Appendix B

B1 Triaxial stress evaluation

B1.1 Measurement procedures

Triaxial stress measurements were conducted in 32 mm diameter (EX) boreholes drilled at the centre of the blind end of an NXCU borehole (Figure B 1). The cell consists of 12 gauges, but theoretically only six are required to determine the 3D stress condition. However, three of the gauges are oriented in the same direction (along the axis of the borehole) and therefore a minimum of seven or eight gauges are required for a proper stress evaluation. The additional gauges are used to determine errors that occur due to poorly stuck gauges. The stress state of the rock mass away from the influence of the borehole are calculated from the measured strains using the elastic constants for the rock and accounting for the stress concentrations of the borehole. In linear elastic, isotropic materials the tangential modulus and Poisson's Ratio at 50% of the UCS strength provides reasonable estimates of the stress condition (Coetzer, 2005). The methodology described in this appendix is based mainly on theory published by Vreede (1981), Vreede (1991) and Ryder (2007).

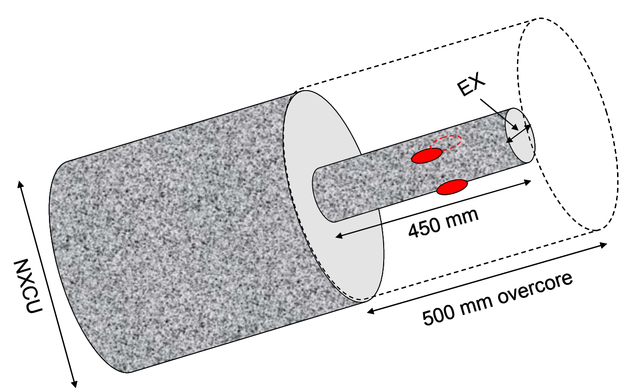

Figure B 1 Sketch showing the NXCU and EX boreholes required for the gluing and overcoring of the triaxial strain cell

B1.2 Borehole strains and field stress components

A right-handed 'borehole coordinate system' was used, where 'Z' is the axis of the borehole in the direction of drilling, 'X' is horizontal and normal to 'Z', and 'Y' is normal to 'X' and 'Z' (Figure B 2). The field stress components in this coordinate system were denoted as Sxx, Syy, Szz, Sxy, Syz and Szx. These are the stresses acting in the rock prior to drilling the borehole.

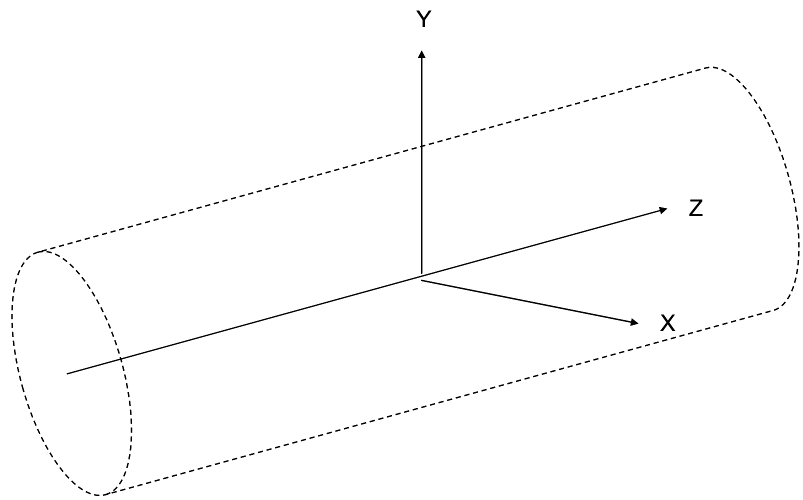

Figure B 2 Borehole coordinate system

After drilling, the stresses on the skin of the borehole change and comprise just three non-zero components, given in polar coordinates by:

$$\sigma_\theta = (1-2C_2)S_{xx} + (1+2C_2)S_{yy} - 4S_2S_{xy}$$

$$\sigma_z = -2\upsilon C_2 Sxx + 2\upsilon C_2 S_{yy} - 4\upsilon S_2 S_{xy} + S_{zz} \qquad \textbf{\textit{Equation B 1}}$$

$$\tau_{z\theta} = -2S_1S_{zx} + 2C_1S_{yz}$$

which depend on the angle θ (counter-clockwise from the X-axis) of the point where the stresses are measured (Figure B 2) and

$C_1 = \cos\theta$ $C_1 = \cos 2\theta$ ***Equation B 2***

$S_1 = \sin\theta$ $S_2 = \sin 2\theta$

By Hooke's Law in a homogeneous elastic continuum, the strains corresponding to these skin stresses are:

$$\varepsilon_\theta = \left(\frac{1}{E}\right)(\sigma_\theta - \upsilon\sigma_z)$$

$$\varepsilon_z = \left(\frac{1}{E}\right)(-\upsilon\sigma_\theta + \sigma_z)$$ ***Equation B 3***

$$\gamma_{z\theta} = \left[\frac{2(1+\upsilon)}{E}\right]\tau_{z\theta}$$

Where:

E = Young's Modulus

υ = Poisson's Ratio

These strains are relieved on overcoring, and are registered by the three rosettes of strain gauges which are centred at three different position angles θ around the rim of the borehole, i.e. θ_1 = 180°, θ_2 = 60° and θ_3 = 300°. Each rosette is made up of four strain gauges (A, B, C and D) oriented at angles α from the Z-axis (Figure B 3).

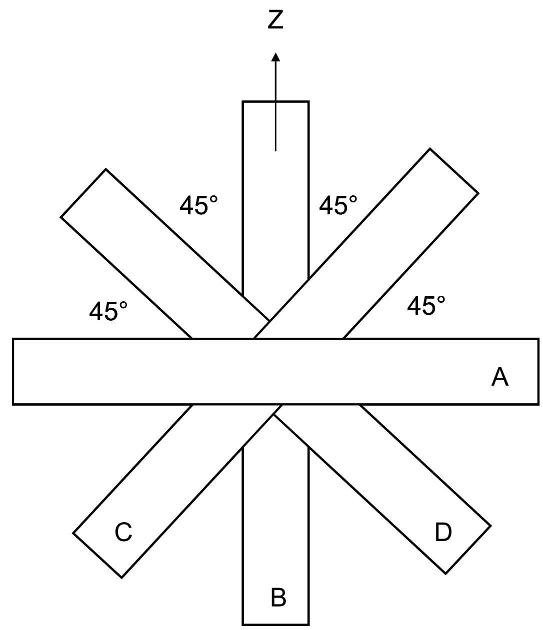

Figure B 3 Rosette made up of four gauges (A – D)

The strain change e_α after overcoring, registered by a particular gauge at angle α, is given by:

$$e_\alpha = \sin^2 \alpha \varepsilon_z + \sin\alpha \cos\alpha \gamma_{z\theta}$$ ***Equation B 4***

or, in matrix notation for the four individual readings e_A to e_D in a rosette:

$$\begin{bmatrix} e_A \\ e_B \\ e_C \\ e_D \end{bmatrix}_4 = \frac{1}{E} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1/2 & -1/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix}_{4,.3} \begin{bmatrix} \varepsilon_\theta \\ \varepsilon_z \\ \gamma_{z\theta} \end{bmatrix}_3$$ ***Equation B 5***

where the subscripts indicate the number of rows, columns in a particular array. After substituting Equations B 3, and B 1 into B5 and evaluating the matrix products, the relationship between the twelve measured strains and the six field stress components is given by:

$$\begin{bmatrix} e_A \\ e_B \\ e_C \\ e_d \\ \vdots \end{bmatrix}_{12} = \frac{1}{E} \begin{bmatrix} 1-f_1C_2 & 1+f_1C_2 & -\upsilon & -2f_1S_2 & 0 & 0 \\ -\upsilon & -\upsilon & 1 & 0 & 0 & 0 \\ f_2-\frac{f_1C_2}{2} & f_2+\frac{f_1C_2}{2} & f_2 & -f_1S_2 & -f_3C_1 & f_3S_1 \\ f_2-\frac{f_1C_2}{2} & f_2+\frac{f_1C_2}{2} & f_2 & -f_1S_2 & f_3C_1 & -f_3S_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{12,6} \begin{bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ S_{xy} \\ S_{yz} \\ S_{zx} \end{bmatrix}_6$$ ***Equation B 6***

where C and S are shown in Equation B 2 and

$$f_1 = 2\left(1-\upsilon^2\right),\ f_2 = \frac{(1-\upsilon)}{2},\ f_3 = 2(1+\upsilon)$$ ***Equation B 7***

There are twelve rows involved in Equation B 6 once the three rosette positions θ = 180°, 60° and 300° are substituted, giving equations for three blocks of four gauges each. In more compact notation Equation B 6 can be written as:

$$C_{12,6}S_6 = F_{12}$$

where F is the vector of measured strain changes, C is the coefficient array, and S is the vector of (unknown) stress components.

B1.3 Least-squares solution and error estimates

Equation B 8 represents an *over-determined* set of simultaneous equations, since twelve equations exist for just six unknowns. A unique ‘best-fit’ solution can nevertheless be obtained (invoking the method of least-squares) by pre-multiplying both sides of Equation B 8 by the transpose Ct of the array C:

$$Ct_{6,12}C_{12,6}S_6 = Ct_{6,12}F_{12}$$ ***Equation B 8***

or

$$A_{6,6}S_6 = G_6$$ ***Equation B 9***

where:

$$A_{6,6} = CtC \text{ and } G_6 = CtF$$

Equations B 9 are called ‘Normal Equations’ and can be solved by applying the inverse Ai of the square A matrix to G.

$$S_6 = Ai_{6,6}G_6 \qquad \textbf{\textit{Equation B 10}}$$

The best-fit stress vector S is evaluated from Equation B 10, and the residual strain errors DE (in $\mu m/m$) evaluated from

$$DF_{12} = C_{12,6}S_6 - F_{12}$$

The standard deviation of these strain errors is estimated from the standard error:

$$SD = \left[\frac{\left(\sum_1^N DF^2\right)}{(N-6)}\right]^{1/2} \qquad \textbf{\textit{Equation B 11}}$$

where N is the number of strain-gauge readings (normally 12), and the value $(N-6)$ reflects the number of degrees of freedom involved. In addition, seven deviations from strain ‘identities’ were evaluated (e.g. $F_2 - F_6$ should be zero, actual value of $(F_2 - F_6)/\sqrt{2}$ displayed as a measure of dispersion of the strains). A second SD is then estimated from $\left[\sum(deviations)^2/7\right]^{1/2}$, which serves as a check on the more accurate value given by Equation B 11. From these published ‘strain errors’, one or more ‘outliers’ (possible gross errors) in the raw strain readings may be indicated. ‘Standard errors’ DS (in MPa) were estimated for the stress components, using the diagonal of Ai per standard statistical theory:

$$DS_j = ESD\left[A_{jj}\right]^{\frac{1}{2}} \qquad \textbf{\textit{Equation B 12}}$$

B1.4 Transformation to general coordinate system

The borehole stresses are transformed from the 'borehole coordinate system' XYZ (Figure B 2) to a 'general coordinate system' X'Y'Z' (Figure B 4), using the hole bearing (clockwise from north) and hole dip δ (positive downward).

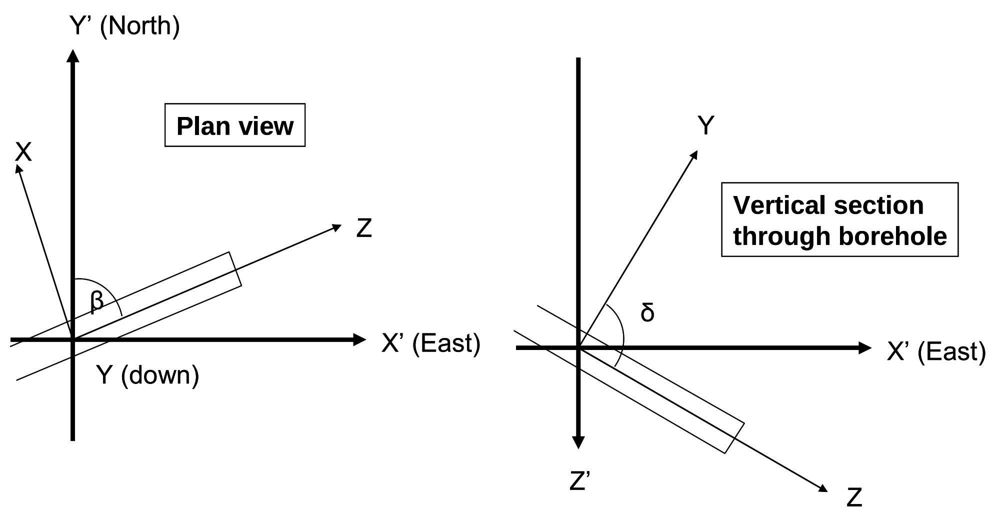

Figure B 4 General coordinate system

The direction cosine array DC (cosine of the angles between the General and the Borehole coordinate systems) is given by:

$$DC = \begin{bmatrix} -Cb & SbSd & SbCd \\ 0 & -Cd & Sd \\ Sb & CbSd & CbCd \end{bmatrix}_{3,3}$$ ***Equation B 13***

where *Cb* = cosβ, *Sb* = sinβ, *Cd* = cosδ and *Sd* = sinδ.

A 3 x 3 stress tensor ST is set up from the stress component vector S, and the rotated tensor ST' evaluated from the following matrix equation:

$$[ST'] = [DC][ST][DC]^t$$ ***Equation B 14***

Principal stresses and their bearings are evaluated using standard methods. The orientation convention applied to these results is clockwise from north and downwards from horizontal are positive.

B2 Biaxial (Doorstopper) stress evaluation

B2.1 Introduction

The Doorstopper measures the stress components in a single plane (2D). The strains are, however, affected by the stress acting along the direction of the borehole axis. This stress is not measured unless two mutually perpendicular or three approximately perpendicular boreholes are employed. Many of the measurements described in the dissertation were not accompanied by direct axial stress measurements, and these stresses had, therefore, to be estimated from a few triaxial measurements and numerical models. The most accurate measurements were thus in the vertical boreholes drilled from the centre of a panel as the axial stress could be assumed to be zero. The triaxial measurements are therefore considered, in most instances, to be more accurate than the Doorstopper measurements, but the Doorstopper is much easier to install, with a higher success rate. In addition, measurements in higher stress conditions are less prone to discing in the Doorstopper than the triaxial measurements. Accurate drilling is required for both methods but the Doorstopper requirements are easier to achieve.

B2.2 Measurement procedures

Doorstopper measurements are conducted at the flattened (blind) end of a NXCU borehole (Figure B 5). The gauges are glued into position with the installation tool shown in Figure B 6 and pressure between the gauges and the rock surface is applied manually by the installer. The glue was allowed to cure overnight and a set of readings were made. The installation tool was extracted from the borehole and the gauges were overcored as shown in Figure B 5. After cleaning and drying, the temperature was allowed to rise to the ambient temperature of the stope and a second set of readings was made. Readings were taken at five minute intervals until the gauges stabilised, i.e. the dummy and active gauges were the same temperature.

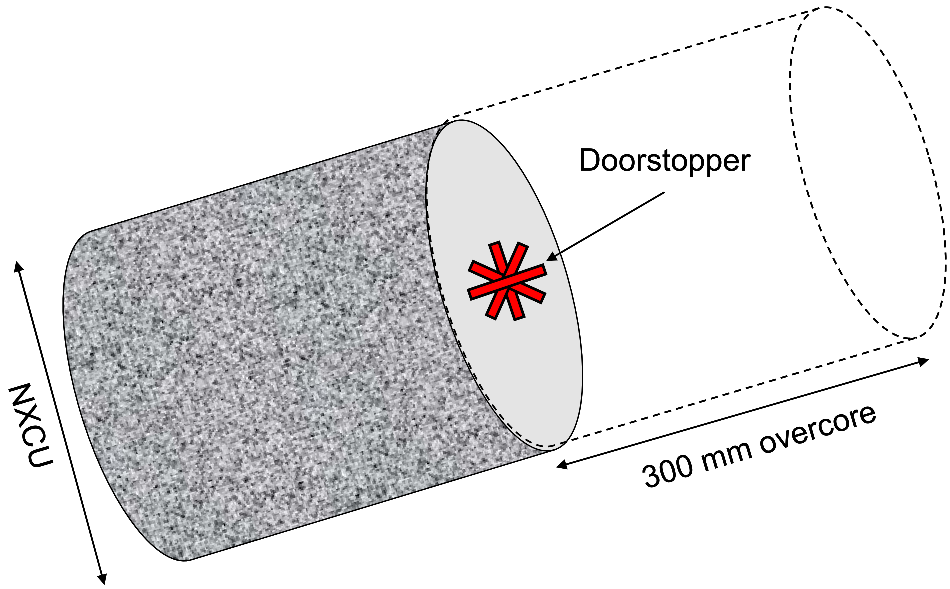

Figure B 5 Sketch showing the NXCU borehole and Doorstopper strain cell, glued to the flattened end of the borehole

Figure B 6 Installation of a Doorstopper in a vertical borehole at the centre of a panel

B2.3 Borehole strains and field components

The arrangement of the gauges for a Doorstopper measurement in a shallow-dipping borehole is shown in Figure B 7. The vertically inclined boreholes were aligned with the vertical gauge on dip at the centre of the panel and along the advanced strike gully (ASG) where measurements were made in the ASG.

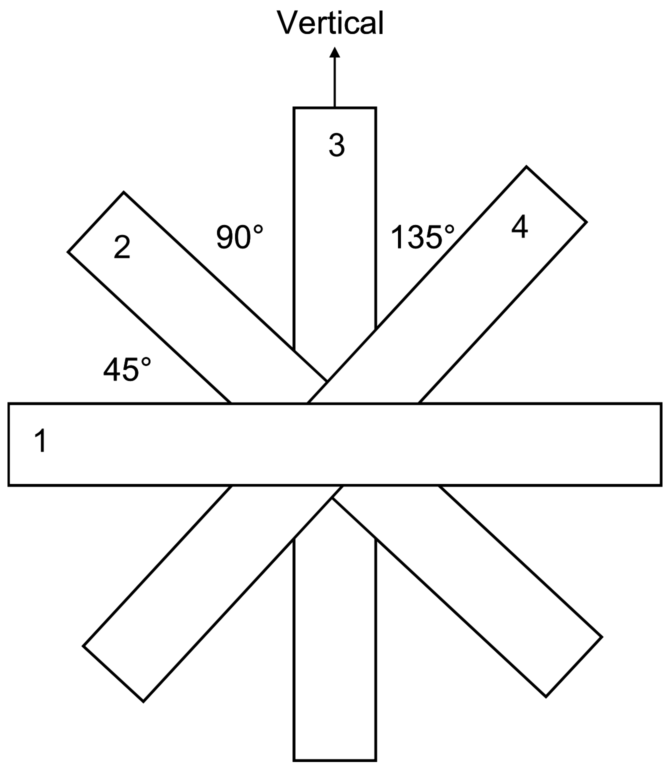

Figure B 7 Gauge orientation looking towards the blind end of the borehole

Standard stress transformation equations were used to determine the stress at the blind end of the borehole (Equation B 15 to Equation B 20). As only three gauges are required to determine the principal stresses and their directions in a plane, the Doorstopper has an extra gauge that can be used to determine the error of the measurements. Errors were calculated by comparing the addition of ε_{G1} and ε_{G3} to ε_{G2} and ε_{G4}. An error of less than 10% was considered reasonable.

$$\varepsilon_1 = \left(\frac{\varepsilon_{G1} + \varepsilon_{G3}}{2} \right) + 0.5\sqrt{\delta_1^2 + \left(\varepsilon_{G1} - \varepsilon_{G3} \right)^2}$$ ***Equation B 15***

$$\varepsilon_2 = \left(\frac{\varepsilon_{G1} + \varepsilon_{G3}}{2} \right) - 0.5\sqrt{\delta_1^2 + \left(\varepsilon_{G1} - \varepsilon_{G3} \right)^2}$$ ***Equation B 16***

$$\delta_1 = \left(\frac{\varepsilon_{G2}}{0.5} \right) - \left(\varepsilon_{G1} + \varepsilon_{G3} \right)$$ ***Equation B 17***

Where: G_1 to G_3 refers to the strains measured on gauges one to three

$$\sigma_1 = \frac{\left(\frac{\mathrm{E}}{\left(1 - \upsilon^2\right)} \right)\left(\varepsilon_1 + \upsilon \varepsilon_2 \right)}{1000} \text{ MPa}$$ ***Equation B 18***

$$\sigma_2 = \frac{\left(\frac{\mathrm{E}}{\left(1 - \upsilon^2\right)} \right)\left(\varepsilon_2 + \upsilon \varepsilon_1 \right)}{1000} \text{ MPa}$$ ***Equation B 19***

$$\theta_1 = ATAN\left(\frac{\delta_{G1}}{\varepsilon_{G1} - \varepsilon_{G3}} \right)$$ ***Equation B 20***

Where:

E = Young's Modulus in MPa
υ = Poisson's Ratio

The strain condition at the blind end of the borehole is affected by the stress acting along the axis of the borehole. Equation B 21 to Equation B 23 were produced by Vreede (1991) to extract the effects of the axial stress from the measurements.

$$\sigma_y = \left(\frac{a}{(b^2 - a^2)}\right)\left[\overline{\sigma_x}\left(\frac{b}{a}\right) - \overline{\sigma_y} + c\sigma_z\left(1 - \frac{a}{b}\right)\right]$$ ***Equation B 21***

$$\sigma_x = \frac{\overline{\sigma_x} - b\sigma_y - c\sigma_z}{a}$$ ***Equation B 22***

$$\tau_{xy} = \frac{\overline{\tau_{xy}}}{(a - b)}$$ ***Equation B 23***

Where: $\overline{\sigma_x}$, $\overline{\sigma_y}$ and $\overline{\tau_{xy}}$ are the measured stresses in the x and y directions.

The a-, b- and c-values were determined from FLAC (Itasca, 1993) modelling. Their relationship to Poisson's Ratio (*v*) is shown in Equation B 24 to Equation B 26. The equations assume that the rock mass was elastic and more work is required to determine the effects of the micro-fracturing on these values.

$$a = 1.33 + 0.1\upsilon$$ ***Equation B 24***

$$b = -0.13 + \frac{\upsilon(1 - 1.5\upsilon + 5\upsilon^2)}{3}$$ ***Equation B 25***

$$c = 0.37 - 1.1\upsilon$$ ***Equation B 26***

B3 References

Vreede, F.A. (1981). A critical study of the method of calculating virgin rock stress from measurement results of the CSIR triaxial strain cell. *Internal CSIR report Me-1679.*

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