Appendix M

SAIDE Open Educational Resources Project

Teaching and Learning Mathematics in Diverse Classrooms

PILOT VERSION

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Draft pilot edition, 2007: Teaching and Learning Mathematics in Diverse Classrooms

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Introduction to the module

This is the first unit of a six unit module entitled *Teaching and Learning Mathematics in Diverse Classrooms.*

The module is intended as a guide to teaching mathematics for in-service teachers in primary schools. It is informed by the inclusive education policy (Education White Paper 6 Special Needs Education, 2001) and supports teachers in dealing with the diversity of learners in South African classrooms.

In order to teach mathematics in South Africa today, teachers need an awareness of where we (the teachers and the learners) have come from as well as where we are going. Key questions are:

Where will the journey of mathematics education take our learners? How can we help them?

To help learners, we need to be able to answer a few key questions:

- What is mathematics? What is mathematics learning and teaching in South Africa about today?
- How does mathematical learning take place?
- How can we teach mathematics effectively, particularly in diverse classrooms?
- What is 'basic' in mathematics? What is the fundamental mathematical knowledge that all learners need, irrespective of the level of mathematics learning they will ultimately achieve?
- How do we assess mathematics learning most effectively?

These questions are important for all learning and teaching, but particularly for learning and teaching mathematics in diverse classrooms. In terms of the policy on inclusive education, all learners – whatever their barriers to learning or their particular circumstances in life – must learn mathematics.

The module is divided into six units, each of which addresses the above questions from a different perspective. Although the units can be studied separately, they should be read together to provide comprehensive guidance in answering the above questions.

Unit 1: Exploring what it means to 'do' mathematics

This unit gives a historical background to mathematics education in South Africa, to outcomes-based education and to the national curriculum statement for mathematics. The traditional approach to teaching mathematics is then contrasted with an approach to teaching mathematics that focuses on 'doing' mathematics, and mathematics as a science of pattern and order, in which learners actively explore mathematical ideas in a conducive classroom environment.

Unit 2: Developing understanding in mathematics

In this unit, the theoretical basis for teaching mathematics – constructivism – is explored. A variety of teaching strategies based on constructivist understandings of how learning best takes place are described.

Unit 3: Teaching through problem solving

In this unit, the shift from the rule-based, teaching by telling approach to a problem-solving approach to mathematics teaching is explained and illustrated with numerous mathematics examples.

Unit 4: Planning in the problem-based classroom

In addition to outlining a step-by-step approach for a problem-based lesson, this unit looks at the role of group work and co-operative learning in the mathematics class, as well as the role of drill and practice in problem-based mathematics classes.

Unit 5: Building assessment into teaching and learning

This unit explores outcomes-based assessment of mathematics in terms of five main questions – Why assess? (the purposes of assessment); What to assess? (achievement of outcomes, but also understanding, reasoning and problem-solving ability); How to assess? (methods, tools and techniques); How to interpret the results of assessment? (the importance of criteria and rubrics for outcomes-based assessment) ; and How to report on assessment? (developing meaningful report cards).

Unit 6: Teaching all children mathematics

This unit explores the implications of the fundamental assumption in this module – that ALL children can learn mathematics, whatever their background or language or sex, and regardless of learning disabilities they may have. It gives practical guidance on how teachers can adapt their lessons according to the specific needs of their learners.

Process of developing the module

The units in this module were adapted from a module entitled *Learning and Teaching of Intermediate and Senior Mathematics*, produced in 2006 as one of the study guide for UNISA's Advanced Certificate in Education programme. The original guide was based on the following textbook:

Van de Walle, JA (2004). *Elementary and middle school mathematics: teaching developmentally.* New Jersey: Pearson Education.

A team of mathematics educators collaborated in the adaptation of the module so that issues related to inclusive education (the teaching of diverse learners), as well as a more representative selection of 'basic' mathematical knowledge could be included.

The team of mathematics educators consisted of the following:

- Constance Babane (University of Limpopo)
- Sam Kaheru / Nicholas Muthambi (University of Venda)
- Norman Khwanda (Central University of Technology)
- Marinda van Zyl / Lonnie King (Nelson Mandela Metropolitan University)
- Sharon Mc Auliffe / Edward Chantler / Esmee Schmitt (Cape Peninsula University of Technology)
- Ronel Paulsen / Barbara Posthuma (University of South Africa)
- Tom Penlington (Rumep at Rhodes University)
- Thelma Rosenberg (University of KwaZulu-Natal)
- Ingrid Sapire (Radmaste at University of the Witwatersrand)

Permissions

Permission has been granted from UNISA to adapt the following study guide for this module:

UNISA (2006). *Learning and teaching of Intermediate and Senior Mathematics (ACE ME1-C)* (Pretoria, UNISA)

Permission has also been sought for the additional materials included in the various units specified below.

Unit 1

RADMASTE Centre, University of the Witwatersrand (2006). Chapters 1 and 2, *Mathematical Reasoning* (EDUC 263).

UNISA (2006). Study Units 1 and 2 of *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern* (EDUC 264).

Stoker, J. (2001). *Patterns and Functions*. ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.

Unit 2

UNISA (2006). Study Unit 3: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

Penlington, T. (2000). *The four basic operations.* ACE Lecture Notes. RUMEP, Rhodes University, Grahamstown.

RADMASTE Centre, University of the Witwatersrand (2006). *Number Algebra and Pattern* (EDUC 264).

Unit 3

UNISA (2006). Study Unit 4: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

Malati (1999). Geometry activities: Geometry Module 3: Representations (nets, models and cross sections), Grades 4 to 7 Learner Materials.

RADMASTE Centre, University of the Witwatersrand. (2006). Mathematical Reasoning (EDUC 263) Chapter 7.

Unit 4

UNISA (2006). Study Unit 5: *Learning and Teaching of Intermediate and Senior Phase Mathematics.*

RADMASTE Centre, University of the Witwatersrand (2006). *Mathematical Reasoning* (EDUC 263) Chapter 6.

Malati (1999). Geometry Module 3: Representations (nets, models and cross sections). Grades 4 to 7 Learner Materials.

Unit 5

UNISA (2006). Study Units 7 to 10: Learning and Teaching of Intermediate and Senior Phase Mathematics.

MM French (1979). Tutorials for Teachers in Training Book 7, SIZE, Oxford University Press, Cape Town.

RADMASTE Centre, University of the Witwatersrand (2005). Data Handling and Probability (EDUC 187) Chapters 3, 8 and 9.

Unit 6

UNISA (2006). Study Unit 6: Learning and Teaching of Intermediate and Senior Phase Mathematics.

University of the Witwatersrand (2006). Module 3 of the Advanced Certificate for Learner with Special Education Needs: Understanding Cognitive, Emotional and Motivational Differences in Development.

Department of Education (2005). Guidelines for Inclusive Learning Programmes. http://curriculum.wcape.school.za/resource_files/20091831_Guidelines_for_Curriculum_June_2005.doc.

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Contents

Unit One: Exploring what it means to ‘do’ mathematics

After working through this unit you should be able to:

- Critically discuss the thinking that the traditional approach to teaching mathematics rewards the learning of rules, but offers little opportunity to do mathematics.
- Explain the phrase ‘mathematics is a science of pattern and order’.
- Evaluate a collection of science verbs (i.e. action words) that are used to reflect the kind of activities engaged by the learners when doing mathematics.
- Construct a list of features of a classroom environment considered as important for learners engaged in doing mathematics.
- Formulate appropriate and interesting activities to help learners explore the process of problem-solving through number patterns and logical reasoning.

1.1 An introduction to mathematics education

The word ‘mathematics‘ comes from the Greek word *máthema* which means ‘science, knowledge, or learning’; the word *mathematikós* means ‘fond of learning’. Today, the term refers to a specific body of knowledge and involves the study of quantity, structure, space and change.

Mathematics education is the study of the practices and methods of teaching and learning mathematics. Not only does the term *mathematics education* refer to the practices in classrooms, but it also refers to an academic discipline.

Before we get into what it means to ‘do’ mathematics, let us have a brief look at the historical background to mathematics education. There have been so many changes in the curriculum and assessment in recent years it is important to understand the context in which there has been the need for these changes.

The history of mathematics education

Mathematics is not a new discipline - it has been around for centuries. Elementary mathematics was part of the education system in most ancient civilisations, including Ancient Greece, the Roman Empire, Vedic society and ancient Egypt. At this time a formal education was usually only available to male children having wealth and status.

In the times of Ancient Greece and medieval Europe the mathematical fields of arithmetic and geometry were considered to be 'liberal arts' subjects. During these times apprentices to trades such as masons, merchants and money-lenders could expect to learn practical mathematics relevant to their professions.

During the Renaissance in Europe mathematics was not considered to be a serious academic discipline because it was strongly associated with people involved in trade and commerce. Although mathematics continued to be taught in European universities, philosophy was considered to be a more important area of study than mathematics. This perception changed in the seventeenth century when mathematics departments were established at many universities in England and Scotland.

In the eighteenth and nineteenth centuries the industrial revolution led to an enormous increase in urban populations, and so basic numeracy skills, such as the ability to tell the time, count money and carry out simple arithmetic, became essential in this new urban lifestyle. This meant that the study of mathematics became a standard part of the school curriculum from an early age.

By the twentieth century mathematics was part of the core curriculum in all developed countries. However, diverse and changing ideas about the purpose of mathematical education led to little overall consistency in the content or methods that were adopted. At different times and in different cultures and countries, mathematical education has attempted to achieve a variety of different objectives. At one time or other these objectives have included the teaching of:

- basic numeracy skills to all school pupils
- practical mathematics to most pupils, to equip them to follow a trade or craft
- abstract mathematical concepts (such as set theory and functions)
- selected areas of mathematics (such as Euclidean geometry or calculus)
- advanced mathematics to learners wanting to follow a career in mathematics or science

Mathematics education in South Africa today

At school level mathematics is often viewed as empowering, and as a means of access to further education, and is offered at all grade levels. However, the level of success in mathematics education in South African schools is very low. In the 1998/99 repeat of the Third International Mathematics and Science Study (TIMSS – R), that was written by Grade 8 learners, South Africa was ranked last of the 38 nations who participated in the study for mathematics. This study included other developing countries. The South African learners scored the lowest across all five topics in mathematics. In the 2003 TIMSS study, South Africa was ranked last of 46 participating nations. This poor performance shows that the majority of South African learners in Grade 8 have not acquired basic knowledge about mathematics and lack the understanding of mathematical concepts expected at that level. This situation is compounded by a huge drop-out rate amongst learners and the fact that many mathematics teachers are not adequately qualified to teach the subject.

Outcomes Based Education (OBE) was introduced into South African schools in 1994. The curriculum changes brought about by OBE are currently being implemented in the FET band in schools nationally. This means that schools are moving out of the old system where mathematics in secondary schools is offered at two levels, namely Higher Grade and Standard Grade. At present many schools only offer Standard Grade mathematics while some schools do not offer mathematics at all. The new curriculum was implemented in grade 10 in 2006, and the first grade 12's to write new curriculum exams will write in 2008. Within the structure of the OBE curriculum all schools now have to offer all learners mathematics up to Grade 12. The choice for learners will be between mathematics and mathematical literacy. Mathematics will suit those learners who wish to further their education in fields which require certain mathematical knowledge. Mathematical literacy provides an alternative which will equip learners with a more contextualised knowledge of mathematics related functions performed in everyday life.

The performance data for the Senior Certificate examination in 2003 showed that:

- Less than 60% of all candidates chose to do mathematics as an exam subject at either Standard Grade or Higher Grade.
- Approximately 35% of all candidates passed mathematics with only a small fraction of these passing on HG.

There are a number of reasons for South Africa's poor performance in mathematics.

South Africa is one of the most complex and heterogeneous countries in the world. Van der Horst and McDonald (1997) point out educational problems which all contribute to the current crisis in education in South Africa. Some of these problems include:

- the challenge of providing equal access to schools
- the challenge of providing equal educational opportunities
- irrelevant curricula
- inadequate finance and facilities
- shortages of educational materials
- the enrolment explosion
- inadequately qualified teaching staff.

These problems imply that change is needed in the South African educational system.

Why is educational change needed in South Africa?

According to Van der Horst and McDonald (1997), as a result of the divisions which existed during the apartheid era, learners were not always taught to appreciate the different aspirations and perspectives of people who were different, and many did not receive adequate educational and training opportunities during the previous era.

This disadvantaged them greatly. This means that educational change must provide equity in terms of educational provision and promote a more balanced view, by developing learners' critical thinking powers and problem-solving abilities.

There is a need for a people-centred, success-oriented curriculum that will grant people the opportunity to develop their potential to the full. The new curriculum in South Africa attempts to adequately cater for these needs. The philosophy that underpins the new curriculum is that of Outcomes Based Education (OBE).

What is outcomes-based education (OBE)?

OBE is a learner-centred, results-oriented approach to learning which is based on the following beliefs:

- All individual learners must be allowed to learn to their full potential.
- Success breeds further success. Positive and constructive ongoing assessment is essential in this regard.
- The learning environment is responsible for creating and controlling the conditions under which learners can succeed. The atmosphere must be positive and learning is active.
- All the different stakeholders in education such as the community, teachers, learners and parents share in the responsibility for learning.

This approach proposes a shift away from a content-based, exam-driven approach to schooling. Instead learners are required to achieve specific learning outcomes for different phases within each subject. Learner centred activities form an integral part of the new curriculum and the emphasis is on encouraging learners to be instruments of their own learning, whether they work individually or in groups.

Outcomes-based education can be described as an approach which requires teachers and learners to focus their attention on two things (Van der Horst, McDonald, 1999):

1. The desired end results of each learning process. These desired end results are called the outcomes of learning and learners need to demonstrate that they have attained them. They will therefore be continuously assessed to ascertain whether they are making any progress.
2. The instructive and learning processes that will guide the learners to these end results.

Activity 1

In the light of what you have read so far, reflect on your mathematics teaching and write answers to the following questions:

1) Write about your experiences as a teacher of mathematics. You should write about a page, describing at least one good experience and one bad experience.
2) Write a short paragraph saying why you have chosen to do an ACE in mathematics education.
3) Have you been aware of the importance of mathematics a subject that can empower the learners in your classes? If yes, say how you have tried to help them be empowered. If no, say how you might encourage them in the future.

4) Write a short paragraph describing what you know about the new NCS for GET or FET curriculum, depending on the level at which you are teaching.
   a) What other course/s have you attended that relate to the new mathematics curricula?
   b) Do you have a copy of any of these curricula? If yes, have you read it?
   c) Do the other teachers at your school who teach mathematics know about the new curricula?

What is mathematics? People’s views

Most people acknowledge that mathematics is an important subject at school. However very few really understand what mathematics is about and what it means to ‘do’ mathematics. People often define mathematics as being about a collection of ‘rules’: arithmetic computations, mysterious algebraic equations or geometric proofs that need to be learnt in order to pass an examination. In general, people tend to feel that they are ‘no good at mathematics and that it is difficult’.

Such people believe that:

- Mathematics requires a good memory
- Mathematics is based on memorisation of facts, rules, formulas and procedures
- You have to have a special brain to do mathematics
- Mathematics is not creative
- There is a best way to do a mathematics problem
- Every mathematics problem has only one correct answer and the goal is to find THE answer
- Mathematics problems are meant to be solved as quickly as possible
- Mathematics is all symbols and no words
- Boys are better at mathematics than girls
- School mathematics is useless
- Mathematics is exact and there is no room for innovation, estimation or intuition.

Much of this restricted (even negative) view of mathematics stems from very traditional approaches to the teaching of mathematics. In such traditional teaching, the teacher ‘tells’ or explains a mathematical concept or idea to learners. The teachers ‘tells’ the learners how to ‘use’ a mathematical idea in a certain way in order to get a correct answer. The learners then practice the method and rely upon the teacher to tell them the correct answers. This way of teaching produces a follow-the-rules, computation-driven, answer-oriented view of mathematics. Learners exposed to this way of teaching accept that every problem has only one solution and that they cannot solve a problem without being told a ‘solution method’ before hand. The ‘rules’ seldom seem to make sense and there is little excitement in lessons, particularly if you cannot remember the rule!

An alternative view of mathematics is that it involves 'making sense' of mathematical ideas, patterns and information. That it involves 'figuring out' how to approach problems; about finding and exploring regularity in patterns and making sense of relationships; about finding patterns and order all around us, for example in art, in buildings and in music. This is the view taken by the new mathematics curriculum in South Africa.

How does the new mathematics curriculum define mathematics?

The introduction to the National Curriculum Statement (NCS) for GET and FET mathematics defines mathematics based on certain characteristics of the discipline:

- Mathematics enables **creative and logical reasoning about problems** in the physical and social world and in the context of mathematics itself.
- It is a distinctly **human activity practised by all cultures**.
- Knowledge in the mathematical sciences is constructed through the **establishment of descriptive, numerical and symbolic relationships.**
- Mathematics is based on **observing patterns** which with rigorous logical thinking, leads to theories of abstract relations
- Mathematical **problem solving** enables us to understand the world and make use of that understanding in our daily lives.
- Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.

This means that mathematics:

- Is something that people create or invent. They can discuss and argue about their creation and in the process reach a shared understanding of what it is;
- Enables creative thinking;
- Enables logical thinking;
- Is based on patterns that lead to abstract ideas;
- Helps us solve problems in the real world;
- Helps us solve problems in the world of mathematics;
- Helps us better understand the world;
- Uses symbols and language to develop reasoning skills and to make meaning;
- Is always changing because the shared understanding reached may be overthrown or extended as the development of mathematics continues.

The NCS goes on to say how mathematics can help and empower learners and describes how the NCS can enable learners to achieve the critical and developmental outcomes for the discipline. It describes how the teacher and learners can 'do' mathematics in the classroom.

Activity 2

1) How do you experience OBE in your classroom practice? Reflect on your own practices and explain how you apply the above principles/guidelines:
   a) Are we developing learners who are confident and independent thinkers?
   b) Are we working at closing the gap between the classroom and real life in all its complexity?
   c) How would you know whether or not the outcomes have been achieved in a lesson?
2) As a teacher of mathematics, there must have been occasions when you have wondered 'What is mathematics?'
   a) Write a paragraph saying what **you** think mathematics is.
   b) Write a paragraph saying what you think mathematicians **do**.
   c) Ask three people in your community (a learner, a teacher who doesn't teach mathematics and another mathematics teacher) what they would answer to a) and b) above. Write their answers down.
   d) Write a paragraph describing the similarities and differences between the responses you received in c).

1.2 What does it mean to 'do' mathematics?

How would you describe what you are doing when you are doing mathematics? In the rest of this chapter we are going to explore what it means to 'do' mathematics. It is fine to come to this point with whatever beliefs were developed from your previous mathematics experiences. However, Van de Walle (2004) emphasizes that it is not fine to accept outdated ideas about mathematics and expect to be a quality teacher.

Fasheh's (1982) description of the teaching of mathematics is very familiar to many learners:

- The classroom is highly organised.
- The syllabus is rigid.
- The textbooks are smartly fixed.
- Mathematics is considered as a science that does not make mistakes. There is one correct answer to every question and one meaning to every word and that measuring is fixed for all people and for all times.
- Direct instruction remains the dominating mode of teaching in mathematics.

In this approach to teaching the learners are not 'doing' mathematics. No wonder many learners find mathematics a dull and unstimulating subject.

Activity 3: Your classroom experience

1) Write down at least five ideas/viewpoints about what it means to do mathematics. You should use only one minute to do this activity. Keep your list somewhere safe so that you can refer to it at a later date.
2) Reflect on your classroom practice in the light of the following questions. Is there a tendency in your mathematics teaching:
   - to reward only formal knowledge?
   - to memorize rules and procedures?
   - to spoon feed (facts, rules, procedure)?
   - to devalue the learners' own way of making sense out of their own experiences, intuition and insight?
   - for learners to remain passive listeners?

As you work through the rest of this study unit, you will be challenged to rethink and reconstruct your own understanding of what it means to know and do mathematics – so that learners with whom you work will have an exciting and more accurate vision of mathematics. Doing mathematics (mathematisation) will be eventful, compelling and creative.

Teachers need to have ideas about how to structure classrooms so that they can help learners develop. Since experience is a powerful teacher, it makes sense for learners to experience mathematical ways of thinking, reasoning, analyzing, abstracting, generalizing-; all modelled on good instruction and doing mathematics (Evan & Lappan: 1994). Therein lies the challenge for South African mathematics teachers.

Contrasting perceptions of school mathematics

More teachers in South Africa are being initiated to Outcomes-based Education (OBE). In the process they are moving towards more co-operative learning, and place a greater emphasis on conceptual learning and problem solving. There is also a greater tolerance for and use of calculators. Often these changes do not go far enough – they are superficial and do not change the nature of what learners do and how they think in the mathematics classroom.

Traditional views of teaching mathematics

Outcomes-based Education seeks to replace the 'traditional' negative image – a fear of mathematics, the spoon-feeding syndrome in teaching it and the assumption of inability to excel in mathematics – with a positive outlook to mathematics as something that is within the reach of all learners and part of the human experience.

As van de Walle (2004: 12) points out, for many:

> mathematics is perceived as a collection of rules to be mastered, arithmetic calculations and algorithms, mysterious algebraic equations and mind-shattering geometric proofs.

The typical traditional approach to teaching mathematics goes something like this:

Mathematics lessons begin with the teacher telling the learners a fact, or giving them the steps in an algorithm or describing a prevalent concept (if at all possible). The teacher then works a textbook example and assigns learners to work exercises from the textbook to help them remember the fact or the process. There are no experiences through which learners discover, invent, or apply mathematics to problems they find meaningful.

This way of teaching mathematics has fostered perspective of mathematics as a collection of mysterious procedures and rules. Even with a hands-on activity, the traditional teacher is still guiding learners, telling them exactly how to use the materials in a systematically prescribed manner. The focus of the lesson is primarily on getting answers, and learners depend on the teacher to determine if their answers are correct. The learners begin to accept that every problem must have a ready and predetermined answer, and that there is only one unique way to solve a problem. This establishes a false perception that they are not expected to solve a problem unless they have been given a solution method (or procedure) ahead of time.

This follow-the-rules, computation-dominated, answer-oriented view of mathematics results in a gross distortion of what mathematics is really about. Apart from the fact that it is not very exciting, only a few learners are good at learning rules and thrive on performing procedures. These are not necessarily the best thinkers in the classroom. The traditional system rewards the learning of rules but offers little opportunity actually for learners to do mathematics.

Activity 4: The traditional approach: three simple examples

Three simple 'problems' (A, B, C) are given to you below (at the Intermediate Phase Level).

Work through these 'problems'.

<u>Problem A:</u>

In servicing a car the attendant used 45ℓ costing petrol at R4,25/ℓ and 2 tins of oil at R8.50 each.

What was the total cost for petrol and oil?

<u>Problem B:</u>

The world's record for the high jump in a recent year was 1.87 metres.

On Mars, this jump would be 2½ times as high. How much higher in metres will it be?

<u>Problem C:</u>

John covers ½ of a journey by car**,** ⅓ of the journey by bicycle, and walks the rest of the way.

a) What part of the journey does he cover by car and bicycle?

b) What part of the journey does he walk?

Indicate by means of a tick (✓) in the blocks given below which of the mathematical come to the fore for learners attempting each of these 'problems'.

Skills acquired by learners	Problem A	Problem B	Problem C
Computational skills			
Using correct order of operations			
Formulating expressions as a mathematical model			
Estimation to assess reasonableness of answer			
Problem-solving thinking skills			
Investigatory skills			
Exploration of rules and logical thinking			
Self-discovery			

Now do the following.

1) Indicate whether the three problems (A, B, C) are typical of the traditional way of questioning learners in S.A.
2) Refer to the skills mentioned above and explain your response briefly.
3) What skills should we teach our learners now in order to prepare them to cope with the 21st century?
4) Make recommendations to your colleagues on the skills we should teach in mathematics.

Mathematics as a science of pattern and order

Van de Walle (2004: 13) uses a wonderfully simple description of mathematics found in the publication 'Everybody counts' (MSEB :1989):

> Mathematics is the science of pattern and order.

This immediately dispels the popular social view that mathematics is dominated by computation and rules that learners need not understand but must apply rigorously. As a science, mathematics is a process of figuring things out (formulating number patterns, investigating, exploring, conjecturing, generalising, inducing, deducing, etc.) or in general, making sense of things.

Van de Walle (2004: 13) continues :

> Mathematics is, therefore, a science of things that have a pattern of regularity and logical order. Finding and exploring this regularity or order and then making sense of it is what doing mathematics is all about.

The traditional view emphasizes procedures and the solving of routine problems, with teachers showing and telling while learners listen and repeat.

The more progressive view is that of the learning of mathematics as a process (irrespective of the content material), emphasising meaningful development of concepts and generalisations, increasing the prospects of real problem-solving, open enquiry and investigation – characterised mainly by teachers challenging, questioning and guiding, with students doing, discovering and applying.

If learners do not understand how things work – cannot see the pattern and order – they often make computational errors. Some examples of this are given in the next activity.

Activity 5: Computational errors and misconceptions

Learners make computational errors as a result of a lack of understanding of how things work. Here are a few examples:

Computational/notational/conceptual errors

(i) $\frac{18}{85} = \frac{1}{5}$ $\frac{1\cancel{8}}{\cancel{8}5} = \frac{1}{5}$ [Learner cancels the 8's]

(ii) $\begin{array}{r} 625 \\ \underline{+\ 25} \\ 875 \end{array}$ [The learner does not understand the concept of place value].

(iii) 0,234 is **bigger than** 0,85 [Since 234 is bigger than 85]

(iv) 3 (4×5) = (3×4) (3×5) [Since 3 (4+5) = 3×4 + 3×5]

(v) 8 ÷ ½ = 4 [Since 8 ÷ 2 = 4 or 8 × ½ = 4]

(vi) $a \times a = 2a$ [Since $a + a = 2a$]

(vii) Half of 8 = 3 [Since half of the figure 8 is 3, if you cut an 8 in half vertically with

a

pair of scissors]

and so on

1) Write down from your own experience a few more examples in which learners make computational errors as a result of a lack of understanding (misconceptions) of how procedures or rules actually work. Consult with other teachers of mathematics.
2) Discuss this with your colleagues and explore solutions to these problems.
3) Suggest the teaching strategies an innovative teacher could use to obviate such misconceptions from developing in learners.

Note: We encourage you to discuss this with your colleagues – we can learn so much from each other.

‘Doing’ mathematics

Engaging in the science of pattern and order requires a good deal of effort and often takes time. The next activity will illustrate this.

Try out the activity, if you can, with a senior phase class.

Activity 6*:* Problem-solving: number pattern activity

(*Ten cities in South Africa need to be directly connected to all other cities by a telephone line. How many direct connections are needed?* (Paling & Warde: 1985).

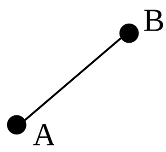
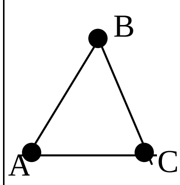
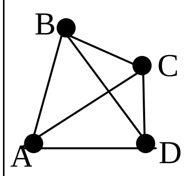
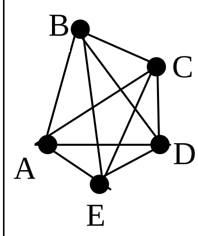
One approach would be to follow the three steps given below – but you are at liberty to use any other problem-solving techniques.

UNDERSTAND THE PROBLEM: e.g. three or more cities are not situated in a straight line. What do we need to do about it?

DEVISE A PLAN: Reduce the problem to simpler terms –start with one city, and then two cities, three cities and so on.

CARRY OUT THE PLAN: Use drawings and write dawn a sequence to establish the pattern, formulate conjectures, test conjectures and generalise.

Using drawings:

Number of cities	1	2	3	4	5
Drawing					
Number of connections	0	1	3	6	10
Establishing a pattern (rule)	0	0 + 1	0 + 1 + 2	0 + 1 + 2 + 3	0 + 1 + 2 + 3 + 4
	$\frac{1(1-1)}{2}=0$	$\frac{2(2-1)}{2}=1$	$\frac{3(3-1)}{2}=3$	$\frac{4(4-1)}{2}=6$	$\frac{5(5-1)}{2}=10$

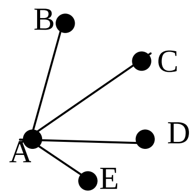
Use the above reasoning to find the total number of connections of 6 cities, 7 cities..., 10 cities. (Use drawings and test your conjecture/rules).

EVALUATE AND EXTEND THE PLAN FOR *n* CITIES: If there are *n* cities how many connections will there be?

Let us use a simpler example again.

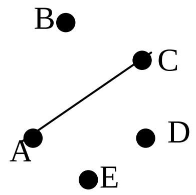
For 5 cities

Each city will be connected to 4 other cities i.e. (5 – 1) = 4



There will be five such cases i.e. from A, B, C, D and E.

From this we have 5 (5 – 1) connections



The connections from A to C is the same as C to A

This is the same for each case. So divide by 2.

We therefore get $\frac{5(5-1)}{2} = 10$

Now write down the number of connections for *n* cities.

Does the process of 'doing' mathematics (mathematising):

- Provide a real problem-solving situation?
- Encourage enquiry, exploration and investigation of numbers?
- Stimulate the learning of regularity and order of numbers?
- Require the teacher guide and pose thought-provoking questions?
- Involve the learners in actively doing mathematics and discovering rules?

Perhaps you are wondering after working through that rather complex example, what mathematics teachers are supposed to do about basic skills? For example, you may be asking, don't learners need to count accurately, know the basic facts of addition, multiplication, subtraction and division of whole numbers, fractions, decimals and so on?

The fact is, that when we teach an algorithm in mathematics (like long multiplication) and then give learners exercises to do in their books, our learners are not 'doing' mathematics. This doesn't mean that teachers should not give learners this kind of exercise, which is simply drill-work, but that drill should never come before understanding.

Repetitive drill of the bits and pieces is not 'doing' mathematics and will never result in understanding. Only when learners are capable of making sense of things by 'doing' mathematics in the classroom, are they being truly empowered.

The verbs of doing mathematics

Farrell and Farmer in *Systematic Instructions in Mathematics* (1980) state that:

> Mathematics is a verb, as well as a noun

Two questions arise from this statement:

1) What do we do when we mathematise?

2) What do we obtain?

Mathematics is about processes (expressed in 'doing verbs') and it is also about products (expressed by nouns). In the table below a few examples are given.

PROCESSES OF MATHEMATICS	**PRODUCTS OF MATHEMATICS**
Generalising	Formula
Computing	Theorem
Assuming	Definition
Solving	Axiom
Proving	Corollary
Testing	Concepts (number etc.)

What verbs would you use to describe an activity in a classroom where learners are doing mathematics? Van de Walle (2004: 13) gives a collection of verbs that can be associated with doing mathematics:

explore	represent	explain
investigate	discover	justify
conjecture	develop	formulate
predict	solve	construct
justify	verify	use

Study these verbs carefully – they describe what action or behaviour is expected from the learners when doing the classroom activity. As Van de Walle notes (2004: 13):

> They are <u>science verbs,</u> that is, verbs indicating the process of <u>making sense and figuring things out</u>.. It is important to note that when learners are engaged in the kinds of activities suggested by the list, it is practically impossible for them to be passive observers and listeners. They will be <u>actively thinking</u> about the mathematical ideas that are involved.

Activity 8: The verbs of doing mathematics

1) Reflect on the collection of 'science verbs' above. Do these verbs clearly indicate the type of action required of the learner during the process of mathematising?
2) Study the NCS for a grade to which you are currently teaching mathematics.
   a) Are the Assessment Standards expressed in terms of the science verbs?
   b) Make a list of the verbs that you find in the Assessment Standards.
   c) Do these verbs clearly indicate the type of action required of the learner during the process of mathematising?
   d) Give an example of a mathematical activity that demonstrates the action involved in each of these verbs.

Now that you have more understanding of what it means to do mathematics and the processes involved, you might like to take another look at Activity 4.

Activity 9: Science verbs used to identify the process skills in mathematics

1) Work through Activity 6 again.
2) Use appropriate science verbs to write down three mathematical process skills that learners could actively acquire through this problem-solving activity.

   Begin with: The learners should be able to ……………

What is basic mathematics?

What is 'basic' in mathematics is always a matter of public discussion and debate. Is it

- mastering the basic operations?
- demonstrating achievement of the Learning Outcomes (in OBE)?
- becoming mathematically literate?
- recognising that mathematics is a part of human creative activity?

Van de Walle (2004: 14) presents a simple, clear and challenging position to this question.

> The most basic idea in mathematics is that mathematics makes sense!

The implications of this statement are:

- Every day learners must experience that mathematics makes sense.
- Learners must come to believe that they are capable of making sense of mathematics.
- Teachers must stop teaching by telling and start letting learners make sense of the mathematics they are learning.
- To this end, teachers must believe in their learners – all of them!

This is a profound challenge – mathematics is for everyone without exception. Everyone has to learn that mathematics makes sense and to make sense of mathematics:

Activity 10: What is basic mathematics?

Van de Walle (2004:15) says

> All learners are capable of learning all of the mathematics we want them to learn, and they can learn it in a meaningful manner that makes sense to them if they are given the opportunity to do so

1) Do you agree with Van de Walle's position on what he considers basic in mathematics? Do you find it realistic or revolutionary? Discuss your opinion with your colleagues.
2) What do you think Van de Walle would say about OBE and its Expanded Opportunity Assessment Strategy (projects, investigations, group-work, oral assessment, peer-assessing and so on)?
3) What do your learners think is basic in mathematics? Ask them, and then compare their ideas with yours.

The following list is what the course developers think is 'basic in mathematics' - fundamental mathematical knowledge for all learners. Essential skills were itemised from each of the LOs for Mathematics from the GET band.

1) Number concept and operations – learning outcome 1 (LO1)
   a) Place value.
   b) Operations (conceptual understanding giving rise to understanding of different strategies for all operations.)
2) Pattern (LO2)
   a) Core concepts – recognition of patterns, using words and drawings to describe and analyse patterns, generalise rules for patterns, generate other patterns
   b) Link working with patterns to everyday situations
3) Space and shape (LO 3)
   a) 2-D and 3-D perception. Up to Van Hiele first three levels: learners should be able to visualise, recognise and describe shapes.
   b) Look at properties of shapes. No calculations of area and volume, simply recognition of shapes, drawing, describing and naming of polygons and polyhedra.
4) Measurement (LO4)
   a) Concept formation using developmental activities and Piaget's conservation tests
   b) The process of quantifying - length, mass, capacity, volume, area, time

   c) The stages in the teaching of measurement

5) Data (LO5)
   a) Collecting, recording, ordering, presenting and analysing
   b) Awareness of statistics presented in everyday life.

This content will be found integrated into the guide, throughout the various units of the guide.

An environment for doing mathematics

It is the job of the teacher to ensure that every child learns to do mathematics, but for this there has to be the right environment.

An environment for doing mathematics is one in which learners are allowed to engage in investigative processes where they have the time and space to explore particular cases (problems). Then they can move slowly towards establishing through discovery and logical reasoning the underlying regularity and order (in the form of rules, principles, number patterns and so on).

Learners create 'a conjecturing atmosphere' in the classroom (James, Mathematics Education. 1992). This atmosphere is one in which the rightness or wrongness of answers is not the issue, but rather an environment which encourages learners to make conjectures (guesses) as to the regularity (sameness) they see and to discuss these conjectures with others without fear of being judged wrong or stupid, to listen to the ideas expressed by others and to modify their conjectures as a result.

The mathematical processes involved in doing mathematics are best expressed by the action verbs. They require reaching out, taking risks, testing ideas and expressing these ideas to others. (In the traditional classroom these verbs take the form of: listening, copying, memorising, drilling and repeating - passive activities with very little mental engagement, involving no risks and little initiative.)

The classroom must be an environment where every learner is respected regardless of his or her perceived 'cleverness', where learners can take risks without fear that they will be criticised if they make a mistake. It should be an environment in which learners work in groups, in pairs or individually, but are always sharing ideas and engaged in discussion.

The following activity illustrates how even the most apparently 'routine' problem can be tackled in a variety of ways.

Activity 11: Doing mathematics: informal methods

Reflect on the following informal strategies attempted by learners at an Intermediate Phase/Senior Phase and then answer the questions below:

MULTIPLICATION AS REPEATED ADDING

43×6:	$40 + 3$
	$40 + 3$
	$40 + 3$
	$40 + 3$
	$40 + 3$
	$\underline{40 + 3}$
	$240 + 18 = 258$

Adding speeded by doubling

43 x 6:	43	
	$\underline{43}$	
	86	(2x43)
	$\underline{86}$	
	172	(4x43)
	$\underline{86}$	
	258	(6x43)

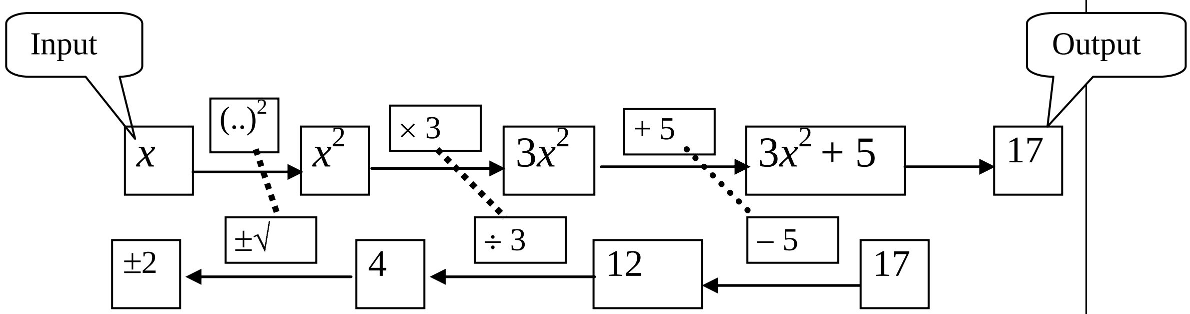
DIVISION AS REPEATED SUBTRACTION:

$564 \div 18$:	10×18	$= 180$
	10×18	$= \underline{180}$
		360
	10×18	$= 180$
		$\underline{540}$
	$\underline{1 \times 18}$	18
	31	556
Remainder:	564	
	$\underline{556}$	
	8	

$564 \div 18 = 31$ remainder 8

USING A REVERSE FLOW-DIAGRAM TO SOLVE AN EQUATION

Solve for x: $3x^2 + 5 = 17$



1) Reverse the flow diagram (see the dotted lines to indicate the inverse operations). Start with the output and apply inverse operations. What do you find?
2) Do you agree that the use of informal strategies where learners wrestle towards solutions is never a waste of time? Motivate your response.
3) Compare the above non-routine strategies with the recipe-type routine methods and explain which offer better opportunities for 'doing mathematics' discussions developing reasons, testing reasons and offering explanations.
4) Have you come across some interesting non-routine methods used by learners in a particular situation? If you have, describe some of these examples. You could also discuss them with your fellow mathematics teachers.

What qualities does the teacher need to create an environment in which learners feel safe and stimulated to 'do' mathematics? The following activity helps to describe these.

Activity 12: An environment for doing mathematics

Read through the following motivational dialogue between Mr Bright and Mr Spark, two mathematics teachers. Describe the features of a classroom environment which you consider as important for learners to be engaged in doing mathematics.

Mr Bright:	A teacher of 'doing' mathematics needs to be enthusiastic, committed and a master of his or her subject.
Mr Spark:	He or she needs to have a personal and easier feel for doing mathematics to create the right environment in the classroom.
Mr Bright:	Teachers should provide activities designed to provide learners with opportunities to engage in the science of pattern and order.
Mr Spark:	Yes, a real opportunity to do some mathematics!
Mr Bright:	We need to develop this technique and discover as much as we can in the process.
Mrs Spark:	Let us invite Mr Pattern and perhaps Mr Order and some of the other mathematics teachers.
Mr Bright:	Yes! We would all be actively and meaningfully engaged in doing mathematics and respect and listen to the ideas put forward by the others.
Mr Spark:	We shall challenge each other's ideas without belittling anyone.

1.3 Exploring pattern in mathematics

In this part of the unit, we invite you to explore patterns in mathematics through a number of different mathematics problems.

The ability to recognise patterns, use words and drawings to describe and analyse patterns, generalise rules for patterns, and generate other patterns is fundamental to understanding mathematics. Learners need to be able to describe patterns observed using words or mathematical symbols. It is important for learners to recognise patterns by looking for common differences. Learners also need to be able to extend numeric and geometric patterns, which helps them recognise a variety of relationships in the patterns and make connections between mathematical topics. Identifying patterns like repeat patterns and growing patterns helps learners become aware of the structures of various pattern types. Teachers need to show their learners how to make generalisations for patterns and understand these generalisations.

Doing activities involving patterns

Read carefully through each problem so that you are sure you understand what is required. None of these problems require any sophisticated mathematics, not even algebra. Don't be passive! Express your ideas. Get involved in doing mathematics!

Activity 13: Start and jump numbers, searching for numbers (adapted from Van de Walle)

You will need to make a list of numbers that begin with a 'start number' and increase it by a fixed amount which we will call the 'jump number'. First try 3 as the start number and 5 as a jump number. Write the start number first and then 8, 13 and so on 'jumping' by 5 each time until your list extends to about 130.

Your task is to examine this list of numbers and find as many patterns as you possibly can. Share your ideas with the group, and write down every pattern you agree really is a pattern.

Here are some suggestions to guide you:

- Look for alternating patterns
- Look for repeating patterns
- Investigate odd and even numbers
- What is the pattern in the units place?
- What is the pattern in the tens place?
- What happens when you go over 100?
- What happens when you add the digits in the numbers?
- Extend the pattern where you see a question mark (?) in the table below

Find the following number patterns (in the columns):

Separate the consecutive terms		Add terms (in columns 1 and 2)	Subtract terms (in columns 1 and 2)	Multiply terms (in columns 1 and 2)
3	8	11	5 (or -5)	24
13	18	31	5	234
23	28	51	5	644
33	38	71	5	1254
43	48	91	5	2064
53	58	111	5	3074
63	68	131	5	?
73	78	151	5	?
?	?	?	?	?
?	?	?	?	?
?	?	?	?	?
?	?	?	?	?
?	?	?	?	?
?	?	?	?	?

Number patterns jump	Add The Digits (from column 1)	Subtract The Digits (from column 1)
3	0 + 3 = 3	3 – 0 = 3
8	0 + 8 = 8	8 – 0 = 8
13	1 + 3 = 4	3 – 1 = 2
18	9	8 – 1 = 7
23	5	3 – 2 = 1
28	10	?
33	6	?
?	?	?
?	?	?
?	?	?
?	?	?
?	?	?
?	?	?
?	?	?

Activity 14: Two machines, one job (adapted from Van de Walle)

Ron's Recycle Shop was started when Ron bought a used paper-shredding machine. Business was good, so Ron bought a new shredding machine. The old machine could shred a truckload of paper in 4 hours. The new machine could shred the same truckload in only 2 hours. How long will it take to shred a truckload of paper if Ron runs both shredders at the same time?

Make a serious attempt to figure out a solution. (You could use drawings or counters, coins and so on). If you get stuck consider:

- Are you overlooking any assumptions made in the problem?
- Do the machines run at the same time?
- Do they run as fast when working together as when they work alone?
- Does it work to find the average here? Explain your answer.
- Does it work to use ratio and proportion here? Explain your answer.

Activity 15: Exploration – product of numbers

Young children learning about their basic number facts could be confronted with the following observation: In a sum, when you make the first number one more and the second number one less, you still get the same answer. For example:

$7 + 7 = 14$ and $8 + 6 = 14$

$5 + 5 = 10$ and $6 + 4 = 10$.

What can you find out about this?

Your task here is to examine what happens when you change addition to multiplication in this exploration. Consider the following examples:

A	B
$7 \times 7 = 49$	$8 \times 6 = 48$
$8 \times 8 = 64$	$9 \times 7 = 63$
$9 \times 9 = 91$	$10 \times 8 = 80$
$10 \times 10 = 100$	$11 \times 9 = 99$
.	.
.	.
.	.

What happens to the product when you increase the first number by 1 and decrease the second number by 1 in column A? Compare the products in column B and identify the pattern. State the pattern in your own words.

How do these results differ when the two factors are 1 apart?

Example:

A	B
$6 \times 7 = 42$	$7 \times 6 = 42$
$7 \times 8 = 56$	$8 \times 7 = 56$
$8 \times 9 = 72$	$9 \times 8 = 72$
.	.

Compare the products in column A to the products in column B and identify the change (if any).

How do these results differ when the two factors are 2 apart or 3 apart?

Example: (the numbers are two apart)

A	B
$6 \times 4 = 24$	$7 \times 3 = 21$
$9 \times 7 = 63$	$10 \times 6 = 60$
$10 \times 8 = 80$	$11 \times 7 = 77$
.	.
.	.
.	.

How do the results differ from the results above?

What if you adjust the factors up and down by 4?

Examples:

A	B
$7 \times 3 = 21$	$8 \times 2 = 16$
$9 \times 5 = 45$	$10 \times 4 = 40$
.	.
.	.
.	.

Find the difference in the products between column A and column B.

Does it make any difference to the results if you use big numbers instead of small ones?

The next activity is about the chances (probability) of getting the colour purple when spinning on two colour wheels. Probability is new in the South African curriculum. It has been introduced as part of the statistics (Data Handling and Probability) curriculum in the GET and FET bands. Weather forecasts give us the daily probability of rain, an example of how we are exposed to the idea of probability on a daily basis. This activity exposes you to some methods for working out probabilities.

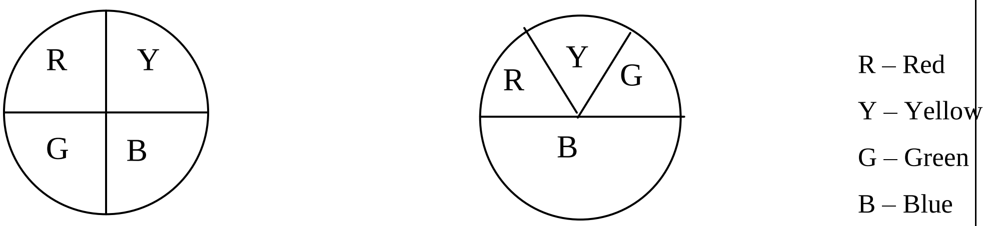
Activity 16: The highest probability of getting purple (adapted from Van De Walle)

Three students are spinning to 'get purple' with two spinners (either by spinning first red and then blue or first blue and then red). They may choose to spin each spinner once or one of the spinners twice. Mathakga chooses to spin twice on spinner A; Jabu chooses to spin twice on spinner B and Sam chooses to spin first on spinner A and then on spinner B. Who has the best chance of getting a red and a blue? (If you mix red and blue paint you get purple paint. This explains why they want to spin red and blue.)

Mathakga chooses to spin A twice.

Two approaches to evaluating her chances of getting both red and blue are investigated.

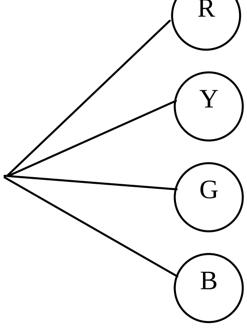
1) Using Tree Diagrams:



Spinner A

Spinner B

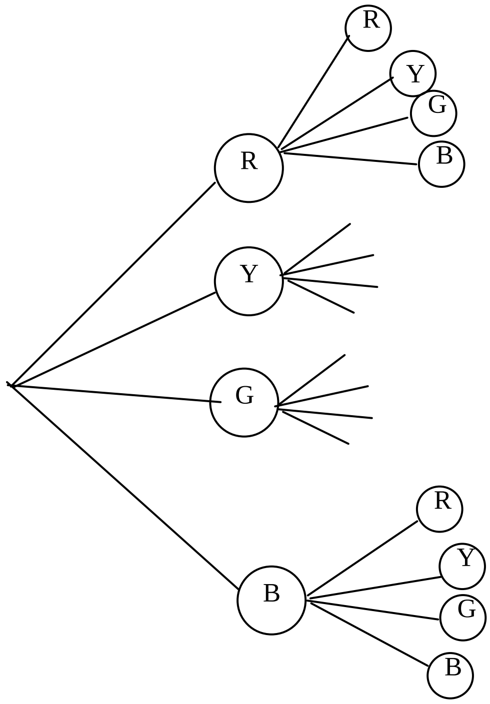
A tree diagram of spinner A



On spinner A, the four colours each have the same chance of coming up i.e. ONE out of FOUR

A tree diagram for spinning twice on spinner A

Spinning twice on spinner A has the following possible outcomes:



4 possible outcomes
(1 of which is R and B)

4 possible outcomes
(None of which are R and B or B or R)

4 possible outcomes
(None of which are R and B or B or R)

4 possible outcomes
(1 of which is R and B)

Total outcomes (R and B or B and R) = 2

Complete: Mathakga's chance of obtaining colours that make purple from two spins on spinner A is ……….. out of………..

If the theoretical probability = $\dfrac{\textit{Number of outcomes of the event}}{\textit{Number of possible outcomes}}$

What is the probability of obtaining the colour purple?

2) Using grids

Use a square to represent all the possible outcomes of spinner A (first spin) and a similar square for spinner A (second spin). Divide the square into four equal parts – each part representing one possible outcome. In the second square draw the lines in the opposite direction.

Spinner A
(first attempt)

R	B	G	Y

R
B
G
Y

Place the one grid over the other so that the two overlap.

R/R	B/R	G/R	Y/R
R/B	B/B	G/B	Y/B
R/G	B/G	G/G	Y/G
R/Y	B/Y	G/Y	Y/Y

Spinner A
(second attempt)

Total outcomes of the events = 2 (circled in the square)

Total possible outcomes = 16

What is Mathakga's chance of obtaining purple from two spins on spinner A?

Did you find the same answer as with the previous approach?

Jabu decided to spin twice on spinner B.

What is his chance of obtaining purple? (You could use the methods presented above to assist you. You could also use a different approach).

Sam chooses to spin first on A and then on spinner B.

What are his chances of obtaining purple?

Who has the best chance of obtaining the colour purple?

Reflecting on doing pattern activities

Even if you worked hard on the above activities, you may not have found all of the patterns or solutions. The important thing, however, is to make an effort and take risks. Engaging with mathematics as the science of pattern and order is rewarding, but requires effort. As you reflect on your experience, ask yourself the following questions:

- What difficulties did you encounter?
- How did you overcome them?
- What methods did you use that were successful?
- Why were they successful?

This will help you to develop your ability to do mathematics and to teach mathematics in a developmental way. Instead of concentrating on explaining rules and procedures, a developmental approach to teaching mathematics is learner-centred and allows learners to grapple with ideas, discuss and explain solutions, challenge their own ideas and the ideas of others. Reflective thinking is the most important underlying tool required to construct ideas, develop new ideas and to connect a rich web of interrelated ideas.

In the rest of this unit we give further examples of pattern activities that you could do and reflect on – first by yourself, and then with your learners.

Repeating patterns

Van de Walle (2004) says that

> Identifying and extending patterns is an important process in algebraic thinking. Simple repetitive patterns can be explored as early as kindergarten. Young children love to work with patterns such as those made with coloured blocks, connecting cubes and buttons.

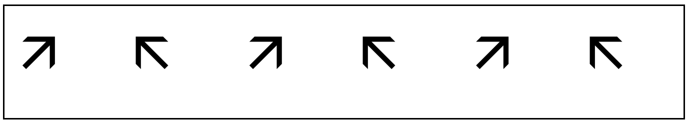
Learning Outcome 2 in the RNCS has several assessment standards that refer to pattern work as it progresses through the GET phase. Patterns may be geometric, numeric or algebraic. Working with patterns develops the logical reasoning skills of the learners, and leads naturally into thinking algebraically. The work on patterns that follows is taken from the RADMASTE materials for the Number Algebra and Pattern module of the WITS GET ACE.

Learners can work independently or in small groups to extend (continue) the patterns given on strips. To do this activity with your class you should prepare enough pattern strips for the whole class.

Van de Walle points out,

> The **core** of a repeating pattern is the **shortest string of elements that repeats**.

Each pattern must repeat completely and never be partially shown. In mathematics there is a convention that if a repetition of a pattern is seen three times, the observer can assume she/he has identified a repeating pattern. Here is an example of a visual pattern strip in which the pattern is repeated three times.



The core of the pattern shown above has two elements. Having each of the arrows in the first two frames in a different colour would highlight the repeating elements.

Here is another easy pattern to continue. The core has three elements, but the detail on the shapes might make it more difficult for young learners to identify.

⊕	∅	∅	⊕	∅	∅	⊕	∅	∅

This pattern below has four elements in its core.

®	®	)	Δ	®	®	)	Δ	®	®	)	Δ

The strip below with letters A and B translates the pattern above it from one medium to another: geometric to variable. Using variables, learners can identify similar types of patterns. The two patterns below are the same type of pattern. They both follow a sequence of a, b, b, a, b, b, a, b, b…

⊙	⬣	⬣	⊙	⬣	⬣	⊙	⬣	⬣
A	B	B	A	B	B	A	B	B

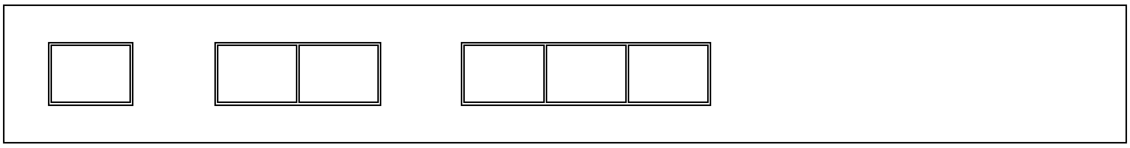
Activity 17

Try this with your class and write a report. Your report should include copies of all of the patterns you used in the lesson.

1) An overhead projector may be used to display a numbered set of different patterns. Teach the learners to use the A, B, C… method of reading a pattern. Half the class can close their eyes, while the other half can read aloud the pattern you point to. The learners who had their eyes closed must then open them and select the correct pattern/s.
2) Suggest an alternative way to present this lesson for a teacher who does not have an overhead projector in his/her class.

Growing patterns

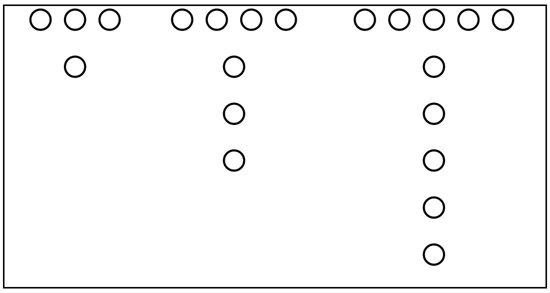
These patterns are geometric, with elements that can be counted. They pictorially illustrate sequences of numbers. You could supply the learners with pattern cards of the type below and have them copy and extend the pattern given in the first three frames. Let them explain why their extension is appropriate, by determining how each frame in the overall pattern differs from the preceding frame. For example, the simple pattern below begins with one brick and increases by one brick from frame to frame, representing the sequence 1; 2; 3, …



Learners should be given time to study the patterns. They can then extend the patterns, giving explanations of why their extension follows from the given sequence. The use of language to explain the extension is important as it develops the learners' mathematical reasoning.

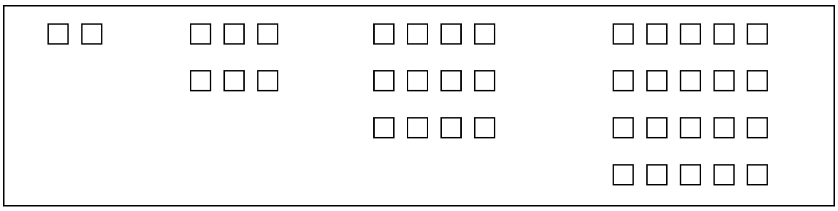
The next pattern is more complex. The pattern illustrated includes two sequences.

- The horizontal shapes increase from three shapes, by one shape from frame to frame, representing the sequence 3; 4; 5, …
- The vertical shapes increase from one shape, by two shapes from frame to frame, representing the sequence 1; 3; 5, …
- So the full pattern grows as a sum of the two: (1 + 3), (3 + 4), (5 + 5), …
- The next picture in the pattern will have 7 dots going down and 6 dots going across, and can be written numerically as (7 + 6).

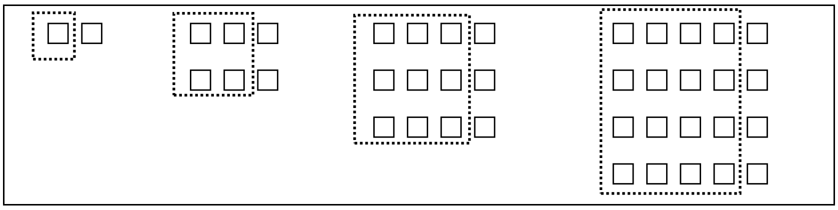


Patterns can be identified (and hence extended) in different ways. This illustrates how different people may see the same pattern in different ways. You need to listen carefully to your learners' explanations to assess whether or not they are valid. They might use different reasoning to you, but still be reasoning correctly. Look at the example of the pattern below. It can be used to illustrate two different relationships.

One explanation could be that from frame to frame we add one row and one column to the display.



Another explanation could be that in each frame we make a bigger square and add a column. The squares have been enclosed in dotted lines to illustrate this explanation.



We can also express this geometric pattern numerically. It is a good idea to use a table to write up the pattern. This helps the observer to identify the pattern, as it presents the information neatly and accessibly.

The frame number and the number of blocks in the frame is tabulated in order to calculate the number of blocks in successive frames. If a general formula can be found, then any term in the sequence can be found, using the general formula.

Let's say we want to find out the number of blocks in the 8th display of this pattern.

Frame number:	1	2	3	4	……	8
Number of blocks:	2	6	12	20	……	?

+ 4 + 6 + 8

Using the table we can look at the number of blocks in each successive display and identify the pattern in which they are growing. Here we see that the displays are growing by adding successive even numbers, starting with 4.

We can use this to write out the numeric pattern.

Frame 1: 2

Frame 2: 2 + 4 = 6

Frame 3: 6 + 6 = 12

Frame 4: 12 + 8 = 20

Frame 5: 20 + 10 = 30

Frame 6: 30 + 12 = 42

Frame 7: 42 + 14 = 56

Frame 8: 56 + 16 = 72

So the 8th frame will have 72 blocks in the display.

Learners up to grade 7 could use the method above to determine the numbers of blocks in the display. Learners in Grade 8 or 9 might be ready to use the second explanation of the visual pattern to generate a simple rule for the successive terms in this pattern. The rule can easily be generalized, and expressed algebraically. This algebraic rule can then be used to find the number of blocks in any display.

Frame number	**Number of blocks**	**Pattern**
1	2	$(1 \times 1) + 1 = 1 + 1 = 2$
2	6	$(2 \times 2) + 2 = 4 + 2 = 6$
3	12	$(3 \times 3) + 3 = 9 + 3 = 12$
4	20	$(4 \times 4) + 4 = 16 + 4 = 20$
5	30	$(5 \times 5) + 5 = 25 + 5 = 30$
6	42	$(6 \times 6) + 6 = 36 + 6 = 42$
7	56	$(7 \times 7) + 7 = 49 + 7 = 56$
8	72	$(8 \times 8) + 8 = 64 + 8 = 72$
General (nth term)	B	$(n \times n) + n = B$

In conclusion to this brief introduction to exploring pattern in mathematics, try out the following activities which are taken from the RUMEP lecture notes on *Patterns and Functions*. First see if you can do them, and then see how your learners manage them. They may surprise you!

Activity 18: Patterns

1) Pascal's Triangle and the Leg-Foot Pattern

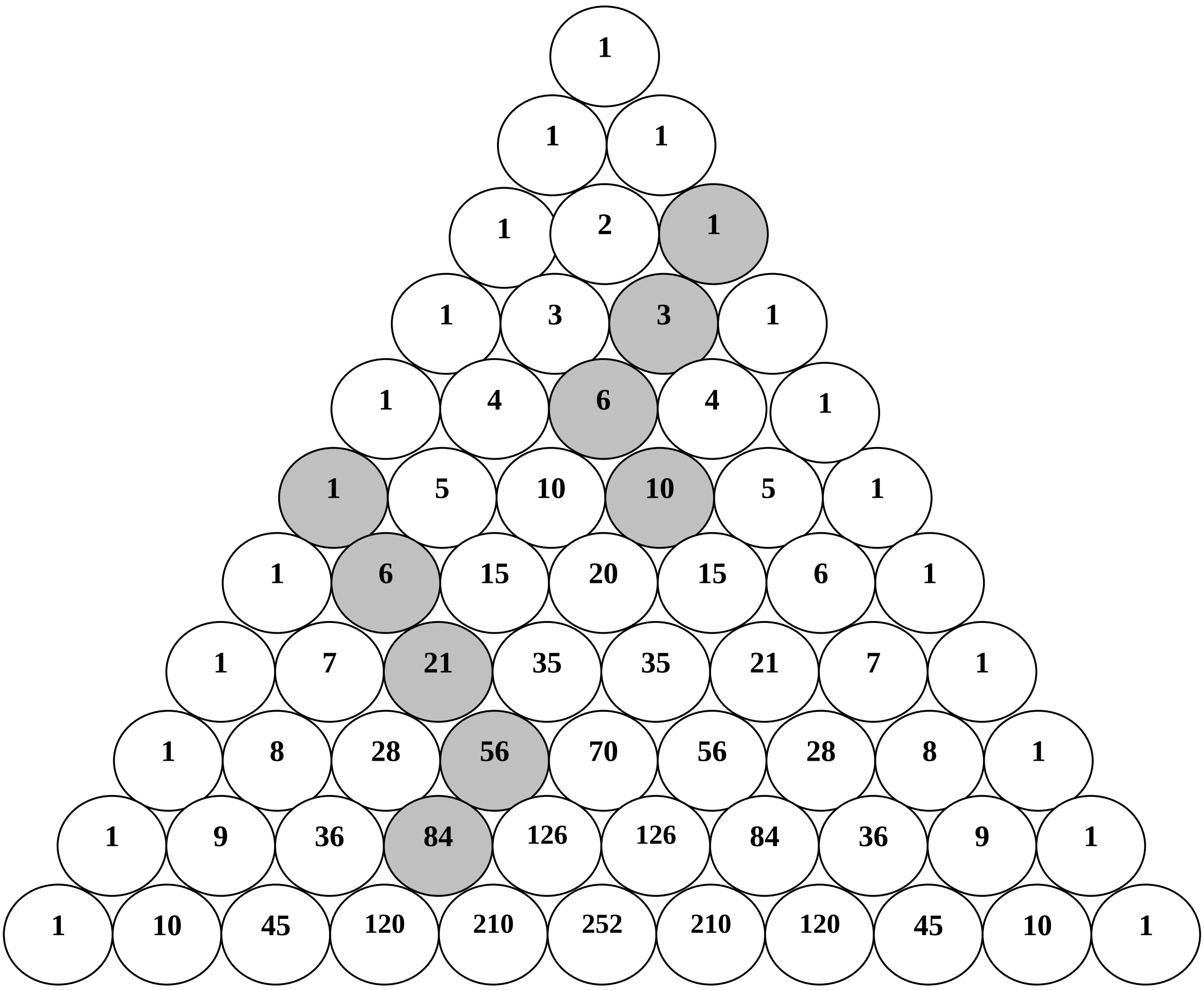
Pascal's Triangle is a fascinating display of numbers, in which many patterns are embedded. In Western writings the Pascal Triangle was named after Blaise Pascal, who was a famous French mathematician and philosopher. Chinese mathematicians knew about Pascal's Triangle long before Pascal was born, so it is also called a Chinese Triangle. It was documented in Chinese writings 300 years before Pascal was born.

Can you shade the circles that make leg-foot patterns in the Pascal Triangle below?

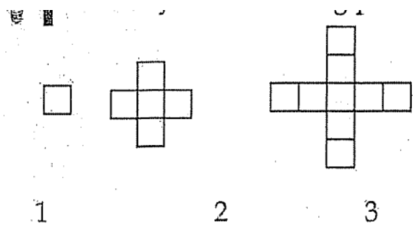
Two examples have been done for you.

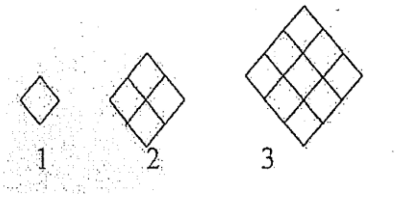
- First look carefully in the given examples to find the pattern.

NB: Always start from the outer edge. If you start on the right edge go diagonally and then turn 1 to the right. Your 'leg' can be any length.



2) Study the following patterns and then extend them by drawing in the next two stages.





a) Draw a table to display the information above:

b) What type of numbers are these?

c) Use the following to show different representations of triangular numbers

i) Square grid

ii) Isometric dotty paper

iii) Square dotty paper

3) The following pattern will help Sipho to calculate the number of blocks he will need to build the stairs of his house.

a) Help him find the number of blocks he will use for 15 steps.

b) Extend this pattern by drawing the next 2 stages of these steps.

c) Enter your data in the following table:

Step	1	2	3	4	5	8	15	20	100
Number of blocks	1	3	6						

d) Find and explain a rule that generates the above pattern.

e) What type of numbers are these?

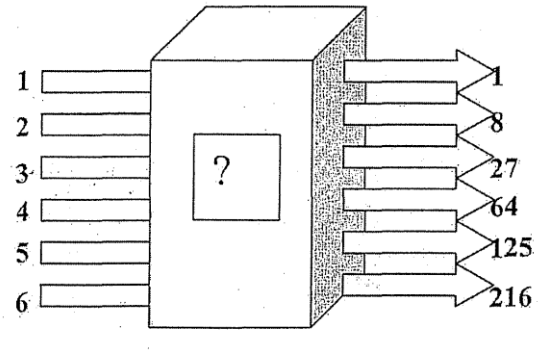
f) Use the following to show different representations of triangular numbers:

i) Square grid

ii) Isometric dotty paper

iii) Square dotty paper

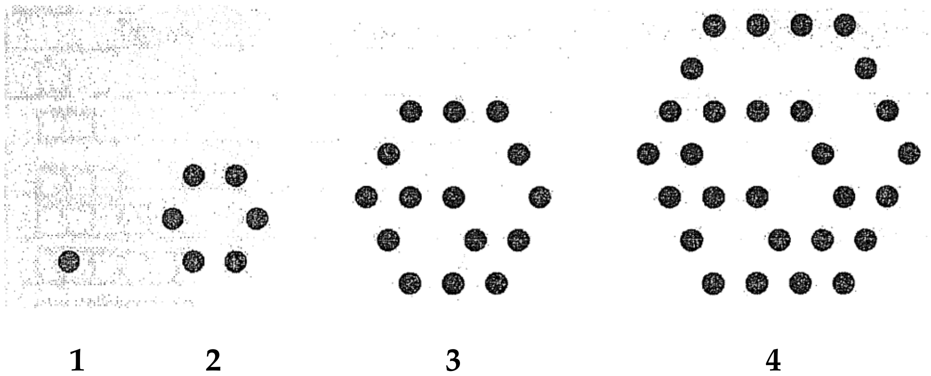
4) The following function machine creates a number pattern.



a) Investigate the rule that it uses. Write up your findings.

b) What type of numbers are these?

5) Study the following number pattern and then complete the table that follows:



Stage	1	2	3	4	5	8	15	20	100
Number of dots	1	6	15	28					

a) Investigate a general rule that generates the above pattern.

b) What type of numbers are these?

Summary

This study unit is designed to change the way in which learners perceive their role in the mathematics classroom – from being passive recipients of mathematics, facts, skills and knowledge to becoming active participants in 'doing' mathematics and doing the spadework for the creation of mathematical problems in the future. Accomplishing such a task has called for the development of stimulating activities and explanations in mathematics that would involve all learners, provide opportunities for more mathematical communication in the classroom, and link creative thinking with mathematical content.

Through the practice of explanation, investigation and 'doing' mathematics in general, learners will begin to experience the full impact of the process of solving problems and thereafter, generating problems. If we are truly committed to the notion that mathematics is for everyone, then we must begin to look for alternative methods to the traditional approach for facilitating learning in the classroom for all learners. Perhaps the creative aspect of 'doing' mathematics might be the key that will open doors to mathematical learning for previously uninterested learners. Simultaneously, active participation in problem-solving and problem investigation might serve to cultivate the talents of learners who maintain an interest in 'doing' mathematics.

Self-assessment

Tick the boxes to assess whether you have achieved the outcomes for this unit. If you cannot tick the boxes, you should go back and work through the relevant part in unit again.

I am able to:

Critically discuss the thinking that the traditional approach to teaching mathematics rewards the learning of rules, but offers little opportunity to do mathematics.	
Explain the term 'mathematics as a science of pattern and order'	
Evaluate a collection of science verbs (i.e. action words) that are used to reflect the kind of activities engaged by the learners when doing mathematics.	
Construct a list of features of a classroom environment considered as important for learners engaged in doing mathematics.	
Formulate appropriate and interesting activities to help learners explore the process of problem – solving through number patterns and logical reasoning.	

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