Abstract

The focus of this thesis is the spectral structure of second order self-adjoint differential operators on graphs.

Various function spaces on graphs are defined and we define, in terms of both differential systems and the afore noted function spaces, boundary value problems on graphs. A boundary value problem on a graph is shown to be spectrally equivalent to a system with separated boundary conditions. An example is provided to illustrate the fact that, for Sturm-Liouville operators on graphs, self-adjointness does not necessarily imply regularity. We also show that since the differential operators considered are self-adjoint the algebraic and geometric eigenvalue multiplicities are equal. Asymptotic bounds for the eigenvalues are found using matrix Prüfer angle methods.

Techniques common in the area of elliptic partial differential equations are used to give a variational formulation for boundary value problems on graphs. This enables us to formulate an analogue of Dirichlet-Neumann bracketing for boundary value problems on graphs as well as to establish a min-max principle. This eigenvalue bracketing gives rise to eigenvalue asymptotics and consequently eigenfunction asymptotics.

Asymptotic approximations to the Green's functions of Sturm-Liouville bound-

ary value problems on graphs are obtained. These approximations are used to study the regularized trace of the differential operators associated with these boundary value problems. Inverse spectral problems for Sturm-Liouville boundary value problems on graphs resembling those considered in Halberg and Kramer, *A generalization of the trace concept*, Duke Math. J. **27** (1960), 607-617, for Sturm-Liouville problems, and Pielichowski, *An inverse spectral problem for linear elliptic differential operators*, Universitatis Iagellonicae Acta Mathematica **XXVII** (1988), 239-246, for elliptic boundary value problems, are solved.

Boundary estimates for solutions of non-homogeneous boundary value problems on graphs are given. In particular, bounds for the norms of the boundary values of solutions to the non-homogeneous boundary value problem in terms of the norm of the non-homogeneity are obtained and the eigenparameter dependence of these bounds is studied.

Inverse nodal problems on graphs are then considered. Eigenfunction and eigenvalue asymptotic approximations are used to provide an asymptotic expression for the spacing of nodal points on each edge of the graph from which the uniqueness of the potential, for given nodal data, is deduced. An explicit formula for the potential in terms of the nodal points and eigenvalues is given.