

The remaining matrices for $n = 1, 3, 4$ may be evaluated by appropriate substitution and integration of equation (F.10).

Thus the total parametric coupling matrix $[W_{ww}(n\Omega t)]$ is obtained from:

$$[W_{ww}(n\Omega t)] = \sum_{n=1}^4 \text{Re}\{[W_{ww}(n\Omega t)]^1 + [W_{ww}(n\Omega t)]^2 + [W_{ww}(n\Omega t)]^3\}$$

The parametric coupling matrix $[V_{vv}(n\Omega t)]$ is obtained in an identical fashion. It is not identical to $[W_{ww}(n\Omega t)]$ since the term $(w_{,s}^2 \bar{v}_{,s})_{,s}$ in equation (F.2) is pre-multiplied by $\frac{1}{2}$, whereas $(w_{,s}^2 \bar{w}_{,s})_{,s}$ in equation (F.3) is pre-multiplied by $\frac{3}{2}$. $[V_{vv}(n\Omega t)]$ can be constructed from the component matrices of $[W_{ww}(n\Omega t)]$ as:

$$[V_{vv}(n\Omega t)] = \sum_{n=1}^4 \text{Re}\{[W_{ww}(n\Omega t)]^1 + [W_{ww}(n\Omega t)]^2 + \frac{1}{3}[W_{ww}(n\Omega t)]^3\}$$

Since the real part of the component matrices constitute the overall parametric coupling matrix, the parametric coupling matrix may be expanded to:

$$[P(n\Omega t)] = \sum_{n=1}^4 \{[P_c]_n \cos(n\Omega t) + [P_s]_n \sin(n\Omega t)\}$$

Thus if the first two harmonics $\Omega, 2\Omega$ of the Lebus cross-over frequency are accounted for, the system is excited parametrically at $\Omega, 2\Omega, 3\Omega, 4\Omega$ by sine and co-sine functions. Thus the parametric matrices relating to each harmonic are impedance functions, and are phase shifted with respect to each other.

F.3 Drift Terms

This section considers the influence of the drift term in the longitudinal response $u^D(s)$, and the static component of w_s^2 in equations (F.2),(F.3) on the lateral variational equations. The longitudinal drift $u^D(s)$ was determined in Appendix E.1 as⁴:

$$u_{,ss}^D = -\frac{1}{4}[w_s w_{,s}^*]_{,s} \quad (F.11)$$

$$u_{,s}^D = -\frac{1}{4}[w_s w_{,s}^*] \quad (F.12)$$

$$u^D(s_1) = -\frac{1}{4} \int_0^{s_1} [w_s w_{,s}^*] ds \quad (F.13)$$

Considering the first harmonic of lateral response w_1 , the static component of $Re(w_{1,s})Re(w_{1,s})$ is denoted $(w_{1,s})_D^2$:

$$(w_{1,s})_D^2 = \frac{1}{2}[w_{1,s} w_{1,s}^*] \quad (F.14)$$

Drift terms do not affect the variational form of the longitudinal equation of motion (F.1). However they do effect the variational form of the lateral equations of motion (F.2-F.3). These equations are reproduced below, where only static or drift components of the linear steady state response are retained.

$$\bar{v}_{,tt} = \mu_l \bar{v}_{t,ss} + \bar{c}^2 \bar{v}_{,ss} + c^2 \{(\bar{v}_{,s} u_{,s}^D)_{,s} + \frac{1}{2}((w_{1,s})_D^2 \bar{v}_{,s})_{,s}\} \quad (F.15)$$

$$\bar{w}_{,tt} = \mu_l \bar{w}_{t,ss} + \bar{c}^2 \bar{w}_{,ss} + c^2 \{(\bar{w}_{,s} u_{,s}^D)_{,s} + \frac{3}{2}((w_{1,s})_D^2 \bar{w}_{,s})_{,s}\} \quad (F.16)$$

Substituting equations (F.12),(F.14) into the above equations leads to:

$$\bar{v}_{,tt} = \mu_l \bar{v}_{t,ss} + \bar{c}^2 \bar{v}_{,ss} \quad (F.17)$$

⁴ w^* represents the complex conjugate of w

$$\bar{w}_{,tt} = \mu_l \bar{w}_{t,ss} + \bar{c}^2 \bar{w}_{,ss} + c^2 \left\{ \frac{1}{2} (w_s w_s^* \bar{w}_{,s})_{,s} \right\} \quad (\text{F.18})$$

Since the lateral in-plane variational equation (F.17) reduces to its linear counterpart, the drift terms exert no influence on the natural frequencies of the in-plane variational modes. However, in the case of the out-of-plane variational equation, the drift terms do exert an influence. Effectively the final term in equation (F.18) modifies the linear lateral natural frequencies of the out-of-plane modes. This influence can be ascertained by applying a normal mode approximation for $\bar{w}$, and carrying out the appropriate integration. This results in a modified modal stiffness matrix with off diagonal terms, coupling the lateral out-of-plane modes.

This matrix is given by:

$$[k_{ij}] = -\frac{\bar{c}^2}{m_{ii}} \int_0^{l_1} \Phi_i \Phi_j'' ds - \frac{c^2}{m_{ii}} \int_0^{l_1} \Phi_i \left\{ \frac{1}{2} (w_s w_s^* \Phi_j')_{,s} \right\} ds$$

where $\bar{c}$, c , m_{ii} , Φ represent the lateral wave speed, the longitudinal wave speed, the modal mass, and the i^{th} linear eigenfunction respectively, and Φ' represents differentiation of Φ with respect to s . The linear eigenfunction satisfying the boundary conditions is:

$$\Phi_i(s_1) = \sin(\delta_i s_1) \quad \text{where} \quad \delta_i = i\pi/l_1$$

Performing the integration leads to:

$$[k_{ij}] = [\omega_i^2] + \sum [A_{ij}]_n$$

$$\begin{aligned} [A_{ij}]_n = & \frac{c^2 \delta_i \delta_j}{2m_{ii}} \left\{ \frac{|W_n| |\gamma_{ln}|}{|\sin(\gamma_{ln} l_1)|} \right\}^2 \left\{ \frac{\sin(\gamma_{ln} - \gamma_{ln}^*) l_1}{(\delta_i + \delta_j + \gamma_{ln} - \gamma_{ln}^*)} - \frac{\sin(\gamma_{ln} - \gamma_{ln}^*) l_1}{(\delta_i - \delta_j - \gamma_{ln} + \gamma_{ln}^*)} \right. \\ & + \frac{\sin(\gamma_{ln} - \gamma_{ln}^*) l_1}{(\delta_i - \delta_j + \gamma_{ln} - \gamma_{ln}^*)} - \frac{\sin(\gamma_{ln} - \gamma_{ln}^*) l_1}{(\delta_i + \delta_j - \gamma_{ln} + \gamma_{ln}^*)} \\ & - \frac{\sin(\gamma_{ln} + \gamma_{ln}^*) l_1}{(\delta_i - \delta_j - \gamma_{ln} - \gamma_{ln}^*)} + \frac{\sin(\gamma_{ln} + \gamma_{ln}^*) l_1}{(\delta_i - \delta_j + \gamma_{ln} + \gamma_{ln}^*)} \\ & \left. - \frac{\sin(\gamma_{ln} + \gamma_{ln}^*) l_1}{(\delta_i + \delta_j - \gamma_{ln} - \gamma_{ln}^*)} + \frac{\sin(\gamma_{ln} + \gamma_{ln}^*) l_1}{(\delta_i + \delta_j + \gamma_{ln} + \gamma_{ln}^*)} \right\} \end{aligned}$$

where $[w_i^2]$ represents a diagonal matrix containing the linear natural frequencies of the out-of-plane modes, and $[A_{ij}]_n$ represents additional stiffness coupling generated between the out-of-plane modes due to the n^{th} harmonic of the lateral response. Since this matrix is a function of the excitation amplitude, the stiffness of the variational system changes with the lateral response amplitude, and consequently the natural frequencies of the out-of-plane variational modes change and detune from the in-plane modes. The modified natural frequencies of the lateral variational modes are thus found by extracting the eigenvalues of $[k_{ij}]$.ie:

$$[\omega_w^2] = Eig[K_{ij}]$$

It is important to note that the detuning of the variational system is a consequence of the assumed linear solution, and is consistent only where this solution is valid. The variation in the natural frequency corresponds to the frequency at which the system would respond, to small disturbances about the linear solution. In constructing the linear steady state solution, it was assumed that the lateral response amplitude was small, and therefore terms of $o(w^3)$ were neglected. This is unlikely to be the case in a region of external lateral resonance.

Appendix G

Longitudinal Damping Estimates

G.1 Damping Mechanisms

A crucial step required in a dynamic simulation concerns the definition of an appropriate damping mechanism. Greenway[1989] examined the logarithmic decrement associated with the free response of the first longitudinal mode of a conveyance at Deelkraal Mine, due to emergency braking. The sheave support structure was strain gauged to measure rope load, providing time data for the event. The data was gathered by Thomas et al.[1987], as a part of a report for COMRO(1987). The logarithmic decrement was extracted from this data, for an empty and full conveyance, decelerated at stations at approximately quarter and three quarters of the shaft depth. The results of these tests are presented in table G.1. In interpreting the logarithmic decrement with respect to the longitudinal oscillations, Greenway[1989] considered the longitudinal equation of motion of the system in the principal modal co-ordinates as:

$$\ddot{q}_i + (a + b\omega_i^2)\dot{q}_i + \omega_i^2 q_i = 0 \quad (\text{G.1})$$

where q_i represents the i^{th} principal mode; the damping coefficient a represents the material damping constant corresponding to a distributed damping force $u_{,t}$, which is proportional to the mass properties; b represents the material damping constant corresponding to the distributed damping force $u_{xx,t}$, which is proportional to the stiffness properties. The latter type of damping amounts to relative damping since it is associated with the relative velocities of the

displacement co-ordinates. Greenway[1989] considered relative damping to be appropriate, and thus set $a = 0$, whilst retaining the coefficient b .

Considering this equation in the form:

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega_i^2q_i = 0$$

where ζ_i represents the modal damping ratio associated with the i^{th} mode and ω_i represents the i^{th} natural frequency. Comparison of the equations leads to:

$$\zeta_i = \frac{b\omega_i}{2}$$

Thus for a constant damping coefficient b , the modal damping ratio in each principal mode is proportional to the undamped natural frequency of that mode. Although the data was limited, Greenway[1989] demonstrated that with respect to this data, the logarithmic decrement depended on the conveyance mass and the vertical length of the rope, whilst the material damping coefficient b was less sensitive to skip mass, but related to the rope length. Greenway[1989] non-dimensionalised the equations of motion, and derived a dimensionless damping parameter $\beta = bc/2l$ where c and l represent wave speed and rope length respectively. This accounted for the variation of the material damping coefficient as a function of rope length.

Proportional relative damping results in a relationship between the modal damping factor of the first mode, and higher modes, such that $\zeta_n = \zeta_1 \frac{\omega_n}{\omega_1}$. Thus the modal damping factor of the higher modes increases in proportion to the ratio of the undamped natural frequencies, and thus higher modes become successively more highly damped. This has lead to the assumption that the fundamental mode dominates the longitudinal response, whilst the higher modes are more strongly attenuated and consequently less important. Greenway[1989] notes that the assumption of proportional modal damping is implemented in his analysis since it appears to be *intuitively* correct. Dimitriou and Whillier[1973] briefly commented that data obtained regarding the free decay response of skips, exhibited a linear decay profile, which is associated with friction, and hence a Coulomb damping mechanism rather than a viscous damping mechanism. This confirmed experimental results of Vanderveldt et al.[1973] regarding the lateral dynamic characteristics of stranded wire rope. Dimitriou and Whillier[1973] concluded that Coulomb damping played a greater roll than viscous damping in mine hoist wire ropes. As is

evident, the aspect of damping in mine hoist ropes has not been given a great deal of attention. It is surmised that this is due to the production losses which would be incurred if an adequate experimental program was instituted to measure and formulate a damping model for the rope. This model is likely to be a complex non-linear function reflecting the rope construction and mean tension. Nevertheless, if accurate simulations are to be performed, a more thorough experimental determination of the damping levels will be required.

It was decided that approximate damping factors would have to be applied due to the limited data available. However, the decision as to the application of a relative proportionally damped model, as opposed to a hysteretically damped model could be investigated. It was this aspect which provided the incentive for performing further site tests.

G.2 Drop Tests

Anglo American Corporation (AAC) was assessing a levelock system on a conveyance at Elandsrand Gold Mine. The levelock system is a hydraulically actuated device which clamps the conveyance or man cage between the guides during unloading and loading cycles. The device releases the clamping force so that the skip or man cage slides without overshoot to its new equilibrium position. As part of the test program, it was decided to assess the cage response due to a pre-load in the cage, followed by a rapid release in the clamping force. This was accomplished by clamping an empty cage in position at the station, inserting a dead weight, and subsequently releasing the clamping force. Once the cage had settled to its new equilibrium position, it was repositioned at the station and clamped; the load was removed and subsequently the clamping force was released. In this manner, the free decay of the longitudinal oscillations could be examined in the presence of a fixed rope length, with various mean tensions and cage mass. Six tests were performed, namely¹:

- Seven ton drop.
- Seven ton lift.
- Seven ton drop.
- Seven ton lift.
- Three ton drop.

¹1 ton= 1000kg

- Three ton lift.

Three different transducers were monitored during the tests. An accelerometer and displacement LVDT were recorded by AAC personnel². Strain gauges were applied to the draw bar structure by University personnel, providing measurement of the rope force at the conveyance. The data from the AAC transducers was recorded directly without any pre-filtering. In order to amplify the response of the higher longitudinal modes, the measurement from the draw bar load cell was filtered prior to amplification. An analogue high-pass filter with a frequency cut-off of 0.5Hz was used.

The time traces from the AAC accelerometer are presented in figure G.2. The repeatability of the data is reflected in figures G.2(a),(d) where two records from different tests are superimposed. Similar results were obtained for the LVDT and draw bar measurement. Frequency response functions were created by treating the time data as being representative of the unit impulse response of the longitudinal system. The frequency response functions were constructed by applying a Fourier transformation to the time data. The frequency response functions³ for the AAC accelerometer records are presented in figure G.3. The I-Deas package was utilised to perform this task, where an exponential window was applied to the time data prior to performing the Fourier transformation. A circle fit method was applied to extract the natural frequency and corrected⁴ damping factor for each FRF. The results from the analysis are presented in table G.2 for the AAC accelerometer. The first mode was curve fitted for the LVDT measurement, whilst the draw bar measurement provided damping estimates for the second and third modes. These results are presented in tables G.3 ,G.4.

²The purpose of the AAC test was to assess the gross motion of the conveyance. The processing of the data to achieve further damping estimates, was proposed as a result of the research carried out in this thesis, which indicated that relative proportional damping overestimated the modal damping factors in the higher modes.

³A spectral resolution of 0.02 Hz was achieved.

⁴The damping factor was corrected to account for the exponential window.

G.3 Discussion

The objective of the test was primarily aimed at determining if a relative proportional modal damping mechanism, as proposed by Greenway[1989] is appropriate to mine hoist ropes. Since the natural frequency of the second mode is approximately four times higher than the first, relative proportional modal damping would imply that the second mode should reflect approximately four times the modal damping ratio of the first mode. Clearly this did not occur. Although an adequate degree of repeatability was achieved with respect to the modal damping factor of the first mode, because the conveyance is close to a nodal point for the higher modes, and consequently the modal coupling is reduced, a higher signal-to-noise ratio occurs with regard to the measured response of the second and higher modes. As a result, difficulty was encountered in curve fitting the second and third modes, as reflected by the scatter of the damping estimates presented in tables G.2,G.4.

This form of test is crude, and one would not expect to achieve highly accurate damping estimates. Nevertheless, the test was useful in that it confirmed that significantly higher damping factors were not measured for the higher modes, and consequently a relative proportional damping mechanism alone would not be appropriate.

The damping factor measured for the first mode reflects a strong dependency on the initial rope tension, or the amplitude of the impulse. Figure G.1 presents a plot of the measured damping factor of the first mode verses skip load. The time traces of the response, and the estimated damping factor reflect that the damping effect is lower when the cage is dropped as opposed to when it lifts. This effect may be the result of a number of different mechanisms, and clearly more extensive and accurate testing techniques would be required to determine these. However, when the skip is dropped, the mean tension in the rope rises; as a result of the rope construction the strands lock more readily, preventing relative interstrand motion, and resulting in a lower damping coefficient. During the lift cycle, overshoot of the cage results in a drop in the mean rope tension, promoting interstrand movement and consequently higher damping factors. Since the higher modes are less strongly coupled to the impulsive force induced in the test, the mean tension changes related to the higher modes would be less significant, and hence have less effect on the modal damping factors⁵. The mechanism proposed would result in a dependence of the damping estimate on the magnitude of the initial impulse applied to the system and in this context the response would be nonlinear. Consequently the

⁵Aerodynamic effects may also be pertinent to the response of the first mode, as a downward forced draft of approximately 10m/s exists in the shaft.

technique adopted to extract the damping estimate would not be valid.

In practice, the disturbances applied to the system during the winding cycle are not as severe as those applied in the drop test⁶. It may be argued that although the damping mechanism may be nonlinear when considering large disturbances, a linear mechanism may apply for smaller disturbances. Figure G.1 illustrates that the damping ratio of the first mode is approximately linearly dependent on the load in the skip or the initial impulse. In this context, the intercept of a line fitted through the data, with the abscissa would represent a *linear* estimate for small disturbances of the fundamental mode. A least squares curve fit of the data provides a *linear* damping estimate of 4.65%, which is of the order of the measurements extracted by Greenway[1989] as presented in table G.1.

A number of options exist regarding a practical approach to deriving a damping model, which reflects to some degree the longitudinal dissipation of the rope. For analytical reasons it is convenient to apply proportional damping models, since the continuous solution can be extracted in closed form. A more flexible method results if the system response is analysed via the normal mode technique, since modal damping factors can be applied independently to each principal mode⁷; however the disadvantage of a normal mode approach is that it requires that sufficient modes are included in the analysis to prevent modal truncation.

The damping estimates can be examined in the context of a general proportional form, where the damping constants a, b in equation (G.1) are not equal to zero. This would require that the damping action is proportional to both the mass and stiffness properties of the rope. In this sense the modal damping coefficient would be:

$$\zeta_n = \frac{1}{2} \left(\frac{a}{\omega_n} + b\omega_n \right)$$

where a, b represent the material damping coefficients, and ω_n represents the natural frequency of the n^{th} mode.

The damping estimates related to the first mode were considered more repeatable than those of the higher modes. Thus the constants a, b were extracted

⁶With regard to the winding cycle, large motion does occur at the conveyance during the acceleration and deceleration phase, and thus one may expect stronger attenuation during a deceleration from the nominal winding speed and visa versa.

⁷A hysteretic damping model would necessitate this approach.

by considering the experimental damping estimates as a function of frequency, pertaining to the first mode only. The constants⁸ were extracted by applying a least squares minimisation procedure. The experimental data and analytical curve fit are presented in figure G.4. Figure G.5 presents the curve fit together with the estimates extracted for the second and third modes. It is clear that although such a damping model may adequately account for the dependence of the modal damping estimate of the first mode on the mean tension, it does not simultaneously satisfy low damping estimates for the higher modes.

Greenway[1993] interpreted the data differently, by globally fitting the data to the damping model. This results in a damping model which neglects the dependence of the first modal damping estimate on the mean tension, whilst adequately accounting for lower damping factors in the higher modes. Figure G.6 presents a plot of the experimental data and curve fit. This approach provides a damping model which adequately describes the trends in an average manner. The advantage of this model is that it can be directly incorporated into existing analyses, and accommodates more correctly the lower damping in the higher modes. The justification for applying such a model would be its convenience rather than being physically correct.

Figure G.7 presents the data curve fitted such that the *linear* modal damping coefficient of the first mode is applied, thus eliminating the dependence of this coefficient on the mean tension.

⁸A negative coefficient was obtained for α , which is not physically sensible.

G.4 Conclusion

Although the test data is sparse, and an extensive test program would be required to assess the dependence of the modal damping estimates on rope length and mean tension, important observations regarding the data can be proposed.

- The assumption of proportional relative damping is not appropriate since it results in large modal damping factors for the higher modes.
- The modal damping estimate is dependent on the mean tension in the rope, which indicates that for large disturbances frictional effects are important. The linear decay of the response in figure G.2(a,b,c) confirms that hysteretic effects are evident.
- It is possible to propose a general proportionally damped model which adequately describes the modal damping estimates in the low as well as higher modes. This model is proposed for convenience in an analytical study, and is not intended to account for the actual damping mechanism.
- A substantial effort by the industry is required to correctly quantify the nature of the damping mechanism, which adequately accounts for mean tension and rope length.

The damping mechanism is complex; although general proportional damping may represent the damping mechanism in an average sense, interstrand friction and hence Coulomb effects are likely to play a roll. Thus it is possible that a more accurate *linear* model would result as a combination of general proportional and hysteretic damping action. These findings are based on limited data, and are not well substantiated. Clearly substantial scope exists for further work which would lead to a more accurate determination of the damping mechanism appropriate to mine hoist ropes.

Table G.1: Deelkraal Data - Greenway[1989]

Depth	LoadedUnloaded	$f_1(Hz)$	δ	b(s)	$\zeta_1 \%$
3/4 down	load	0.278	0.163	0.03	2.6
3/4 down	empty	0.388	0.230	0.03	3.7
1/4 down	empty	0.734	0.173	0.12	2.7

Table G.2: Elandsrand - Drop test measurements - AAC accelerometer

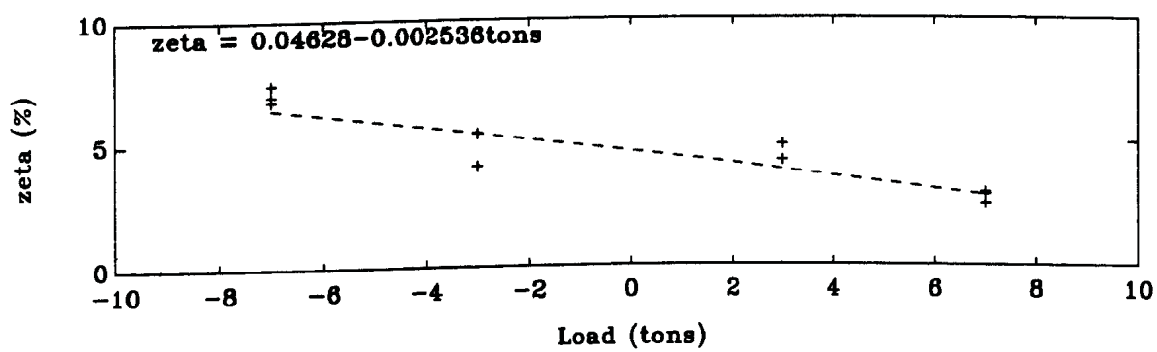
Depth	Load	$f_1(Hz)$	$f_2(Hz)$	$\zeta_1 \%$	$\zeta_2 \%$
75level	+7 ton	0.289	1.05	2.90	1.81
75level	-7 ton	0.337	1.102	6.76	2.20
75level	+7 ton	0.289	1.074	2.50	1.29
75level	-7 ton	0.343	1.114	6.75	1.26
75level	+3 ton	0.329	1.105	4.86	1.42
75level	-3 ton	0.34	1.148	5.37	0.65

Table G.3: Elandsrand - Drop test measurements - AAC LVDT

Depth	Load	$f_1(Hz)$	$\zeta_1 \%$
75level	+7 ton	0.289	2.94
75level	-7 ton	0.337	7.43
75level	+7 ton	0.289	2.53
75level	-7 ton	0.345	6.97
75level	+3 ton	0.331	4.23
75level	-3 ton	0.338	4.07

Table G.4: Elandsrand - Drop test measurements - WITS load cell

Depth	Load	$f_2(Hz)$	$f_3(Hz)$	$\zeta_2\%$	$\zeta_3\%$
75level	+7 ton	1.048	1.927	2.25	2.42
75level	-7 ton	1.066	1.908	0.22*	2.60
75level	+7 ton	1.091	1.947	1.282	1.85
75level	-7 ton	1.114	1.981	0.701*	1.94
75level	+3 ton	1.108	2.021	0.798	-
75level	-3 ton	1.118	-	0.951	-

Figure G.1: Drop Test : Modal damping ratio ζ_1 verses conveyance load

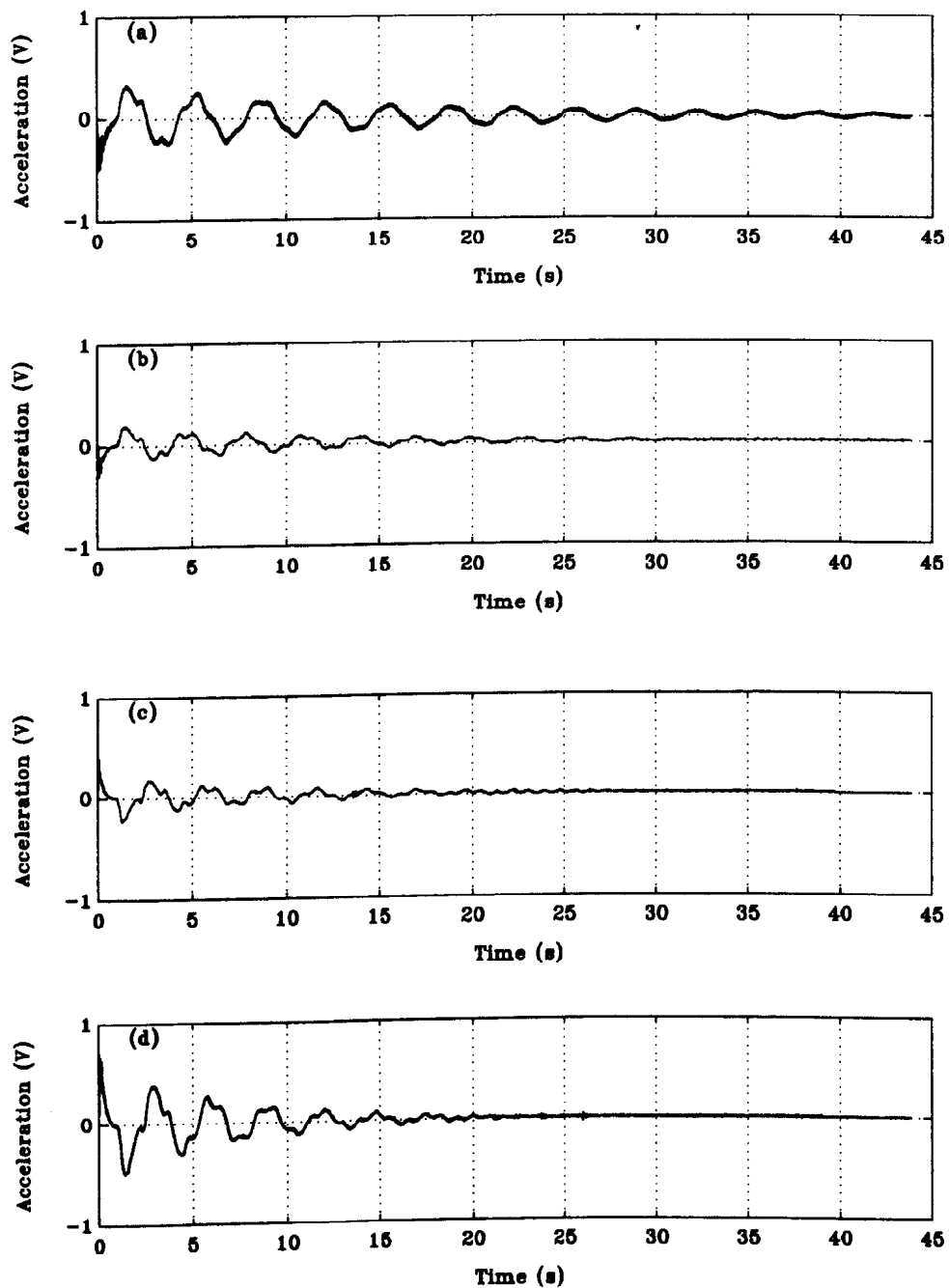


Figure G.2: Drop test free response: AAC accelerometer record

(a) 7 ton Drop (b) 3 ton Drop
(c) 3 ton Lift (d) 7 ton Lift

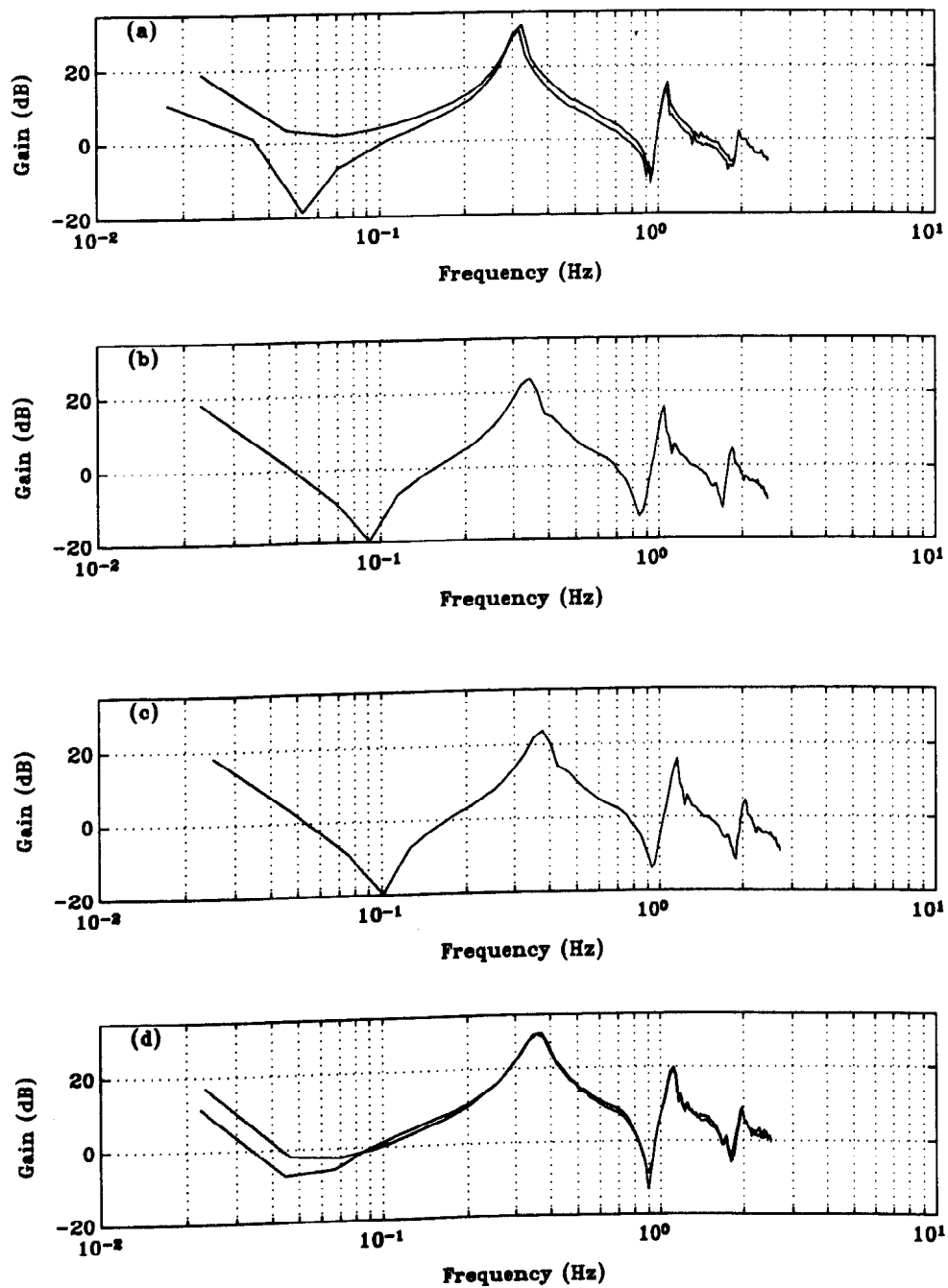


Figure G.3: Frequency response functions: AAC accelerometer record

- (a) 7 ton Drop (b) 3 ton Drop
(c) 3 ton Lift (d) 7 ton Lift

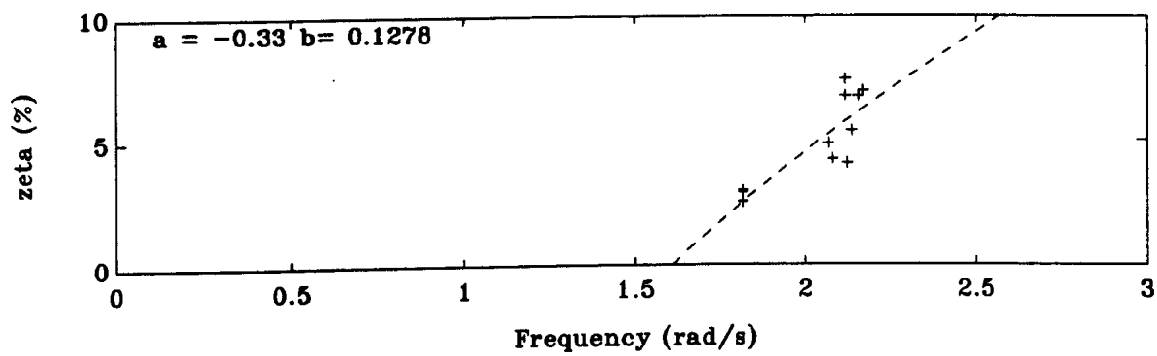


Figure G.4: Curve fitted data: Based on first modal damping estimates.

$$\zeta_1 = \frac{1}{2} \left(\frac{a}{\omega_1} + b\omega_1 \right)$$

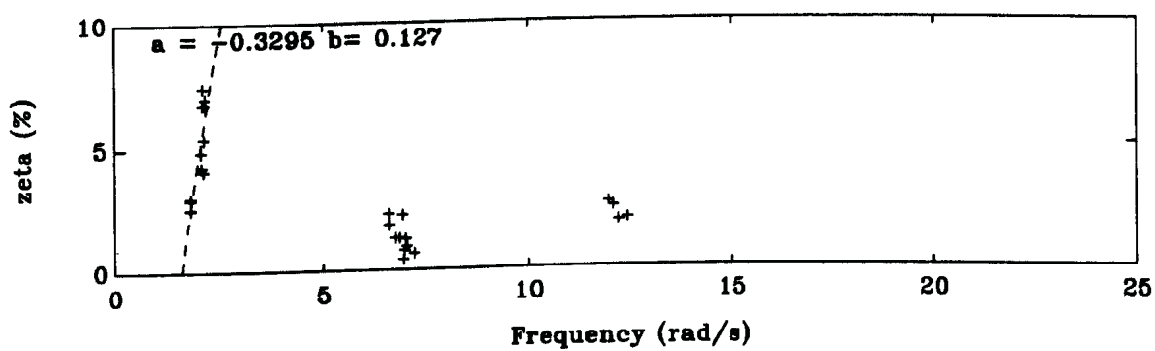


Figure G.5: Damping data and curve fit for the first longitudinal mode.

$$\zeta_1 = \frac{1}{2} \left(\frac{a}{\omega_1} + b\omega_1 \right)$$

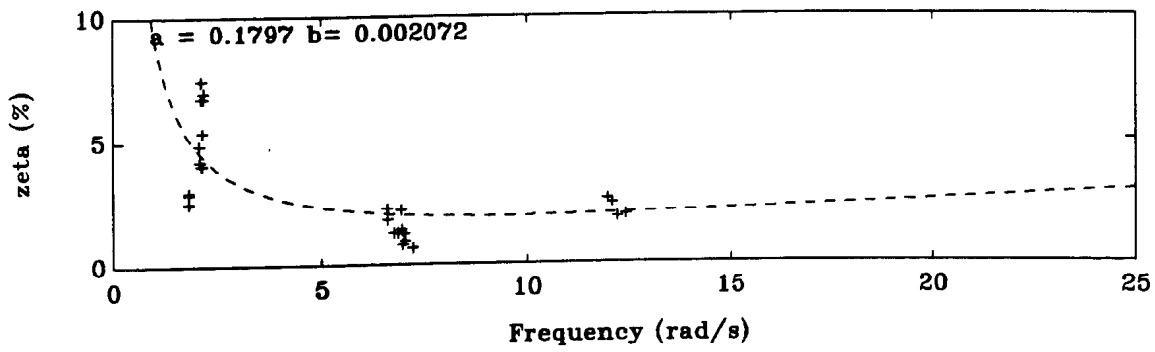


Figure G.6: Greenway (1993): Global longitudinal damping estimate

$$\zeta_n = \frac{1}{2} \left(\frac{a}{\omega_n} + b\omega_n \right)$$

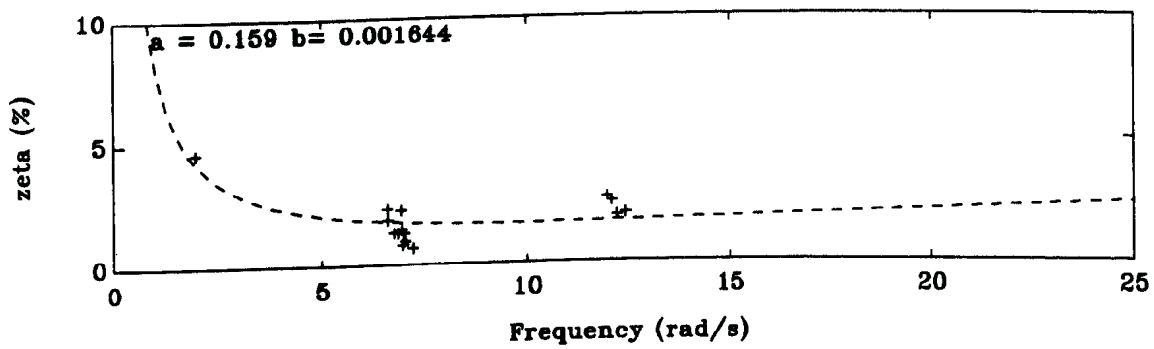


Figure G.7: Global longitudinal damping estimate based on *linear* ζ_1

$$\zeta_n = \frac{1}{2} \left(\frac{a}{\omega_n} + b\omega_n \right)$$

Appendix H

Lateral Damping of the Catenary

Information concerning the lateral damping coefficients applicable to mine hoist ropes is sparse. Vanderveldt et al [1973] examined the lateral damping coefficients obtained from a number of $\frac{3}{8}$ -in diameter 65-inch long wire cables of different construction and material. The study concluded that:

- The dimensionless damping coefficient decreases with increasing axial load, and in the case of the cables tested, this coefficient varied between 0.05-0.15 % of critical.
- The major component of the damping mechanism is due to coulomb friction.
- The structural strength, heat treatment or alloy used is unrelated to the damping mechanism.
- The rope construction and geometry significantly influence the damping effort, however this is not strongly dependent on the number of wires, but rather as speculated, a feature of wire/strand interaction and hence construction geometry.

Although these results define broad dissipation characteristics of wire cable, mine hoist ropes are of larger diameter, experience higher tensions, and longer amplitude wavelengths than those tested by Vanderveldt et al. Mankowski [1988] experimentally simulated the energy loss of a typical mine hoist cable undergoing non-planar whirling motion. The experimental apparatus measured the power loss in a cable which was spun about its geometric axis, whilst

being deflected to induce curvature along its length. This was achieved by supporting a length of cable in thrust bearings, inclined so as to simulate cable sag. The rotation of the cable about its geometric centre effectively simulated the flexural effects of an irrotational cable undergoing non-planar whirl, and hence the power loss per cycle was directly related to the time rate of change of rope curvature¹. Mankowski obtained an empirical relationship between power dissipation, rotational frequency, amplitude and catenary span empirically as:

$$P(n)_{Loss} = (2n)SSRF_n C_1(1 + C_2 A_n) \quad \text{Watts}$$

Where P_n is the internal power loss for the n^{th} mode, A_n is the amplitude of vibration of the n^{th} mode, F_n is the frequency of vibration (Hz), SSR is the sag to span ratio defined as $\frac{2nA_n}{span}$. Mankowski determined the coefficients C_1, C_2 for a cable which was of a similar construction and size to that employed at Kloof as $C_1 = 42.75J$ and $C_2 = 0.34m^{-1}$. The units of these coefficients were related to the damping capacity and curvature characteristic of the rope.

The energy losses achieved were small, as reflected in table H.1, where the calculated energy loss associated with the modal amplitudes observed at Kloof Mine are tabulated. Mankowski[1990] extended this work to investigate the effect of the power dissipation associated with a transverse kinetic shock travelling up the rope. Although the research provides valuable results, the experimental method is limited, since although various sag to span ratio's were achieved in the testing rig, the mean axial tension applied was significantly lower than that found in practice (the mean axial tension was varied between 1-30KN, whereas in practice the axial tension in the catenary varies between 120-300 KN). As it has been established that the dissipation mechanism is likely to be dominated by friction, the dissipation measured is likely to be dependent on the mean tension, which cannot be simulated correctly by the experimental apparatus. Furthermore, as is shown subsequently, it appears that aerodynamic drag may be as significant in practice as the damping mechanism associated with the time rate of change of curvature. Although Mankowski defines an empirical formula for the particular rope tested, relating power loss to the steady state amplitude and frequency of motion, this particular format would be difficult to apply in a non-stationary simulation of the system. However, the latter analysis of the power dissipation relating to transverse kinetic shocks may be valuable if a wave mechanics approach is adopted in the simulation.

¹The time rate of change of curvature is defined as $y_{xx,t}$, where y represents the amplitude of the lateral motion. This implies a damping mechanism which is proportional to the stiffness properties of the rope.

H.1 Aerodynamic Damping

Since the lateral damping mechanism in the rope is not well defined, and Mankowski's[1988] tests indicate that the energy dissipation related to the time rate of change of the rope curvature is low, it is likely that during rope whip, aerodynamic drag may be significant, and warrants consideration. The results of Mankowski's tests, which were conducted on a similar rope to that employed on the Kloof mine are tabulated in table H.1, together with an equivalent viscous modal damping coefficient².

The following relations were applied to convert Mankowski's results into an equivalent viscous and dimensionless damping coefficient:

$$P = \frac{1}{2}\omega_n^2 C_{eq} X_n^2$$

$$\zeta = \frac{C_{eq}}{2m\omega_n}$$

$$m = 0.5\rho_R l_c$$

where:

P represents the average power dissipation, X_n represents the modal amplitude, m represents the effective modal mass of the catenary, ρ_R represents the linear mass density of the rope, l_c represents the catenary length.

H.1.1 Aerodynamic Drag

Aerodynamic drag is proportional to the square of the rope velocity. Since the transport cable velocity is significant compared to the lateral velocity of the rope, the drag characteristic of the rope in the lateral direction is likely to be difficult to assess accurately. An upper bound to the modal drag force is estimated by neglecting the axial transport velocity of the rope, and treating the rope as a cylinder in steady state cross flow. Treating the rope as a cylinder,

²The purpose of presenting Mankowski's data in this form, is primarily to provide a comparative basis with respect to the equivalent aerodynamic dissipation, tabulated in table H.2.

with a drag coefficient of 1, the drag force per unit length of rope is calculated as:

$$F_d = \frac{1}{2}\rho d|V|^2 \text{sign}(V)$$

where V represents the absolute lateral velocity of the rope, $\rho \approx 1\text{kg/m}^3$ represents the air density, and d represents the rope diameter. The modelling of this drag force in modal space would be complex, as it would require the calculation of the absolute velocity, as well as a projection of the drag force onto the in and out of plane axes. For this reason, the following calculations are performed for motion in a single plane.

H.1.2 Planar Motion

In modal space the planar velocity of the rope is expressed as:

$$V(s)^2 = \left[\sum_{i=1}^n \sum_{j=1}^n \Phi_i(s) \Phi_j(s) \dot{q}_i \dot{q}_j \right]$$

$$\Phi_i(s) = \sin\left(\frac{i\pi}{l_c}s\right)$$

Considering only direct coupling between the lateral catenary modes and the drag force, ie $i = j$, the drag force for the i^{th} mode is:

$$F_d^i(s) = \frac{1}{2}\rho d \sin^2(i\pi s/l_c) \dot{q}_i^2 \text{sign}(\dot{q}_i)$$

where d represents the cross-sectional diameter of the rope.

The modal damping force is obtained by orthogonalising the drag force in modal space:

$$Q_d^i = i \int_0^{l_c} |F_d^i(s)| \Phi_i(s) ds$$

$$Q_d^i = \frac{1}{2} \frac{\rho d i}{m} \int_0^{l_c} [\sin^3(\frac{i\pi s}{l_c}) ds] \dot{q}_i^2 \text{sign}(\dot{q}_i)$$

$$Q_d^i = \frac{4\rho d}{3\pi\rho_R} \dot{q}_i^2 \text{sign}(\dot{q}_i)$$

The equivalent viscous damping coefficient and average power loss during a cycle are calculated as³:

$$C_{eq}^{Aero} = \frac{16\rho d l_c \omega_n X_n}{9\pi^2} \quad (\text{H.1})$$

$$P^{Aero} = \frac{1}{2} \omega_n^2 C_{eq}^{Aero} X_n^2$$

$$\zeta^{Aero} = \frac{C_{eq}^{Aero}}{2m\omega_n} = \frac{16\rho d X_n}{9\pi^2 \rho_R} \quad (\text{H.2})$$

H.1.3 Non-Planar Motion

When the motion is non-planar, the absolute velocity vector of the rope is composed of motions in both planes. In this case the drag force is:

$$F_d = \frac{1}{2} \rho d |V|^2 \text{sign}(V)$$

where:

$$V(s)^2 = [\sum_{i=1}^n \sum_{j=1}^n \Phi_i(s) \Phi_j(s) (\dot{q}_i \dot{q}_j + \dot{r}_i \dot{r}_j)]$$

³For velocity squared damping, where the damping force is defined as $F_d = c_2 \dot{X}^2 \text{sign}(\dot{X})$, it can be shown [1978] that the equivalent viscous damping coefficient is defined as $C_{eq} = \frac{8}{3\pi} \omega_n C_2 X$, where X is the amplitude at frequency ω_n

The drag force in the v, w plane would be obtained as:

$$F_d^v = F_d \cos(\phi)$$

$$F_d^w = F_d \sin(\phi)$$

$$\phi = \tan^{-1}\left(\frac{V_w}{V_v}\right)$$

If one considers single mode behaviour in both planes simultaneously ie $i = j$, the modal force in each plane would simplify to:

$$Q_d^i = \frac{4\rho d}{3\pi\rho_R}(\dot{q}_i^2 + \dot{r}_i^2)\cos(\phi)$$

$$R_d^i = \frac{4\rho d}{3\pi\rho_R}(\dot{q}_i^2 + \dot{r}_i^2)\sin(\phi)$$

$$\phi = \tan^{-1}\left(\frac{\dot{r}_i}{\dot{q}_i}\right)$$

where Q_d^i, R_d^i represent the modal components of the aerodynamic drag force in the v, w directions respectively.

If the the motion is circular ie $\dot{q} = \dot{r}$, then the relationships presented for the equivalent viscous and non-dimensional damping coefficients in the previous section would increase by $\sqrt{2}$.

H.2 Equivalent Aerodynamic Power Losses

Table H.2 represents the average aerodynamic power loss, for planar single mode motion ($C_d = 1$). The equivalent viscous and dimensionless damping coefficients are calculated via relations H.1,H.2 with the same parameters as used by Mankowski. For non-planar circular whirl, these values should be increased by $\sqrt{2}$.

It is likely that the drag coefficient is lower than 1 due to the axial transport velocity, and hence this calculation represents an upper bound to the aerodynamic dissipation. Nevertheless, even if a drag coefficient of 0.1 is assumed, dissipation through aerodynamic drag may be as significant as the mechanism proposed by Mankowski, especially in the fundamental mode.

Table H.1: Damping coefficients - Mankowski[1988] -Kloof Mine

Mode No.	Frequency Hz	Amplitude m	Power Loss Watts	Equivalent Viscous Damping C_{eq} (N s m)	Equivalent Modal Damping ζ %
1	1.12	1	7.6	0.306	0.007
2	2.24	0.575	46.27	1.413	0.016
3	3.36	0.5	92.3	1.65	0.012
4	4.48	0.375	171.8	3.08	0.017

Table H.2: Damping coefficients - Aerodynamic -Kloof Mine

Mode No.	Frequency Hz	Amplitude m	Power Loss Watts	Equivalent Viscous Damping C_{eq} (N s m)	Equivalent Modal Damping ζ %
1	1.12	1	112.9	4.56	0.1
2	2.24	0.575	171.8	5.25	0.06
3	3.36	0.5	381.3	6.84	0.05
4	4.48	0.375	381.3	6.84	0.04

H.3 Damping Model Applied in the Simulation

It is clear that in reality the damping mechanism is complex and falls beyond the realms of this study. It is commonly found in dynamic analyses that the most tenuous step an analyst can make is the assumption of one damping mechanism over another. An inappropriate damping mechanism may change the character of the response, as well as the amplitudes and consequently the correlation between simulation and reality dramatically. This issue is further complicated in this study, since the system is non-linear, and it is proposed that the mechanism leading to whip is related to the parametric stability of the system, which is known to be sensitive to damping effort. In fact, although viscous damping stabilises regions of simple parametric resonance, it can be shown to have the reverse effect on regions of combination parametric resonance (Nayfeh and Mook [1983]).

A numerical parametric study of damping would require substantial computational effort, which would be negated in the absence of an experimental research effort. For this reason, a relative proportional viscous damping mechanism is applied, where the aspect of the aerodynamic drag is left for future consideration.

Appendix I

Video Measurement System

Catenary vibration on mine hoist ropes has received attention in the past through computer simulation and analytical modelling. However, due to instrumentation difficulties associated with the rope transport speed, measurements quantifying vibration levels or frequency content have not been performed. This appendix describes the video measurement system which was developed to measure the vibration of mine rope catenaries in situ.

I.1 Video System

Motion analysis methods have been introduced in recent years, through the development of frame grabber cards and high resolution CCD cameras (750×750 lines). The principle of operation is simple; the motion of the object is recorded via the video camera and recorder onto video tape. The recorded image is replayed through the frame grabber card, frame by frame, where the image is digitised and the object is tracked via a numerical algorithm. The resolution of the system is determined by the resolution of the recorder and frame grabber card. The frequency bandwidth of the system is dependent on the framing speed of the video equipment used. Standard video equipment is capable of a framing speed of 25 frames per second, or a bandwidth of 12.5 Hz. It is possible to double the framing speed at the cost of a lower frame resolution, by utilising a time lapse format video recorder, which formats the even and odd fields sequentially. Thus the bandwidth is increased to 25 Hz, at the expense of the effective image resolution (approximately 300×300 lines). This approach was applied, as a bandwidth of 25 Hz was considered adequate. Apart from its passive nature, this form of instrument has significant advantages over many other

methods when low frequency measurements with minimal drift are required. It is at a disadvantage when one considers its relatively small bandwidth and the possibility of aliasing the motion, which cannot be overcome. Thus it is crucial that the response bandwidth lies within the system bandwidth of 25Hz. Since catenary vibrations on mine hoist ropes lie well within this bandwidth, the video measurement system is ideally suited to this task.

I.2 Rope Tracking

This application requires the tracking of a travelling rope in a plane normal to its axial direction. Zoom lenses are utilised to magnify the image to obtain the maximum resolution. The rope is viewed against the sky, and by adjusting the contrast the rope appears as a black bar on a white background. Since it is possible to electronically superimpose two images from different cameras, it is possible to track the 2D motion of the rope in a plane normal to its axis. The set-up configuration, and resulting video image after mixing the two signals is illustrated in figure I.1, I.2. The tracking programme locates the vertical and horizontal edge of the combined image, as illustrated in figure I.2. The pixel co-ordinates are stored on disk, and provide a direct measurement of the rope motion.

I.3 Hardware Configuration

The hardware configuration of the the system developed is listed below. The measurement and recording equipment required on site consists of two CCD cameras, a camera synchronisation circuit, a signal mixer, a time lapse recorder and a video monitor. The site test work revealed that the set up time and measurement process was short and required no interference to normal mining operations. The analysis phase required the time base corrector unit, which is necessary to enhance the quality of the video synchronisation signal from the video recorder, to the quality required by the frame grabber card. This unit also compensates for signal drop out, thus providing a high quality freeze capability. Once the vertical and horizontal edges of the rope had been located via the frame grabber and software, the video recorder was automatically advanced one frame. This measurement was then stored in a data file and the process was repeated. Typically 20 000 frames were processed for a complete winding cycle.

- (i) Two Pulnix TM-765 CCD high resolution black and white video cameras.
- (ii) 70-300mm Zoom lenses.
- (iii) Two channel video camera synchroniser.
- (iv) Two Channel Video mixer.
- (v) National Panasonic Time lapse Recorder AG-6720
- (vi) National Panasonic Time Base Corrector
- (vii) Data Translation DT2853 Frame grabber card.
- (viii) IBM compatible PC 386

I.4 Dual Camera Measurement

It was originally envisaged that the system would measure the two dimensional motion of the rope, by focussing each camera at a point on the rope, and surveying the geometrical location of the cameras and their angle of inclination from the horizontal plane. During an on site commissioning test, it became apparent that although accurate measurements may be possible in a laboratory environment, the achievement of accurate geometrical positioning on site is unlikely. This is mainly due to the fact that in order to focus both cameras at the same point on the rope, the rope would have to be marked, wound out and held stationary whilst the cameras were focussed. Consequently the winding process would be interrupted, leading to a loss in production. It was also clear that triangulation of the geometrical location of the cameras and rope would require accurate surveying. In addition to this, it was also necessary to monitor a series of winding cycles, so that optimum zoom adjustment of the cameras was achieved to maintain the largest image within the field of view, maximising the sensitivity of the system. Since the zoom on each camera could be adjusted independently, a further variable would be introduced. Nevertheless, with these reservations in mind, it was found in fact that with little effort, the approximate two dimensional motion of the rope could be successfully obtained.

The two camera system was also applied to track the catenary motion and the lateral motion of the vertical rope at the shaft collar simultaneously. This measurement provided evidence of the autoparametric excitation of the vertical rope by the catenary.

As a result, the measurement system was applied to ascertain the real time motion, and hence the frequency content of the motion, as a function of shaft depth, based on a single or dual camera measurement. This information could then be employed to identify the natural frequencies of the catenary, and the location of maximum modal response relative to shaft depth.

I.5 Commissioning Tests

During the commissioning tests, a total of three test measurements were performed at Elandsrand Gold Mine. The purpose of the measurements was primarily directed at commissioning and evaluating the performance and versatility of the video system. The first two tests were performed on the Siemens Rock Vent Shaft catenaries, whilst due to availability problems, the third test was performed on one of the adjacent rock winders. The test configurations are listed below.

I.5.1 Measurements Performed

- (i) Test 1 - This consisted of simultaneously measuring the overlay and underlay motion of the catenaries on the Siemens RV winder. The cameras were positioned vertically below each catenary.
- (ii) Test 2 - This consisted of focussing one camera on the underlay catenary of the Siemens RV winder, whilst the other camera was focussed on its corresponding vertical rope at the shaft collar.
- (iii) Test 3 - The purpose of this test was to evaluate the feasibility of monitoring the 2D motion of the rope by locating each camera at a base distance from the vertical plane of the catenary, and aiming the cameras at the same portion of rope, as illustrated in figure 1. The cameras were inclined at approximately 40° from the horizontal plane. The test was performed on an adjacent rock winder as the Siemens RV winder had stopped winding due to a bin blockage.

I.5.2 Measurement Results

The results of the measurements for each test are presented graphically in figures I.3, I.4, I.7. These figures present the displacement time traces, which