CHAPTER 4

METHODOLOGY

4.1 Introduction

The methodology of the study consists of two aspects. The one aspect refers to what characterises this study, which follows a component of Dindyal’s (2003) approach on the exploration of students’ algebraic thinking in geometry and their related conceptual difficulties with aims related to mathematics education. The other aspect of the methodology refers to general research process in (mathematics) education, which includes the research methodology, the instruments for data collection and the approach to data analysis.

4.2 Dindyal’s Research Approach

Dindyal set out to investigate how students in high school use algebraic thinking in geometry. Therefore, his aim was to investigate the link between algebra and geometry in the curriculum at the high school level. He considered three aspects of algebraic thinking – symbols and algebraic relations, representations, patterns and generalisations. He used five theoretical perspectives to analyse his data: operational and structural conceptions of algebraic expressions and theory of reification; theory of representations and representation systems; theory about thinking as communicating; socio-cultural theory or emergent perspective; and theory of knowledge connectedness (Dindyal, 2003, p. 163).

The first perspective he drew on Sfard’s (1991) theory about operational and structural conceptions of mathematical constructs. This perspective allowed him to analyse how students used symbols and algebraic relations while solving geometric tasks. The theory of representations (Dindyal, 2003 quoting Golding, 2002, Dreyfus, 1991, and Hilbert and Carpenter, 1992) provided him with some perspectives to make inferences about students’ thinking from the different representations that they used, especially on the notions of internal and external representations. Sfard’s (2001) participationist theory (as quoted by Dindyal, 2003) provided a perspective to study thinking through communication. In Dindyal’s study the students communicated [their thinking] with their teacher, among themselves and with the
researcher. Emergent theory (Cobb and Yackel, 1996 as quoted by Dindyal) complemented Sfard’s participationist perspective by giving credence to both the social and individual aspects of learning and also provided the flexibility of emphasizing one aspect more than the other. Finally, knowledge connectedness (Dindyal quoting Anderson, 1990 and Lawson and Chinnappan, 2000) provided a perspective on how the availability of knowledge components in mathematical tasks could be related to aspects of memory (low activation or high activation).

For the present study due to the nature of the data collected so as the subject’s context I drew on the Prawat’s framework on learning and transfer (see Chapter 3) to analyse my data. As this framework allowed me to analyse how students with specific characteristics approached the tasks rather than how the different tasks were carried out by students which was the case of the Dindyal’s work. This framework comprises some perspectives similar to perspectives used by Dindyal (theory about thinking as communicating, socio-cultural theory or emergent perspective, and theory of knowledge connectedness). The theory about thinking as communicating in Dindyal’s work may be explained by the term awareness in the Prawat’s framework. This awareness takes place when students articulate (communicate) their own thoughts in different ways and in different settings. The theory of knowledge connectedness is accounted for by the Prawat’s term organization. According to Prawat organization is equivalent to connectedness of key concepts and procedures which provide the glue that holds cognitive structures together. In addition, he affirmed that the adequacy of this structure determines the accessibility or availability of information later. Lawson and Chinnapan’s (2000) study illuminated my research on “knowledge connectedness” from a concept-mapping perspective.

As to the socio-cultural theory or emergent perspective, Prawat termed it as disposition. He distinguished two types of disposition: performance and mastery dispositions. More details on these dispositions referred to Chapter 3 and Section 3.2.

4.3 On General Research Process in (Mathematics) Education

In this section I give an overview of the research methods used in this research. Details of the methods for each of the phases of the research will be presented in the following chapters. The overview shows the overall design – the structure, which indicates the different phases. For each phase, the following are presented: the research methodology, the research problem
being addressed, the subjects, the data collection instruments and the approach to data analysis.

4.3.1 The Overall Research Design

The main body of the research was done in three phases, which can be analysed separately. The three phases were:

- The Pilot Study
- The Main Study – Euclidean Geometry Course
- The Main Study – Analytic Geometry Course

With regard to the research problem being addressed, the latter two studies responded to the same questions – the main research questions, which are stated below:

1. How do first-year university students solve geometry problems? To what extent do they use algebraic knowledge and thinking in solving such problems? What kinds of meanings do students make of different algebraic and geometric concepts involved in problem solving situations?
2. To what extent does algebraic knowledge and thinking aid students’ conceptual understanding and problem solving performance in geometry?
3. To what extent is geometric work linked to algebraic thinking in the first year university geometry course at the Universidade Pedagógica in Maputo?

The key issue in this study was to investigate, in an exploratory way, how and to what extent students make use of algebraic knowledge and thinking as they work on geometry problems at University level. Specifically, in this study, the focus was on how first-year university students at Universidade Pedagógica in Maputo-Mozambique bring their thinking and knowledge of algebra in understanding and working with geometry. To examine how students conceptually understood and performed in geometry, I looked at them solving specific geometric tasks while thinking aloud, in other words to see them in action. Obviously, cognitive processes cannot be observed directly, but can only be inferred from the observations of some actions within social interaction. Even then, the correspondence between an action and the mental processes leading to that action will be quite difficult to ascertain. In order to minimise this difficulty I found it worthwhile to take a theoretical
perspective (symbolic interactionism) which helped me to understand social interaction. I applied the three aspects recommended by this perspective. First, I attempted to maintain a certain openness of mind, that is, I tried to see their actions or activities from their point of view. Second, I witnessed as close as possible the activities carried out by my subjects through semi structured interviews, classroom observations, concept maps, and informal meetings outside the classroom during one academic year. Third, I tried to create an environment of trust and rapport between my subjects and myself. To attain this goal I presented myself as a student who was carrying out an investigation and wanted them to become my partners so that we might ameliorate the teaching and learning of geometry in Mozambican context and at the same time to help them reflect of their career as future teachers. Thompson (1994) as quoted by Steele and Johanning (2004) summed up the notion of action within cognitive processes saying that “action (is) not an observable behaviour, but an activity of the mind (which is) tied to the experience of individuals as they interact with the physical world around them” (p. 66). (The words in bracket were added).

There is an intrinsic relationship between conceptual understanding and problem solving performance. One, who conceptually understands a certain area, may perform well in the same area in the problem-solving environment. Mousley (2003), quoting Knapp et al (1995, pp.128, 133) provided evidence supporting this link.

Children receiving instruction focused on multiple mathematics topics and conceptual understanding performed significantly higher in advanced mathematical skills… than their counterparts in classrooms that focused on arithmetic skills only. Similar evidence appeared with results on a test of mathematical problem-solving ability. The evidence regarding the retention of learning over a 12-month period … tells a similar story… Students exposed to instruction aimed at meaning and understanding performed significantly better than their counterparts exposed to conventional instruction in two of three subjects’ areas (mathematics and writing).

Mousley in the above quote showed that instruction focused on multiple mathematics topics (e.g. algebra and geometry) and conceptual understanding allows one to perform well in mathematics. On the contrary instruction stuck on instrumental understanding of arithmetic skills one might perform poorly. In the former case instruction is rich in meanings represented by several concepts in different domains of mathematics while in the latter case
instruction is poor in meanings as it is limited to only one domain of mathematics. In the present study I was exploring conceptual geometric understanding of five first year students at the Universidade Pedagógica in Maputo – Mozambique while they were bringing their thinking and knowledge of algebra in understanding and working with geometry.

Several factors may affect the ways in which students understand and perform mathematical tasks. These factors may be of contextual, structural, and conceptual in nature. Hence, this type of study can best be approached through research methodologies that can account for multiple perspectives. Qualitative research methodology provides such perspectives. The idea was to collect extensive data with an open mind. As the study progressed, the data were continually examined for emerging patterns and insights. This is a type of qualitative methodology called ‘grounded theory’ (Savenye and Robinson, 2004).

4.3.1.1 The Pilot study

The Pilot Study was designed to explore students’ algebraic and geometric knowledge and thinking and their ability to access and use algebraic knowledge and thinking in geometry problem solving. I constructed a 50- minute pilot test (Appendix 1) and administered to 26 first-year students at Universidade Pedagogica (UP) in October 2004 (UP mainly aims at training secondary and high school pre-service teachers for different subjects including mathematics). After sorting them according to their background, I selected 7 students for deeper analysis of their written responses. The tasks of the pilot test were developed by me with adaptations from different sources (Lewis, 1964 and Alvarinho et al, 1992) and validated by the lecturer of Euclidean and Analytic Geometry courses at UP. This method enabled me to rearrange some aspects concerned with appropriateness of the tasks in terms of interplay between algebraic and geometric thinking for the next stages (the Main Study – Euclidean Geometry Course and the Main Study – Analytic Geometry Course) as well as to find out the first-year students’ academic and professional background at the beginning of their degree at UP.
4.3.1.2 Main study – Euclidean Geometry course

In the Main Study - Euclidean Geometry Course, a 60-minute Diagnostic test (Appendix 2) was administered in the start of the Euclidean Geometry course to 32 first-year university students at Universidade Pedagogica (UP) in March 2005 (later the class increased up to 40 students because some of them came late for several reasons). These students constituted the only first-year university class pursuing secondary and high school pre-service mathematics teacher course. This test assessed the students’ proficiency in school geometry and algebra when they enter UP. The Diagnostic test was jointly constructed with the lecturer of Euclidean Geometry. Students’ responses to this Diagnostic test were examined and analysed (Section 5.3.1). Afterward, I selected 14 students (8 low and 6 high achievers) for Interviewing Phase 1. These students were individually interviewed using Free Recall and Hinting Task on their Diagnostic test responses (Appendix 3). Interviewing Phase 1 ran in two slots because of technical issues: the first slot was carried out from the 12th to the 20th of April and the second slot from the 5th to the 6th of May. One of the students dropped out from interviewing. Thereafter, I selected 8 students (5 low and 3 high achievers) students for in depth follow up study. This was a purposely selected sample as during interviewing process I found their conversation interesting for the purpose of this study. After analyzing, the transcripts of the 8 students I came up with four contextual background categories of students (Section 5.3.5.2). Accordingly, I selected 5 students out of 8 as my target sample. All categories were represented in this target sample.

From the 27th of June to the 5th of July I administered Elaboration and Concept Mapping Task (Appendix 4) meant for the 8 students. However, only 3 students took the task. The 5 students who missed this activity did accomplish it at the commencing of the 2nd semester, which was at the end of August. Elaboration Task consisted in constructing tasks from some concepts or theorems previously given. The Concept Mapping Task was used in the form of yielding true propositions using the concepts presented. The lecturer of Euclidean Geometry recommended these concepts finding them as some of the most critical for the course.

4.3.1.3 Main study – Analytic Geometry course

In the Main Study, the Analytic Geometry Course, I worked with the same target students. They all passed Euclidean Geometry course and they enrolled and attended Analytic Geometry course in Semester II. On the 29th of September the lecturer administered the first
test (Appendix 5) to the class and I collected the scripts of the eight target students for Interviewing Phase 2 which took place in the first two weeks of November. This delay was due to the availability of the students for that purpose. Free Recall and Hinting Task was used in Interviewing Phase 2. In this interviewing, I posed an open-ended question on the relationship between Euclidean and Analytic Geometry and its influence on geometric conceptual understanding.

4.3.1.3 Summary of the overall research design

Table 4.1 presents a summary of the overall research design.

Table 4.1: Summary on the overall research design

<table>
<thead>
<tr>
<th>Phase</th>
<th>Pilot Study</th>
<th>Main Study-Euclidean Geometry Course</th>
<th>Main Study-Analytic Geometry Course</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Methodology</strong></td>
<td>Grounded Theory using Case Studies</td>
<td>Grounded Theory using Case Studies</td>
<td>Grounded Theory using Case Studies</td>
</tr>
<tr>
<td><strong>Objectives</strong></td>
<td>Preparing and refining of task-based instruments for the next phases</td>
<td>Exploring how students bring algebraic knowledge/thinking as they work with geometry at university level</td>
<td>Exploring how students bring algebraic knowledge/thinking as they work with geometry at university level</td>
</tr>
<tr>
<td><strong>Subjects (sample)</strong></td>
<td>7 out of 26 students</td>
<td>-40 students for class observation</td>
<td>-40 students for class observation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-8 out of 40 students for the first series of interviews</td>
<td>-5 target students for the second series of interviews</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-5 target students out of 8 for Elaboration and Concept Mapping Task</td>
<td></td>
</tr>
<tr>
<td><strong>Data Collection Instruments</strong></td>
<td>Pilot test</td>
<td>- Diagnostic test</td>
<td>- Lecturer’s test</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Interviews</td>
<td>-Interviews</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-Audio/video tapes</td>
<td>-Audio/video tapes</td>
</tr>
<tr>
<td></td>
<td></td>
<td>- Elaboration Task and concept maps</td>
<td></td>
</tr>
<tr>
<td><strong>Data Analysis Approach</strong></td>
<td>Qualitative analysis: Identification of individual students’ responses regarding use of algebraic knowledge and thinking in working with geometric tasks</td>
<td>Qualitative analysis: Students’ profiles regarding their thinking pattern(s) as their meanings are reshaped as they exploit available tools in algebra to move the focus of their attention onto new objects and relationships in geometry.</td>
<td>Qualitative analysis: Students’ profiles regarding their thinking pattern(s) as their meanings are reshaped as they exploit available tools in algebra to move the focus of their attention onto new objects and relationships in geometry.</td>
</tr>
</tbody>
</table>
From this summary of the overall research design, we can see that the entire research produced qualitative data. Before I present the details of the methodology of the three studies, I find it necessary to explain the logic of the design of these three studies as a whole. That is, I first need to explain why I undertook these studies in the way I did - in terms of methods. I then need to establish the level of validity of the conclusions made.

These two issues lead me to discuss methods and validity in qualitative research.

4.3.2 On the Overall Research Methodology (Methodological Framework)

Several factors may affect the ways in which students understand and perform mathematical tasks. These factors can be contextual (e.g. class size), structural (e.g. syllabuses and textbooks), and conceptual (e.g. actual algebraic and geometric concepts of the courses) in nature. Hence, this type of study can best be approached through research methodologies that can account for multiple perspectives. Qualitative research methodology provides such perspectives. The qualitative research methodology largely used in this study is that of Grounded Theory informed by the theoretical perspective Symbolic Interactionism.

4.3.2.1 Symbolic interactionism

This theoretical perspective, on the one hand suits the present study because of its three central principles: (1) human beings act toward things on the basis of the meanings that the things have for them, (2) this attribution of meaning to objects through symbols is a continuous process, and (3) meaning attribution is a product of social interaction in human society (Woods, 1992, p. 338 quoting Blummer, 1969). These principles account for the key concepts of this study ‘thinking’, ‘understanding’, and ‘knowledge connectedness’. As we have seen in Chapter 3 these concepts are intertwined and take place in one’s mind [influenced by social interactions]. In order for an individual to come to know the nature or the meaning of something (i.e. ‘understands’ according to Collins English Dictionary, (1979)), s/he should first exercise her or his mind to make a decision about that something (i.e. ‘thinks’ according to the same source) (Principle 1). Besides, in particular, making meaning in mathematics involves making connections among different concepts in mathematics, that is “… the forging of connections across domains [in mathematics through continuous complex processes and symbols]” (Noss and Hoyles, 1996, p. 30) (Principle 2). Mathematics is considered as a language (Chapter 3). Hence, in order for one to know the
language of mathematics, in other words, to understands mathematics, one must be involved in social processes (interactions) through discursive practice in particular settings (Principle 3). On the other hand, Symbolic Interactionism “provides opportunities for researchers and teachers to join together in doing research, thus promoting professionalism and helping to effect change from the inside, as it were. The researcher’s theoretical and methodological knowledge and teacher’s practical knowledge of teaching make a strong combination, the one enriching the other” (Woods, 1992, p. 389 quoting Pollard, 1984; Woods, 1985,1986, 1989; Hustler, Cassidy, and Cuff, 1986; and Woods and Pollard, 1988).

Symbolic interactionism is characterized as a theoretical perspective where the conduction of a research should be subject to rigorous scrutiny: “exploration in depth to the extent of ‘saturation’; attention to sampling across people, places, and time; the use of multimeods (triangulation); naturalistic methods that are relatively free from researcher interference in the sense of misconstructing others’ meanings and that penetrate through layers of reality to the innermost core; reflectivity and the keeping of a research diary and compilation of a research biography, so that the personal element is seen as part of the research, not separate and hidden from it; the search for contrary cases and alternative explanations; and tightness of fit among data collection, analysis, and theory” (ibid., p. 369).

Layder (1993) corroborates this view stating that “this humanist strand of symbolic interactionism favours research methods and strategies such as participant observation, in-depth or semi-structured interviews, and documentary evidence, which seek to tap the subjective understandings of the people who are the subjects of the research” (p. 38).

This study has an exploratory approach style and seeks “the subjective” geometric conceptual understanding of first-year university students when they bring and use algebraic knowledge and thinking in working with geometry tasks. Hence, Layder’s contention also suggests symbolic interactionism as an appropriate theoretical perspective for the present study.

4.3.2.1.1 Layers of reality

Symbolic interactionism acknowledges that social reality has many layers of meaning. Wood (1992) quoting Berger (1966, p. 34) states that “the discovery of each new layer changes the perception of the whole”. This statement underpins what Noss and Hoyles (1996) claim that algebra is viewed as the official language of communication and expression for the most part
of mathematics (including geometry). That is, algebra constitutes one of a possible myriad of generalisations expressible in widely different forms. Hence, algebraic thinking (a way of abstracting) can be seen as a way of layering meanings on each other, connecting between ways of knowing and seeing, rather than as a way of replacing meaning of each other. Particularly, in this study it was expected that learners would exploit available tools in algebra to move the focus of their attention onto new objects and relationships in geometry so that their meanings become reshaped towards “the perception of the whole” [reality]. The central idea, therefore, was that algebra can be a particularly fruitful domain of “situated abstraction”, a setting where we may see the externalized (or internalized) face of mathematical objects and relationships (specifically geometric objects and relationships) through the window of algebraic thinking and accompanying linguistic and notational structures (symbols).

In terms of a theoretical perspective, symbolic interactionism allowed me, as a researcher, to be aware of three important aspects to take into consideration while carrying out a research: first, I should maintain “a certain openness of mind, not prejudging the matter under investigation, nor necessarily settling for first or even second appearances.” In the contrary, I should “press on to the next ‘peak’ until the summit is reached and the whole mountain can be viewed in perspective”. Second, I should realize that “this kind of exploration cannot be undertaken in a day or a week [meaning that this study should be of type of a longitudinal survey] (Ary et al., 1996). Nor can it take place outside the actual situation of the object of study or by proxy.” Third, I should create an environment of trust and rapport between subjects and myself.

These three aspects of symbolic interactionism are of paramount importance to understand social interaction as shown in Wood’s quote:

In summary, to understand social interaction, it is necessary to witness it as close as possible and in depth in all its manifestations and all the situations in which the form under examination occurs. Because social interaction is constructed by the people engaged in it, one should try to see it from their point of view and appreciate how they interpret the indications given to them by others and the meanings they assign to them and how they construct their own action. In addition, because this is a process, it must be sampled over time (pp. 349, 350).
According to Dindyal’s (2003) review, “little or no research has been carried out on the influence of algebra in the geometry curriculum, in particular, algebraic thinking in geometry has not been explored” [enough] (p. 10). The proposed study attempted to make a contribution to this important and emerging research field. Specifically, this study was exploring what kinds of meanings tertiary-level students showed for different critical algebraic and geometric concepts that they were working with over one academic year. Besides, it was concerned with what resources (contextual, structural, and conceptual) promoted or hindered first-year university students’ use of algebraic thinking in geometry in Mozambique and what their implications were for students’ geometric conceptual understanding.

Accordingly, this study attempted to make a contribution to the “substantive theory” (the theory developed for an empirical area) of this emerging research field within Mozambique context from the empirical data collected. Within symbolic interactionism, the research methodology which accounts for the generation of theory inductively “grounded” in the empirical world is known as *Grounded Theory*.

### 4.3.2.1.2 Grounded theory (generation of theory)

Grounded Theory shares the assumption that the social world must be discovered using qualitative methods and employing an exploratory orientation (Layder, 1993). This feature of Grounded Theory fully characterizes the nature of the present study: to actively explore the central idea of how algebra might be a particularly fruitful domain of “situated abstraction”, a setting where we might see the externalized (or internalized) face of mathematical objects and relationships (specifically geometric objects and relationships) through the window of algebraic thinking and accompanying linguistic and notational structures (symbols) within Mozambique context. In this regard, Eggen (2004) affirms “the importance of stating contextual factors is important within Grounded Theory for data analysis…” (p. 214).

Besides, this present study is not concerned with description pure and simple as ethnography places great emphasis on the description. In order to achieve this purpose, it is important to know the main features of Grounded Theory: 1) “the researcher interested in developing grounded theory is an active sampler of theoretically relevant data” (Layder, 1993, p. 44), this process is similar to data reduction; 2) “the constant selection and control over comparison groups is part of the dynamic and emergent design of the research process and encourages the
development of properly grounded theory” (ibid., p. 45); 3) “theory should be viewed as a constant and flexible accompaniment to the incremental collection of data and the unfolding nature of the research” (ibid., p. 45); and 4) the heart of Grounded Theory is “to generate theory that fits the data” (ibid., p. 45).

Moreover, this qualitative methodology combines theory and empirical evidence with the context-close-ness of significance for the illumination of phenomena in practical fields like the one mentioned above through assorted approach (ibid). Strauss and Corbin (1991) as quoted by Sanger (1994) enlighten this combination between theory and empirical evidence with the notion of “theoretical sensitivity”:

Theoretical sensitivity represents an important creative aspect of Grounded Theory. This sensitivity represents an ability not only to use personal and professional experience imaginatively, but also literature. It enables the analyst to see the research situation and its associated data in new ways, and to explore the data’s potential for developing theory (p. 179).

In my study I used theory in combination with my personal and professional experience as a mathematics teacher. To ensure that my creative elements (my imagination) were systematized I used ATLAS.ti 5.5, a computer program which helped me in the coding process through my qualitative data. Hence it assisted me in patterning and clustering of words and phrases.

It is important to highlight how I conducted my data analysis and data gathering utilizing Grounded Theory. I used the seven types of creativity suggested by Sanger (1994) as follows:

a) **Labels and Categories**

I labeled and categorized parts of the data that I found connected with the focus of my study and a posterior to guide me in finding significance in data.

b) **Methodological Imports**

I read, listened, and watched the data for each of my subjects. I summarized each piece of data in order to find the appropriate selection from the data (data reduction method). Sanger compares this methodological treatment of the data to the
homoeopathic medicine where “the practitioner attempts to treat the whole person by providing a poison which elicits the total range of symptoms that the patient is presenting- albeit in extraordinarily insignificant dosages. These tiny doses can be read by the body, which raises its armies of immunity to the poison- and thus to the prevailing illness, which had, hitherto, completely besieged the body’s power to diagnose what was wrong.” (p. 181)

c) Theoretical Import

I carried out an intensive reading across a wide range of literature and it helped me find novel insights and refine my imagination.

d) Novel Methods

I used the multiple methods to collect data (interviews, observation, documentation analysis, and concept maps). However, I found interesting the method of prompting during interviewing: researching learning within peer to peer relationship. This is a sort of “peer debriefing” technique (Lincoln and Guba, 1985). Accordingly, most of the students were talkative and participating, thus creating a sound conversational environment as if it were a group discussion amongst peers. This environment spread out trough the entire process of data gathering. Besides, during concept mapping the students were asked to yield and solve tasks for their virtual students as though they were the teacher (Elaboration Task): researching learning within the elaboration task process. This method enabled the students “to reflect critically upon what they regarded as normal, habitual or ritualistic” (p. 183).

e) Reporting

I wrote throughout the various phases of research. I made plans, called designs. I made descriptive notes. I transcribed, analysed, and regularly reported before my fellows and experts in mathematics education. I received feedback and accordingly this led to different and sometimes new conceptions in my writing.

f) Metaphors
I looked at the metaphors (symbolic meaning) in the data. I found it very insightful as “data can always be looked at from numerous perspectives, especially when they are symbolic in nature” (Krippendorff, 1980, p.22) which is the case of this study.

g) Aliens Structures

I was open to other theoretical frameworks. My concern was “ways of seeing adopted by other fields of enquiry or simply frameworks that made the researcher think in new ways” (Sanger, 1994, p. 184) in order to fit the data.

Grounded Theory views the coding procedures in the light of the constant comparative method. According to Eggen (2004) this method comprises four steps:

The four steps of the constant comparative method are comparing incidents applicable to each category; integrating categories and their properties; delimiting the theory; and writing the new empirically informed theory (p. 213).

Coding procedures may be divided in open, axial, and selective coding (A. Strauss and Corbin, 1998 as quoted by Eggen, 2004). Open coding is the process of discovering new properties and dimensions in the data. Axial coding is the process of linking properties and dimensions to categories and sub categories. In turn, selective coding is applying already defined categories, either defined because of open coding or defined because of previous theory development, to other sections of the data set. Theory development and verifying using this strategy is not a linear process. In practice, I have been working back and forth between these procedures until the stage of theoretical saturation was reached.

4.3.3 Validity and Reliability

This is a qualitative research study and as in any research study the criteria of quality are very important. It is imperative to ensure that the methods used in the study are reliable and its conclusions valid. In order to meet this goal, qualitative research uses different sets of criteria compared with the positivist paradigm. Standards such as internal validity, external validity, reliability, and objectivity do not seem to fit in qualitative research (Confrey and Lachance, 2000 quoted by Dindyal, 2003). In order for qualitative research to produce more reliable and valid findings, two criteria, suggested by Silverman (2000), were used. The first relates to the “constant comparative method”. In this method, the researcher attempts to find a range of
cases through which to test provisional hypotheses. In this study I produced profiles of the target students. The constant comparative method was used across profiles for each of the subjects so that I could find out recurrent patterns throughout the study. This helped me to cluster the students according to corresponding patterns and their contextual backgrounds. Accordingly, I drew some emergent themes, configurations, and explanations. This procedure suits the four steps of the constant comparative method mentioned by Eggen above.

The second criterion relates to “triangulation of methodology”. The strength of qualitative research rests in its multi-method focus and the use of multiple methods to secure in-depth understandings of the phenomenon being studied. I used different data sources such as interviews, class observations, syllabus content analysis, and students’ artefacts (tests and concept maps).

In addition, I had my field notes critiqued by a colleague as an additional check on bias. Moreover, two individuals, one who is fluent in English and Portuguese (Appendix 8) and the other who is an expert in the field of Mathematics Education edited my thesis.

In the meantime, it is important, on one hand, to note that it is a matter of limiting observers’ biases, not eliminating them. On the other hand, qualitative researchers tend to view reliability as a fit between what they record as data and what actually occurs in the setting under study, rather than the literal consistency across different observations. They also view validity as the degree to which a study generates theory, description, or understanding (Bogdan and Biklen, 1982).

4.3.4 On the Pilot Study

The type of research: Grounded theory using case studies

The Pilot Study was conducted to test the tasks used in the pilot instrument (a test) whether they would measure some indicators of algebraic and geometric thinking through case studies. After analyzing some students’ written responses I expected to construct appropriate instruments for the gathering of data in the following stages of the research.

The design
In order to develop task-based instruments to be used in the study, a 50-minute pilot test was administered to 26 first-year students at Universidade Pedagogica (UP) in October 2004. The tasks were developed by me with adaptations from different sources and validated by the lecturer of Euclidean and Analytic Geometry at UP. This test aimed at exploring students’ algebraic and geometric knowledge and thinking and their ability to access and use algebraic knowledge in geometry problem solving.

For analysis of students’ written responses seven test scripts were sampled based on their different academic, professional backgrounds and the richness of their responses.

The subjects

The subjects of this phase initially consisted of 26 first-year university students at UP. After sorting them according to their background, I selected 7 students for deeper analysis of their written responses (Table 5.1). They had completed the Euclidean Geometry course and were attending the Analytic Geometry course. These subjects came from different academic and professional (pedagogic) backgrounds as shown in Table 4.2.

Table 4.2: Academic and professional background of subjects (Pilot Study)

<table>
<thead>
<tr>
<th>Number of students/Category</th>
<th>Academic background</th>
<th>Professional background</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 / (I)</td>
<td>Grade 12</td>
<td>None</td>
</tr>
<tr>
<td>1 / (II)</td>
<td>Grade 12</td>
<td>Grade 7+ 2 years lower primary school teacher training course</td>
</tr>
<tr>
<td>2 / (III)</td>
<td>Grade 12</td>
<td>Grade 10+2 years upper primary school teacher training course</td>
</tr>
<tr>
<td>1 / (IV)</td>
<td>Grade 12</td>
<td>Grade 10+3 years technical college teacher training</td>
</tr>
<tr>
<td>8 / (V)</td>
<td>Grade 10</td>
<td>Grade 10+2 years upper primary school teacher training course</td>
</tr>
<tr>
<td>4 / (VI)</td>
<td>Grade 10</td>
<td>Grade 10+3 years technical college teacher training</td>
</tr>
</tbody>
</table>
According to the differences in terms of teaching and learning experiences and contexts it was also expected differences in the approaches the subjects used in the problem solving situations particularly in geometric tasks.

The data collection instrument

The pilot test consisted of two tasks. The Task 1 (see below) aimed at assessing the underlying cognitive processes in geometry (visualization, construction, and reasoning processes) so as in algebra (symbolization, relations, modeling and generalisations). The students were expected to construct a triangle under given condition; to visualize some relations between segment lines DE and BC; and finally to reason how to prove those relations found. For proving I expected the students to use either vector algebra (Analytic Geometry) or Euclidean Geometry knowledge as they might have treated both topics either at school or at the university according to the syllabuses. On the one hand in Euclidean Geometry domain the students might have used the theorem on bisection and parallelism as an auxiliary theorem and using indirect proof (see the solution of this task in Lewis, 1964). In this solution symbolization of segments and the relations of their lengths play a fundamental role. On the other hand in Analytic Geometry domain the students might have added vectors and known the concept of co-linearity of vectors and interpreted it as the parallelism of the respective segments. After proving the students were expected to generalize the result to any other triangle under the same conditions. This aim might be assessed when they encounter a task where they should use this theorem. As we can see these concepts (symbolization of segments, relations of the segment lengths, vector addition, and co-linearity of vectors) evoke algebraic thinking (symbolization, relations, modeling and generalisation).

Exercício 1

Seja dado um triângulo [ABC], onde o segmento DE bisecta os lados AB e AC.

a. Construa o triângulo [ABC] e o segmento DE.

b. Conjectura a(s) relação(ões) entre os segmentos DE e BC.

c. Demonstre essa(s) relação(ões).

[Task 1:
In triangle \([ABC]\), DE bisects AB and AC.

a. Sketch \([ABC]\) and DE.
b. What relation(s) can you write connecting DE and BC?
c. Prove the relation(s).]

The purpose of Task 2 (see below) was to assess how students construct and visualize polygons and the respective diagonals; how they see an algebraic model in order to generalize the total number of diagonals of \(n\) sided polygon; and how they handle the generalized formula to find the number of diagonals of any polygon.

**Exercício 2**

Seja dado um polígono com \(n\) vértices.

a. Deduza a fórmula do número total de diagonais do polígono.
b. Calcule o número total de diagonais dum polígono de 70 vértices.

[**Task 2:**

A polygon possesses \(n\) vertexes.

a. Write a formula, which indicates the total number of its diagonals.
b. Determine the number of diagonals of 70-vertex polygon.]

**The data collection procedure**

In this phase I only used a 50-minute pilot test to collect data. The lecturer of the Analytic Geometry course introduced me to the students and explained the purpose of the test to the students before administering it. The students showed seriousness as they answered the tasks using all the given time and more.

4.3.5 **On the Main Study – Euclidean Geometry Course**

**The type of research: Grounded theory using case studies**

**The design**
For Main Study, in the Euclidean Geometry Course I used a multiple methods of data collection (tests, interviews, concept maps, video recording, and field notes). This strategy (triangulation) reduces the risk of chance associations and of systematic biases due to a specific method and allows a better assessment of the generality of the explanation that is developed (Mouton, 1996). All methods are used to explore how students bring algebraic knowledge/thinking as they work with geometry at university level. Qualitative approaches (mainly Symbolic Interactionism- Grounded Theory and Case Studies) are used to construct and explain students’ profiles regarding their thinking pattern(s) as their meanings are reshaped while they exploit available tools in algebra to move the focus of their attention onto new objects and relationships in geometry.

**The subjects**

The subjects of this phase constituted 32 first-year university students at UP (the only existing class at UP for secondary and high school mathematics teacher training course) who engaged in the Diagnostic test. They came from different academic and professional (pedagogic) background (Table 4.3).

Table 4.3: Academic and professional background of subjects (Euclidean Geometry)

<table>
<thead>
<tr>
<th>Number of students/Category</th>
<th>Academic background</th>
<th>Professional background</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 / (I)</td>
<td>Grade 12</td>
<td>None</td>
</tr>
<tr>
<td>12 / (II)</td>
<td>Grade 10</td>
<td>Grade 10+2 years primary school teacher training course</td>
</tr>
<tr>
<td>3 / (III)</td>
<td>Grade 10</td>
<td>Grade 10+3 years vocational college</td>
</tr>
<tr>
<td>2/ (IV)</td>
<td>Grade 10</td>
<td>Grade 10+3 years secondary school teacher training course</td>
</tr>
<tr>
<td>1/ (V)</td>
<td>Grade 10</td>
<td>Grade 10+3 years technical college teacher training</td>
</tr>
<tr>
<td>1 / (VI)</td>
<td>Grade 12</td>
<td>Grade 10+3 years secondary school teacher training course</td>
</tr>
<tr>
<td>1 / (VII)</td>
<td>Grade 12</td>
<td>2nd year of electronic engineering</td>
</tr>
</tbody>
</table>
Fourteen students (8 low and 6 high achievers) out of thirty-two were selected for Interviewing Phase 1 after analyzing their Diagnostic test responses (Section 5.3.1). After interviewing, I selected eight target students (5 low achievers- LA and 3 high achievers- HA) amongst the thirteen (one student missed the interviewing) students as a provisional target sample (Table 4.4). Due to a data-driven approach adopted by this study I chose a sub sample of five case studies amongst the eight, one of which represented the contextual categories emerged during the process of analysis (Section 5.3.5.2 and Boyatzis, 1998). The raw data collected from this sub sample was the basis for developing the code which is supported by the data reduction method (Boyatzis, 1998) for the rest of the study. I selected these five students as my target sample because I found their interviews interesting for the purpose of my study.

Table 4.4: Academic and professional background of the provisional target sample

<table>
<thead>
<tr>
<th>Number of students/Category/Performance</th>
<th>Academic background</th>
<th>Professional background</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 / (I)/HA</td>
<td>Grade 12</td>
<td>None</td>
</tr>
<tr>
<td>1 / (I)/LA</td>
<td>Grade 12</td>
<td>None</td>
</tr>
<tr>
<td>1 / (VII)/HA</td>
<td>Grade 12</td>
<td>2nd year of electronic engineering</td>
</tr>
<tr>
<td>1 / (II)/LA</td>
<td>Grade 10</td>
<td>Grade 10+2 years primary school teacher training course</td>
</tr>
<tr>
<td>2/ (IV)/LA</td>
<td>Grade 10</td>
<td>Grade 10+3 years secondary school teacher training course</td>
</tr>
<tr>
<td>1 / (III)/LA</td>
<td>Grade 10</td>
<td>Grade 10+3 years vocational college</td>
</tr>
</tbody>
</table>

This narrowing down allowed me to carry out an in-depth exploration of how these five students bring their algebraic thinking and knowledge in working with geometry tasks.

The data collection instruments

Diagnostic test

The Diagnostic test tasks are described as follows:
Task 1:
1. Enuncie e demonstre o famoso “Teorema de Pitágoras” (sobre as medidas dos lados de um triângulo retangular).

[1. What is the Pythagorean Theorem? Please, demonstrate this theorem. (Use the right triangle).]

Task 1 had the purpose of determining how students use algebraic symbols and relations (use of variables, use of formulae, and use of other symbols, such as abbreviations and acronyms) while they demonstrate Pythagorean Theorem. At the same time this task was constructed to see how students translate verbal language into algebraic symbols and relations, representations, and to find out how they generalize the theorem for any right triangle. They are several proofs for Pythagorean Theorem, ranging from the synthetic approaches (where at each step what you say has a meaning in relation to a figure) and the analytic approach (using proportion, area formulae, algebraic expressions to facilitate transfer to a numeric or algebraic framework, which allows blind calculation) (Douady, 1998). In both approaches the presence of algebra is strongly noticed in analytic approaches than in the synthetic approaches. The fundamental idea used in synthetic approach proofs is the concept of area of squares and congruency of triangles. Besides, the students need to know how to construct and visualize squares and triangles and compare their areas towards the conclusion $a^2 = b^2 + c^2$ (Fig 4.1).

![Figure 4.1: Pythagorean Theorem](image)

Task 2:
2. Considerando a figura abaixo. Subdivide a figura em quatro partes congruentes.
In Task 2 the students could use construction and visualization processes to solve it (synthetic approach). However, they could also divide the figure into 12 congruent squares. Thereafter, they could realize that with an even number of congruent squares they could construct 4 congruent parts (analytic approach). That is 12:4=3, which means that the 4 asked congruent parts should entail 3 congruent squares each. In turn, through visualization and construction they could come up with the solution (Fig 4.2). This task may appear to be solely geometric. However, the task extends to algebraic thinking because through the analytic approach one is asked to find a pattern for the congruent figures which must entail three congruent squares.

![Division into small congruent squares](image)

**Figure 4.2: Division into small congruent squares**

**Task3:**

3. Observe the cube \([ABCDEFGH]\) below. Can you fit any additional pyramid(s) congruent to \([ABCDH]\)? If ‘yes’, draw it (them).
In Task 3, the students could similarly proceed as in Task 2. Using a synthetic approach they would visualize and construct 3 congruent pyramids inscribed in a cube or they could use an analytic approach comparing the volume of a pyramid $V_p = \frac{1}{3} A_g h$ with the volume of a cube $V_c = A_g h$, where $A_g$ is the ground area and $h$ the altitude of the pyramid of the cube. Then, they could realize that at most three inscribed congruent pyramids fit in the cube, namely [ABCDH], [ABFEH], and [BCGFH] through visualization and construction processes.

Task 4:

4. Um quadrilátero tem duas diagonais, um pentágono tem cinco, um hexágono tem ... diagonais. Determine (uma expressão para) o número total de diagonais de um polígono de $n$ lados.

Task 4 requires students to see number patterns of the diagonals of polygons and generalize the number of diagonals of any polygon. To get to this point, students need to know how to use variables (in this case the number of diagonals and the number of sides or vertexes of a polygon). To organize their thoughts they can use a table or a sequence of arrays of values $(n; d)$, where $n$ stands for the number of sides or vertexes of a polygon and $d$ for the number of its diagonals.

*Interviewing Phase 1*

During Interviewing Phase 1 I intended to find out about “knowledge connectedness” of the students from different perspectives. That is why I constructed some general questions (Appendix 3) in this regard using the same classification of questions as used by Chinnappan et al (1999). The first type of question is called Free Recall Task, which permits to derive estimates of knowledge that is functionally available to students. The second type of question is labeled as Hinting Task. This type examines the levels of connectedness of knowledge schemas, that is, “the degree to which new nodes of information is connected with one
another to form a single well-defined structure” (Chinnappan et al, 1999, p. 168). This task indexes the level of the “internal connectedness” of a schema (Lawson and Chinnappan, 2000). They explain that:

A schema with components that are effectively organized is one for which minimal levels of cueing are required for activation. When a greater level of hinting support is needed for access, we argued that the knowledge schema is either less extensive or less well connected. (p. 31)

The students were asked to explain specific aspects of their solutions in the Diagnostic test, task by task, sequentially, and wherever possible, their thinking process either through Free Recall Task or Hinting Task.

_Elaboration and Concept Mapping Task_

During this activity I also intended to find out about “knowledge connectedness” of the students from different perspectives. That is why I constructed Elaboration and Concept Mapping Task (Appendix 4). As to “external connectedness” (relationships among knowledge schemas), Elaboration Task was designed. Elaboration Task aimed to assess how different knowledge schemas that are relevant to a particular problem can be related, one to the other (Lawson and Chinnappan, 2000, p. 31). The Concept Mapping Task was used in the form of yielding true propositions using the concepts presented. The concept maps formed an entry point for probing students’ understanding of relationships among concepts as well as information about how they perceive and use those concepts to explain other related concepts (Chinnappan et al, 1999). Research has shown that concept-mapping procedure is particularly suitable for use as an interview technique in research on mathematical knowledge development (Hasemann and Mansfield, 1995).

_The data collection procedure_

_Diagnostic test_

A 60 minute Diagnostic test was administered to 32 first-year university students at UP in the beginning of Euclidean Geometry Course in March 2005. The Diagnostic test aimed at assessing the students’ proficiency in school geometry and algebra when they enter UP so as to select 14 target students for the follow-up study. Before administering the Diagnostic test
the lecturer of Euclidean Geometry explained the purpose of the Diagnostic test to the students and he gave some instructions how to proceed with the test. He also used that opportunity to introduce me to them. He said that my research work was related to the course they were taking, namely teacher training, as the purpose of my study was to attempt to improve the teaching of geometry. He added saying that their co-operation would be worthwhile to achieve that purpose and they must not feel stressed when taking the test, as it was not intended to assess the students for the course. The lecturer gave me the opportunity to address the students and I added stating that they must not be afraid of putting their names on the test sheet as this would allow me to select some students for follow study and I assured them that their names would not be displayed in the final report or be mentioned in public. During the test I noticed that the students were very co-operative, as they posed questions when they did not understand the question. Besides, most of them took more than the due time set of 60 minutes. Nobody handed over their test scripts before 60 minutes.

Interviewing Phase 1

After analyzing the students’ responses to the Diagnostic test (Section 5.3.1) fourteen students (8 low achievers and 6 high achievers) were selected for Interviewing Phase 1. The lecturer told the students that some of them would be selected for the interviewing phase. And he read their names and asked them to write down their time preference in a table previously constructed. He added saying that the non-selected students could also write down their names and the time they preferred if they wished to be interviewed. The lecturer gave me the opportunity to address the students. I contended that the selection of the students was on the basis of the time available, as I had a schedule for my research and I needed to meet some goals. I also said that if I could I would have selected all of them.

Interviewing Phase 1 ran for about three weeks. The interviews lasted from about one hour to one and half hours depending on the interviewees’ responses.
These students were individually interviewed using Free Recall and Hinting Task on their Diagnostic test responses (Appendix 3). These interviews were audio recorded and later transcribed.

The lecturer asked to take part in some interviews with the students. I explained politely to him that his presence might not make the students comfortable to express their opinions because they might fear him. In the language of research it means that the presence of the lecturer might bias the data collected in those interviews where he would be present. However, the most suitable venue available for me and for the students to administer the interviews was in the lecturer’s office as the UP infrastructures were undergoing some improvements and there was a lot of noise. Once I used the interior of my own car to interview a student because even in the office of the lecturer the noise was very disturbing.

Occasionally, one of the interview sessions was changed in terms of schedule. Instead of having one interview I had two. Because of this we collided with the lecturer’s schedule and he wanted to work in the office while I was interviewing a student. He apologized because of his presence in the office and he assured us that he would not interfere with our work. After finishing the interviews and when the students had gone away he commented that he liked the interview, especially when the student was answering the question on how he found the interview. I noticed that the student was not intimidated by the presence of the lecturer and he was a very interested and active interviewee (Section 5.3.5.7). Once again, the lecturer passed by the office while I was interviewing a student. When the interviewing finished he commented stating that he could listen to a passage of the interview which was very important data for his institution. He heard the student saying that he had never had a demonstration of the Pythagorean Theorem. Meanwhile, the school syllabuses cover that aspect. I explained to him that some students said that when they were schooling geometry was the last topic in the school syllabuses. They also said their teachers did not teach the topic stating time constraints. He ended the conversation by saying that my thesis would be a worthwhile document for his Institution.

_Elaboration and Concept Mapping Task_

This activity ran during students’ holidays. Just before the students’ holidays I planned it with the eight target students. This activity took place in my office at Eduardo Mondlane University. In the meantime, only three students participated in it. The other five attended it
in the beginning of the second semester for several reasons. The students accomplished this activity individually and they had to produce true propositions using the concepts presented. This task was related to the Concept Mapping Task. Besides they had to solve an Elaboration Task where some concepts and theorems were given to students in order to construct a task from that knowledge and to solve it mentioning the concepts and the theorems used during the solution. This activity ranged from forty seven minutes to two hours.

4.3.6 On the Main study – Analytic Geometry Course

The type of research: Grounded theory using case studies

The design

The same design used on the Main Study - Euclidean Geometry was used on the Main Study–Analytic Geometry with some changes (only used a test, interviews, video recording, and field notes) (Section 4.3.5).

The subjects

The subjects for this part of the study were the five target students selected during the Main Study – Euclidean Geometry.

The data collection instruments

Lecturer’s Test

The lecturer of the course administered the first test to the students (Appendix 5). I collected the scripts of the five target students to analyse their written responses. The first test consisted in some equivalent sub-tests. Each student had to take only one sub-test. That is why I described some tasks of the sub-test I and sub-test II.
1. Consider the following vectors \( \vec{u} = 8\vec{i} + 2\vec{j} \) and \( \vec{v} = 3\vec{i} - 4\vec{j} \).
   a) Determine the dot product of these vectors.
   b) Determine the angle between these vectors.
   c) Represent the vector \( \vec{w} = (-2; 9) \) through the basis \( \vec{u} \) and \( \vec{v} \).

This is a task of vector algebra. The first two questions require one to know the formulae of the dot product and the angle between two vectors. In the latter question one needs to know the definition of a basis for a vector and that any other vector is expressible through this basis.

2. Knowing that the vectors \( \vec{a} \) and \( \vec{b} \) form (the angle) \( \beta = \frac{2}{3}\pi \) and (their lengths are)

   \[ \|\vec{a}\| = 3 \text{ and } \|\vec{b}\| = 4 \], determine:
a) \( (\vec{a} + \vec{b})^2 \)
   b) \( \vec{a} \times \vec{b} \)

This is also a task of vector algebra. In the first question the student should know the properties of the dot product before applying the definition of it. And in the second question they had to apply the definition of the norm of the cross product.

3. Determine the angle formed by the segments AB and CD, given (the points) A(-3; 2; 4), B(2; 5; -2), C(1; -2; 2), and D(4; 2; 3).

This is an application of vector algebra to a geometry task. The students should determine the vectors \( \vec{AB} \) and \( \vec{CD} \). Thereafter they should determine the angle between them which is, consequently, the angle between the segments AB and CD.
The latter two tasks are also an application of vector algebra to geometry.

4. Determine the general equation of the plane \( \pi \) passing through point \( A(3,3,-4) \), where \( \frac{3}{13}, \frac{12}{13}, \frac{4}{13} \) are the direction cosines of the normal vector.

The students should know the general equation of a plane \( Ax + By + Cz + D = 0 \) and the elements present in it and they should apply the givens in it. Or they should use the point-normal form of the equation of a plane \( A(x-x_o) + B(y-y_o) + C(z-z_o) = 0 \).

5. The distance between the origin (of a coordinate system) to the plane \( \delta \) is 5 and the normal vector coordinates are \((-2; 6; 3)\). Determine:
   a) Determine the general equation of this plane;
   b) Determine the equation of this plane in the form of a vector formula.

To solve this task the students should use the distance formula between the origin of the coordinate system and the plane \( \delta \). This requires one to make algebraic transformations using absolute value. Then the students should apply the values of the parameter \( D \) (in the general equation of a plane \( Ax + By + Cz + D = 0 \)) obtained from these transformations and the given to get the general equation of the plane. For the latter question the students should know the elements involved in the equation of the plane in the form of a vector formula and they should determine the coordinates of two non-collinear vectors parallel to this plane or lying on it and they should replace them in the formula \((x, y, z) = (x_0, y_0, z_0) + \mu(x_1, y_1, z_1) + \lambda(x_2, y_2, z_2)\), where \( \mu, \lambda \in \mathbb{R} \).

Interviewing Phase 2

Similarly, I used Free Recall and Hinting Task (Appendix 3) adapted to the students’ written responses to the lecturer’s test.
The data collection procedure

Lecturer’s Test

The lecturer of the course administered the first test to the students. I was not present when the students took this test. I collected the scripts of the five target students for analysis and for further interviewing.

Interviewing Phase 2

I individually interviewed the five target students regarding their written responses to the lecturer’s test. The interviews lasted between 41 minutes to 1.30 h depending on the responses the students gave during the interview. These interviews were audio recorded and later transcribed.

4.3 Summary

This chapter focused on the methodology for the study. It detailed how the sample of the target students for the study was selected and how the data were collected. The use of the instruments (tests, elaboration task, and concept mapping task) and interviews with the target students were also highlighted. The criteria for quality in this research were mentioned and briefly the method of data analysis was explained. The next chapter focuses on the data analysis and interpretation.