A Review of Contemporary Trends in Mathematics Teaching in Britain

BY

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I WANT to make it clear at the beginning that we do not know the answers to many of the questions concerning mathematics teaching. In England, we are in tumult trying to get mathematics teaching improved. I want to make it clear that there is an enormous amount of heartbreak about the teaching of mathematics in England. There is some poor mathematics teaching and there is some that is good. We want to make it clear to you that we do not know the way. We are making suggestions to you based on our experiences. You will have the opportunity of rejecting them. Perhaps some of you may wish to accept some of them, but we are never going to say: "This is how it should be done," because we only wish we really knew. We have not come as smart Alecs who know the answers—we have come to learn from you, as well as, perhaps, to give you some ideas that you will think over so that you may say: "Well, I would like to try that in my school", or you may say: "Well, the man's mad! The last thing we are going to try is anything that he suggests."

I think it was the publication of a book called *The Teaching of Mathematics in Primary Schools* by the Mathematics Association that started people in England thinking about maths teaching. It came about this way. There were complaints from people in industry and other walks of life that children were leaving school incapable of doing the very elementary things in mathematics required for their jobs. With that the Mathematical Association produced this publication. They started the right way with the teaching of mathematics in the primary school. I was delighted to hear that point in the introduction, that it is no good starting in secondary schools until we have got the primary and junior schools in order. This publication aroused interest on the part of all types of teachers in various schools. Some of them said: "Hello, here's something new!" It was nothing new at all—there is nothing new. One of the most regrettable things we have is a phrase called "Modern Mathematics" which is not modern in any shape or form. It is more appropriate to use the term "Mathematics for the Modern Age"; then I think we may see it in better perspective. As a result of this book, conferences and study groups were organised all over the country and they are still taking place, as are in-service training schemes for teachers, especially for those who have not had any mathematical training at all. They have to teach the subject; they teach it as they were taught, because that is the only way they know. We have done our best to try and get this in-service training going in order that some of the new thoughts may be got over to these people. The B.B.C. on sound and television and I.T.V. have played their part by presenting daily programmes for children of all ages and all types of schools, also for parents and teachers. Everybody is now starting to jump on to a band-wagon that we hope will achieve success.

The Nuffield Foundation, as you have heard, is starting a project next September in which I shall be taking part, a rather interesting project in that it is going to be an experiment on the teaching of mathematics for ages 5 to 13, which is an interesting age group.

Numerous text-books are being produced, many of which should be destroyed. Structural equipment is coming out in showers daily; again, much of it should be destroyed.

It has been very difficult to get people in England to start to re-think about mathematics. This has been the trouble—in England, children take an examination in some areas at 11—it is the Day of Judgement—it has almost reached the stage where your social standing will depend on whether your child gets through, which is a bad state of affairs. Enlightened authorities like Leicestershire and Birmingham have abolished the 11-plus examination and in all those areas where they have abolished it, mathematics teaching has improved out of all recognition. But you can imagine the teacher who says: "Not on your life! Nothing new for us! Every year I have produced eight for the grammar school with my methods and as long as I can produce eight year by year, I'm safe. But if ever I try this new modern stuff they are bringing out and I only get seven, I'm done!" You may be interested in this, because the changes that have taken place and are taking place have been brought about where this examination has been abolished.

It is evident that where we really widen our wings in the teaching of mathematics, not only are the examination results better, but everyone is happier—teachers and children.

Nevertheless, everybody is on this wagon for a change of thought because they are dissatisfied with the standards that have been. But the greatest thing about it all is that the reform has come from within the schools themselves, and this has been due undoubtedly to the freedom that the head teachers have in our primary schools to organise their timetable and also to organise the syllabus that they use. I think that the syllabus has probably had a bad effect on the teaching of mathematics, and I regret very much that in England they are bringing in an examination called the C.S.E. for secondary schools. Whereas the primary schools are trying to drop examinations, the secondary schools want to bring them back and I do not know where it is going to end. It is very regrettable that once a teacher gets a syllabus, he never wants to move from it. Half the damage that has been done in England has been done by this syllabus. It has not allowed width at all. Give the teachers that piece of paper; ordain all the orders in which it shall be taken, and they will not move from it. In the primary schools where they can make their own syllabus (and they can in the secondary schools if they wish) teachers are stretching out from it and not being kept "cabin'd, cribb'd, confin'd" in a watertight compartment. I remember going into a school (and all my references are to England and all the other counties but Staffordshire, Leicestershire and Birmingham) where they had reached the horrible state where the teacher, working to the syllabus, which was made out in terms, had to get to decimals by Christmas. She was determined to get there by Christmas and by the middle of November, she was there—decimals were to start; so they started decimals and they kept on with decimals. She said, "We have *done* decimals", and you had only to ask three or four questions and you would know they had *assassinated* decimals. They had more than done them, and if there was anything the children never wanted to see again, it was decimals.

This is a point that I think has caused a lot of trouble. You cannot produce the same syllabus for every school, because schools are different and you arrange the syllabus according to the type of school you have and the type of children you have. What is far more important is how the material is presented. The key to all I shall ever say is the quality of the teacher and the vision and the enthusiasm of the teacher. That will prevail whatever system is in force. But it is this freedom of a syllabus that has played a great part in what we would call a revival, a new life in the mathematics teaching in England, which undoubtedly in some areas has been highly successful.

What are the chief characteristics that we find in this changed pattern? We are in the second age of discovery in England. We now believe that if a child is allowed to discover, to experiment, then what he discovers will remain with him. We are against the injection process—the idea of pumping, pumping, pumping, without the children knowing what they are doing at all. This is evident —the more we pump into certain types of people, the less they will ever know. We spend considerable time in our country on what is called "revision". It is utter nonsense. The teachers are not revising at all. They are re-teaching again what was forgotten the first time. The regrettable part about it is that they are re-teaching it the second time the same way as they taught it the first, and I shudder for the boys in some of our secondary modern schools who are revising division of fractions for the thirty-ninth time in their career because something has to be turned upside down.

So you will find now in the schools of England an enormous amount of discovery taking place. What are the children discovering? They are discovering laws. They are discovering the laws of mathematics, the basic laws of arithmetic. They are not looking at addition and just totting up—they are seeing what laws govern these things. We are going slowly—we have to go slowly: it is through going too fast that the trouble arises. A school where discovery of some particular pattern is taking place is a hive of industry. If you will only allow children to discover things, they will discover some of the things *you* did not know. Many children have come to me with what they had found out about some pattern in number that I knew nothing about, and I was honest enough to say, "Thank you for telling me, I didn't know." There is nothing disgraceful in saying to a child, "Sorry, I don't know the answer", as long as you can say, "But I know where to find it".

We want children to become thinking beings. If they can discover laws, if they can discover and can apply them to the next step, we are gaining ground. It is rather remarkable that many of the children who leave our schools cannot do certain computations after they have spent all their lives on it. If that happens, there is something wrong with the method of presentation. And if they have reached the low standard that some of ours have reached in England, then any method you like to bring in cannot be worse. It is like allowing a child to discover and not always to be saying, "This is what teacher says", and "This is what teacher wants me to do", that has led to a delight

in the subject which so many people and so many children in England just did not like at all.

But let me give a word of warning. This word "discovery" can be abused. There once was in England what was called the "play way". If anybody misunderstood the play way, it was the teachers of England. They concentrated on the word "play" and forgot the "way". It is the same with discovery. I have seen children cutting up cardboard and string. "What are you doing?" "Discovering." Discovering nothing—there was nothing for them to discover. The teachers said, "Ah, but this is modern, Therefore, we must do it." So nowadays in England, they use their discovery lessons to discover something.

The planned learning situations and the right tools are essential for success. Do not think that by introducing discovery into the mathematics lesson that it is going to be easy for the teacher. It is harder. The easiest way to teach mathematics is to get S. & S. Book I, start at No. 1 and then go right through page by page. This is the easiest way—if you have not lost the answer book. (That was the excuse I was once given by a teacher when she had not marked the books for about three months.)

The second point is that we are now in the age of child discussion. If a child wants to explore for himself, if a child makes discoveries, then what he wants to do is to communicate them to everyone else. The main means of communication in life is speaking, so we have lessons where the children discuss. The only way any of you can be sure whether your children know what they are doing is if they can talk to you about it. You have no proof at all if they can write it down correctly, particularly in computation. But if they are able to discuss with you what they are doing and why they are doing it, then you are succeeding. Unless we get our children to speak and to talk freely, I do not think we are going to get very far—but remember that it is the child who does the talking and discussing: the teacher is there to guide. We want to discuss the wrong questions and the wrong answers, not only think about the right ones.

I ask you to consider carefully this idea of having a day in your school when you drop writing altogether and you have a discussion. The days have gone when we put a date on the page as an insurance policy against an inspector coming in to see whether we have done our work. He knows we are working and he knows that the fruits of the work will be revealed in discussion and talk with children and teachers.

If you put the situation on the board and get the children to talk about it and to extract the relevant data and then do some of the mathematics in order to simplify it and put it into its simplest form so that they can understand it, it is amazing how much joy the children get out of it. There is one thing about this discussion that is certain—the children you least expect to answer will probably answer the most and give you the most intelligent answers.

When I take a lesson in a school, I talk with the children, and all this requires careful preparation. None of my ways are easy. The teachers suddenly say to me, "I can't understand it. That boy has never said a word before." This discussion has developed a changed classroom relationship. Children through this become confident in their own abilities. Teachers obtain a deeper understanding of their children, of their individual children, and they understand more how these children learn.

It is a good thing that discovery and discussion have come about in the teaching of mathematics. It is rather amazing that this change came first in the visual arts: we gave children clay, sand, stone, wood, plaster, colours of all different kinds and then we stimulated them to make their own creations as they saw them, and we saw the enjoyment they got from that. As a matter of fact, physical education preceded mathematics in this respect in England. I should like to read from a document on physical education published in 1953. "If the teacher provides the general framework and then allows the children to make their own discoveries in their own time, he not only opens up for them a different sort of opportunity, but makes a different demand on them. The teacher's understanding of individual children and their needs is absolutely essential." That is really what we are trying to do in mathematics—discover, talk about what we have discovered, and record what we have discovered. I could almost sum up the present situation in England by saying that it is the age of exploration, the age of discussion and the age of recording. It is not universal—we still have the dark spots, but as a result of courses and of discussions which are taking place, the improvement is great.

But we must again beware (and I always like to give you the "beware" side of this) of unthoughtful participation. We must ever keep in mind the appropriate environment for the learning of mathematics. I think we must keep in mind the appropriate discoveries a child should make; I think we should keep in mind the mathematics that is appropriate to the individual child.

You are aware of the change in relationship between mathematics and science which was mentioned earlier today. Science demands today the people who can not only answer problems, but can think problems. We have to think them as well as answer them, and I think that the way in which we are doing our discussion is leading to this. In England, you find certain children get on quite well with computation as a result of the mathematics training they have had under the old régime, but the moment you introduce the situation or the problem, they have had it. They can do a long multiplication sum, but the moment you put "sheep" in it, that is the end. Nowadays we try to make the situations as real as possible. But we still have men digging trenches of remarkable dimensions—and yet we have got mechanical diggers!

The manipulation in our new text-books is quite simple. It is the thinking side that we are after. We want children to become thinking beings. We want true education, and true education is what a child achieves in himself. In these text-books there are illustrations, problems and situations which will require quite easy computation, but the children have to think about it and discuss it before they do it. So you will understand that today we do not concentrate any longer on long multiplications that are totally unnecessary. We start with the situations and draw out the simple stuff we want to do instead of "mental, mechanical, problems". Please do not misunderstand me. I believe in number facts. I believe in computation. I believe children have to know their tables, but I believe they have to understand them, see them in situations and know when they are used. I want the children to know that eight sevens and seven eights are not two different things, that there is some relationship between them.

When we come to the secondary field, we begin to include study of structure of formal languages, such as sets, groups and logic. We consider this essential because the computer must be fed with details that are expressed in a formal language. If only we can see this formal language as being of value in developing a unity of thought, then perhaps it will serve some purpose. It is this unity of thought that is one of the changing ideas in England. Children do their geometrical work and their algebraic work and their arithmetic work as a single activity.

There was a time when the great glory of going to a secondary school in England was starting algebra or doing a bit of surveying. Now this kind of work is being done in the primary schools and teachers in the secondary schools have to think about the way in which they can advance the new material. Modern mathematics, the whole idea of number, starts in reception class and we can use set language (which is a different thing from set theory) in reception classes in order to build up our ideas of number. What was once the prerogative of the university in this set language is now found in the reception class.

Let me warn you, however, of the opposite. Many people talk about elementary fractions. We have now come to the conclusion that fractions are not elementary. There is an enormous amount behind the concept and the teaching of fractions, and it is concepts that we are after. In my opinion, concepts cannot be taught. Facts can, concepts can not, and a child will get the feeling of a concept only when we have provided him with a wide variety of interests and experiences.

The secondary schools are moving into another field because the primary schools have taken on much of what we call "modern" stuff. This is due entirely to the fact that the Sealeys and the Flavells and the Biggs and the Skemps of England have seen the light before others. I hope you notice I have not mentioned the Fletchers, because I am a humble man.

We have realised that if we do not get our infants' and our primary schools right, then we are going to waste our time in the secondary schools and I should say this is the same the world over. It is their attitude to mathematics that we have to get right in the early stages.

Everybody wants to pass the blame from one to the other. In September, at the beginning of the school year, I go into certain secondary schools in certain areas in England and they say to me, "Just look at these, just look what they have sent me this term!" The head gives me a list of results. He has been stupid enough to test some children the first day after a summer holiday and use that as an indication of their ability. He says, "What are they doing in primary schools?" I go along to the primary school and the teacher says, "I'm glad you've come. What are they doing in the infant schools? Look what they've sent me up!" He says, "They're playing too much! We know what's wrong, they're playing too much!" So you have the secondary complaining about the primary, the primary complaining about the infants. You go to the infants' teachers, who to me are the salt of the earth, and what can you say to them? They call in the psychologist who says, "It's pre-natal"—and there is no argument.

The trends in England are these at the moment. Even with the very beginners, let us get the attitude right; let us get them to enjoy what they

do. Let them discover. Give them as much equipment as possible. But in the wrong hands structural equipment can be dangerous and I would say to you all: if you do not know much about this, never try it. One of the greatest dangers is to do something to pretend to be up with the Jones's. You have heard of Cuisenaire. Everything has value, everything is useful, but it is only one of the useful things in developing concepts which come from a variety of experiences. I can tell you a remarkable story about Cuisenaire. Do not go away from here and say, "This man is against it." I am not. It has its value. It has its mathematics. But I did go into a school where they were concentrating on Cuisenaire and they asked me, much against my will, if I would ask some questions. I said, "Tell me the story of seven. Anything you like about the story of seven." They all gave the same answer—"A black rod." Here is a case where it has been abused. This was not the fault of the equipment, but the fault of the teachers who were using it.

We believe very strongly in England that it is much better for a child to do one thing three ways than three things one way. I was delighted with certain parts of your secondary syllabus that I read where this kind of attitude was advocated. Who are we to say that it must be done one way? It must be done in as many ways as possible. You can show them which is the most economical way, but never, never prevent a child from using some particular way he wants to use.

We always say, "Whatever you are doing in your computation, will you please make up stories about it." One of the changes in my area has been that about every fifth page in the exercise book consists of English sentences in which children describe situations. If a child has discovered a particular number fact like this:—

$$7 - 2 = 5$$

he makes up a situation about it and he writes about the situation. If he cannot write, then he talks about it—I am going to weary you to death with this idea of talking, because it is only in this way that you really know what you are getting at. If ever I put that on the board and say, "Tell me a story about it," I invariably get the same answer: "There were 7 birds on a wall and 2 flew away, there are 5 left." I like it, because I say, "There were 7 birds on a wall and I fired a gun . . . 2 flew away, how many do you think would be left?" They are too cunning in our part of the world. They will say, "None, they've been killed." If they only say things like that, it is well worth while.

Whatever we are doing, we try to allow children to do sums in many different ways and to make up their own sums. Let the children make up their own sums—the amazing thing about this is that they will make them harder than you would set them. It is an amazing thing that children, if allowed to make up their own sums and work their own calculations for a certain time, get into large numbers and enjoy large numbers. Children do like working in large numbers.

I have brought a few examples which may give you an idea as to what we are doing in this regard. There was a class of ten-year-olds where they made up their own books called "Facts and Fancies". They had to find an interesting fact and then make up certain problems about it. There was one particular boy who wrote as his fact: "The sun is 93 million miles from the earth", and his fancy was: "How long would a man running at a speed of 1 mile in 4 minutes take to run to the sun?" He worked out the problem and found the answer to be 707 years, 278 days, 8 hours and a bit: his conclusion was: "But he would have been dead long before that." There may be some of you who think it a waste of time for a boy to make up that, work it out and form his conclusion because—we shall never get to "area" if we keep on like that. But I promise you that not only will you get to "area" with greater speed: you will go on and on in the long run, if such things are accomplished.

Another example is a group of ten-year-olds who were asked, as a discovery, to find the number of grains of wheat demanded by the inventor of the game of chess who asked for his reward as one grain of wheat on the first square, two on the second, four on the third and so on, doubling the number on the previous square right up to the sixty-fourth square. The children were not daunted by this, but they obtained such a large number that they decided to have it checked by a computer. There was a computer firm in the neighbourhood and it was six months before they received a reply, and so they wrote as their conclusion: "We got our answer nearly right, but the computer took longer than we did."

Another example: for some reason best known to himself, a teacher put on the board—

$$2/11\text{d.} \times 6$$

The children were asked to do that in several ways. One child who supposedly had an I.Q. of 83 (notice the word I have used there—"supposedly") did this sum in four ways and for the last way, he put down 2/11d. six times and added it up. Then came his story: "One day, my father went to put 2/11d. on a horse and the

man timsed it by 6, so there was 17/6d. to come back to my dad, because the horse won. I don't like this story I've written, because my dad would put 3/– on a horse, not 2/11d." (If you follow my method, you will not need welfare officers. You will know all that is going on at home by their arithmetic stories.) Then he wrote: "For 2/11d. I could get a pair of stockings, but I don't want 6 pairs at once, so that would be silly. I have seen a cake for 2/11d. and I could buy 6 if I was having a party. The last way I added it, it takes longer." (I am reading it as he wrote it.) "I couldn't have multiplied 2/11d. plus 2/11d. plus 3/11d., but as they were all 2/11d., I could."

This is the trend in our maths teaching: activity, discussion, recording, making up their stories, practice coming after. We have to have practice—do not let me leave you this morning with the idea that I am against tables, number facts and all those things. I am with them solidly.

Another great change that has come about is the introduction of graphical work right from reception centre onwards. I never want to see any graphs done that have not got white papers with them. I believe that when a child has drawn a graph, he should produce a piece of written work that describes what he reads into the graph and what he can predict from the graph. When I see a lot of blank graphs on school walls, I get worried, because I think they have been put there just for somebody to see. But when I see them up on the walls with white papers and predictions (which may well be wrong), then my heart glows because we are making our mathematics a thinking subject. You will find the infants having their histograms of height. You will find your children making their pictorial graphs of TV programmes, shoe sizes, occupations, number of lorries that are passing by the school in an hour, the daily paper father reads, the brands of cigarettes that the fathers smoke. You find them in the primary school lengthening one axis and halving the other, getting the idea of caricatures and how they are formed. You get children in schools who are making their open sentences like this:

$$\square = 2 \times \square$$

and from the results of that, they are drawing their graphs and are then producing their white papers on them. You find children today who are graphing on one-dimensional vectors the sets of certain number facts. I would say that England at the moment has gone over very enthusiastically to teaching through graphical work. It is amazing the standard you can reach.

We are experimenting in the secondary schools with calculating machines and with teaching machines, both in linear and the branching types. The trouble is writing the programmes. We are all right with the machine; it is when we start to write the programmes we fall into trouble, I think.

We are trying our best to develop mathematics as a thinking subject from the earliest stages right through, but we shall fail if we rush. Never be disturbed about how fast you are getting along if you are spending time in ensuring that children understand what they are doing. If you rush along and they are not understanding, you are only making a rod for your own back.

Tradition dies hard and we have to struggle. Nevertheless, a new era in England in mathematics has been born and in due season will bear fruit and I feel that the progress being made in England today in mathematics is because the child can express himself in his language, not the teacher's, hence he can show his understanding or otherwise. He has started his mathematical training with the building of a vocabulary, the true vocabulary. I am tired of hearing people telling children that when you multiply by ten, you add a nought. As you know, you do nothing of the kind. I am tired of hearing them moving decimal points all round the earth. I am tired of hearing that area is length times breadth. The other reasons are that he expresses himself about things he understands and discoveries he has made, and pattern means something to him. He has had the experience of recording orally or in writing or in diagramatic form, thus allowing him to discover relationships—that is what mathematics is surely, the classification of all pattern. From that pattern, to see a relationship which eventually can be presented in the abstract is the whole basis of the mathematics that we desire to put over. He has met logic in his early days.

How are you going to be guided in your mathematical principles? May I ask you always to keep these three things in front of you? What you are doing in mathematics, does it make the child and teacher happy? Does it develop the children to face new challenges with confidence? Is the content and presentation mathematically respectable? If you can answer those three, you can judge for yourselves.

We are only just starting in England; we are going slowly; it is the enthusiast that we need. I have to organise in-service training of some groups of teachers for a term just to try and

inspire them on what we want to do, but of this I am certain, where they are carrying out some of the things I have suggested the children thoroughly enjoy themselves. I am not saying, "This is how it must be done." What I am saying is, "When Fletcher did it this way, Fletcher and his pupils enjoyed themselves." Let me make that very clear, because after all, the child is what matters.

We are all interested in finding an answer, any way that will promote those three things I have suggested to you. We are in one common cause because it is our duty to present to our people and to our children a subject and a study of a subject that began long before Pythagoras and will continue long after Einstein—the youngest and the oldest of them all.