IDENTIFICATION OF AN INDUSTRIAL SCALE
SEMI-AUTOGENOUS GRINDING CIRCUIT
FOR CONTROL PURPOSES

30 November 1988

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A dissertation submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Master of Science in Engineering.

Department of Metallurgy
DECLARATION

I declare that this dissertation is my own unaided work. It is submitted for the degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other university.

Mark Robert Giddy

This 30th Day of November 1988
Optimisation of the control of an autogenous grinding circuit requires a better understanding of system dynamics than exists at present. To this end extensive operating data were acquired on an industrial semi-autogenous milling circuit and a model was developed to explain the circuit responses to changes in input variables. This model represents the trends in the data fairly accurately and showed that:

- When an excess of slurry was present in the load (i.e. more than the volume of the voids) a pool of slurry forms at the toe of the load.
- The complexity of the power dependence on the variables describing the mill load defied dynamic modelling with the limited knowledge available.
- The key parameters determining grinding efficiency vary constantly as the grinding environment is disturbed indicating a need for adaptive control.
- The flow through the mill is adequately modelled by a single well mixed tank.

It was also clear that maximisation of the power with respect to the mass alone was insufficient, and that control of the feed dilution water rate is essential.

Mill discharge slurry rheology was studied to gain an insight into conditions in the load. An energy balance using temperature measurements around the mill discharge sump provided a very reliable estimate of the mill discharge density. This estimate was not successfully used for control although this potential should be exploited. Attempts to commission an on-line measurement of discharge slurry viscosity failed.

Several theories and ideas were tested in a production environment subject to a multiplicity of disturbances and conclusive qualitative results were obtained. An improved understanding of milling circuit dynamics and a rationalisation of where control can be improved has resulted from this work.

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Nomenclature
SI units are used throughout unless otherwise specified.

Upper case symbols
A area
B_i frac material broken from class j into class i
C_r heat rate and water rate to the cyclone
C_d orifice discharge coefficient
C_p specific heat of the solids
C_v specific heat of the water
D_m mill diameter
D,D_W total mill discharge rate and discharge water rate
E mill power draft
F_i feed rate of size class i
F_W feed dilution water rate
G transfer function
H enthalpy
H_L distance of the load surface from the axis
J energy
L mill length
M_i load mass in size class i
N_r/N_c mill rotational speed and critical speed
N_G no. of holes/m²&sp2. in the grate
ΔP pressure drop
P, PW: Total circuit product rate and product water rate
Q: Volumetric flowrate
R: Mill radius
Re, v: Recovery to the cyclone U/F of fluid, solids, and by volume
Re: Reynold's no.
Si: Selection function for size class i
SW: Sump dilution water rate
Split: Split ratio of cyclone U/F to O/F volumetric rates
T: Temperature
U, UW: Total mass rate and water rate to cyclone underflow
V: Volume
Vs: Sump volume
Wg: Gland service water rate to discharge pump

Lower case symbols

d: Diameter
d50c: Corrected particle size which has equal chance of reporting to U/F or O/F
hL: Depth of the load = R-HL
k, k': Proportionality constants
m: Rosin-Rammler parameter to determine the classification efficiency
q: Energy transfer rate
t: Time
v: Belt speed

Introduction
\[ y_i \] \text{ fraction of particles in size class } i \\
\[ w_k \] \text{ rate of energy input by the pump} \\
\text{Greek symbols} \\
\[ \alpha \] \text{ half the angle subtended at the centre by the load surface} \\
\[ \delta \] \text{ hole diameter in discharge grate} \\
\[ e \] \text{ error - deviation from setpoint} \\
\[ \phi \] \text{ fraction solids (by mass unless volume is specified with a subscript)} \\
\[ \mu \] \text{ viscosity} \\
\[ \rho \] \text{ density (the solids s.g. is 2.67 throughout)} \\
\[ \theta \] \text{ angle of repose of the load} \\
\[ \tau \] \text{ torque} \\
\[ \omega \] \text{ angular velocity} \\

\text{Subscripts} \\
\[ b \] \text{ quantity related to the feed belt} \\
\[ c \] \text{ cyclone property corrected for material that bypasses classification} \\
\[ G \] \text{ grate property} \\
\[ L \] \text{ refers to the load} \\
\[ s \] \text{ solids} \\
\[ v \] \text{ volumetric quantity} \\
\[ w \] \text{ water} \\
\text{Process stream names or symbols are used as subscripts to describe quantities relating to the stream.}
1. Introduction

1.1. The Autogenous Grinding Control Problem

The major difficulties that arise in approaches to autogenous grinding circuit control strategies result from:

- The highly interactive nature of the system
- The non-linear and time-varying response of mill power draft - the only cost and efficiency indicator - to the system inputs
- The large material holdups and equipment volumes and consequent time lags
- The presence of unmeasured and unmeasurable disturbances
- The inability to measure many key variables on line, particularly particle size distributions for all but the finest size classes (-75 μm).

In order to realise the objective of the grinding circuit, for example maximum throughput at constant fineness of grind, control strategies have to address the above problems. Current strategies tend to control important performance such as mill power draft or final product particle size although a direct approach, i.e. control of the grinding environment or indicators close to the grinding process itself, would obviously be more desirable. The latter approach does require the solution of some of the above problems, however, and will always be complicated by those that remain insoluble.

1.2. The State of the Art in Autogenous Grinding Control

Initially ball mill control techniques (covered by Lynch, 1977) were modified for the autogenous problem. These strategies aim to maintain a constant cyclone overflow size distribution with the basis for control being adjustment of the percent solids in the cyclone feed by dilution water control. Autogenous and semi-autogenous grinding processes differ from ball milling in that the feed size distribution is very variable resulting in continuous changes in the concentration of grinding media in the load. Continuous monitoring and adjustment of the solids feed rate to the mill is thus required and the need for an automatic control strategy using this variable was implied.
Because the condition of the load is subjected to continuous changes in the solids feed, a control variable is sought which will reflect variations in the load. The power drawn is a strong function of load volume (Austin (1973), Kjøs (undated) and Harris et al. (1983)) and is easy to measure, making it a popular choice. Williamson (1962) and Flock (1975) both implemented control strategies aimed at maximising power draft by manipulating feed rate. Maximum power draft appears to be optimal since the rate of product formation is strongly correlated with power input.

Figure 1: Schematic Load - Power Relationship.

Initially these control strategies had to revert to manual control under adverse conditions. This resulted from the non-linearity of the load power relationship (Figure 1). A drop in power can be caused by both over and under feeding the mill. Thus measurement of the rate of change of power draft is insufficient. Combining this with a measure of rate of change of mill loading can more uniquely define the operating regime of the mill.

Several indicators of mill loading have been used, e.g. oil back pressure on support bearings (Olsen et al., 1976 and McManus et al., 1978) and noise level (Linnen et al., 1974). Unfortunately these do not provide an absolute measure of load mass or volume, but only represent the relative rates of

1 Direct mass measurement using load cells to support the mill provides an accurate measure of the load mass and can be used to establish “normal” peak mass values under different conditions, but variations in load bulk density will still result in different power draw at the same mass.

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change of these values. However, this is sufficient for control purposes since these indicators allow the establishment of the load condition relative to that at which peak power would be drawn. The controller varies the mass by manipulation of the solids feed rate so that if the mill is underloaded (i.e. an increase in mass causes an increase in power) the feed rate is increased until overloading (i.e. a decrease in power drawn) is experienced at which point the controller cuts back on the feed rate to decrease the mass. This approach is termed "peak seeking" control with the objective being to maintain maximum power draft under all conditions.

Implementation of this strategy is still insufficient to utilise the circuit to capacity. The power is not a function only of the mill loading, but of all the factors determining the load environment making it an ambiguous variable to use for control purposes. Operation may be at the local peak power, but the grinding rates remain low because slurry conditions are sub-optimal or the media have been depleted. This indicates the need for closer monitoring and control and an improved understanding of the mill environment and of the grinding process itself.

Krogh (1979) used the feed dilution water to the primary mill to balance grinding between the primary and secondary mill. Also, if an overload condition was reached in the primary (i.e. power dropped while loading increased), the solids feed rate would be reduced while water addition was stepped up to decrease the load quickly. When the power draft was maximised the controller would make changes to the feed dilution water at regular intervals. If this change caused the power consumed per ton of ore to drop, the change was repeated, otherwise the change was reversed.

Pauw et al. (1984) found a similar dependence of power on slurry density in the load to that of power on load volume for a pebble mill. A two-stage peak seeking controller was thus implemented: first the optimum pebble loading was found (i.e. pseudo peak power) and then the true power peak was searched for by making step changes to the water addition rate.

The two previous strategies suffer from very long time lags between perturbing the system and observing the response. Only if approximately steady state circuit operation occurs during the perturbation and measurement periods, will the action resulting from the delayed response be correct. In autogenous
practice approximately steady conditions are unpredictable, and assuming them falsely could well result in the wrong action which would completely disturb the load from its previous "steady" condition.

Pauw's method has a two-stage controller using one output to independently determine the manipulation of two inputs. This is possible because pebble milling is a two phase operation:

i) normal operation with a slurry feed, and

ii) addition of grinding media at intervals to maintain their concentration

In autogenous milling, continuous control of the mill loading is required since its constancy cannot be relied on with rapid build-up or depletion of the load occurring with feed size changes.

A major advance in approaches to the control of milling circuits is the implementation of multivariable control. A decoupled multivariable controller has been included in the grinding circuit at East Driefontein G.M. (Hulbert et al., 1982) manipulating the water addition rates to the feed and sump to maintain constant cyclone feed rate and feed density and a constant product fineness. This has proven very effective in stabilising the circulating load and classifier performance, and consequently has increased throughput and efficiency by reducing over-grinding.

However, this strategy does not aim to improve or understand the grinding process itself. A massive statistical analysis of data containing responses to step changes in the inputs provides the system model, with the system itself being regarded as a "black box", ignoring most existing knowledge of the process. Also it cannot cope with the strong non-linearities in the power vs. input variable relationships near the operating point, so power is not used in the control strategy. Further, the extensive dynamic testing required for the design of the decouplers must be independently performed for each new circuit. Even having done this, the strategy may not cope with the unmeasured system disturbances e.g. spillage return, additional water streams and equipment wear, so that ideally some form of continuous on-line identification is required.

The alternative approach is to understand the grinding process and to utilise indicators of conditions in the load as control variables wherever measurable

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e.g. mill discharge percent solids, load slurry rheology and angle of repose. Work is being performed to produce reliable estimates of these quantities e.g. Moys (1985) has used conductivity probes in the mill shell. These probes pick up the positions of the toe and shoulder of the load, which, when averaged over several revolutions, provide indications of the load angle of repose, expanded load volume and even an estimate of the slurry viscosity in the load (Liddell & Moys, 1988). These measurements allow determination of the optimum degree of filling and the influence of slurry rheology on power draft. The dynamic behaviour of the load can also be quantitatively described under various conditions.

Comprehensive studies of grinding mechanisms on both laboratory scale (Dilger, 1969) and industrial autogenous and semi-autogenous mills (Stanley, 1974 and Austin et al., 1976) have produced mechanistic models of performance. These cannot yet be utilised for control purposes because it is not yet possible to provide the measured particle size distributions required as inputs, but by understanding the grinding process simpler physical models may be developed to describe operational trends in control variables.

1.3. Project Objectives

The present study concerned observation of the dynamics of an industrial semi-autogenous mill at Elandsrand Gold Mine operating in closed circuit with a hydrocyclone classifier (Chapter 2), the investigation of improved measurements relating to the mill environment (Chapter 4) and the development of a simple physical model (Chapter 5) to describe the trends observed and to provide an understanding of this multivariable interactive system.
2. The Grinding Circuit

2.1. Description of the Circuit at Elandsrand G.M.

The milling section of the plant consists of three primary mills fed with "run of mine" (ROM), i.e., uncrushed, ore. Each of these operates in closed circuit with a hydrocyclone classifier whose overflow passes through a secondary bank of cyclones with the underflow from that stage serving as feed to the secondary ball mill (Figure 2).

![Schematic Overall Milling Circuit Flowsheet](image)

Thus in this setup the primary mills are tonnage mills i.e. maximum throughput is the operating objective, while the secondary or regrind mill achieves the desired fineness of grind. This implies that small fluctuations in product fineness from the primary circuit are tolerable. Realising steady operation at maximum grinding efficiency should also result in a fairly constant product fineness.

The primary mills are the slipper pad type with open end grate discharge (as opposed to trunnion overflow). They are 4.85 m in diameter and 9.15 m long, and are each driven by a 3 MW motor such that they turn at 68-92% of critical speed (approx. 18 rpm). This study was restricted to the No. 3 primary mill.

The ore feeds by gravity from a silo via a chute onto a variable speed conveyor. The mill discharges into a small (~7 m³) sump and the slurry is pumped...
directly to a cyclone (diameter 0.915 m) which returns its underflow to the mill feed hopper (Figure 3).

Dilution water is added to the mill feed hopper and to the mill discharge sump. Additional water enters the system as feed moisture, belt wash water and gland service water for the discharge pump. Spillage is returned to the discharge sump and in the event of the ball mill being off line the secondary underflow returns to the primary mills.

![Figure 3: No. 3 ROM Mill Circuit.](image)

An approximate steady state mass balance provides an idea of the stream flowrates and densities.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Flow</th>
<th>Mass Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>95</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>25</td>
<td>10</td>
</tr>
<tr>
<td>D</td>
<td>335</td>
<td>15</td>
</tr>
<tr>
<td>S</td>
<td>180</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>105</td>
<td>10</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>13</td>
</tr>
<tr>
<td>U</td>
<td>305</td>
<td>31</td>
</tr>
</tbody>
</table>

The Grinding Circuit
2.2. Existing Control Loops

2.2.1 Instrumentation

The solids feed rate is monitored by a nuclear weightometer. This measures belt loading and multiplies this signal by belt speed to produce a feed rate. The feed dilution water rate is measured by a magnetic induction flowmeter as is the cyclone feed rate. A nuclear densitometer measures the feed density to the cyclone and an ultrasonic device provides the sump level indication. An orifice plate flowmeter was installed in the sump dilution water line (Figure 3).

Oil back-pressure measurements on each of the ten slipper pads are summed according to their position to provide a relative measurement of the mill mass, i.e., an indication of the trend in load mass but with no absolute significance. A transducer on the high tension switch gear registers the mill power draft. There are automatic control valves on the dilution water lines and variable speed drives on the feed conveyor and the mill discharge sump pump.

2.2.2 Circuit Control

There are five possible manipulated variables in the circuit, namely

- feed rate
- feed dilution water rate
- sump dilution water rate
- discharge pump speed
- steel ball addition rate

At present the major circuit control element is a microprocessor-based "peak seeking" algorithm which aims to maintain maximum power draft. The relative rates of change of the mass and power signals are considered and the approach to peak power draft is gauged. The controller then decreases or increases the mass setpoint depending on the position on the mass/power curve. A further cascaded control loop then manipulates the feed rate to keep the mass on setpoint. A block diagram of the control loop is shown in Figure 4.
Figure 4: A Block Diagram of the Peak Seeking Control Loop

The Grinding Circuit
Figure 4: A Block Diagram of the Peak Seeking Control Loop
Initially the feed solids rate was very erratic, never attaining a mean value. The actuator on the belt drive responds non-linearly to the linear control output and has different characteristics on accelerating and retarding the belt (see examples in Appendix A). This response of the final control element was further complicated by having no feedback on the belt speed but only on the feed rate, resulting in a bivariate random disturbance for the controller to contend with: the randomly varying solids profile on the belt and the near random variation in belt speed. This situation was improved by cascading an extra feedback loop on the belt speed (Figure 4), although the actuator response is still a problem.

It is common practice to ratio the feed water to the solids feed rate. This is a hangover from ball milling practice and is often incorrect in autogenous applications e.g. consider a feed consisting of a high percentage of large rocks that is increased to compensate for a low power draft. The feed water then increases correspondingly such that the fine material that was present in the load would be flushed out, aggravating the low power draft condition since the available capacity for attrition grinding of interstitial slurry is not utilised and sub-optimal conditions in the load slurry rheology prevail.

In the Elandsrand circuit feed water is ratioed to the load mass with a view to maintaining a constant slurry density in the load, assuming that the slurry volume at peak power is constant. Variation in the relative mass is accounted for by manual discharge density sampling to determine the ratio setpoint. This is subject to similar problems to those experienced with a feed-ratio scheme, although the fluctuations in the feed are integrated by the mass.

The longer time response introduced by this latter scheme can be a problem and the technique employed for discharge density sampling is not reliable, varying from one operator to the next. The magnitudes of adjustments made to the feed water rate on the basis of these samples are random and there is no feedback on the action taken. No satisfactory control output to determine feed water rate is continuously monitored at this stage.

A rough idea of the steel addition rate is maintained by counting the number of ball loads added. The "set point" for the number of loads to be added is determined by long term monitoring of the average power draft, mill throughput rate and visual inspection of the steel level in the loads. If these drop sub-
slightly more steel than usual is added. This approach is not very reliable and does not always serve to improve the grinding conditions. Rationalising the steel addition rate is important, not only for increased efficiency, but also because steel is costly and overloading causes increased wear by the action of steel on steel and on the liners.

The sump dilution water is used to maintain the sump level. This is essential in the small sump as it can rapidly overflow or drain and cause the pump to surge. Because the holdup in the sump is small a fast acting controller is desirable. Unfortunately the valve response time is of a similar order of magnitude to the sump residence time. Thus tuning the controller for fast actions results in an oscillatory response verging on unstable behaviour. This in turn affects cyclone performance adversely and thus the entire circuit. This behaviour did prevail prior to this testwork. That "normal" operation was assumed under these conditions clearly shows the need for monitoring variables and assessing control loops. The controller action was slowed down necessitating manual start up to find the approximate steady state value but thereafter the controller coped with small fluctuations resulting from variations in discharge rate and water line pressure.

The variable speed pump proved to have an even slower response to changes in level than the valve when applied to the level control problem. It was also used in attempts to control the cyclone feed rate and density. These control actions tended to interfere with the sump level control and indicated the interactive nature of the system and the suitability of a decoupled multivariable controller for this problem. In addition this variable speed drive was not a suitable final control element having a slow, non-linear response, different characteristics on accelerating and decelerating and losing capacity at the high torques imposed on the fluid coupling when pumping dense slurries. These mechanical problems should be overcome to provide the extra manipulated variable for tighter circulating load control.

The inadequate final control elements described above illustrate the importance of careful instrument selection at the design stage; to change these later requires large capital expenditure and often structural changes to the existing plant.
2.3. Disturbances

- Solids Feed
The major disturbance to the system is the practically unmeasurable feed particle size distribution. Fortunately with Witwatersrand quartzite ores it has been found that the grindability or hardness of the ore does not vary much so the effect of ore hardness can be ignored. However, not knowing the feed size distribution, especially in autogenous grinding, will hamper any control strategy design. Knowledge of the feed particle sizes will allow steel addition to be performed on a rational basis and allow the use of a model for predicting breakage conditions in the load, and probably provide an estimate of the optimum water addition rate.

Lynch and Duckworth (1982) outline test work using an online particle size analyser for the feed, but this instrument only provided the concentration of one size class and was not marketed commercially in this application. However, while such a measure remains unavailable, control objectives must be achieved by alternate means.

- Steel Addition
The random fashion in which steel is added to the mill makes it an unknown disturbance. Although estimation of the steel content of the load and knowing when to add steel is difficult, close monitoring of steel added could be used in conjunction with a model to predict ball size distribution and wear characteristics in the load (Vermeulen and Howat, 1985). This would help to describe the influence of this variable and reduce the number of unknowns.

- Unmeasured Inputs (Disturbances)
The performance of any mill control system is affected by its ability to deal with the following disturbances:

Spillage is returned to the sump causing the water addition rate to drop and the density to the cyclone to increase, increasing the cut size and possibly causing roping.
Secondary cyclone underflow is returned to the primary mills when the ball mill is off-line, increasing the fines in the load so that the grinding rate drops and these particles remain in circulation. The high circulating load maintains the mass so that an overload condition is sensed, causing a decrease in the solids feed rate, aggravating the situation.

The control system should be informed of the existence of these disturbances by the use of digital inputs linked to the spillage pumps and the ball mill drives.

At high load slurry viscosities or when the mill is overloaded, a portion of the load no longer tumbles, but centrifuges causing a sharp drop in power and less grinding occurs. This behaviour provides a strong motivation for on-line sensing of mill discharge percent solids or viscosity to indicate the onset of centrifuging.

- Unnecessary Disturbances

The major disturbances are feed chute and hopper chokes. These are cases of the feed being stopped by large rocks or small rocks packing together in the outlet of the chute and hopper, or dry fines backing up the feed chute. Improvement of the feed arrangement is essential so that this source of disturbance can be eliminated, as no control strategy can compensate for this and random periods of no feeding upset the load environment such that steady operation is rarely achieved. Constant running of the mill is desirable, but on this plant the mills are regularly stopped, defeating controller action and the approach to steady operation.

Finally, operator-induced disturbances should be eliminated. Closing manual valves and bypassing final control elements should only be resorted to in the event of controller failure.

These "unnecessary" disturbances must be contended with prior to attempting to optimise operation with improved control.
3. Observation of Dynamic Circuit Behaviour

3.1. Data Acquisition

On-line monitoring of as many variables as possible is essential to understand the dynamic response to changes in the system inputs. A Hewlett-Packard data logger was utilised for this purpose. Initially this was controlled by an HP-41CV calculator but later a HP-85 micro-computer was used for its increased speed and storage capacity.

Sampling times should be very much less than the smallest system time constants. This was possible with the facilities used, but storing and manipulating the vast quantities of data gathered in a relatively short time proved a problem. Thus measurements were taken at one second intervals and these were averaged over a one minute period to reduce the amount of data. Although information is lost by this smoothing technique it eliminates uninformative system noise and still provides representative data, since the mill, which is the dynamic element of interest, has a mean slurry residence time of the order of ten minutes.

<table>
<thead>
<tr>
<th>Component</th>
<th>Residence Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rock in the Mill</td>
<td>70 min</td>
</tr>
<tr>
<td>Slurry in the Mill</td>
<td>10 min</td>
</tr>
<tr>
<td>The Discharge Sump</td>
<td>1 min</td>
</tr>
<tr>
<td>The Cyclone Feed Line</td>
<td>0.5 min</td>
</tr>
</tbody>
</table>

Table 3.1 Mean Residence Times

Data logging is only worthwhile if accurate information is gathered. Calibration checks on measuring instruments were performed (where possible) to minimise errors resulting from drift. Unfortunately it was not possible to check all of the instruments owing to the scale of the operation.

Measurements of power draft, solids feed rate and sump level could be checked and calibrated. The magnetic-induction flowmeters were presumed to be accurate within the manufacturer's specification. The density gauge was standardised with water and manual density sampling provided similar results to the measured values within the accuracy of the sampling technique. Slightly high readings were anticipated because the meter is mounted in a vertical
line which would concentrate the solids but trends should be reliably represented.

A British Standard (BS1042) orifice plate flowmeter was used in the sump waterline to allow use of the standard discharge coefficient correlation. However, the correlation value produced very high, sharply fluctuating flow rates, indicating that in situ calibration was required. Calibration of this instrument was thus performed by integrating the measured flow rates through the orifice plate and the magnetic flowmeter in the cyclone feed line over a range of values for a sufficient period that fluctuations in sump level could be ignored. (Appendix B)

The calibration technique is only subject to a small error but unfortunately could not cover the full range of flow rates, so that at flow rates outside normal operation very inaccurate values are predicted. This instrument proved unreliable in any case because the solids in the water (being thickener overflow water, fine solids are often present) tended to pile up against the front of the plate, checking the pressure tappings, changing the flow characteristic and causing orifice wear. Despite these flaws, some informative trends from sump water addition rate measurements were observed.

3.2. Experimental Design

Dynamic tests were generally performed on the open loop system although controller action was also monitored. The sump level control loop, i.e. manipulating the sump water, was of necessity maintained at all times. This left only two effective system inputs: the water and solids feed rates. In each test one of the inputs was perturbed while attempts were made to keep the other constant, and all the measurements on the circuit were monitored. Variable size step tests were used to identify the responses, with care being taken not to affect the circuit operation adversely e.g. flow rates and densities were maintained near normal operating values to keep the cyclone from roping and the mill from pulping, although these conditions can occur during normal operation.

Prior to each test a settling period of approximately half an hour was allowed while it was attempted to maintain constant inputs while remaining at the
local power peak. After a step input, once again a period of constant operation was desirable so that the response could be monitored without interference.

Very few step tests were performed on the solids feed since the feed rate was subject to random variation and was very hard to maintain at a constant level. The effect of steps in the feed water rate was often obscured by solids feed rate changes unless the changes were reasonably large, in which case they substantially perturbed the circuit operation.

Data gathered in this fashion are graphically presented in Chapter 6 with one example of tests and responses shown in Figure 5.

**Description of Run No. 1 (Figure 5)**

The 'peak seeking' controller was on manual for this test. The sump level controller remains active. This data set is chosen for the clarity with which responses may be observed due to the unusually fine and evenly spaced rate.

The feed rate is manually increased to find the mass at which the peak power is achieved. The power rises at first to (a) and then falls as an overload condition is reached. The feed is then cut back and a setpoint of 100 t/hr is specified for the duration of the record. This allows the mass to grind down. The mass remains fairly constant until (b) despite several partial feed chokes. The power draft reaches a higher level as the mill recovers from the overload condition. It is interesting that peak power is reached at a similar mass level to that at (a).

A step increase is introduced to the feed water rate at (b) since the mass, power and feed have stabilised. This increases the mass slightly and then causes a drop as the fines are flushed out. The subsequent sharp drop to half the initial water rate then causes the mass to build up and the power to drop as the mill re-enters an overload condition, illustrating the influence load slurry rheology has on the mill discharge rate and hence power draft.

When the water is increased at (c), a complete change in operating environment is experienced and the power draft and mill mass fall concurrently clearly indicating an underload condition. The power begins to recover but the fines are flushed from the mill so rapidly that it quickly passes to an under-
Figure 5: Plant Operating Data
load condition. During this period a higher feed rate would have been possible without overloading the mill. The need for a balanced slurry rheology to achieve peak power and optimum throughput is again highlighted.

Note how clearly the step changes in the feed water rate are reflected in all the circulating load parameters. The higher discharge rates (after (c)) cause less water to be added to the sump and consequently, a higher cyclone feed density results. The cyclone feed rate also drops with the higher dynamic pump head.

A further behaviour pattern which is apparent in this clear data set is the response of the discharge rate to the increases in feed water rate. The manner in which an initial pulse is discharged (especially at (c)) of approximately one residence times length suggests that a pool of excess slurry is formed and rapidly discharges before the long term response is observed. The observation of plug flow behaviour in this slurry pool is successfully exploited while modelling the circuit responses (Chapters 5 & 6).
4. Investigation of Alternate Control Variables

The measured variables listed in Section 2.2 can be combined in various control loops in an attempt to optimise the grinding circuit performance. Several of the key variables remain unknown, however, and will persistently disturb the circuit and prevent attainment of the optimum operating condition.

Knowledge of the feed size distribution would provide a rationale for steel addition and feed dilution water control (Section 4.1). Continuous monitoring of cyclone performance would ensure that:

- the desired product was consistently achieved
- excess energy was not being consumed for overgrinding
- oversize material was not being sent downstream.

Information regarding the grinding environment itself, e.g. load volume, slurry rheology within the load or load angle of repose, would improve understanding of the load dynamic behaviour and ease trouble-shooting. Consequently, the inputs could be manipulated to optimum advantage.

An indication of the cyclone overflow size distribution has been used successfully for control (Hulbert et al., 1982) but little progress has been made towards identifying changes in the feed size or load condition in industrial applications. Testwork thus concentrated on measurement of the rheological properties of the mill discharge slurry to gain insight into one aspect determining the grinding environment.

4.1. Mill discharge density

Present manual sampling of mill discharge density on a two hourly basis to set the feed water rate is completely ineffective. Samples are not representative, techniques vary from one operator to the next and two hours elapse before any feedback on the control action occurs, assuming the system has not been disturbed during this period. An accurate continuous measurement of discharge density is necessary to

- Control feed water rate (with a tolerable dynamic lag)
- Provide a indication of slurry conditions in the load
Allow for optimisation of slurry density in the load with respect to power.

Slurry viscosity is a strong function of % solids so that at high densities, viscosity is very high, increasing the holdup and thus the fineness of grind. This suggests that it might be possible to correlate particle size distribution with discharge viscosity providing an extremely informative grinding efficiency indicator.

Slurry viscosity and holdup are key determinants of grinding efficiency. It is desirable to coat the media with slurry so that as the media cascade and cataract with normal mill operation, the adherent layers of slurry are subjected to continuous compressive grinding while they in turn cause abrasion of the media. If the slurry consistency is too fluid, it is likely that the slurry volume will exceed the volume of the voids in the media causing a slurry pool to form. This cushions the impact of the cataracting media on the toe of the load and reduces the power draft since the centre of gravity of the load will align more closely with the mill axis reducing the angular moment. In addition, steel ball and liner wear rates are much higher when the slurry viscosity is too low.

At high slurry densities hold-up is higher and the excess slurry in the load can cause expansion of the load with consequent loss of the forces between the particles and inhibition of the tumbling action of the media. Onset of premature centrifuging of the load is also likely at high slurry viscosities.

Control of % solids in the load slurry would thus clearly improve grinding efficiency and knowledge of the discharge slurry density would allow prediction of trends in this variable. Direct measurement of discharge density was not possible as the discharge flows directly to the sump via an open launder. Thus estimation from other measurements was investigated.

4.1.1. Density Estimation using a Measurement of Sump Dilution Water Flow Rate

An orifice plate, providing a measure of the dilution water rate, was commissioned to allow estimation of the discharge density. Combining this with the cyclone feed measurements and completing the mass balance allows evaluation of the solids mass fraction in the discharge stream.
where $C$ is the mass flowrate to the cyclone
$SW$ is the sump dilution rate
$W_g$ is the gland service water rate
$V_s$ is the volume in the sump
$oc$ is the slurry density in the sump assuming perfect mixing
$d$ are the solids mass fractions in the cyclone feed and mill discharge

The only unknown is the gland service water which may be assumed to be fairly constant in the absence of a measurement since the gland service water circuit is independent of the other water utilities. It is only subject to fluctuations resulting from starting other pumps and leaks in the line.

If the gland service water flowrate estimate was the only error, this technique would be acceptable since this flowrate is only about 3% of the sump dilution water rate. However, as was pointed out previously (Section 3.1), the sump dilution water rate measurement is also subject to error. In this circuit the dilution water rate can be as much as four times the water in the mill discharge stream magnifying any measurement error in the discharge density estimate.

An error analysis (Appendix E) shows that a 10% measurement error in the sump water rate can produce a maximum error of nearly 33% in the estimated discharge density. Time lags between measuring elements as well as unsteady state behavior serve to complicate the calculation further.

A magnetic flowmeter in this application should provide a sufficiently representative trend in discharge density for use as a control output, since it would be accurate over the entire range of flow rates and although the error still seems large (12%) this is an estimate of the maximum error and would rarely be experienced under normal operation. Also since only the trends and not the absolute values of the variables are required for control, constant errors, e.g., instrument drift, can be tolerated.
4.1.2. Temperature Measurement

An interesting idea (Moys, 1986a) for sump water rate and discharge density estimation is to utilise the temperatures of the streams around the sump in an energy balance (derived in Appendix E).

The mass balance provides the discharge solids rate

\[ D_{\phi_D} = C_{\phi_C} \]

while the water rate in the mill discharge is given by

\[ D(1-\phi_D) = (ws - q_{pipe} + C_{pw}C_{\phi_C} (T_{D} - T_C) + C_{pw}(1-\phi_C)C(T_{W}T_{D}) / (T_{W}T_{D})C_{pw} \]

where \( C_{\phi_C}, C_{pw} \) are the solids and water heat capacities respectively \( C_{\phi_D}, D_{\phi_D} \) are the mass flow rates of solids in the cyclone feed and discharge streams and \( T_D, T_w, T_C \) are the discharge, sump dilution water and cyclone feed temperatures.

Despite surface energy losses (\( q_{pipe} \)) and shaft work input by the pump (\( ws \)) this technique provides an accurate estimate of discharge density (Appendix E), because of the negligible error in the temperature measurements.

Measuring the discharge temperature has other uses too. It may be possible to relate this measurement directly to slurry residence time and fractional circulating load. If the residence time is below average the temperature will be below its equilibrium value and above in the case of a low circulating load. The equilibrium value can be estimated by assuming that the power drawn is totally utilised for raising the temperature of the load, since energy utilised in breakage and noise production is relatively minimal. Temperatures of the feed streams need to be measured too.

Slurry viscosity is a function of temperature as well as % solids so that a measurement of temperature will help to characterise this.

A preliminary investigation into the viability of temperature measurements on the full scale mill was carried out and initial results looked promising, motivating further work on this idea.
4.1.3 Implementation of Temperature Measurements

The bulk of this work was done at Western Deep Levels No. 3 Gold Plant. This is a newer installation and the problem of poor feed control experienced at Elandsrand has been solved by using a wider feed chute opening. This results in a higher belt loading so that the variable speed drive can make large changes in feed rate with small changes in belt speed. The wider feed chute also results in a substantial reduction in feed chokes.

The same type of mill is used at both plants but at Western Deep two stage pumping is used to feed the cyclone with a variable speed second stage pump providing closer control of the cyclone feed flowrate. Only one milling stage is used at No. 3 Plant so that a finer cyclone overflow product is required and a lower mill throughput tolerated.

Resistance thermometers in stainless steel probes were inserted in the mill discharge launder, the sump dilution water line and the cyclone overflow launder (Figure 6). The last of these is not the ideal position since temperatures as close to the sump as possible are desired. The alternative of temperature measurement inside the sump required a special long probe and was affected by imperfect mixing during transient phases. Measurement at the cyclone feed pump suction or delivery would result in very high wear of the probe which would then require frequent replacement.

Figure 6: The No 3 Plant Milling Circuit Showing the Temperature Probe Locations

Alternate Control Variables
The energy input by the pump, the surface losses and gland service water addition compound the error in the estimate but these energy contributions are small relative to the stream energies and fluctuations in the above variables have a negligible effect on the trends in the density estimate.

4.1.4 Operating Data

An offset between the estimated discharge density and weighed samples was observed, so, in order to check the estimate, the system was disturbed and the transient response monitored. The same data logging apparatus as before was used with manual density samples being drawn from the stream at various time intervals.

As can be seen in Figures 7 & 8 the trends in the measured values are clearly reflected in the estimate. It must be noted here that the estimate values have been reduced by 10% for the comparison. This offset is fairly large and can be ascribed to the cyclone feed density measurement which cannot be accurately calibrated in this application; a small measurement error is magnified in the calculated values. In addition an offset observed in the cyclone overflow temperature measurement, as compared with that measured in the sump, affects the estimate.

Description of Run A (Figure 7)

The mill feed rate is on manual control as is the feed dilution water rate. Sump level and flow to the cyclone are automatically controlled.

An hour is allowed for the circuit to settle and approach a steady operating point. A step decrease is then made to the feed water rate and manual density measurements are made around this point. The density response appears to have almost reached its final value when the feed chute chokes, disturbing the system (c).

The feed solids and water rates are increased to correct the underload condition which has occurred. At approximately 100 minutes (b) the feed water is returned to ratio control by the solids feed rate which is the normal strategy. It is interesting to note that during the subsequent feed choke a decreasing discharge slurry rate or by inference an increasing load slurry density is calculated. A correspondingly sharp drop in power is observed with the two

Alternate Control Variables
Figure 7: Run A - Plant Operating Data Showing Temperature Measurements. Temperature measurements are used to estimate discharge density, rate and sump dilution water rate.
Figure 7: Run A - Plant Operating Data Showing Temperature Measurements. Temperature measurements are used to estimate discharge density, rate and sump dilution water rate.
concurrent behaviours suggesting centrifuging of the load; i.e. a portion of the load adhering to the mill shell for the full rotation and thus effectively reducing the inside diameter (Power $\propto D^{1.5}$). This data also shows how this control strategy is not suited to maintaining optimum load slurry rheology. Feed water control reverts to manual at approximately 190 minutes.

A settling period is allowed prior to introducing a large step increase in the feed water (a). Manual samples are again taken around this point and a clear response to this change is observed in all the variables downstream of the mill. The initial rise in discharge density occurs as the slurry in the load is flushed out prior to the long term fall in density. The period required to flush out the slurry inventory (about 3 mins) is much less than the mean residence time of the slurry in the mill (10 mins) supporting the postulate that a pool of slurry exists and is displaced in plug flow. This idea is discussed in detail in Section 6.1.

The measured discharge density data points lead the estimate by approximately 1 minute. This can be ascribed to the residence time in the sump and cyclone feed time which is approximately 80 seconds. These time lags can be accounted for if further work is performed utilising this technique. The unusual behaviour of the sump dilution water temperature (it is usually constant) results from the supply tank receiving make-up water from a colder source. This does not affect the variables being estimated by the energy balance.

The rock fed to the circuit in this case was generally coarse with a lack of fines. This could explain the relative insensitivity of the circuit to the feed chokes as compared with changes to the feed dilution water rate. The influence of the solids feed rate on downstream variables depends strongly on the ore size distribution.

Description of Run B (Figure 8)

The low solids feed rate at the start of this record has caused an underload condition to develop. This is rectified by a step increase in the solids feed rate. Manual discharge density samples are taken and since these are constant a step increase is introduced to the feed water at (a).
Figure 8: Run B - Plant Operating Data with Temperature Measurements
A plug of slurry is flushed from the mill and the discharge density drops as expected. The response of power draft to this change is interesting as it mirrors that of discharge density or by inference load slurry density. As the slurry density rises the power draft falls only to rise again as the dense plug is flushed from the mill.

The peak-seeking controller is enabled at (b) and the feed water returned to ratio control. The inputs to the mill are steadily decreased until (c) where the controller is disabled and a step increase is applied to the feed water rate.

The temperature estimate of discharge density rises steadily through this period and responds to the increase in water as before at (a). At (d) a larger step decrease in the feed water rate results in an immediate decrease in discharge density before the expected rise. This is the opposite response to a step increase in feed water but is not as easily explained. The manual samples indicate the same response. In addition the mill power draft response again mirrors the density clearly illustrating the strong dependence on load slurry rheology.

4.1.5 Conclusion

This technique for discharge density estimation has been shown to have potential as an additional control output in the milling circuit. It may be possible to control the discharge density by feed water manipulation alone, but it is clearly influenced by several variables. Large step changes in the feed water rate such as those introduced during data logging produce a clear response in the discharge density. Controller action applied to this situation attempts with small incremental changes to maintain a specific density. Under ideal circumstances the response to these would be apparent and hence controllable, but in many instances the influence of the circulating load, feed size distribution, solids feed rate and the rheological history of the slurry in the load, which are all in a constant state of flux, will mask the response and lead to possible incorrect action.

This does not imply that control is not viable but highlights the need for sophisticated control algorithms in this application. In Chapter 5 it is shown that a very simple model of the mill circuit allows prediction of the mill discharge temperature from measured inputs. Alternatively, the model could be used...
to determine which input has the overriding influence on discharge temperature and thence density. Discharge density control is also complicated by the fact that the optimal density is not constant but, like the peak power draft, varies with changing grinding environment. It is recommended that further observations be made of the response of the temperature measurements with a view to implementing control action and to determine a means of specifying optimal density as related to other variables. This study will also provide more quantitative insight as to conditions within the load.

4.2. Mill Discharge Viscosity

Knowledge of the slurry rheology within the load is clearly invaluable for optimisation of grinding environment control. The properties of the discharge slurry may reasonably be assumed to be indicative of those within the load and, as discussed in the previous section, monitoring of discharge density assists in understanding the grinding circuit behaviour.

Mill discharge viscosity measurement would be even more informative however since slurry viscosity is very much more sensitive to changes in solids mass fraction than density. In addition variations in slurry hold-up and distribution in the load are directly caused by the slurry viscosity so that the relationship of these variables to slurry density is incidental.

The effects of varying slurry viscosity in the load have been reported by several authors to influence both charge motion and particle breakage rates (Tucker 1982) showed that as solids content increased grinding efficiency passes through a maximum. Klimpel and Manfroy (1976) found predictable and consistent correlations of slurry viscosity with selection and breakage parameters and suggest that optimising slurry viscosity can result in increased throughput and/or fineness of grind. For grate discharge mills Moys (1986) showed slurry viscosity to be the major factor affecting hold-up in the mill and in conjunction with Liddel (1986) showed that in the higher percent solids operating range, viscosity influences the load behaviour more than at lower percent solids where friction between the media provided by the particles in the slurry is the dominant factor. Liddel (1986) showed that mill slurry viscosity varied from 10 to 275 cp for a change in percent solids from 60% to 75%.
In order to investigate viscosity measurement an on-line viscometer was borrowed from Debex for a trial period (Reeves, 1984).

The operating principle of the viscometer is shown in figure 9. A slurry sample is fed to the instrument and flows by gravity into the measurement chamber where the torque required to maintain the bobbin's angular velocity is measured, and the sample is returned to the process stream.

The mill discharge slurry is very coarse (top size ±25mm) and contains milled steel balls, wood chips and other tramp material. Fluid viscosity is unaffected by rapidly settling particles allowing a representative sample for this measurement to be obtained after screening at 5mm.

Continuous sampling of the mill discharge proved to be the major problem in commissioning this instrument. Lengths of wire in the sample caused blockages of even the two-inch sample line, while small pieces of wire that bypassed screening, and particles packing together caused chokes in the instrument itself. Cleaning using magnetic separation was tried but failed in this application because of the high content of steel fines, suggesting that intensive screening of the sample, at possibly a finer top size (e.g. 1mm), is the only
solution. The screen must be self-cleaning and the sample sufficiently large to produce enough screen underflow for the instrument.

The scale of this study was insufficient to cope with these problems requiring mechanical modifications to provide a more suitable sampling point, purchase of arotating or vibrating screen and a sample return pump. The slurry could be characterised, however, and the effects of % solids and temperature on the viscosity investigated using a recirculating slurry sample.

Results (figure 10) indicate that the slurry shows dilatant behaviour at higher shear rates and that the apparent viscosity shows a distinct response to small

![Characterisation of the Slurry Rheology](image)

**Figure 10: Viscosity Relationships with Frac Solids and Temperature**

The relationship between shear stress and mass fraction of solids is shown at various bobbin angular velocities. At lower bobbin speeds an instrument anomaly produced unacceptable results. These results are omitted here, to enhance the clarity, but they do appear in appendix E. The relationship derived by Mooney (1951) is overlaid for comparison with the measured data.

*Alternate Control Variables*
changes in % solids. At low shear rates or rotational speeds the results are highly improbable and it is likely that the instrument is at fault. An instrument on this size scale is unsuitable for accurate viscosity measurements in this milling application. The distinct response to % solids changes shows that the viscosity measurement would be extremely informative and that a measuring technique is worth pursuing.

The effect of particle size distribution on the viscosity should also be investigated because intuitively viscosity is a surface phenomenon and just as with increasing percentage solids the particle interactions increase they should also increase as the concentration of fine particles increases. The on-line sample necessary for this measurement would also allow accurate sampling of density and temperature so that viscosity variations resulting from these influences may be identified.

The data gathered suggests that continuous viscosity measurement be further investigated since dynamic trends in this variable would help to quantify much of the dynamic behaviour of the grinding circuit. This viscometer is not recommended for control purposes in its present form as it is not robust enough.
5. Development of a Mechanistic Circuit Model

Although insufficient knowledge is available to consider comprehensive and accurate modelling of the mill circuit, sensible postulates may be made about representative average conditions in the mill and combined with published models formulated under more controlled conditions to represent the trends observed in the output responses measured in the field. Choosing key variables to parameterise the model allows the magnitude and rate of response of the measured outputs to be more accurately represented.

5.1. Why a Mechanistic Model?

There are two approaches that could be followed in the development of a mathematical model to simulate a process. The first is a statistical or "black box" approach. The type of model developed does not utilise a priori knowledge of the process. The model equations are developed using regression techniques on a large quantity of data containing the effects of some selected inputs on process performance. The model is applicable only to a narrow range of operating conditions. Also, in this type of model, relationships between inputs and outputs must be linear or linearisable. The relationship of power draft to the inputs is strongly non-linear and thus cannot be included (unless different transfer functions are applied to various states of operation), yet power has been observed to be the key indicator of grinding efficiency.

The second approach to modelling is the mechanistic one. This is based on the mechanisms which govern the process and utilises the basic understanding of the process gained during the data acquisition phase. The model promotes understanding of the process because the equations are closely related to the actual events occurring in the mill. This also means that the model has wider application and is more flexible to changes in conditions of operation.

The mechanistic approach is the clear choice in this case since the study aims to improve the understanding of the milling process and to develop a model with applicability to other milling circuits. Qualitative observations concerning the process had been made during the data acquisition phase, e.g. peak re-
relationships between both feed water and feed solids rates and mill power draft and it was necessary to verify and quantify these observations. The relationships between input and output variables and power drawn by the mill provide the key to optimal grinding control, thus it was essential that these be included in the model.

5.2. Description of the Model Developed

5.2.1 Introduction

The model takes its feed streams from the measured data and adds these to a cyclone underflow generated by a previous cycle. This combined stream is added to the load currently in the mill. The mill unit model produces a discharge stream which is perfectly mixed with the sump dilution water and fed to the cyclone unit model. The circuit is completed with the cyclone splitting its feed into the overflow product and a new underflow to join a later set of feed streams.

5.2.2 Mill Solids and Water Feedrates

Three size classes of rocks are assumed in the feed:
- Media size rocks (+150mm)
- +grate size -media size rocks (+25-150mm)
- -grate size rocks (-25mm)

Feed moisture content is given at -4% by mass in the industry but since the ore sometimes had long residence times in the stockpile, a feed moisture content of 2% of the feed was chosen as a more reasonable estimate and this moisture was added to the feed dilution water.

5.2.3 The Mill Unit Model

The mill itself is modelled as a perfect mixer in series with a pure time lag. The breakage and selection function concepts (Whiten, 1974) are used in the mass balance equation to describe the load:
\[
\begin{align*}
\frac{dM_1}{dt} &= F_1 - S_1 M_1 \\
\frac{dM_2}{dt} &= F_2 + S_1 B_{21} M_1 - S_2 M_2 \\
\frac{dM_3}{dt} &= F_3 + S_2 (1 - B_{21}) M_1 + S_2 M_2 - D_{iu} + U_{iu} \\
\frac{dM_4}{dt} &= F_W + U (1 - \phi_u) - D (1 - \phi_u)
\end{align*}
\]

Where \( M_i \) are the masses in each size class and \( M_4 \) is the mass of water in the load. \( F_i \) are the feed rates to each size class and \( F_W \) is the feed water rate. \( D_{iu} \) and \( U_{iu} \) are the discharge and underflow solid rates (\( \phi_u \) is the mass fraction solids). \( S_1 \) and \( S_2 \) are the selection functions, i.e. parameters describing the rate at which particles are selected for breakage, for classes 1 and 2 and \( B_{21} \) is the breakage function i.e. the fraction of material broken out of class 1 that reports to class 2.

These equations were solved using a fifth order Runge-Kutta routine employing an Adams Moulton predictor corrector.

It was found that the fourth equation could be replaced by the overall water balance, i.e. \( \frac{dM_4}{dt} = F_W + S_W - P_W \) where \( P_W \) and \( S_W \) are the product and sump water rates, with little change in the model output. This is expected since the model considers the mill to be the only major dynamic element in the circuit (see system time constants in section 3), and it was observed that water passes through the system almost in open circuit (figure 5).

The steel inventory in the mill is specified as a mass fraction (15) of the load mass at time \( t=0 \). In the model the steel content of the load influences the load density and thus the power draft.

### 5.2.4 Power Draft Model

It was attempted to model the power draft using the relationship derived by Hogg & Fuerstmann (1972):

\[
E = k_\nu D^{2.5} \left( \frac{N}{N_c} \right) \left( 1 - \frac{H_L}{R^2} \right)^{2/3} \mathrm{kw}
\]

Where the term \( (1 - \frac{H_L}{R^2})^{2/3} \) is dependent on the fractional filling of the mill \( V_L/V_{cuit} \), \( N \) is the rotational speed of the mill, \( N_c \) the critical speed, \( \eta_c \), 

Development of a Mechanistic Model
the load bulk density, L, the mill length, D, the inside diameter and e the angle of repose. A derivation of this equation appears in Appendix F.

The derivation shows that this model has as its basic assumption that the surface of the load is flat and that no cataracting of media occurs. This is true for mills operating at low rotational speeds and has been found to still hold well for higher speeds (70-80% of critical) in some cases. Liddell & Moys (1988) illustrated the deficiencies in this model and others at higher fractional fillings and speeds.

In agreement with these results it was found that in this case (the mill under consideration running at around 90% of critical speed) the power model tended to be dominated by the \( V_I / V_{mic} \) term with \( \rho_e \) being strongly dependent on \( V_I \) with the assumption of a constant steel mass. This resulted in good agreement between the model and measured data during underloaded phases, but the model did not capture the turning points signalling overload conditions.

Since the power models investigated proved unsuitable here and the situation could not be rectified the measured power was taken as the mill model power. This ensured that the influence of the power draft on the selection functions and discharge size distribution in the model had a sound basis.

The selection functions are set proportional to the power draft to simulate faster breakage at increased power draft. The product size distribution is determined using a form of Bond's relationship (1952):

\[
E = W_I (1/V_{p_{ao}} - 1/V_{f_{ao}})
\]

rearranged to give

\[
V_{p_{ao}} = 1/(E/k + 1/V_{f_{ao}})^2
\]

where \( V_{p_{ao}} \) and \( V_{f_{ao}} \) are the 80% passing sizes for the mill feed and product respectively, \( W_I \) is the Bond Work Index while \( k \) is a similar constant fitted from approximate plant data (28 000 kW.m°.5).

5.2.5 The Mill Discharge Model

Moys (1986b) derived a physical model to describe the resistance to flow through the grate which was found to predict hold-up in a pilot mill reliably.

Development of a Mechanistic Model
This model differs from that developed by Hogg & Rogovin (1982), based on the assumption of axial flow of slurry only in the pool of slurry in the load, in that flow through a stationary bed is considered. This simplification was found necessary when grate discharge mills, rather than trunnion overflow mills in the former case, were described.

The model can be used for almost any grate geometry and uses the equations for flow through orifices and packed beds to describe the slurry flow through the grate.

Moys’ model was used to evaluate the discharge rate and slurry hold-up in the mill. The discharge rate, \( D \), is given by:

\[
D = k'N_{o}^{2/3}N_{A}^{1/3}R_{h}L^{2}
\]

where \( k \) is dependent on slurry viscosity which when correlated with density, gives

\[
k = k'(\rho_{crl} - \rho)^{b}
\]

and \( N_{o} \) is the no. of holes in the grate per m\(^2\)

\( a \) is the hole diameter

\( R \) is the mill radius of the grate area

\( h_{L} \) is the level of slurry behind the grate evaluated from slurry volume in the load.

\( \rho_{crl} \) is the density at which \( a \rightarrow a \)

A detailed derivation appears in Appendix F.

5.2.6 The Circulating Load Model

The mill discharge is added to the sump dilution water. The latter flowrate is determined by a PI controller maintaining a constant level in the sump. The sump is considered to be perfectly mixed in all places so that the cyclone feed density may be regarded as being the same as that of the sump contents.

The cyclone feed flowrate can then be predicted by evaluating the momentum balance assuming a constant pump power drift and iterating for the cyclone feed pressure using Pitz’s model equation (1976). This assumption of constant pump power proved unreasonable so an empirical relationship between cyclone feed density and flowrate was derived from the measured data.
The cyclone feed rate was found to vary approximately inversely with the slurry density. This results from the dynamic head and friction factor increasing with density, and pump efficiency deteriorating under conditions of high loading.

This provided a reasonable representation of the measured data but the presence of inaccuracies which subsequently manifest themselves in the modelled cyclone underflow and then the mill load, detract from the representativeness of the model as a whole. Thus the flow model was scrapped and the measured data used in its place. This is justifiable since the cyclone feed flow measurement is accurate and readily available on many installations. Specifying the cyclone feed flow rate and predicting the mill discharge rate allows evaluation of the sump dilution water rate by completion of the mass balance:

\[ SW = C - D_o - D(1 - \eta) \]

This balance assumes steady state operation with no accumulation in the sump. This is generally the case since the sump dilution water is manipulated to control the level in the sump. Occasionally this control loop did not function owing to an instrument fault and the sump water valve position was kept constant. It is interesting that under these conditions level control was still reasonably well maintained despite disturbances from both discharge rate and percent solids. The reason for this is apparent from the flat nature of the discharge pump characteristic curve i.e. small changes in the dynamic head produce large changes in flowrate. Thus a small drop in level causes a corresponding increase in head which substantially reduces the flow stabilising the level rather than draining the sump.

Thus the accumulation of material in the sump can generally be ignored but the consequences of the above behaviour are to produce a varying cyclone feed rate which reduces classification efficiency and hence product and underflow consistency.

5.2.7 The Hydrocyclone Classifier

In order to describe the operation of the cyclone, slurry samples of mill discharge, cyclone overflow and underflow were ground to allow description of the classifying action of the plant cyclones.
These measurements were used to construct the fractional and corrected fractional recovery to underflow curves and hence to evaluate the average dsoc and sharpness of classification parameters. The basic forms of the Plitt model equations (1976) are fitted to the data and are used in the model to describe the variation in the above parameters with respect to cyclone feed rate and density.

The model equations used are:

\[
\begin{align*}
\text{dsoc} &= 20.25 \exp \left( 5.3 \, v \right) / C^{0.5} \\
\text{Split} &= 7.19 \times 10^{-4} \exp \left( 3.3 \, \psi \right) / C^2 \\
\eta &= \sqrt{\frac{85}{(v + 0.05)}}
\end{align*}
\]

for C in m³/s;

\( \psi \) is the volume fraction solids while \( \eta \) is the Rosin-Rammler parameter for sharpness of classification.

Figure 30 in Appendix F shows that the corrected classification curves are effectively represented by the equation:

\[
y_c = 1 - \exp \left( 0.693 \left( \frac{d}{d_{50}} \right)^{-3} \right)
\]

5.2.8. The Milling Circuit

All unit models are combined, with the product from one unit serving as the feed for the next. The inputs to the model are the measured feed solids and feed water rates. These inputs are specified on a time base and the model output is then sequentially calculated over short time intervals (every 6 seconds). The model outputs can then be plotted against time so that they can be compared with the measured data.

5.3. Parameter Identification

Using parameters representing the influence of all aspects of the model, the predicted mass was regressed against the measured mass to identify the system from the measured data. The mill mass is chosen as the key variable since the other variables such as circulating load and mill discharge rate determine
These measurements were used to construct the fractional and corrected fractional recovery to underflow curves and hence to evaluate the average $d_{50c}$ and sharpness of classification parameters. The basic forms of the PIlt model equations (1976) are fitted to the data and are used in the model to describe the variation in the above parameters with respect to cyclone feed rate and density.

The model equations used are:

$$d_{50c} = 20.25\exp \left( \frac{63}{v} \right) / C^{0.5}$$

$$\text{Split} = 7.19 \times 10^{-5} \exp \left( \frac{33.4}{v} \right) / C^2$$

$$m = \sqrt[3]{85/(\phi v + 0.05)}$$

for $C$ in m$^3$/s;

$\phi_v$ is the volume fraction solids while $m$ is the Rosin-Rammler parameter for sharpness of classification.

Figure 30 in Appendix F shows that the corrected classification curves are effectively represented by the equation:

$$y_c = 1 - \exp \left( 0.693 \left( \frac{d}{d_{50c}} \right)^m \right)$$

5.2.8. The Milling Circuit

All unit models are combined, with the product from one unit serving as the feed for the next. The inputs to the model are the measured feed solids and feed water rates. These inputs are specified on a time base and the model output is then sequentially calculated over short time intervals (every 6 seconds). The model outputs can then be plotted against time so that they can be compared with the measured data.

5.3. Parameter Identification

Using parameters representing the influence of all aspects of the model, the predicted mass was regressed against the measured mass to identify the system from the measured data. The mill mass is chosen as the key variable since the other variables such as circulating load and mill discharge rate determine
the mass. Thus the representativeness of the model mass is indicative of the reliability of the other model elements.

Initially it was necessary to regress by eye to find the correct order of magnitude for the parameters and to detect to which parameters the mass was most sensitive. This approach clearly showed how interactive and unstable the system is and clearly indicated the need for multivariable regression. The flexibility of the model was also exploited at this stage with attempts to vary the model structure to see if it could be improved:

- the mill was modelled by two tanks in series of the same and then different sizes
- the number of size classes was decreased to two (± grate size) and increased to four with an extra intermediate class.

The two tanks model incorporated a resistance between the tanks causing the slurry level in the first tank to be higher than that in the second. This simulates the slurry gradient that must exist in the mill for mass transport. This investigation was hampered by the increased model complexity making parameter fitting more difficult, but this configuration integrated the response to changes in the inputs excessively. This resulted in a much smoother output than that measured in practice (Appendix C) and showed that the single tank described the output better.

Two size classes provided a very poor model response and no noticeable improvement over three classes was gained with four, at the expense of an extra parameter. Thus both the above model variations were not adopted.

Originally it was proposed that there was a lag in series with and owing to the mixed tank (the mill), since it was assumed that hold-up in the mill would result in the circulating load responses to an input change lagging behind those in the mill (e.g. load slurry density). This lag was found to be minimal, however, but the responses of load mass and power draft to the inputs were noticeably delayed resulting in a pure time lag being required prior to the mixed tank. The solid response lag is owing to the distance of the feed weighometer from the mill - a change in belt loading is detected 1.5v minutes prior to reaching the mill (i.e. the distance from the mill and vs the belt velocity at the time). The water must pass through a plug flow region prior to entering the mixed zone of the pool.
The parameters selected for optimisation were:

- The feed size distribution (2 parameters)
- The breakage and initial selection function values (3 parameters)
- The magnitude of the resistance of the discharge grate and the dependence of the discharge on slurry viscosity (2 parameters)
- The dependence of the cyclone flow split model on % solids in the feed (1 parameter)

An advanced quasi-Newton method was then used to minimise the sum of squared errors between the calculated and measured masses. Accurate fitting could not be expected because there are too many unknown elements but good representation of the major trends is desired at this stage. Examples of the model response are shown in the following chapter.
6. Analysis and Discussion of the Dynamic Model Response

The model parameters were regressed using one set of data and it was pleasing to see that the "best fit" values could then be used with all the sets of measured inputs to provide reasonably representative predictions of the response. In Section 6.3 (Sensitivity Analysis) it is shown how the parameter fitting should really be dynamic since in practice conditions change all the time. The results in Figures 11 - 19 show good trend prediction, however, with correct slopes and responses occurring at the right times.

The model inputs (feed water and solid rates) do not correspond with the data precisely, because the moisture associated with the solids (2% by mass) is deducted and added to the feed water in the model.

6.1. Comparison of the Measured Data with the Model Response

Run No.1 - Figure 11

This data set is described in Figure 5 on page 17. The model responses are superimposed on the data plots for comparison.

Reasonable trend prediction is apparent with one clear exception: the model load mass responds poorly to the low feed water rate from 70-90 minutes. The predicted solids discharge rate and cyclone feed also do not illustrate the rapid changes found in practice. Varying the discharge rate dependence on slurry viscosity and other parameters showed no substantial improvement in these basic fault areas.

Having highlighted a serious shortcoming of the model, possible faults in the model structure were identified. It was found that the predicted slurry hold up in the load changed very little despite substantial changes in the feed water rate. Further, the data indicates that a "plug" of slurry was discharged before the inventory of slurry became depleted and the true response to a feed water increase was apparent (e.g. change (c) at 90 mins).
Figure 11: Run No.1 - Plant Operating Data and Dynamic Model Response
Figure 12: Run No.1 - Plant Operating Data and Dynamic Model Response
Incorporating the Model for the Slurry Foci in the Load
Hogg & Rogovin (1982) postulated that the load consists of two distinct regions. Part of the slurry in the load is associated with the media while a liquid pool contains the remainder of the slurry. The pool serves as the means for mass transport from the feed to the discharge end of the mill. The layer of slurry adjacent to the mill shell is drawn into the load by the rotating motion displacing the slurry filling the interstices. This results in intermixing between the regions. These dynamics are easily envisaged in trunnion overflow mills; the model deficiencies discussed above suggest similar behaviour in the grate discharge mill under observation.

Moys (1987) proved that all available slurry would be used to fill the voids in the media before a pool formed at the toe of the load. The slurry fraction in the load model could thus be split between the media voids and the pool. A pool is thus formed when excess slurry is present e.g. immediately subsequent to a step increase in the feed water rate. The model describing the discharge rate could then cater for a much higher discharge rate during periods when the load slurry volume increased to form a pool. The hole area of the grate available to the pool is greater than that for a proportional amount of slurry in the load, since the latter only occupies the voids. When a pool is formed the high discharge rate (proportional to the level in the pool) rapidly depletes this extra slurry and normal discharge resumes.

Inclusion of these ideas in the model results in the much improved predicted responses shown in Figure 12. Clearly this representation of the dynamic behaviour of the slurry in the load needs refinement but it provides very sound qualitative support for the theories proposed above.

Run No. 2 - Figure 13

This record shows the response of the load mass and power draft to changes in feed rate at a fairly constant dilution water flowrate. The peak seeking controller is disabled and it is clear that the mill is overloaded during periods of feeding and recovering towards the peak while the feed is choked.

The point of real interest is the period after (c) where the mill recovers to a peak power condition which passes to an overload once feeding resumes until
Figure 13: Run No.2 - Plant Operating Data and Dynamic Model Response
(d) where the mill is suddenly underloaded. None of the variables that were monitored or estimated explain this phenomenon illustrating the multiplicity of disturbances that occur in the milling circuit. Possible causes are a step change in feed size distribution, a change in ore consistency and/or moisture content to list but a few.

This event is also noteworthy because it occurred during the normal course of operation showing that sudden shifts in the state of the grinding environment are not only the result of manually input step changes to the inputs.

This situation is clearly relatively simple to model with a fairly constant discharge rate being maintained. The initial rise in discharge rate is not simulated by the model at all and is probably the response to a change immediately prior to the record.

Run No. 3 - Figure 14

This is a very interesting data set with clear responses in all measured variables to input changes and illustrations of the power - mass - solids feed - water feed peak relationships and interactions.

During this data logging period no controllers were operational with the exception of one to maintain a constant solids feed rate. The sump level controller could not function since the slurry feed arrangement had been altered and interfered with the level measurement. Thus a constant valve position was selected. Operation under these conditions is only possible because of the flat nature of the discharge pump curve causing a large reduction in flow for a small increase in the system head allowing the level to be maintained in the sump despite variations in discharge rate and density. The changes in the sump dilution water rate resulted from other flow demands on the supply header and not control action.

The solids feed to the mill was extremely dry and coarse resulting in many chokes and the erratic behaviour shown. During a feed choke the controller acts to increase the conveyor belt speed to a maximum to maintain the feed rate. This results in massive overshoot of the set point (100 t/hr in this case) once the choke has been cleared and a subsequent negative deviation, often resulting in a further choke, to counteract the initial overshoot.
Figure 14: Run No.3 - Plant Operating Data and Dynamic Model Response
The feed water rate was manually manipulated with step changes being introduced at (a), (c), (e) and (f).

After an initial stabilisation period the feed water was cut off at (a). An immediate increase in power draft and discharge percent solids (estimated from the mass balance) occurred. This change in water rate moved the grinding environment to a completely different operating regime where the mill is overloaded and the initial response indicates recovery from that overload.

The mill discharge rate falls sharply with the increasing slurry viscosity and a peak in the solids feed rate causes a sharp increase in mass.

This aggravates the overload condition and the power draft falls. It is likely that premature centrifuging would have occurred during this period, i.e. a layer of slurry was sufficiently viscous to adhere to the shell causing an effective reduction in mill diameter.

The substantial reduction in cyclone feed pressure during this period results in reduced classification efficiency which, coupled with a low feed solids concentration to produce a very dilute underflow. This compensates for the lack of feed dilution water and one and a half mean residence times (~15 mins) after (a) the discharge rate begins to recover. The mill feed hopper choke at (c) allows the circulating load to stabilise and the power rises as the mass falls to the point where the peak power condition (with respect to mass) is passed and the mill becomes underloaded.

A low feed water rate is provided subsequent to (c) to avoid a recurrence of the drastic responses with no feed water.

At (d) it is interesting to note the magnitude of the drop in power draft during this period of low feeding. This response provides an indication of the steepness of the power vs. mass curve for a small deviation away from the peak.

The step increase in feed dilution water at (e) produces an increase in the solid discharge rate and a reduced discharge density. The effect of these changes on the circulating load is apparent although no marked response of
load mass or power draft is observed. Any changes resulting from the water flow rate decrease at (f) are masked by other disturbances around this time.

The model represents the trends in load mass with complete reliability although the magnitudes of the responses to some changes are quite different. The major difference results from the model not simulating the rate of cyclone underflow restoring the mill discharge and resulting in the rapid mass decrease at (c). This is predictable since the cyclone model is based on a range of variations in conditions around a "normal" operating point and would not cope with these unusual parameters. It is of interest that the model mass response to the low feed rate prior to (d) is substantially more marked than the measured response. The model response seems to be more appropriate when the magnitude of the response of the power draft is taken into consideration and it is remembered that the measured mass is merely a relative indication.

Run No. 4 - Figure 15

The power and mass rise together initially to approach the peak operating condition. A sharp increase in feed rate causes a small improvement in power draft before the mill becomes overloaded at (a). This condition persists until (b) when a recovery phase is entered. The peak condition is achieved at very much lower mass and power levels than the previous peak (a). The mill is slightly overloaded as the mass rises from (c) with a recovery during the period of low feeding prior to (d). No noticeable response to the small changes in dilution water rate is observable.

The model mass response is the most interesting aspect of this data set with a surprising discrepancy between the model and data for most of the records although trends in both responses are similar.

It is important to remember that the mass measurement is not absolute but only an indication of the variation in oil back-pressure on the mill's slipper pads. In this instance it appears that the model could be more representative of the absolute load mass. This may be concluded from:

- The difference in mass levels at peak power at (a) and subsequently at (c).
Figure 15: Run No. 4 - Plant Operating Data and Dynamic Model Response
Figure 15: Run No.4 - Plant Operating Data and Dynamic Model Response
In all other instances where the model mass differs substantially from
the data this can be ascribed to a change in feed size distribution or
selection function (see sensitivity analysis in section 6.3) which results in
a change in discharge rate which is not the case here.

The model response for the solids discharge rate and hence the cyclone feed
density is more dependent on the feed variations than the measured variables.
These model responses suggest that the model feed size distribution is finer
than that experienced in practice. This reinforces the proposal that the model
mass is more representative than the pressure indication in this case since a
coarse feed is less likely to be milled so rapidly. This data also strongly moti­
rates more accurate measurement of load mass and a measurement of the
feed particle size distribution.

Run No. 5 - Figure 16

The mill feed rate is very erratic initially with many chokes and subsequent
surges. The mill overloads with the power draft falling to (a) where a slight
recovery results from the drop in feed. With more stable feed conditions after
(a), water changes can be introduced.

The recovery in power at (b) can be ascribed to the feed choke with the sub­
sequent steady decline in power draft being the result of the lower water rate
increasing hold-up in the load. This trend is broken by the water being in­
creased again at (c). This increase is accompanied by a fall in mass as the plug
of slurry is discharged.

The discharge solids rate is estimated from the mass balance around the sump
resulting in unlikely values at times e.g. (a) and (d). Neither of these changes
are simulated by the model and it is likely that the sump level was not main­
tained at these points.

Run No. 6 - Figure 17

The sump dilution water flowrate is very low during this record. This results
in a high cyclone feed density and consequently a high cyclone d50 size and a
low circulating load. The solids and water feed rates are at their normal le­
vels, but the water rate is clearly too high under the circumstances as is evi­
denced by the low power draft. With little circulating slurry, the feed dilution
water is merely flushing the solids from the mill and not producing a load

Dynamic Model Response
Figure 16: Run No. 3 - Plant Operating Data and Dynamic Model Response
Figure 17: Run No.6 - Plant Operating Data and Dynamic Model Response
slurry rheology conducive to efficient grinding. A step decrease in the feed dilution water flowrate at (a) results in an immediate increase in power as the mass builds up with increasing load slurry viscosity. The slight decrease in discharge rate allows a reduction of cyclone feed density and a probable increase in circulating load. The power continues to rise and conditions are unaffected by an increase in feed dilution water at (b) although this influences the circulating load parameters. Feed chokes cause both the power and mass to drop showing the mill to be still underloaded. The response of the circulating load variables to the load changes at (c) and (d) are very clear and are well simulated by the model as well.

A very good model response is observed throughout although some trends in the load mass and solids discharge rate are exaggerated, such as the mass loss resulting from the combination of a high water rate and the feed chokes after (b). This response is reflected in the pulse of solids discharged at (b).

Run A - Figure 18

This data set is fully described under section 4.1.2 regarding mill discharge density estimation. It is presented here to illustrate the model response to data from a different installation and to observe how the model temperature estimates compared with the measurements.

Very simple temperature models were formulated with energy inputs described by the measured feed solids and water temperatures and the power drawn by the mill being considered to be transferred to the load purely to increase the temperature.

Imperfect mixing in the mill is ignored and the discharge temperature is assumed to be the same as that in the load. Energy accumulation in the sump as well as energy input by the pump are ignored since these terms are very much less than the others in the energy balance.

It is surprising that the model temperatures are lower than those measured since energy losses have been largely ignored. This can be ascribed to a low initialisation estimate of cyclone underflow temperature resulting in a cooler circulating load throughout the record.
Figure 18: Run A - No. 3 Plant Operating Data and Dynamic Model Response
Showing Temperature Measurements
The model mimics the temperature trends accurately and simulates the other circuit variables reliably. No adjustments to parameter values were made to allow modelling of the data from the later testwork.

It is difficult to glean any useful information regarding the causes of the variations in temperature measurements by direct comparison with the circuit inputs or the mill power draft. It appears that at this stage the temperature measurements are most useful for density and flowrate estimation and consequent control (as opposed to direct control of a temperature), since these variables are better understood in terms of single input relationships.

The circulating fraction in this circuit is substantially higher than that of the Elandsrand mill (100% as compared with approximately 60%) since there is no regrind mill, and the model consequently predicts a low solids discharge rate and cyclone feed density. The higher circulating load allows a more rapid recovery from the choked feed condition (b) where the model is slow to respond. As the cyclone feed density decreases the dso value decreases as does the O/P: U/F split. With the higher circulating load more solids are available to be recovered to the underflow restoring the slurry inventory in the la.

The model only anticipates a rise in discharge rate once the choke has been relieved.

Run B - Figure 19

This data set is also described in section 4.1.2 and the figure illustrates the simulation of the measured temperature by the model. The model trends are reasonable with a more plausible mass indication than that of the oil pressure signals.

The same flaws as were apparent in the data set of "Run A" are present in these model responses, particularly at (a) where the change in cyclone performance is not captured by the model.

6.2. Shortcomings of the Model

The major flaw in the model results from the lack of a good description of power, or rather of the independent variables determining power, e.g. volume, load density and angle of repose. This resulted in the power being reli-

Dynamic Model Responses
Figure 18: Run B - No 3 Plant Operating Data and Dynamic Model Response

Showing Temperature Measurements

Dynamic Model Response
tively poorly modelled with a few occurrences of trends being the reverse of what they were in reality. This resulted in measured power draft values being used to replace the power model. This was suitable in the present application, where on-line power measurements were available, but precludes the use of the milling circuit model as a stand-alone simulator.

Accurate prediction of the mill power draft and the measurements that will make this possible provide the key to comprehensive understanding of the grinding environment. The investigations by Moys and co-workers (1985 and 1988) using conductivity bolts in the mill shell show that the position of the load in the mill, the load slurry viscosity and the onset of centrifuging can be measured. With this additional information being fed from a continuous operation to an expert system it would be possible to build up sufficient experience to anticipate the influence a disturbance would have on the power draft and work to correct it so as to maintain near optimal operation.

The cyclone model could also be improved. The bulk of previous experimentation on cyclones covers units smaller than 350mm in diameter and although the form of Plitt’s model (1976) is correct the parameters must be adjusted for application to large cyclones (915 mm) requiring size distribution data and another flow measurement.

Obviously having no on-line size distribution measurements is a deficiency of the data, but it is promising to see the reasonably reliable response obtained from the model despite this. Measuring equipment for particle size is costly and maintenance intensive, and even then is limited to an indication of the fraction of particles less than only one chosen size. On-line particle size distribution measurement is the subject of ongoing research. Success in this area will allow identification of the major disturbances influencing the grinding circuit as well as ensuring that the size reduction objective is achieved without loss of unliberated metal or energy wastage through overgrinding.

6.5. Sensitivity of the Model Response to Parameter Variations

For all the data sets considered, the measured load mass appears to have been most effectively modelled, i.e. data trends are reflected well, and when regressing by eye, it was found that the model mass stabilised as a consequence of
of all other elements being reasonably represented. This is to be expected since in closed circuit, any aspect of the model that is abnormal will result in a rapid filling or emptying of the mill.

For these reasons the mass has been isolated to allow parameter sensitivities to be studied. The vertical scale in figures 20 & 21 has been exaggerated for clarity. A few key parameters have been selected to illustrate the influence that they have on the load mass model response. From the cases studied it becomes clear that no single combination of parameters is ideal since the data may be well represented while one set of conditions prevail but other parameters will suit new conditions. The response labelled "original parameters" represents those parameter values that were regressed and provide a reasonable average representation of all the data under consideration. This response is shown for comparison with the alternate cases. In all cases only the parameter in question was changed while all others retained their original values.

From Run No. 1 - Figure 20

1. The resistance to flow through the mill discharge grate depends on the viscosity of the slurry. In this instance the dependence of the discharge flow rate on viscosity is increased. The slurry hold up in the load is generally greater than that obtained with the original parameters. While the feed water rate is low, however, (b-c) the higher slurry viscosity causes the rate at which the mass builds up to increase, more like the measured data, although the subsequent mass loss after the step up in the water rate is too rapid.

2. The discharge rate constant is the proportionally constant relating the slurry discharge rate to the volume of slurry in the pool. Reducing this constant causes the model to approach the case with no separate pool. This is clear from this graph with broader peaks reflecting the averaged response to the changes in the inputs.

3. The selection functions determine the rate of breakage out of each size class. The model represents these as base parameters which increase or decrease with the mill power draft as it varies about the mean. Here the base parameter value for size class 2 has been increased. This results in more rapid production of fines which decreases the load mass progressively. The first
SENSITIVITY OF LOAD MASS TO VARIOUS PARAMETERS

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Figure 22: Sensitivity Analysis on the Model Parameters - Load Mass Data
peak (a) where the mass decreases rapidly after the feed pulse is best captured by this parameter change.

4. The feed size distribution is specified and remains constant for all data sets. The feed fraction in the coarse size range increased here with a corresponding reduction in fines. This causes the load mass to become progressively higher but it decreases sharply when the final step increase in the water rate is sufficient to result in a pool which depletes the fine fraction present.

From Run No. 3 - Figure 21

This example shows very clearly how different combinations of parameters are appropriate for various conditions.

Again the measured data and the model response using the "best fit" original parameters are shown.

1. The increased dependence of the discharge rate on slurry viscosity exaggerates the model mass response to input changes.

The initial mass increase (a) resulting from the sharp decrease in feed water is well captured by this parameter set but no improvement is shown over the original parameters during the subsequent mass decrease. The model mass remains parallel to the original response while the water remains constant but decreases more rapidly after the feed water is increased again.

2. Increasing the grinding rate or class 2 selection function shows a substantially improved representation of the large mass decrease (b) showing that the actual grinding rate was very high during this period and that the more constant rate proposed for the model is too simple. The high grinding rate is no longer appropriate after the water increase (c) where the higher concentration of sub grate size material is flushed out too rapidly.

Analysis of the effects of these various parameter changes shows how the magnitude and rate of the model response can be altered to better represent certain transient behaviour. The dynamic and interactive nature of the mill circuit variables is clearly illustrated, but it is reassuring to note that the reliability of the model for trend prediction is apparent in all cases discussed.
SENSITIVITY OF LOAD MASS TO VARIOUS PARAMETERS

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**Figure 21:** Sensitivity Analysis on the Model Parameters - Load Mass Data

From Run No 5
If the model were more accurate and the estimated power draft more reliable, the model could actually be used to predict the controller actions to be taken. An on-line parameter identification stage would precede an investigation of many possible control actions to see which would provide maximum power draft in the short term – it is unreasonable to assume that the input size distribution will remain constant for very long – and then implement adaptive control utilising the best control strategy for the current situation. This concept is idealistic at this stage but does serve as a goal to motivate further, more accurate modelling and improved understanding of the system.
7. Conclusions and Recommendations

A large set of dynamic data describing various states of operation of an industrial autogenous grinding circuit were acquired. Field observations and qualitative knowledge gleaned from the data were combined with physical models from the literature to derive a simple model of the grinding circuit to help understand the responses resulting from changes to the inputs. This model provided insight into several features of the milling circuit:

- the strong interaction between variables
- the complexity of the model required to accurately describe power draft
- the variation in key parameters during operation

Parameters were successfully identified from the plant data so that reasonably accurate representation of operating trends was obtained.

Many of the responses observed have been qualitatively explained although some remain enigmatic as a consequence of having limited knowledge of the circuit variables. The chief success of the model resulted from incorporation of the description of the excess load slurry being in a pool at the toe of the load with more rapid flow-through characteristics. This behaviour is apparent in most of the data sets and an indicator of the onset of pool formation, i.e., an increased discharge rate:throughput ratio, would be valuable. This condition obviously reduces grinding efficiency by increasing the circulating load while absorbing little extra energy. The power draft is in fact reduced by the presence of a pool as it results in a centre of gravity shift towards the axis of the mill.

It is unfortunate that the perturbations required to observe distinct responses were often large. This could mean that the normal operation was disturbed so that the observations are not representative of normal operation. Instances of large uncontrolled disturbances (Run No. 2) do occur in the normal course of events, however, suggesting that all states should be considered when trying to understand the circuit dynamics.

The sensitivity analysis on the model parameters proved very valuable, illustrating the dependence of the behaviour on these parameters and indicating
the variable nature of the circuit dynamics and the need for adaptive control or on-line parameter identification.

It is interesting that a single CSTR provided the best mill unit model. A mill with a high L/D ratio attempts to achieve a closer approach to plug flow to obtain maximum conversion in a single pass. The model shows that this is not being achieved in practice and this is borne out by the very high power utilisation (~40 kW-hr/t-75μm material produced) in comparison with the industry standard for this mill type (30.5 kW-hr/t-75μm). An optimising control strategy is clearly required to improve this situation.

The possible use of discharge slurry properties such as density, viscosity and temperature, as control outputs indicating conditions in the load itself, was investigated. It was concluded that estimation of density from other measured variables, rather than direct measurement, would provide a sufficiently accurate trend for use as a control variable. Temperature measurements around the mill circuit were only investigated as a means of estimating discharge density, but they could prove to be useful measurements in their own right and further work on this aspect is recommended. The density estimate using temperature measurements proved to be very reliable, representing trends accurately, but could not easily be applied as a control output. Determination of the optimum density is difficult since it varies with conditions in the load. Optimising control of the discharge density will help to maximise the mill power draft but ideally a two phase control strategy such as that implemented by Pauw et al. (1984) (see section 1.2) is required. A more intelligent controller would be required in this case since the water and solids feed rates must be optimised simultaneously. More knowledge of conditions in the load is required to allow accurate prediction of the effect an input variable change will have on power so that the controller can anticipate the required action.

Sampling difficulties prevented on-line viscosity measurements being made, but the clarity and reproducibility of responses to changes in slurry percent solids show that viscosity is a good indicator of slurry conditions and that the Debex instrument is capable of reflecting these. A technique for sampling of the mill discharge must be sought and dynamic viscosity data gathered since this appears to be a most informative output variable.
The work on discharge density as well as other observations highlighted the need for control of the slurry rheology within the load. Efforts to improve grinding efficiency should concentrate on this area.

Tighter control and a better understanding of the mill circuit may be achieved by application of new technology. Conductivity probes mounted in the mill shell provide an accurate average value of the load angle of repose in a laboratory mill (Mays, 1985). Implementation of this idea on an industrial mill could be very informative. Measurement of the size distribution of the mill feed would allow prediction of the load contents and would help to provide a basis for steel and water addition rates.

Finally, experience in industry has stressed the need to ensure that building blocks of the control strategy are not also the stumbling blocks. Field equipment - actuators and measuring instruments - must be suited to the application, rather than altering the control strategy to cope with poor devices. Good plant operator training and awareness is also very important, although more sophisticated control should allow for fewer, more skilled operators.
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Appendix A - Improvement of the Solids Feed Rate Control

The mill solids feed rate was completely random during the initial testwork making nonsense of attempts to analyse step tests.

The causes of this problem were several:

- The feed chute opening was less than the larger rock sizes resulting in multiple chokes.
- The variable speed drive has different characteristics on accelerating and decelerating and was a slow final control element.
- Output to the final control element was determined by the measured feed rate which is the product of the solids density on the belt and the belt speed.

Nothing could be done in the first two cases but in the third an extra feedback loop to control belt speed could be cascaded to the feed rate control loop. This modification eased control loop tuning and reduced the number of random disturbances that the controller had to contend with to only the solids density on the belt.

Plant operating records show the improved performance (figure 22). Note that the initial period in the latter set does represent an exceptionally constant feed as it is unusual to have a period of more than half an hour without a feed stoppage or choke.

![Figure 22: Record of Feed Weightometer Output](image-url)
Appendix B - Orifice Plate Flowmeter Calibration

The pressure drop across the orifice plate is measured and may be related to flow by the equation:

\[ Q = C_D A_0 \left( \frac{2/5 \Delta P}{1 - (A_0/A_p)^2} \right) \]

For an incompressible fluid:

\( C_D = \) Coefficient of discharge
\( A_0 = \) Area of orifice
\( A_p = \) Area of pipe
\( \Delta P = \) Pressure drop across the orifice
\( Q = \) Volumetric flowrate
\( d_p = 200\text{mm} \)
\( d_o = 150\text{mm} \)
\( Q_{ave} = 0.05\text{m}^3/\text{s} \)

\( C_D \) is correlated with Reynolds number for British standard orifice plates.

\[ Re = d_p \nu / \mu = 4.2 \times 10^5 \]

Thus \( C_D = 0.61 \) and is constant for \( Re > 10^5 \)

The variations in flowrate are quite substantial however, and this equation provided unacceptable fluctuations in the flow measurement. This was ascribed to the location of the orifice being non-ideal and to the presence of settled solids in the pipe altering the flow characteristics. Thus, it was necessary to calibrate the meter on site. The magnetic induction flow meter in the cyclone feed line was used for this purpose. The sump water was pumped through the magnetic flow meter and the flowrate integrated over a period of time - sufficient that sump volume fluctuations could be ignored (see figure 23).
Three sets of measurements were averaged to give the following sets of results:

Figure 23: Calibration Circuit

<table>
<thead>
<tr>
<th>Water flow rate m³/s</th>
<th>Discharge coefficient Cₚ</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.052</td>
<td>0.364</td>
</tr>
<tr>
<td>0.069</td>
<td>0.368</td>
</tr>
<tr>
<td>0.070</td>
<td>0.373</td>
</tr>
<tr>
<td>0.082</td>
<td>0.383</td>
</tr>
<tr>
<td>0.088</td>
<td>0.398</td>
</tr>
</tbody>
</table>

Figure 24: Orifice Plate Calibration Curve
A curve was fitted to this data (figure 24). Calibration at low flow rates was not possible using this approach since the pump required a minimum flow-rate. The measurement error increases substantially outside the above range of values.
Appendix C - Responses to Alternate Model Configurations

This is the same experimental data from Run No. 1 (shown in figures 5, 11 & 12). The model used in this case has the same circuit with the exception that two perfectly mixed tanks in series are used to describe the mill. There is a hypothetical resistance between the tanks, similar to the discharge grate, to simulate the gradient that must exist for flow through the load.

The complexity of the model is greatly increased with two extra discharge rate parameters and a parameter for the relative sizes of the two tanks. This made parameter estimation more difficult, but the response shown indicates that the order of the model is too high, integrating the changes over a long period, and filtering the response. This is to be expected, since the hold up introduced by the discharge grate increases the model order, making this a fourth order model, whereas qualitative observation indicates that the responses are second order.
Figure 25: Plant Operating Data Showing the Two Tanks Model Response
Appendix D - Error Analysis of Discharge Density Estimation Techniques

Error Estimation

When a value is calculated from data values, each with their characteristic error, it is important to have an estimate of the error propagated in the calculated value.

For the true values

\[ A = f(a_1, a_2, a_3, \ldots, a_n) \]

\[ A + \Delta A = f(a_1 + \Delta a_1, a_2 + \Delta a_2, a_3 + \Delta a_3, \ldots, a_n + \Delta a_n) \]

A Taylor series expansion of \( f \) gives:

\[ A + \Delta A = f(a) + \frac{\partial f}{\partial a_i} \Delta a_i + \ldots \]

Ignoring higher order terms, on the assumption that the errors are small, gives

\[ \Delta A = \frac{\partial f}{\partial a_i} \Delta a_i + \ldots \]

Error in Discharge Density Calculated from Flow Measurements

For the case of discharge density estimation (or % solids in the discharge)

\[ \phi_0 = \frac{C_{kC}}{C \cdot SW - W_g} \text{ at steady state} \]

\[ = \Delta \phi_0 = \frac{\phi_0(C \cdot SW - W_g) - C_{kC}}{(C \cdot SW - W_g)^2} \Delta C + \frac{C}{C \cdot SW - W_g} \Delta \phi_0 \]

\[ = \frac{C_{kC}}{(C \cdot SW - W_g)^2} \Delta SW - \frac{C_{kC}}{(C \cdot SW - W_g)^3} \Delta W_g \]
The average values and maximum errors in the measurements are:

\[ C = 1 \pm 2\% = 0.1 \pm 0.002 \text{m}^3/\text{s} \] Magnetic induction flowmeter specification.

\[ \Phi_{\text{Hyd}} = 0.20 \pm 5\% = 0.20 \pm 0.01 \] The specification suggests 1% accuracy but variation in the concentration of solids in the vertical pipeline and the non-uniformity of the solids size distribution are expected to produce a higher error.

\[ W_g = 0.002 \pm 20\% = 0.002 \pm 0.0004 \text{m}^3/\text{s} \] This error may be very large since the average value is the result of several spot measurements and the variance is unknown.

\[ SW = 0.05 \pm 15\% = 0.05 \pm 0.0006 \text{m}^3/\text{s} \] This error is also unknown but is assumed to be large at times.

For the maximum error deviation in the % solids to the cyclone is taken as negative:

\[ \Delta \Phi_{\text{Hyd}} = -0.017 - 0.026 - 0.125 - 0.006 = -0.174 \]

The average value of \( \Delta \Phi_{\text{Hyd}} = 0.53 \)

\[ \rightarrow \] The maximum error is -33%.

If a magnetic induction flowmeter were used to measure the sump water rate the error would be reduced to 12% which would be tolerable considering that only a trend is required for control purposes, so that constant errors can be ignored and that maximum error conditions are rare during normal operation.

Discharge Density Estimation from Temperature Measurements.

An energy balance over the sump gives:

\[ H_{\text{in}} + W_{\text{pump}} = H_{\text{out}} + W_{\text{pipe}} \]

\[ H_{\text{in}} = \Phi_{\text{Hyd}} \int_{T_b}^{T_d} \left( C_{\text{pw}} \frac{dT}{\Delta T} + (1 - \Phi_{\text{Hyd}}) D \right) \]

\[ + W_g \int_{T_b}^{T_d} \left( C_{\text{pw}} \frac{dT}{\Delta T} + SW \right) \]

\[ + \int_{T_b}^{T_d} \frac{T_d}{T_b} \]

Appendices
\[ H_{\text{out}} = \phi_{\text{C}} C \left[ \frac{T_{\text{C}}}{C_{\text{pw}}} \frac{dT}{T_{\text{b}}} + (1 - \phi_{\text{C}}) C \right] \]

\[ C_p \text{ is assumed constant over the temperature range. } T_{\text{b}} \text{ is the base temperature.} \]

The mass balance equations are:

\[ \phi_{\text{D}} D = \phi_{\text{C}} C \]

and 

\[ (1 - \phi_{\text{D}}) D + SW + W_g = (1 - \phi_{\text{C}}) C \]

Thus the discharge water rate may be solved for

\[ D(1 - \phi_{\text{D}}) = (w_{\text{c,pp}} - q_{\text{pipe}} + C_{\text{ps}} \phi_{\text{C}} (T_{\text{D}} - T_{\text{C}})) + C_{\text{pw}} (1 - \phi_{\text{C}}) C (T_{\text{W}} - T_{\text{C}}) / (T_{\text{D}} - T_{\text{C}}) \]

\[ \Delta (D(1 - \phi_{\text{D}})) = \Delta w_{\text{c}} - \Delta q + (C_{\text{pw}} (1 - \phi_{\text{C}}) (T_{\text{C}} - T_{\text{W}}) - C_{\text{ps}} \phi_{\text{C}} (T_{\text{D}} - T_{\text{C}})) \Delta C \]

Assuming errors in the temperature measurements are negligible, average values and errors are:

\[ w_{\text{c}} = 200 \pm 0.50 \text{ kW} \]
\[ q = 2 \pm 0.2 \text{kW} \]
\[ C = 0.130 \pm 0.002 \text{kW/s} \]
\[ \phi_{\text{C}} = 0.4 \pm 0.02 \]
\[ C_{\text{ps}} = 1.5 \text{MJ/kg} \]
\[ C_{\text{pw}} = 4.2 \text{MJ/kg} \]
\[ T_{\text{D}} = 45^\circ \text{C} \]
\[ T_{\text{W}} = 30^\circ \text{C} \]
\[ T_{\text{C}} = 36^\circ \text{C} \]

\[ \Delta (D(1 - \phi_{\text{D}})) = (0.050 + 0.002 + 0.0253 + 0.030)/63 \]

\[ = 0.0017 \]
\[ D(1 - \phi_D) = 0.0166/s \Rightarrow \text{Error} = 10\% \]
\[ \Delta(D\phi_D) = \phi_C\Delta C + \phi_C \Delta C = 0.00364 \]
\[ D\phi_D = 0.050/s \Rightarrow \text{Error} = 7\% \]
\[ \Delta\phi_D = 0.041 \]
\[ \phi_D = 0.76 \Rightarrow \text{Error} = 5.4\% \]

This is thus the best estimation technique although \( T_{W_H} \) was assumed equal to \( T_w \) since \( T_{W_H} \) is actually unknown. The gland service flows are low and already has a high margin of error ascribed to it, so this assumption should have little influence on the overall error. The error may be decreased by monitoring pump power draft and ambient temperature for more accurate estimates of \( q \) and \( w_r \).
Appendix E - Viscometer data

In order to characterise the slurry rheology and investigate the sensitivity of the Debex instrument to changes in % solids and temperature a mill discharge slurry sample was allowed to circulate in the instrument.

Density samples were taken from the circulating slurry and the slurry temperature was measured. The bobbin speed was then varied over the given range to change the shear rate. The shear stress could then be monitored from the electronic converter which changed the measured motor current to a torque.

Water (~5%) was then added to the sample to reduce the density, and when well mixed, further density and temperature measurements were made. The shear rate/stress characteristic was then investigated for the new density.

Several sets of readings were taken for each sample, and very consistent results were obtained.

Density measurements were made by first weighing the wet samples and later, the dried solids.

The results (figure 26) show an inexplicable increase in shear stress at low shear rates. This occurrence has been found with other slurries. Debex are aware of this symptom and are trying to explain it, but results have not yet been forthcoming.

This unusual behaviour does not affect the relative viscosity readings at a fixed bobbin speed however, and the high sensitivity of the measurement to changes in % solids recommends the use of this instrument for on-line results.
Characterisation of the Slurry Rheology

Figure 28: Rheology Characterisation
Appendix F - Model Development

Power model

The approaches to calculating power draft have been either i) to calculate the paths of individual media particles and integrate to find the energy required to raise the media over all possible paths (Hogg and Fuerstenau, 1972), or ii) to assume that the axial moment of the driving force must balance the axial moment of the load centre of gravity (Austin et al., 1984). Both result in a similar equation for power, and assume low speeds of rotation; i.e. a planar load surface with no catareching.

![Coordinate System Used in the Development of the Model for Power Consumption](image)

Figure 27: Coordinate System Used in the Development of the Model for Power Consumption

i) Referring to figure 27 consider a radial element at $r$ of thickness $dr$. The energy required to raise these particles through the bed is

$$dJ = ag \, dm$$

where $a = \text{vertical height through which the particles are raised}$

$$= 2(r^2-H)^{0.5} \sin \theta$$

$dm =$ \text{mass of particles in the element}$

$$= 2\pi a \, u \, r \, dr$$
But the total energy consumed by all the particles during one revolution is

\[ J = \int u_a \, df \]

or substituting for \( df \)

\[ J = 4\pi \rho p L \sin \theta \int_{0}^{R} \left( r^{2} - H_{L}^{2} \right) \, dr \]

\[ = 4/3 \pi \rho p L \sin \theta \left( R^{3} - H_{L}^{2} \theta \right) \]

Then the power consumed, \( E = J \omega /2\pi \)

where \( \omega = (NN_{0})/R^{0.5} \), the fraction of the critical speed

\[ E = \frac{R^{2}N_{0} \left( 1 - H_{L}^{2} \right) \theta^{3}}{4 \rho \pi L} \sin \theta \]

where the term in \( H_{L} \) represents the degree of filling in the mill.

ii) For angular moments about the mill axis \( r = RF \) - see sketch ignoring frictional forces (figure 28).

![Figure 28: Balancing Moments About the Mill Axis](image)

\[ mg \times \frac{x}{x_{m}} \times \text{mill capacity} \times \text{mill diameter} \]

\[ = k(V_{m})_{3} \pi D^{2} \pi \sin \theta \]

and \( E = \omega \), where \( \omega = (N_{0}N_{i})D^{1.5} \)

Appendices
\[ E = k_f(V_L) R_a D^2 \varphi L (N/N_0) \sin \theta \]

A function with peak characteristics is used to describe the parabolic relationship between power and load volume e.g.

\[ \sin(\pi V_L/V_{\text{max}}) \]

The peak characteristics are accounted for in the model (i) by \( H_L \) which has a peak at 50% mill filling (\( H_L = 0 \)).
The Discharge Rate

This model (Moys, 1986b) evaluates the resistance to flow through the grate using static load considerations.

Figure 29: Area of Static Load

The load area behind the grate (figure 29) is given by

\[ A = 2 \int_{H_l}^{R} \sqrt{R^2 - x^2} \, dx \]

Flow through the grate:

Consider one hole, diameter \( s \) at a height \( x \) below the axis. Using the form of the orifice equation:

\[ q(x) = k \delta(x - H_l)^n \quad k = f(\text{viscosity}) \]

Total flow, \( Q = \int_{H_l}^{R} N_G q(x) \, dA(x) \)

\[ = N_G \int_{H_l}^{R} k \delta(R - x)^n \sqrt{R^2 - x^2} \, dx \]

Analytical solution of this integral for non-integer values of \( n \) is possible, but using a numerical solution an expression is derived which expresses the sol-
olution as function of $n$. $n$ is usually fitted from data but in this case is set equal to 0.5 as in the orifice equation.

$$Q = 2k\delta^2N_G R^{0.5} h_L^2, \quad k = k'(p_{c,r}-p)^b$$

Where the area behind the grate can be found setting $n = 0$ in the integral solution.

$$A = R^2\pi/2(h_L/R)^{1.5}$$

$$h_L = (2V_L/(\pi R^2))^{2/3} R$$
Sump and Pump Models

The sump is pyramidal and an expression relating volume to height was derived from physical measurements:

$$V(h) = (0.311h^3 + 53.276h^2 + 2970h) \times 10^{-6} \text{ m}^3 \quad \text{for } h \text{ in cm.}$$

The maximum sump volume is 7.3 m$^3$ at $h = 230$ cm.

Note that the strongly non-linear relationship of volume to height, the controlled variable, makes it difficult to have good level control.

The model uses PI control with the same tuning parameters as those used in the field, but achieves much better control since the non-linear valve characteristics are ignored. This only has the effect of smoothing the model predictions around the sump.

The discharge pump is a 'D' frame Hydroseal centrifugal pump with a 775 mm diameter rubber impeller, running at approximately 450 rpm. The cyclone feed line is 250 mm ID pipe, 29 m long with 13 m of vertical displacement. The pressure drop across the cyclone is approximately 65 kPa.

The pump efficiency decreases with increasing density and an empirical model for the flow rate gives:

$$C = (a_{pc} + b) + C_{ss}$$

where $C_{ss}$ is the steady state or average cyclone feed rate. Problems were experienced in fitting these constants because very little consistent data were available. This is ascribed to different states of impeller wear, motor speed, belt tension, etc.
Cyclone Model

Pitt (1976) derived a model for the hydrocyclone which, although empirical, has a mechanistic basis. A broad database was used but unfortunately most previous work had been performed on small diameter cyclones. Thus application to this circuit model predicted too low a split (underflow to overflow volumetric ratio) and too high a classification efficiency.

Thus using this model structure and limited plant data, parameters were fitted to better describe operation.

Cyclone Dimensions:

Dc = 915 mm
Do = 300 mm
Dh = 95 mm
Di = \sqrt{4A/V} = 444 mm
b = 2400 mm
r = 2.67 m
pw = 1.00 m

\[ d_{50c} = 20.25 \exp(6.34\phi_c)/C^{0.5} \]

Split = \[7.19 \times 10^{-4}\exp(3.3\phi_c)/C^2\]

\[ m = \sqrt{35/(\phi_c + 0.05)} \]

for C in m³/s

\(\phi_c = 1 - 0.693 \phi(d/d_{50c})^{10} \): Rosin-Rammler equation describes corrected classification curve (Pitt, 1976)

\[ y = (1 - R_t)\phi_c + R_t \]: Assuming that the solids fraction bypassing classification equals the fluid recovery, \(R_t\)

Where \(R_t = (R_o - R_{so})/(1 - \phi_c)\)

and \(R_o = \text{Split/Split} + 1\) the volume recovery to the underflow

\(R_{so}\) the solids recovery must then be iteratively solved for.
Plant Classification Data

Figure 30: Cyclone Operating Characteristics used to Fit Plitt Model Parameters
Model Algorithm

1. Initialise load & circuit conditions.
2. Input solids rate, feed water rate (possibly sump water rate).
3. Use Adams Predictor-Corrector method to solve load model equations over a short time interval.
4. Use results from load to calculate volume of solids, volume of slurry, and slurry density.
5. Calculate discharge slurry rate from level of slurry behind the grate and slurry viscosity. Discharge density = load slurry density.
6. Calculate sump volume from mass balance. PI controller determines water rate to maintain level.
7. Density in sump is density of discharge. Flow rate is a function of density.
8. Is Sump Volume Correct?
   - If N, go to step 9.
   - If Y, go to step 10.
9. Impose Rossin Rammler size distribution on cyclone feed; fineness is proportional to slurry hold-up in the mill. Calculate underflow from cyclone equations.
10. Calculate load mass and power draft. Allowance is made for centrifuging by reducing the effective diameter.

Is the mass of slurry in the load correct?
Model Algorithm

1. Initialize load and circuit conditions.
2. Input solids rate, feed water rate (possibly sump water rate).
3. Breakage rates = power draft.
4. If a minute has passed, print the results.
5. Use Adams Predictor-Corrector method to solve load model equations over a short time interval.
7. Use results from load to calculate volume of solids, volume of slurry, and slurry density.
8. Calculate discharge slurry rate from level of slurry behind the grate and slurry viscosity. Discharge density = load slurry density.
9. Calculate sump volume from mass balance. PI controller determines water rate to maintain level.
10. Density in sump is density of discharge. Flow rate is a function of density.
11. Is sump volume correct?
   - Y: Impose Rosin-Rammler size distribution on cyclone feed; fineness is proportional to slurry hold-up in the mill. Calculate underflow from cyclone equations.
   - N: Calculate load mass and power draft. Allowance is made for centrifuging by reducing the effective diameter.

Appendices