FRACTIONAL CURRENT EFFICIENCY VS FEED FLOWRATE

FIGURE 14 - Effect of Feed Flowrate

Remaining feed and system parameters are the same as in Table 6. The section indicated on the right-hand side of the arrow cannot be realised in practice as Mn(OH)₂ precipitation would occur with consequent drop in fractional current efficiency.
3.2.2 - Influence of pH on Fractional Current Efficiency

Figure 15 Shows the effect of changing the catholyte pH on fractional current efficiency. As expected, the current efficiency increases with pH up to a point where manganese hydroxide \( \text{Mn(OH)}_2 \) precipitates. When this condition occurs it is not practical to increase the pH any further as the overall plant performance will decrease drastically.

Figure 16 shows the results as published by MINTEK (6). The lower experimental current efficiencies were previously explained as being due to the way in which the MINTEK operations were performed. Higher current efficiencies are in fact found on industrial plants with a similar trend to that predicted.

<table>
<thead>
<tr>
<th>Fractional Current Efficiency</th>
<th>Catholyte pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.617</td>
<td>7.2</td>
</tr>
<tr>
<td>0.666</td>
<td>7.4</td>
</tr>
<tr>
<td>0.685</td>
<td>7.5</td>
</tr>
<tr>
<td>0.702</td>
<td>7.6</td>
</tr>
<tr>
<td>0.718</td>
<td>7.7</td>
</tr>
<tr>
<td>0.731</td>
<td>7.8</td>
</tr>
<tr>
<td>0.752</td>
<td>8.0</td>
</tr>
</tbody>
</table>
FIGURE 15 - FRACTIONAL CURRENT EFFICIENCY VS CATHOLYTE PH

Feed Conditions
- \( T_f = 39^\circ \text{C}; \) \( \text{pH}_f = 6.88 \)
- \( X_M = 32.16 \text{ kg/m}^3 \)
- \( X_a = 28.57 \text{ kg/m}^3 \)

Precipitation of \( \text{Mn(OH)}_2 \)
FIGURE 16 - Influence of pH on Current Efficiency as published by MILTER (6)

EMCOR acidified to pH 7 by H₂SO₄ and brought to pH 9.2 by NH₃
FIGURE 16 - Influence of pH on Current Efficiency as published by MINTEK (6)

EMCOR acidified to pH 7 by H₂SO₄ and brought to pH 9.2 by NH₃

Current efficiency, %

pH

7 8 9
3.2.3 - The Influence of Ammonium Sulphate

Ammonium sulphate is added to the electrolyte to increase the conductivity and also to buffer the solution, as mentioned in section 1.3.2.

Figure 17 shows this influence, where a minimum amount of ammonium sulphate is required before the buffering action of this compound is felt, at a given catholyte pH. The fractional current efficiency decreases with increasing concentration of ammonium sulphate. Therefore the optimum lies in the interrelated effect of these two variables, as shown in section 3.2.7.

TABLE 9 - Ammonium Sulphate Concentration versus Fractional Current Efficiency

<table>
<thead>
<tr>
<th>Concentration of Ammonium Sulphate (as NH₃ - kg/m³)</th>
<th>Fractional Current Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.747</td>
</tr>
<tr>
<td>15</td>
<td>0.735</td>
</tr>
<tr>
<td>17</td>
<td>0.730</td>
</tr>
<tr>
<td>20</td>
<td>0.723</td>
</tr>
<tr>
<td>26</td>
<td>0.709</td>
</tr>
<tr>
<td>28,57</td>
<td>0.702</td>
</tr>
<tr>
<td>30</td>
<td>0.699</td>
</tr>
<tr>
<td>35</td>
<td>0.687</td>
</tr>
<tr>
<td>40</td>
<td>0.674</td>
</tr>
</tbody>
</table>
FIGURE 17 - FRACT. CURRENT EFF. VS AMMONIUM SULPHATE CONC.

Feed conditions
pH = 6.88
T_f = 39°C
X_M = 32.16 kg/m³

Precipitation of Mn(OH)_2
3.2.4 - The Effect of Total Impurity Level on the Fractional Current Efficiency

Figure 18 reveals that ultrapure electrolytes may not substantially increase the current efficiency. According to the model, by extrapolating the curve to 0 ppm of total impurity level, the maximum efficiency obtainable is 0.74 compared to 0.716 for a total impurity level of 1 ppm, as shown in Table 10. To reduce the impurity level from 3 ppm to 1 ppm can be achieved with relative ease in practice, but purification below a ppm would probably increase the cost of this operation to a level far beyond the possible benefits that could be achieved through marginally higher current efficiencies.

It should be noted that it was only possible to consider the effect of total impurity level, and not the effect of the individual impurities.

<table>
<thead>
<tr>
<th>Fractional Current Efficiency</th>
<th>Total Impurity Level (ppm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.711</td>
<td>0.06</td>
</tr>
<tr>
<td>0.728</td>
<td>0.50</td>
</tr>
<tr>
<td>0.716</td>
<td>1.00</td>
</tr>
<tr>
<td>0.702</td>
<td>1.56</td>
</tr>
<tr>
<td>0.693</td>
<td>2.00</td>
</tr>
<tr>
<td>0.660</td>
<td>4.00</td>
</tr>
<tr>
<td>0.631</td>
<td>6.00</td>
</tr>
</tbody>
</table>
FIGURE 18 - FRACT. CURRENT EFF. VS TOTAL IMPURITY LEVEL

Feed Conditions as in Table 6
3.2.5 - The Effect of Feed Temperature on Fractional Current Efficiency

Temperature is known to be a very important parameter in the electrolytic deposition of manganese. Table 11 and Figure 19 show that, for every feed condition, there is an optimum temperature at which the current efficiency will be maximized. However, that optimum lies very near the point of manganese hydroxide \( \text{Mn(OH)}_2 \) deposition, as shown by the dotted line. The model, as it can be seen by Figure 19, shows a steeper loss of fractional current efficiency at higher temperatures due to the lowering of the lowest pH at which precipitation of \( \text{Mn(OH)}_2 \) will occur. The MINTEK (6) results are also plotted on the same graph, and again, the current efficiencies are always lower than the ones obtained at Delta, and predicted by the model. However, the trend is the same as well as the optimum temperature reported.

**TABLE 11 - Fractional Current Efficiency versus Temperature**

<table>
<thead>
<tr>
<th>Feed Temperature (°C)</th>
<th>Fractional Current Efficiency</th>
<th>Catholyte pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>0.679</td>
<td>7.5</td>
</tr>
<tr>
<td>31</td>
<td>0.688</td>
<td>7.6</td>
</tr>
<tr>
<td>33</td>
<td>0.695</td>
<td>7.6</td>
</tr>
<tr>
<td>35</td>
<td>0.700</td>
<td>7.6</td>
</tr>
<tr>
<td>39</td>
<td>0.706</td>
<td>7.63</td>
</tr>
<tr>
<td>43</td>
<td>0.685</td>
<td>7.5</td>
</tr>
<tr>
<td>45</td>
<td>0.670</td>
<td>7.45</td>
</tr>
<tr>
<td>47</td>
<td>0.659</td>
<td>7.4</td>
</tr>
<tr>
<td>49</td>
<td>0.640</td>
<td>7.3</td>
</tr>
</tbody>
</table>
Figure 19 - Fract. Current Eff. vs Feed Temperature

Feed Conditions as in Table 6, other than temperature

- Model Predictions
- + Mintek Data (CD)

Model Results with pH = 7.6 where precipitation of Mn(OH)$_2$ would occur

Model Results where precipitation of Mn(OH)$_2$ would not occur
3.2.6 - The Effect of Cathodic Current Density on Fractional Current Efficiency

Figure 20 shows that there is an optimum current density at which the fractional current efficiency will be maximized. Both model prediction and the MINTEK (6) results agree on the figure of 550 A/m². When plotting the model prediction results, the catholyte pH was always chosen as the maximum value that could be used without precipitating manganese hydroxide.

TABLE 12 - Cathodic Current Density versus Fractional Current Efficiency

<table>
<thead>
<tr>
<th>Cathodic Current Density (A/m²)</th>
<th>Fractional Current Efficiency</th>
<th>Catholyte pH</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.688</td>
<td>7.35</td>
</tr>
<tr>
<td>450</td>
<td>0.695</td>
<td>7.4</td>
</tr>
<tr>
<td>500</td>
<td>0.703</td>
<td>7.50</td>
</tr>
<tr>
<td>555</td>
<td>0.706</td>
<td>7.63</td>
</tr>
<tr>
<td>600</td>
<td>0.685</td>
<td>7.67</td>
</tr>
<tr>
<td>650</td>
<td>0.656</td>
<td>7.70</td>
</tr>
<tr>
<td>700</td>
<td>0.624</td>
<td>7.72</td>
</tr>
</tbody>
</table>
3.2.7 - The Limiting Catholyte pH versus the Ammonium Sulphate Concentration

The limiting catholyte pH, i.e. the highest pH at which no manganese hydroxide precipitation will occur, is plotted against the concentration of ammonium sulphate, in Figure 21, showing the lines of fractional current efficiency. If no conductivity improvements were present by the addition of $(\text{NH}_4)_2\text{SO}_4$, the lower the concentration of this compound and the lower the pH would produce the highest current efficiencies. However it was found that the energy requirements outweigh this increase in cathodic current efficiency, and a minimum amount of ammonium sulphate is required.

Practice shows that an amount of 25 - .5 kg/m³ (given as NH₃) is required for the increase of electrical conductivity of the solution.

<table>
<thead>
<tr>
<th>Ammonium Sulphate Concentration (kg/m³)</th>
<th>Limiting Catholyte pH</th>
<th>Fractional Current Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 (as NH₃)</td>
<td>7.58</td>
<td>0.745</td>
</tr>
<tr>
<td>15</td>
<td>7.60</td>
<td>0.730</td>
</tr>
<tr>
<td>20</td>
<td>7.613</td>
<td>0.723</td>
</tr>
<tr>
<td>25</td>
<td>7.625</td>
<td>0.714</td>
</tr>
<tr>
<td>30</td>
<td>7.635</td>
<td>0.704</td>
</tr>
<tr>
<td>35</td>
<td>7.646</td>
<td>0.694</td>
</tr>
<tr>
<td>40</td>
<td>7.657</td>
<td>0.683</td>
</tr>
</tbody>
</table>
Figure 21 - Limiting Catholyte pH vs Ammonium Sulphate Conc.

$T_e = 39^\circ C$

$\text{Mn(OH)}_2 \text{ will not precipitate}$

$\text{Mn(OH)}_2 \text{ will precipitate}$

Constant Current Efficiency Lines

Limiting Catholyte pH

Ammonium Sulphate (kg/m$^3$ as NH$_3$)
4 - REFERENCES

(1) Oliver and Boyd: 'Manganese Steel', Hadfield Ltd, London, 1956


(7) Jacobs, J. H., Churchward P E, Knickerbocker R G: 'The Effect of certain Variables on the Electrodeposition of Manganese' A paper presented at the 86th General Meeting held at Buffalo, N.York, October 14, 1944 (US Bureau of Mines)


APPENDIX A

Computer Model
Computer Model (continued)

[Text is not legible and cannot be transcribed accurately]
Computer Model (continued)
APPENDIX B

Typical Mass Balance Calculations

Given: \( T_F = 39^\circ C = 312 \, K \)
- \( pH_F = 6.88 \)
- \( X_M = 32.16 \, \text{kg/m}^3 = 0.5854 \, \text{kmole/m}^3 \)
- \( X_A = 28.57 \, \text{kg/m}^3 = 1.6806 \, \text{kmole/m}^3 \)
- \( Q_F = 0.000552 \, \text{m}^3/s = 1.9872 \, \text{m}^3/h \)
- \( E_a = 0.99 \)
- \( E_C = 0.702 \)

Basis - 1 hour

Total Mass input = 1185 x 1.9872 = 2354,8320 kg/hr

TABLE 14 - Feed Input - Mass Balance

<table>
<thead>
<tr>
<th>Component</th>
<th>Concentration</th>
<th>kmoles/hr</th>
<th>Kg/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Mn}^{2+} ) (aq)</td>
<td>0.5399</td>
<td>1.0610</td>
<td>58,2895</td>
</tr>
<tr>
<td>( \text{H}^+ ) (aq)</td>
<td>1.3182 x 10^{-7}</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( \text{NH}_4^+ ) (aq)</td>
<td>1.6114</td>
<td>3.2022</td>
<td>57,6396</td>
</tr>
<tr>
<td>( \text{MnNH}_3^{2+} ) (aq)</td>
<td>0.05147</td>
<td>0.1023</td>
<td>7,3581</td>
</tr>
<tr>
<td>( \text{NH}_3 ) (aq)</td>
<td>0.01772</td>
<td>0.0352</td>
<td>0.5986</td>
</tr>
<tr>
<td>( \text{SO}_4^{2-} ) (aq)</td>
<td>1.39106</td>
<td>2.7643</td>
<td>265,5445</td>
</tr>
<tr>
<td>( \text{HSO}_4^- ) (aq)</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Impurities</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( \text{H}_2\text{O} ) (aq)</td>
<td>54.9509</td>
<td>109,0963</td>
<td>1965,4017</td>
</tr>
</tbody>
</table>

TOTAL

116,2613   2354,8320

* The ion concentrations were calculated as in Appendix B.1
### TABLE 15 - Products - Mass Balance

<table>
<thead>
<tr>
<th>Components</th>
<th>kmoles/hr</th>
<th>kg/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn(s)</td>
<td>0.7333</td>
<td>40,2864</td>
</tr>
<tr>
<td>H₂(g)</td>
<td>0.3113</td>
<td>0.6275</td>
</tr>
<tr>
<td>O₂(g)</td>
<td>0.5171</td>
<td>16.5452</td>
</tr>
<tr>
<td>MnO₂(s)</td>
<td>0.0145</td>
<td>1.2609</td>
</tr>
<tr>
<td></td>
<td><strong>1.5762</strong></td>
<td><strong>58.8200</strong></td>
</tr>
</tbody>
</table>

These figures are calculated using the equations below

\[
P_{\text{Mn}} = \frac{E_c I}{2 F} \cdot 54.94 \times 10^{-3} \text{ (kg/s)} \quad (2.58)
\]

\[
P_{\text{H}_2} = \frac{(1 - E_c) I}{2 F} \cdot 2.0139 \times 10^{-3} \text{ (kg/s)} \quad (2.59)
\]

\[
P_{\text{O}_2} = \frac{E_a I}{4 F} \cdot 31.9988 \times 10^{-3} \text{ (kg/s)} \quad (2.60)
\]

\[
P_{\text{MnO}_2} = \frac{(1 - E_a) I}{2 F} \cdot 86.9388 \times 10^{-3} \text{ (kg/s)} \quad (2.61)
\]

In this typical set of results, using \( I = 56000 \, \text{A} \)

\[
P_{\text{Mn}} = 0.0112 \, \text{kg/s} = 40.2864 \, \text{kg/hr}
\]

\[
P_{\text{H}_2} = 1.7431 \times 10^{-4} \, \text{kg/s} = 0.6275 \, \text{kg/hr}
\]

\[
P_{\text{O}_2} = 0.0046 \, \text{kg/s} = 16.5452 \, \text{kg/hr}
\]

\[
P_{\text{MnO}_2} = 3.5026 \times 10^{-4} \, \text{kg/s} = 1.2609 \, \text{kg/hr}
\]
<table>
<thead>
<tr>
<th>Components</th>
<th>kmoles/hr</th>
<th>kg/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>H₂O(catholyte)</td>
<td>0.03276</td>
<td>0.59026</td>
</tr>
<tr>
<td>H₂O(anolyte)</td>
<td>0.04051</td>
<td>0.7297</td>
</tr>
<tr>
<td>NH₃(catholyte)</td>
<td>0.10699</td>
<td>1.8188</td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.18026</td>
<td>3.13876</td>
</tr>
</tbody>
</table>

These losses are calculated using the equations in section (2.6), i.e. equations (2.50) to (2.55). These yield the following figures for the typical values:

\[
\begin{align*}
Y_3 &= 1.09817 \times 10^{-3} \\
Y_3 &= 2.3720 \times 10^{-3} \\
T_c &= 40^\circ C = 313K \\
y_o &= 0.07264
\end{align*}
\]

Thus

- kmoles H₂O lost from catholyte = 0.03276 kmoles/hr = 0.59026 kg/hr
- kmoles NH₃ lost from catholyte = 0.10699 kmoles/hr = 1.81883 kg/hr
- kmoles H₂O lost from anolyte = 0.04051 kmoles/hr = 0.7279 kg/hr
Anolyte Discharge

Section 2.7 explains how the mass balance is performed in order to calculate the anolyte discharge. Equation (2.57) is the overall mass balance equation, and yields, with an anolyte density of 1160 kg/m³, and the values from Tables 15 and 1:

\[ \rho_a Q_a = 2293.1732 \text{ kg/hr} \]

\[ Q_a = 1.9769 \text{ m}^3/\text{hr} \]

For each component, namely manganese, ammonia, and sulphate, a mass balance is also performed to determine the total amounts of the above species present in the anolyte. These are:

**Manganese Balance**

\[ 54.94 \times Z_M Q_a = 54.94 \times X_M Q_f - P_{Mn} - P_{MnO_2} \]

**Ammonia Balance**

\[ 17.0 \times Z_A Q_a = 0.17.0 \times X_A Q_f - L_{NH_3} \]

**Sulphate Balance**

\[ (Z_6 + Z_7) \times Q_a = X_6 Q_f \]

In this typical example, these equations yield:

\[ Z_M = 0.2089 \text{ kmole/m}^3 \]

\[ Z_A = 1.6356 \text{ kmole/m}^3 \]

\[ Z_6 + Z_7 = 1.3983 \text{ kmole/m}^3 \]
Using the method explained in Appendix B.2 the various anolyte species concentrations are determined, and then, their individual discharge rates calculated, as shown in Table 17 below.

<table>
<thead>
<tr>
<th>Component</th>
<th>Concentration (kmole/m³)</th>
<th>kmoles/hr</th>
<th>kg/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn²⁺(aq)</td>
<td>0.20887</td>
<td>0.4129</td>
<td>22.6856</td>
</tr>
<tr>
<td>H⁺(aq)</td>
<td>0.22841</td>
<td>0.4515</td>
<td>0.4551</td>
</tr>
<tr>
<td>NH₄⁺(aq)</td>
<td>1.63560</td>
<td>3.2334</td>
<td>58.2015</td>
</tr>
<tr>
<td>NH₃(aq)</td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>MnNH₃⁺₂(aq)</td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>SO₄²⁻(aq)</td>
<td>0.88344</td>
<td>1.7465</td>
<td>167.7689</td>
</tr>
<tr>
<td>HSO₄⁻(aq)</td>
<td>0.51488</td>
<td>1.0179</td>
<td>98.8029</td>
</tr>
<tr>
<td>Impurities</td>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>H₂O (aq)</td>
<td>54.67238</td>
<td>107.9782</td>
<td>1945.2591</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>114.8404</strong></td>
<td><strong>2293.1732</strong></td>
<td></td>
</tr>
</tbody>
</table>

The following Table can be constructed using all the previous results, from Tables 14 to 17.
<table>
<thead>
<tr>
<th>Description</th>
<th>IN</th>
<th>OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Feed Input</strong></td>
<td>Kmoles/hr</td>
<td>kg/hr</td>
</tr>
<tr>
<td>Mn(^{2+})(aq)</td>
<td>10610</td>
<td>58,2895</td>
</tr>
<tr>
<td>H(^+)(aq)</td>
<td>0,0000</td>
<td>0,0000</td>
</tr>
<tr>
<td>NH(_4^+)(aq)</td>
<td>3,2022</td>
<td>57,6396</td>
</tr>
<tr>
<td>MnNH(_2^+)(aq)</td>
<td>0,1023</td>
<td>7,3581</td>
</tr>
<tr>
<td>NH(_3)(aq)</td>
<td>0,0352</td>
<td>0,5986</td>
</tr>
<tr>
<td>SO(_4^{2-})(aq)</td>
<td>2,7643</td>
<td>265,5445</td>
</tr>
<tr>
<td>H(_2)O(aq)</td>
<td>109,0963</td>
<td>1965,4017</td>
</tr>
<tr>
<td><strong>Products</strong></td>
<td>116,2613</td>
<td>2354,8320</td>
</tr>
<tr>
<td>Mn(s)</td>
<td>0,7333</td>
<td>40,2864</td>
</tr>
<tr>
<td>H(_2)(g)</td>
<td>0,3113</td>
<td>0,6275</td>
</tr>
<tr>
<td>O(_2)(g)</td>
<td>0,5171</td>
<td>16,5452</td>
</tr>
<tr>
<td>MnO(_2)(s)</td>
<td>0,0145</td>
<td>1,2609</td>
</tr>
<tr>
<td><strong>Losses</strong></td>
<td>1,5762</td>
<td>98,8200</td>
</tr>
<tr>
<td>H(_2)O(catholyte)</td>
<td>0,03276</td>
<td>0,59026</td>
</tr>
<tr>
<td>H(_2)O(anolyte)</td>
<td>0,04051</td>
<td>0,72970</td>
</tr>
<tr>
<td>NH(_3)(catholyte)</td>
<td>0,10699</td>
<td>1,81880</td>
</tr>
<tr>
<td><strong>Anolyte Discharge</strong></td>
<td>0,18026</td>
<td>3,13876</td>
</tr>
<tr>
<td>Mn(^{2+})(aq)</td>
<td>0,4129</td>
<td>22,6956</td>
</tr>
<tr>
<td>H(^+)(aq)</td>
<td>0,4515</td>
<td>0,4551</td>
</tr>
<tr>
<td>NH(_4^+)(aq)</td>
<td>3,2334</td>
<td>58,2015</td>
</tr>
<tr>
<td>SO(_4^{2-})(aq)</td>
<td>1,7465</td>
<td>167,7669</td>
</tr>
<tr>
<td>HSO(_4^{2-})(aq)</td>
<td>1,0179</td>
<td>98,8029</td>
</tr>
<tr>
<td>H(_2)O(aq)</td>
<td>107,9782</td>
<td>1945,2591</td>
</tr>
<tr>
<td><strong>TOTAL(to the nearest integer)</strong></td>
<td>116</td>
<td>2335</td>
</tr>
</tbody>
</table>
Calculating the Catholyte Conditions

The catholyte temperature is estimated using the approximations:

\[ T_c = 0.7 T_f + 0.3 T_A \]  

(2.69)

Once the temperature is known, then the catholyte conditions are determined using the diaphragm model and the equilibrium ratio equations (2.2), (2.4) and (2.6). The method is the same as for the feed conditions, as explained in Appendix B.1.

Using equation (2.13), with \( Y_2 \approx 0 \) the following values for \( Y_a \) and \( W_2 \) are found:

\[ Y_a = 1.9723 \, \text{kmole/m}^3 \]
\[ W_2 = -1.779453 \times 10^{-4} \, \text{kmole/s} \]

Now \( Y_2 = 10^{-7.6} \), as this is the figure that balances \( W_2 \) with the amount of \( H^+ \) required to produce \( H_2 \) and to react in the equations below:

\[ \text{NH}_3 + H^+ \rightleftharpoons \text{NH}_4^+ \]  

(2.3)

\[ \text{MnNH}_3^{2+} + H^+ \rightleftharpoons \text{Mn}^{2+} + \text{NH}_4^+ \]  

(2.1)

Now \( Y_M \approx 2M \) and having this, the pH and \( Y_a \) it is possible to find the individual concentrations of the species using the same method as in Appendix B.1.

The typical catholyte concentrations then become:

\[ Y_1 = 6.13138 \, \text{kmole/m}^3 \]
\[ Y_2 = 10^{-7.6} \, \text{kmole/m}^3 \]
\[ Y_3 = 1.784995 \, \text{kmole/m}^3 \]
\[ Y_4 = 0.0775 \, \text{kmole/m}^3 \]
\[ Y_5 = 0.10982 \, \text{kmole/m}^3 \]
\[ Y_6 = 1.10136 \, \text{kmole/m}^3 \]
\[ Y_7 = 2.6 \times 10^{-2} \, \text{kmole/m}^3 \]
Solving for the Feed Ion Concentration

The following equations apply to the feed conditions, as they relate the individual ion concentrations to the total amounts of manganese and ammonia.

**Manganese**

\[ X_M = X_1 + X_4 \]  \hspace{1cm} (B.1)

**Ammonia**

\[ X_a = X_3 + X_4 + X_5 \]  \hspace{1cm} (B.2)

The electrical neutrality equation, as previously explained, holds for the feed solution as well, and it is

\[ 2X_1 + X_2 + X_3 + 2X_4 - 2X_6 = 0 \]

As \( X_2 = 10^{-pH} \approx 0 \) when compared to the other concentrations, the above equation simplifies to

\[ 2X_1 + X_3 + 2X_4 - 2X_6 = 0 \]  \hspace{1cm} (B.3)

**Solution Equilibria**

At the feed, the equilibrium conditions as stated below apply

\[ \text{Mn}^{2+} + \text{NH}_4^+ \rightleftharpoons \text{MnNH}_3^{2+} + \text{H}^+ \]
where

\[ K_c' = \frac{X_2}{X_1 \times X_3} = \exp(-34.57 + 0.051 T) \]  

(B.4)

\[ NH_3 + H^+ \xrightarrow{K_a'} NH_4^+ \]

and

\[ K_a' = \frac{X_3}{X_2 \times X_5} = \exp(40.32 + 0.064 T) \]  

(B.5)

The precipitation of manganese hydroxide \( \text{Mn(OH)}_2 \) is detrimental to the electrodeposition process

\[ \text{Mn}^{2+} + 2 \text{H}_2\text{O} \xrightarrow{K_s'} \text{Mn(OH)}_2 + 2 \text{H}^+ \]

where

\[ K_s' = \frac{X_2^2}{X_1 s} = \exp(-65.17 + 0.1025) \]  

(B.6)

Defining

\[ \Delta X_1 = \frac{X_2^2}{K_s'} - X_1 \]

Then the precipitation of manganese hydroxide will occur if

\[ \Delta X_1 < 0 \]  

(B.7)

Given equations (B.1) to (B.5) then it is needed to solve for \( X_1, X_3, X_4, X_5 \) and \( X_6, X_2 \) is known from the pH of the solution.
Method

\[ x_2 = 10^{-pH} \]

Write \( x_3, x_4 \) and \( x_5 \) as functions of \( x_1 \). From equation (B.5) put \( x_5 = f(x_3) \)

\[ x_5 = \frac{x_3}{K_a X_2} \quad \text{(B.8)} \]

Subtracting equation (B.2) from (B.1)

\[ x_a - x_m = x_3 + x_5 - x_1 \]

Substituting (B.8) in the above equation we obtain

\[ x_a - x_m = x_3 + \frac{x_3}{K_a X_2} - x_1 \]

i.e.

\[ x_a - x_m = x_3(1 + \frac{1}{K_a X_2}) - x_1 \]

Now let

\[ U = \frac{1}{1 + \frac{1}{K_a X_2}} \]

then

\[ x_3 = \left[ (x_a - x_m) + x_1 \right] . U \quad \text{(B.9)} \]

From equation (B.1)

\[ x_4 = x_m - x_1 \quad \text{(B.10)} \]

Substituting in equation (B.4) we can solve for \( x_1 \) as follows:
Let

\[ v = (X_a - X_M) \cdot u \]

and

\[ w = \frac{U \cdot K'_C}{X_2} \]

and

\[ z = \frac{K'_C \cdot v}{X_2} + 1 \]

the obtaining the following equation in \( X_1 \)

\[ w \cdot X_1^2 + z \cdot X_1 - X_M = 0 \]

which can be solved for \( X_1 \)

\[ X_1 = \frac{-z \pm \sqrt{z^2 + 4 \cdot w \cdot X_M}}{2 \cdot w} \]

(3.11)

Then equations (B.8), (B.9), (B.10) and (B.3) give \( X_4 \), \( X_3 \), \( X_5 \) and \( X_6 \) respectively.

Typical set of results

Using the following feed conditions

\[ T_f = 39^\circ\text{C} = 312 \text{ K} \]
\[ X_M = 32,16 \text{ kg/m}^3 = 0,3854 \text{ kmole/m}^3 \]
\[ X_a = 28,57 \text{ kg/m}^3 = 1,6806 \text{ kmole/m}^3 \]
\[ p_{\text{H}_2} = 6,66 \]

The ion concentrations of the feed will be

\[ X_1 = 0,3333 \text{ kmole/m}^3 \quad X_5 = 1,772 \times 10^{-2} \text{ kmole/m}^3 \]
\[ X_3 = 1,6115 \text{ kmole/m}^3 \quad X_6 = 1,3905 \text{ kmole/m}^3 \]
\[ X_4 = 0,0514 \text{ kmole/m}^3 \quad X_2 = 1,3183 \times 10^{-7} \text{ kmole/m}^3 \]
\[ \Delta X_1 = 2,7833 > 0 \text{ i.e. Mn(OH)}_2 \text{ will not precipitate} \]
APPENDIX B.2

Solving for the Anolyte Concentrations

The following are the chemical reactions taking place in the anolyte, as previously cited.

\[ \text{HSO}_3^- \xrightarrow{K_h'} H^+ + \text{SO}_4^{2-} \]

where

\[ K_h' = \frac{Z_2 Z_6}{Z_7} = \exp(8.339 - 0.0294 T) \quad (B.12) \]

All the other species \( \text{NH}_3 \) and \( \text{MnNH}_3^{2+} \) do not exist in the anolyte, due to the pH. However, the component balances and electrical neutrality equation still apply.

**Manganese Balance**

\[ Z_1 = Z_M \quad (B.13) \]

**Ammonia Balance**

\[ Z_3 = Z_a \quad (B.14) \]

**Sulphate Balance**

\[ S = \frac{Q_a X_6}{Q_6} \quad (B.15) \]

and

\[ S = Z_6 + Z_7 \quad (B.16) \]

**Electrical neutrality**

\[ 2Z_1 + Z_2 + Z_3 - 2Z_6 - Z_7 = 0 \quad (B.17) \]
Method for solving

The above equation (B.17) is solved for $Z_6$, using equation (B.16) as an intermediate, i.e.

$$Z_6 = 2Z_1 + Z_2 + Z_3 - S$$  \hspace{1cm} (B.18)

then, using equation (B.16) again and solving for $Z_7$ as a function of $Z_2$ gives

$$Z_7 = S - Z_6 = 2S - Z_1 - Z_2 - Z_3$$  \hspace{1cm} (B.19)

Substituting equations (B.18) and (B.19) in equation (B.12) a quadratic in $Z_2$ is obtained as follows

$$Z_2 = \frac{-U + \sqrt{U^2 + 4V}}{2}$$  \hspace{1cm} (B.20)

where

$$U = 2Z_1 + Z_3 - S + K_h'$$  \hspace{1cm} (B.21)

and

$$V = (2S - 2Z_1 - Z_3) K_h'$$  \hspace{1cm} (B.22)

Once $Z_2$ is obtained from equation (B.20) then it is easy to calculate the other ion concentrations using equations (B.18) and (B.19), since $Z_1$ and $Z_3$ are obtained from equations (B.13) and (B.14) directly.

Typical Set of Results

In our example, the analyte conditions become

$Z_{H^+} = 11,475 \text{ kg/m}^3$

$Z_a = 27,303 \text{ kg/m}^3$
and the individual ion concentrations are

\[ Z_1 = 0.2089 \text{ kmole/m}^3 \]
\[ Z_2 = 2.2841 \times 10^{-1} \text{ kmole/m}^3 \]
\[ Z_3 = 1.6356 \text{ kmole/m}^3 \]
\[ Z_6 = 0.8834 \text{ kmole/m}^3 \]
\[ Z_7 = 0.51487 \text{ kmole/m}^3 \]
\[ Z_0 = 54.6724 \text{ kmole/m}^3 \]

Thus

\[ \text{pH}_a = 0.6413 \]

and

\[ T_a = 42.5^\circ C \]
APPENDIX C

Typical Energy Balance Calculations

The overall energy balance in kJ/a is

$$Q_f \cdot X_f \cdot H_j + \frac{\sum \alpha_i}{1000} = Q_a \cdot Z_j \cdot E_j + \sum M_j \cdot H_j + q_0 + q_1$$  \hspace{1cm} (2.64)

where

$$H_j = \Delta H^0_{f_j} + (T - 298) \cdot C^*_{pj}$$  \hspace{1cm} (2.65)

Table 5 in section 2.8 gives Molar Enthalpies of Formation and Molar Specific Heats for the necessary species.

The energy removed by the cooling coils is given by

$$q_c = 4184 \cdot Q_o \cdot (T_2 - T_1)$$  \hspace{1cm} (2.67)

The plant data from Delta allowed the calculation of an empirical relationship between the feed and anolyte temperatures to estimate the catholyte temperature

$$T_c = 0.7 \cdot T_f + 0.3 \cdot T_a$$  \hspace{1cm} (2.69)

Using Table 14, from Appendix B, in conjunction with Table 5 in section 2.8, it is possible to calculate the energy input in the feed stream, using equation (2.66).
The power input term is

\[ \frac{V I}{1000} \times 3600 = \frac{6.52 \times 56000 \times 3600}{1000} = 1.314432 \times 10^6 \text{ kJ/hr} \]

because the voltage is \( V = 6.52 \text{V} \) and the current 56000A.

Thus the total energy input term is

\[ 35,588446 \times 10^6 \text{ kJ/hr} \left( 9,8856794 \times 10^3 \text{ kW} \right) \]
For an anolyte temperature of 42,5°C, a similar procedure is now followed to estimate the energy taken by the discharge stream. Tables 17 (Appendix B) and 5 (section 2.8) are used for this calculation, as shown below in Table 20.

### Table 20 - Energy Output through the Discharge Stream

<table>
<thead>
<tr>
<th>Component</th>
<th>( \dot{n}_j ) ( \bar{z}_j ) mol/hr</th>
<th>( \dot{n}_j ) ( \bar{z}_j ) ( \Delta H ) ( \bar{p}_j ) ( 10^3 ) kJ/hr</th>
<th>( \dot{n}_j ) ( \bar{z}_j ) ( \Delta H ) ( \bar{p}_j ) ( 10^3 ) kJ/hr</th>
<th>( \dot{n}_j ) ( \bar{z}_j ) ( H_j ) ( 10^3 ) kJ/hr</th>
<th>( \dot{n}_j ) ( \bar{z}_j ) ( H_j ) ( 10^3 ) kJ/hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Mn^{2+} ) (aq)</td>
<td>0,4129</td>
<td>91,0444</td>
<td>-1,3295</td>
<td>89,7149</td>
<td></td>
</tr>
<tr>
<td>( H^+ ) (aq)</td>
<td>0,4515</td>
<td>0,0000</td>
<td>0,0000</td>
<td>0,0000</td>
<td></td>
</tr>
<tr>
<td>( NH_4^+ ) (aq)</td>
<td>3,2334</td>
<td>428,7488</td>
<td>-6,6204</td>
<td>422,1284</td>
<td></td>
</tr>
<tr>
<td>( Mn_2(2H_2O)_6^{2+} ) (aq)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( NH_3 ) (aq)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( SO_4^{2-} ) (aq)</td>
<td>1,7465</td>
<td>1587,9178</td>
<td>9,1997</td>
<td>1597,1175</td>
<td></td>
</tr>
<tr>
<td>( HSO_4^- ) (aq)</td>
<td>1,0179</td>
<td>904,6077</td>
<td>1,2647</td>
<td>905,8724</td>
<td></td>
</tr>
<tr>
<td>Impurities</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>( H_2O ) (aq)</td>
<td>107,9782</td>
<td>30860,170</td>
<td>-141,7214</td>
<td>30718,4486</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>114,8404</td>
<td>33733,2819</td>
<td>(9,3704 x 10^3 kJ/hr)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Once the discharge stream enthalpy is estimated, there are other outlet enthalpies, namely the produced products and the ammonia and ammonia losses. Once again, using Table 5 and Tables 15 and 16, the energy term for these compounds is estimated and tabulated below (Table 21).
### Table 21 - Energy Term from Products and Losses

<table>
<thead>
<tr>
<th>Component</th>
<th>( M_j ) [kg/4hr]</th>
<th>( \Delta H_j^0 ) [kJ/hr \times 10^{-3}]</th>
<th>( M_j C^0_j (T-298) ) [kJ/hr \times 10^{-3}]</th>
<th>( \Delta H_j ) [kJ/hr \times 10^{-3}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_2 (g)</td>
<td>0.3113</td>
<td>0.0000</td>
<td>-0.37398</td>
<td>-0.1588</td>
</tr>
<tr>
<td>O_2 (g)</td>
<td>0.5171</td>
<td>0.0000</td>
<td>-0.3077</td>
<td>-0.3077</td>
</tr>
<tr>
<td>MnO_2 (s)</td>
<td>0.0145</td>
<td>7.5415</td>
<td>-0.0127</td>
<td>7.5288</td>
</tr>
<tr>
<td>H_2O (g) catholyte</td>
<td>0.03276</td>
<td>7.9214</td>
<td>-0.0167</td>
<td>7.9047</td>
</tr>
<tr>
<td>H_2O (g) anolyte</td>
<td>0.04051</td>
<td>9.7953</td>
<td>-0.0241</td>
<td>9.7712</td>
</tr>
<tr>
<td>NH_3 (g) catholyte</td>
<td>0.10699</td>
<td>4.9215</td>
<td>-0.0546</td>
<td>4.8670</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>29,2312</strong></td>
</tr>
</tbody>
</table>

The energy removed by the cooling coils is

\[
q_c = 4184 Q_c (T_2 - T_1)
\]

where \( Q_c = 30.05 \text{ m}^3/\text{hr} \)

\( T_2 = 306 \text{ K} \)

\( T_1 = 292 \text{ K} \)

Then \( Q_c = 489.08 \text{ kW} = 1760.688 \text{ kJ/hr} \)
Thus the Energy Balance will be

**TABLE 22 - Overall Energy Balance**

<table>
<thead>
<tr>
<th></th>
<th>Energy IN (kJ/s)</th>
<th>Energy OUT (kJ/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet Stream</td>
<td>9520,5</td>
<td></td>
</tr>
<tr>
<td>Power Input</td>
<td>365,1</td>
<td></td>
</tr>
<tr>
<td>Discharge Stream</td>
<td></td>
<td>9370,4</td>
</tr>
<tr>
<td>Products and Losses</td>
<td></td>
<td>8,1</td>
</tr>
<tr>
<td>Energy Removed from</td>
<td></td>
<td>489,1</td>
</tr>
<tr>
<td>cooling coils</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energy lost to</td>
<td></td>
<td>18,0</td>
</tr>
<tr>
<td>surroundings</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>9885,6</td>
<td>9885,6</td>
</tr>
</tbody>
</table>
APPENDIX D.1

Modelling the Ammonia-Manganese Complex Reaction Equilibrium Ratio

The following are a Table of results obtained by directly measuring the concentration ratio for the reaction below at various temperatures

\[
\text{Mn}^{2+} + \text{NH}_4^+ \rightleftharpoons \frac{\text{MnNH}_3^{2+} + \text{H}^+}{K'_C}
\]

where

\[
K'_C = \frac{Y_2 Y_4}{Y_1 Y_3}
\]

and has been modelled as

\[
K'_C = \exp(-34,57 + 0,051 T) \quad \text{for} \quad 295 < T < 330
\]

**TABLE 23** Experimental and Model values for the Manganese-Ammonia Complex Reaction Equilibrium Ratio

<table>
<thead>
<tr>
<th>°C</th>
<th>T (K)</th>
<th>Experimental</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K'_C$</td>
<td>$\ln K'_C$</td>
</tr>
<tr>
<td>32</td>
<td>305</td>
<td>$5,181 \times 10^{-9}$</td>
<td>-19.08</td>
</tr>
<tr>
<td>36</td>
<td>309</td>
<td>$6,61 \times 10^{-9}$</td>
<td>-18.80</td>
</tr>
<tr>
<td>40</td>
<td>313</td>
<td>$8,19 \times 10^{-9}$</td>
<td>-18.62</td>
</tr>
<tr>
<td>44</td>
<td>317</td>
<td>$10,10 \times 10^{-9}$</td>
<td>-18.41</td>
</tr>
<tr>
<td>48</td>
<td>321</td>
<td>$12,61 \times 10^{-9}$</td>
<td>-18.19</td>
</tr>
</tbody>
</table>
FIGURE 22

Comparison Between Experimental and Model Values for the Equilibrium Ratio of the Manganese-Ammonia Complex Formation Reaction
APPENDIX D.2

Modelling the Ammonium-Ammonia Equilibrium Ratio

The reaction between the ammonium ion and ammonia in the liquid solution has significance in the catholyte. The equilibrium ratio between the two species was modelled from the data supplied by Weast (16).

\[ \text{NH}_3 + \text{H}^+ \rightleftharpoons \text{NH}_4^+ \]

\[ K_a' = \frac{Y_3}{Y_5 Y_1} \]

The model found to best fit the data in the range \( 295 < T < 330 \) is

\[ K_a' = \exp (40.32 - 0.064 T) \]

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Experimental</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K' )</td>
<td>( \ln K'_a )</td>
</tr>
<tr>
<td>30</td>
<td>303</td>
<td>1.239 \times 10^9</td>
</tr>
<tr>
<td>35</td>
<td>308</td>
<td>0.885 \times 10^9</td>
</tr>
<tr>
<td>40</td>
<td>313</td>
<td>0.637 \times 10^9</td>
</tr>
<tr>
<td>45</td>
<td>318</td>
<td>0.467 \times 10^9</td>
</tr>
<tr>
<td>50</td>
<td>323</td>
<td>0.345 \times 10^9</td>
</tr>
</tbody>
</table>
FIGURE 23

Comparison Between Reported and Model Values for the Ammonium - Ammonia Reaction Equilibrium Ratio

\[ \ln K' \]

Temperature (K)

305.0
310.0
315.0
320.0
325.0
APPENDIX D.3

Modelling the Solubility Product of Manganese Hydroxide

Manganese ion when reacting with the hydroxide ion producing manganese hydroxide, which precipitates if saturation is reached, does so according to the following reaction:

\[ \text{Mn}^{2+} + 2\text{OH}^- \rightarrow \text{Mn(OH)}_2 \]

The solubility product \( K'_{sp} \) is defined as

\[ K'_{sp} = [\text{OH}^-]^2 [\text{Mn}^{2+}] \]

However, in this study the hydrogen ion is required to be related to all other variables and \( [\text{H}^+] \). This is done by the solubility product in terms of \( [\text{H}^+] \). This is done by defining the following equilibrium ratio

\[ K'_s = \frac{[\text{H}^+]^2}{[\text{Mn}^{2+}]} = \frac{K'_w^2}{K'_{sp}} \]

where \( K'_w = [\text{OH}^-][\text{H}^+] \) is the equilibrium concentration product for the dissociation of water. Thus in terms of our nomenclature

\[ K'_s = \frac{Y_2^2}{Y_1} \]

The model used for this newly defined equilibrium ratio is

\[ K'_s = \frac{Y_2^2}{Y_1} = \exp(-56.7 + 0.1025 T) \]

and by letting \( \Delta Y_1 = \frac{Y_2^2}{K'_s} - Y_1 \) precipitation of manganese hydroxide will occur if \( \Delta Y_1 < 0 \)
<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Experimental</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ln $K'_e$</td>
<td>ln $K'_e$</td>
</tr>
<tr>
<td>32</td>
<td>-37.07</td>
<td>-34.11</td>
</tr>
<tr>
<td>36</td>
<td>-36.75</td>
<td>-33.50</td>
</tr>
<tr>
<td>40</td>
<td>-36.33</td>
<td>-33.09</td>
</tr>
<tr>
<td>44</td>
<td>-35.92</td>
<td>-32.68</td>
</tr>
<tr>
<td>48</td>
<td>-35.51</td>
<td>-32.27</td>
</tr>
</tbody>
</table>

The experimental values in Table 25 are those determined by Bryson (unpublished data). These were evaluated with a 0.91 Molar MnSO$_4$ solution with no (NH$_4$)$_2$SO$_4$ present. The expression used in the model was adjusted because:

(i) The model results using the above expression fitted observed behaviour on the plant.

(ii) Jones (31) found that his results could best be simulated using

$$K'_e = \exp (-66.00 + 0.1025 T)$$

(iii) The high sulphate content of the plant solution, compared to that used by Bryson, would result in lower manganese ion activity.
Figure 24
Equilibrium Ratio for Precipitation of Manganese Hydroxide

Temperature (K)

-33.0  -35.0  -37.0

Model Predictions

Experimental Results

Bryson's data with 0.91 molar MnSO4 and no (NH4)2SO4.
APPENDIX D.4

Modelling the Sulphate-Bisulphate Equilibrium Ratio

At the low anolyte pH's, the ionic species present in the electrolyte change drastically from those present in the feed and catholyte due to the sulphate-bisulphate equilibrium

\[ \text{HSO}_4^- \rightleftharpoons \text{SO}_4^{2-} + \text{H}^+ \] (2.8)

For the purpose of this dissertation, it was found that the equilibrium ratio

\[ K'_n = \frac{z_2 z_6}{z_7} = \exp (8.339 - 0.0294 T) \] (2.9)

describes the situation to an accurate level. Figure 25 below shows this relationship in a graphic way.
APPENDIX E

Theory on Development of the Diaphragm Model

Transport of Ions in Solution (28)

Dilute Solution Theory

The flux of each ionic species may be expressed as

\[ N_j = -z_j u_j F B_j \nabla V - D_j \nabla B_j + B_j v \]

(E.1)

In this equation \( N_j \) represents the flux of species \( j \) and indicates the direction and rate of movement of the species. The three terms on the right side of the equation represent the three mechanisms of mass transfer:

(i) motion of a charged species in an electric field
(ii) molecular diffusion due to a concentration gradient
(iii) convection due to the bulk motion of the solution.

The current density of the solution can be expressed in terms of the fluxes of all the ions

\[ i = F \sum_{j} z_j N_j \]

(E.2)

The material balance for each species is

\[ \frac{dB_j}{dt} = -\nabla N_j + r_j \]

(E.3)

where \( r_j \) is the production rate of the \( j \)th species. Equations (E.2) and (E.3) are basic physical laws. All electrolytic solutions are electrically neutral in the bulk, except in a thin layer near the electrodes. Hence the following "accurate assumption" can be made.
In equation \( E.1 \) an approximation has already been made as the driving force for the diffusion should be an activity gradient. Activities are equivalent to concentrations under dilute conditions, and concentrations are the ones considered in the referred equation. Another simplification made is that the bulk velocity was taken to be the velocity of all the species. Thus, when using the equation \( E.1 \), one must be cautious due to its limitations.

Combining equations \( E.1 \) and \( E.2 \) the following is the result

\[
i = -F^2 \nabla \sum_j z_j^2 u_j B_j - F \sum_j z_j D_j \nabla B_j + F \sum_j z_j B_j
\]

\( (E.5) \)

From equation \( E.4 \) the last term is zero, and in case of no concentration gradients then

\[
i = K \nabla V
\]

\( (E.6) \)

where \( K = F^2 \sum_j z_j^2 u_j B_j \) is the solution conductivity.

The fraction of current carried by species \( j \), \( t_j.i \), can be written in the case of no concentration variation as

\[
t_j.i = -F^2 \frac{z_j^2 u_j B_j \nabla V}{\sum_j z_j^2 u_j B_j^2} \cdot i
\]

\( (E.7) \)

Hence

\[
t_j = \frac{z_j^2 u_j B_j}{\sum_j z_j^2 u_j B_j} \]

\( (E.8) \)
which is called the transport or transference number.

In a binary electrolyte (29) only two ions are present. Then

\[
\tau_+ = \frac{z_+ u_+ B_+}{z_+^2 u_+ B_+ + z_-^2 u_- B_-} \tag{E.9}
\]

and

\[
\tau_- = \frac{z_- u_- B_-}{z_+^2 u_+ B_+ + z_-^2 u_- B_-} \tag{E.10}
\]

If these equations are added then the following is concluded

\[
\tau_+ + \tau_- = 1 \tag{E.11}
\]

The Nernst-Einstein equation provides a relationship between \( D_j \), the diffusion coefficient, and \( u_j \), the mobility of species \( j \) (28)

\[
D_j = RT u_j \tag{E.12}
\]

**Concentrated Solution Theory**

Equation (E.1) should, for concentrated solutions, be replaced by

\[
B_j \frac{\nabla u_i}{\nabla} = \sum_j k_{ij} (v_j - v_i) = RT \sum_j \frac{B_j B_i}{k_{ij} D_{ij}} \tag{E.13}
\]

where \( u_i \) is the electrochemical potential of species \( i \), \( k_{ij} \) is the friction coefficient that accounts for interactions between the various ions. \( D_{ij} \) is a "diffusion coefficient" allowing for interaction between species \( i \) and \( j \), and \( v_j \) and \( v_i \) are the average velocities of species \( i \) and \( j \) (28).
As the ions present in the electrolytes, except for the hydrogen and hydroxide ions, have concentrations ranging from 0.2 to 3 Molar, the flux equation for dilute solutions should not be applied here. Therefore it is necessary to develop the concentrated solution flux equation.

If one considers the flux equation in only one direction perpendicular to the diaphragm plane, calling it the x-direction, then

\[
\frac{d\mu_i}{dx} = \frac{RT}{B_i} \sum_j \frac{B_j}{D_{ij}} (v_j - v_i)
\]

As the model is considered under steady-state assumption, the following conditions will hold

\[
N_i = B_i v_i
\]

\[
I = F \sum_j z_j N_j
\]

\[
\frac{dN_i}{dx} = 0
\]

\[
\sum_j z_j B_j = 0
\]

For convenience the electrochemical potential of an ion \( i \), \( \mu_i \), is expressed as a sum of a chemical potential and an electrical potential as follows

\[
\mu_i = \mu_{i0} + RT \ln \frac{B_i}{B_i^0} + z_i F(E-E^0)
\]

Then differentiating the above equation with respect to \( x \) and rearranging

\[
\frac{B_i}{RT} \frac{d\mu_i}{dx} = \frac{dB_i}{dx} + \frac{B_i}{R} \frac{z_i F}{RT} \frac{dE}{dx}
\]

Assuming our system is a multicomponent one made up of water, manganese, ammonium and sulphate ions, having a total of \( n+1 \) and giving the subscript 0 for water, E.14 will become
Extracting the first term from the summation one would obtain, after some rearrangement

\[ v_i = v_o - \sum_{j=0}^{n} \frac{B_j \delta_i}{B_o R T D_{ij}} (v_j - v_i) \]  \hspace{1cm} (E.22)

Defining

\[ D_i = \frac{B_i B_T}{B_o} \]  \hspace{1cm} (E.23)

and

\[ K_i = \sum_{j=1}^{n} \frac{B_j D_{ij}}{B_o D_{ij}} (v_j - v_i) \]  \hspace{1cm} (E.24)

Then equation (E.22) becomes

\[ v_i = v_o - \frac{D_i}{R T} \frac{dV}{dx} + K_i \]  \hspace{1cm} (E.25)

Using (E.15), (E.20) and (E.25) on gets

\[ N_i = B_i (v_o + K_i) - D_j \frac{d\delta_i}{dx} - \frac{B_i D_i z_i F}{R T} \frac{dV}{dx} \]  \hspace{1cm} (E.26)

Note that when \( K_i = 0 \), the equation reduces to the same form as for the dilute solution flux equation (E.5). Substituting (E.26) into (E.16) and using the electrical neutrality condition (E.18) one obtains

\[ I = \sum_{j=1}^{n} z_j F B_j K_j - \sum_{j=1}^{n} \sum_{j=1}^{n} z_j F D_j \frac{d\delta_j}{dx} - \sum_{j=1}^{n} \frac{B_i D_i z_i^2 z_j^2}{R T} \]  \hspace{1cm} (E.27)

Now it is required to eliminate the term \( \frac{dV}{dx} \), as it is difficult to measure. That is done by solving for the electrical potential gradient term in (E.27) and substituting in equation (E.26) to obtain:
Bryson and Lawrence (18) further developed this condition, by finding that the following assumptions hold

\[
\frac{dV}{dx} = \frac{n}{R T} \sum_{j=1}^{n} B_j D_j z_j^2 e^{2 \int_{0}^{x}} \left( z_j F B_j K_j - z_j F D_j \right) dB_j \frac{d}{dx} \]

(E.29)

Then equation (1.27) reduces to

\[
\frac{dV}{dx} = \sum_{j=1}^{n} B_j D_j z_j^2 \left( \frac{F}{R T} - \frac{I}{R T} \right) \]

(E.30)

Furthermore, \( B_j \) can be replaced in (E.30) by the arithmetic mean concentration \( B_j^* \) without introducing a significant error.

\[
B_j^* = \frac{1}{2} (Y_j + Z_j) \]

(E.31)

Thus

\[
\frac{F}{R T} \frac{dV}{dx} = \frac{-I}{R T} \sum_{j=1}^{n} B_j^* D_j z_j^2 \]

(E.32)

Substituting into equation (E.26) it leads to

\[
N_i = B_i (V_0 + K_i) - D_i \frac{dB_i}{dx} + \sum_{j=1}^{n} \frac{B_i D_i z_j^2}{F \sum_{j=1}^{n} B_j^* D_j z_j^2} \]

(E.33)
This equation multiplied by the area leads to the equation used in the model, (2.10)

\[ AN_j = W_j = Q_c, \quad \left(1 - \frac{I F D_j z_j B_j(x)}{R T K} - \varepsilon A D_j \frac{dB_j}{dx}\right) \]

(2.10)
APPENDIX F

Fitting the Kinetic Model

Equation (2.27) gives the kinetic model in the form

\[ E_0 = \frac{1}{1 + \frac{\lambda_2}{\lambda_1} \left[ 1 + \left( \frac{\lambda_4 - \lambda_2}{\lambda_2} \right) \frac{\lambda_3}{\lambda_1} \frac{Y_8}{Y_1} \right] \frac{Y_2}{Y_1}} \]  \hspace{1cm} (2.27)

Let

\[ \eta = 1 + \left( \frac{\lambda_4 - \lambda_2}{\lambda_2} \right) \frac{\lambda_3}{\lambda_1} \frac{Y_8}{Y_1} = A + B \frac{Y_8}{Y_1} \]  \hspace{1cm} (F.1)

and solving for \( \frac{\lambda_2}{\lambda_1} \) we will obtain

\[ \frac{\lambda_2}{\lambda_1} = \frac{1}{\eta} \left( 1 - E_0 \right) \frac{Y_1}{Y_2} \]  \hspace{1cm} (F.2)

This equation is of the form used to obtain the fitting of the Delta Manganese data, as well as the MINTEK's.

Fitting the Impurity Level

From the sets of data the constants \( A \) and \( B \) from the kinetic model expression (2.27) incorporating the impurity level were fitted as well as possible. The equation has a linear form

\[ \eta = A + B \frac{Y_8}{Y_1} \]  \hspace{1cm} (F.1)

The values for the constants used in the model are

\[ A = 0.72919681 \]  \hspace{1cm} (F.3)
\[ B = 1.3479702 \times 10^{-3} \]
Figure 26 shows the plot of $\eta$ versus $Y_\delta/Y_\eta$, from which the constants resulted. The following are worth noting:

(i) the readings are very spread, giving a large variance

(ii) the impurities were expressed as a total impurity level, as there was not enough data to study the effects of the individual impurities.

(iii) as the expected trend was linear it was decided to fit the best straight line through the points, thus explaining the operational data.

Fitting the Current Density Effect

From the MINTEK data (6) it can be found that the relationship between the cathodic current efficiency and current density is quadratic, going through a maximum. By plotting the results tabulated in Table 26 yields Figure 27, in which the MINTEK results are compared to the model predictions. Please note that here no correction factor had been taken into account due to the fact that the operational data from Delta was consistently higher than that of MINTEK, when considering the fractional current efficiency. The way the equation was rearranged to fit the MINTEK data is shown on Figure 27.

It can be observed that in the area around the optimum, the fitted model agrees with the experimental readings within 5%. Only at very low current densities the error involved is in the order of 10%.

The model used has the following form, which was obtained from equation (F.2)

$$\left(\frac{1 - \xi}{\xi}\right) = 49,78009(i-550)^2 - 472,4760(i-550) + 3,8947157 \times 10^6 \quad (F.4)$$
FIGURE 26 - Fitting the Impurity Model

+ Experimental
* Fitted Model
FIGURE 27  FITTING CURRENT DENSITY EFFECT

\[ y = \frac{(1 - E_c)}{E_c} \frac{y_1}{y_2} \]

Current Density (A/m²)
TABLE 26 - Fitting the Current Density Effect on the Kinetic Model

<table>
<thead>
<tr>
<th>i (A/m²)</th>
<th>MINTEK $Y = \frac{(1 - E_c)}{E_c} Y_1 \over Y_2$</th>
<th>MODEL $Y = \frac{(1 - E_c)}{E_c} Y_1 \over Y_2$</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>9.2393663 x 10⁶</td>
<td>10.158143 x 10⁶</td>
<td>9.04</td>
</tr>
<tr>
<td>250</td>
<td>7.2632819 x 10⁶</td>
<td>8.516666 x 10⁶</td>
<td>14.71</td>
</tr>
<tr>
<td>300</td>
<td>5.939526  x 10⁶</td>
<td>7.1240903 x 10⁶</td>
<td>16.63</td>
</tr>
<tr>
<td>350</td>
<td>5.3627937 x 10⁶</td>
<td>5.9804145 x 10⁶</td>
<td>10.30</td>
</tr>
<tr>
<td>400</td>
<td>4.9356732 x 10⁶</td>
<td>5.0856391 x 10⁶</td>
<td>2.95</td>
</tr>
<tr>
<td>450</td>
<td>4.4397642 x 10⁶</td>
<td>4.4397647 x 10⁶</td>
<td>0.00</td>
</tr>
<tr>
<td>500</td>
<td>4.252856  x 10⁶</td>
<td>4.0427289 x 10⁶</td>
<td>5.00</td>
</tr>
<tr>
<td>550</td>
<td>3.8947157 x 10⁶</td>
<td>3.9947157 x 10⁶</td>
<td>0.00</td>
</tr>
<tr>
<td>600</td>
<td>4.0708548 x 10⁶</td>
<td>3.8955421 x 10⁶</td>
<td>1.88</td>
</tr>
<tr>
<td>650</td>
<td>4.3452690 x 10⁶</td>
<td>4.3452690 x 10⁶</td>
<td>0.00</td>
</tr>
<tr>
<td>700</td>
<td>4.632906  x 10⁶</td>
<td>4.9438963 x 10⁶</td>
<td>5.65</td>
</tr>
</tbody>
</table>

Fitting the Temperature Effect

The expected trend for the temperature is of the exponential type. The model was found to be

$$\frac{(1 - E_c)}{E_c} Y_1 \over Y_2 = A \exp \left( \frac{B}{T} \right) + C$$

where

- $A = 4.2589946 \times 10^{-12}$
- $B = 12.587794 \times 10^4$
- $C = 2.6000000 \times 10^6$

These values were found by fitting a straight line to the logarithmic plot as shown in Figure 28. The best straight
line was fitted to the MINTEK (6) results and then the model has been run at the same conditions and compared to the results. These are tabulated in Table 27 and plotted in Figure 28. The results agree within 2.5%.

TABLE 27 - Fitting the Temperature Effect

All results at 550A/m²

<table>
<thead>
<tr>
<th>t (°C)</th>
<th>MINTEK</th>
<th>MODEL</th>
<th>%Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Y=ln[(1-ε₂/ε₀)^Y₁-2.6x10^6]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>16.448937</td>
<td>16.780153</td>
<td>2.01</td>
</tr>
<tr>
<td>25</td>
<td>16.089197</td>
<td>16.024885</td>
<td>0.40</td>
</tr>
<tr>
<td>30</td>
<td>15.671057</td>
<td>15.73732</td>
<td>0.77</td>
</tr>
<tr>
<td>35</td>
<td>14.803890</td>
<td>14.690579</td>
<td>0.77</td>
</tr>
<tr>
<td>40</td>
<td>14.088244</td>
<td>14.036014</td>
<td>0.37</td>
</tr>
<tr>
<td>45</td>
<td>13.482339</td>
<td>13.406624</td>
<td>0.56</td>
</tr>
<tr>
<td>50</td>
<td>12.487183</td>
<td>12.789822</td>
<td>2.36</td>
</tr>
<tr>
<td>55</td>
<td>12.193645</td>
<td>12.198196</td>
<td>0.03</td>
</tr>
<tr>
<td>60</td>
<td>11.638676</td>
<td>11.619157</td>
<td>0.16</td>
</tr>
</tbody>
</table>

By joining (F.3), (F.4) and (F.5) together, the general effect of λ₂/λ₁ was derived and the form used in the computer model is as in equation (2.32) with the value of γ as 1.36x10⁶ (equation 2.33).
FIGURE 28  FITTING THE TEMPERATURE EFFECT

*--* MODEL PREDICTIONS

+ + MINTEK DATA (6)

Y as in Table 27

TEMPERATURE (K)

18.0

14.0

12.0

295.  305.  315.  325.  335.
APPENDIX G

Modelling the Water Losses

Equation (2.53) was derived from the data in the Steam Tables (23) to estimate the mole fraction of water vapour versus temperature. The following equation was found to give the best fitting in the range $310 < T < 320$

$$y_o = z_o = -1.267 + 0.00428 \ T$$ (2.53)

The comparison between model predictions and experimental data are presented both in Table 28 and Figure 29.

**TABLE 28 - Fitting the Mole Fraction Equation from Steam Tables (23)**

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>Experimental Partial Pressure (bar)</th>
<th>Experimental Mole Fraction</th>
<th>Model Mole Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>37</td>
<td>310</td>
<td>0.0628</td>
<td>0.06198</td>
</tr>
<tr>
<td>38</td>
<td>311</td>
<td>0.06624</td>
<td>0.06537</td>
</tr>
<tr>
<td>39</td>
<td>312</td>
<td>0.0700</td>
<td>0.06908</td>
</tr>
<tr>
<td>40</td>
<td>313</td>
<td>0.07375</td>
<td>0.07279</td>
</tr>
<tr>
<td>41</td>
<td>314</td>
<td>0.07787</td>
<td>0.07685</td>
</tr>
<tr>
<td>42</td>
<td>315</td>
<td>0.08198</td>
<td>0.08091</td>
</tr>
<tr>
<td>43</td>
<td>316</td>
<td>0.08649</td>
<td>0.08536</td>
</tr>
<tr>
<td>44</td>
<td>317</td>
<td>0.09100</td>
<td>0.08981</td>
</tr>
<tr>
<td>45</td>
<td>318</td>
<td>0.09595</td>
<td>0.09470</td>
</tr>
<tr>
<td>46</td>
<td>319</td>
<td>0.10090</td>
<td>0.09958</td>
</tr>
<tr>
<td>47</td>
<td>320</td>
<td>0.10625</td>
<td>0.10486</td>
</tr>
</tbody>
</table>

Mole fraction = \( \frac{p}{p_{atm}} \) = \( \frac{p_{(bar)}}{1.01325\text{(bar)}} \)
FIGURE 29 - Comparison Between Experimental Results and Model Predictions for the Mole Fraction versus Temperature in the Estimation of Water Losses.

* Model Predictions
+ Experimental Results

Mole Fraction

Temperature (K)
Author Rodrígues J C P
Name of thesis Modelling the performance of Manganese Eletrowinning cells 1983

PUBLISHER:
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