AN ECONOMIC MODEL OF AN
OPEN PIT MINE

A thesis presented to the
Department of Mining Engineering
of the
University of the Witwatersrand
for the Degree of
Master of Science.

Submitted by -

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June, 1969.
ERRATA

Page 33:  \[ A = \frac{(Y^* X Z) - (Y^* Y))}{((NS**Z) - (X**Z))} \] should be:
\[ A = \frac{((Y^* X Z) - ((X) * (X^* Y)))}{((NS**Z) - (X**Z))} \]

Page 50:  First line - 41000 should read 48000.

Page 85:  Bottom - V is substituted for PV in the formula
\[ PV - W = P - (d^*V^*T) \] without comment and is changed back again on page 86, again without comment.

Page 59:  GMC = ....... should read:
\[
\begin{align*}
GMC &= Gm & \text{if} & & \text{Gmc} \leq Gm \\
Gmc &= Gc & \text{if} & & \text{Gmc} \leq Gc \\
Gmc &= \text{otherwise}
\end{align*}
\]
The phrase "different than" is used in pages 21 and 120.

Page 118:  Second last line: "past" should read "last".
ACKNOWLEDGMENTS

I would like to thank my supervisors, Professor R. P. Fleeman and R. A. Section Clee of the Mining Department for their guidance and help with this work throughout.

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CHAPTER 1 INTRODUCTION

The model described by this thesis is an economic planning tool designed to analyse alternative actions for the valuation of any mineral deposit suitable for exploitation by open pit mining methods.

The model devises a pilot scheme to make capital out of the given mineral deposit according to a specified proposal. The proposal is selected by the user and in the projection of:

1) the production capacities of the mine, concentrator and refinery,

2) the open pit production method

- the sequence of excavation
- the dimensions of the pit benches,
- the angles of the safe and working wall-slopes,
- the number of ore-working faces and exposed ore benches required to supply the plants listed in subparagraph one;

3) the scheduling and proportioning of equity and loan capital to meet the capital requirements.

This proposal directs the model to create a course of action devised to exploit the given deposit. The best proposal is selected by manipulating the alternatives to find the combination which yields the greatest return on invested capital. In this way, the model determines the course of action which will generate the optimum return from the mine.

In addition to the proposal, the following information on the user's deposit is required to execute the model:

1) mineral content,

2) topography,
geologic and physical features, and

peculiar costing elements.

The model is an instrumental aid for working upon these facts according to the selected mining proposal. These input data are resolved by the model from the complex mine structure into simple components for the exact determination of an open pit mining solution. Consequently, the model is constructed to be a "total system" analogue of an open pit mine. The "total system" analogue is a duplication of the interdependent economic, mathematical, mining and design concepts forming the integral open pit mining concept. In this way, the dependence of the computations on one another were accounted for so that the model would be a realistic duplicate of an actual open pit mine.

The model produces the following information which makes up the elements of the open pit mining solution:

1) A mineral grade cut-off point is computed to make the ore or waste decision. This cut-off grade depends on the mineral content of the material exposed for mining at any given time and the current value of costs and selling prices at such time. Consequently, it is computed on an annual basis.

2) The final pit surface is designed. This surface is the result of mining out the given deposit. The surface is designed to achieve the maximum total profit.

3) The pit production is simulated to give the excavation time of the material within the final pit surface, and annual topographic maps of the pit.

4) Production, cost and revenue schedules are produced on an annual basis.

This solution is produced to study the way in which the deposit's exploitation reacts to the alternative mining proposals. The reaction is measured by computing the profitability of the solution from an analysis of the cash generated.

The model's practical application lies in the investigation of its solution to exploit the given mineral deposit and...
this solution's profitability. In this way, the user seeks to become intimately acquainted with how the particular deposit behaves through adjustment of the mining proposal. Two opposite extremes of the range of situations which can be evaluated by the Model are:

1) the examination of a long term proposal that may cover the entire life of the mine, and

2) the examination of a short term proposal, for example, a two or three year project to mine an additional section of an existing mine.

To make these investigations, the mutual relationship between the constituent parts of the mine structure are simulated to establish the peculiar nature of the given deposit. The simulation is a computer analogue designed to account for the physio-economic differences between deposits. The pit and deposit are simulated with a computer analogue of both a three dimensional network of lines and specially devised spatial data sets. This analogue is used to imitate the physical structure of the deposit's mineral distribution and topography; the pit surface; the important geologic features; and the spatial distribution of any density functions.

The three dimensional network of lines creates a grid of interrelated blocks or intersections for which certain values are calculated to describe the deposit. The grid-intersections are the geometric centre of the grid-blocks formed from perpendicular planes bisecting the lines between adjacent grid-intersections. The following elements of the mine and deposit are simulated using this grid system and the spatial data sets:

1) Topography and Geology: Computer programming techniques are devised to make the grid system resemble the topography and important geologic features.

2) Mineral Distribution: The deposit's mineral distribution is simulated by forecasting the division, arrangement and dispersal of mineral values through the grid system.

3) Value Distribution: The cash value of each grid-block is calculated with the standard cost method. The
DECLARATION

I, ROBERT HOWARD ROBINSON, do hereby declare that this dissertation is my own work and has not been presented to any other university for the purpose of obtaining a degree.

[Signature]

Robert H. Robinson
grid-block values include all costs and revenue accrued by the production of the minerals occurring in the particular grid-blocks. The standard cost method is devised by assigning a standard cash value to the grid-block in proportion to its mineral content. Then, this standard is adjusted according to the specific conditions under which the particular grid-block is mined. For example, the standards could be adjusted to vary with location, metallurgical penalties or time.

4) Pit Production Method: The pit production is simulated by imitating the real sequence of grid-block excavation. A record is kept of each grid-block excavation time. The annual production schedules and topographic maps are computed with this time record.

The simulation of the deposit's physical structure and pit production method tailors the Model to the user's deposit. Only simple modifications are required to adapt the simulations to a different deposit or pit production method. This flexibility was an objective in the design of the Model to ensure that no loss of accuracy would occur from the physio-economic differences between mineral deposits.

The Model is capable of producing an open pit mining solution when exploration drilling has delimited the extent of mineralization. The profitability of this solution is measured with a cash flow analysis. The solution and its potential value are examined in terms of the user's needs. This analysis can be rerun repetitively as additional information becomes available from further drilling. The solution computed by the Model becomes more accurate as the exploration data increase. The exploration work can be controlled by making the decision to continue or not at each analysis of the ever-increasing exploration data. Ultimately, the final decision is made to mine the deposit or not, when the exploration data yields enough information to achieve the desired accuracy level.

The Model is described with a detailed account of its characteristics in the following Chapters. The literature survey in Chapter 2 describes the current practices of mine planning and pit design. The Model is presented by describing the construction and derivation of its parts in Chapters 3 and 6. Chapters 4, 5 and 8 list the Model's computer programmes. These programme listings follow the
chapters, in which their computations are discussed. An example of an application of the Model is presented in Chapter 7. Chapter 9 contains a criticism, the limiting provisions, and some recommendations on the Model. Finally, the list of references appears in Chapter 10.

The computer programmes used in the Model were written in the FORTRAN language which consists of the set of statements described in "IBM System/360 FORTRAN IV Language" (2). FORTRAN is a body of expressions which communicate the computations required from the computer. The means used to initiate and control the programmes used in this thesis are described in "IBM System/360 Operating System Job Control Language" (3). The job control language is the method of commanding the computer to execute the action required on a particular programme. The Chapters listing computer programmes contain the following information:

1) a list of steps required to prepare the programmes for execution;
2) a chronological description of the programme computations;
3) a programme list and notes describing how the programmes are tailored, for different deposits;
4) a glossary to define the FORTRAN variables; and
5) a list of the job control language for the Mine Plan Programme only.

The following discussions assume that the reader is familiar with FORTRAN and the job control language.
CHAPTER 2 LITERATURE SURVEY

A broad view of the customary mine planning and pit design methods is presented in "Surface Mining" (3). It is the most recent and inclusive summary of the generally accepted surface mining practices. The two articles in the book which are applicable to this thesis are 'Preliminary Evaluations' by S. M. Sundeen, page 54, and 'Pit Planning and Layout' by A. Soderberg and D. O. Rausch, page 141.

Sundeen describes the factors required to evaluate an open pit mining venture. He states: "The following outline lists the basic factors which must be taken into account for a preliminary evaluation of a prospective mine:

1) Geology,

2) Mining conditions,

3) Economic analysis,

4) Ore treatment requirements,

5) Geography,

6) Legal status of land and mining rights, and

7) Historical, political and sociological factors."

The evaluation of any potential deposit is updated continuously as additional data becomes available from exploration of the factors in the above list. Consequently, the point where a preliminary evaluation develops into a comprehensive evaluation is peculiar to the individual making the analysis. The model described by this thesis is capable of taking over the continuously expanding evaluation at the point when exploration drilling has marked the boundaries of mineralization.

The first five items in Sundeen's list are used by the Model...
to produce the optimum open pit solution for the user's deposit. This solution and the return on capital investment are compared with the user's company policy. The user adopts a course of action according to the results of the comparison between company policy and the model's evaluation. The model is not able to analyze the early data of the continuous evaluation, which is predominantly socio-political. Therefore, the last two items in the list are included in the model's evaluation indirectly, as far as they affect the costs, revenue, pit design, plant production capacities and cash flow analysis.

The article 'Pit Planning and Layout' by Soderberg and Rausch (9, page 141) describes the current practice in designing the final surface of an open pit mine. This design is contrived by finding the physical limits of the pit beyond which the mineralization ceases to be profitable. The final pit surface designed with this logic constitutes all the points which define the ore boundary. These pit limits are found in the following manner:

The entire deposit is divided into sections, for example, the deposit illustrated in Fig. 2. The sections' end boundaries conform generally to the final pit shape. The side boundaries are formed by the cuts which divide the deposit into sections. To simplify the calculations, the sections are assumed to be two-dimensional by eliminating the direction perpendicular to the sides. These consecutive two-dimensional sections are assumed to be small enough parts of the deposit to approximate the final pit surface.

An economic limit is found for each section by moving its end boundaries in increments to the break-even point between profit and loss. The pit limits of each section are defined when the configuration of end boundaries produces the break-even point. Finally, the limits of adjacent sections are smoothed so that they do not exceed the allowable safe wall-angle. The resulting pit limits become the final pit surface.

This 'pit limits' concept of pit design is a marginal examination for the boundaries beyond which the deposit's mineralization ceases to be profitable. The design of the final pit surface at the marginal limit between ore and mineralization is not the ideal objective. The objective should be to find the final pit surface yielding the maximum total profit. Another obstacle in the 'pit limits' design
FIG. 2.1 DESIGN SECTIONS OF THE 'PIT LIMITS' CONCEPT
concept is that the boundaries are determined separately for each section. This design is rather like examining each leg, in turn, of a three-legged stool to find out which leg supports the stool. If the legs were examined by isolating them individually from the system, the conclusion would be that each leg alone holds the stool upright. This is foolish, of course, but the right conclusion, that all three legs are necessary to hold the stool upright, can be made only if the stool is examined as a total system. Similarly, there are inherent errors in the sectional approach to pit design. The section's large size and the two-dimensional assumption for their magnitude over-simplifies the design, which creates the following errors:

1) The sections do not simulate the pit shape as the pit is excavated. Each section is evaluated independently on its own mineralization. Consequently, the economic potential of any mineralization occurring in a section must include the cost of exposing that mineralization. However, the sides of the sections are formed with planes that do not coincide with pit walls of the excavation. The sides are commonly vertical as shown in Fig. 21. The difference between these sides and the actual pit walls is not considered in the evaluation of each section. Therefore, the potential value of a particular section does not include the exact overburden stripping costs. Consequently, the designed pit limits will be greater than they should be.

2) The economic boundaries found for each section must be smoothed to get the final pit surface. Often the boundaries of adjacent sections differ enough to make the wall-angles between sections exceed the allowable extreme. If this is the case, the section boundaries are smoothed to achieve the safe wall-angles. This smoothing alters the pit limits and the final surface is no longer the result of the design concept.

3) The sections are too bulky to simulate the deposit's structural features. The sections are not capable of readily responding to the influence of structural features on the pit design, pit production method, or the cash value and ore reserve calculations. For example, topography, fault zones, or an irregular mineral distribution are difficult to manage with the regular shape of the sections. Additionally, the sec-
tions would be difficult to construct for deposits requiring variable intermediate and final wall-angles. Consequently, the pit design loses accuracy through the cumbersome shape of the sections.

In the final analysis, it can be understood that the 'pit limits' design concept is unacceptable for two reasons: Firstly, the marginal limit objective is improper because it is not a basis for maximizing profits. Secondly, the sections used as a design tool are inflexible and not representative of an actual pit.

M. T. Pana (7) suggests the following pit design method which overcomes some of the disadvantages of the 'pit limits' design concept:

1) Construct a three-dimensional grid system to simulate the deposit. The grid system forms evaluation units or grid-blocks. The grid-blocks are smaller, more versatile and more representative evaluation units than the sections of the 'pit limits' design concept.

2) Estimate the mineral grades for each grid-block by interpolation from the bore-hole samples.

3) Evaluate each grid-block "by weighting the recoverable (minerals) against the total costs including a minimum acceptable profit but excluding stripping related" (7) to the grid-block.

4) Classify the grid-block as ore if the evaluation is positive, or as waste if negative.

5) Construct removal increments from a family of overlapping cones. The geometry of the pit is "approximated by a series of interconnecting cones. The material contained within each cone is defined as a removal increment." (7)

6) Total the grid-block values occurring in each removal increment.

7) Include the removal increments in the pit with a total positive value and exclude those with a total negative value.
The group of positive removal increments forms the final pit surface.

Pana's pit design method solves the following two disadvantages of the 'pit limits' design concept: Firstly, the burden of waste stripping costs is distributed to the mineralization exactly. The three-dimensional removal increments simulate the pit shape, whereas the two-dimensional sections do not. Secondly, the grid system responds to deviations in the deposit's mineral distribution, structural features and topography.

However, Pana's pit design method does not provide for a maximum profit objective. H. Lerchs and I. F. Grossman (5) state "The objective is to design the contour of a pit so as to maximize the difference between the total value of ore extracted and the total extraction cost of ore and waste." Lerchs and Grossman (5) have derived the theory, in the form of a graph algorithm, to design the final pit surface which will yield the maximum profit from a given deposit. The theory is an arithmetical set of rules designed to find the maximum value of the points in a special graph. This graph is described and defined by one (6) as follows:

"The term 'graph' denotes something quite different from the graphs you may be familiar with from analytical geometry or function theory. The kind of graph you probably have dealt with consisted of the set of all points in the plane whose co-ordinates (X, Y), in some co-ordinate system, satisfy an equation in X and Y. The graphs (in the present context) are simple geometric figures consisting of points and lines connecting some of these points; they are sometimes called 'linear graphs'. It is unfortunate that two different concepts bear the same name, but this terminology is now so well established that it would be difficult to change it."

Definitions (See Fig. 2-2):

Graph: A figure consisting of points (vertices) and segments connecting some of these vertices. (The connecting segments may be straight line segments or curved segments and are called edges.)

Directed Graph: A graph with directed edges. The directed edges are called arcs."
Lorchs and Grossman (5) construct one of these graphs for an open-pit mine in the following manner:

"Let the entire pit be divided into a set of volume elements $V(I)$. This division can be quite arbitrary, but may also be obtained by taking for $V(I)$ the unit volumes defined by a three-dimensional grid. Associate to each volume element $V(I)$ a mass

$$m(I) = p(I) - c(I)$$

where $p(I)$ and $c(I)$ are the mine (revenue) and the extraction cost of element $V(I)$. Let each element $V(I)$ be represented by a vertex $X(I)$ of a graph. Draw an arc $(X(I), X(J))$ if $V(J)$ is adjacent to $V(I)$, that is, $V(I)$ and $V(J)$ have at least one point in common, and if the mining of volume $V(I)$ is not permissible unless volume $V(J)$ is also mined. We thus obtain a directed three-dimensional graph $G = (X, A)$ with a set of vertices $X$ and a set of arcs $A$. Any feasible contour of the pit is represented by a 'closure' of $G$, that is, a set of vertices $Y$ such that if a vertex $X(I)$ belongs to $Y$ and if the arc $(X(I), X(J))$ exists in $A$, then the vertex $X(J)$ must also belong to $Y$. If a mass $m(I)$ is associated to each vertex $X(I)$, and if $M(Y)$ is the total mass of a set of vertices $Y$, then the problem of optimal pit design comes to finding in a graph $G$ a closure $Y$ with maximum mass or, shortly, a maximum closure of $G$" (See Fig. 2*2).

This maximum closure is found by following a set of rules which identify the vertices or grid-blocks existing within the final pit surface.

The graph algorithm designs the final pit surface with the cash value and extraction cost given for each grid-block. This given information represents an assumption of the economic circumstances under which the deposit exists. Lorchs and Grossman (5) observe, however, that "besides pit design, planning may bear on questions such as:

1) what upgrading plants to install;
2) what quantities to extract, as a function of time;
3) what mining methods to use;
4) what market to select; and

5) what transportation facilities to provide.

There is an intimate relationship between all the above points, and it is meaningless to consider any one component separately. A mathematical model taking into account all possible alternatives simultaneously would, however, be of formidable size and its solution would be beyond the means of present knowhow.

This deficiency in mathematical knowledge is overcome by technological development of computer, mathematical, simulation and design methods. These methods are programmed to constitute the Model described by this thesis. They are techniques devised to duplicate the physio-economic environment of an open pit mine. The Model designs, evaluates and partially plans the items in the preceding list, in the following manner:

1) The mine environment is given by the user in terms of the product market prices, the mine's cost structure, the approximate production capacity and the pit production method.

2) The Model performs the following functions:

   A. selects the exact production capacities of the upgrading plants,
   B. computes annual production targets and cut-off grades,
   C. designs the final pit surface,
   D. simulates the pit production method to give a time and location record of the annual production, and
   E. computes the profitability of the venture.

For some mining situations, it will be possible to vary the environmental conditions listed in the first subparagraph. Those conditions, which are variable, are changed one at a time to find the ideal solution for exploiting the user's deposit. The ideal solution is chosen by the user from the combination of environmental conditions yielding the greatest profit, as computed by the Model.

There is one last open pit design reference that should be
mentioned. P. J. Hartman and C. G. Verma (1) have written a paper describing an open pit design model which also is based on the Lerchs - Grossman algorithm. Their model performs only the design function for the final pit surface. Additionally, they made some suggestions for the methods of grade prediction and costing that are employed in the model described by this Thesis.
Chapter 3  Simulation of the Surface Mineral Deposit

3.1 Opening Discussion

The surface mineral deposit given by the user is simulated to provide a record of:

1) the important structural, geologic and physical features, and
2) the spatial distribution of minerals.

The simulation uses a computer analogue of a three-dimensional grid system superimposed on the user's deposit. This computer analogue is devised with three-dimensional FORTRAN arrays. The array subscripts correspond to the intersections of the grid system's fixed network of lines. For example, the point 500ft south, 400ft east and 200ft deep of the grid system illustrated in Fig. 3.1 would furnish the subscripts (5,4,4) of the array address (I,J,K). The grid systems are discussed further in Section 3.2.

The structural, geologic and physical features requiring simulation are all those which could affect the development of an open pit mining solution. For example, the method of mineral grade prediction must be sensitive to the mineralization zones which have evolved from different geologic processes. Consequently, it is necessary to identify the boundaries of these zones so that the prediction of mineral values in a particular zone is not influenced by mineral samples coming from another zone. These zones could be bounded by faults, intrusions, the water table and so forth. The boundaries delimiting these mineralization zones are simulated by reporting the presence or non-presence of each feature at the intersections of the grid system's line network. The simulation of these surfaces is described in Section 3.3.
Every new deposit evaluated with the Model will have a different configuration of structural, geologic and physical features. Therefore, the Model's computer programs have to be tailored to the user's deposit. The structural simulation techniques and computer programmes are discussed in detail to explain how the programmes are modified for different deposits.

The spatial distribution of minerals is a record containing the accumulation of recoverable minerals at each intersection of the grid system's line network. (The recoverable minerals are those capable of being extracted from the deposit with a profit.) Each estimate of the recoverable mineral accumulation is a point density in the spatial distribution of minerals. The two computer programmes discussed in Section 3.4 are used to estimate these mineral accumulations from the samples taken out of the exploration bore-holes.

3.2 SELECTION OF A GRID SYSTEM

The grid system is an imaginary network of lines superimposed on the specified deposit. The network of lines in a three-dimensional arrangement forms a set of reference points, planes and sections. This spatial reference set is used to define the essential nature of the given deposit at the reference points. Any data collected to describe a point in the deposit is stored in a data set by referencing that data with the co-ordinate location of the given point. Thus, the spatial distribution of any value is defined with reference to the fixed co-ordinates of the grid system.

The spatial features of the grid system used to reference data sets are defined below. These definitions are illustrated in Fig. 3.1.

1) Grid Co-ordinates: The grid co-ordinates are the number of unit interval distances from the origin along each axis.

2) Grid-intersections: The grid-intersections are the points common to the grid system's fixed network of lines.
Fig. 3.1 A RECTANGULAR AND REGULARLY SPACED GRID SYSTEM
3) **Grid-blocks:** The grid-block boundaries are the planes intersecting the midway points between grid-intersections. The grid-blocks represent a unit mass of the deposit in its unmined state. The grid-intersections are at the geometrical center of the grid-blocks.

4) **Grid-section:** A grid-section constitutes all the grid-blocks occurring between the parallel planes which are perpendicular to any of the three co-ordinate directions, are centered on any particular grid co-ordinate, and have a gap equal to the unit interval distance.

5) **Grid-level:** A grid-level is a horizontal or nearly horizontal grid-section.

The grid co-ordinates are scaled down distances from the actual distances to create small integer values suitable for FORTRAN subscripts. These subscripts become the addresses for the spatial data sets stored in FORTRAN arrays. The scaling is in proportion to the unit interval distances.

### 3.2.1 GRID SYSTEM TYPES

The grid system is designed by planning the unit interval distances between grid co-ordinates and the angular separation between the axes. There are four basic grid system types which are constructed from the combinations of:

1) **regular or irregular unit interval distances,** and

2) **rectangular or non-rectangular axes.**

The most commonly used grid system is constructed with regularly spaced unit interval distances and rectangular axes. These unit interval distances are equally spaced along each axis although they need not be the same interval for all axes. The illustration in Fig. 3.1 is one of these regularly spaced and rectangular grid systems. The zero grid-level is designed to insure that the highest point on the deposit's surface is within the grid system.

The regularly spaced and rectangular grid system is a good simulation for massive deposits with uniform mineral quantities. For these deposits, grade prediction can be calculated by estimating the mineral quantities in any
particular grid-level solely from the assays occurring on that grid-level. However, this horizontal interpolation is not acceptable for deposits of which the grid system must portray any of the following characteristics:

1) zonal mineral trends,

2) erratic mineral quantities, and

3) some specific structural features.

Soderburg and Hausch (9, page 141) have warned that "Horizontal interpolations of ore values must be made with caution. Some ore bodies have a zonal trend with a definite strike direction; others have ore values trending at dips up to vertical, with definite cut-offs in values normal to the dip zones. In such cases, horizontal interpolation of values will be misleading." For these situations, Soderberg and Hausch recommend that the grade prediction calculations be made with vertical grid-sections and that the information be transferred to the horizontal grid-sections. However, this merely changes the direction of the interpolation calculations; and therefore the same problems still exist, except in a different dimension.

Another solution to this representation problem can be formulated by constructing grid systems with different configurations from the traditional regularly spaced and rectangular grid system. The answer is to design the grid system to represent the mineral bearing rock structures. These grid systems are constructed by:

1) specifying the co-ordinate intervals equal to the rock structure spacing, and

2) orienting the grid axes parallel to the major mineral trends.

The following two examples illustrate the type of grid systems which can be constructed to overcome the representation problem. Firstly, consider a regularly spaced but non-rectangular grid system:

The vein deposit and grid system illustrated by the cross-section view in Fig. 9.28. The grade prediction of minerals is calculated individually for each grid-level parallel to
CROSS-SECTION OF A VEIN DEPOSIT

FIG. 3.2A A NON-RECTANGULAR GRID SYSTEM USED TO SIMULATE A VEIN DEPOSIT

CROSS-SECTION OF A LAYERED DEPOSIT

FIG. 3.2B AN IRREGULARLY SPACED GRID SYSTEM USED TO SIMULATE A LAYERED DEPOSIT
the vein. The mineral quantities of a particular grid-level are estimated from the assay composites taken in that grid-level. In this way, the zonal mineral trend of the vein deposit is simulated to avoid estimating mineral quantities from assays taken in different mineral bearing rock structures.

Secondly, consider a rectangular grid system of which the horizontal grid axes are equally spaced but the vertical axis is irregularly spaced:

The stratified deposit and grid system illustrated by the cross-section view in Fig 3-2. The co-ordinate intervals in the vertical direction have been constructed to correspond to the layer thickness of the rock strata. The mineral quantities of a particular grid-level can be estimated from the assay composites taken from the same grid-level. In this way, assay composites taken from different rock layers will not influence the grade prediction calculations.

The construction of the grid system to represent the deposit is important to insure the accuracy of the grade prediction calculations.

3. 2. 2 DIMENSIONS OF THE GRID SYSTEM

The grid system and the co-ordinate interval dimensions must be compatible with both the deposit size and the storage capacity of the computer used to execute the Model. Obviously, the grid system must be large enough to contain entirely any pit that could be derived from the deposit. The number of grid co-ordinates along the axes determines the total number of grid-intersections. The data sets created to describe the pit and deposit have a mesh for each of these grid-intersections. Consequently, the unit interval distances must be large enough to enable the computer to accommodate the data sets. The computer capacity required for the largest program in the Model is twenty-two bytes per grid-intersection for the data sets describing the pit and deposit, and an additional 210,000 bytes for the program and remaining data sets. These capacity estimates are slightly generous to allow for uncertain events. The total capacity could be trimmed for a particular trial of the Model.

The unit interval distances are designed to yield grid
blocks whose boundaries contain any convenient unit amount of the pit production capacity. This grid-block unit is adopted as a measurement standard for the mine's tonnage and volume computations. The grid-block dimensions should be small enough so that these unit tonnages and volumes can accurately measure the mine production and simulate the pit surface. Therefore, the vertical and transverse unit interval distances should be an integer divisor of the bench height and width as put forward for solution by the given mining proposal. Additionally, the longitudinal unit interval should approximate an integer divisor of the ore working face advance per year. In other words, the production capacity per year of the smallest excavator planned for the mine should be a multiple of the material within the grid-block boundaries. It is not essential that the grid-blocks be designed in this way; however, it does facilitate the pit production simulation.

3.2 3 SPATIAL DATA SETS

The data sets used to describe the deposit and pit are addressed by using the grid co-ordinates to direct the attention of the computer to the storage location of the data set members. Accordingly, there is a storage location corresponding to each value or quantity computed at the grid-intersections or grid-blocks. In this way, the density of any spatial distribution is given at the points defined by the grid system.

There are two types of data sets used to store these spatial distributions: The first type is a FORTRAN array which is a set of variables identified by a single variable name. A particular variable in the array may be referred to by its position in the array. Each variable in this array consists of the name of the array immediately followed by numbers enclosed in parentheses, called subscripts." (2, page 18). For example, consider the array named 'VALUE' which has three subscripts corresponding to the three co-ordinate directions of the grid system illustrated in Fig. 3. If it is desired to address the point density of 'VALUE' at grid co-ordinate I = 1, J = 3 and K = 2, this would be:

```
VALUE(1, 3, 2)
```

"Ordinary mathematical notation might use VALUE i, j, k to represent any element of the array VALUE." (7, page 19) In
FORTAN, this is written as \texttt{VALUE(I,J,K)}.

The second data set type used to store the spatial distributions is called a direct access data set. These data sets are kept on external storage devices, for example, a magnetic disk, instead of the internal computer storage. These data sets provide a permanent record and minimize the internal computer storage requirements. Additionally, they allow the user to specify the location within a data set from which data is to be read, or into which data is to be written" (2, page 53). The FORTAN statements 'READ' and 'WRITE' "cause transfer of data into or from internal storage" (2, page 53). The 'READ' and 'WRITE' statements have the following format:

\begin{verbatim}
READ(a'r) list
WRITE(a'r) list
\end{verbatim}

where: \texttt{'a'} represents the data set reference number followed by an apostrophe.
\texttt{'r'} represents the position of a record within the data set associated with \texttt{'a'}.
\texttt{'list'} is a series of variable or array names.

For example, the following 'WRITE' statement will cause the internal value of 'GRADE' to be written on data set ten in location 'LOC':

\begin{verbatim}
WRITE(10'LOC) GRADE
\end{verbatim}

If it is desired to 'WRITE' the point density of 'GRADE' at grid co-ordinate \( I = 2, \ J = 4 \) and \( K = 1 \), the record location 'LOC' is found with the following equation:

\[
LOC = MI + NJ \times (K-1) + NI \times (J-1) + 1
\]

Where: \texttt{'MI'} and \texttt{'NJ'} are the maximum grid co-ordinates in the \( I \) and \( J \) co-ordinate directions.

For the grid system illustrated in Fig. 3-1:

\[
LOC = 7 + 3 \times (3-1) + 7 \times (4-1) + 2
\]

\[
LOC = 149
\]

When the 'WRITE' statement is executed with the corresponding values of 'LOC' and 'GRADE', it will cause the record to be transferred from internal storage to the appropriate
location on the direct access device.

Thus, the two data set types--array and direct access--are created to simulate the spatial distribution functions of the pit and deposit. The importance of these data sets is in the construction of a technique to correlate the computer storage location of the data set members with their location in the deposit.

3.2.4 SOME PROGRAMMING CONSIDERATIONS

The grid co-ordinates and the actual distances represent two different measurement scales for the user's deposit. The two scales are mixed in some of the computer programme expressions. Consequently, scaling factors are included in these computations to convert between actual distances and the grid co-ordinates. This is simply a matter of scaling the distances in proportion to the unit interval distances between grid co-ordinates. However, the scale conversion must take into account the angular separation of the axes, if a non-rectangular grid system is used. The way in which these scale conversions are made is explained in Section 4.1.

Complicated grid system designs should and probably can be avoided to keep the computer programs simple. The irregularly spaced grid systems are an unnecessary complication, if a regular grid can be designed by reducing all the unit interval distances to equal the smallest significant rock layer. Then, the thicker rock layers are simulated by building up the equal unit interval distances. Additionally, the non-rectangular grid can be used conveniently for the Grade Prediction Program, and then substituted by a regular grid for the remaining computations of the Model. This change is made easily by transferring the point densities at the grid intersections in the non-rectangular grid to the same position in the rectangular grid.

3.3 SIMULATION OF THE DEPOSIT'S STRUCTURAL FEATURES

Most deposits have structural features that interfere with some of the Model's computations, as explained previously in Section 3.1. These structural features are the rock formations, topography, fault zones, water table, and so forth.
that will influence, in any way, the development of an open pit mining solution. To overcome this problem, the model is provided with a decision capability to branch the computer programmes to the computations which are appropriate for the particular physical conditions. This is accomplished by programming the model to recognize the regions distinguished by the structural features.

The following two examples demonstrate the need to simulate the structural features:

1) Consider an intruded rock structure that has a different mineral composition than the remainder of the deposit. This difference makes it necessary to insure that assays taken in the two zones are not used together to produce the mineral grade predictions.

2) Consider a pit surface whose safe wall-angles depend on the relative orientation of the pit walls to the bedding planes of the rock formations. The variable safe wall-angles make it necessary to define the pit zones limited by a particular safe wall-angle.

The shape of any structural features can be simulated to make a computer analogue of the pit and deposit. In addition to these physical or structural features, there may be legal restrictions on the pit, for example, property lines. The same simulation techniques are employed to make certain that the pit surface does not extend beyond the property lines, as demonstrated in Subsection 6.5.2.

3.3.1 INDICATOR ARRAYS

There are two techniques available for simulating the shape of the structural features and legal boundaries. The first technique is an indicator array. These arrays are similar to the spatial density arrays described in Subsection 3.2.3. They have an array member corresponding to each grid-intersection. The array members are pointers which indicate the presence of the particular structural features. The deposit's topography is a good example for illustrating the indicator arrays. A zero value is given to the array members representing grid-intersections above the deposit's surface. Any positive integer value is assigned to the remaining array members. The positive array members represent grid-intersections below the deposit's surface.
The input data used to create the topographic file are the intercepts of the deposit's surface with the grid system. The intercepts are read into a data set called 'SCORK'. 'SCORK' is an array with one subscript which identifies the horizontal grid co-ordinate location of the intercepts. The single subscript defines this horizontal location by specifying an array member for each intercept. The vertical co-ordinates of the intercepts become the array members. The 'SCORK' variable is given in Table 3.1 for the grid system illustrated in Fig. 3.1.

<table>
<thead>
<tr>
<th>Surface Intercepts</th>
<th>Input</th>
<th>INTERCEPT</th>
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<tbody>
<tr>
<td>I</td>
<td>J</td>
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</table>

where:

\[ N = N_1 \times (I - 1) + I \]

'\(N_1\)' is the maximum value of 'I'.

| T A B L E  3 . 1 |

The indicator array 'TCPG' is created from 'SCORK' by...
executing the following FORTRAN statements:

```fortran
READ(S,1) NI,NJ,NK
1 FORMAT (31H)
    NH=NI+NJ
READ(S,10) (SCOR(N),N=1,NK)
10 FORMAT (7H)
DO 104 K=1,NK
    N=0
    DO 104 J=1,NK
    DO 104 I=1,NI
    IF (SCOR(N)=K) 105,103,103
103 II=I
    GO TO 104
104 TOPO(I,J,K)=II
```

'NI', 'NJ' and 'NK' are the maximum grid co-ordinates in the 'I', 'J' and 'K' co-ordinate directions, respectively. The '7' in statement number ten is equal to 'NI'.

This topographic indicator array is constructed to control all computations of the spatial density functions. The density computations are by-passed for any particular array member, if the 'TOPO' array indicates the air space above the deposit's surface. For example, assume the model is executing the computations to estimate the mineral quantities at each grid-intersection. Obviously, it is a redundant exercise to make this estimate for the grid-intersections representing air space. Consequently, the model is programmed to recognize these grid-intersections and to assign them a zero mineral quantity without going through the computations.

All indicator arrays are created and used in a similar way to the 'TOPO' example.

3.3.2 POLYNOMIAL EQUATIONS

The second technique for simulating structural features is the derivation of polynomial equations to represent the surfaces and boundaries. The polynomial equations are constructed on the co-ordinates of the grid system. They are only an effective alternative to the indicator arrays for surfaces with few irregularities.
Consider, for example, a fault through which there has been relative rock movement. As a result of this rock movement, the fault defines a boundary on either side of which the grade predictions computations must be independent of each other. The fault is simulated with a polynomial equation to identify the relative location of the bore-hole samples to the boundary. This fault could be simulated with a plane, as illustrated in Table 3.2.

<p>| Bore-hole Intercepts with the Fault Plane: | Grid Co-ordinate Intercept |</p>
<table>
<thead>
<tr>
<th>Bore-hole No.</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29.50</td>
<td>18.25</td>
<td>4.40</td>
</tr>
<tr>
<td>26</td>
<td>17.50</td>
<td>14.00</td>
<td>14.60</td>
</tr>
<tr>
<td>38</td>
<td>3.00</td>
<td>12.00</td>
<td>3.80</td>
</tr>
</tbody>
</table>

Matrix of the Bore-hole Intercepts:

\[
\begin{bmatrix}
I & J & K \\
29.50 & 18.25 & 4.40 \\
17.50 & 14.00 & 14.60 \\
3.00 & 12.00 & 3.80
\end{bmatrix}
\]

Matrix Solution of the Fault plane:

\[66.3e1 - 277.5e3 J - 37.625e5 K + 3274.075 = 0\]

TABLE 3.2

The grid co-ordinate location of the bore-hole samples compared with the fault plane equation. Then, the mineral quantities on each side of the fault are predicted from only those samples coming from the same side of the fault. In this way, samples existing on different sides of the fault are prevented from affecting the grade predictions unfavourably.

3.3 SOME PROGRAMMING CONSIDERATIONS

The employment of these structural simulation techniques is discussed and demonstrated in the next section. The following list itemizes their advantages and disadvantages.
The indicator arrays:
1) are simple to programme,
2) are rapidly executed by the computer,
3) can accurately simulate irregular surfaces, but
4) require a great amount of internal computer storage space.

The polynomial equations:
1) require little internal computer storage space, but
2) are difficult to derive and programme, and
3) are not accurate for highly irregular surfaces.

The large storage requirements for the indicator arrays is partially alleviated by allocating two computer storage locations per array member instead of the usual four. As a result, the maximum value that any of these variables can adopt is 32,000. This allocation is executed by dimensioning indicator arrays with the FORTRAN specification 'INTEGER*2'.

3.4 SPATIAL DENSITY OF THE DEPOSIT'S MINERAL VALUES

Description of the two computer programmes used to estimate the spatial density of the mineral distribution for the user's deposit is contained in this section. The spatial mineral distribution is prerequisite to the development of the open pit mining solution described in Chapter 6. The following subsections correspond to the two computer programmes. They describe the input, output and basic computations, and how these computations cope with interference from the given deposit's structural features.

3.4.1 COMPUTATION OF THE ASSAY COMPOSITES

The Assay Composite Programme divides each bore-hole into the grid-levels defined by the FORTRAN array addresses on the vertical axis of the grid system. The assays of each grid-level from each bore-hole are averaged in proportion to
their sample lengths to furnish composite values. The composites are identified by their bore-hole, grid-level and mineralization zone. Then, they are passed to the Grade Prediction Programme.

The mineral samples from the exploration bore-holes are input data to the Assay Composites Programme. The format for these data consists of the proportional mineral content per bore-hole length for each recoverable mineral. Table 3 presents the input data from the bore-hole illustrated in Fig. 3. The vertical unit intervals are one hundred feet. The samples are reported from the top of each borehole to the bottom and their lengths are measured in feet. The first sample length includes the distance from the zero grid-level to the deposit's surface, and additionally, any barren rock from the surface down to the first reported mineralization. The first sample length is reported in this manner so that the measurements conform to the same base. In this way, the cumulative sample lengths of any bore-hole will correspond to a relative location in the grid system.

<table>
<thead>
<tr>
<th>Sample No</th>
<th>Sample Length</th>
<th>Proportional Mineral Content</th>
<th>Mineral 1</th>
<th>Mineral 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>340.6</td>
<td>0.00</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2</td>
<td>87.2</td>
<td>0.0025</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>3</td>
<td>204</td>
<td>0.0028</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>4</td>
<td>50.2</td>
<td>0.0041</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>5</td>
<td>39.8</td>
<td>0.0038</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>6</td>
<td>47.6</td>
<td>0.0049</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>7</td>
<td>74.6</td>
<td>0.0052</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>8</td>
<td>249.2</td>
<td>0.0053</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>9</td>
<td>196.4</td>
<td>0.0049</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>10</td>
<td>158.6</td>
<td>0.0036</td>
<td>0.0010</td>
<td>0.0</td>
</tr>
<tr>
<td>11</td>
<td>295.9</td>
<td>0.0075</td>
<td>0.0026</td>
<td>0.0001</td>
</tr>
<tr>
<td>12</td>
<td>62.8</td>
<td>0.0041</td>
<td>0.0035</td>
<td>0.0001</td>
</tr>
<tr>
<td>13</td>
<td>35.8</td>
<td>0.0036</td>
<td>0.0012</td>
<td>0.0001</td>
</tr>
<tr>
<td>14</td>
<td>19.0</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

TABLE 3*3
The input bore-hole samples are averaged in proportion to their length to give a single mineral value for each grid-level and bore-hole. This average becomes the 'assay composite' for the given grid-level. For instance, the assay composite of Mineral 1 for the fifth grid-level illustrated in Fig. 3 is calculated as shown in Table 3. Note that the lengths of samples two and five are not equal to their reported length. These sample lengths are truncated by the upper and lower boundaries of the fifth grid-level. Sample No. 2 is 427.8 ft. and Sample No. 5 is 530.2 ft. from the zero grid-level. Consequently, only the bottom 27.8 ft. of Sample No. 2 and the top 1.6 ft. of Sample No. 5 are included in the average for the fifth grid-level's assay composite.

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Mineral Content</th>
<th>Sample Length</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.0025</td>
<td>27.8</td>
<td>0.00695</td>
</tr>
<tr>
<td>3</td>
<td>0.0028</td>
<td>20.4</td>
<td>0.00571</td>
</tr>
<tr>
<td>4</td>
<td>0.0041</td>
<td>50.2</td>
<td>0.02058</td>
</tr>
<tr>
<td>5</td>
<td>0.0038</td>
<td>1.6</td>
<td>0.00060</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>100.0</strong></td>
<td><strong>0.3385</strong></td>
</tr>
</tbody>
</table>

**Assay Composite =**  
\[
\frac{0.3385}{100.0} = 0.0034
\]

The assay composites for all the grid system types discussed in Subsection 3.2.1 are computed as illustrated in the above example. There is one simple difference in the programming for irregularly spaced grid systems: The vertical unit intervals for these grids are stored in a FORTRAN array to replace the constant interval size of the regularly spaced grid systems. This modification is pointed...
out in Section 4.1.

When the assay composites have been computed, it is necessary to define the geologic region in which they occur. This is required to identify the different mineralization zones, as explained in Section 3.3. The Assay Composites Programme executes this identification with the following routine:

```
I=0
DO 10 N=1,NBH
DO 10 K=1,NF
I=I+1
IF (3274*10*663*CORI(N)-3875*CORJ(N)-37.625*Z) 6,7,7
   IF (K-10) 1,1,2
   IF (K-10) 8,8,9
   M=1
   GO TO 3
   M=2
   GO TO 3
   M=3
   GO TO 3
   M=4
3 ASAY1(K,N,M)=SAC0(K,1,M)
ASAY2(K,N,M)=SAC0(K,2,M)
WRITE(1,1) ASAY1(K,N,M),ASAY2(K,N,M)
10 CONTINUE
```

Definitions:

'I' is a counter for the storage location of the direct access data set number eleven.

'N' is a subscript which identifies the bore-holes.

'NBH' is the total number of bore-holes.

'K' is a subscript which identifies the grid-level.

'NF' is the total number of grid-levels.

'CORI(N)' and 'CORJ(N)' are the I and J bore-hole coordinates in feet.

'Z' is a subscript which identifies the mineralization zones.
'SACO(K,1, M)', and 'SACO(K,2, M)' are the unzoned assay composites for two minerals, respectively.

'ASSAY1(K,N,M)', and 'ASSAY2(K,N,M)' are the assay composites for two minerals, respectively. These data sets include the subscript 'N' to identify the mineralization zones.

The preceding routine sorts the assay composites into four mineralization zones. The four zones are delimited by a fault and the water table. The first IF statement locates the assay composites relative to the fault. The IF statements '6' and '7' locate the assay composites relative to the water table which occurs at the tenth grid level.

Note that the expression in the first IF statement has mixed scales. 'CCPI' and 'CORJ' are measured in feet and the remaining terms in grid co-ordinates. The mixed scales are compensated for by the constants. Expressions with mixed scales occur frequently in the simulation routines; therefore, care must be exercised, if they are modified.

The programme accommodates as many minerals as required by specifying the appropriate dimensions for the FORTRAN variables. The dimensions are listed and explained in Section 4.8.

The above example serves to illustrate the method of sorting assay composites into their mineralization zones. In this example, the given fault and water table must be replaced with the new boundaries of any different deposit. The construction of the surface expressions for these new boundaries was explained in Section 3.3. A simple example will serve to illustrate how the substitution is made. The fault expression in the first IF statement could be replaced by the following indicator array, if the fault surface is highly irregular:

\[
\text{FAULT(CORI(N),CORJ(N),K)}
\]

where 'CORI' and 'CORJ' are measured in grid co-ordinates instead of feet, as above. This 'FAULT' indicator array would be created the same way as explained for the 'TOPO' array in Section 3.3.

The output data from the Assay Composites Programme are the assay composites for each potentially recoverable mineral.
The "WRITE" statement in the above routine causes the assay composites to be stored in a direct access data set for the Grade Prediction Programme.

3.4.2 INTERPOLATION OF THE SPATIAL MINERAL DISTRIBUTION

An interpolation method is used to estimate the point densities of the spatial mineral distribution at the grid intersections. The computations described in this Subsection are executed by the Grade Prediction Programme listed in Chapter 5.

The interpolation method was suggested by R.C. Weaver (11), and it was chosen over the trigonal and polygonal methods. The interpolation method offers search parameters that adapt it to the peculiar grade prediction conditions of different mineralization zones. These parameters define the search for the assay composites to be included in each estimate of a point mineral density. The capability for varied execution makes the interpolation method more desirable than the trigonal and polygonal methods in terms of increased accuracy and adaptability. Nevertheless, the interpolation method is limited to massive and relatively uniform mineral deposits.

The point densities of the spatial mineral distribution are averages of the surrounding assay composites on the same grid-level and in the same mineralization zone. The assay composites are weighted by the inverse proportion of their distance from each point density. The basic interpolation formula follows; it is written in FORTRAN notation:

\[
X = 0 \\
Y = 0 \\
\text{DO } 10 \ N = 1, \text{NAC} \\
X = X + (G(N)/D(N)**P)) \\
10 \ Y = Y + (1/D(N)**P)) \\
\text{GRAD}(I,J,F) = X/Y
\]

where:

\(\text{NAC}\) is the total number of assay composites.

\(G(N)\) is the proportional mineral content of assay compo-
site 'N'.

'D(M)' is the distance between assay composite 'N' and grid-intersection (I,J,K).

'P' is a weighting factor.

'GRAD(I,J,K)' is the estimated point mineral density at grid-intersection (I,J,K).

The preceding routine is executed at each grid-intersection (I,J,K) to estimate the point densities of the spatial mineral distribution 'GRAD'. This estimate is repeated for all recoverable minerals reported in the user's deposit. The assay composites 'G' included in each estimate are identified by the search parameters mentioned previously. These parameters are:

1) A maximum distance limit: The assay composites for each estimate are limited to those within a maximum distance of the particular grid-intersections. This is a circular limit so that each estimate is made only from the assay composites occurring on the same grid level or mineral horizon. The grid-levels will correspond to the mineral trends or horizons, if the grid system is constructed as described in Section 3.2.

2) A minimum distance limit: If an assay composite exists within a minimum distance of a particular grid-intersection, this grid-intersection's point mineral density assumes the value of the assay composite.

3) A weighting factor: The reciprocal distance between grid-intersection and assay composites is weighted with a power factor 'P', as demonstrated in the preceding routine.

4) A maximum number of assay composites: The number of assay composites per estimate is limited to those closest to the particular grid-intersections, up to the maximum.

5) A minimum angular separation: The minimum angular separation between assay composites must be exceeded; if not, the assay composite furthest from the particular grid-intersection is deleted from the average.
The function of these parameters can be demonstrated with the grid-intersection illustrated in the center of Fig. 3.4 and the surrounding assay composites. Of the seven assay composites illustrated, the following ones are rejected from the estimate of the grid-intersection's point mineral density:

1) AC6 and AC7, because they exceed the maximum distance limit.

2) AC2, because it is further from the grid-intersection than AC3 and the angular separation between the two is less than the minimum.

3) AC1, because it is the furthest from the grid-intersection of the remaining assay composites; and without it, the maximum number of assay composites per average is not exceeded.

These search parameters are defined by the user before the Grade Prediction Programme is executed. They can be determined with a trial and error solution in the following manner: A random sample is taken of the assay composites from each different mineralization zone. The mineral values of each random sample are estimated with the remaining assay composites in the same zone with assumed search parameters. Alternatively, the assay composites could be used to estimate the mineral values of a previously mined-out section, if one exists. The random samples' estimated values are compared with their actual values using a correlation test. The search parameters are redefined until the correlation test becomes significant.

The correlation test is made from a regression curve of the estimated and actual values. The actual values are the independent variables on the curve and the estimated values are the dependent variables. A least square regression line is calculated from this data. If a straight line relationship can be assumed between the variables, the regression line is expressed in FORTRAN as:

\[ EV = A + (B \times AV) \]

where:
- 'EV' is the estimated value,
- 'AV' is the actual value, and
- 'A' and 'B' are constants.
AC = ASSAY COMPOSITE

GRID-INTERSECTION

MAXIMUM DISTANCE LIMIT

FIG. 3.4 THE SURROUNDING ASSAY COMPOSITES OF A PARTICULAR GRID-INTERSECTION
The constants $A$ and $B$ can be calculated with the following FORTRAN routine:

```
X=0
Y=0
X2=0
XY=0
DO 10 N=1,NS
   X=X+AV(N)
   Y=Y+EY(N)
   XY=XY+EV(N)*AV(N)
10 X2=X2+EY(N)**2
A=((X**2)-(X*Y))/( NS**2)-(X**2)
B=((NS*XY)-(X*Y))/(NS*X2-(X**2))
```

where:

- 'X' is the summation of the actual values,
- 'Y' is the summation of the estimated values,
- 'X2' is the summation of the actual values squared,
- 'XY' is the summation of the product of the actual and estimated values, and
- 'NS' is the total number of samples.

The degree of association between the estimated and actual values is measured to find out how well the current search parameters estimate the actual mineral quantities. The relative intensity of this association is given by the coefficient of correlation. The correlation coefficient can be computed with the following FORTRAN routine:

```
C=0
D=0
DO 10 N=1,NS
   C=(A*(B*AV(N)))-N**2
10 D=(EV(N)-Y)**2
R=SQRT(C/D)
```

where:

- 'NS' is the total number of samples,
- 'A' and 'B' are the constants from the regression line.
'AV(N)' are the actual values.

'(A (R*AV(N)))' are the estimated values given by the regression line.

'\( \bar{E} \)' is the mean of the estimated values.

'EV(N)' are the estimated values, and

'R' is the correlation coefficient.

The correlation coefficient 'R' is dimensionless and varies between -1 and +1. The plus sign shows positive linear correlation and the minus sign shows negative linear correlation. Either extreme of the coefficient is perfect correlation, and any value in between is less than perfect. Additionally, a zero correlation indicates no association between the estimated and actual mineral quantities.

The search parameters are varied until the correlation analysis indicates that the parameters have produced a good association between the estimated and actual mineral quantities. However, the association may not be linear as assumed, in which case the correlation coefficient may approximate zero. If so, the linear relationship is replaced with a non-linear regression equation.

The number of random samples required and a significant correlation coefficient are calculated for each mineralization zone. Student's distribution can be used for this in the following manner: Firstly, a correlation coefficient is calculated on an assumed sample size. Next, the assumed size is compared with the minimum sample size required for a significant correlation coefficient. If the assumed size is larger than the minimum, then the correlation coefficient is significantly different than zero on a particular confidence level. This minimum is computed from the following statistic, expressed in FORTRAN:

\[
T = \frac{R \cdot \text{SQRT}(N-2)}{\text{SQRT}(1 - (R^2))}
\]

where:

'R' is the given correlation coefficient from the assumed sample.
FIG. 3.3 VERTICAL CROSS-SECTION OF A BORE-HOLE
'S' is the minimum sample size, and 'T' is Student's statistic.

The minimum sample size 'S' must be such that the right hand side of the above expression is equal to the critical value 'T' from Student's distribution on a given confidence level and with 'N-2' degrees of freedom. When 'S' has been calculated, it is compared with the assumed sample size. The correlation coefficient 'R' is significant if the assumed sample size is greater than 'S'. Otherwise, the assumed sample size is enlarged until it is greater than the minimum size.

The search parameters are defined for each of the mineralization zones in the preceding manner. M. R. Spiegel's "Theor; and Problems of Statistics" (10) is recommended for further discussion of the sampling theory of correlation.

The mineralization zones are recognized by the Grade Prediction Programme with a duplicate of the simulation routine used to sort the assay composites. This routine is demonstrated in Subsection 3.4.1. The simulation routine recognizes the distinct structural features to identify the mineralization zone of the point mineral densities being estimated. Then, the routine identifies the zones of the assay composite. The assay composites, whose zones are not the same as the particular point mineral density, are excluded from the estimate. The mineralization zones used for demonstration purposes in Chapters 4 and 5 will be undoubtedly different for any other deposit. Consequently, this simulation routine must be tailored to the user's deposit by creating the appropriate indicator arrays or polynomial equations, as explained in Section 3.3.

In addition to the mineralization zones, it is necessary to simulate the deposit's surface for the Grade Prediction Programme. This simulation is made with the 'TOPO' indicator array used as an example in Section 3.3. The interpolation for the point mineral density at any given grid-intersection is executed only if the 'TOPO' array indicates that the point is below the deposit's surface. Otherwise, the interpolation computations for that point are by-passed and the Programme moves to the next grid-intersection.

Thus the spatial mineral distribution is computed. The point
mineral densities estimated at the grid-intersections are stored in a permanent record. This record is a direct access data set which is kept to provide the succeeding Mine Plan Programme with some of its input data.
distances between grid co-ordinates and the direction of the axes, as explained in Section 3.2. The grid system dimensions that must be planned to execute the Assay Composites Programme are:

1) the angular separation of the co-ordinate axes,
2) the relative position of the zero grid-level with the deposit's surface, and
3) the unit interval distances of the more nearly vertical axis.

The zero grid-level is defined to correspond with the highest point on the given deposit's surface. Elevations are measured from the zero grid-level with depth being the positive direction. The vertical unit intervals are given as explained in Step 1.

STEP 2:

The dimensions must be specified for the FORTRAN arrays and direct access data sets. The dimensions:

1) are defined for the FORTRAN arrays in Section 4.2,
2) are given for the DEFINE FILE statement in Section 4.3 for the direct access data sets.

STEP 3:

A scale factor must be defined to convert between actual vertical distances and grid co-ordinates. This scale factor is called 'KINT' in the Assay Composites Programme:

A) 'KINT' is a constant and equal to the unit interval distance between the vertical grid co-ordinates for a regularly spaced and rectangular grid system.

B) 'KINT' is a variable with an array member equal to each interval distance between grid co-ordinates for an irregularly spaced vertical axis. 'KINT' is dimensioned as illustrated in Fig. 4.1 for this situation.

C) 'KINT' is a constant and equal to the vertical distance between the more nearly horizontal axis of the non
The scale factor 'KINT' for the above deposit

KINT(1)=50
KINT(2)=165
KINT(3)=30
KINT(4)=130
KINT(5)=150

Fig. 4.1 Scale factor 'KINT' for an irregularly spaced grid system
-rectangular grid systems. It is illustrated in Fig. 4.2 for a non-rectangular grid.

STEP 4:
The appropriate number of variables must be added at Lines 086 and 162 for each potentially recoverable mineral reported in the deposit.

STEP 5:
The boundaries of the distinct mineralization zones are simulated with indicator arrays or polynomial equations, as described in Section 3.5. These simulated surfaces are substituted into the Programme's simulation routine between Lines 092 and 101. In this way, the Programme is tailored to locate the specific mineralization zones of the given deposit's assay composites. The substitution of these zones is described in Section 3.4.1.

Care must be exercised to insure that the surface simulations are made with the correct units. Simple scale factors are used to convert actual distances to grid co-ordinates or conversely. The constant scale factor described in Step 3.A is used to convert scales on regularly spaced axes. The array described in Step 3D is used to convert scales on irregularly spaced axes. Finally, an equation is derived to convert from actual rectangular co-ordinates to non-rectangular grid co-ordinates. This equation for the grid system illustrated in the cross-section view of Fig. 4.2 is:

\[ K = \frac{\tan(a) \times (I \times SF1 + \text{---})}{\tan(a)} \]

\[ K = \frac{\text{---}}{SFK} \]

where:

'\( K \)' is the grid-level;

'\( I \)' is the horizontal grid co-ordinate;

'\( V \)' is the vertical distance in feet;

'SFI' is a scale factor equal to the interval distance.
FIG. 4.2 CROSS-SECTION OF A NON-RECTANGULAR GRID
between the I grid co-ordinates in feet.

'SFK' is a scale factor equal to the interval distance between the K grid co-ordinates in feet.

'a' is the angle between the horizontal and the K co-ordinate axes.

The scale factors are included in any of the FORTRAN expressions that have terms with mixed scales; these expressions have been noted in Section 4.3.

4.2 SEQUENCE OF PROGRAMME COMPUTATIONS

LINE 001 - 003:
Specification statements.

LINE 004 - 015:
Input statements.

LINE 014:
The horizontal bore-hole co-ordinate locations, in feet, are stored on a magnetic disk for rapid access in subsequent programmes.

LINE 016 - 022:
DO loop 16 composites the bore-hole samples 'SAMP' between grid-levels to give an average mineral grade 'SACO' for each recoverable mineral (H), bore-hole (N) and grid-level (K).

LINE 020 - 022:
These statements calculate the cumulative distance 'CUMINT' from the zero grid-level to each bore-hole sample.

LINE 029 - 031:
These statements simulate the deposit's surface by assigning any negative value to 'SACO', if the assay composite should represent air space. Whenever 'SACO' is used, its array
members are tested for negative values and the programme branches past the appropriate calculations, if the array member is negative.

LINES 032 - 075:

These statements contain an implied computational loop to identify the bore-hole samples 'SAMP' to be included in each assay composite 'SACO'.

LINES 076 - 081:

The bore-hole samples 'SAMP(N,L,C(LL))', 'SAMP' AND 'SAMPB' occurring on the given grid-level (K) are averaged in proportion to their sample lengths 'INT(N,L,C(LL))', 'INTA' AND 'INTB' to yield that grid-level's assay composite 'SACO'.

LINES 083 - 105:

The assay composites 'SACO' are given an additional array dimension (K) to identify their mineralization zones.

4.3 A LIST OF THE ASSAY COMPOSITES PROGRAMME

The following variables were dimensioned for the open pit proposal described in Section 7.1. They must be changed for any new deposit, as defined in Section 4.4.

LINES 001 - 002:

REAL SAMP(2,60,18),INT(60,18),C0STI(60),C0RJ(60),
1CMINT(17),INTA,INTB,INTC(17,2,60),
2ASAY1(19,60,4),ASAY2(19,60,4),KINT
002 INTEGER LC(20),SURFAC(60).

-----------------------------------------------COMMENT-----------------------------------------------

The lower case constants in the following line are: ii = NAM * NK, jj = (number of recoverable minerals + 1) * 4, kk = NAM.
A dimension must be added to the constant 'KINT', if the vertical grid co-ordinate intervals are irregularly spaced. The intervals, in feet, between grid co-ordinates are stored in the new array 'KINT(K)' for DO loop 16. 'KINT(K)' must be substituted for 'KINT' wherever it appears in the program.

```
READ (5, J) KINT, NMIN, NBR, Nsamp, NK, NZ
DO 801 K = 1, NBR
801 READ (5, 1) SAMP(1, N, L), SAMP(2, N, L), INT(N, L),
1SAMP(1, N, L+1), SAMP(2, N, L+1), INT(N, L+1),
2SAMP(1, N, L+2), SAMP(2, N, L+2), INT(N, L+2)
1 FORMAT (F7*4, F6*1, 2F7*4, F6*1, 2F7*4, F6*1)
DO 1003 N = 1, NBR
1003 READ (5, 2) SURFAC(N), CORI(N), CORJ(N)
X = CORI(N)
Y = CORJ(N)
DO 515 L = 1, Nsamp
515 SAMP = 0
DO 515 L = 1, Nsamp
515 SAMP = 0
130 SACO(K, A, N) = 10*0
GO TO 16
131 SUBZ = 0
DO 51 LL = 1, Nsamp
51 LC(LL) = 0
I = 0
```
034  L=1
036  IF (K*K+KINT-CUMINT(L)) 6,8,7
038  INTA=INT(N,L)-CUMINT(L)*K*KINT
039  IF (INTA-KINT) 14,9,9
040  SAMP=SAMP(N,N,L)
041  GO TO 10
042  IF (L-NSAMP) 101,100,100
043  L=L+1
044  IF (INT(N,L)) 52,52,5
045  INTA=INT(CUMINT(L)+K*KINT
046  IF (INTA-KINT) 53,112,112
047  SAMP=0
048  GO TO 10
049  100 IF ((K*K+KINT-CUMINT(L))-KINT) 111,111,112
050  111 INTA=K*KINT-CUMINT(L)
051  SAMP=0
052  SUM=INT(N,L)
053  SUM=INT(N,L)+SAMP(N,N,L)
054  IF (SUM-KINT) 150,150,151
055  112 SACO(K,N,M)=0*0
056  GO TO 16
057  IF (INT-D(L,L)) 9,9,110
058  9 SACO(K,N,M)=SAMP(N,N,L)
059  GO TO 16
060  110 INTA=INT(N,L)
061  SAMP=SAMP(N,N,L)
062  10 SUM=INT(A)
063  150 LL=0
064  11 IF (SUM-KINT) 12,151,13
065  12 IF (L-1) 13,13,102
066  102 SUM=SUM+INT(N,L-1)
067  LL=LL+1
068  LC(LL)=L-1
069  L=L-1
070  I=I+1
071  GO TO 11
072  13 SUM=SUM+INT(N,L)
073  I=I-1
074  151 INTA=KINT-SUM
075  SAMP=SAMP(N,N,L)
076  IF (I) 120,120,140
077  140 DO 15 LL=1,I
078  IF (LC(LL)) 120,120,15
079  15 SUM=SUM+SAMP(N,N,LC(LL))*INT(N,LC(LL))
080  120 SUM=SUM+SAMP(INTA+SAMP)*INTA
081  SACO(K,N,M)=SUM/INT
A variable 'ASAYn' is added for each recoverable mineral at lines 086 and 102. In other words, ASAY1(K,N,K)=0, ASAY2(K,N,K)=0, **ASAYn(K,N,K)=0 is added at line 086; and ASAY1(K,N) = SAC1(K,1,N), ASAY2(K,J,M) = SAC2(K,2,N), **ASAYn(K,N) = SACN(K,n,N) is added at line 102, where 'n' equals the number of recoverable minerals.

The given mineral deposit was divided into four mineralization zones by a fault (line 092) and a water table (lines 093 and 094). The portion of this program from lines 092 and 101 is tailored to the user's deposit, as explained in Section 3.4.1. Note the mixture of scales in line 092 and 'C01' and 'C02' are measured in feet but 'Y' is measured in grid co-ordinates. This difference is accounted for by including scale factors in the expression's constants.

```
082 16 CONTINUE
083 DO 1112 K=1,NK
084 DO 1112 N=1,NH
085 DO 1112 M=1,NZ

086 ASAY1(K,N,K)=0
087 1112 ASAY2(K,N,K)=0
088 I=0
089 DO 1010 N=1,NH
090 DO 1010 K=1,NK
091 I=I+1

092 IF (1274*0.075+0.663*C01(N)-1*3875*C02(N)
1-37.625*K) 1006, 1007, 1007
093 1006 IF (X-10) 2001, 2001, 2002
094 1007 IF (X-10) 1003, 1008, 1009
095 1008 N=1
096 GO TO 2003
097 1009 M=2
098 GO TO 2003
099 2001 M=3
100 GO TO 2003
```
CHAPTER 4 ASSAY COMPOSITES PROGRAMME

The computer programme used to execute the assay composites computations described in Section 3.4.1 is listed in Section 4.3. The remaining Sections of Chapter 4 supplement this list with:

1) a description of how the Programme is prepared for execution in Section 4.1,

2) a chronological description of the Programme's computations in Section 4.2,

3) comments within the Programme list in Section 4.3, and

4) a glossary of the FORTRAN variables in Section 4.4.

4.1 Preparation of the Programme for Execution

The Assay Composites Programme, as listed in Section 4.3, was written for a regularly spaced and rectangular grid system. Additionally, its physical simulation routine was written for the deposit described in Section 7.1. The steps required to adapt the Programme to another deposit's:

1) grid system,

2) size and

3) mineralization zones

are listed below. These steps are cross referenced with the Programme list in Section 4.3 to locate the exact position of the modifications.

STEP 1:

The grid system is defined by choosing the unit interval
This glossary defines the FORTRAN variables of the Assay Composites Programme. The glossary is not complete; some transfer variables used to facilitate programming are not included in the list. The entries appear in the following format:

NAME(i,j,k) - { } definition { };

• i - the name of the first subscript
• j - the name of the second subscript
• k - the name of the third subscript

This arrangement of each variable's definition is described as:

1) The variable's FORTRAN name appears first.

2) The first set of brackets displays the number of subscripts or does not appear, if the variable is not an array.

3) The variable's precision is displayed between the second set of brackets with the following abbreviations:

• 'D' is double precision, with eight storage locations;
• 'R' is real precision, with four storage locations;
• 'I' is integer precision, with four storage locations; and
• 'I*2' is integer precision, with two storage locations.
This glossary defines the FORTRAN variables of the Assay Composites Program. The glossary is not complete; some transfer variables used to facilitate programming are not included in the list. The entries appear in the following format:

\[ \text{NALE} (i, j, k) - ( ) \text{ definition ( )}; \]

\[ i - \text{the name of the first subscript} \]
\[ j - \text{the name of the second subscript} \]
\[ k - \text{the name of the third subscript} \]

This arrangement of each variable's definition is described as:

1) The variable's FORTRAN name appears first.

2) The first set of brackets displays the number of subscripts or does not appear, if the variable is not an array.

3) The variable's precision is displayed between the second set of brackets with the following abbreviations:

- 'D' is double precision, with eight storage locations;
- 'R' is real precision, with four storage locations;
- 'I' is integer precision, with four storage locations; and
- 'I2' is integer precision, with two storage locations.
4) A statement of the variable’s significance appears next.

5) The third set of brackets displays the subroutines in which the name appears, if the same name is used for different variables. The brackets do not appear, if there is no conflict.

6) Finally, the dimensions of the variable’s array are named. These dimensions only appear as required.

-----------------------------GLOSSARY-----------------------------

ASA(i, j, k), ASAY(i, j, k), ...ASAYn(i, j, k) - (R) the assay composites for ‘n’ recoverable minerals;

i - grid-level
j - bore-hole identification number
k - mineralization zone identification number

CORI(i), CORJ(i) - (") the rectangular I and J co-ordinate locations, respectively, of the bore-holes in feet;

i - bore-hole identification number

CUXINT(i) - (R) the cumulative distance from the zero grid-level to each bore-hole sample;

i - sample identification number

INT(i, j) - (R) the sample length;

i - bore-hole identification number
j - sample identification number

INTA, INTB - (R) two variables used to adjust the top and bottom sample lengths between the grid-level boundaries, when the boundaries do not coincide with the cumulative sample length ‘CUXINT’;

KINT - (R) the interval distance between the vertical grid co-ordinates.

LC(i) - (P) a counter of the number of samples per grid-level per bore-hole;
i - sample identification number

NBH - (I) the total number of bore-holes

NK - (I) the total number of grid-levels

NMIN - (I) the total number of recoverable minerals

NSAMP - (I) the total number of samples in the bore-hole with the most samples

NZ - (I) the number of mineralization zones

SACO(i, j, k) - (R) assay composites;

   i - grid-level
   j - mineral identification number
   k - bore-hole identification number

SAMP(i, j, k) - (R) the proportional mineral content of each bore-hole sample. This array must have all sample values from the zero grid-level to the bore-hole bottom including zero values for air space, waste rock and unused array members, as explained in Section 3.4.1;

   i - mineral identification number
   j - bore-hole identification number
   k - sample identification number

SURFAC(i) - (I) the bore-hole collar elevation in grid co-ordinates;

   i - bore-hole identification number
CHAPTER 5 GRADE PREDICTION PROGRAMME

The Grade Prediction Programme computes the spatial density of the given deposit's mineralization as described in Subsection 3.4.2. A list of the source programme is given in Section 5.3. The remaining Sections of Chapter 5 supplement these lists with:

1) a description of how the Programme is prepared for execution in Section 5.1,
2) a chronological description of the Programme's computations in Section 5.2,
3) comments within the Programme list in Section 5.3, and
4) a glossary of the FORTRAN variables in Section 5.4.

5.1 PREPARATION OF THE PROGRAMME FOR EXECUTION

The Grade Prediction Programme, as listed in Section 5.3, must be tailored to a new deposit's:

1) grid system,
2) size, and
3) mineralization zones.

The following steps are cross-referenced with the Programme list to locate the exact position of these modifications:

STEP 1:

The grid system was defined partially for the Assay Composites Programme as noted in Section 4.1. The grid system dimensions planned to execute that Programme were the angular separation of the co-ordinate axes, the relative
position of the zero grid-level with the deposit's surface and the co-ordinate intervals on the more nearly vertical axis. The co-ordinate intervals of the two horizontal axes are planned for the Grade Prediction Programme to complete the definition of the grid system. The ideal grid system dimensions are discussed in Subsection 3.2.2. Briefly, these dimensions must be small enough to give grid-block volumes that can accurately approximate the topography and pit surface. However, they also govern the total number of grid-blocks which must be less than the maximum INTEGER*2 value of 32000. The limit of the grid system size and the co-ordinate intervals are given to the Programme by reading the values of the following constants (See line 005, Section 5.3):

1) 'IS' and 'JS' are the co-ordinate intervals of the I and J axes, respectively. These scale factors are used to reduce the actual distances to small integer values which are suitable for subscripts of FORTRAN arrays. These scale factors are constant regardless of the grid system type. For the irregularly spaced grid systems, only the axis perpendicular to the mineral trends needs to be irregularly spaced. The scale factors 'IS' and 'JS' specify the co-ordinate intervals in the plane perpendicular to the irregular axis. Therefore, they may be constants. For the non-rectangular grid systems, the input data are modified to read the bore-hole locations in grid co-ordinates instead of actual distances. The scale factors are unnecessary with this change. In which case, they may be deleted from the Programme expressions containing them or they may be set equal to one.

2) 'NI', 'NJ' and 'NK' are the maximum number of co-ordinate intervals in the I, J and K co-ordinate directions, respectively.

The co-ordinate intervals 'IS' and 'JS' must be mutually tolerant with the total grid size 'NI', 'NJ' and 'NK' to produce less than the maximum number of grid-blocks. This limit is 32000 and is imposed by the maximum value permitted for INTEGER*2 variables. For example, assume that the given deposit is 3000 ft by 5000 ft by 1600 ft, and that the desired grid system is regularly spaced with co-ordinate intervals of IS = 100 ft, JS = 100 ft, and the vertical interval equal to 50 ft. These dimensions produce a grid
with 41000 grid-blocks which exceeds the maximum 32000. The vertical scale factor is already defined to be suitable for the Assay Composites Programme. Therefore, the 'IS' and 'JS' scale factors must be increased to reduce the number of grid-blocks to the maximum allowable limit.

STEP 2:

The following search parameters must be defined for each mineralization zone, as described in Subsection 3.4.2:

1) 'NCLUS' - the maximum number of assay composites per estimate,
2) 'MAXD' - the maximum distance limit,
3) 'BEAR' - the minimum angular separation,
4) 'P' - the inverse proportional weighting factor, and
5) 'MIND' - the minimum distance limit.

These data are read in at line 007 in the programme list. The interpolation in a particular mineral horizon automatically will be made only from the assay composites in that horizon, if the grid system is constructed as explained in Step 1 of Sections 4.1 and 5.1.

STEP 3:

The dimensions must be specified for the FORTRAN arrays and direct access data sets.

A) The dimensions are defined for the FORTRAN arrays in Section 5.4.

B) The dimensions are given for the DEFINE FILE statement in Section 5.3 for the direct access data sets.

STEP 4:

The boundaries of the mineralization zones simulated in the Assay Composites Programme are also simulated in this Programme with the same indicator arrays and polynomial equations. These surfaces are simulated between lines 052 and 062 of the Programme list in Section 5.3. The derivation
and substitution of these surfaces are described in Sections 3.3 and 3.4.1

STEP 5:

The appropriate number of variables must be added for each potentially recoverable mineral as described by the comment in the Programme list at Line 008.

5.2 SEQUENCE OF PROGRAMME COMPUTATIONS

---

LINES 001 - 039:
Specification and input statements.

LINES 040 - 186:
The nested DO loops 901 and 45 estimate the point mineral densities 'GRAD1' and 'GRAD2' at each grid-intersection (I, J, K) from the assay composites 'ASAY1' and 'ASAY2'.

LINE 048:
The 'LOC' expression calculates the location in a direct access data set of the member (I, J, K).

LINE 050:
This IF statement branches the Programme around the remaining grade prediction computations, if the grid-intersection (I, J, K) is one of those above the deposit's surface.

LINES 051 - 075:
DO loop 17 calculates the radial distance \( r \) between the grid-intersection (I, J, K) and the assay composites (C0RI, CORJ) in the same mineralization zone.

LINES 052 - 054:
These statements define the mineralization zone \( z \) in which the grid-intersection (I, J, K) occurs.

LINES 055 - 062:
The computations for the radial distance between grid-intersection and assay composite are bypassed, if the particular assay composite is not in the same zone as the given grid-intersection (I,J,K).

LINES 063 - 065:

If there is an assay composite 'ASAY1' and 'ASAY2' within a minimum distance 'MIND' of the grid-intersection (I,J,K), the point mineral density 'GRAD1' and 'GRAD2' of that grid-intersection is given the value of that assay composite.

LINES 076:

This statement selects the appropriate maximum number of assay composites 'NCLUS' per interpolation for the mineralization zone 'K'.

LINES 077 - 089:

DO loop 24 ranks the radial distances 'R' from the grid-intersection (I,J,K) to the nearest 'LMAX' assay composites in increasing order.

LINES 090 - 100:

The radial distances 'R' are lost in DO loop 24. Consequently, DO loop 26 is required to recalculate them, but only for the nearest 'LMAX' assay composites. In this way, the assay composites existing outside the maximum distance limit 'LMAX' are deleted from the grade prediction at the grid-intersection (I,J,K).

LINES 101 - 143:

DO loop 29 calculates two bearings 'B' and 'B1' from the grid-intersection (I,J,K) to the selected (NC(L)) assay composites. The two bearings are computed from north and south base directions, respectively. The reason for this is that two assay composites close to and on either side of a base line will have a small angular separation but a large difference between their bearings. Therefore, the two bearings are required to test the angular separation close to the opposite base line.

LINES 145 - 169:
DO loops 33 and 37 test the angular separation between the selected assay composites (NC(L)). If the angular separation is less than the minimum 'HSTAR', the assay composite furthest from the grid-intersection (I,J,K) is deleted from the grade prediction by reassigning its radial distance a zero value.

LINES 165 - 173:

The point mineral densities (GRAD1,GRAD2) at the grid-intersection (I,J,K) are zero, if there are no assay composites within the maximum distance limit 'MAXD'. The 'WRITE' statement in Line 171 is included to display these grid-intersections. Normally, the grid-intersections on the outer edges of the grid system will be out of range of the search parameters, if the grid system extends past the exploration drilling. However, definition of the search parameters should be re-examined, if a grid-intersection that is within the deposit is out of range.

LINES 174 - 184:

The assay composites that have not been excluded by previous tests are averaged in inverse proportion to their radial distances 'R' by the power 'P' to estimate the point mineral densities 'GRAD1' and 'GRAD2'.

5.3 A LIST OF THE GRADE PREDICTION PROGRAMME

The parameters of the following DEFINE FILE statement are:

1) \( i = \text{MIN}\)\( \text{NJ}+\text{NK} \),
2) \( j = \text{(number of recoverable minerals)} \times 4 \),
3) \( k = \text{NBH}+\text{NK} \),
4) \( n = (\text{number of recoverable minerals} + 1) \times 4 \), and
5) \( \text{mm} = \text{NBH} \)

---

004 DEFINE FILE 10(ii, jj, L, JJ), 11(kk, ll, L, JJ), 112(\text{mm}, B, L, JJ)
005 \( \text{READ} (5, 1) \text{NI, NJ, NK, NBH, N,Z, JS} \)
006 1 \text{FORMAT (74)}
007 \( \text{READ} (5, 1004) (\text{NCLUS(M)}, \text{MAXD(M)}, \text{BEAR(M)}, \text{P(M)}, \text{bind(m)}, m=1,4) \)
008 1004 \text{FORMAT (I2, F6.1, F7.4, F3.1, F4.2)}

---

The appropriate number of variables and expressions must be added to the Programme for each recoverable mineral. Parallel expressions are created for \( \text{ASAYN(K,m,M)} \), \( \text{ASSAYn, GRADn} \) and \( \text{ASAYn} \), where 'n' identifies each recoverable mineral, at Lines 012, 018, 020, 049, 073, 169, 180, 184 and 185.

---

009 DO 1112 K=1,NK
010 DO 1112 N=1,NBH
011 DO 1112 I=1,NZ
012 ASSAY1(K,N,N)=0
013 1112 ASSAY2(K,N,M)=0
014 I=0
015 DO 1010 N=1,NBH
016 DO 1010 K=1,NK
017 I=I+1
018 \( \text{READ} (11'1) \text{ASSAY1, ASSAY2, K} \)
019 IF (ASSAY1) 1010, 1010, 1006
020 1006 ASSAY1(K,N,M)=ASSAY1
021 ASSAY2(K,N,M)=ASSAY2
022 1010 \text{CONTINUE}
023 DO 903 K=1,NBH
024 \( \text{READ} (12'N) \text{X, Y} \)
025 CO3(N)=X
026 803 CO3(N)=Y
027 \( 'N=N!+NJ \)
028 \( \text{READ} (5, 1051) \text{(SCORK(N),N=1,NN)} \)
029 1051 \text{FORMAT (3012)}
030 DO 1054 K=1,NK
031 N=C
032 DO 1054 J=1,NJ
033 DO 1054 I=1,NI
034  \texttt{N=N+1}
035  \texttt{IF ('SCORK(N)-K) 1055,1053,1053}
036  \texttt{1055 II=I}
037  \texttt{GO TO 1054}
038  \texttt{1053 II=0}
039  \texttt{1054 TOPO(I,J,K)="}
040  \texttt{ECGRAD=0+}
041  \texttt{DO 901 K=1,NK}
042  \texttt{DO 901 J=1,NJ}
043  \texttt{DO 45 I=1,NI}
044  \texttt{GRAD1=0+}
045  \texttt{GRAD2=0+}
046  \texttt{DO 161 N=1,NBH}
047  \texttt{161 N(N)=0}
048  \texttt{LOC=NJ*NI*(K-1)+NI*(J-1)+1}
049  \texttt{FIND (10*LOC)}
050  \texttt{IF (TOPO(I,J,K)) 46,46,999}
051  \texttt{999 DO 1 N=1,NBH}
052  \texttt{IF (11096*3+2*652*COR1(N)-5*55*COR(J(N))-150*5*K) 1011,1012,1012}

---

\textit{Lines 052 - 062 are tailored to the user's mineralization zones.}
070  1045  R(N) = SQRT((ABS(CORJ(N) - IS*I))**2 + (ABS(CORJ(N) - IS*I))**2)
071  1046  GO TO 17
072  1016  GRAD1=ASAY1(K, M, N)
073  1017  GRAD2=ASAY2(K, M, N)
074  1018  GO TO 46
075  17  CONTINUE
076  LMAX = NCLS (M)
077  L2 20  L = 1, LMAX
078  DO 1018  Y = 1, NBH
079  IF (R(N)) 1018, 1016, 19
080  1018  CONTINUE
081  19  SMEL=R(N)
082  20  DO 23  M = 1, NBH
083  IF (R(N)) 23, 23, 21
084  21  IF (R(N)-SMEL) 22, 22, 23
085  22  NC(L)=N
086  23  CONTINUE
087  171  R(NC(L))=90000000
088  24  CONTINUE
089  26  DO 26  L = 1, LMAX
090  27  IF (ABS(CORJ(NC(L)) - IS*I) < MINW(M)) 180, 180, 181
091  180  R(NC(L))=ABS(CORJ(NC(L)) - JS*J)
092  181  GO TO 184
093  182  R(NC(L))=ABS(CORJ(NC(L)) - IS*I)
094  183  GO TO 184
095  184  IF (R(NC(L))-SAXD(M)) 26, 26, 25
096  25  R(NC(L))=0
097  26  CONTINUE
100  29  DO 29  L = 1, LMAX
101  27  IF (R(NC(L))) 29, 29, 27
102  28  IF (CORJ(NC(L)) = JS*J) 1102, 1100, 1101
103  1101  IF (CORJ(NC(L)) - IS*I) 106, 103, 103
104  1102  IF (CORJ(NC(L)) - IS*I) 105, 304, 104
105  1103  B(NC(L))=ATAH ((ABS(CORJ(NC(L)) - JS*J)) / (ABS(CORJ(NC(L)) - IS*I))
106  1104  GO TO 28
107  1105  B(NC(L))=4*7123+ATAH ((ABS(CORJ(NC(L)) - IS*I))
108  1106  GO TO 28
109  1107  B(NC(L))=3*1459+ATAH ((ABS(CORJ(NC(L)) - JS*J)) / (ABS(CORJ(NC(L)) - IS*I))
110  1108  GO TO 28
111  1109  CONTINUE
111 GO TO 28
112 106 B(NC(L)) = 1*57080*ATAN ((ABS(CORI(NC(L)))-IS*I))/
1(AES(CORI(NC(L)))-JS*J))
113 GO TO 28
114 300 IF (CORI(NC(L)))-IS*I) 301,308,302
115 301 B(NC(L)) = 3*14159
116 GO TO 28
117 302 B(NC(L)) = 0*0
118 GO TO 28
119 303 B(NC(L)) = 1*57080
120 GO TO 28
121 304 B(NC(L)) = 0*71238
122 28 IF (CORI(NC(L)))-JS*J) 201,310,202
123 201 IF (CORI(NC(L)))-JS*J) 203,313,204
124 202 IF (CORI(NC(L)))-JS*J) 206,314,205
125 203 B1(NC(L)) = ATAN ((ABS(CORI(NC(L)))-JS*J))/
1(AES(CORI(NC(L)))-JS*J))
126 GO TO 29
128 204 B1(NC(L)) = 1*5708*ATAN ((ABS(CORI(NC(L)))-IS*I))/
1(AES(CORI(NC(L)))-JS*J))
129 205 B1(NC(L)) = 3*14159*ATAN ((ABS(CORI(NC(L)))-JS*J))/
1(AES(CORI(NC(L)))-JS*J))
130 GO TO 29
131 206 B1(NC(L)) = 0*71238*ATAN ((ABS(CORI(NC(L)))-IS*I))/
1(AES(CORI(NC(L)))-JS*J))
132 GO TO 29
133 310 IF (CORI(NC(L)))-IS*I) 311,303,312
134 311 B1(NC(L)) = 0*0
135 GO TO 29
136 312 B1(NC(L)) = 3*14159
137 GO TO 29
138 313 B1(NC(L)) = 1*5708
139 GO TO 29
140 314 B1(NC(L)) = 0*71238
141 GO TO 29
142 308 WRITE (6,303)
143 29 CONTINUE
144 309 FORMAT (' TILT')
145 DO 33 L=1,LMAX
146 DO 33 T=1,LMAX
147 IF (R(NC(L))) 33,33,1120
148 1120 IF (R(NC(T))) 33,33,121
149 121 IF (ABS(B(NC(T))-B(NC(L)))-BEAR(T)) 30,33,33
150 30 IF (R(NC(T))-R(NC(L))) 31,33,32
151 31 R(NC(L)) = 0
This glossary defines the FORTRAN variables of the Grade Prediction Program. The glossary is not complete; some variables used to facilitate programming are not included in the list. The format of the entries is defined in Section
---GLOSSARY---

\textbf{ASAY1}(i,j,k), \textbf{ASAY2}(i,j,k), **ASAYn(i,j,k) - (R) the assay composites for 'n' recoverable minerals;}

\begin{itemize}
  \item \textit{i} - grid-level
  \item \textit{j} - bore-hole identification number
  \item \textit{k} - mineralization zone identification number*
\end{itemize}

\textbf{ASSAY1}, \textbf{ASSAY2}, **\textbf{ASSAYn} - (R) a variable used to transfer assay composites from external storage to internal storage for 'n' recoverable minerals*

\textbf{B}(i) and \textbf{B1}(i) - (R) the bearings from north and south, respectively, between grid-intersection and assay composites;

\begin{itemize}
  \item \textit{i} - bore-hole identification number*
\end{itemize}

\textbf{BEAR}(i) - (R) a search parameter which is the minimum allowable angular separation between assay composites measured in radians;

\begin{itemize}
  \item \textit{i} - mineralization zone identification number*
\end{itemize}

\textbf{CORI}(i) and \textbf{CORJ}(i) - (R) the I and J co-ordinate locations of the bore-holes in feet;

\begin{itemize}
  \item \textit{i} - bore-hole identification number*
\end{itemize}

\textbf{GRAD1}, \textbf{GRAD2}, **\textbf{GRADn} - (R) the point mineral densities at a grid-intersection for 'n' recoverable minerals*

\textbf{IS} and \textbf{JS} - (I) the scale factors for converting between actual distances and grid co-ordinates*

\textbf{LOC} - (I) the location of a member in a direct access data set*

\textbf{MAXD}(i) - (R) a search parameter which is the maximum distance between grid-intersection and assay composites;

\begin{itemize}
  \item \textit{i} - mineralization zone identification number*
\end{itemize}
MIND(i) - (3) a search parameter which is the minimum distance between assay composite and grid-intersection;

\[ i \text{ - mineralization zone identification number}\]

NBH - (I) the total number of bore-holes

NC(i) - (I) a variable used to identify the assay composites to be included in any particular point mineral density estimate;

\[ i \text{ - bore-hole identification number}\]

NCLUS(i) - (I) a search parameter which equals the maximum number of assay composites to be included in each point mineral density estimate;

\[ i \text{ - mineralization zone identification number}\]

NL, NJ and NK - (I) the maximum J, J and K grid co-ordinates, respectively

NZ - (I) the total number of mineralization zones

P(i) - (2) a search parameter which is the inverse proportional weighting factor;

\[ i \text{ - mineralization zone identification number}\]

R(i) - (3) the radial distances between grid-intersection and its surrounding assay composites;

\[ i \text{ - bore-hole identification number}\]

TOPO(i,j,k) - (I*2) an indicator array used to simulate the deposit's surface;

\[ i,j,k \text{ - the I, J and K grid co-ordinates, respectively}\]
CHAPTER 6  OPEN PIT PLANNING SYSTEM

The theories described in this Chapter are used to work out how to conduct the open pit mining operations on the user's surface deposit. Economic, mathematical and mine design theories are combined with computer programming techniques to yield a pit planning system based on maximum total profit and present value. This system is executed with the Mine Plan Programme listed in Chapter 8. The MP Programme produces a set of schedules and plans that represent an optimum open pit mining solution for the user's deposit. The flow diagram in Table 6.1 illustrates the computational sequence of the MP Programme. Chapter 6 is organized more or less parallel to this diagram.

6.1 INPUT DATA

There are two types of input data required to initiate the MP Programme. These data types are distinguished by being variable or fixed. They are described generally in the following Subsections and are listed specifically in Chapter 8.

6.1.1 VARIABLE INPUT DATA

The variable input data are the information requiring decisions by the user. These data are collectively called the 'INPUT MINING PROPOSAL' in Table 6.1. This proposal is the selection of all conceivable plant production capacities of the pit, concentrator and refinery, and an open pit mining method.

PLANT PRODUCTION CAPACITIES:

A routine has been written into the MP Programme which chooses the plant production capacities yielding the greatest present value. This choice is made from the possibilities given by the user. The user selects any number of plant
TABLE 6.1

capacities, and gives the investment capital and overall
recovery efficiency of each combination. The investment capital must be discounted to the first production year at the opportunity cost of investment capital. Table 6.2 illustrates an example of thin input data.

### Table 6.2: Capital Cost of 27 Plant Capacity Combinations

<table>
<thead>
<tr>
<th>EFFICIENCY</th>
<th>OVERALL (M)</th>
<th>RECOVERY (T)</th>
<th>(M)</th>
<th>(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>85%</td>
<td>20.0</td>
<td>30.0</td>
<td>32.0</td>
<td>37.75</td>
</tr>
<tr>
<td>90%</td>
<td>22.5</td>
<td>32.5</td>
<td>32.5</td>
<td>37.75</td>
</tr>
<tr>
<td>95%</td>
<td>25.0</td>
<td>35.0</td>
<td>35.0</td>
<td>37.75</td>
</tr>
</tbody>
</table>

PIT CAPACITY = 7 MILLION TONS PER YEAR
CAPITAL COSTS IN MILLIONS OF CASH UNITS
(T) = capacities in thousands of tons per year
(M) = capacities in millions of tons per year

The plants are defined in the following manner according to production processes and costing methods rather than by location:

A) The pit: The pit production is given as the total output including ore and waste. The pit production processes are all those common to both ore and waste excavation.

B) The concentrator: The concentrator is that part of the production processes which distinguishes the ore from the raw material of the pit. As opposed to refining,
concentration is the group of processes that can be charged without regard to the ore's mineral content. For example, it will include the difference between the productive effort of ore and waste excavation, crushing, grinding, floating, leaching, control sampling, and so forth. However, if the cost of any of these component processes becomes a function of mineral grade, it becomes part of the refinery production.

C) The refinery: The refinery is the group of purification processes that are distinguished from the concentrator production by the manner in which the costs are charged. Refining includes the processes that depend on the ore's mineral content. For example, it will include smelting and marketing.

OPEN PIT MINING METHOD:
The second part of the 'INPUT MINING PROPOSAL' requires the following decisions from the user:

1) Pit layout:
   a. the minimum angles between vertical, and the safe and working pit walls;
   b. the height and width of the pit benches, and c. the number of exposed ore benches and working faces per ore bench.

2) Ore Excavation Sequence

Safe Wall-Angles:
The safe wall-angles define the minimum allowable angles to the vertical that the final pit surface may have to prevent it from sliding as a result of the slope stability conditions. Any number of safe wall-angles may be specified for changing slope stability conditions through the user's deposit. For example, safe wall-angles can be defined for each region of the pit illustrated in Fig 6. The three regions are delimited by their orientation with the weak linear strength direction of the deposit's bedding planes.

Working Wall Angles:
The working wall-angles define the allowable minimum angle
to the vertical that the pit walls must exceed until they intersect with the final surface of the pit. This specifies the overall angle of the intermediate operating slopes which are required to expose the ore benches. Any number of these working wall-slopes may be specified for different working conditions through the pit. The relationship between the safe and working pit walls is illustrated in Fig. 6.2.*

**Bench Height and Width:**

The size of the excavating equipment planned for the pit will determine the bench height and width. These dimensions are specified to equal the excavating equipment’s working space requirements. The computer simulation of the pit is considerably simpler, if the vertical and transverse grid-block dimensions are made equal to the bench height and width as recommended in Subsection 3.2.2. Otherwise, the grid-blocks have to be built up to equal the bench dimensions.

The pit walls formed by staggering the benches at an offset equal to their height and width produces a fixed wall-angle as shown in Fig. 6.3A. However, a fixed wall-angle does not permit any freedom for planning:

1) **the stripping ratio of ore to waste and**

2) **different intermediate and final pit walls in the various parts of the pit.**

Therefore, the MP Programme is written to allow the user to specify the bench dimensions independently of the wall-angles. The Programme designs the pit walls at the specified angles by grouping the benches as follows:

1) **The benches are grouped horizontally,** as illustrated in Fig. 6.3B, when the pit walls are flatter than the slope fixed by a single offset of the benches. The horizontal bench grouping does not create any operational problems. Its only effect is to increase the horizontal working space on the grouped benches.

2) **The benches are grouped vertically,** as illustrated in Fig. 6.3C, if the pit walls are steeper than the slope fixed by a single bench offset. The bench grouping causes local slope variations of the pit walls alterna-
ting between steeper and flatter wall-angles than that
specified by the user. The locally alternating slopes
result in the desired overall wall-angle of the pit
surface as illustrated in Fig. 63C. However, the user
must be confident that the pit walls can stand at the
local slope exaggerations, if he specifies a wall-angle
less than the single bench offset.

The pit wall design function of the MP Program permits the
following specifications of the 'INPUT MINING PROPOSAL' to
be made independently:

1) the bench height and width as defined by the excavation
equipment working space requirement,

2) the working wall-angles as determined from the desired
stripping ratios and the slope stability conditions, and

3) the safe wall-angles as defined by the slope stability
conditions.

This design capability frees the user from matching the
working space requirements to the stripping ratios within
the limitations of the slope stability conditions.

Number of Exposed Ore Benches and Working Faces per Exposed
Ore Bench:

The number of exposed ore benches and ore working faces per
exposed ore bench is interrelated with:

1) the production capacity of the ore excavating equip-
ment,

2) the production capacity of the concentrator, and

3) the bench height and width.

The total number of ore working faces must be capable of
providing the concentrator input production. For a particu-
lar concentrator, this number equals the concentrator
capacity divided by the production capacity per ore working
face. However, a large range of concentrators may have been
selected for analysis by the MP Program. Consequently, it
may be difficult to match the number of ore working faces to
the whole range of concentrators. If this is so, then:

1) assume the number of ore working faces,
2) execute the MP Programme to select the best concentrator capacity,
3) check this concentrator capacity against the number of ore working faces, and
4) if necessary, adjust the number of ore working faces and re-execute the MP Programme.

Ore Excavation Sequence:

The simulation of the pit production requires the following decisions:

1) where should ore excavation begin?
2) in which direction should the ore working faces advance from the starting point?
3) which grid-blocks are ore and which are waste?
4) in which order should the ore grid-blocks be removed?
5) what overburden should be removed to expose the ore?
6) how much ore and waste should be excavated in any given year? and
7) in which year should the grid-blocks be mined?

The last five decisions are made by the MP Programme to maximize the present value of the user's deposit. The first two decisions are made by the user and are part of the 'INPUT MINING PROPOSAL'. The starting point of the ore excavation should be the most advantageous in relation to the location of the waste dumps, plant facilities, property lines, and so forth. The ore working face advance should be parallel to the long axis of the pit. This will minimize the number of changes required for the haulage roads. The pit production simulation routine automatically changes the direction of the working face advance on alternate ore benches in both the vertical and horizontal directions. This
minimizes the movements of the ore excavation equipment.

The 'INPUT MINING PROPOSAL' for a particular deposit can be developed with the MP Programme as a planning aid. The unknown or undecided items can be assumed and an open pit mining solution computed. Then, the assumptions are changed to compute another solution. The cycle of input assumption -- computation -- input adjustment -- re-computation is continued until the best open pit mining solution results. This rapid execution facility of the Programme provides a means for quickly assessing the effects of changing the mining proposal.

6. 1. 2 FIXED INPUT DATA

The fixed input data is characterized by being constant for any given deposit. There are the following three categories:

1) spatial mineral distribution,
2) economic data, and
3) physical data.

Spatial Mineral Distribution:

The spatial mineral distribution is supplied by the computer programs described in Chapters 4 and 5. Recall that the Grade Prediction Programme stores the spatial mineral distribution in a direct access data set. This data set is read by the MP Programme as part of its input data.

Economic Data:

Elements of the deposit's cost structure must be supplied by the user. The cost structure is simulated in the MP Programme with the standard costing method. This method is the computation of each grid-block's net cash value to the venture from standard costs and prices. These standards are adjusted and totalled individually to give the total net cash value of each grid-block according to the peculiar conditions under which it is mined. The standard costs and prices, and the peculiar costing conditions are the cost structure elements that must be supplied by the user.

The standard costs and prices are given by the user in unit
terms and adjusted according to each grid-block's:

1) **time of excavation** - for the time inflation of costs and prices,

2) **location** - for the difference in the excavating costs in various parts of the pit, and

3) **mineral content** - for the differences in the metallurgical characteristics of the ore.

The adjusted standards are used to compute the particular grid-block's net cash value along with its unit weight, mineral content and overall recovery efficiency. The net cash value computations are discussed in more detail in Section 6.5.

The 'ANNUAL UNIT COSTS' defined in Table 6.3 are the arithmetical means of all the adjusted 'STANDARD UNIT COSTS' assigned in each year of the mine's life span. The annual unit costs represent an estimate of the yearly pit, concentrator, and refinery costs. The standard and annual unit costs are defined differently for two separate applications in the MP Programme. Consequently, Table 6.3 is presented to detail the differences in their definitions to avoid any confusion.

**Physical Data:**

The physical data is the last category of the fixed input. This data includes the deposit's surface topography and any geologic or structural features that will interfere with the development of an open pit mining solution. The following computational routines of the MP Programme are the ones which could be affected:

1) **costing** — for example, by the location of the water table,

2) **final pit surface design** — for example, by variable slope stability conditions, and

3) **pit production** — for example, by rock movements along a fault plane.

The MP Programme is written to recognize any of these
features so that it can modify the open pit mining solution for their occurrences. The programming techniques described in Section 1.1 are used to simulate these structural features. Application of the structural simulation techniques is explained later when the various computational routines that use them are discussed.

6.2 OBJECTIVES OF THE OPEN PIT PLANNING SYSTEM

The objectives of the open pit planning system are:

1) to design the final pit surface that will yield the maximum total profit and
2) to furnish a solution for exploiting the pit that will maximize the mine's present value.

This planning system is a development following from the previous work in the field. The latest and most advanced system (1,5,7) designs the final pit surface with the maximum total profit. This thesis introduces the time dimension and the exploitation solution. In particular, the mine's cash flow and the time value of money are included in the open pit planning system.

The objectives can be described with the following analytical development:

Let \( C(x, y, z) \) be defined in a closed three-dimensional region \( R \) encompassing the user's deposit. The region \( R \) is subdivided into subregions, called parallelepipeds, by constructing a grid consisting of planes parallel to the \( xy \), \( yz \) and \( xz \) planes. The function \( C(x, y, z) \) denotes the net cash value of \((x, y, z)\) which is a point or grid-block in the three-dimensional region \( R \).

Let \( P(C(x, y, z), t) \) be defined as the net cash value density function \( C(x, y, z) \) extended to include the time period \( T \). The function \( P(C(x, y, z), t) \) denotes the net cash value of grid-block \((x, y, z)\) for the year \( t \) during which it is mined. The domain of the function \( P(C(x, y, z), t) \) is restricted:

1) to the family of pit surfaces whose walls are not steeper than the minimum safe wall-angle at any point,
2) to the path specified by the user for the sequence of ore excavation, and
3) by the prevailing cost and price inflation rates and the opportunity cost of investment capital.

The objective is to find:

1) the pit volume \( V \) of the region \( R \), and
2) the mine production schedule \( t \) of the time period \( T \)

that will maximize the mine's present value \( PV \) of the following multiple integral with the preceding restrictions:
\[
PV = \int_0^T \int_V \mu(\{x,y,z\}, t) \, dV \, dt 
\]

The three innermost integrals are evaluated first to find the final pit surface yielding a maximum total profit. Then, the remaining integration is performed to find the exploitation solution that will maximize the mine's present value. Unfortunately, a simple relationship does not exist for the function \( \mu(\{x,y,z\}, t) \). Additionally, the limits for the time period \( T \) are unknown, or in other words, the mine's life span cannot be defined before the integration is executed. Therefore, the multiple integral's solution is executed indirectly with numerical, economic, mathematical and mine design methods which were devised to make up the NP Programme.

The solution is executed with the converging iteration illustrated in Table A. The iteration converges towards a definite open pit mining solution following from certain initial assumptions.

So to initiate the solution, it is assumed that the entire deposit is mined in the first year. This assumption is made for the data sets that are time dependent, namely the two time-space distributions of minerals and net cash values, respectively, and the annual unit production costs.

To avoid confusion, it is important to define these preceding data sets:

A) Time-space (T-S) distributions: The T-S distributions provide the dispersion of mineral quantities and net cash values by their spatial location and by the period in which they are mined. The time and space dimensions are used to account for the differences in the mineral quantities and net cash values available for exploitation at any moment in the pit's life span. The two T-S distributions are defined somewhat differently. Firstly, the T-S net cash value distribution is a four dimensional density function. Each point density is defined by its three-dimensional location in the user's deposit and by the production year of the material represented by that point. Recall that the T-S net cash value distribution was defined previously as \( \mu(\{x,y,z\}, t) \). Secondly, the T-S mineral distribution is the deposit's tonnage per mineral grade interval per time
The location of these tonnages is given by their association with the time in which they are mined. The two distributions are described more thoroughly in Sections 6.3 and 6.5.

B) Annual unit production costs: These costs are the annual excavation, concentration and refining costs per throughput tonnage of the respective production plants. These terms were defined earlier in Table 6.1. Normally, they are estimated from the T-S net cash value distribution as described in Subsection 6.1.2. However, the estimates cannot be made until the T-S net cash value distribution has been computed. These computations are the fourth step in the iterative cycle as illustrated in Table 6.1. Meanwhile, the unit production costs are required for the preceding cut-off grades, production targets and plant capacities selection computations. Consequently, the estimates are assumed for the first cycle of the iterative solution until they are replaced in the next cycle.

The assumptions required to initiate the MP Programme for the preceding data sets are:

1) The time dimension is assumed to be the first production year for the T-S distributions.
2) The space dimensions of the pit are assumed to include the entire region encompassed by the grid system for the T-S distributions.
3) An estimate of the unit production costs for the first production year is given by the user.

The time assumption represents a pit life span of one year; therefore, the unit production costs assumption is made only for that year. The three assumptions are replaced with better estimates as the solution progresses through the iterations.

The whole process can be conceived as a closed loop in which all computations depend upon each other. Obviously, it is necessary to break into this loop to initiate the solution. It is with the above assumptions that this is achieved. In the end, the iteration converges as corrections to the assumptions become compatible with the final pit surface.
design and exploitation solution. Table 3.1 illustrates the process graphically:

The routines illustrated there can be described by means of an analogy with the functions and operations implied by the multiple integral developed previously:

$$PV = \int_{T} \left( \int_{N} \int f(x, y, z, t) \, dV \right) \, dt$$

The assumptions, as listed above, are made for the limits of the outermost integral. Based on these assumptions, it is possible to compute the path T for the integral. This path is restricted by the ore excavation sequence given by the user. The remaining decisions required to formulate the path are computed by the following routines to maximize the mine's present value:

1) PLANT CAPACITIES SELECTION,
2) CUT-OFF GRADES COMPUTATIONS, and
3) PRODUCTION TARGETS COMPUTATIONS.

The point densities of the function $P(C(x, y, z), t)$ are computed in the routine T-5: NET CASH VALUE DISTRIBUTION COMPUTATIONS assuming the time dimension. Then, the operation

$$\int_{T} \left( \int_{N} \int f(x, y, z, t) \, dV \right) \, dt$$

is executed to find the pit yielding the maximum total profit over the region $E$, i.e., the grid system, by the routine FINAL PIT SURFACE DESIGN. Thereafter, the operation

$$f(T)$$

is performed by the routine PIT PRODUCTION SIMULATION along the path T. As the solution progresses through the iteration, the initial assumptions are replaced each cycle by better estimates. This is achieved:

A) with the times computed by the routine PIT PRODUCTION SIMULATION.
B) with the final pit surface (for the space dimensions) computed by the routine FINAL PIT SURFACE DESIGN, and

C) with the annual unit production costs estimated by the routine T-S NET CASH VALUE DISTRIBUTION COMPUTATIONS.

The solution converges when the results of the preceding iteration cycle equal the results of the ultimate cycle. These results are listed as the OUTPUT in Table 6.1.

6.3 TIME-SPACE DISTRIBUTIONS OF MINERALS

The FORTRAN data set for the T-S mineral distribution is named:

ORERES(M,N)

where the subscript:

'M' identifies the mineral grade increment, and

'N' identifies the pit increment or the time period during which the minerals will be mined.

Each array member is the weight of pit material with a mineral content 'M' which will be mined in the 3th period or pit increment; 'M' is given by the point densities of the spatial mineral distribution 'GRAD' computed by the Grade Prediction Program (listed in Chapter 5); 'N' is given by the excavation times computed by the routine PIT PRODUCTION SIMULATION (see Table 6.1); 'N' identifies a horizontal cross-section of the deposit when the ORES data set is assumed in order to start the iterative solution. Subsequently, 'N' identifies a period in the mine's life span.

6.3.1 INITIAL ASSUMPTION OF THE T-S MINERAL DISTRIBUTION

As noted previously, the subscript 'N' of the ORES data set is unknown at the start of the iterative solution. A pit surface has not been designed and its dimensions are unknown. Therefore, all the material within the grid system is included in ORES. The pit production has not been simulated and the time dimension is unknown. Therefore, this is estimated by totalling the mineral quantities of succes-
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