples that match the specific purpose or goal of their lesson and that highlight significant or critical features. Additionally if a teacher finds that the resources from the textbook are inadequate or do not match their lesson goals, they should include supplementary resources so that the example set allows their goals to be met.

6.5 Conclusion

The MDI framework together with the complementary additions succeeded in recognising the possibilities that each teacher's choice of exemplification and explanatory talk had in mediating the object of learning. The two teachers’ attention to exemplification and explanatory talk was different, yet in both classes task demand remained low. However by examining each component of exemplification and explanatory separately I was able to pinpoint the strengths and weaknesses of each teacher, and determine the possible reasons for the low task demand. Additionally I was able to examine the teachers’ explanations through their discourse and relate the factors from naming, legitimations and scaffolding that impacted the intended cognitive demand of the presented examples. The authors Adler and Ronda (2015) of the MDI framework developed the theoretical and analytical tool in the hopes of contributing to the mathematical description of practice and with the intention of developing the ongoing professional development of teachers. In using and adapting their framework, I have learned much about the selection and sequencing of examples in algebraic expressions, as well as the pertinence of teachers' explanations in the mathematical classroom.

6.6 Reflection and Implications for my own Teaching

When I set out to do this study I was motivated in part by my own difficulties that I had experienced in selecting and sequencing examples. I hoped that this research would make a personal contribution to my continued development as a mathematics teacher. This research has accomplished this task by enhancing my awareness of the difficulties that teachers experience with the selection and sequencing of examples. However the greatest significance that this research has presented me with is the interplay and connection between the exemplification and explanatory talk components. After conducting this study, I feel that I have gained a new reflection tool, one in which I can pinpoint my strengths and weaknesses in a classroom lesson. I have become aware of the example sets that I use, and if my learners are continuously struggling with the work and I find that I have to continue to provide scaffolding techniques, I stop and think about the example set. I question if I have created the connections to fusion examples by first exposing the learners to similar and contrasting examples, or if I need to go back and continue to present learners with more of these examples.

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APPENDIX A

EXERCISE 6.8
1) Learto is *p* years old. Her sister is twice her age. a) How old is her sister? b) How old will Lerato be in 10 years? c) How old was her sister three years ago? d) What will their combined ages be in *q* years time?
2) Nigella sells a cupcake that cost her *x* cents at a profit for *y* cents. What is her profit?
3) What is the next natural number after the natural number t?
4) What is the sum of three consecutive natural numbers, t being the smallest?
5) Write down an expression for: a) Any real number. b) Any odd number. c) Use any three values for the variable to check your answers.
6) If the length of a swimming pool is *q* meters and the breadth is *n* metres, what is: a) the area of the pool? b) the perimeter of the pool?
7) A ticket to the movies costs Rx and popcorn costs Ry. a) How much money does Mpho need to treat herself and three friends to a movie and popcorn? b) How much change can she expect from Rz? c) Rina, one of the three friends, refuses the popcorn. How much can Mpho expect to pay now?
8) Nicholas notices that there are *x* people present at the school assembly. Of these, *y* are male teachers. Double the number of male teachers plus 15 are female staff. How many learners were present at the assembly, expressed in terms of *x* and *y*.
9) In your wallet you have *p* R5 coins, the number of R2 coins is double the number of R5 coins, and you have *q* 50c pieces and *r* 20c pieces. Write down an expression for the amount of money in your wallet.
10) What is the average speed of a cyclist (in km/h) if he covers *p* km in *q* hours and *r* minutes.

APPENDIX B

Ethics Clearance WITS

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

28 July 2016

Student Number: 300330

Protocol Number: 2016ECE012M

Dear Danielle Barkay

Application for Ethics Clearance: Master of Science

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Teachers use of examples and the impact of language in maintaining (or not) the intended cognitive level.

The committee recently met and I am pleased to inform you that **clearance was granted**.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

MMabety

Wits School of Education

011 717-3416

APPENDIX C

Ethics Clearance GDE

For administrative use only:
Reference no: D2017 / 082
enquiries: Diane Buntting 011 843 6503

GAUTENG PROVINCE
EDUCATION
REPUBLIC OF SOUTH AFRICA

GDE RESEARCH APPROVAL LETTER

Date:	29 July 2016
Validity of Research Approval:	29 July 2016 to 30 September 2016
Name of Researcher:	Barkay D.
Address of Researcher:	43 West Road; Midrand; 1685
Telephone / Fax Number/s:	084 560 4469
Email address:	danileelbarkay@bryanston.com
Research Topic:	Teacher's use of examples and the impact of language in maintaining (or not) the intended cognitive level
Number and type of schools:	ONE Secondary School
District/s/HO	Johannesburg North

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved. A separate copy of this letter must be presented to the Principal, SGB and the relevant District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted. However participation is VOLUNTARY.

The following conditions apply to GDE research. The researcher has agreed to and may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

CONDITIONS FOR CONDUCTING RESEARCH IN GDE

1. *The District/Head Office Senior Manager/s concerned, the Principal/s and the chairperson/s of the School Governing Body (SGB.) must be presented with a copy of this letter.*
2. *The Researcher will make every effort to obtain the goodwill and co-operation of the GDE District officials, principals, SGBs, teachers, parents and learners involved. Participation is voluntary and additional remuneration will not be paid;*

2016/08/01

APPENDIX D

Information and consent forms

LETTER TO THE PRINCIPAL 2016ECE012M

Dear Mr Skelton

My name is Danielle Barkay, I am a postgraduate student in a Masters of Mathematical Education degree at the university of the Witwatersrand. I would like to invite you and your school to participate in my research study, which I am conducting as part of my degree. I am researching the teacher's use of examples and the impact of language in maintaining (or not) the intended cognitive level.

To help you with this decision, I have provided a brief explanation of the research study and what it entails.

My research involves videotaping four mathematics classes, two grade 8 classes and two grade 9 classes as well as the two teachers who teach these classes for the duration of the introduction of algebraic expressions and equations. Data collection methods will include the video recording as mentioned above as well as post teacher interviews. The whole process is expected to take three weeks and I wish to assure you that my presence will be of minimal disruption to the everyday operations of your school and of the four classrooms where I will collect the data from.

Parental consent will be requested for the learners and an information sheet will be provided for the parents and learners. This information sheet will discuss the purpose of the project, explain that the learners' mathematics class will be video recorded and invite the learners to take part in the research study.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study. The collected data will be stored on my password protected personal computer and the learners tests will be kept in a locked desk drawer.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Danielle Barkay

43 West Road, Midrand 1685

daniellebarkay@bryanston.com

INFORMATION SHEET LEARNERS 2016ECE012M

Dear Learner

My name is Danielle Barkay, I am a postgraduate student in a Masters of Mathematical Education degree at the university of the Witwatersrand. I would like to invite you to participate in my research study, which I am conducting as part of my degree. I am researching the teacher's use of examples and the impact of language in maintaining (or not) the intended cognitive level.

My project involves working with grade 8 and 9 teachers and learners to analyze the examples and mathematical language used by your teacher when assisting you in learning algebraic expressions. Participating in this project will include the video recording of your mathematics class during the teaching time of algebraic expressions (roughly two weeks)

The video camera will be placed in the back of the classroom and the focus of the video recordings will be on your teacher and what is written on the board. You will not be directly filmed or identified in the video. If you ask a question or participate in a class discussion, I will keep your identity anonymous by creating a pseudonym (fake name).

Remember participation in my study is voluntary, which means that you don’t have to do it. Also, if you decide halfway through that you'd prefer to stop, this is completely your choice and will not affect you negatively in any way.

All information collected during the research project will be kept confidential and I will use pseudonyms so that you remain anonymous at all times. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join me in the study.

I look forward to working with you!

Thank you

Danielle Barkay

43 West Road, Midrand 1685

daniellebarkay@bryanston.com

084 560 4469

Learner Consent Form 2016ECE012M

Please fill in the reply slip below if you agree to participate in my study investigating the teachers use of examples and the impact of language in maintaining (or not) the intended cognitive level.

My name is: ____________________________

Permission to observe you in class	**Circle one**
I agree to be observed in class.	YES/NO
Permission to be videotaped	**Circle one**
I agree to be videotaped in class.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be videotaped
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

INFORMATION SHEET PARENTS 2016ECE012M

Dear Parent

My name is Danielle Barkay, I am a postgraduate student in a Masters of Mathematical Education degree at the university of the Witwatersrand. I would like to invite you to participate in my research study, which I am conducting as part of my degree. I am researching the teacher's use of examples and the impact of language in maintaining (or not) the intended cognitive level.

My project involves working with grade 8 and 9 teachers and learners to analyze the examples and mathematical language used by your child's teacher when assisting them in learning algebraic expressions. This will entail videotaping some lessons of your child's class. I assure you that I will be committed to ensuring that there are minimal disruptions during the data collecting period.

To help you in this decision, a brief description of the study is provided. Your child's mathematics class will be videotaped during the teaching period of algebraic expressions. The whole process is expected to take two weeks. During this time, a video camera will be placed at the back of the classroom, away from your child. None of the learners will be directly filmed and only the back of their heads will be visible.

Participation in this study will not be advantage or disadvantage your child in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study. Your child’s name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please fill out the consent form and return it to the school, should you give permission for your child to participate. Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

Danielle Barkay

43 West Road, Midrand 1685

daniellebarkay@bryanston.com

084 560 4469

Parent's Consent Form 2016ECE012M

Please fill in the reply slip below if you agree to participate in my study investigating the teachers use of examples and the impact of language in maintaining (or not) the intended cognitive level.

I, ________________________ the parent of ______________________

Permission to observe my child in class **Circle one**

I agree that my child may be observed in class. YES/NO

Permission to be videotaped **Circle one**

I agree my child may be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she can withdraw from the study at any time.
- he/she can ask not to be videotaped
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

TEACHER INFORMATION SHEET 2016ECE012M

Dear Teacher July 2016

My name is Danielle Barkay, I am a postgraduate student in a Masters of Mathematical Education degree at the university of the Witwatersrand. I would like to invite you to participate in my research study, which I am conducting as part of my degree. I am researching the teacher's use of examples and the impact of language in maintaining (or not) the intended cognitive level.

To help you in this decision a brief description of this project is provided. Participating in this project will include:

1) Videotaping your grade 8 and 9 mathematics lessons that focus on the targeted algebraic concepts.
2) A post lesson interview (audio recorded) that will be conducted after the videotaping and learner assessment has been concluded (half an hour).

During the video recording of the lesson, the video camera will be placed at the back of your classroom so that it is out of your way and will not distract you or your students. The video camera will focus on the front of the classroom where you will be standing and the board will be clearly visible in the frame. I will take full responsibility of setting up the video camera and collecting it when the lesson has ended.

Lastly, I would like to invite you to participate in a post lesson interview. In this interview we will discuss the different examples that you used during your lessons. Remember, participation in my research is voluntary,. Also, if you decide halfway through that you'd prefer to stop, this is completely your choice and will not affect you negatively in any way.

All the information collected during the research project will be treated confidentially and I will use pseudonyms so that your name and identity will be kept confidential at all times and in all academic writing about the study. All data collected will be stored securely for five years after the project has been concluded and will then be destroyed.

Participation in this study will not disadvantage you in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study. If you agree to participate in this research study, please complete the attached consent form and return it me. If you require any further information, please contact me on the details that I have provided below.

Thank you very much for your help and I look forward to working with you.

Yours sincerely,

Danielle Barkay

43 West Road, Midrand 1685

daniellebarkay@bryanston.com

084 560 4469

Teacher’s Consent Form 2016ECE012M

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called: An analyis of the influence of the example's cognitive level on the teacher's mathematical language.

I, ___________________________ give my consent for the following:

Permission to observe you in class **Circle one**

I agree to be observed in class. YES/NO

Permission to be audio taped **Circle one**

I agree to be audio taped during the interview or observation lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Permission to be interviewed **Circle one**

I would like to be interviewed for this study. YES/NO

I know that I can stop the interview at any time and don’t have to

answer all the questions asked. YES/NO

Permission to be videotaped **Circle one**

I agree to be videotaped in class. YES/NO

I know that the videotapes will be used for this project only. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________