In order to specify an equation for sweat rate in terms of the central drive and local drive from information available in the literature, it is important to note the following.

(i) Sweat rate = evaporation rate + drippage rate
+ rate of accumulation of sweat on the skin
= \( E_{gm} + D + \dot{Q} \)  \hspace{1cm} (5.4)

Quantitative statements of sweat rate are determined either from weight loss measurement or from evaporative heat loss measurement. The former gives only a measure of evaporation rate and drippage rate. Only during steady-state sweating, usually achieved after long periods of exposure, is the rate of accumulation of sweat on the skin zero. In very hot and dry environments, the sweat may evaporate from within the sweat pores, never reaching the surface, there is, under these circumstances alone, no accumulation of sweat on the skin and certainly no drippage of sweat. Measurement of evaporative heat loss would in this case reflect true sweat rate.

(ii) The sweat rate should not have been suppressed by sweat gland fatigue or hidromeiosis.

It is essential to be able to describe sweat rate in terms of central and local skin temperature drives over the entire range of these drives without inclusion of the effects of sweat gland fatigue and hidromeiosis both of which act in time. To achieve the required range of driving signal without suppression of sweating is difficult.
The second hour data of Wyndham and Atkins (1968) was used to describe the sweat rate drive in terms of $T_{mb}$ and the local effect of an elevated skin temperature. Although the data was recorded after two hours of exposure, and sweat gland fatigue and hidromeiosis could already have been operative, the form of the equation is assumed to be correct. The parameters in the equation are therefore particular to the second hour data and their values may require adjustment to represent sweat rate drive without the effects of fatigue and hidromeiosis.

A logistic function of the following form was fitted to Wyndham and Atkins' (1968) data using a non-linear parameter estimation technique (Draper and Smith 1966).

$$S_r = \frac{he^{(T_s-q)/m}}{(1-e^{-1/m})}$$  \hspace{1cm} (5.5a)

$$T_t = \frac{1}{(n-T_{mb})} - \frac{1}{(T_{mb}-p)}$$  \hspace{1cm} (5.5b)

The best fit values of the parameters were

- $h = 1.47 \times 10^{-4}$
- $j = 6.05$
- $q = 31.2$
- $m = 10$
- $n = 39.5$
- $p = 35.0$

The values of parameters ($n$) and ($p$) represent mean body temperature where sweat rate reaches a maximum and minimum respectively. The term $he^{(T_s-q)/m}$ is consistent with Nadel et al. (1971) in that the maximum sweat rate that can be attained is dependent upon the local effect of mean skin temperature.
Figure 5.3 shows the functional relationship (equation (5.5)) between sweat rate, weighted body temperature driving signal and mean skin temperature. The data of Wyndham and Atkins (1968) are superimposed.

5.2.2 Sweat gland fatigue

The block under discussion is shown in Figure 5.4.

When an individual is exposed to heat, and his rate of weight loss measured at frequent intervals, the characteristic response is a rapid increase followed by a progressive decline in the rate of weight loss.

Haldane and Hancock (1929) and Hancock et al. (1929) introduced the concept of sweat gland fatigue to explain firstly the decrement of sweat rate and secondly the increment in salt concentration of the sweat with time of exposure. Ladell (1945) observed that sweating began its decline as early as the second hour although the rectal temperature was still increasing. The observed decline in sweat rate with time was borne out by MacDonal and Wyndham (1950) and Gerking and Robinson (1946) and was interpreted as fatigue of the sweat glands or of the controlling mechanism.

The term hidromeiosis is the general word used for the depression of the rate of sweating (Sargent 1962) but it has become restricted to mean a particular type of reduction in sweating associated with wetting the skin (Kerslake 1972). It is the above definition of hidromeiosis which will be used.

In practice it is very difficult to obtain a quantitative assessment of the relative contributions of the two mechanisms of sweat suppression, namely hidromeiosis and fatigue of the gland itself. Bulmer and Forwell (1956) measured the sweat and sweat sodium concentration at three succes-
Figure 5.3 SWEAT RATE VERSUS WEIGHTED BODY TEMPERATURE DRIVING SIGNAL AT DIFFERENT LEVELS OF SKIN TEMPERATURE. BASED ON WYNDHAM AND ATKINS (1968)

(from Cohen and Stewart 1982)
<table>
<thead>
<tr>
<th>Dsr</th>
<th>S*CN(0,0)</th>
<th>Dsrf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweat rate drive</td>
<td>S*CD(0,0)</td>
<td>Fatigued sweat rate drive</td>
</tr>
</tbody>
</table>

Figure 5.4 THE SWEAT GLAND FATIGUE BLOCK
sive half-hour periods under different heat stress conditions. Sweat samples were obtained from an arm bag and hidromeiosis must have been present during the latter collections of samples. Results showed that the rate of sodium reabsorption, a measure of the activity of the gland, was greatest during the first period of measurement and progressively less in subsequent periods. The time constant associated with the observed reduction in sweating was about 110 minutes, similar to time constants associated with hidromeiosis.

Wyndham et al. (1966) plotted weight loss rate against rectal temperature for each hour of a five hour experiment. Measurements in the first two hours were coincident but weight loss rates declined progressively in the third, fourth and final hours. At high rectal temperatures the reduction in sweating between the first and final hour was roughly proportional to the weight loss rate in the first hour. The results are consistent with the mechanism of hidromeiosis although they attributed the reduction in sweating to sweat gland fatigue. Gerking and Robinson (1946) conducted experiments in hot humid and hot dry environments. They concluded that the decline in sweating was 21 per cent greater in moist heat than in dry heat. The greatest declines occurred when initial rates of sweating were greatest. If a rate of 10 g/min was never exceeded, the level of secretion could be maintained in hot humid conditions. The corresponding rate for hot dry conditions was 20 g/min. Sargent (1962) found a linear correlation between initial rate (first hour) of sweat production and the magnitude of the decline in the sixth hour.

Peter and Wyndham (1966) assessed glandular activity and the density of active glands by means of plastic impressions. They found that the number of active sweat glands diminished during exposure to warm humid environments. Hidromeiosis was discounted since they observed that
sweating was suppressed even though the skin did not "look" wet. Contrary to this Candas, Libert and Vogt (1980) have made similar measurements using plastic impressions and concluded that suppression of sweating is a result of a decrease in the mean output of each gland rather than a decrease in the number of active glands. Brown and Sargent (1965) deduced that sweat gland fatigue could not be the main mechanism in sweat suppression since, on transferring an experimental subject from a hot humid environment where sweating had diminished, to a hot dry environment, sweating was soon reinstated to a higher level.

The body of literature consulted gives no conclusive evidence, either way, as to the existence of the sweat gland fatigue phenomenon. It was therefore decided that a block which would cater for the sweat gland fatigue effect would be included in the model. If, at a later stage it was found that this block did nothing to enhance the accuracy or completeness of the sweat rate model, it could be discarded.

If we argue that as soon as the sweat glands become active, sweat gland fatigue is initiated, evidence in the literature suggests a time constant of approximately 120 minutes for the decay in sweat production (rounded off to the nearest half hour from the 110 minutes observed by Bulmer and Forwell (1956)). A transfer function which exhibits this response is a phase lead network represented in the Laplace domain as (Shinners 1972)

\[
\frac{D_{grf}(S)}{D_{gr}(S)} = \frac{S+C_n(0,0)}{S+C_d(0,0)}
\]

\[
= 1 + \frac{C_n(0,0)-C_d(0,0)}{S+C_d(0,0)}
\]

(5.6)

where \(C_d(0,0) > C_n(0,0)\)

and \(\tau = 1/C_d(0,0)\)
Figure 5.5a  RESPONSE TO A PHASE LEAD NETWORK TO A UNIT STEP INPUT

Figure 5.5b  RESPONSE OF THE PHASE LEAD NETWORK TO A SECOND UNIT STEP INPUT HEIGHT (h) AT A TIME = 4 TIME CONSTANTS
The above transfer function shifts the phase of the output so that it leads the phase of the input. The resulting output to a unit step input can be seen in Figure 5.5a. The final value or d.c. gain after a unit step input is given by the Final value theorem (Shinners 1972)

\[
\lim_{s \to 0} \frac{s + C_n(0,0)}{s + C_d(0,0)} = \frac{C_n(0,0)}{C_d(0,0)} \quad (5.7)
\]

Similarly the initial value or high frequency gain is given by the Initial value theorem

\[
\lim_{s \to \infty} \frac{s + C_n(0,0)}{s + C_d(0,0)} = 1 \quad (5.8)
\]

Should another unit step input be applied at any time the response would be given by Figure 5.5b.

Choosing a value of 0.4 for the d.c. gain (Wyndham et al. 1966) and a time constant of 120 minutes (7200 seconds), the transfer function of the sweat gland fatigue block is given by

\[
\frac{D_{sr}(S)}{D_{sr}(S)} = \frac{s + 1/18000}{s + 1/7200} \quad (5.9)
\]

which is implemented as

\[
\frac{D_{sr}(S)}{D_{sr}(S)} = 1 + \frac{1/18000 - 1/7200}{s + 1/7200} \quad (5.10)
\]

and represented in block diagram form in Figure 5.6. (See Figure 5.16 for the true shape of response when the input is not a unit step input.)
Figure 5.6 IMPLEMENTATION OF THE SWEAT GLAND FATIGUE BLOCK FOR SIMULATION
5.2.3 Skin wettedness

The components of the model which will be covered in this section are given in Figure 5.7.

Gagge (1937) first proposed the concept of skin wettedness and defined it as the ratio of the actual evaporation rate to the maximum evaporation rate possible under prevailing conditions,

\[ W = \frac{E}{E_{\text{max}}} \]  

(5.11)

The physical interpretation is that the skin is divided into mosaics of wet and dry patches. The total wet area of the body \( A_W = A_S W \) and the dry area \( A_D = A_S (1 - W) \). Evaporation can only proceed from wet areas where there is a vapour pressure gradient thus

\[ E = W h_e (P_s - P_a) \]  

(5.12)

Mcle (1948) proposed a somewhat different mechanism of skin wettedness. The skin was considered to have an overall vapour pressure \( \psi_P P_a \), and not to consist of wet and dry patches. Evaporative heat loss was determined by

\[ E = h_e (\psi_P P_a - P_a) \]  

(5.13)

The former concept will be used in the formulation of the model.

As sweating commences, if it is not evaporated immediately, it appears as globules on the skin. These globules constitute the wet mosaics. As sweating increases, the droplets coalesce and must eventually begin to run down and drip off the body. A stage of diminishing returns must be reached when any further accumulation of water on the skin surface does not contribute to an increased wetted area, but may only increase the thickness of the
Figure 5.7 SKIN WETTEDNESS BLOCK COMPONENTS
layer of water already on the skin. The relationship between accumulated water on the skin and the skin wettedness could be expected to have an exponential form which explains the abovementioned diminishing returns effect.

\[ W = 1 - e^{-\alpha Q} \]  

Equation (5.14) indicates that at low values of \( Q \) the rate of increase of \( W \) with \( Q \), \( (dW/dQ) \), is high. At higher values of \( Q \) however, \( (dW/dQ) \) is low.

An initial guess at a value of \( 1/33 \) was chosen for \( \alpha \) such that when 33 g of water \( Q \) is spread over the skin surface the wettedness is 0.63. A plot of Equation (5.14) is shown in Figure 5.8.

Peiss et al. (1956) demonstrated that sweating from the palm and fingers was suppressed when submerged in dilute saline or water. The suppression in sweating was thought to have been caused by epidermal swelling and obstruction of the sweat pore. Since many observations have shown that reduction in sweat rate is greater in humid heat than in dry heat, Peiss et al. (1956) postulated that as the skin became increasingly hydrated, the keratin ring that surrounds the pore of the duct swells, occludes the duct, thus preventing the outflow of sweat. At any time, the probability of an active sweat duct becoming obstructed is constant, the rate of blockage is therefore proportional to the number of glands which remain active. The resultant decline in sweating would be exponential. Fox et al. (1963) and Brehner and Kerslake (1964) both showed that the sweat rate would decline to zero if the skin was kept fully wet. Kerslake (1972) proposed that the sweating would not decline to zero when evaporation of sweat was possible. The evaporation of sweat would cause an increase in the salt concentration on the skin. The osmotic gradient between gland and surface could become reversed.
Figure 5.8  SKIN WETTEDNESS AS A FUNCTION OF THE AMOUNT OF SWEAT ON THE SKIN

\[ w = 1 - e^{-\frac{q}{33}} \]
which would promote sweating and act to increase the time constant of the decline. The evaporation rate would asymptote to a value close to $h_e(0.82p_g-p_a)$.

Hertig (1960) has advanced the idea of "depressed irritability of the receptors". As the skin approaches a fully wet state, water diffuses in small quantities into a critical layer located below the epidermis. The water dilutes the chemical environment thus decreasing the "irritability" of the sweat receptors. Both of the hypotheses outlined above agree that suppression is small when the skin remains dry or only partially wet.

Time constants for the decay of sweating with time assuming that the ultimate sweat rate is equal to zero have been suggested in the literature. Brebner and Kerslake (1964) determined the time constant after heating their subjects in a water bath to a core temperature of greater than 38°C and then transferring them to a saturated environment at 37°C. Time constants for three subjects lay between 90 and 140 minutes. Fox et al. (1969) showed that time constants were approximately 60 minutes for unacclimatized men and 90 minutes for acclimatized men.

Nadel and Stolwijk (1973) attempted to quantify the effect of skin wettedness on the central drive for sweating. So as to avoid sweat gland fatigue, their experiments were of extremely short duration (16 to 28 minutes). Considering that during immersion experiments where the skin is completely wet from the start, hidromeliosis was only established after 15 minutes (Fox et al. 1963, Brebner and Kerslake 1964), hidromeliosis must take considerably longer in conditions in which the skin is not completely wet from the start. Candas, Libert and Vogt (1980) found that hidromeliosis was only noticeable after 73±13 minutes into experimentation. Nadel and Stolwijk's results in terms of the dynamic effect of hidromeliosis must therefore be doubted. Theirs is really a steady-state determination
of the effect of skin wettedness on the sweat rate drive from instantaneous measurement and non steady-state results. They (Nadel and Stolwijk 1973) proposed the following mathematical relationship to predict whole body sweating.

\[ S_r = \phi \left[ a (t_{re} - 36.7) + b (\bar{T}_b - 34) \right] e^{(\bar{T}_b - 34)/10} \quad (5.15) \]

The wettedness suppression factor \( \phi \) is a function of the wettedness divided by the drive as shown in Figure 5.9. Stewart (1981) derived empirical equations relating fourth hour sweat rates, for unacclimatized men, to mean body temperature, mean skin temperature and skin wettedness.

\[ S_r = \left( 94.24/ W^{0.57} \right) (T_{mb} - 36.5)^0.3 e^{(\bar{T}_b - 34)/10} \quad (5.16) \]

The implication of the above equation is that the fourth hour sweat rate is dependent only upon fourth hour skin wettedness although the extent of the decline in sweat rate has been shown to depend on the past history of sweating (Ladell 1945, Gerking and Robinson 1946, Brown and Sargent 1965).

From the literature studied, it is apparent that the skin wettedness effect (\( W_{ef} \)) begins as the skin starts to get wet, consistent with the concept of wet and dry mosaics on the skin. \( W_{ef} \) builds up with increasing wettedness and has its greatest effect when the skin is fully wet and continues to remain fully wet. \( W_{ef} \) begins to decrease as the wettedness descends from one to zero. The skin will only stay fully wet as long as the sweating drive can compensate for hidromeiosis.

The transfer function suggested by the above qualitative discussion is written in the Laplace domain as

\[ \frac{W_{ef}(S)}{W(S)} = \frac{C_n(2,0)}{S + C_d(2,0)} \quad (5.17) \]
Figure 5.9 SWEAT RATE SUPPRESSION FACTOR AT DIFFERENT LEVELS OF SKIN WETTEDNESS TO CENTRAL SWEATING DRIVE RATIOS (FROM NADEL AND STOLWIJK 1973)
The time constant is equal to \( 1/C_d(2,0) \) and \( C_n(2,0) \) is equal to \( C_d(2,0) \). The d.c. gain is equal to \( K_{sr} \) and the high frequency gain is small. The response of Equation (5.17) to a unit step in wettedness is shown in Figure 5.10.

The value of \( K_{sr} \) was chosen as follows. After four time constants, the wettedness effect should have reduced the sweat rate from its maximum value to zero. This argument implies that \( K_{sr} \) must be approximately equal to the maximum sweat rate of the experimental subject which was \(+15 \) g/min during experimentation.

The specification of the time constant requires some explanation. Two characteristic responses of evaporation rate plus drippage rate are shown in Figure 5.11. Environmental conditions and work rates were different on these two days. Weight loss rate due to thermal sweating reached a maximum at similar times in both cases and then began to decline. A crossover occurred at 120 minutes, the day with the higher peak decaying to a lower fourth hour value. The cross-over or "memory" effect is consistent with the literature and in control engineering terminology is called hysteresis. The way in which the cross-over is achieved using the transfer function as specified in Equation (5.24) is as follows. As the wettedness effect is increasing, the time constant \( 1/C_d(2,0) \) is assigned a value of 5000 seconds consistent with the 60 to 110 minutes found in the literature. When the wettedness effect begins to decrease as a result of a decrease in the skin wettedness, the time constant \( 1/C_d(2,0) \) is made ten times longer as an initial guess at slowing down the recovery time. The result is a slowing down in the rate of decrease of the wettedness effect, and as such the body is penalized for having had a high skin wettedness for a long time. The response of the \( W_{ef} \) block to a positive unit step in wettedness at time zero and a negative unit
Figure 5.10 RESPONSE OF THE WETTEDNESS EFFECT BLOCK TO A UNIT STEP IN WETTEDNESS
Figure 5.11 DEMONSTRATION OF THE MEMORY OR "HYSTERESIS" EFFECT ON SWEAT RATE

Note: Day X —— X has lower rectal and skin temperatures than day 0 —— 0, yet the 4th hour sweat rate is higher.
step at time equal to ten time constants is shown in
Figure 5.12.

The $W_{ef}$ block for increasing $W_{ef}$ can be written as

$$\frac{W_{ef}(S)}{W(S)} = \frac{1/5000}{S+1/5000}$$

(5.18)

and for decreasing $W_{ef}$ can be written as

$$\frac{W_{ef}(S)}{W(S)} = \frac{1/50000}{S+1/50000}$$

(5.19)

5.2.4 Sweat drippage

The components of the model to be postulated in this
section are shown in Figure 5.13.

In a review of his work on heat transfer from cylinders,
Kerslake (1972) showed that variations in the local
coefficients of evaporative heat transfer caused water to
drip from a vertical cylinder "sweating" uniformly, when
about one third of its surface was fully wet. He showed
that the dripping phenomenon could be described in terms
of the efficiency of sweating,

$$\tau_S = E/\lambda S_r$$

and the ratio $\lambda S_r/E_{max}$

The derived relationship between the above two quantities
is given as the solid lines in the two graphs presented in
Figure 5.14. Also shown in the same figure are the data
obtained by Galamidi and Stewart (1978) and Candas et al.
(1979). It is important to consider the manner in which
the data were obtained. In the former, experimentation
was carried out on a single nude acclimatized man exposed
to a range of hot humid conditions at three rates of block
Figure 5.12  RESPONSE OF THE WETTEDNESS EFFECT BLOCK TO A POSITIVE UNIT STEP AT TIME = 0 AND A NEGATIVE UNIT STEP AT TIME = 10τ
Figure 5.13  SWEAT DRIPAGE MODEL COMPONENTS
stepping. Measurements of dripped sweat were made after 90 minutes of exposure when sweat rate as measured by weight loss rate was constant. Dripped sweat was collected in oil pans on the floor. In the latter, two air temperatures and wind speeds were used in experiments conducted on four men in a reclining position. Measurements of total weight loss rate and drippage rate were taken between one and two hours of exposure, when readings were constant. Figure 5.14 illustrates the good agreement between both sets of data and Kerslake's curve. Galamidi and Stewart (1978) derived mathematical expressions to represent this curve.

For $\lambda S_r/E_{\text{max}} < 0.46$
\[ E = \lambda S_r \]  
(5.20a)

For $0.46 \leq \lambda S_r/E_{\text{max}} \leq 1.7$
\[ E = \eta_s \lambda S_r \]  
(5.20b)

where $\eta_s = e^{-0.4127(\lambda S_r/E_{\text{max}}-0.46)^1.168}$  
(5.20c)

For $\lambda S_r/E_{\text{max}} > 1.7$
\[ E = E_{\text{max}} \]  
(5.20d)

The determination of the evaporative efficiency of sweating allows one to calculate the steady-state amount of sweat dripping off the body under conditions of constant sweat rate and evaporation rate. Thus

$S_{\text{rgm}} = E_{\text{gm}} + D$ when $\dot{Q} = 0$  
(5.21)

$E_{\text{gm}} = \eta_s S_{\text{rgm}}$  
(5.22)

$D = (1-\eta_s)S_{\text{rgm}}$  
(5.23)

It does not however allow one to calculate the quantity of dripped sweat when dynamic changes are occurring, i.e. when $\dot{Q} \neq 0$. 