A THESIS ENTITLED.

An analysis of the economic aspects of insulation in buildings, with special reference to the stresses in framed structures, resulting from changes in temperature, and humidity.

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1. INTRODUCTION.

The consideration of the problems of econimic insulation against temperature variation has only come to the fore in recent years. The outstanding developments in methods of central heating which have grown up with the XXth Century have brought with them a desire to investigate the manner in which heat losses occur in different types of buildings.

After the Great War, shortage of capital, lack of fuel, and the general want of suitable material for building caused a demand for general construction on much cheaper lines and with the use of much lighter material.

One difficulty of this new method of building was, that very little data on the insulating properties of the materials chosen for use was available, and what little data there was, was largely incorrect.

The consumption of fuel required to heat buildings during the winter months reached enormous proportions because of this, and it was quite impossible to lower the consumption without discomfort.

In direct contrast, during the summer months, the rays of the sun penetrated to such an extent that adequate cooling became an even more urgent problem.

In certain countries in which long bright cold nights are a feature of the winter climate, walls and especially flat roofs of buildings radiate heat rays continuously into the atmosphere for 24 hours every day. The atmosphere being of comparatively infinite proportions absorbs these heat rays, providing only short periods of sun radiation in exchange. In fact in mid winter in extreme latitudes the sun only shines for such a short period out of the 24 hours that the heat

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loss radiated from buildings reaches serious proportions.

Progress during the last few years has however gradually brought with it increasing appreciation of the importance of insulating walls, floors and roofs. Against temperature changes, noise, draughts and dampness.

It is necessary to refer to these four simultaneously because they have so many factors in common that they are virtually inseparable.

For instance, any building which will absorb water must obviously be unprotected from temperature changes. A humidity of 100% in the air is sufficient to reduce insulating properties by 50% in building materials and temperature changes, of such a nature as to produce condensation reduce them still further.

This difficulty has already been found in S.Africa, especially on the high-veldt where sudden cold following upon heavy rain causes the relative humidity of the air to rise above the normal figure of 60% - 80% giving rise to condensation.

Again - materials which allow the passage of air through them cannot possibly be sound proof. One has to pay particular attention to this fact when chosing so-called modern windows and doors built in light materials. Well-fitting doors and windows are to a certain extent self insulating because they tend to prevent the passage of air, sound and water.

Any material must subscribe to a certain specification if it is to have adequate insulating properties.

It is extremely difficult to find materials which are non-hygroscopic and the usual practise therefore is to treat normal materials with one or two waterproof layers in the form of a protective covering.

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The property of whether or not any material will allow the passage of air is one of the most important factors in finding the perfect insulation material for both heating or ventilating problems.

When considering external walls, it sometimes happens that too much importance is paid to temperature and noise insulation whilst too little is paid to insulation from dampness.

In recent years reinforced concrete has come into general use in S.African building trade. Scarcely a building exists to-day which has no reinforced concrete in it. Simplification in the methods of construction brought about by the use of re-inforced concrete has resulted in a big increase in the uses to which it is put, without scientific development keeping pace with all the factors arising from these new uses. As a result defects occur in constructional work which sometimes have serious consequences.

Frequently the cause of these defects lies in the system of preparation of the materials.

For example:-

The reinforced concrete Engineer in conjunction with the contractor has designed and placed in position respectively the reinforcing steel. They are followed by the general Contractor who pours in the surrounding concrete. During this procedure it is possible for careless workmen, generally natives, to move the reinforcing steel, with the result that it no longer occupies the position intended by the designer. This is very often the case with cantilever designs.

Frequently too, the cause of defects can be traced to a poor mixture, which has not been kept up to the composition standard called for by the designer -

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or is not sufficiently well mixed.

In this country however, stresses set up by fluctuations of temperature as well as by super imposed loads are more important causes of cracks than the technical errors mentioned above. It is as a general rule the calculations appertaining to super imposed load stresses which are accurately checked, whilst those set up by temperature changes, because they are more complicated, are frequently neglected altogether.

It can be said therefore, that the reinforced concrete Structure has on the whole not been fully adapted to meet the physical conditions of S.Africa.

Expansion joints and insulation against temperature extremes have not been taken into account.

In the case of cracks actually caused by temperature variation it has often happened that either bad soil or poor foundations have been blamed.

Every building is subjected to periodic heating and cooling.

Divers circumstances control this fact.

For example:-

Most houses are heated inside in the winter whilst being simultaneously cooled outside by the wintry air.

Again- there is a tremendous difference between winter and summer temperatures and often an even greater difference between day and night temperatures especially in winter.

For instance the outer surface of any building exposed to direct rays from the sun often reaches a temperature of 70°- 75°C having absorbed heat from the sun whilst at night, temperatures are down only a few degrees.

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2. TEMPERATURE CONDITIONS IN SOUTH AFRICA

What exactly is the danger to buildings from temperature fluctuation?

Only two causes will be dealt with here.

A. That the temperature of the entire building will rise and fall with the surrounding air.

In other words winter and summer air temperatures will be winter and summer building temperatures and in each case maximum summer and minimum winter temperatures have to be taken into account. It is already known that in S.Africa the temperature difference between day and night, especially in winter, is fairly large. This big temperature difference has to be taken into consideration when materials are being used which have very little or no heat accumulation capacity, as for instance corrugated iron. The only way to compensate for this is to make use of materials with great capacity for the accumulation of heat. For example:- To build heavier walls, to apply proper insulating, etc.

The selection of building materials with the properties of great heat accumulation factors on the one hand and of cold resisting factors on the other play an important part in the precautionary measures which have to be taken against thermal expansion and contraction - prevention of condensation and the creation of economic heating and cooling conditions.

The actual meaning of heat accumulation and absorption and cold resisting or cold accumulating materials will be discussed later in the paper.

The average temperatures throughout the year in Johannesburg according to data compiled by the Union Observatory over the last 54 years are as follows:-

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	Mean Max.	Mean Min.
January	25 °C	13.5° C
February	24.5° C	13.0° C
March	23 °C	12.5° C
April	21 °C	10 °C
May	18 °C	7.5° C
June	16 °C	5 °C
July	15.5° C	4.5° C
August	18.5° C	6.5° C
September	21.5° C	9 °C
October	23.5° C	11 °C
November	24 °C	12 °C
December	24.5° C	13 °C

(The thermometer used in the determination of the above figures was mounted in a Stephenson Screen).

The maximum difference between mean max. and mean min. in the last 34 years in Johannesburg can be seen from the above to be 25°C - 4.5°C = 20.5°C.

The highest temperature ever to be recorded since 1904 was on the 21st December, 1935 and was 34°C. The lowest was on the 23rd July, 1926 when it stood at 7.5°C. The maximum difference ever recorded is therefore 41.5°C.

Other critical temperatures in the Union of S. Africa are as follows:-

<u>Johannesburg.</u>	Observatory	Max. 31° C, min. -2° C
	Other suburbs	Max. 31/32°C, Min. -2/-3°C
<u>Pretoria.</u>	Arcadia and Sunnyside	Max. 33/34°C, Min -5/-6°C
<u>Middelburg(Tvl).</u>		Max. 32/33°C, Min. -6°C
	and once in June 1905	-9°C
<u>Barberton.</u>		Max. 35/36°C.
<u>Bethal.</u>		Max. 31/32°C
<u>Klerksdorp.</u>		Max. above 38°C.

A glance at some of the temperatures quoted above is sufficient to show that it is not out of place to take a working figure of 25-30°C into any calculations on thermal expansion, and for insulating problems, the figure of 25°C max. and 0°C min. should prove acceptable.

It is extremely important, when determining the tem-

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perature of solids, fluids or gases, to ensure that there is a heat balance between the thermometer and the specimen under test.

The transfer of heat brings with it problems of heat conductivity and heat radiation.

This means that not only the temperature of the thermometer and the specimen must be the same but also conductivity and radiation of heat must be mutually equal if a true heat balance is to be maintained.

The exchange of heat between the thermometer and the specimen under test is influenced by the external heat conductivity of the thermometer. This in turn is affected by the size, shape, surface and the material of which the thermometer is manufactured and also by heat capacity, thermal conductivity, ventilation and denseness.

The temperature shown on any thermometer (t_T) is always more or less below the effective temperature (t_e) of the specimen and this is greatest when its internal heat conductivity is very high.

The **inertia** of the thermometer, can be written thus:- $n = t_e - t_T = \alpha \frac{dt_T}{dz}$

α being the coefficient of inertia of the thermometer. and $\frac{dt_T}{dz}$ the temperature fluctuation of the thermometer per minute.

t_e being assumed constant then by integration:-

$$t_e - t_T = e^{-\frac{z}{\alpha}} (t_e - t_{Ta})$$

t_{Ta} being the temperature of the thermometer at the beginning.

The rapidity of exchange of temperature between the thermometer and the specimen under test can be increased in three ways.

1. By enlarging the surface of the thermometer (A).
2. By reducing its heat capacity (C).

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3. By increasing its external heat conductivity (λ).

Thus $d_\omega = \frac{S}{\lambda A}$

After a certain period has elapsed a system which has been perfectly sealed thermally is fairly well balanced in thermodynamical terms. It is however, possible in the future that an even better balance will be obtained by methods not yet generally available.

In taking the temperature of the atmosphere certain great difficulties arise since the result is influenced by direct and indirect radiation as well as by the colour and surface of the thermometer used.

The adverse influence of these factors can be offset by enclosing the thermometer in a form of cylinder with a double wall. Recently a so-called screen has also been used (Invented 70 years ago by Stevenson and generally known as the "English Screen" or "Stevenson Screen") which as good as eliminates all kinds of radiation.

The temperature of the air is therefore by this means reduced to a question of measuring the kinetic energy of the air molecules alone.

There are various methods of mounting a thermometer for the purpose of taking air temperatures without radiation. None of them are absolutely exact.

If the screen is exposed to intense radiation the thermometer will read higher than it should.

If the screen is exposed to intense cold the thermometer will read lower than it should.

Only in cloudy or windy weather are the figures given by the thermometer sufficiently accurate to be accepted as exact.

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B. THE TEMPERATURE INCREASES ON ONLY ONE SIDE OF THE structure.

On account of the more or less insulating capacity of the material used and the relatively short heating period, heat is not transferred from one side of the structure to the other. This means that one side remains at a relatively lower temperature whilst the other is overheated. The sun may easily cause this effect but it is also possible to create it artificially by the close proximity of boiler rooms, cold storages etc.

In one case the temperature of an overheated concrete slab was taken by the writer when in Hungary. This slab was exposed to the sun on one side and showed 66°C on the surface. The ambient air temperature was 25°C and on the reverse side of the slab the temperature was only 28°C. (See Fig.1).

The high surface temperature was due to the heat absorption of the concrete. The amount of heat which penetrates below the surface depends on the insulating capacity.

Heat Energy radiated by the sun - so called "Solar Constant" is given as c.a. 1,200 gr. cal. per sq. centimetre per minute. The Solar Constant is defined as the amount of Solar Energy which would fall on 1 sq. centimetre in one minute when placed at right angles to the rays of the sun just above the Earths surface. It is not strictly constant in value and there are slight variations of this figure in actual practise.

All instruments for measuring the Solar Constant work on the same principle.

A hollow body is used with one very small opening which allows entry of the sun's rays to the measuring

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instrument. There are various different arrangements of this scheme.

One consists of a cavity with an absolutely black surface inside. This black surface absorbs all the rays which reach it through the opening. All the absorbed heat is taken up by a fluid of known specific heat. (See Fig.2.).

Care has to be taken to maintain a uniform increase in the temperature of the fluid and this is done by constantly stirring it. From the temperature of the fluid at the beginning of the experiment and the temperature at the end, as well as from the known Constants of the instrument it is possible to determine the intensity of the radiated energy.

Another method is to use a hollow body with a very large heat capacity.

Inside this is a small plate absolutely black with a very small heat capacity. This plate is exposed to the rays of the sun. (See Fig. 3).

The determination of the intensity of radiation is then calculated from the temperature of the black plate and from the other constants of the apparatus. The choice of a suitable opening for the entrance of the rays of the sun is an important factor in this piece of apparatus - too big an opening or unsatisfactory insulation will cause loss of heat and so must be avoided.

The Solar Constant has also recently been measured by means of so - called Pyrheliometers. The earliest known form of this instrument was due to Pouillet. It consisted of a blackened shallow cylindrical Calorimeter, containing water, with a thermometer in it. The sun's rays are normally allowed to fall on the flat base of the calorimeter. Knowing the thermal capacity

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of the calorimeter and its contents it is possible to calculate the heat received per unit area and per second.

In Angstroms instrument there are two strips of blackened platinum. One strip is exposed to the rays of the sun and the second strip is heated to exactly the same temperature electrically - under these conditions the electrical energy dissipated by the second strip is equal to that received by the first.

As mentioned elsewhere the Solar Constant is not a true constant but is subject to slight fluctuations.

Its average value for every month in the year is as follows:-

January	2.003	gr. cal. per cm^2 per min.
February	1.989	gr. cal. per cm^2 per min.
March	1.960	gr. cal. per cm^2 per min.
April	1.929	gr. cal. per cm^2 per min.
May	1.899	gr. cal. per cm^2 per min.
June	1.887	gr. cal. per cm^2 per min.
July	1.887	gr. cal. per cm^2 per min.
August	1.894	gr. cal. per cm^2 per min.
September	1.918	gr. cal. per cm^2 per min.
October	1.935	gr. cal. per cm^2 per min.
November	1.980	gr. cal. per cm^2 per min.
December	2.001	gr. cal. per cm^2 per min.

Observations by the Medical School, Johannesburg gave the figures of 1.84 - 2.02 gr. cal. per cm^2 per minute. These figures agree fairly well with those obtained in Potsdam (Germany) between 1907 - 1923.

The intensity of radiation is weakened in its passage through the strata of air surrounding the earth.

Not only is the intensity reduced considerably but the composition of the radiation also changes. Heat

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rays of certain wavelengths are absorbed by vapour and carbonic acids. The radiation is also broken up by the presence of gas molecules and dust in the air.

Effective heat by radiation from the sun on a flat surface placed at right angles to the direction of the rays depends on :-

1. The position of the sun. The nearer the sun to the horizon, the thicker each stratum of air through which the radiation has to pass.
2. The proportion of dust and smoke in the air.
3. The altitude of the site chosen for the test above sea level.
4. The humidity of the air.

The effective heat energy at right angles to the rays of the sun has been measured in Johannesburg during the winter and on the 12th July at 1 p.m. the value was found to be .462 gr. cal. per cm^2 per minute. i.e. about one quarter of the solar constant. This figure is in close agreement with figures obtained in Potsdam (Germany) also in winter where .463 gr. cal. per cm^2 per minute was obtained with the sun 10° over the horizon.

The higher the sun, the more intense does its effective radiation become. In S.Africa the effective heat energy is greater in winter than in summer because of the low humidity of the air. Johannesburg receives an especially intense radiation because of its altitude.

The actual radiation can be measured by Photometry, Calorimeter Photometric Methods and Sun Radiation Autographs. Measurements by Photometry incorporate the amount of heat absorbed together with other effects, bringing in the intensity of radiation in terms of the wavelength of the received rays.

Simply explained, Photometry is a means of measuring the effect of radiation in the form of its 'light' value.

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This 'light' value is often termed the 'Brightness' which depends for the greater part on the wavelength. Light itself occupies only a short range in the wavelength spectrum.

Apart from the 'Brightness' of equal importance is the Chemical action and the electrical effect of the radiation.

Both methods of measurement bring in the Ultra Violet Rays. The terms used in Photometry such as candles, foot candles, lumen, lux, etc., cannot be discussed here for lack of space does not permit. An intensive study can be made by reference to "Handbuch der Meteorologisten Instrumente" by E.Kleinschmidt, Berlin.

It is worth while to mention the photo electric effect of radiation, which will cause electrons to be released from the parent atom and enables them to generate minute electrical current on their own.

The surface of a building is very rarely directly at right angles to the rays of the sun - on those rare occasions when it is at right angles it is only for a very short period of time daily. The intensity of radiation on any surface of a building is therefore subject to wide variations. It also varies with the zone - with the geographical latitude and longitude - with the month of the year and with the hour of the day.

The surface of any building rises in temperature, due to absorption of radiated heat energy, to a larger or smaller extent, far above the ambient air temperature. A temperature of 50° to 60° C is more than sufficient to over heat a wall or a roof with a low capacity for accumulating heat.

Tests made at the Union Observatory have shown that very high surface temperatures occur through radiation from the sun.

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The results of tests made in 1904/5 were as follows:-

Colour.		Surface Temperature.
Black	Summer up to	60°C
	Winter up to	43°C
Bright	Summer up to	38°C
	Winter up to	21°C
Diff.	Summer up to	43°C
	Winter up to	41°C

Despite the intensive winter radiation the lower surface temperature is explained by the fact that there is a much shorter period of sun radiation during the day and a much greater reflection from the surface into the colder surroundings.

The influence of colour on the question is shown by tests made in Germany by K.Schropp and J.S.Cammerer. The difference between the surface temperature and the temperature of the surroundings (over - temperature) is according to J.S. Cammerer as follows:-

Colour.	Maximum Over Temperature	Black In comparison with
Black	54	100%
White and Bright	21	38%
Red	30	55%
Yellow	27	50%
Green	31	57%
Blue	38	70%
Bright aluminium	20	37%

Messrs.Schropp and Cammerer also determined the possible maximum over-temperatures with surfaces placed at right angles to the rays of the sun and the surroundings.

The list above shows that white surfaces absorb only 40% of the quantity of heat absorbed by black surfaces. White has therefore a good effect but the extent of its effective value depends on the quantity of smoke

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and dust etc., which accumulate on the surface after a certain period of time.

There is a small influence on the surface temperature which is caused by the temperature on the opposite side of the material and this depends on the insulating capacity of the material itself, but it is so small that it is not worth mentioning.

A dark surface absorbs about 90% of the total radiated heat which falls upon it. The final temperature is far above the temperature which may penetrate the wall from an adjoining room. The over heated surface re-radiates a certain portion of the heat absorbed, back into the surroundings.

Only 5-25% of the heat absorbed actually penetrates into the material but even this is sufficient to cause a rise of temperature in the whole structure which may have serious consequences.

Surface over-heating is an important factor in the consideration of thermal expansion in buildings.

Surface temperature depends upon:-

1. Intensity of radiation brought about by atmospheric conditions.
2. Insulating Capacity of the building as a whole.
3. Heat absorption Capacity of the surface.
4. Ambient Air Temperature and air movement in the immediate neighbourhood of the surfaces.

Any form of building with a low insulating capacity will be overheated in no time, vide — Corrugated iron roof. Building materials of high insulating capacity can themselves be a danger if subjected to one sided heating.

Loss of heat by re-radiation into the cool atmosphere at night has disadvantages because it introduces one sided cooling which is equally dangerous.

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On the other hand if cooling down of walls and roofs is expedited too many difficulties arise through the fact that heating conditions in the winter months become uneconomical. (See section on "heat accumulation").

3. LOSS OF HEAT BY NIGHTLY RADIATION.

Not many experts are engaged in research work devoted to loss of heat by re-radiation into the atmosphere. This problem is more important than one would think especially in countries where bright cold nights are experienced and particularly on the high veldt with its dry continental climate with big variations between day and night temperatures.

The walls of buildings and particularly flat roofs are re-radiating heat all night into the cold and infinitesimally large atmosphere; its temperature being theoretically -273°C; without receiving any direct radiation from the sun in exchange.

The extent of the radiation may be judged from the difference between early morning and noon temperatures. In desert areas the fluctuation in daily temperature is as much as 20 - 40°C.

Flat roofs give up more heat than walls overnight. The re-radiation from walls is balanced by re-radiation from surrounding buildings. In the tropics it is usual to sleep either on or under a flat roof simply because the temperature is at least bearable owing to the nightly cooling.

Viewed from the technical angle there are many arguments for and against the use of flat roofs on the continent.

It is possible to offset the incidence of night re-radiation by using materials with very large heat accumulating capacity or even by insulating the structure with similar materials.

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When designing heating establishments it is necessary to calculate for a bigger margin of safety. In cities, due to protection from surrounding buildings 10% extra is advisable whilst in the open country 20% or more is necessary.

The heat energy radiated into the atmosphere from solids can be determined by simple calculation once the temperature equivalent of the re-radiation is known.

The temperature equivalent is that temperature which has to replace the universal figure of -273°C in the formula taking into account atmospheric conditions such as the presence of clouds etc. Values for this equivalent temperature have been calculated and proved by practical tests by Dines, Angstrom and Homen., they are as follows:-

Bright weather : from -4[illegible]°C to -30°C.

Cloudy weather : up to +[illegible]0°C.

Generally speaking, these figures are lower in winter than in summer on account of the lower humidity.

Suppose the surface temperature of a flat roof is 15°C and the temperature equivalent -55°C then the radiated heat energy will be:-

$$E = C\left[\left(\frac{273+t_1}{100}\right)^4 - \left(\frac{273+t_x}{100}\right)^4\right] = 170 \text{ kg.cal/hour p.sq M.}$$

Where C=Coefficient of radiation.

and -273°C = the absolute temperature (-273.2°C to be exact).

Absolute temperature is the absolute zero temperature conceived by mankind - it is the point at which material particles have no motion at all.

t_1 = the temperature of the flat roof.

t_x = the temperature equivalent.

The radiated energy is proportional to the fourth power of the absolute temperature (Law of Stefan - Boltzmann).

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The quantity of heat lost by radiation always increases as the temperature rises although the difference between the surface temperature of the roof and the temperature equivalent may remain the same.

For example $2^4-1^4=15$

$3^4-2^4=65$

$4^4-3^4=175$ and so on.

The radiating surface being warmer during the day than during the night it stands to reason that the re-radiated heat energy will also be more in the daytime than at night. The quantity of heat received from the sun is always more than that given back in re-radiation which is the reason why surfaces do not cool down during the day.

4. HEAT AND COLD ACCUMULATION.

The heat accumulation capacity of a building as already mentioned, plays an important part in the amount by which any building may be over-heated or over-cooled as well as in the economic heating and cooling of such a building.

The work of heat and cold accumulation in any building is the same as the work done by the flywheel of an engine. The flywheel - due to the kinetic energy stored in it, helps the engine over the dead centre position. Heat accumulation or storage of heat helps a building through the period when winter heating is being stopped, and the inside has to be kept cold during the summer. Buildings with walls and roof of great heat and cold accumulation capacity will be cold in summer and pleasantly warm in winter. The rooms in this type of building will not cool down quickly after heating has been stopped. Conversely it will take a long time to warm up the rooms in this type of building once they have been cooled down. This is a disadvantage which has to be taken into consideration when using so-called light insulating materials in buildings with a minimum heat accumulation capacity.

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Two materials having the same insulating capacity very often have quite different heat accumulation capacities.

Light insulating materials may often be equal to heavier materials as regards insulating capacity when subjected to continuous heating but they will cool down very much quicker when subjected to periodic heating.

In summer the nights pass so quickly, that buildings with great heat accumulation capacity are reheated by the sunshine the following morning before they have fully cooled down from the previous day. The lost heat energy is then made good again by heat absorption and retained by heat accumulation. The position is reversed in winter when with the long nights, sunshine falls on the building for such a short period that it never gets a chance to recover from the cold conditions.

The influence of temperature fluctuations during the day or night on different heat accumulating materials is not worth mentioning.

Heat accumulation capacity of a body is defined as its specific heat multiplied by its own weight. The result is multiplied by the difference between its own temperature and that of its surroundings.

e.g. $H = S.C. (t_a - t_b)$

The weights and specific heats of most of the important building materials can be found in the following list.

Name.	Average weight	Specific Heat 0 C - 100 C
Aluminium	2700	0.22
Zinc	7100	0.094
Copper	8900	0.094
Iron	7850	0.115
Concrete	2200	0.21
Bitumen	1000	0.22

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Gypsum	1000	0.20
Sand	1600	0.22
Slag.	750	0.18
Brick	1600	0.22
Clay	1900	0.22
Wood	400 - 800	0.57
Stone	1600 - 2700	0.20 - 0.[illegible]
Kieselguhr	300 - 600	0.21 - 0.25
Tarred Cork	180 - 300	0.31 - 0.36
Water	1000	1.00
Ice	900	0.50
Engine Oil	900	0.40 - 0.45
Air flue gas	1.3	0.241
Superheated steam	-	0.47 - 0.55

The heat accumulation capacity has to be considered from the following view points.

1. Its influence upon heating and cooling conditions in order to create economical winter heating and summer cooling.
2. Condensation.
3. With an eye to the prevention of too large a fluctuation between day and night temperature of the whole building.

5. THE TRANSFERENCE OF HEAT.

The temperature of a room which has been warmed is higher than the surface temperature of the surrounding walls. Once the heating has been stopped, assuming that the temperature outside the walls is lower, then the room temperature will drop to the surface temperature of the walls.

The air and surface temperature will also gradually drop proportionately until they reach the outside temperature, when heating is started the same procedure occurs only it is reversed and for drop of temperature read rise

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of temperature in the above paragraph.

The surface temperature of a wall under the influence of sunshine will be higher than the temperature of the surrounding air - If the temperature on one side of a wall is higher than on the other, then heat will be transfered from the warmer side to the cooler.

With no heat accumulation materials in use, once heating has stopped, or if a sudden drop in temperature occurs, condensation will take place.

All buildings are periodically heated and cooled down. This can be done from the inside (as in winter) or from the outside (as in summer). In the first instance a portion of the heat will be accumulated by the surrounding walls, floors and roof, and the balance will gradually seep through to the outer atmosphere. After some time a steady flow of heat will be reached, when all the heat will seep through and no more will be accumulated.

In the second instance, there is a continuous supply of heat from outside the building. The quantity of heat absorbed will depend on the insulating capacity and the heat accumulation of the building, assuming a constant surface temperature outside (see also radiation, surface temperature etc).

When studying economic heating and cooling problems three periods have to be considered.

The first period is the heating of the room and its surroundings.

The second period covers the room at constant temperature and all superfluous heat seeping through walls.

The third period is the cooling of the room, walls, floors and roof to the outside temperature.

If a start is made by cooling the room (such as in cold storage) then the reverse position takes place. The difference is in the second period when cold will be con-

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sumed by the surroundings and all superfluous cold will seep through the walls.

The second period will here be taken first for consideration because this represents stationary conditions in the flow of heat. Afterwards the first and third periods will be discussed.

A. Steady Flow of Heat.

In the case of two rooms with temperature ϑ_1 & ϑ_2 which are separated by a wall with surface temperatures t_1 & t_2 heat will be transfered from the hotter to the colder room.

The passage of the heat through walls with parallel faces occurs in three ways.

1. By convection.
2. By conduction.
3. By radiation.

1. In heat transfer by Convection the body which carries the heat also moves from one place to the other. Because there is movement of the infinitesimally small particles of fluids or gases so heat will be transferred. Transfer of heat by convection is natural when the body carrying heat is being moved by temperature fluctuation. Next to a warm wall, air will rise giving place to cold air which in turn will take heat from the wall. Next to a cold wall the air will drop and so in its circular route the air is continually taking new heat energy from the warm wall.

Convection is termed artificial when the heat carrying body is given movement by some outside agency such as a fan.

When heat is passing through a wall the convection conditions on both sides of the wall are of great importance. Due to convection it becomes possible that; when passing heat through a wall; the surface temperature

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of the wall on the warm side is cooler and on the cold side is warmer, than the adjoining air temperature. (See fig.4).

The coefficient of insulation of a brick wall is 72.6% and the resistance to convection in the most unfavourable case only 27.4% of the total resistance to the passage of heat. If a wall of glass is used the coefficient of insulation is only 2.2% and the resistance by convection 97%.

The lifting power which sets the air in motion increases with the temperature and also with the height of the wall along which it moves (as an example take the lifting power in a tall factory chimney). It also depends on the material of which the wall is built. The friction is less if walls are smooth and under these conditions the flow of air is more intense, the loss of heat is also lower because there is less contact with rough surfaces.

The definition of the coefficient of convection is that quantity of heat expressed in kilogram - calories which is taken by 1 sq. metre surface of a wall from its surroundings when the temperature difference is 1°C. The flow of heat being taken to be steady.

Radiation is already expressed in this definition of the coefficient of convection and its value depends on the absolute temperature and the coefficient of radiation.

The formula compiled by Lorentz - Nusselt gives some very usefull values, but these are based on absolutely still air conditions.

$\alpha = C + 2.2\sqrt[4]{\vartheta - t}$ for vertical self supporting walls.

$\alpha = C + 2.8\sqrt[4]{\vartheta - t}$ for horizontal construction.

where C = the coefficient of radiation of the wall in question.

/T....

ϑ = the temperature of its surroundings.

t = the temperature of its surface.

and a = $\frac{1}{\alpha}$ resistance against convection.

These figures do not agree with figures obtained in practice. "Still air" is very seldom met with in practise.

The smallest motion of the air tends to increase convection and reduce its resistance to heat transfer.

In channel sections and in openings for insulating purposes which are not more than 1 - 2 c.ms. in extent and which are surrounded by rough materials; friction is increased; circulation and therefore convection, is reduced far below the values given by the Lorenz - Nusselt formula.

It is perhaps best therefore to set aside formulas in dealing with this type of problem and to depend on scientific tests. For the purpose of calculation of loss of heat and cold by convection the fol-lowing approximate values may be used. These are the practical values obtained by scientific tests.

For loss of heat through external walls and floors assuming the possible presence of wind.

α = 25 or a = 0.04.

For internal walls with no likelihood of wind.

α = 7 or a = 0.14.

For horizontal construction with the flow of heat from the bottom upwards.

α = 9 or a = 0.11

In corners of walls.

α = 5 or a = 0.20

In order to make exact calculations, especially when the temperature is very high, the quantity of heat lost by conduction, circulation and radiation has to be considered as well as the coefficient of convection.

/The.....

The following lists will be found very useful for this purpose.

<u>Coefficients of convection between solids and air</u> -
<u>Radiation being neglected along vertical walls.</u>

Velocity of the wind M/sec	Along smooth surfaces.	Along rough (Jurgens) surfaces.
0.0	4.6	5.0
0.5	7.0	7.5
1.0	8.7	9.2
2.0	11.9	12.7
3.0	14.3	15.2
4.0	17.8	19.0
5.0	21.8	25.1
10.0	37.0	39.5
25.0	76.0	80.7

<u>Values (c') proportional to the radiation - (Cammerer).</u>

for c = 4.0 (oxidised metals except aluminium).

for c = 4.6 (building materials).

Surface temperature	TEMPERATURE OF THE AIR					
	0°C		20°C		40°C	
	C=4.0	C=4.6	C=4.0	C=4.6	C=4.0	C=4.6
0°C	3.2	3.7	-	-	-	-
20°C	3.6	4.2	4.0	4.6	-	-
40°C	4.0	4.6	4.4	5.1	4.9	5.7
60°C	4.5	5.2	4.9	5.7	5.4	6.2
80°C	5.0	5.8	5.4	6.3	5.9	6.8
100°C	5.5	6.3	6.0	6.9	6.5	7.5
150°C	7.0	8.1	7.6	8.7	8.2	9.4
200°C	8.9	10.2	9.5	10.9	10.0	11.5

Thus $\alpha = \alpha' + c'$ and the total resistance against the propagation of heat by convection.

$$a = \frac{1}{\alpha} = \frac{1}{\alpha' + c'}$$

For example:-

What would be the resistance to the transfer of heat

/between.....

between plastered brickwork with a surface of temperature of 40°C and air at a temperature of 0°C - velocity of the wind being 5 M sec^{-1} ?

$$a = \frac{1}{d' + c'} = \frac{1}{23,1 + 4,6} = 0,036$$

There is very little resistance to the transfer of heat between running water and saturated steam and for insulating calculations they may be neglected.

In the case of superheated steam - except when condensation enters into the question - the resistance is much more; but on account of the velocity of the flow of heat it is still inconsiderable as compared with the insulating capacity. The resistance to the passage of heat by convection in flue gas is 25% more than the resistance of air under the same conditions.

2. In heat transfer by <u>Conduction</u> the heat passes from the hotter to the colder part of the system without any relative movement between the two parts. This is therefore only a question of transport of energy. Conduction depends purely on the nature of the substance in use, in other words upon its specific thermal conductivity. The definition of thermal conductivity is that quantity of heat, expressed in kilogramme calories which can be passed in an hour through a cross section of 1 sq. metre of a wall of 1 meter thickness and a temperature difference of 1°C between faces.

The fundamental formula relative to heat movement in a homogeneous body is given by Fourier as:-

$$dQ = -\lambda \, dA \frac{\partial t}{\partial x} z$$

where Q = the heat transferred in kg. cal.

λ = thermal conductivity

A = The cross section at right angles to the transfer.

t = the temperature of an infinitesimally small

/particle.....

particle of the body.

x = The distance between such two points

z = the point of time for which the temperature is $t°C$.

By using this formula it is possible to determine the quantity of heat transferred either by a constant or a changeable flow.

The differential equation for the heat transfer is:-

$$\frac{1}{a}\cdot\frac{\partial t}{\partial z} = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial v^2}$$

x, y & v being the three coordinates of the direction in which heat is being transferred.

In technical calculations the lateral movement of heat may be neglected.

The formula for heat transfer in one direction (x) will be :- $\frac{1}{a}\cdot\frac{\partial t}{\partial z} = \frac{\partial^2 t}{\partial x^2}$

This is the case when heat is passing through walls, floors and roofs etc.

If the surface temperature on both sides of an infinitely large wall is constant then the differential equation will be :-

$$\frac{\partial^2 t}{\partial x^2} = 0$$

By integration then:-

$$t = Wx + U$$

This means that assuming a steady flow of heat the drop of temperature will be directly proportional to the thickness of the wall. (δ) (See fig. 5).

The value of the constants (W and U) can be determined from the initial and limiting values.

Limiting values: Given the surface temperature on both sides of the wall. x being equal to 0.

Then $t = U = t_1$

and for $x = \delta$

 /t......

$$t = W\delta + t_1 = t_2$$

or $Wt = t_2 - t_1$

The amount of heat (dQ) passed through the area (dA) in a unit of time will now be in accordance with the fundamental equation of Fourier

$$dQ = \frac{\lambda}{\delta} dA (t_1 - t_2)$$

Should the wall be constructed of several layers with a different coefficient of conduction of heat then the flow of heat at right angles to the layers will be :- (See Fig.6).

by addition

$$dQ \left(\frac{\lambda_1}{\delta_1} + \frac{\lambda_2}{\delta_2} + \ldots\ldots\ldots \frac{\lambda_m}{\delta_m}\right) = (t_1 - t_2)\, dA$$

$$\text{or} \qquad dQ = \frac{t_1 - t_2}{\Sigma \frac{\delta}{\lambda}} \cdot dA$$

which can be reduced to the following form.

$$dQ = \frac{\lambda_m}{\delta_m} \cdot dA (t_1 - t_2)$$

δ_m being the total thickness of the wall

λ_m being the average coefficient of conduction of heat.

$$\lambda_m = \frac{\delta_m}{\Sigma \frac{\delta}{\lambda}}$$

The average coefficient of conduction parallel to the layer is

$$\lambda'_m = \frac{\Sigma (\delta \lambda)}{\delta_m}$$

3. In Radiation the passage of heat from one body to another occurs without the cooperation of the matter between them. The heat rays are electro magnetic rays emitted from any solid body.

In the form of light, only a very small portion of the electro magnetic rays can be observed by the human eye. Actually it is only those rays whose wavelengths

/lie....

lie between $\lambda = 0.365\mu$ and 0.75μ, - 1μ being equal to .001 mm.

The process of heat radiation is termed "Emission".

The radiation of heat is really a form of energy.

Emission always takes place at the cost of other forms of energy. i.e. it creates a loss of calories or chemical or electrical energy.

The propagation of heat rays takes place at the same speed as the propagation of light rays, namely, 300,000 kilometres per second.

Heat radiation is subject to geometrical laws just as they are used in optics.

The propagation of heat along a straight line can be subjected to reflection and refraction and the heat rays can be focussed by means of a lens. (objective).

Mention of the radiation of heat should immediately infer the presence of very high temperatures.

It can be proved that radiation is in existence all the time, even at very low temperatures and that it is independant of the body carrying the heat, such as air. It has been proved by Stefan and Boltzmann that heat radiation depends to a very large extent on the existing temperature and that the energy radiated is proportional to the fourth power of the absolute temperature.

This law is only valid for the radiation from completely black bodies which absorb theoretically all the heat which strikes their surface.

In their proof they used a completely evacuated hollow cylinder with a frictionless piston fitted into it. The bottom of the cylinder had a temperature (T) and was completely black. The walls of the cylinder and the piston were arranged to give complete reflection. The piston remaining still and the temperature (T) constant, it followed that the radiation was the same in all directions.

/In.......

In accordance with Boltzmann's kinetic theory of gasses the pressure caused by radiation was the same along each of the coordinates of the system and in any one direction would be p = u/3.

With the addition of a very little heat (dQ) to the bottom of the cylinder the piston began to move.

The first law of thermodynamics says that heat and work are equivalent.   Consequently dQ = dM + pdN. dM being the internal and pdN the external work.

The change of Entropy (dS) caused by a pressure of p = u/3 and by internal work of $M = uN$ is.

$$dS = \frac{dQ}{T} = \frac{d(uN + pdN)}{T} = \frac{N}{T}\frac{du}{dT}dT + \frac{4u}{3T}dN.$$

Entropy is the thermal ability which any substance may have to transform its heat into mechanical energy.

The equation for the change of Eutropy should be an exact differential viz:

$$dS = \left(\frac{\partial S}{\partial T}\right)_N dT + \left(\frac{\partial S}{\partial N}\right)_T dN$$

which is

$$\left(\frac{\partial S}{\partial T}\right)_N = \frac{N}{T}\frac{du}{dT} \quad \text{and} \quad \left(\frac{\partial S}{\partial N}\right)_T = \frac{4u}{3T}$$

Differentiating the first equation in terms of N and the second in terms of T the result will be.

$$\frac{\partial^2 S}{\partial T \partial N} = \frac{1}{T}\frac{du}{dT} = 4/3\left(\frac{1}{T}\frac{du}{dT} - \frac{u}{T^2}\right)$$

$$\frac{du}{dT} = \frac{4u}{T} \qquad u = \sigma T^4$$

The result is the law of Stefan and Boltzmann which was determined by Stefan by practical experiment in 1879 and confirmed by Boltzmann by theoretical mathematics in 1884.

The total Radiation of an area (A) will be.

$$R = A\sigma T^4$$

σ being the coefficient of radiation.

This law may be applied to other bodies having a

/smaller.....

smaller coefficient of radiation as well but the results are only approximate.

σ is only a very small figure i.e. 5,709 x 10^{-5} ergs per sq. cm per second, whilst the fourth power of the absolute temperature is obviously a very large figure.

It is therefore advisable to put the above equation into the following form.

$$R = A\,\sigma\,10^{8}\left(\frac{T}{100}\right)^{4} \quad \text{where } C = \sigma.10^{8}$$

$$R = AC\left(\frac{T}{100}\right)^{4} = E.$$

C being a new coefficient of Radiation.

E being the radiation capacity of the material.

It is known that some substances are more and others less transparent to visible rays. The same is true in the case of heat radiation.

Substances which transmit heat rays without raising their temperature are known as diathermics, those which do not transmit but absorb are known as adiathermics.

If two adiathermic bodies (1 & 11) are placed with faces parallel to each other (See Fig.7) and separated by a diathermic substance (for instance:- air) what is the amount of heat exchanged by radiation?

The backs of both the bodies are capable of total reflection so that any radiation which might influence any of the substances is kept out.

The capacities of radiation are respectively E_1 & E_2

The coefficient of absorption are respectively O_1 & O_2

The absolute temperatures are T_1 & T_2.

The coefficients of radiation per unit area are σ_1 & σ_2

The amount of heat radiated by substance No.1 is:-

$$R_1 = \sigma_1 . T_1^4 = E_1 \text{ (Law of Stefan \& Boltzmann)}$$

Substance 11 will absorb an amount of $E_1 O_2$ & $E_1\,(1-O_2)$ will be reflected back to No.1 substance.

The latter will once again absorb an amount given by $E_1\,(1-O_2).O_1$ of this & $E_1.(1-O_2).(1-O_1)$ will be reflected back to No.11 substance.

/Substance.....

Substance No.11 will again reflect to No.1 the amount of $E_1(1-O_2)\ (1-O_1).(1-O_2)$ and No.1 will absorb. $E_1.(1-O_2)\ .O_1(1-O_1).(1-O_2)$ of this.

Replacing the factor $(1-O_1).(1-O_2)$ by q, then the amount of heat absorbed by substance No.1 from its own radiation (Q_1') assuming a steady flow of heat, will be:-

$$Q_1' = E_1(1-O_2)O_1\left[1+q+q^2+q^3+\ldots\ldots\ldots q^n\right]$$

$$q < 1.$$

$$1+q+q^2+q^3\ldots\ldots\ldots q^n = \frac{1}{1-q}$$

and $$Q_1' = \frac{E_1(1-O_2)O_1}{1-(1-O_1)(1-O_2)}$$

It is also possible to apply the law of Kirchoff which states that.

The ratio between thermal emission and thermal absorption is for all substances, a function of the wavelength (μ) and of the absolute temperature (T) this ratio is the same for all substances having an equal wave length.

$$E_1 = O_1e_1 = .O_1\sigma T_1^4 = \sigma_1 T_1^4$$

and $$E_2 = O_2e_2 = .O_2\sigma T_2^4 = \sigma_2 T_2^4$$

Replacing E_1 & E_2 by the above values in the equation for Q_1'

Then $$Q_1' = \frac{\sigma_1 T_1^4\left(1-\frac{\sigma_2}{\sigma}\right)\frac{\sigma_1}{\sigma}}{1-\left(1-\frac{\sigma_1}{\sigma}\right)\left(1-\frac{\sigma_2}{\sigma}\right)}$$

The same equation can be drawn up for substance No.11

Assuming stationary conditions then substance No.1 will receive from substance No.11 by radiation an amount of heat given by

$$Q_1'' = \frac{\sigma_2 T_2^4\ \frac{\sigma_1}{\sigma}}{1-\left(1-\frac{\sigma_1}{\sigma}\right)\left(1-\frac{\sigma_2}{\sigma}\right)}$$

and the total amount of heat exchanged will be

$$Q = Q_1 - Q_1' - Q_1'' = \frac{T_1^4 - T_2^4}{\frac{1}{\sigma_1}+\frac{1}{\sigma_2}-\frac{1}{\sigma}}$$

/Suppose.....

Suppose now that substance No.1 is completely enclosed by a larger substance No.11 and that the two are exchanging heat by radiation (See Fig.8).

Then assuming that both surfaces A_1 & A_2 are completely black and that they have no angles of re-entry, then the heat energy radiated by substance No.1 is.

$$Q_1 = \sigma T_1^4 A_1$$

But the radiation of substance No.11 will only strike a certain portion of substance No.1 and it will be totally absorbed there.

So. $Q_1'' = \sigma T_2^4 \varphi_2 A_2$

is called by Mollier the rates of the two angles and is introduced on the following grounds.

One elementary body in a flat surface will radiate in all directions in the shape of a hemisphere. These rays will only strike a limited portion of any other body placed with its face to the elementary body (See Fig.9).

This limited portion will form the base of a cone.

The ratio between the cone and the hemisphere is called φ.

The product of the surface (A) of a certain body and the average value of the ratio of the angles (φ) calculated for each surface element will give the total value of rays radiated towards any other body.

If the rates of the angles of specimen No.1 are placed opposite those of specimen No.11 then it is clear that

$$\varphi_1 A_1 = \varphi_2 A_2$$

Since the smaller surface has the bigger ratio of angles.

As in the present case all the rays emitted by specimen No.1 reach specimen No.11. therefore $\varphi_1 = 1$, and consequently $A_1 = \varphi_2 A_2$.

The total amount of heat passed from specimen 1 to specimen 11 will be $Q = Q_1 - Q_1'' = \sigma_1 A_1 (T_1^4 - T_2^4)$

The same definition is also valid for two bodies with

/different....

different radiation capacities. A is only dependant upon the form of the radiating body and is independant of the material.

Should the value of σ be very small.

and $C = 10^8\sigma$ $C_1 = 10^8\sigma_1$, $C_2 = 10^8\sigma_2$,

then the amount of heat supplied by radiation by specimen No.1 (constants being $A_1 T_1 C_1$) to specimen 11 (constants being $A_2 T_2 C_2$) will be

$$S = \frac{A_1\left[\left(\frac{T_1}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C}}$$

C_1 & C_2 are the coefficient of radiation of the two bodies placed face to face and C is the coefficient of radiation of a completely black body.

The value of C is given by Kurlbaum as :-

$\sigma = 1.28.10^{-12}$ gram cal cm.$^{-2}$ sec^{-1} $(^\circ C)^{-4}$

$C = 4.61$ Cal unit m^{-2} h^{-4} $(^\circ C)^{-4}$.

4. <u>Passage of heat.</u> When heat is passing through a wall it is necessary to take into account all three methods of heat transfer - viz. convection, conduction and radiation.

Suppose that the surface area of a wall built from homogeneous material is A sq. metres, and the temperature of the air on either side of the wall is ϑ_1 and ϑ_2 .

Assuming $\vartheta_1 > \vartheta_2$ then there will be a transference of heat through the wall from the hotter to the colder side. (See fig. 10).

The surface temperatures of the wall being $t_1 > t_2$, then the wall will take over heat by radiation and convection from the warmer air represented by.

$$Q = \alpha A(\vartheta_1 - t_1)$$

This is the coefficient of heat transmission and indicates the amount of heat received by a surface of 1 sq. metre per hour from a temperature of 1°C including convection and radiation. (See folio $\frac{27}{28}$).

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The same amount of heat will be transferred from the hotter to the colder side of the wall by conduction.

$$Q = \frac{\lambda A}{\delta}(t_1 - t_2)$$

where λ is the a thermal conductivity of the material (See folio 26)

and δ the thickness of the wall in metres.

On the opposite side heat will also pass from the wall into the colder atmosphere by radiation and convection.

and $Q = \alpha_2 A\ (t_2 - \vartheta_2)$

α being the coefficient of heat transmission again.

By combining the three equations

$$Q = A(\vartheta_1 - \vartheta_2)\frac{1}{\frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}} = k.A(\vartheta_1 - \vartheta_2)$$

$$\frac{1}{k} = \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}$$

k is defined as the capacity for transmission of heat which is the amount of heat per sq. metre which will pass in an hour under steady flow conditions through a wall 1 metre thick, the temperature difference being 1°C on both sides.

The above equation indicates that by an increase in convection and thermal conductivity or a reduction in the thickness of the wall, the value of k will increase accordingly.

In order to make the calculation of the reciprocal in the above equations easier Prof. Jacob & Dr. Moller introduced into their version of the equations the heat insulating coefficient $\sigma = \frac{1}{\lambda}$ and the resistance against convection $a = \frac{1}{\alpha}$ which results in

$$Q = \frac{A(\vartheta_1 - \vartheta_2)}{a_1 + \delta\sigma + a_2} = \frac{A(\vartheta_1 - \vartheta_2)}{R}$$

where R the total resistance to heat transference or in other words, the insulating capacity.

In the case of a wall built up from layers of a different insulating capacity (viz: a concrete wall with

/surface....

surface insulation) the total insulating capacity equals the sum of the insulating capacity of the single layers.

and $R = a_1 + \sum \delta_1 r_1 + \delta_2 r_2 \ldots\ldots\ldots\ldots + \delta_m r_m + a_2$

If the wall is constructed in such a way that there are also layers of air in between (viz: a wall of hollow blocks) it is necessary to follow the theory of Flencky and to work out an insulating figure equivalent to the insulating capacity of the air space.

This figure must include the convection, conduction and radiation of the air.

$$\lambda = \lambda_0 + \lambda_c + \delta c C_k$$

where λ_0 = the thermal conductivity of the air under still conditions.

λ_c = a coefficient deducted from the convection.

$\delta c C_k$ = ditto, deducted from the radiation.

δ in this case represents the thickness of the air, and

C_k the coefficient of the mutual radiation.

c = is a factor dependant on the temperature.

5. Thermal conductivity.

The knowledge of the correct coefficient of thermal conductivity λ is of great importance in thermal calculations.

Most building materials are more or less porous.

The passage of heat through porous materials brings under consideration four different methods of heat transference:

1. Conduction due to the particles being in contact with each other.
2. Conduction due to the air in the pores.
3. Radiation between the walls from the pores and in the neighbourhood of bigger pores.
4. Convection.

The insulating capacity of non-crystalline bodies is about the same in normal temperatures - tests show

/that...

that the insulating capacity of a compact limestone for instance is .955 at 0°C and .938 at 100°C.

The insulating capacity of gold is exactly the same from 0°C to 200°C.

The insulating capacity of crystalline bodies on the other hand does change with temperature. This is to be expected from their structure which is on the principle of micro-laminations and the action of thermal expansion tends to loosen the contact between the laminations.

The following tables give the coefficients of heat conduction and insulation of crystals. (Taken from Eucken).

Temperature	-100°C		0°C		+100°C	
	λ	σ	λ	σ	λ	σ
Sodium Chloride Crystals	10.6	.09	6.0	.17	4.2	.24
Quartz crystal (parallel to axis).	19.0	.05	11.7	.09	7.7	.13
Quartz Crystal (at 90° to axis)	10.0	.10	6.2	.16	4.8	.21
Limestone Crystal (at 90° to axis)	5.8	.17	3.7	.27	3.1	.32

From this it is seen that the insulating capacity of crystals increases with temperature. There are crystalline rocks also which have the same property.

Nelson extracted the following figures for the coefficient of heat conduction of white marble having a weight of 2710 kg per cubic metre.

at 50°C λ = .455

100°C λ = .490

150°C λ = .595

This shows that it is incorrect to believe that the insulating capacity of material is proportional to its weight per unit volume.

The coefficient of heat conduction of a quartz crystal is three times as much as that of limestone having the

/same....

same weight per unit volume.

The insulating capacity of the pores can not in any case be taken as having a constant value.

It changes with the temperature, size and form of the pores as well as with the humidity of the material.

The heat energy radiated from the warmer side of the pores to the cooler wall amounts to (see also page 34-35)

$$Q = Ac'\left[\left(\frac{273+T_1}{100}\right)^4 - \left(\frac{273+T_2}{100}\right)^4\right]$$

where C the coefficient of radiation = 4.5 the valid figure for porous materials.

The amount of heat radiated increases with the temperature.

The difference in temperature of two porous walls placed face to face will increase with a dr - - tempera-ture if the porosity is increased.

At the same time friction between the air and the walls of the pores decreases with the result that transference of heat by radiation and convection increases.

The insulating capacity of porous materials decreases as the pores are enlarged.

<u>The coefficient of thermal conductivity of air enclosed in pores amounts to</u> :- (from Cammerer).

<u>Average temperature.</u>	<u>Diam. of pores in mm.</u>					
	0	.1	.5	1.0	2.0	5.0
0 C	.0203	.0207	.0221	.0238	.0273	.0378
100 C	.0263	.0272	.0307	.0351	.0439	.0704
300 C	.0368	.04	.0530	.0691	.1018	.1983
500 C	.0460	.054	.0860	.1255	.205	.4435

Continuous layers of air reduce the insulating capacity of the material on a big scale; (hollow bricks etc) especially where there is a difference of atmospheric pressure between the two sides of the wall.

Pressure causes air circulation in the hollows resulting in convection.

/A.....

A test in still air by Dr. Moller in Budapest on a given material gave the insulating capacity as $\nu = 12.5$. With a wind velocity of 10^{-1} metres it dropped to $\nu = 4.2$ being 66% lower.

Insulating capacity is also decreased by the pressure of solid particles in the material such as fibre in wood etc.

Since the thermal conductivity of water is 25 times that of air it follows that the thermal conductivity of porous materials is increased by the presence of moisture which enters the pores through capillary attraction.

A very small amount of water reduces the insulating capacity on quite a large scale.

Insulating figures obtained in laboratories with the aid of the Poensgen Lamellen Set cannot be used in practise.

It is also wrong to increase such figures by 25% and to imagine that reality has been reached by doing so.

In the first instance there is already a difference of 20 - 25% between the samples sent for approval and the material actually used in the building.

Many Manufacturers send faultless samples to be tested and take special care with the manufacture of test specimens. Cammerer investigated the insulation of steam pipes under absolutely dry conditions and found 10% - 19% and sometimes even 25% lower figures than those obtained in laboratory tests.

All porous material absorbs moisture from the air, even after the material has dried there will still be a small moisture content in the pores.

A well burned brick may dry out after a certain period if it contains no salts capable of absorbing water.

There are however many building materials which contain water absorbing colloids, these belong to the so-

/called.....

called hydrophil gel (i.e concrete, chalky sandstone - sandstone mortar - pumice gravel concrete and wood).

These materials will never dry out completely in the open air.

They can be dried out in a temperature of over a 100°C, but will immediately absorb moisture again in the open air.

There is a considerable difference in the properties of water absorption and water retention of various building materials.

A well burned hard hollow brick with a low insulating capacity may be more satisfactory in practise than a light pumice gravel concrete stone with a greater humidity. The same thing may apply in the use of special insulating materials.

One cork slab for instance may absorb 15% moisture whilst another will only absorb 5%.

Every material must be judged separately on its merits and only by carefully checking its behaviour under practical conditions can its true value be determined.

Messrs. Schmidt & Grossman made a series of tests with the object of proving the points mentioned above. They determined the insulating capacity of a brick wall 28 cms thick, plastered on both sides. The humidity varied during the tests from 25% to 5%.

The results obtained according to the "Forschungsheim" in Munchen were as follows:-

Heat conductivity of brickwork.

Time taken to dry.	Humidity in volumes %	λ Coefficient of heat conduction.	γ Insulating coefficient.
0 Months	25 %	1.2	.83
4.5 Months	3.4%	.84	1.19
6.4 Months	1.9%	.74	1.35
9.1 Months	1.0%	.65	1.54
12.5 Months	.5%	.6	1.67
Artificially dried		.38	2.63

/Conformation...

Confirmation of these figures has been obtained by Dr.Hofbauer who investigated the insulating capacity of external and internal walls in a building 50 years old. Dr Hofbauer's figures for the internal walls gave σ as 1.4 and for the external walls σ as 2.6.

It is incorrect to increase figures for insulation obtained in the laboratory by 20% in order to obtain figures for practical work. In reality there is always 1 - 2% of humidity in brickwork which will reduce the insulating capacity by as much as 95 - 70%.

The same thing applies in the case of concrete. Poensgen & Grober found for dry concrete a thermal conductivity of $\lambda = 0.6$ to 0.65.

The concrete applied to most buildings in this country is, on account of its humidity, twice as good as a conductor of heat as it would be in absolutely dry conditions. It is therefore not an exaggeration to use $\lambda = 1.2$ and $\lambda = 0.83$ for concrete.

6. Testing insulating materials.

There are many different methods and ideas about what concerns correct testing of materials and determination of thermal conductivity. All of them have however the same leading thought.

One side of the test piece has to be supplied with a certain amount of heat electrically generated.

The actual quantity of heat can be checked by measuring the voltage and current intensity of the source of supply.

The surface temperatures are measured when the flow of heat is stationary.

The first known method of testing is due to Nusselt who used a spherical hollow body filled with the material under test.

In the centre was an electrically heated element.

/The....

The thermal conductivity was obtained by checking the amount of heat supplied and measuring the temperatures at different points in the material.

Grober did the same thing but used two plate shaped testing pieces one on top of the other, and of the same thickness. A heating element of the same shape was placed between the two pieces and the outside cooled by running water.

The surface temperatures were measured by soldered wires on the thermo-electric principle.

The only difficulty in this experiment was the lateral loss of hea from the plates.

Poensgon improved on the scheme by covering in the sides with a frame of the same material as that under test.

This covering frame was then kept at the same temperature as the material under test. (See Fig. No.11).

In this way the test pieces are not influenced by isothermal curves which find a path in the covering frame.

The supply of heat and cooling water in this experiment has to be controlled in such a way that the temperatures remain constant (i.e. t_1°C on the warmer side and t_2°C on the cooler remain the same).

The amount of heat supplied being Q the equation for the conduction of heat becomes.

$$Q/2 = \frac{\lambda A (t_1 - t_2)}{\delta}$$

where A the exact area of the test pieces

and δ their thickness.

The usual method of controlling the cooling water is to arrange for the water flowing away from either surface to have the same temperature. If this is done then the amount of heat passing through the two slabs is the same.

/The....

The coefficient of thermal conductivity will be

$$\lambda = \frac{Q\ \delta}{2A\,(t_1 - t_2)}$$

Any plate shaped or annular subject can be tested in this way, and it is possible to determine the thermal conductivity at any temperature as the test depends only on the heat and water supply limitations.

There is only one disadvantage in this and that is, that special testing pieces only can be examined.

Actually of course the material under test dries out during the process and the figures obtained do not cover reality conditions.

The Forschungsheim für Wärmeschutz endeavoured to get closer to reality.

The walls between two rooms formed the subject of their tests and those walls were actually part of the testing equipment. The rooms concerned were warm and cold respectively and the wall dimensions were 1.5 x 1.0 metres. The maximum temperature normal to continental climates was used - namely +20°C on one side and 0°C on the other. Air was kept at a different humidity in these rooms during the tests in order to avoid drying up the walls.

The writer built up test apparatus in Holland for the W.V.Betondak (Concrete Roof Ltd) in Gorinchem which supplied insulating data which proved fairly useful in practise.

Fig. No.12 shows the complete equipment. Tests are carried out once a stationary flow of heat has been reached. This is checked by means of recording distance thermometers which show air and surface temperatures on both sides of the test piece. It is assumed that the temperature of the room in which the test is conducted will also remain constant.

The first portion of the experiment is to determine the loss of heat in the upper box which is heated by an

/electric .....

electric current. The amount of heat supplied is Q_1WE/hour = Qx0.86 watt.

If the temperature in the bottom box is kept the same as it is in the upper box, the loss of heat through the test piece is avoided as all the surplus heat will vanish through the walls of the upper box.

The amount of heat lost per hour through the walls of the box is also Q_1WE = Q_1.86 watt.

If the temperature of the bottom box is now cooled down to the temperature of the room and the upper box is heated again until a stationary flow of heat is reached between the two boxes, then the amount of heat supplied will be Q_2WE/hour.

The amount of heat passed through the test slab is then $$QWE = Q_2WE - Q_1WE$$
and the coefficient of heat conduction is
$$QWE = \frac{\lambda}{\delta} A(t_1 - t_2)$$

so $\lambda = \frac{QWEd}{A(t_1 - t_2)}$ where t & t are the surface temperatures.

The same test can be carried out if heat is passing from the lower to the upper component.

The advantage of this arrangement is, that it is possible to determine insulating coefficients at lower temperatures with no danger (at any rate in the beginning) of the material drying out quickly.

Furthermore, if the temperatures of the air are measured on both sides it is possible to determine the coefficients of heat transmission on both sides.

since $Q = \alpha_1 A(t_2 - \vartheta_2)$ and $Q = \alpha_2 A(\vartheta_1 - t_1)$

This coefficient of heat transmission, as already mentioned, consists of a factor of convection (α') and of the factors proportional to the radiation (C) with regard to a certain temperature in one of the boxes and the area of its walls.

/A.....

A hollow block floor built into the above equipment gave the following results.

$t_1 - t_2 = 32°C.$ $Q_1 WE = 210 \times 0.86$ $Q_1 WE - Q_2 WE = 103.2$ Cal

$Q_2 WE = 330 \times 0.86$

The thickness of the floor was 0.21 metres and the area 1 sq. metre.

so $\lambda = \frac{103.2 \times 0.21}{32} = 0.68$

This figure is very low due to the fact that the temperature on one side had to be raised to 48°C in order to get a difference of 32°C between the two sides. This meant that the floor, especially on the one side where the pumice gravel insulating blocks were built in, was dried out to a greater extent.

It is of great importance in making these tests to see that the thermocouples are properly fixed to the surface of the test pieces. Fixing has to be done in such a way that the presence of the couple does not disturb the normal distribution of temperatures in the test piece itself.

Fig. No.13 shows the correct and incorrect methods of fixing.

When testing powdered material a box made of a material such as sheet metal, the heat conductivity of which is well known, is used as a container.

The dimensions and shape of the box being the same as those of one of the testing pieces.

The simplest form in which testing apparatus can be made is the "Cone of Nusselt" (See fig. No.14).

The main advantage of this apparatus in testing powdered material is that all the heat produced by the heater in the centre of the cone has to pass right through the material and none is lost.

Loss of heat can be measured in practise in quite a different way - Hencky developed a primitive method which

//involved....

involved fixing an auxiliary slab, with a well known thermal conductivity (cork slab) against the wall to be tested once the steady flow of heat condition has been reached, the amount of heat passed through the cork-slab (B) is the same as that passed through the wall (A).

The constants for the wall are λ, δ, and for the auxiliary slab λ_1, δ_1 (See Fig. No.15).

Then $Q = (t_1 - t_2)\frac{\lambda}{\delta} = (t_2 - t_3)\frac{\lambda_1}{\delta_1}$

t_1 and t_3 being the air temperature on both sides of the wall, and t_2 the common temperature of the wall and testing slab.

and $$\lambda = \lambda_1 \frac{\delta}{\delta_1}\left(\frac{t_2 - t_3}{t_1 - t_2}\right)$$

If the temperature difference between the two surfaces of the wall under test is Δt then the amount of heat passing through the wall will be

$$Q = \Delta t\, A\, \frac{\lambda}{\delta}$$

This is a cumbersome method of testing materials as, on account of the possibility of wind on either side of the wall, it sometimes takes days before the stationary flow of heat condition is reached.

A method of checking heat flow through flat walls, pipes etc., has been developed by E Schmidt in Danzig and is called the "Heat Flow Meter". It is an improvement on the scheme suggested above.

In his apparatus a 60 x 6 cm rubber mat, 3m/m thick, was used. When this is fixed against the wall under test there is a small temperature difference between the two faces of the rubber mat $(t_2 - t_3)$. This temperature difference is measured with the aid of built in thermocouples. The thermo electric tension on the clamps of the heat flow meter is a measurement of the heat passed through the wall. This heat flow meter is

/used....

used with a galvanometer scaled in kg cal/sq metre per hour, in order to avoid complicated calculations when taking readings.

A rubber frame is placed round the heat flow meter in order to avoid lateral loss of heat in the same way as was described for the Poensgen Apparatus.

The real purpose for which Schmidt prepared this heat flow meter was to control loss of heat in pipes and in pipe insulation which involves loss of heat to the extent of 50 up to some hundreds of kg cal/sq M/hour. The loss of heat through walls is usually much lower than this.

The coefficient of heat transmission of external brickwork 38 cms thick is 1.33 kg cal/sq M.hour /°C so that the amount of heat lost with a difference in temperature of 10°C would be 13.3 kg cal/sq M/hour. The heat flow meter in its original form is not sensitive enough to measure such a small quantity of heat.

Hoffbauer increased the sensitivity of the apparatus by using thermo electric elements consisting of 1000 soldered joints. Such elements consist of iron and constantan wire crossing over at 1000 points each of which is soldered together (See Fig. No.16). When using copper and constantan wires the sensitivity can be increased up to 0.05°C.

In the Poensgen Apparatus the heat flow meter itself forms with one testing slab a complete unit, which enables the meter to be properly calibrated.

b. <u>Cooling.</u> From the view point of economic heating and cooling it is of considerable importance to find out the following facts:-

a. The amount of heat which is lost by periodical heating.

b. The time taken to cool down after heating has ceased.

/C.....

c. The influence of heat accumulation

d. The time taken to warm up the system to the correct temperature.

Experience shows that in many cases the stationary state is reached, during the heating up process.

It can be assumed that cooling starts at the moment heating stops and that this is the stationary state.

The quantity of heat given out to the surroundings while any system is cooling depends on the insulation and the heat accumulation capacity, sothese two points can never be neglected.

Cooling does not take place in the entire system simultaneously once heating has stopped. The passage of heat through a wall with a limited heat conduction and heat accumulation capacity requires time.

As an example take a building which has external walls only, and which has after a certain period already reached the stationary state.

It is not necessary to consider the influence of corners of walls, windows etc., for the present purpose.

On the inside the surface temperature of the wall is in the stationary state $t_{ins.\ st.}$

The coefficient of heat transmission on the outside of the wall is α_{outst} and its value in relation to the thermal conductivity, infinitely large.

The temperature on the outside is arranged to be equal to zero, then, when the zero point of the co-ordinate system is inside the wall the stationary state can be represented by the following equation. (See fig. No.17).

$$t_{st} = t_{ins.\ st.} - \frac{t_{ins.\ st.}}{\delta} x$$

and the flow of heat when it is in stationary state

$$Q_{st} = \frac{\lambda}{\delta} \cdot t_{ins.\ st}$$

The heating system in this example consists of a warm current of air. The room is absolutely empty and any

/heat.....

heat content of the air can therefore be neglected.

Once heating has been stopped inside, all the heat which reaches the outside must be taken from the accumulated heat in the wall.

Heating has been stopped at the moment $Z = 0$. Up to this moment the same amount of heat has been flowing through each part of the wall - after this no more heat will be added to the air inside.

Imagine that the wall is divided into equal layers 1.2.3.... so that the heat given out by layer 2 passes into layer 3 and was in the first instance taken from layer 1. ($Q_{st} \Delta Z$).

Layer 1 will therefore cool down by this amount since no more heat is flowing to the inside surface of the wall the drop in temperature which should cause transference of heat is equal to zero, and the cooling curve will have a maximum value at this point (See Fig. No.18).

The maximum drop of temperature must be always on the outside of the wall, and the stationary state always exists there for the longest period. A gradually increasing portion of the wall gradually participates in the cooling process.

Lastly, no portion of the wall is entirely free from cooling. The temperature of the wall will be reduced continually as shewn in fig.No.19, where the curves have been drawn in accordance with the laws relating to cooling (Schmidt). The speed at which the temperature will drop depends on the heat conductivity of the wall and on its heat contents.

As the flow of heat is more intense at the beginning, it is natural to expect that cooling will also be quicker then. This is the case in practise but as the temperature drops so does the speed of cooling.

 /Curves.....

Curves of temperature and flow of heat approach zero in an asymtote formation.

Fig. No.20 shows how flow of heat depends on time in graphical form.

The amount of heat given out (Q_0^z) by the system up to the time z is equal to the integral

$$\int_0^z Q\,\delta z$$

What is the cooling curve for a wall with parallel faces?

The equation for the flow of heat through a wall in the direction of x has already been mentioned (See page 26.)

$$\frac{1}{a}\cdot\frac{\partial t}{\partial z} = \frac{\partial^2 t}{\partial x^2}$$

The only solution of the equation which is taken into consideration is

$$t = bx + c + \sum_{m=1}^{\infty} A_m \{\cos m_m x + p_m \sin m_m x) e^{-a m_m^2 z}$$

The constants b & c are found from the conditions applying to the state at the end of the process (at $z = \infty$ — the infinite series disappear).

If a system happens to be cooled down in such a way that no over-temperature is left, as against atmospheric temperature, in other words, so that the two temperatures are both zero then the equation can be put in the following form.

$$t = \sum_{m=1}^{\infty} A_m u_m e^{-a m_m^2 z} = A_1 u_1 e^{-a m_1^2 z} + A_2 u_2 e^{-a m_2^2 z} \ldots\ldots etc.$$

Where the solution of u_m results in a sine and cosine function for a straight wall.

$$u_m = (\cos m_m x + p_m \sin m_m x)$$

Cooling starts from the moment the stationary state is reached and the drop of temperature commences on the warmer side of the wall. The temperature difference between the two faces of the wall being at a maximum at the beginning it stands to reason that cooling will be at its

/fastest.....

fastest in this state. The elapsed time τ between the stationary state and the moment when all the parts of the wall participate in the cooling process is Z_{μ}.

The drop of temperature during further cooling will be in accordance with the equation.

$$t = A\, u_1\, e^{-a m_1^2 (z - z_{\mu})}$$

A... represents a cosine curve which is at a tangent to the straight line of the stationary state at δ. (See Fig. No. 21).

The maximum value is on the warmer side of the wall at $Y = 0$.

A_m, m_m & p_m are constants of three infinite series. m_m & p_m are calculated from the prescribed limit conditions for the wall or its insulation.

A is determined by the initial condition - namely that, at the start of the cooling process ($Z = 0$) the equation of t has to represent a defined temperature distribution.

Suppose that the heat contents W_e of a system at the time Z_m and the state Au_1 are required,

Then assuming that the intensity of heating currents on the outside of the wall remain stationary up to the time Z, it is immediately possible to determine Z_u from the difference between the heat contents at the beginning (W_{st}) and those at the time Z. (See fig. No. 21)

$$\text{As} \qquad Z_m = \frac{Wst - We}{Qst}$$

C. <u>Heating of walls.</u>

The process of heating a wall is the exact opposite of cooling. Instead of a drop in temperature there is a rise and the initial state is in this case the final or stationary state, reached in theory after an infinitely long period.

/Whatever.....

Whatever has been said of the cooling process is also valid for the heating process.

The movement of heat starts where the conditions of a system have been changed and the heat will gradually spread over the whole system.

Once the whole system participates in the heating process there is no further change in the character of the heating curve. This process is described mathematically on page 47. etc.

The stationary state is reached after an unlimited time represented by $Z = \infty$.

The infinite series disappear at this moment and the temperature at this state is represented by a straight line.

$$tst = bx + C$$

$$t = tst - \sum_{m=1}^{\infty} A_m U_m e^{-a m_m^2 z}$$

The way in which the stationary state is reached depends on the manner in which the heat is supplied during the heating period, and from the conditions of the supply of heat the constants A_m, m_m and p_m are determined.

An approximate method of calculating the cooling of a system from the stationary state and the heating up of a system from the stationary state of heat flow has been worked only by Dr.W.Esser.

The primary fact, on which his method is based, is that the motion of heat starts only at the point where temperature conditions change.

The heat only spreads over the entire system after a certain period Dr.Esser considers that there is a certain dividing line before this, separating one area in which heat has already spread and another area which has not yet been effected by the flow of heat (See Fig. No.22).

The curves which represent the distribution of temperature in straight walls before the stationary state has

/been....

been reached are sine and cosine curves, as already mentioned. (In the case of pipes they are Bessel curves).

At the dividing line these curves are at a tangent to the straight line indicating the stationary state.

When the movement of temperature reaches the cooler side of the system (in this case a wall) it will be represented by a cosine curve at a tangent to the straight line representing the stationary state at $X = \delta$. This curve does not change its character again during further cooling. The state known as a free movement of heat has been reached. The rapidity with which cooling takes place beyond this point depends only on the flow of heat and the heat contents of the system.

At the beginning of this state the flow of heat is represented by Qst and the heat contents of the system by W_e. Dr. Esser has introduced at this point a coefficient which when multiplied by. Wst results in We ($We = \psi . Wst$).

The time required to reach this state is

$$Z_u = (1-\psi)\frac{Wst}{Qst}$$

With the location of ψ and the values of the stationary state known; the whole heating and cooling system may be determined.

ψ has certain geometrical proportions but is mathematically determined.

The values are given in the 2nd part of the handbook written by Dr. W. Esser and Dr. C. Kirscher (German).

The simplified theory given above can also be enlarged and compared with test results made in actual practise by reference to this book.

For all thermal calculations in connection with buildings the formula for the stationary flow of heat can be used with safety, it is not intended here, to discuss the complicated problems of heat transfer under changing intensity of heat flow.

/All.....

All calculations for cooling and heating of walls are complicated mathematical problems as they are based on the investigation of conditions in infinitely large walls. In practise these calculations do not give much satisfaction. Despite this the theoretical and experimental results have to be composed in order to explain the difference between the heat content of walls with and without lateral loss of heat.

A wall with a surface of 10 sq. metres and 30 cm thick may have a cross section of 4 sq. metres in which lateral loss of heat can occur. The position of the wall is also of great importance in cooling problems. Cooling the external wall of a multi-storey building is slower because neighbouring rooms prevent the loss of lateral heat. Cooling the walls of a corner room will of course be quicker. In small houses where each room has at least two or three external walls cooling is very uneconomical. The following reasons explain why this is so. Firstly the size and number of windows (K = 6.0 - 8.0 for capacity of heat conduction for glass in steel window frame). Secondly the heat accumulation of the walls, furniture and internal walls. Thirdly the system of heating and fourthly the temperature of adjacent rooms.

J.S.Cammerer by tests in which he used brick and chalky sandstone 21 cm thick, found that in 8 hours the difference between inside and outside surface temperatures was reduced by 70%. What is the position of the temperature of a room situated behind a wall which is being cooled?

During the supply of heat in the room the room temperature is say 6°C above the temperature of the surrounding walls (outside temperature supposed to be ±0°C, inside +20°C). As soon as heating has ceased, the air, having no heat accumulation capacity worth mentioning, will cool down and the 6°C difference will soon disappear.

/See......

(See fig.No.23). As shown in the diagram the temperature of the room will drop in a few minutes to that of the surrounding walls, and this will also drop as already discussed. The drop in surface temperature on the inside of the wall was in this particular case 7°C in 8 hours. These tests were carried out with electric heaters having no appreciable heat accumulation capacity. If stoves possessing a varying amount of heat accumulation capacity had been used, the temperature of both the room and its surroundings would have been considerably higher after 8 hours.

In the case of S.African buildings the accepted thickness of walls is 25 cm (9") and the use of an adjustable heater with variable heat accumulation capacity is to be recommended.

Houses built of so called light building materials warm up in a very short period and even slow heating stoves with great heat accumulation capacity will not effect this. When heating is stopped however they will of course cool down very quickly, and for this reason it is advisable to use stoves with a very high heat accumulation capacity.

Fig. No.24 shows the heating periods required by walls built ~~from~~ of different materials with the same insulating capacity but different heat accumulation capacity. These tests were made by Schmidt in Germany. The importance of protecting buildings from over heating cannot be over emphasised. As already mentioned, the surface of a wall or roof exposed to the sun can reach an over temperature of 50°- 60°C approx. The high temperature penetrates the material and makes the house inside unbearable.

The highest temperatures are usually reached in the afternoon especially in houses built of materials with low heat accumulation capacity and having large unprotected windows.

/Inside....

Inside houses with a great heat accumulation capacity however this state is only reached during the night assuming there is no ventilation in the house. Houses can be protected from over heating either by means of a ventilated loft over the house, or by nightly ventilation of the rooms or by windows protected with blinds, or by inside insulation with materials having a large heat accumulation capacity. About 90% of the rays of the sun radiated through glass windows are retained inside the house and act as a heater.

6. CONDENSATION.

The insulating capacity of any material is influenced to a very large extent by its hygroscopic qualities. Absolutely dry material can only be found in the laboratory as under normal testing processes most materials lose whatever moisture they originally possessed.

The insulating capacity of a wall is often reduced by a half or even more due to the absorption of moisture.

Walls can be become permanently humid in various ways by

1. Hygroscopic absorption of moisture from the atmosphere.
2. Heavy showers on the outside surfaces.
3. Moisture retained from the construction period.
4. Moisture permeating into them from the ground.
5. Condensation on the inside surfaces.

Hygroscopic absorption of moisture represents as a rule the minimum source from which insulating capacity is effected. The quantity of moisture absorbed depends on the humidity of the surrounding atmosphere and on the treatment of the surface the material.

Hygroscopic moisture content is roughly 3% of the volume in concrete and o.3% of the volume in brick.

These figures again confirm that the so-called hydrophile gels (concrete, cement, etc.) absorb more moisture

/from......

from the atmosphere than bricks which are more porous.

The quantity of water which penetrates into the material during heavy showers is dependant on the porous nature of the material and on the treatment of its surface.

A surface which is completely waterproof is however not a solution to this problem and cannot be recommended because, whilst it prevents the entry of water it also prevents the material from drying out inside.

Quite useful protection against heavy showers can be obtained from slates, tiles and waterproof layers with openings which allow evaporation to take place inside.

A material which is very porous naturally absorbs more rain water but on the other hand it dries out much more quickly.

Buildings constructed with concrete or built in a similar manner tend to retain more moisture from the period of construction than those built from stone or bricks. They are in consequence less favourable from the point of view of insulation.

Hollow blocks, especially in the case of materials having large holes inside dry out quicker.

In general it may be said that most walls, floors, etc., lose their moisture within one year and only in the case of very heavy walls is this period exceeded.

Materials of a hollow nature with large holes such as pumice gravel concrete etc., are better than closed materials when it is a question of prevention of absorption of moisture from the ground. In any case today this question is of little import as it is only in the case of improperly insulated foundations that any moisture can find its way into the superstructure.

Of much greater importance is condensation on the inside surfaces of walls, the origin of which is closely connected with insulating problems.

/Damp......

Damp walls are obviously impracticable on the grounds of ill health.

Paintings - wall paper - furniture and clothes all suffer if condensation takes place.

In factories and stores especially in textile industries provision stores, grain silos, salt magazines, cold storage premises etc., condensation can cause a tremendous amount of damage.

In the air water vapour is always present in either a smaller or greater degree. The actual content is constantly changing but it never exceeds a certain maximum value.

The maximum value depends on the temperature and is called the absolute humidity of the air.

It is expressed in grammes of water vapour per cubic metre of air.

If the temperature falls to such an extent that the air can no longer retain its water vapour content, then the vapour condenses and forms into drops of water.

An every day illustration of this characteristic can be seen when the air is cooled down in the neighbourhood of a glass of cold water and drops of moisture in the form of a mist are formed outside the glass.

At higher temperatures air can absorb more water.

The ratio of the absolute humidity to the maximum possible water vapour content under conditions of saturation at the same temperature is called the relative humidity.

It is usually expressed as a percentage.

As an example:-

1 Cubic metre of air at atmospheric pressure and a temperature of 20°C will contain a maximum of 17.3 grams of water vapour.

If it is found that the same quantity of air only contains 13 grams of water its relative humidity is:-

/13/17.3......

13/17.3 x 100 = 75%

Dew point occurs at the temperature at which air is saturated with water vapour, and so a minimum drop in temperature will cause condensation to begin. The higher the relative humidity, and the closer to the saturation point of air, the less cooling is required to create condensation. In the following table, figures are given for the maximum humidity of the air at various temperatures, and for dew points at temperatures which occur frequently in practise.

Water Vapour Content and Dew Point of the Atmosphere.

Temperature.	Water vapour content in grammes per cubic metre.	Relative humidity. 40	50	60	70	80	90
		Dew point in C.					
-5	3.27	-15.8	-13.2	-11.2	-9.4	-7.7	-6.3
0	4.84	-11.3	- 8.7	- 7.5	-4.6	-2.9	-1.4
2	5.56	- 9.7	- 7.0	- 4.8	-2.8	-1.0	-0.5
4	6.36	- 8.1	- 5.3	- 3.1	-1.1	0.8	2.5
6	7.26	- 6.5	- 3.7	- 1.3	0.7	2.7	4.4
8	8.27	- 4.9	- 2.1	- 0.4	2.6	4.6	6.4
10	9.40	- 3.3	- 0.4	2.2	4.5	6.5	8.3
15	12.82	0.8	4.0	7.4	9.3	11.4	13.2
20	17.29	5.3	8.5	11.5	14.0	16.2	18.1
25	23.10	9.7	13.1	16.2	18.8	21.1	23.1
30	30.40	14.1	17.7	20.9	23.5	25.9	28.0
35	39.40	18.5	22.3	25.4	28.2	30.6	33.0
40	50.70	22.8	26.8	30.1	33.0	35.6	37.9
45	64.50	27.2	31.4	34.8	37.7	40.4	42.9
50	83.30	31.6	36.0	39.5	42.5	45.3	47.8

The table above shows quite clearly that a very small drop in temperature can cause condensation at the higher temperatures and greater relative humidities.

/As....

As a further example:-

Suppose air is present with a relative humidity of 80% at a temperature of 25°C. A drop of only 3.9°C will cause condensation.

As a general rule, the relative humidity of the air lies between 60 and 80% and the drop in daily temperature 6 to 10°C particularly in the early summer.

When this condition is prevalent, condensation commences on the outside of buildings, and is absorbed through the pores of the building material (plaster, bricks etc) to be evaporated again later.

This is the reason why condensation on the outside of buildings is rarely noticeable.

Changes in the percentage of moisture present causes corrosion of building materials such as plaster paint etc. Building materials which are practically non-porous such as Basalt, smooth concrete, glazed tiles etc., are unable to absorb moisture and so when condensation occurs they become wet.

Condensation will occur inside a building on surrounding walls, floors and roofs, if their respective temperatures happen to be lower than the dew point of the air inside the building.

Example; Is it necessary in S.Africa to take precautionary measures against condensation if a house is built of 28 cm wall thickness constructed from bricks and plastered on both sides? Assume the outside temperature to be ± 0°C and the inside temperature 25°C (a warmed room in winter) with a strong wind blowing outside.

The temperature difference between the inside and outside of the house is 25°C.

The total resistance against heat conduction will be

Convection outside = 0.04

Brickwork .28x1.35 = 0.38

/Convection.....

$$\text{Convection inside} = \underline{0.14}$$

$$R = 0.56$$

The drop of temperature on the inside of the walls will be:-

$$\Delta t_2 = \frac{0.14}{0.56} \times 25 = 6.25°C$$

The surface temperature of the inside of the walls will be:- $t_2 = +25 - 6.25 = 18.75°C$

For 60% relative humidity the dew point is only 11.5 C so no condensation is likely to occur inside the building.

Outside $\Delta t_1 = \frac{.04}{.56} \times 25 = 1.8°C$

The surface temeprature is $0 + 1.8°C < 11.5°C$

There will therefore be condensation outside the building which is absorbed by the plaster or good bricks. (These insulating figures and coefficients are taken from Dr.Moller - Csiszar, on 'Isolatie' published in the Netherlands).

A very slight movement of the air is sufficient to raise the surface temperature on the warm side by 1°C or drop it on the cold side by 1°C.

In the same way one will observe that a 4½" (12cm) brickwork is sufficient under South African conditions to prevent condensation inside the building.

As the motion of the air accelerates the evaporation of any moisture formed by condensation, it is possible by air circulation and or ventilation to prevent condensation up to a certain limit.

The surface temperature of a wall depends on its total insulating capacity as well as on the ratio of the inside convection to the total insulating capacity.

Convection inside the building can be taken as fairly constant. Hencky has provided a formula for the calculation of maximum heat conduction capacity (k) without the occurrence of condensation.

/The......

The amount of heat passing through 1 sq. metre per hour

$$Q = \alpha_1 (\vartheta_1 - t_1) = k (\vartheta_1 - \vartheta_2)$$

and $$k = \alpha_1 \frac{\vartheta_1 - t_1}{\vartheta_1 - \vartheta_2}$$

where t_1 is the dew point of the air, at a temperature of ϑ_1 and relative humidity m.

Values for k have been worked out by Dr.Moller - Csiszar and are as follows.

Relative humidity	k.Capacity for heat conduction cal/sq. m/h/°C.	
	Outside temp. -10°C Inside temp. +20°C	Outside temp. 0°C Inside temp. +20°C
90%	0.350	0.525
80%	0.746	1.12
70%	1.4	2.1
60%	1.93	2.9
50%	2.64	3.95
40%	3.06	4.58

Example:-

What is permissable minimum thickness for brickwork, both sides of which are plastered and where $\lambda = 0.75$ with no condensation present wet on the outside? Inside temperature +25°C. Outside 0°C, dew point 16.2 at 60% relative humidity.

$$k = 7 \times \frac{25-16.2}{25} = 2.5 = \alpha_1 \frac{\vartheta_1 - t_1}{\vartheta_1 - \vartheta_2}$$

$$\frac{1}{k} = \frac{1}{\alpha_1} + \frac{\delta}{\lambda} + \frac{1}{\alpha_2}\,; \qquad \frac{1}{2.5} = \frac{1}{7} + \frac{\delta}{0.75} + \frac{1}{25}$$

$$\delta = 0.165\ m = 16.5\ cm$$

For the above typical S.African conditions a minimum thickness of brickwork of 16.5 cms ($6\frac{1}{2}$") is required to avoid inside condensation.

If minimum insulation is to be applied to avoid condensation then k for internal walls of brick with both sides plastered should be 2.5 for S.African conditions in normal cases the relative humidity of the air being between 40 and 75%.

/The.....

The average humidity of the air in Johannesburg was, for the past 16 years, according to the records of the Union Observatory as follows:-

Month.	Humidity	Month	Humidity.
January	71%	July	46%
February	73%	August	45%
March	71%	September	46%
April	61%	October	54%
May	55%	November	62%
June	50%	December	68%

The humidity of the air is much higher in buildings which have not dried out properly, in rooms crowded with people, in kitchens, bathrooms and winter gardens.

In some textile industries it is necessary for the air to have a constant humidity of 80 - 85%.

In these instances condensation is avoided by the application of adequate insulation.

Water conduits, cold water tanks and refrigerating chambers all have to be insulated in order to avoid condensation. When the humidity of the air is even higher than this it is not possible to find adequate insulation and in such instances it is necessary to resort to air circulation and ventilation. A very good but nevertheless expensive solution of the problem is the construction of double floors, with warm air circulating between them.

So called panel heating which is built into ceilings and walls represents another adequate protection against condensation. In order to cover all the possible causes of condensation it is necessary, to know the temperature of the air and to measure its humidity and dew point.

The method of taking the temperature has already been referred to on page 5 etc.

One of the simplest ways in which the humidity of the air can be measured is to pass air through a tube which is

/filled......

filled with a hygroscopic substance. The volume of the air passed is then measured. The hygroscopic substance can be either calcium chloride or concentrated Sulphuric Acid. The tube has to be accurately weighed in 1/100th parts of a gramme before and after passing the air. After the experiment, the quantity of air passed is known, and the difference in volume (weight of moisture absorbed) is also known, so the humidity can be expressed as a percentage.

Another primitive method is to measure the pressure of the air in a closed vessel before and after the absorption of moisture by a hygroscopic substance placed in it.

The modern method of measuring the humidity of air gives direct readings in % and the instrument used is known as a hygrometer.

Wet and dry bulb hygrometers consist of two thermometers both of which are identical. The bulb of one of these thermometers is wrapped in a damp cloth. The water evaporates from this cloth and the bulb cools down withdrawing heat from the thermometer. The greater the saturation of the air, the less evaporation and consequently, the less the cooling. The difference in the readings of the two thermometers gives the absolute humidity. There is one difficulty in this method of measurement and that is that the air in the immediate neighbourhood of the wet bulb soon becomes saturated because of the evaporation. The result is therefore a very high absolute humidity. In order to prevent this happening, the air near the hygrometer is kept in a continuous state of motion.

Another type of hygrometer employes a stretched human hair entirely free from grease. The hair becomes longer in moist air. One end of the hair is fixed and the other end is wound round a pointer which moves as the hair extends. The scale along which the pointer moves is calibrated

/empirically....

empirically and indicates the absolute humidity of the air in %. The principle of the dew point hygrometer is that one part of the instrument is cooled down until moisture condenses on it, and the temperature is then measured. This is of course the dew point so that the/ temperature taken is a/ direct measurement.

7. THERMAL EXPANSION.

It is well known that when a solid body is heated, the resulting increase of volume takes place in all directions simultaneously.

The so-called isotropic bodies expand in the same ratio in all directions; whilst crystaline structures often expand in one direction more than another.

Isotropic bodies however will only be considered in this case.

Making allowance for the fact that all buildings and structural erections consist of oblong or rod shaped bodies, such as beams, slabs, columns etc: it is proposed to deal only with expansion in one direction, namely linear expansion.

The deffinition of the coefficient of linear expansion (β) of a solid body is taken to be the ratio of its increase in length per degree (Centigrade) rise in temperature, to its length at 0°C (i.e. $\beta = \frac{\Delta l}{l}$ where l is the length of the bod at 0°C.

A rod of length l will therefore expand by an amount of '$l\beta$' for 1°C rise in temperature or '$l\beta t$' for a rise to t°C.

The overall length of the rod would then be $l + l\beta t$.

As an example take a rod of length l_o at 0°C which would expand to l_t at t°C.

Then $l_t = l_o\ (1+\beta t)$.

In order to determine the thermal expansion of concrete many tests have been made in various countries.

/The.....

The "Konigliche Materialprufungsamt zu Berlin Lichtervelde - West" (Germany) carried out some very accurate tests on a concrete prism in air and in water and in both.

During the tests the prism was heated whilst in air and water and also cooled to below freezing point in both instances. A continual check being kept on expansion and contraction.

The following facts emerged from the tests.

1. No change of dimensions worth mentioning were noticed 24 hours after the shuttering had been removed.

2. Whilst the hardening was taking place under water the concrete expanded, but whilst hardening in the open air the concrete contracted.
   During the first few days of the hardening process under water it was noticed that the expansion was somewhat greater than the contraction noticed under the open air conditions.

After six months however there was no noticeble change in the dimensions.

The contraction during hardening in the open air had therefore been influenced by the temperature and humidity of the surrounding air and also by the motion of the air over the surface of the concrete.

3. After hardening in the open air the specimen under test was immersed in water and it was noticed that it once again expanded to its original length as measured at the time of removal of the shuttering.
   It was found however that the older the concrete the quicker the time required to expand.
   On the other hand the specimen had been hardened in water was now exposed to the open air and in the course of the contraction which took place

/it.....

it was found that the fresher the concrete the quicker the contraction. Here again contraction ceased as soon as the original dimensions were reached and in no case did it continue to the dimensions reached by the specimen first hardened in the open air.

4. Expansion due to heat and contraction through cold conditions increase with the age of concrete. Both are influenced by the humidity of the concrete and by delay or anticipation of the hardening process caused by warming or cooling. Resulting from the numerous tests the assumption was made that the coefficient of thermal expansion is fairly proportional to temperature rise.
The accepted figure for the coefficient of linear thermal expansion of concrete whether the hardened in air or under water has been given as $\beta = .00001$.
This means that the increase in length of a 1 metre concrete rod will be .00001 metres per degree centigrade.

It has been stated in Holland that for concrete the figure should be .0000095 under the condition of three days, setting under water and 28 days in the open air.

During the period however the specimen was heated from 15°C to 42°C in the air and taking proportional expansion into account the above was an average value.

According to tests made by the German Cammerer the coefficient of thermal expansion of re-inforcing steel should be taken as .0000094 the expansion also being proportional to temperature increase.

These figures hold good for normal temperatures, i.e. not in access of 75°C or below -20°C.

In practice, as the figures for steel and concrete are practically the same, it is safe to take a figure of 0,00001 or $1X10^{-5}$ for calculations in both materials

/and.....

and herein lies the secret of the continued use of steel as a reinforcing factor in all forms of concrete constructions.

As already mentioned, there are two cases for consideration :

1. Uniform and simultaneous increase in temperature throughout the entire building.
2. One sided heating or cooling of a building.

1. The calculation of thermal expansion due to simultaneous and uniform increase of temperature throughout the whole structure is the most simple and the most customary.

Many structural Engineers are satisfied with a check on stresses caused by thermal expansion under maximum and minimum temperature conditions only. This is quite satisfactory if there is no danger of over heating the surface from sun effects. This method of calculation however is restricted to figures covering temperatures at the extremes of winter and summer conditions.

The expansion and contraction figures for winter and summer are however far more important, especially in the case of very large buildings.

Taking into account a maximum possible temperature difference of 30°C the change in length of a building with a 50 metre frontage would be

$$L\beta t = 50 \times .00001 \times 30 = 0.015 \text{ metres or } 1.5 \text{ cm } (5/8")$$

An expansion or contraction of 1 cm is quite sufficient to break a column firmly connected to a 50 metre beam or slab, or even to move a brick pillar from its place supporting such a beam.

The change of length in structural steel work might be as much as 20% more so that it becomes imperative to provide proper insulation in such cases.

When deformation is prevented it gives rise to stresses in the structure. These can sometimes be excessively high.

/Expansion.....

Expansion and contraction give rise in horizontal slabs and beams to compression or tensile stresses. The method of calculating these stresses is as follows:-

A prismatic rod is fixed in such a manner that both ends are immovable. (See Fig.No.25).
The length between the two ends B & C at a given temperature is L metres.

The change of length after the temperature has become $t°C = \pm L\beta t$.

This alteration in length causes either compression or tensile stresses to be set up acting freely all along the longitudinal axis of the rod.

From static engineering principles it is already known that the specific change of length of a bar influenced by normal stresses is $\varepsilon = \frac{\sigma}{E} \quad \frac{P}{AE}$

and the total expansion (λ) of a length L will be

$$L\beta t = \frac{PL}{\frac{EA}{E}}$$

Therefore $L\beta t = \frac{PL}{EA}$ and $P = EA\beta t.$

The compression or tensile stress per unit of cross section will then be $\sigma = \frac{P}{A} = \frac{EA\beta t}{A} = E\beta t.$

From this it will be seen the stress set up are undependant of length and of the area of the cross section.

The stresses set up in a steel structure arising from temperature fluctuations per square M. and per degree centigrade would be obtained as follows.

$E = 2.150.000$ kg/cm $\qquad \beta = \frac{1}{85.000}$

$\sigma = 2.150.000 \times \frac{1}{85.000}$ 25 kg/cm per °C.

at 30°C this would be 30 x 25 750 kg/cm

From this it is seen that a change of temperature of 30°C is quite sufficient to raise the stresses in a steel structure by c.a. 50%.

Although all calculation on structures are made with

/due....

due regard for certain safety factors and low stresses laid down by law it is obvious that stresses such as those emphasised above cannot be neglected unless the design of the building is such that freedom of movement can be obtained without danger to stability.

If there is no freedom of movement then horizontal forces set up in horizontal portions of the structure will resolve themselves co-axially and will set up transverse forces in the longitudinal axis of the supporting columns.

This will cause an addition to the bending moment of the supports and an eccentric pressure on the foundations and bearing soil will be the result.

If the expansion or contraction of the horizontal portions of the structure $f = L\beta t$ then the vertical portions of the structure (See Fig.No.26) will suffer from the influence of the force P and an elastic deformation will occur $f_0 = \frac{Pl^3}{3EI}$

or $P = f\,\frac{3EI}{l^3}$

Where l = the free length of the vertical structure and f_0 = the expansion or contraction of the horizontal structure which in turn is equal to the displacement of the point O under the influence of the force P.

$f = L\beta t = \frac{Pl^3}{3EI}$ L being the length of the horizontal structure.

and $P = \frac{L\beta t 3EI}{l^3}$ $\qquad M = Pl = \frac{L\beta t 3EI}{l^2}$

Beams consisting of a single slab are very seldom met with in reinforced concrete structures. The customary arrangement is in the form of a frame where slabs, beams and columns are all joined to a stiff frame work. It is in these solid joints which go to make up the stiff framework of the structure, especially in the joints between beams and columns that bending moments of great importance

/are.....

are set up as well as compressive and tensile stresses by expansion and contraction.

The expansion is reduced by columns fixed tightly to beams and therefore offering resistance. Each one takes up a certain bending moment. H (horizontal) and V (vertical) in the form of a force acting on the top of each column at the point where it is fixed. To simplify the case take an example of the top of a frame structure with the distance between columns the same. This is usually the case in a well designed structure where the difference rarely exceeds 10 - 15%. The inside structure does not change its temperature to any extent worth mentioning, as most buildings are internally heated during winter. In summer however the temperature is fairly constant inside, and is protected from overheating by the external structure. (See Fig.No.27).

The constants of the structure which now have to be considered are as follows:-

Temperature factor $C = \frac{E.I.\beta.t}{l} = \frac{T}{6}$

The elasticity modulus of the material E kg/cm^2.

The coefficient of expansion β

The inertia modulus of all the sections I

The assumption is made that the framework is absolutely stiff and that there are no loose joints.

The frame being symmetrical it will expand equally in both directions. It is therefore sufficient to investigate only one half of the structure.

There is one portion of the structure which is statically undetermined namely the beam A-D.

This beam is influenced by three stanchions at the points B, C & D. The influence is expressed in three horizontal forces (H_1, H_2 & H_3) and in three vertical forces (V_1, V_2 & V_3) as well as three turning moments at the points B, C & D denoted by (m_1, m_2 & m_3).

/The.....

The displacement from the effects of thermal expansion at the points B, C & D respectively will be as follows:-

B/ Horizontal displacement $\Delta_1 = \beta l t$

Vertical displacement $V_1 = 0$

Angular displacement due to turning moment $m_1 = \alpha_1$

C/ Horizontal displacement $\Delta_2 = \beta 2 l t$

Vertical displacement $V_2 = 0$

Angular displacement due to turning moment $m_2 = \alpha_2$

D/ Horizontal displacement $\Delta_3 = \beta 3 l t$

Vertical displacement $V_3 = 0$

Angular displacement due to turning moment $m_3 = \alpha_3$

Fig. No.28 shows all the moments which have to be considered.

The equation referring to the points of torsion set up by the moments m_1 m_2 & m_3 can be written from the following considerations.

The angle of torsion on the one side at any given point of a continuous beam fixed at one end is equal to the Algebraical sum of all the "forces" acting on the beam up to that point from the fixed end.

Under the heading of "forces" acting on one part of the beam it is to be understood that the volume of the area under the moment diagram belonging to that portion must be divided by the product of stiffness (EI).

1. Then $EI\alpha_1 = (m_1+m_2+m_3)\,l - \frac{1}{2}v_1 l^2 - 3/2v_2\, l^2 - 5/2v_3\, l^2$

2. and $EI\alpha_2 = (m_1+2m_2+2m_3)\,l - \frac{1}{2}v_1 l^2 - 2v_2\, l^2 - 4v_3\, l^2$

3. $EI\alpha_3 = (m_1+2m_2+3m_3)\,l - \frac{1}{2}v_1 l^2 - 2v_2\, l^2 - 9/2v_3\, l^2$

(See Fig.No.29).

It is now necessary to write up the equations for the vertical displacement of points B, C, & D respectively.

The displacement in any direction of any given point on the one side of a continuous beam which is fixed at one end

/is.....

is equal to the Algebraical sum of the static moments of the 'forces' acting from the fixed end up to that point. The meaning of 'forces' being the same as defined above.

4. Then $EIV_1 = \frac{1}{2}(m_1 + m_2 + m_3)l^2 - 1/3v_1 l^3 - 5/6v_2 l^3 - 4/3v_3 l^3 = 0$

5. and $EIV_2 = \frac{1}{2}(3m_1 + 4m_2 + 4m_3)l^2 - 5/6v_1 l^3 - 8/3v_2 l^3 - 14/3v_3 l^3 = 0$

6. and $EIV_3 = -\frac{1}{2}(5m_1 + 8m_2 + 9m_3)l^2 + 4/3v_1 l^3 + 14/3v_2 l^3 + 9v_3 l^3 = 0$

It is next necessary to consider the three stancheons supporting the continuous beam B.C.& D. (See Fig.No.30).

These stancheons are fixed into the ground floor frame at the bottom end.

At the point B.C.& D. they are attached by $m_1 H_1 V_1$ & $m_2 H_2 V_2$ and $m_3 H_3 V_3$ all acting in the opposite direction to those on the beam (See Fig.No.30).

The displacements at the points B.C. & D. will be as follows:-

B/ Horizontal displacement $\Delta_1 = \beta l t$

Vertical displacement $V_1 = 0$

Angular displacement caused by the moment $m_1 = \alpha_1$

C/ Horizontal displacement $\Delta_2 = \beta 2 l t$

Vertical displacement $V_2 = 0$

Angular displacement caused by the moment $m_2 = \alpha_2$

D/ Horizontal displacement $\Delta_3 = \beta 3 l t$

Vertical displacement $V_3 = 0$

Angular displacement caused by the moment $m_3 = \alpha_3$

All the displacements are in the same direction as before.

Applying the previous theorems the displacement equations are

7. $EI\alpha_1 = \frac{1}{2}H_1 l^2 - m_1 l$

8. $EI\alpha_2 = \frac{1}{2}H_2 l^2 - m_2 l$

9. $EI\alpha_3 = \frac{1}{2}H_3 l^2 - m_3 l$

10. $EI\Delta_1 = 1/3H_1 l^3 - \frac{1}{2}m_1 l^2 = EI\beta l t$

11. $EI\Delta_2 = 1/3H_2 l^3 - \frac{1}{2}m_2 l^2 = 2EI\beta l t$

12. $EI\Delta_3 = 1/3H_3 l^3 - \frac{1}{2}m_3 l^2 = 3EI\beta l t$

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In order to eliminate the angles $\alpha_1, \alpha_2 \& \alpha_3$ and so leave nine equations with nine unknowns the values of $EI\alpha$ are replaced as follows:-

$EI\alpha_1$ from equation (7) into equation (1)

$EI\alpha_2$ from equation (8) into equation (2)

$EI\alpha_3$ from equation (9) into equation (3)

with the following result.

1a. $(2m_1 + m_2 + m_3)\ell - \frac{1}{3}H_1\ell^2 - \frac{1}{2}v_1\ell^2 - 3/2v_2\ell^2 - 5/2v_3\ell^2 = 0$

2a. $(m_1 + 3m_2 + 2m_3)\ell - \frac{1}{3}H_2\ell^2 - \frac{1}{2}v_1\ell^2 - 2v_2\ell^2 - 4v_3\ell^2 = 0$

3a. $(m_1 + 2m_2 + 4m_3)\ell - \frac{1}{3}H_3\ell^2 - \frac{1}{2}v_1\ell^2 - 2v_2\ell^2 - 9/2v_3\ell^2 = 0$

Thus equations 1,2,3,7,8, & 9 are now replaced by 1a,2a,& 3a, and these form with 4,5,6,10,11,& 12 the nine equations with nine unknowns.

Either ℓ or ℓ^2 can be taken out of all these equations by division and in order to simplify the factors and reduce the coefficients to complete numbers, the following transposition has to be made.

$m_1 = x_1$	$H_1\ell = y_1$	$V_1\ell = Z_1$
$m_2 = x_2$	$H_2\ell = y_2$	$V_2\ell = Z_2$
$m_3 = X_3$	$H_3\ell = y_3$	$V_3\ell = Z_3$

The temperature factor $\frac{EI\beta t}{\ell} = C$ also has to be introduced.

By substitution:-

$3x_1 + 3x_2 + 3x_3 - 2Z_1 - 5Z_2 - 8Z_3 = 0$ 1

$9x_1 + 12x_2 + 12x_3 - 5Z_1 - 16Z_2 - 28Z_3 = 0$ 11

$15x_1 + 24x_2 + 27x_3 - 8Z_1 - 28Z_2 - 54Z_3 = 0$ 111

$-3x_1 + 2y_1 = 6c$ 1V

$-3x_2 + 2y_2 = 12c$ V

$-3x_3 + 2y_3 = 18c$ V1

$4x_1 + 2x_2 + 2x_3 - y_1 - Z_1 - 3Z_2 - 5Z_3 = 0$ V11

$2x_1 + 6x_2 + 4x_3 - y_2 - Z_1 - 4Z_2 - 8Z_3 = 0$ V111

$2x_1 + 4x_2 + 8x_3 - y_3 - Z_1 - 4Z_2 - 9Z_3 = 0$ 1X

Equations V11, V111 and 1X can be multiplied by 2 and added to equations 1V, V and V1 respectively leaving

/6......

6 equations with 6 unknowns (X_1, X_2, X_3 and Z_1, Z_2 and Z_3) and the result is

that $3X_1 + 3X_2 + 3X_3 - 2Z_1 - 5Z_2 - 3Z_3 = 0$ 1.

and $9X_1 + 12X_2 + 12X_3 - 5Z_1 - 16Z_2 - 23Z_3 = 0$ 2.

$15X_1 + 24X_2 + 27X_3 - 8Z_1 - 28Z - 84Z_3 = 0$ 3.

$3X_1 + 4X_2 + 4X_3 - 2Z_1 - 6Z_2 - 10Z_3 = 6c$ 4.

$4X_1 + 9X_2 + 8X_3 - 2Z_1 - 8Z_2 - 16Z_3 = 12c$ 5.

$4X_1 + 8X_2 + 13X_3 - 2Z_1 - 8Z_2 - 18Z_3 = 18c$ 6.

The values of the unknown will be as follows:-

$X_1 = C\ 4.4$ $y_1 = C\ 9.6$ $Z_1 = -C\ 5.5$

$X_2 = C\ 9.7$ $y_2 = C20.5$ $Z_2 = -C10.2$

$X_3 = C\ 9.6$ $y_3 = C23.4$ $Z_3 = +C16.2$

(See Fig.No.31).

The moments caused by temperature fluctuations over the entire beam and stanchcons will be as follows:-

$M_D = m_3 - X_3 = +3.6c$

$M_{C1} = m_3 - v_3 l = -6.5c$

$M_{C2} = M_{C1} + m_2 = + 3.2c$

$M_{B1} = M_{C2} - (v_3 + v_2) l = -2.8c$

$M_{B2} = M_{B1} + m_1 = +1.6c$

$M_A = M_{B2} - (v_3 + v_2 + v_1) l = -0.8c$

$M_J = m_3 - H_3 l = -13.8c$

The calculation of the unknown moments at the junction points in more complicated statically undetermined structures is based on the equations of Mohr. These cover the work done in deformation as well as upon virtual displacement. The equations ought to be built up in such a way that it is possible to control the influence of each girder and stile separately.

The magnitudes of the statically undetermined forces at each junction point are the moments of the girders and therefore there must be as many equations.

Two cases must be distinguished from each other in these problems.

/1.......

1/ If the points of junction between girders and stiles are immovable (most reinforced concrete structures) the number of points of junction of the girder are equal to the degree of static undetermi ation.

2/ If the points of junction and so the whole frame can be moved horizontally. The undetermined static forces being one less than the number of junctions, the whole structure becomes unstable and in order to simplify the calculations they have to be made statically determined.

The moments at the points of junction can be put in the following plain form:-

$$\pm \text{ B e ik}$$

where B = the temperature factor $T = \frac{6EI\beta T}{s}$
$\frac{6EI}{s}$ is the reduction factor, S being equal to l or h depending on whether it concerns a girder or a stile eik is a constant for the structure and in frame structures where stiles are all of the same length it only concerns the girders (R).

The eik = R = $\mathcal{L}\mathcal{H}$

$\mathcal{L} = \frac{6}{5}$ x the reaction from the volume of the area of the moment diagram which is applied as a 'force' and belongs to the girder in question.

$\mathcal{H}$ = the stiffness coefficient of the same girder its value being $\frac{I_c S}{I S_c}$

The elasticity equation is set up as follows:-

$$X_1\delta_{11} + X_2\delta_{12} + \dots\dots\dots + X_M\delta_{1M} = \delta_{10}$$
$$X_1\delta_{21} + X_2\delta_{22} + \dots\dots\dots + X_M\delta_{2M} = \delta_{20}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$X_1\delta_{M1} + X_2\delta_{M2} + \dots\dots\dots + X_M\delta_{MM} = \delta_{M0}$$

where $X_1 = M_A$, $X_2 = M_{B1}$, $X_3 = M_{B2} \dots\dots X_M = M_{MM}$.

and the solution will be

$$X_1 = +\delta_{10}m_{11} + \delta_{20}m_{21} - \dots\dots\dots - \delta_{M0}m_{M1}$$
$$X_2 = +\delta_{10}m_{12} + \delta_{20}m_{22} - \dots\dots\dots - \delta_{M0}m_{M2}$$
$$\dots\dots\dots\dots\dots\dots\dots\dots$$
$$X_M = -\delta_{10}m_{1M} - \delta_{20}m_{2M} + \dots\dots\dots + \delta_{M0}m_{MM}$$

/The.....

The plus or minus signs of the moments are given automatically in the theory of their determination by the condition that the magnitudes of each influence $M_{iK} = \frac{Z}{N}$ ought to be a positive number.

A list of the magnitudes can be made up as follows:-

$$\begin{array}{ccccc} M_{11} & M_{21} & M_{31} & \cdots\cdots & M_{M1} \\ M_{12} & M_{22} & M_{32} & \cdots\cdots & M_{M2} \\ M_{13} & M_{23} & M_{33} & \cdots\cdots & M_{M3} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{1M} & M_{2M} & M_{3M} & \cdots\cdots & M_{MM} \end{array}$$

M is the magnitude of the influence created from the area of the moment diagram acting as a 'Force' on the girder it concerns.

Z and N are compound stiffness coefficients formed from the simple stiffness coefficients $\mathcal{K}_1, \mathcal{K}_2 \ldots \mathcal{K}_M$ and $\mathcal{K}_I$ $\mathcal{K}_{II} \ldots \mathcal{K}_{Z}$ Compound magnitudes of influence can be formed in a similar way from the simple magnitudes (k for stiles and g for girders).

This method of calculation is a very complicated one and is not altogether necessary.

If precautionary measurements (expansion joints - insulating etc); which are discussed later in this paper; are taken into consideration then the calculation of thermal expansion at uniformity and also simultaneously increase of temperature may be neglected.

2/ The temperature increase on one side of the structure only. One sided heating is of great importance in reinforced concrete structures especially in the case of flat roofs, canopies, balconies etc. In South Africa the average daily sunshine which is 8.7 hours, has to be reckoned with. The maximum sunshine period occurs in August (9.9 hrs) and the minimum in January (7.2 hrs). This data has been supplied by the Union Observatory - Johannesburg.

The stresses set up by one sided heating can be very dangerous in structures. That temperature fluctuations

/can....

can cause very high stresses is proved by the numerous cracks in buildings. The real reason for all these cracks is very rarely thought about.

One sided heating is a problem which frequently arises in respect of walls and roofs of boiler rooms and roofs exposed to sunshine. It is also to be blamed for cracks in concrete chimneys, smoke tubes and boiler lagging.

Cracks are sometimes to be found in reinforced concrete floor slabs, especially after a severe winter when on the one side of the floor the adjoining room has been well heated.

In making calculations for one sided heating or cooling it is necessary to assume that heat conduction in homogeneous material is rectilinear. The fundamental formula has to be drawn up for a slab through which a steady flow of heat is passing.

If the temperature of the one surface is t_1 and of the other t_2 then the temperature at any point on the slab is (see Fig.no.33)

$$t_x = \frac{t_1 - t_2}{\delta} x + t_2$$

$$\text{If } t_2 = 0 \text{ then } t_x = \frac{t}{\delta} x$$

The flow of heat through walls and floors is fairly constant under the influence of one sided over heating, the elements of a slab expand unevenly with the result that the slab will bend if it is free from eternal forces.

The convex side of the slab will always be on the side exposed to the higher temperature and the radius of curvature will be the same overall (See Fig.No.33).

There will however be no stresses raised in the slab, and after the deformation the radius of curvature will be

$$R = \frac{\delta}{\beta t}$$

where β = the coefficient of expansion of the material.

/The....

The thicker the slab the less the curvature. Any increase of t or β will increase the curvature.

β is fairly constant for reinforced concrete, and therefore any deformation is entirely dependant on the thickness of the slab and the temperature difference. ✓

If the slab is divided along a line parallel with its faces into two or more layers the ratio $\frac{S}{t}$ remains the same and R will also be the same. There are also no stresses set up in the slab under these conditions.

The same remarks apply equally to beams or other sections of structure provided the temperature conditions are the same and that there are no external forces to take into account.

Most reinforced concrete structures are not however free from external forces.

Slabs are fixed into beams and beams into columns or walls and so on.

To reach the true situation in such cases it is necessary to bend back the slab or beam so far, that it will comply with all the conditions (See Fig.No.34). The conditions are of course imposed by the fixing points of the slabs or beams.

In the course of bending back, stresses will be set up determined by the moments acting on the edges of the slabs or beams, tending to return them to their original straight positions.

The moment M can be found from the following equation

$$\frac{1}{R} = \frac{M}{EI}$$

where E = the modulus of that portion of the structure influenced by M.

The bending stresses can be determined from the following considerations.

Suppose the temperature on one side of the girder is t°C above that on the other side and that the drop of

/temperature.....

temperature is rectilinear.

The temperature at the top and bottom of the girder will then be $+\frac{1}{2}t$ and $-\frac{1}{2}t$ respectively (See Fig.No.35).

The girder will therefore tend to expand at the top and contract along the bottom or in other words it will bend.

The radius of curvature is R and from Fig.No.36 the angle covered by a tangent to the bent girder and the original horizontal position will be :-

1. $\gamma_t = \frac{\frac{1}{2}l}{R} = \frac{l}{2R}$

In fig. No.35 is a small element taken from the girder shown in fig. No.36.

$$d\varphi = \frac{\beta t \, dl}{\delta} \quad \text{and} \quad \frac{d\varphi}{dl} = \frac{\beta t}{\delta}$$

2. also $\frac{1}{R} = \frac{\beta t}{\delta}$

from equation No. 1 and No.2 $\sphericalangle \gamma_t = \frac{\beta t l}{2\delta}$

What is the position of a slab or beam subjected to a temperature fluctuation of t°C if not able to expand or contract on account of external moments acting on the edges?

The moment will be :- $\sphericalangle \gamma_t = \frac{\beta t l}{2\delta}$ and $\sphericalangle \varphi = \frac{Ml}{6EI} + \frac{Ml}{3EI} = \frac{Ml}{2EI}$;

$\frac{\beta t l}{2\delta} = \frac{Ml}{2EI}$ hence $M = \frac{EI\beta t}{\delta}$

Str.max. $= \frac{EI\beta t}{\delta} : \frac{I}{\frac{1}{2}\delta} = \frac{1}{2}E\beta t$

As an example take a continuous beam with 3 supports (see fig. No.37).

The distances between the supports are l_1 and l_2.

The modulus of inertia for No.1 girder is I_1, and for girder No.2- I_2, E is the modulus of elasticity.

If the beam is cut through over the centre support then the angles formed through deformation caused by thermal expansion of the two beams will be:

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By applying moments M_1, M_2 and M_3 over the three supports the girder will automatically close again at the centre support. At both ends of beam No.1 the acting moments M_1 and M_2 will cause angles φ_1 and φ_2 and the equation for these angles is well known from the statics.

$$\sphericalangle \varphi_1 = \frac{M_1 l_1}{6EI_1} + \frac{M_2 l_1}{3EI_1}$$

and

$$\sphericalangle \varphi_2 = \frac{M_3 l_2}{6EI_2} + \frac{M_2 l_2}{6EI_2}$$

The angles φ_1 and φ_2 should be equal to γ_1 and γ_2 therefore

$$\frac{M_1 l_1}{6EI_1} + \frac{M_2 l_1}{3EI_1} + \frac{M_3 l_2}{6EI_2} + \frac{M_2 l_2}{3EI_2} = \frac{\beta t l_1}{2\delta_1} + \frac{\beta t l_2}{2\delta_2}$$

If the dimensions of the sections of all the beams are the same then this equation can be put in the following form

$$M_1 l_1 + 2M_2(l_1 + l_2) + M_3 l_2 = \frac{3EI\beta t}{\delta}(l_1 + l_2)$$

and if all girders are the same length

$$M_1 + 4M_2 + M_3 = \frac{6EI\beta t}{\delta}$$

As a further example take a girder supported by 5 columns, the columns being equally spaced. The modulus of inertia of the girder is the same throughout. (See fig.No.38)

If the modulus of inertia of the end columns is very small in comparison with the modulus of inertia of the girder the moments over the end supports may be neglected.

The equation will be :-

$$0 + 4M_1 + M_2 = \frac{6EI\beta t}{\delta} \qquad \text{hence } \frac{EI\beta t}{\delta} = C$$

and $$M_1 + 4M_2 + M_1 = \frac{6EI\beta t}{\delta}.$$

and $$\begin{aligned} 4M_1 + M_2 &= 6C \\ 2M_1 + 4M_2 &= 6C \end{aligned} \qquad \text{hence } M_1 = \frac{9}{7}C\,; \quad M_2 = \frac{6}{7}C.$$

The maximum stresses set up by temperature fluctuations will be :-

$$St_{r.max.} = \frac{M_{max}}{W} = \frac{M_1}{W} = \frac{9}{7}\cdot\frac{E.I.\beta.t}{\delta} \Big/ \frac{I}{\frac{1}{2}\delta}$$

/or......

or $Str._{max.} = \frac{9}{14} E.\beta.t$

This shows that stresses set up by temperature fluctuations are independant of the cross section and length of girders.

It is not an exageration to say that a temperature fluctuation of 25°C causing one sided overheating can set up a compressive stress of 50 kg/cm² in a reinforced concrete structure

$$C = \frac{210000 \times 10^3 \times 100 \times 25}{100000 \times 10 \times 12} = 43750.$$

Since if $E = 210,000$ kg/cm² and $\beta = 1:100,000$

$M = 9/7 . C = 56250$ kg.cm and $Str.max = \frac{2M}{b \cdot x (h_1 - \frac{x}{3})} = 55$ kg/cm²

This proves that one sided over heating can cause many cracks in reinforced concrete structures especially where the structure is exposed to the rays of the sun.

In such cases it is wise not to neglect wellapplied insulation as a preventative against damage from temperature fluctuations.

8. CONTROL OF THERMAL CONDITIONS IN BUILDINGS.

The following points of view have to be borne in mind when the insulating problems of buildings are under consideration.

1. To create a pleasant and bearable temperature for those who are going to inhabit the buildings, to create the right temperature in factories, cold storages etc., during both summer and winter.
2. To keep the air dry (food stores etc:) and to create the right humidity where equired (for instance in silk weaving industries). Generally to give satisfactory effect to the numerous prescriptions existing for industrial buildings.
3. To prevent condensation which only causes corrosion and the formation of mould etc.,. Humidity will also reduce the insulating capacity of walls, floors, roofs and insulating materials.

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The thermal conditions in buildings can be influenced in three different ways.

1. By insulation and ventilation.
2. Heat and cold accumulation.
3. Expansion joints.

The first and also the most important precautionary measures which have to be taken are proper insulation and proper ventilation. Attention to this major point can give proper regulation of temperature and prevent over heating and condensation in buildings. A different kind of insulation has to be applied when buildings are heated or cooled permanently or even temporarely. In such special cases as churches or halls for particular purposes, where use is only made of such buildings for perhaps a few hours daily or a few days weekly, the designer has to consider lighter insulating materials with a higher insulating capacity.

In this way the heating of the room can be accelerated and so the periodic use of fuel is more economical.

On the other hand buildings which must be protected against heating, have to be insulated with heavy materials with great heat accumulation capacity.

The difference between insulating materials of low and great heat accumulation capacity has already been referred to in earlier paragraphs.

Heat accumulation does not come into the question when continuous heating or cooling are under consideration. In any case Insulation of buildings does not prevent loss of heat, it merely postpones the cooling period.

A reinforced concrete or other type of building with proper insulation is equipped in such a way that over heating from the sun is postponed long enough for the night to come again. When rooms are heated, correct insulation postpones the cooling period until heating commences again.

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The use of good insulating materials is not in itself satisfactory as much depends on the greatest care being taken with the application.

As already mentioned, humidity can reduce insulating capacity (in many cases by more than 50%).

In such cases the insulating material must be protected against water penetration.

Buildings exposed to temperature fluctuations have to be insulated if cracks are to be avoided through expansion and contraction, further they have to be provided with expansion joints, properly designed and executed.

Again th insulating material must be made waterproof.

Insulation is at its minimum just before condensation starts. The whole problem of insulation can be illustrated by two examples taken from practise.

1. Iron Roofs. (No insulation & no heat accumulation).

In a factory in Budapest (Hungary) which was provided with big windows and an iron roof exposed to the rays of the sun all the workmen suddenly became ill on account of the intolerable heat inside the factory.

The outside temperature was 20 - 22°C and the inside temperature had risen to 35°C.

The total resistence of the roof to the passage of heat was:-

Iron sheets	0.00
0.2 cm layer of air between the iron sheet and the boarding found to be at a temperature of 50°C -.002 x 12	0.024
2 cm roof boarding (wood) : .02 x 8.3	0.166
2 cm reed mat, with plaster, forming the ceiling : .02 x 2.5	0.05
	R = 0.24
The convection inside could be put at	0.14
	$\frac{1}{k}$ = 0.38

/R......

$R = 2.63$

The surface temperature of the roof outside was + 60°C (oxidised black metal sheets) and in the beginning the inside temperature was + 20°C.

Assuming a steady flow of heat, the supply of heat was :- $Q = 2.63 \times (60 - 20) = 105$ cal/m²/hr.

The amount of heat supplied became gradually smaller as the room became hotter otherwise a critical situation would have arisen earlier.

What could be done in a case like this with improvement in insulation?

With the application of two layers of exp. cork each 2 cms thick under the roof and with a layer of air of 2 cms in between the increase in insulating capacity will be :-

$$R' = \underbrace{0.04 \times 25}_{\text{Cork}} + \underbrace{0.02 \times 7.1}_{\text{Air}} + \underbrace{0.01 \times 3.4}_{\text{Plaster}} = 1.176$$

$$\frac{1}{k} = a + R + R' = 0.14 + 0.24 + 1.176 = 1.556$$

$$k = 0.645$$

$$Q = 105 \frac{0,643}{2,63} = 25.7 \text{ cal/m}^2\text{/h}$$

It can be said therefore that this treatment would reduce the original heat supply by 1/4. ✱

Taking into account the fact that the heat accumulation capacity of cork and plaster is not a fixed figure and by treating the surface of the roof with aluminium paint to prevent heat absorption, the heat supply can be cut down to 1/8 of the original supply.

All this, combined with an adequate system of ventilation will improve conditions in the factory to such an extent that they would be normal.

Without air circulation however the air inside would still be unbearable, because only 3 cubic metres (4.2 kg) of air are present (as against 1 sq metre of roof surface. The specific heat of air being .31 it would require 1.3 cal. only to raise the temperature of 3 cubic metres of air

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by 1°C. Taking into consideration the insulating scheme already mentioned the temperature in the room would rise by $\frac{25.7}{1.3} = 19.8$°C in an hour.

To this has to be added the supply of heat from the bodies of th workmen (80 cal/ hr) and on the other hand the heat which exudes from walls and floors can be deducted.

If the air is changed 4 times per hour the increase of temperature would drop to $\frac{19.8}{4} = 4.95$°C per hour. This is no longer an unbearable increase in temperature. Evaporation from the human bodies is accelerated by the air circulation and the skin is pleasantly cooled down. The question of heat accumulation can now be investigated and possible improvements suggested.

The heat accumulation capacity of the roof per sq metre was:-

5 kg sheet metal	a	.12 =	0.6 cal
12 kg roof boarding and reed mat	a	.6 =	7.2 cal
24 kg. plaster	a	.22 =	5.28 cal.
			13.08 cal.

The roof temperature outside was + 60°C and inside + 40°C. This means an average temperature of + 50°C. It requires 13 X 30 = 390 cal to warm up the roof from + 20°C to + 50°C and this can be radiated by the sun in 1 hour. The roof can be improved by changing the reed mat and plaster for a 10 cm concrete slab with 6 cm fossil meal slab and a 0.5 cm layer of bitumen as insulation.

Th heat accumulation capacity will be:-

3 kg.	bitumen	a 0.22
30 kg.	fossil meal	a 0.22
240 kg.	concrete	a 0.22
273 kg.	x 0.22 = 60.06 cal.	

This means that 4.6 times as much heat is now required

 /to.....

to warm up the building to the same temperature as before.

Before this high temperature can be reached the sun will be past its peak and will be on the wane.

2. Stresses created in a reinforced concrete roof by one sided heating.

The following example which has been supplied by Theophile Schaerer Esq., an Architect of Johannesburg has been investigated by the writer. (Thanks are due to Mr. Schaerer for this opportunity).

The problem was that of cracks in the slabs as well as the beams in the reinforced concrete roof of a factory.

The general situation is shown in Fig.39.

The roof was not loaded in this case nor was there any insulation for protection against damage from the rays of the sun. The problem was therefore to investigate the structure from the angle of one sided over-heating from the rays of the sun. Suspicions were first of all aroused by the fact that the Northern portion of the building, on which the rays of the sun would naturally rest longest, showed the most damage. The thickness of slab in the roof was 12 cms (5"). The average dimensions of the beams showing damage (5 & 5a) were 84 X 23 cms (33" x 9").

The roof was covered with bituminous waterproof layers which owing to their dark colour have a very high heat absorption capacity.

The assumption can be made that over-temperature on the outside surface of the roof reached 30°C as against the air temperature of 20°C. The actual surface temperature of the roof can be taken thus 30°C + 20°C = 50°C

The commencing temperature inside the factory was say 10°C. In the first instance the concrete slabs were investigated (See fig. No.39). The weakest section of the slabs is at right angles to beams 5 & 5a because distribution bars of only 1/4" (dia.) were used, most of them

/ at......

at the bottom. Compressive stresses set up by overheating on one side should therefore lie at the bottom and tensile stresses at the top of the slab.

The total resistance of the roof to heat transfer is:-

$$R = \underbrace{0.12 \times 1.22}_{\text{Concrete}} + \underbrace{0.005 \times 8.33}_{\text{water proofing}} = .147 + .042 = .189$$

Convection inside the building (See chapter 5/1).

$$a = \frac{1}{\alpha} = \frac{1}{4.6 + 4.9} = 0{,}105.$$

$\frac{1}{k} = .105 + .189 = .294 \quad k = 3.4 \quad Q\ 3.4 \times (50-10) = 136$ Cal/per hour.

This heat supply is very high, but, because the room beneath was high and also had intense ventilation, it was not troublesome.

What was the possible surface temperature of the concrete?

By convection	$100\,\frac{0.105}{0.294}$	= 35.8%
Concrete	$100\,\frac{0.147}{0.294}$	= 50%
Waterproof layer	$100\,\frac{0.042}{0.294}$	= 14.2%
		100%

The rise in temperature through the roof from bottom to top is:-

By convection	40 X .358	= 14.32°C
Through the concrete slab	40 X .5	20 °C
Through waterproofing	40 X.142	5.68 °C
		40 °C

Temperature of the concrete slab.

Underneath	10 + 14.32°C	= 24.32 C
On top	24.32°C + 20°C	= 44.32 C
Temperature difference between top and bottom		= 20.- °C

Before the roof was covered with waterproof layers the temperature difference was as much as 25 - 30°C.

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The maximum stresses created by this temperature are as follows. (See fig. No.40).

$$4M_1 + M_2 = 6c$$

$$M_1 + 4M_2 + M_3 = 6c$$

$$2M_2 + 4M_3 = 6c$$

where $M_1 = 1.270.C$

$M_2 = 0.925.C$

$M_3 = 1.040.C$

$C = 43750$ kg.cm (See chapter 7/B)

$M\ max = 1.27 \times 43750 = 55500$ kgcm

$St.max = 54.3\ kg/cm^2$ which is a very high figure.

What is the position in the beams?

Here the temperature difference between top and bottom on account of less convection (greater thickness and lower bottom temperature mean less convection) was 30°C.

$$C = \frac{EI\beta t}{\delta} = \frac{210.000 \times 84^3 \times 23 \times 30}{12 \times 100.000 \times 84} = 850.000\ kg.\ cm$$

$$M = 1.27 \times 850.000 = 1.080.000\ kg\ cms$$

$$Str.\ max. = \frac{2 \times 1080000}{71 \times 26.7 \times 23} = c.a.\ 50\ kg\ cms.$$

The maximum additional tensile stress is thus 50 kg/cm² on the top of the beams over the supports and this means that the steel has to carry twice as much as the original calculations made for dead and moving loads.

It is however much worse along the top reinforcement. Here only 2 bars 5/8" diameter have to carry the stresses set up by one sided over heating. In fact most of the cracks occured 1/5th from the supports where the heavy top reinforcement has been stopped. The surface reaches a very high temperature during the day and the nightly re-radiation will therefore be intense. This will bring about one sided cooling of the structure especially during the winter months, when the radiation from the sun is more

 /intense.....

intense for shorter periods than the re-radiation overnight.

The heat accumulation of the roof is:-

270 kg concrete slab X .21 = 56.00 cal.

3 kg. water proofing X .22 0.66 cal

56.66 cal

The average temperature of the roof was $\frac{24+50}{2}$ =

To heat the roof from 20°C up to 37°C, 56.66 X 17=960 calories are required.

In Johannesburg the radiation is ca 0.462 gramme cal/sq cm/ minute which is 277 cal/sq M/hour.

It therefore takes $3\frac{1}{2}$ hours to heat up the roof from 20°C to 37°C which is fairly fast.

The one sided over heating and the high additional stresses caused there by can be prevented by putting 15 cm (6") of slag concrete over the concrete roof but under the water proof layer. (See fig,No.41).

The total resistance of the roof to heat through will now be.

$$R = \underbrace{0.12 \times 1.22}_{\text{Concrete}} + \underbrace{0.15 \times 2.0}_{\text{slag concrete}} + \underbrace{0.005 \times 8.3}_{\text{waterproofing}} = 0.147+0.3+0.042=0.489$$

$1/k$ = 0.105 + 0.489 = 0.594 k = 1.68.

and q = 1.68 X (50 - 10) = 67.2 cal/per hour.

The possible surface temperature become.

by Convection $100\frac{0,105}{0,594}$ = 17.7%

Concrete $100\frac{0,147}{0,594}$ = 24.8%

Insulation

(Slag concrete) $100\frac{0.3}{0,594}$ = 7 %

100 %

and the rise of temperature from bottom to top becomes

/by.....

By convection	40 X 0.177	= 7.1°C
through concrete slab	40 X 0.248	= 9.9°C
through slab concrete	40 X 0.505	= 20.2°C
through waterproofing	40 X 0.07	= 2.8°C
		40°C

In this case the temperature difference between the two sides of the concrete slab has been reduced by more than half, and consequently the stresses are reduced as well.

The improved heat accumulation of the roof will be:-

270 kg. concrete slab	X 0.21	= 56 cal
190 kg slag	X 0.2	= 38 cal
3 kg waterproofing	X 0.22	= 0.66 cal
		94.66 cal

Average temperature of the roof $\frac{17+50}{2}$ = 33.5°C

In order to heat the roof from 20°C up to 33.5°C 94.66 X 13.5 = 1300 cal is required.

If 4 cms of exp. cork were used for insulation instead of breez concrete the heat required to warm up the roof would be ca 1900 cal.

In this case 7 hours intense radiation would be necessary to warm up the roof, not to mention that the surface temperature of the concrete is also reduced considerably.

What is the difference between the application of heat accumulating and insulating material on the cold as opposed to the warmer side (See fig.No.42).

In the first case if the insulation is placed on the warmer side the heat accumulation will be lower and the roof will be warmed up quicker. The difference between the surface temperatures of the concrete structure will be lower however and over heating of the surface will be prevented.

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The second case is more favourable from the point of view of the heating of the entire roof.

Heating and cooling will take longer, but the structure is not protected against over heating.

Where there is no fear of surface over heating from the sun, it is more economical to keep the temperature constant by putting the insulation on the cooler side of the roof or wall. In the case of cold storages it is more economical to put insulation inside the cold chambers.

From the angle of one sided over heating or over cooling it is best to apply the insulation on the side where either the one or the other is most to be feared.

If hollow blocks were used for insulation an even bigger reduction could be made.

In the New Hospital at Middelburg Transvaal (for which writer was appointed reinforced concrete engineer by the Architect MR.G.Moerdyk) the roof has ~~been~~ insulated with breez concrete and hollow blocks as well.

This was done in order to prevent any possibility of cracks, which might normally be expected as a result of the bright sunshine.

Furthermore the whole of the concrete frame structure has been insulated with 11.5 cm ($4\frac{1}{2}$") brickwork which is sufficient for vertical walls which are not struck at right angles by the sun.

9. EXPANSION JOINTS.

As well as the precautionary measures mentioned before, care must be taken in large buildings to examine the problem of contraction and expansion caused by temperature difference between summer and winter. As already mentioned, if there are expansion joints at the right places and these are properly carried out, then there is no danger from this cause.

It is not only necessary to make expansion joints at

/the....

the points where expansion and contraction due to thermal conditions are likely to cause changes in calculated dimensions but also where portions of the building are constructed from two different materials which join each other and where a new building adjoins an old one (due to different bearing soil pressures).

It is also advisable where a high building joins a lower one.

These expansion joints have to go right through the whole building. In ordinary circumstances expansion joints can be made at an average distance of 30 metres where greater temperature variations are to be expected however this distance should be cut down to 15 metres.

This is valid for unprotected flat concrete roofs, unprotected balconies, verandahs etc.

In Middelburg Transvaal expansion joints at 15 - 20 metres were made by the writer.

These joints go right through the building from top to bottom except in the foundations which are fairly deep and are therefore already protected from temperature fluctuations.

In high buildings it is always best to increase the number of expansion joints in the upper floors.

Additional joints should be put into canopies, balconies and flat roofs. Expansion joints must go right through the structure, that is:- right through the brickwork, plaster etc: and they should be shown on the elevation.

In order to cover the cracks in the covering plaster at the expansion joints it is advisable to make deep joints in the plaster at those places. In this way the crack will be straight and therefore hidden. (See fig.No.43).

Plenty of room should be allowed for joints to enable the structure to move and also in case of fire. At Middelburg Hospital the space is 13 m/m (½").

/Cantilevers.....

Cantilevers at the joints have to be correctly calculated in order to avoid deflection. Where heavily loaded beams rest on cantilever consoles, every effort has to be made to allow movement at the joints, eye joints etc. (See Fig.No44).

In flat concrete roofs exposed to a great quantity of heat, numerous and heavy distribution bars have to be applied in order to carry the stresses caused by thermal expansion and contraction and by shrinkage of the concrete due to quick drying out.

Fig. No. 45 shows various methods of carrying out expansion joints in different structures.

Fig.A shows double beams with one resting on the column itself and the other loose on a console belonging to the same column. Between the two beams there is a proper joint enabling the two parts of the building to move.

Fig. B shows the solution in Middelburg Transvaal where the joints go right through the new Hospital building.

Fig.C shows a solution with double columns and double beams. The opening between the adjoining parts has to be filled up.

Fig.No.46 shows several different solutions of this.

In the Middelburg Hospital ordinary damp course has been used as shown in Fig.A. This damp course goes through the concrete and brickwork. The filling can consist of bitumen or lead. The opening can be covered by either an iron or copper strip (Fig.B).

Console bearings must be strong enough to carry any horizontal movement of the beams which rest on them.

In the Middelburg Hospital there are beams which do not actually rest on the consoles. They are an exception as they are cantilever beams (See fig.No.45).

There is an open space between cantilever and console which is filled with a few layers of damp course.

/Once.....

Once again care has to be taken where floors join walls (Fig.47). shows the solution at Middelburg. The deal floors are able to move under the floor plinths.

Care must also be taken when waterproofing roofs over expansion joints. It is better to put one extra layer over the joints and best of all to make a special moveable cover out of Zinc or copper which can take up expansion or contraction. (See fig.No.48).

Lack of space prevents further discourse on this matter. Each must be treated separately and there are no special regulations concerning the execution of expansion joints.

The views held by the designer play the major part in the scheme. If first principles are taken into consideration by the designer on the lines of those discussed in this thesis, a solution suitable to the particular circumstances always presents itself. In such instances there is no likelihood of any damage from cracks caused by thermal expansion and contraction due to uniform or simultanoous increases in temperature.

10. EXPANSION & CONTRACTION BY HUMIDITY.

Finally the stress caused by expansion and contraction under the influence of humidity must not be overlooked. These stresses are similar to those set up by temperature fluctuations.

Deformation of bodies under the influence of humidity, is different from deformation caused by temperature fluctuations. The passage of heat for instance through concrete is gradual but there is quite a sharp separation between the dry and the moist portions of it.

The drop of temperature may be gradual, but there is no question of gradual change of humidity in concrete during the setting process.

If at the beginning of the setting period a concrete slab is cut through a wet outer coating will be exposed sur-

/-rounding....

rounding wet core.

The stresses set up are the same as when cooling a slab on both surfaces. In the core compression is set up whilst outside a tensile stress causes expansion.

If the cross section of the outer coat and the core are the same then the stress will also be the same.

At the beginning of the setting process, that is when the drying of the surfaces first starts, a maximum tensile stress is set up which will cause cracks due to contraction. This is the reason why it is so important to protect the outer surfaces from drying too quickly, either by keeping them wet, or by keeping them covered as a protection from the rays of the sun. It is also very useful to place contraction netting in the concrete as near as possible to the surface, but sometimes this is not possible (concrete ornaments with no reinforcement) and then the only way to stop cracks is to prevent the concrete from drying up too quickly during the setting period. There is no formula for the calculation of stresses set up in concrete by expansion and contraction caused by humidity. Only the change of length for dry and wet conditions is known (see also chapter 7) which amounts to .2 - .5 mm per metre (for contraction or expansion).

Assuming that E = 210.000 kg/cm² for contraction.

and E = 80.00 kg/cm² for expansion.

the result will be stresses of 40 kg/cm² set up in the concrete.

This illustrates the fact that high stresses can be set up by a change of humidity (quick drying on the outside) as well as by temperature fluctuation in reinforced concrete structures.

It has been clearly shown therefore that; removal of shuttering from columns after a few hours; removal of side shuttering from beams in one or two days; and also

/that.....

that concrete construction in bright sunlight with intense rays beating down on freshly placed concrete can cause damage of an important nature. These conditions being particularly applicable to South Africa it can be said that the root cause of many cracks in reinforced concrete buildings can be traced to expansion and contraction set up by temperature fluctuation and changing humidity.

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B I B L I O G R A P H Y.

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Cammerer, J.S.- Heizungs-und Wärmeschutztechnische Untersuchungen an Siedlungbauweisen.

Cammerer, J.S.- Konstruktive Grundlagen des Wärme und Kälteschutzes,. Berlin.

Boon, A.A.- Gewapend Beton,. Leiden.

Bosch, M. ten,- Die Wärmeübertragung,. Berlin.

Edser Edwin,- General Physics for Students,. London.

Esser, W. Dr.Ing.- Kirscher, O. Dr.Ing.- Die Berechnung der Anheizung und Auskühlung ebener und zylindrischer Wände,- Berlin.

Gregor, A.- Der practische Eisenhochbau,. Berlin.

Grimsehl, E.- A Textbook of Physics,. London.

Gröber, H. Dr.Ing.- "Rietschel" Heiz und Lüftungstechnik, Berlin.

Hartgerink, G.J.- Natuurkunde for den Technicus,.

Hencky, K, Dr.Ing.- Die Wärmeverluste durch ebene Wände. München-Berlin.

Gröber - Erk.- Die Grundgesetze der Wärmeübertragung,Berlin.

Kalender für Gesundheits & Wärmetechnik,- 1938,. Berlin.

Hutchinson's Technical & Scientific Encyclopedia,. London.

Kleinlogel, A.- Mehrstielige Rahmen,. Berlin.

Kleinlogel, A.- Bewegungsfugen im Beton und Eisenbetonbau. Berlin.

Kleinschmidt, E.- Handbuch der Meteorologischen Instrumente,.Berlin.

Klopper, J.- Leerboek der toegepaste Mechanica,. Delft.

Knoblauch, O. & Hencky, K.- Anleitung zu genauen Technischen Temperaturmessungen,. München-Berlin.

Möller, K. Dr.Ing.- Építési zsebkönyv,. 1934,. Budapest.

Möller, K. Dr.Ing. & Geiszár, J.- Isolatie,. Leiden.

Poensgen, R.- Ein technisches Verfahren zur Ermittlung
der Wärmeleitfähigkeit plattenformiger Stoffe.
Berlin, V.D.I.

Raisch, E.- Die Luftdurchlässigkeit von Bausteffen und
Baukonstructionsteilen.

Szily, R.- Statika,. Budapest.

PAPERS.

Deutscher Ausschluss für Eisenbeton.

Die Wärme.

Gesundheits Ingenieur.

Gewapend Beton..

Zeitschrift des Vereines Deutscher Ingenieure.

Zeitschrift für Wärmewirtschaft und Dampf-Kesselwesen.

Zeitschrift für die Gesamte Kälte-Industrie.

Zeitschrift für Physik.

APPENDIX.

CHAPTER No. 1. -------

CHAPTER No. 2.

t_T = temperature shown on any thermometer.

t_e = temperature of a specimen.

α = inertia of the thermometer.

$\frac{dt_t}{d\tau}$ = temperature fluctuation of the thermometer p.m.

t_{Ta} = temperature of the termometer at the beginning.

τ = time required to take the temperature.

A = surface of the thermometer.

C = heat capacity of the thermometer.

λ = external heat conductivity of the thermometer.

CHAPTER No. 3.

$-273°C$ = absolute temperature.

C = coefficient of radiation.

t_1 = surface temperature of a specimen.

t_x = temperature equivalent replacing the absolute temperature.

CHAPTER No. 4.

H = heat accumulation capacity of body.

C = specific heat of a body.

S = weight per unit of volume.

t_a = avarage temperature of a body.

t_b = temperature of the surroundings of a body.

CHAPTER No. 5.

t = temperature general .

t_1 and t_2 = surface temperatures

ϑ_1 and ϑ_2 = temperatures of surroundings.

Δt = temperature difference between two surfaces.

C_1 & C_2 = coefficient of radiation = $\sigma_1 10^8$ and $\sigma_2 10^8$

c' = values proportional to the radiation.

R = total radiation (for radiation).

E_1 & E_2 = capacity of radiation.

φ = ratio of angles (radiation).

T_1 & T_2 = absolute temperatures.

u = total radiation of an absolutely black body.

δ = thickness of a partition between two rooms.

δ_m = total thickness of a compound wall.

A = sectional area of a body at right angles to the heat transfer.

x, y & v = coordinates - distances.

p = u/3 pressure caused by radiation in any direction.

Q = amount of heat transferred.

α_1 & α_2 = coeeficients of convection between air and soli

$\frac{1}{\alpha} = a$ = resistance against convection.

λ = thermal c onductivity.

O_1 & O_2 = coefficients of heat absorption.

k = capacity of heat transmission.

$\frac{1}{k} = r$ = resistance against heat transmission.

R = total resistance to heat transference. (passage of heat)

z = a given time.

U & W, & C = integral constants.

S = entropy.

M = internal work equivalent to heat.

N = external work equivalent to heat.

CHAPTER No. 6. --------

CHAPTER No. 7.

β = coefficient of thermal expansion.

L & l = given length.

Δl = expansion p.unit length.

t = rise of temperature.

l_0 = length at 0°C.

l_t = length at t°C.

ε = coefficient of expansion under the influence of a force.

V, H & P = forces.

E = modulus of elasticity.

A = sectional area.

σ & St = stresses.

m & M = moments.

f = deflection.

C' = temperature factor. $= \frac{E.I.\beta.t}{l} = \frac{T}{6}$

I = modulus of Inertia.

W = modulus of section.

α = angle of turning (due to thermal conditions)

γ = dito (due to external forces)

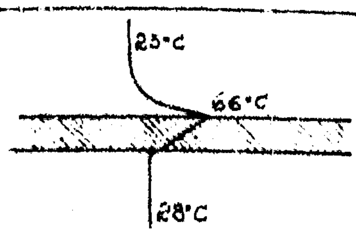
h = length of a stilo.

R = radius of curvature.

CHAPTER No. 8. -------

CHAPTER No. 9. -------

CHAPTER No. 10. -------



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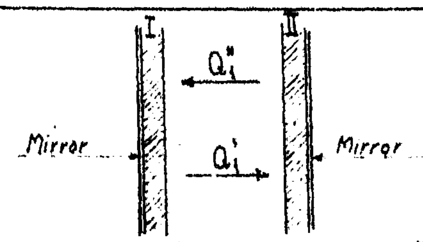

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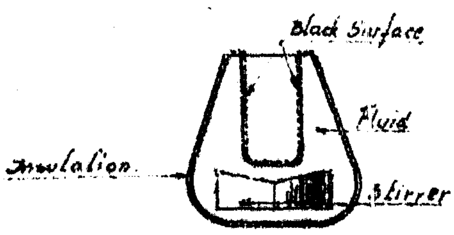

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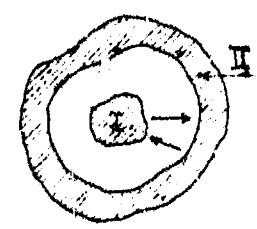

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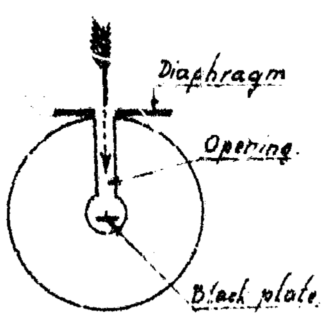

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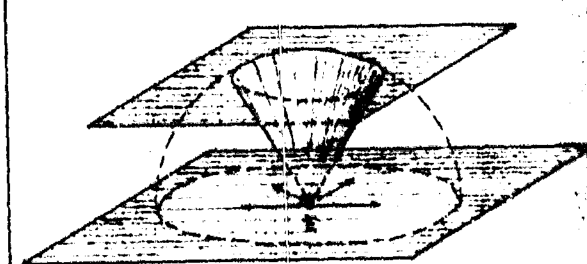

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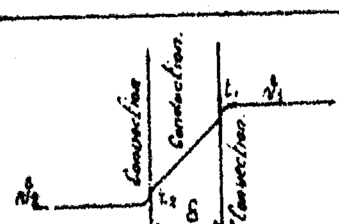

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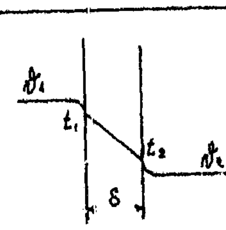

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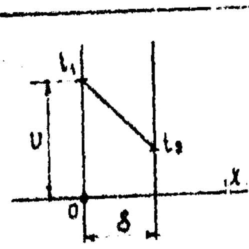

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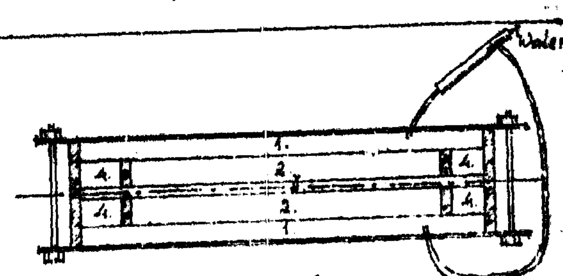
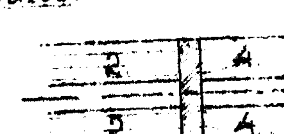


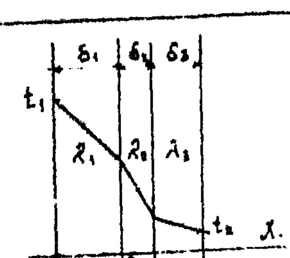
Poensgen Apparatus

1. water.
2. testing slab
3. electric heater.
4. covering frame



Isothermal curves in frame

FIG. Nº 11. PAGE 42.



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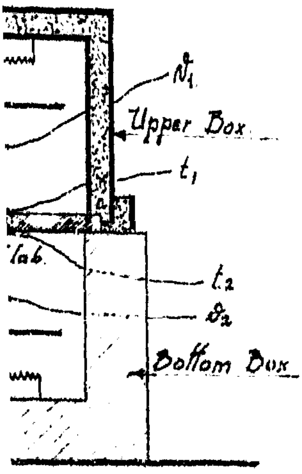

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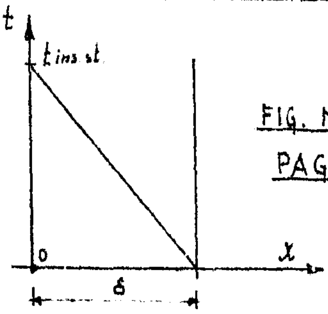

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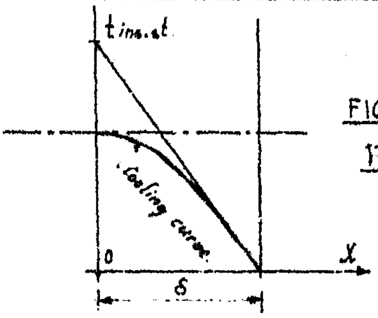

FIG. № 18.
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Correct

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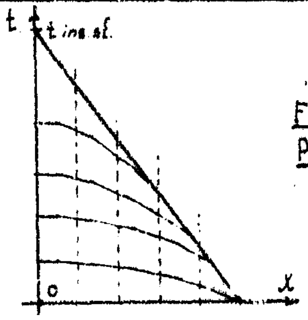

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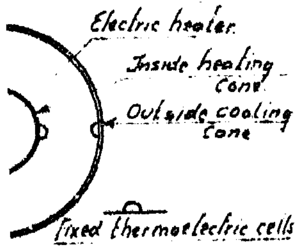

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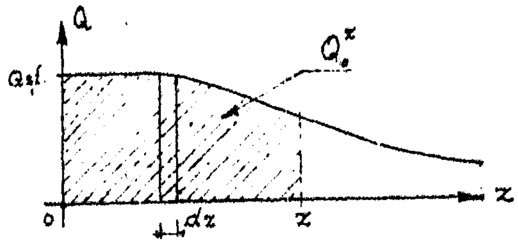

FIG. № 20. PAGE 50.

FIG. № 15.
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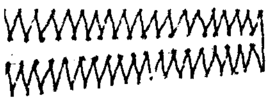
lab



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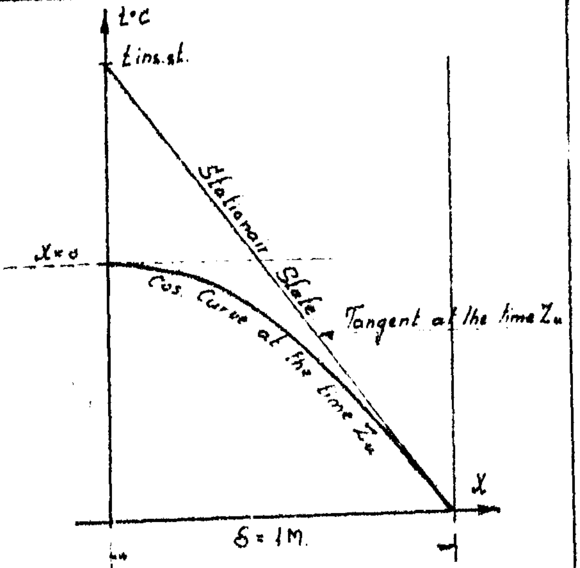
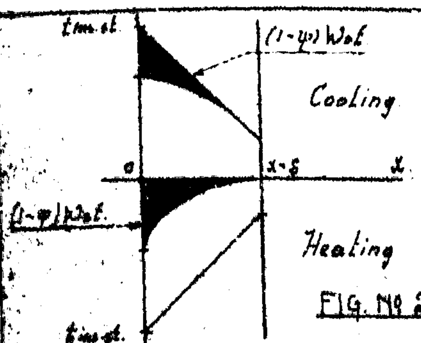


FIG. № 21. PAGE 51.



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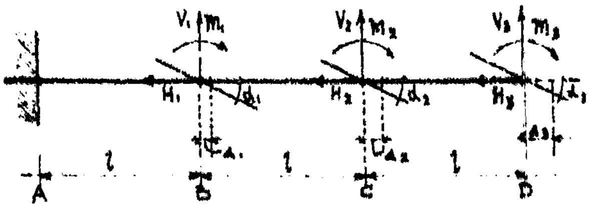

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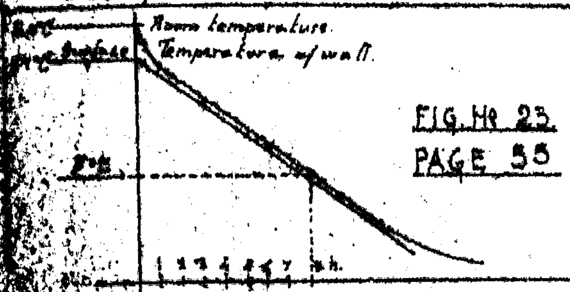

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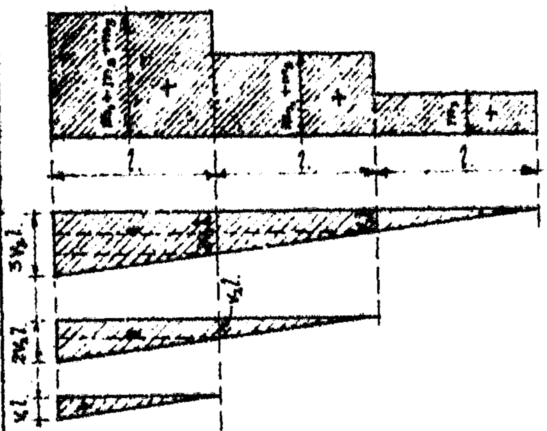


FIG. Nº 29. PAGE 71B

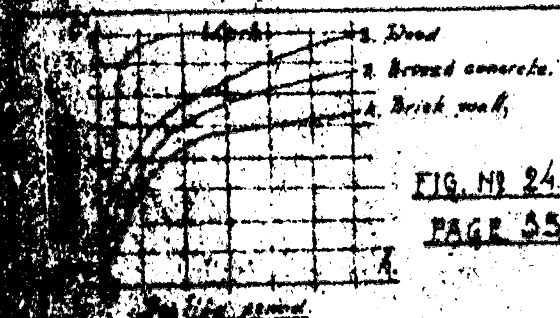
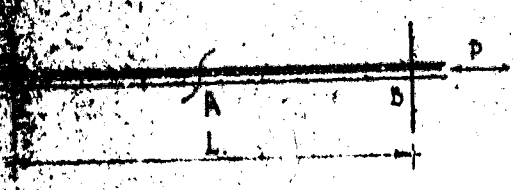


FIG. Nº 24. PAGE 55



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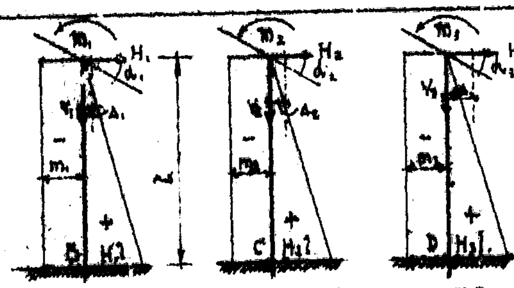

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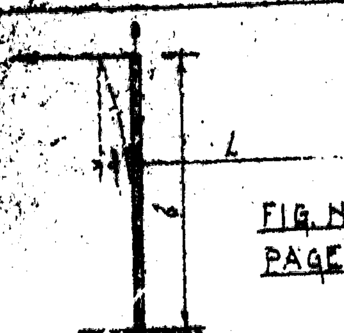

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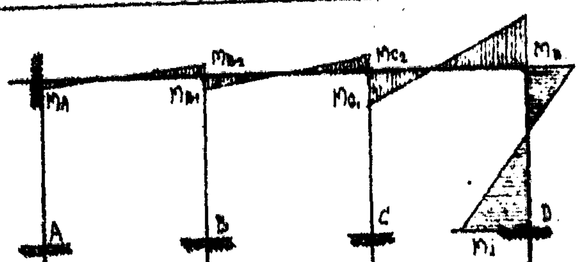

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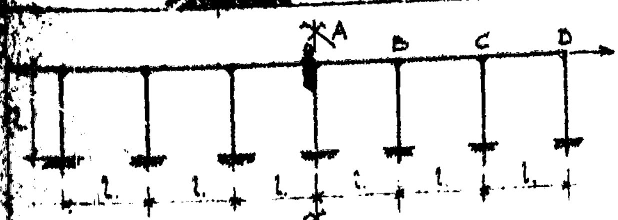

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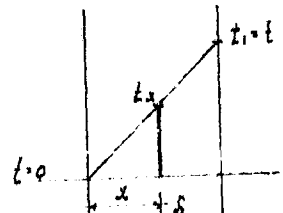

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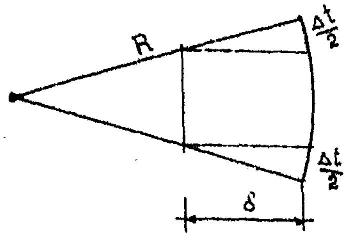

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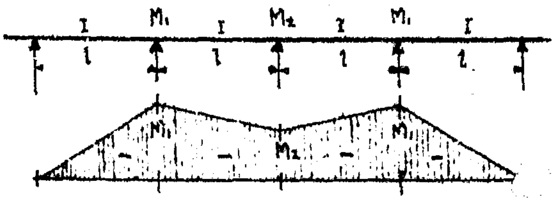

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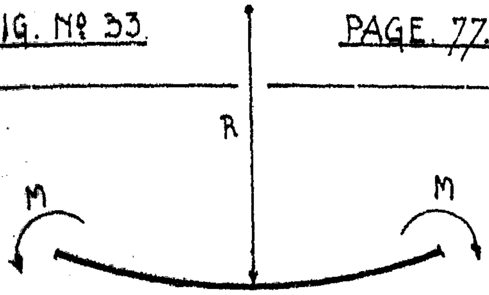

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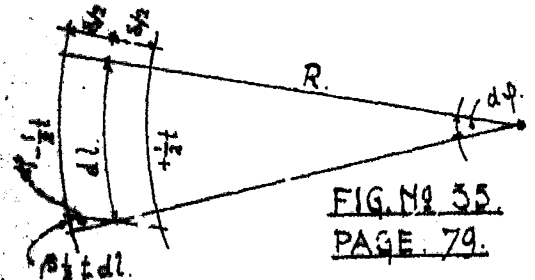

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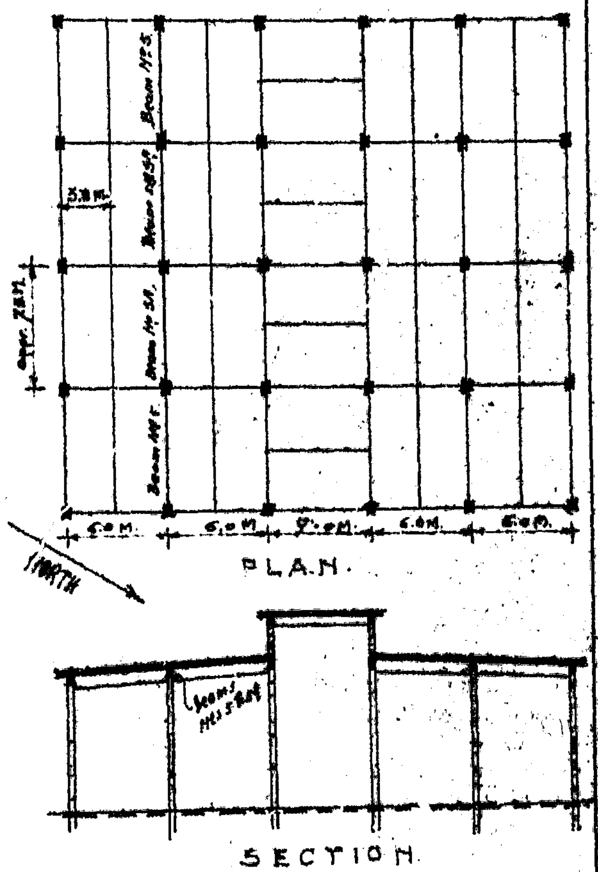

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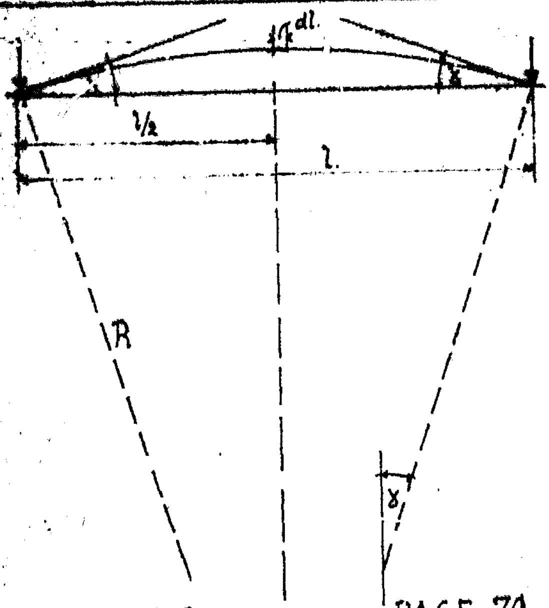

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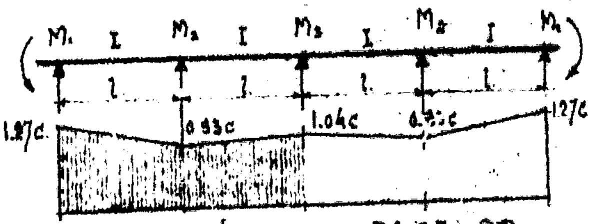

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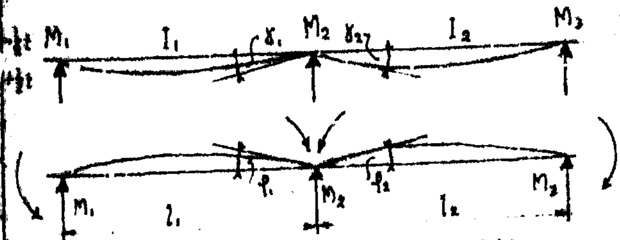

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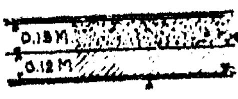

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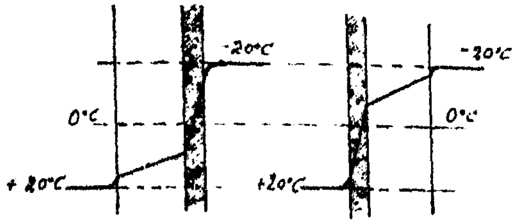

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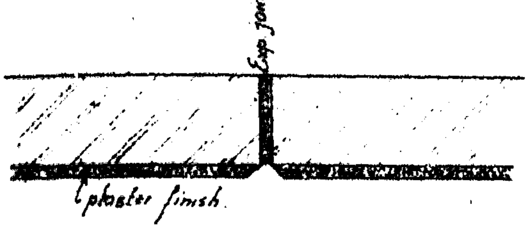

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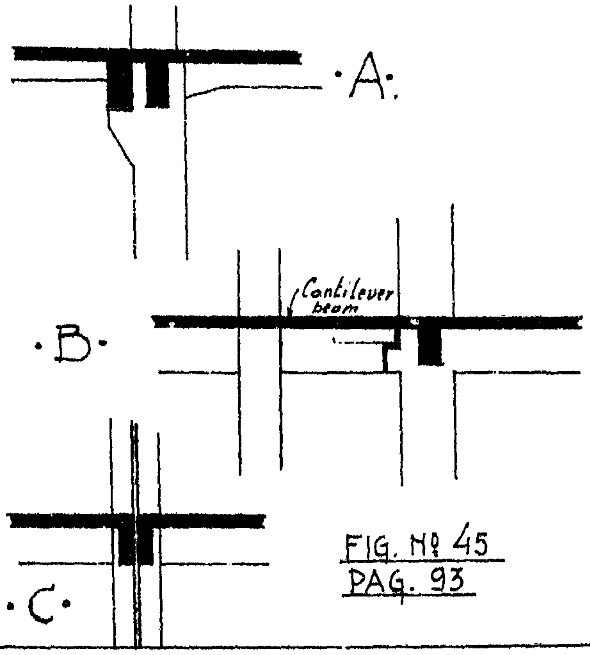

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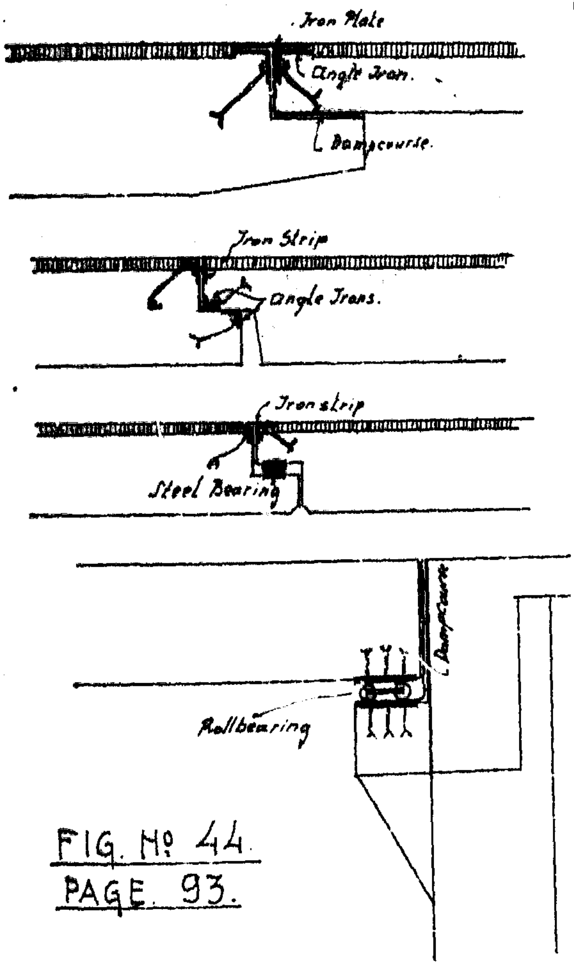

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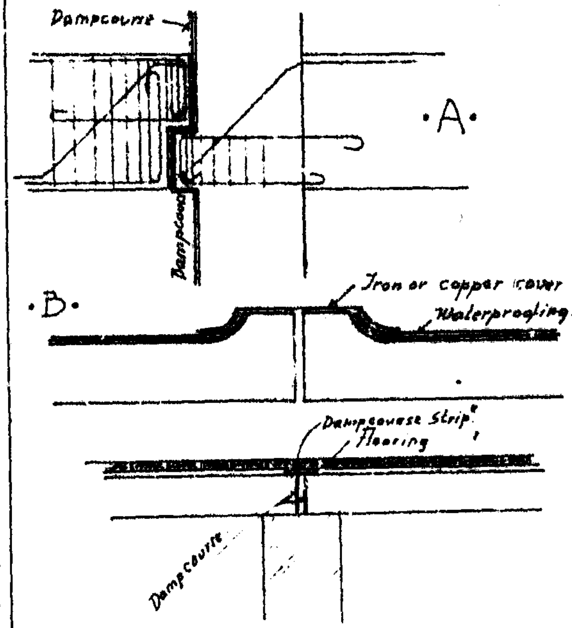

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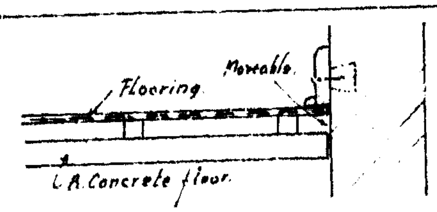
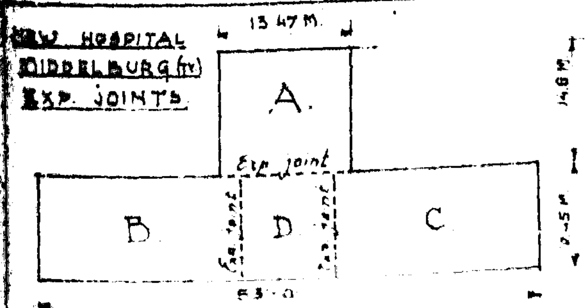


FIG. Nº 47 PAGE. 94.



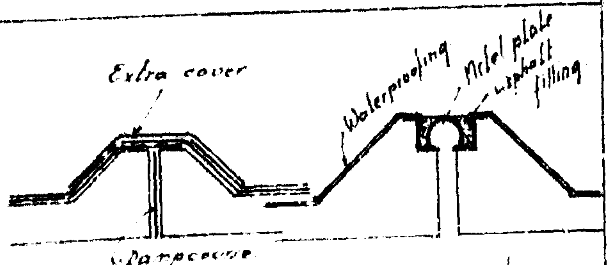


FIG. Nº 48 PAGE 94