THE ANALYSIS OF TURBULENT FLOWS USING A DIGITAL COMPUTER
WITH SPECIAL REFERENCE TO THE PLANE MIXING LAYER

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THE ANALYSIS OF TURBULENT FLOWS
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SUMMARY

Hot-wire anemometer measurements (single wire and X array) were made in the mixing region produced by two parallel streams of air moving at different velocities. Correlation function, power spectrum function and amplitude probability density function profiles were obtained across the mixing region using digital analysis.

The system developed for digital analysis of turbulence consisted of recording the analogue signals on magnetic tape, followed by digitising to produce a digital record on another magnetic tape for processing on an IBM 360/50 computer.

The computer programs can handle a single time series, two time series (to calculate cross correlations and cross spectra) and separate the two signals from an X array probe into the u and v velocity components. These time series were then analysed, using Fast Fourier Transform, to give correlation function, power spectrum function and Reynolds shear stress estimates.

Using a digital program developed by Drayer (1973) an intermittency signal associated with a turbulence signal was obtained. The turbulence signal was separated (using a rough procedure) into signals for the turbulent and non-turbulent zones. Estimates of the correlation functions and power spectra in the two zones were then obtained which showed expected trends. This experiment further illustrates the versatility of the digital approach to turbulence analysis.

Finally a check on Phillips (1967) theory concerning the maintenance of Reynolds shear stresses was made. The value of 0.23 obtained for the constant A is in good agreement with that of 0.2 obtained by Wynnanski and Fiedler (1970).
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# CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Arrangement of the Dissertation</td>
<td>1</td>
</tr>
<tr>
<td>1.2 The Aim and Scope of the Project</td>
<td>1</td>
</tr>
<tr>
<td>1.3 Previous work Using the Digital Computer</td>
<td>5</td>
</tr>
<tr>
<td>1.4 Previous Work on the Two-Dimensional Mixing Region</td>
<td>7</td>
</tr>
<tr>
<td>2. INTRODUCTION TO DIGITAL FOURIER ANALYSIS</td>
<td>8</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>8</td>
</tr>
<tr>
<td>2.2 Fourier Representation</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Aliasing</td>
<td>10</td>
</tr>
<tr>
<td>2.4 Leakage</td>
<td>11</td>
</tr>
<tr>
<td>2.5 The Wiener-Khinchine Relationship</td>
<td>17</td>
</tr>
<tr>
<td>2.6 Method of Analysis</td>
<td>20</td>
</tr>
<tr>
<td>2.6.1 Single Series Analysis</td>
<td>20</td>
</tr>
<tr>
<td>2.6.2 Two Series Analysis</td>
<td>21</td>
</tr>
<tr>
<td>2.6.3 Zone Averages</td>
<td>22</td>
</tr>
<tr>
<td>2.7 Averaging Over Samples</td>
<td>23</td>
</tr>
<tr>
<td>3. STATISTICAL DESCRIPTION OF TURBULENT FLOW USING FOURIER ANALYSIS</td>
<td>25</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>25</td>
</tr>
<tr>
<td>3.2 The Hot-Wire Anemometer</td>
<td>26</td>
</tr>
<tr>
<td>3.3 Correlation Analysis</td>
<td>30</td>
</tr>
<tr>
<td>3.3.1 Introduction</td>
<td>30</td>
</tr>
<tr>
<td>3.3.2 Longitudinal and Lateral Length Scales</td>
<td>31</td>
</tr>
<tr>
<td>3.3.3 The Autocorrelation</td>
<td>34</td>
</tr>
<tr>
<td>3.3.4 The Periodogram</td>
<td>36</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>3.3.6</td>
<td>Moving Axis Autocorrelation</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>3.3.7</td>
<td>The Cross Spectrum</td>
</tr>
<tr>
<td>3.3.7</td>
<td>The Cross Correlation</td>
</tr>
<tr>
<td>3.4</td>
<td>The Measurements Taken</td>
</tr>
<tr>
<td>3.4.1</td>
<td>Single Wire Measurements</td>
</tr>
<tr>
<td>3.4.2</td>
<td>Two Wire Measurements</td>
</tr>
<tr>
<td>3.4.3</td>
<td>Conditional Measurements</td>
</tr>
<tr>
<td>4.</td>
<td>THE COMPUTER PROGRAMS</td>
</tr>
<tr>
<td>4.1</td>
<td>General</td>
</tr>
<tr>
<td>4.2</td>
<td>Data</td>
</tr>
<tr>
<td>4.3</td>
<td>Results</td>
</tr>
<tr>
<td>4.4</td>
<td>The Programs</td>
</tr>
<tr>
<td>4.5</td>
<td>Method of Analysis</td>
</tr>
<tr>
<td>4.6</td>
<td>The General Program NLBONE</td>
</tr>
<tr>
<td>4.6.1</td>
<td>Quantities Calculated</td>
</tr>
<tr>
<td>4.6.2</td>
<td>Output of NLBONE</td>
</tr>
<tr>
<td>5.</td>
<td>EXPERIMENTAL APPARATUS AND PROCEDURE</td>
</tr>
<tr>
<td>5.1</td>
<td>The Turbulent Shear Layer</td>
</tr>
<tr>
<td>5.2</td>
<td>The Analogue Apparatus</td>
</tr>
<tr>
<td>5.3</td>
<td>The Experimental Procedure</td>
</tr>
<tr>
<td>6.</td>
<td>EXPERIMENTAL RESULTS</td>
</tr>
<tr>
<td>6.1</td>
<td>Calibration of the Hot-Wire Anemometer Probes</td>
</tr>
<tr>
<td>6.2</td>
<td>Directional Sensitivity of the Hot-Wire X-Array Probe</td>
</tr>
<tr>
<td>6.3</td>
<td>Measurements in the Free Shear Mixing Layer</td>
</tr>
<tr>
<td>6.3.1</td>
<td>General</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Single Hot-Wire Traverse 30 cm Downstream of the Splitter Plate</td>
</tr>
</tbody>
</table>
6.3.3 Cross-Wire Traverse 30 cm Downstream of the Splitter Plate

6.3.4 Zone Properties of the Turbulent Shear Layer 30 cm Downstream of the Splitter Plate

6.4 Check of Phillips' Theory on the Maintenance of Reynolds Shear Stress

7. DISCUSSION ON THE ERRORS
   7.1 The Analogue System
   7.2 The Digitising Process
   7.3 Sampling Errors
   7.4 Analysing a Limited Time of Signal
   7.5 Computational Errors
   7.6 Characterising the Digital System
      7.6.1 Resolution
      7.6.2 Dynamic Range
      7.6.3 The Minimum Detectable Signal (M.D.S.)
   7.7 The Statistical Accuracy of the Results

8. CONCLUSIONS
   8.1 The Analysis System
   8.2 Turbulence Results
   8.3 Suggestions for Further Work

9. NOMENCLATURE

10. REFERENCES

APPENDIX I : The Hot-Wire Response Equations

APPENDIX II : The Computer Programs

APPENDIX III : Analogue Circuit for u- and v- velocity fluctuations
<table>
<thead>
<tr>
<th>Figure Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Aliasing</td>
<td>11</td>
</tr>
<tr>
<td>2-2</td>
<td>Frequency Folding</td>
<td>13</td>
</tr>
<tr>
<td>2-3</td>
<td>Physically Realisable One-Sided Power-Spectral Density Function</td>
<td>19</td>
</tr>
<tr>
<td>2-4</td>
<td>Single Series Method of Analysis</td>
<td>21</td>
</tr>
<tr>
<td>2-5</td>
<td>Two Series Method of Analysis</td>
<td>22</td>
</tr>
<tr>
<td>3-1</td>
<td>Single Hot-Wire Sensor</td>
<td>25</td>
</tr>
<tr>
<td>3-2</td>
<td>Inclined Hot-Wire Sensor</td>
<td>27</td>
</tr>
<tr>
<td>3-3</td>
<td>X-Array Hot-Wire Probe</td>
<td>29</td>
</tr>
<tr>
<td>3-4</td>
<td>Longitudinal and Lateral Length Scales</td>
<td>32</td>
</tr>
<tr>
<td>3-5</td>
<td>Log-log Plot of Periodogram</td>
<td>37</td>
</tr>
<tr>
<td>3-6</td>
<td>Wire Configuration for Moving Axis Autocorrelation</td>
<td>39</td>
</tr>
<tr>
<td>3-7</td>
<td>Cross Correlation $R_{uu}(r, \tau)$</td>
<td>39</td>
</tr>
<tr>
<td>3-8</td>
<td>Moving Axis Autocorrelation</td>
<td>40</td>
</tr>
<tr>
<td>5-1</td>
<td>The Wind Tunnel</td>
<td>53</td>
</tr>
<tr>
<td>5-2</td>
<td>The Analogue Equipment</td>
<td>58</td>
</tr>
<tr>
<td>5-1</td>
<td>Calibration Curve for Cross-Wire Probe</td>
<td>63</td>
</tr>
<tr>
<td>5-2</td>
<td>Calibration Curve for Single-Wire Probes</td>
<td>63</td>
</tr>
<tr>
<td>5-3</td>
<td>Periodogram of the Analogue Sine Wave</td>
<td>67</td>
</tr>
<tr>
<td>5-4</td>
<td>Amplitude Probability Density Function of the Sine Wave</td>
<td>68</td>
</tr>
<tr>
<td>5-5</td>
<td>Autocorrelation of Analogue Sine Wave</td>
<td>68</td>
</tr>
<tr>
<td>5-6</td>
<td>Mean Velocity Profile and r.m.s. Values of the $u$ Fluctuations (30 cm downstream)</td>
<td>70</td>
</tr>
<tr>
<td>5-7</td>
<td>Log-log Plot of Periodogram (Single Wire)</td>
<td>72</td>
</tr>
<tr>
<td>Figure Number</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6-8</td>
<td>Normalized Autocorrelation (Single Wire)</td>
<td>73</td>
</tr>
<tr>
<td>6-9</td>
<td>Taylor's Microscale Profile (30 cm downstream)</td>
<td>76</td>
</tr>
<tr>
<td>6-10</td>
<td>Skewness and Kurtosis Profiles (30 cm downstream)</td>
<td>75</td>
</tr>
<tr>
<td>6-11</td>
<td>Amplitude Probability Density Function (Single Wire)</td>
<td>78</td>
</tr>
<tr>
<td>6-12</td>
<td>R.M.S. Values of Signals from Two Wires of X-Array Probe</td>
<td>81</td>
</tr>
<tr>
<td>6-13</td>
<td>Calculated and Analogous Values of u- and v Velocity Fluctuations</td>
<td>86</td>
</tr>
<tr>
<td>6-14</td>
<td>Periodogram of the u- Velocity Fluctuations (Log-log)</td>
<td>87</td>
</tr>
<tr>
<td>6-15</td>
<td>Periodogram of the v- Velocity Fluctuations (Log-log)</td>
<td>87</td>
</tr>
<tr>
<td>6-16</td>
<td>Amplitude Probability Density Function of the u Fluctuations</td>
<td>89</td>
</tr>
<tr>
<td>6-17</td>
<td>Amplitude Probability Density Function of the v Fluctuations</td>
<td>90</td>
</tr>
<tr>
<td>6-18</td>
<td>Autocorrelations of the u- Velocity Fluctuations</td>
<td>91</td>
</tr>
<tr>
<td>6-19</td>
<td>Autocorrelations of the v- Velocity Fluctuations</td>
<td>91</td>
</tr>
<tr>
<td>6-20</td>
<td>Co-Spectrum, Quad-Spectrum and Cross Correlation ($R_{uw}$) at $y = -0.018$</td>
<td>92</td>
</tr>
<tr>
<td>6-21</td>
<td>Co-Spectrum, Quad-Spectrum and Cross Correlation ($R_{uw}$) at $y = 0.015$</td>
<td>93</td>
</tr>
<tr>
<td>6-22</td>
<td>Co-Spectrum, Quad-Spectrum and Cross Correlation ($R_{uw}$) at $y = 0.039$</td>
<td>94</td>
</tr>
<tr>
<td>6-23</td>
<td>Co-Spectrum, Quad-Spectrum and Cross Correlation ($R_{uw}$) at $y = 0.099$</td>
<td>95</td>
</tr>
<tr>
<td>6-24</td>
<td>Velocity Signal with Superimposed Intermittency Signal</td>
<td>99</td>
</tr>
<tr>
<td>6-25</td>
<td>Conventional Periodogram of u Fluctuations</td>
<td>99</td>
</tr>
<tr>
<td>Figure Number</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>6-26</td>
<td>Turbulent Zone Periodogram of the $u$ Fluctuations</td>
<td>99</td>
</tr>
<tr>
<td>6-27</td>
<td>Non-Turbulent Zone Periodogram of the $u$ Fluctuations</td>
<td>100</td>
</tr>
<tr>
<td>6-28</td>
<td>Conventional Autocorrelation of the $u$ Fluctuations</td>
<td>100</td>
</tr>
<tr>
<td>6-29</td>
<td>Turbulent Zone Autocorrelation of the $u$ Fluctuations</td>
<td>101</td>
</tr>
<tr>
<td>6-30</td>
<td>Non-Turbulent Zone Autocorrelation of the $u$ Fluctuations</td>
<td>101</td>
</tr>
<tr>
<td>6-31</td>
<td>Conventional Amplitude Probability Density Function</td>
<td>102</td>
</tr>
<tr>
<td>6-32</td>
<td>Turbulent Zone Amplitude Probability Density Function</td>
<td>102</td>
</tr>
<tr>
<td>6-33</td>
<td>Non-Turbulent Zone Amplitude Probability Density Function</td>
<td>103</td>
</tr>
<tr>
<td>6-34</td>
<td>Graph of Convected Integral Time Scale and Mean Velocity (30 cm downstream)</td>
<td>107</td>
</tr>
<tr>
<td>6-35</td>
<td>Cross Correlations ($R_{uu}(r, \tau)$) for Various Wire Separations at $y = 0.037$</td>
<td>108</td>
</tr>
</tbody>
</table>
ANALYSIS OF TURBULENCE USING A DIGITAL COMPUTER

CHAPTER I

1. INTRODUCTION

1.1 Arrangement of the Dissertation

In this introductory chapter the aim and scope of the project are described with reference to certain investigations. The second chapter gives an introduction to digital Fourier analysis while chapter three discusses the parameters used to characterise turbulent flow. Chapter four describes the digital analysis system produced, while chapter five describes the experimental apparatus and procedure adopted. In chapter six the experimental results are presented, chapter seven indicates the various errors involved in the analysis and chapter eight presents the conclusions.

1.2 The Aim and Scope of the Project

Turbulence is a random process and its analysis is limited to a knowledge of relationships between statistical measures, such as averages of variables, products of variables, correlations and joint probabilities. The structure of turbulent flow is described by Mollo-Christensen (1971) in the following manner.

Turbulence is generated in intermittent bursts which are associated with distortion of the velocity profiles. The bursts contain velocity fluctuations of a range of scales and the intermittency (time fraction of no production) increases with Reynolds number.

It is therefore possible to have a situation where averages might hide rather than reveal information about the turbulence process. Intermittent events tend to be hidden in time averages unless properties occurring only in the intermittent event are measured.
Kibens (1968) discussed and showed the importance of separating an intermittent turbulent signal into two zones, turbulent and non-turbulent. The results for the two dimensional mixing region of Hygnanski and Fiedler (1970) show differences in the rms values of the velocity fluctuations and shear stress values in the turbulent bursts and the non-turbulent region between bursts. Mollo-Christensen (1971) showed that the production of turbulence is dominated by that generated in the turbulent bursts. Obviously it is essential to analyse the turbulent and non-turbulent regions separately when studying turbulence situations where the intermittency is significant. In digital analysis an intermittency signal usually consists of a series of ones or zeros, depending on whether the signal is turbulent or not at a particular point.

Intermittency signals and zone averages are generally obtained using a number of hot-wire probes and very sophisticated analogue circuits (Kibens (1968), Hygnanski and Fiedler (1969)). Measurements of zone correlation functions and zone power spectra are extremely difficult to obtain using analogue methods. Complex analogue systems such as those used by the authors referenced above are too expensive to construct or maintain in small University Departments.

Digital computer facilities are readily available and most students obtain training in computer programming. An investigation into the utilization of digital computers for the analysis of turbulence data is therefore of interest. This investigation is not into on-line digital analysis but describes the tape recording of relevant data, digitising it and then using digital methods of analysis.
There are of course various other considerations. In favour of analogue methods one can say that:

1) Computer time may be expensive.
2) Large quantities of data are required to obtain statistically stable answers. This introduces problems of data handling as well as increasing the cost of analysis.
3) Analogue methods give the answer immediately.

However these advantages quickly disappear if a number of analogue instruments are used in series and the impedances are difficult to match. The maintenance of many different instruments soon requires the help of an electronics expert. In certain situations digital analysis can be an attractive proposition:

1) With the introduction of the Fast Fourier transform algorithm by Cooley and Tukey (1965) accurate estimates of the periodogram and velocity correlations can be obtained in small computation times. The algorithm reduces the number of multiplications in a Fourier transformation from the conventional $N^2$ to $N \log_2 N$ where $N$ is the size of the sample being transformed. Due to the decrease in multiplications there is a decrease in round-off error. Using the Fast Fourier transform it is quicker to calculate the autocorrelation going via the frequency domain than to convolve in the time domain. The frequency resolution possible using digital analysis is sometimes better than that using analogue filters.
When making comparative studies the same set of data can be used for each study. This generally makes interpreting the results much easier.

When the analysis procedure is to be changed the associated program is updated. This eliminates modifying electronic equipment and dealing with the associated problems.

Intermittency decisions are easier to make as the complete signal is available to be studied. Overall properties can be used to make the decisions. Once these have been made it is simple to separate the fluctuations into turbulent and non-turbulent regions. Conditional and point averages as described by Kibens (1968) can then be calculated.

When separating X-array probe signals into the velocity fluctuations, differences in the directional sensitivity and calibration curves of the two sensors can be accounted for.

The scope of the project involved the following steps. Firstly, a system to tape record, digitise and store the resulting data in a form compatible to the IBM 360/50 computer at the University of the Witwatersrand was evolved. Secondly, a suite of programs to compute the various functions used in turbulence analysis was written. The programs are simple to use and can be combined to form any desired analysis path. A general analysis program utilising the above programs was also written. Thirdly, measurements were taken in the mixing region produced by two parallel streams of air moving at different velocities. The data was analysed using the prepared digital system and it compared favourably with results obtained by Wygnanski and Fiedler (1970) in a similar flow situation. Using properties of the correlation functions it was possible to obtain an idea of the accuracy of the analysis and also the statistical stability of the averaged results.
Finally, using programs developed by Drayer (1973) which determine the intermittency signal associated with a turbulent signal, the digital analysis approach was used to calculate the correlation functions and power spectra in the turbulent and non-turbulent zones. These particular quantities would be very difficult to obtain using analogue methods and serve to show the versatility of the digital approach to the analysis of turbulence data.

1.3 Previous Work Using the Digital Computer

Digital analysis procedures have been used to good advantage in a great variety of different situations.

The most comprehensive use of the digital computer for the analysis of hot wire anemometer signals is the system described by Kaplan and Laufer (1969). Other workers, Frenkel and Klebanoff (1971) and Van Atta and Chan (1969) have used digital Fourier analysis for measuring correlation functions of turbulent quantities. Similar systems have also been used in the fields of acoustics and vibration analysis (Singleton and Poulter (1967) and Villasenor (1968)). A theoretical discussion on the application of the Fast Fourier transform is given by Cooley, Lewis and Welch (1970).

1) Kaplan and Laufer (1969) used a digital computer to analyse the intermittently turbulent region of the boundary layer. They used 10 hot-wire anemometers simultaneously and recorded the signals on an instrumentation tape recorder. An analogue to
digital converter connected directly to the digital computer made intermediate storage of the digitised data unnecessary. Programs were written to calculate the intermittency function for each hot-wire signal and then produce conditional and point averages. It was possible to perform calculations and digitise data simultaneously by using the computer's interrupt system. Care was taken not to let the calculations overtake the digitising and vice versa. In the above application the computer was used to perform relatively simple calculations on large quantities of data.

11) Van Atta and Chen (1968, 1969) used the digital computer to Fourier analyse turbulent velocity data and then calculate various time correlations of the longitudinal and lateral velocity fluctuations in grid generated turbulence. The Fast Fourier transform algorithm was used to obtain the Fourier series coefficients of the velocity or velocity squared etc. These were then combined and inverse Fourier transformed to give the correlation function. Obviously one particular series can be used for various different order correlations and time can be saved by careful designing of the analysis.

111) Frankel and Clebanoff (1971) used digital computing methods to study the statistical behaviour of turbulent velocity derivatives in a nearly isotropic turbulent field downstream of a grid. The first and second derivatives of the velocity fluctuations were obtained using analogue methods as numerical differentiation is not very accurate. The data was digitised at two different rates so that a reasonable range in time delays could be obtained.
iv) A statistical reduction program, designed to reduce and analyse random data processes without regard to record length, is described by Krause et al (1970). Statistical certainty is constantly checked and updated until desired statistical accuracies are realised or the data source is exhausted. This discussion is mainly theoretical.

v) Further applications of and discussion on Fast Fourier analysis can be found in the IEEE Transactions on Audio and Electroacoustics (1967 and 1969).

1.4 Previous Work on the Two Dimensional Mixing Region

A well-known self-preserving shear flow was chosen to compare the results obtained. The two-dimensional incompressible mixing region can be treated theoretically and has been studied by Liepmann and Laufer (1947) and recently by Hygnanski and Fiedler (1970). Townsend (1956) briefly discusses the above turbulent flow and compares it to other well-known flow situations.

Hygnanski and Fiedler (1970), using some very elaborate analogue circuits performed a particularly comprehensive investigation. They took intermittency into account and measured conditional and point averages. Third and fourth order products of the velocity fluctuations, spatial derivatives of these fluctuations and space-time correlations were also made.
CHAPTER II

2. INTRODUCTION TO DIGITAL FOURIER ANALYSIS

2.1 Introduction

Chapters II and III try to clarify the problems and assumptions involved in the digital analysis of hot-wire anemometer signals. It was thought appropriate to discuss first the digital procedures and then move on to the more specific calculation of the various properties used to characterise turbulence flows.

A short theoretical discussion is presented to cover the proposed analysis. Various pitfalls encountered in digital Fourier analysis are discussed and the method of calculating correlations and spectra is described.

2.2 Fourier Representation

Any function \( x(t) \), defined in the region \( 0 \to T \) can be represented by the Fourier series

\[
x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{2\pi nt}{T} + b_n \sin \frac{2\pi nt}{T} \right)
\]

where

\[
a_n = \frac{2}{T} \int_{0}^{T} x(t) \cos \frac{2\pi nt}{T} \, dt \quad n = 0,1,2,\ldots
\]

\[
b_n = \frac{2}{T} \int_{0}^{T} x(t) \sin \frac{2\pi nt}{T} \, dt
\]

If \( x(t) \) and \( \frac{dx}{dt} \) are piecewise continuous and \( x(t) \) is assumed periodic in \( t \) with period \( T \), then the series converges to \( x(t) \) if \( t \) is a point of continuity and it converges to

\[
\frac{1}{2}(f(t+0) - f(t-0))
\]

if \( t \) is a point of discontinuity. A detailed analysis of the Fourier series can be found in Papoulis (1962).
Assuming that the function $x(t)$ is now sampled at times $kh$
giving $N$ discrete points.

Where

$$T = kh$$
$$t = kh$$

and

$$k = 0, 1, 2, 3, \ldots, N - 1$$

the above expression becomes

$$x(kh) = \alpha + \sum_{n=1}^{N/2} a_n \cos \frac{2\pi nk}{N} + \sum_{n=1}^{N/2-1} b_n \sin \frac{2\pi nk}{N} \quad (2-1)$$

For ease of manipulation the above can be represented as a complex
function

$$X(kh) = \sum_{n=0}^{N-1} A_n e^{2\pi i nk/N}$$

where the time series must now be a complex series $X(kh)$. In practice
only real series are obtained and hence the imaginary parts of $X(kh)$
will be zero. For a real time series only $N/2$ of the complex
coefficients are unique, thus accounting for the different limits of $n$
in the two summations.

It is useful to see the relationships between the coefficients in
the two notations.

$$X(kh) = \sum_{n=0}^{N-1} A_n e^{2\pi i nk/N}$$

expanding the exponential:

$$X(kh) = \sum_{n=0}^{N-1} A_n (\cos \frac{2\pi nk}{N} + i \sin \frac{2\pi nk}{N})$$
Now $X$ and $A$ are complex

Therefore
$$X(k) = \sum_{n=0}^{N-1} (A_n^* + iA_n^2) \left( \cos \left( \frac{2\pi nk}{N} \right) + i\sin \left( \frac{2\pi nk}{N} \right) \right)$$

The sampled time series considered is real and hence the imaginary part of the right-hand side is zero.

Therefore
$$X(k) = \sum_{n=0}^{N-1} (A_n^* \cos \left( \frac{2\pi nk}{N} \right) - A_n^2 \sin \left( \frac{2\pi nk}{N} \right) + i(0))$$

Comparing equations (2-1) and (2-2) we have

$$a_n = R(A_n)$$
$$b_n = -I(A_n)$$

NOTE: $a_0 = 2a_0$

Obviously it is much more compact to use the complex notation, and it will be used in this presentation.

We now move on to two of the problems that arise when dealing with digitised data.

2.3 Aliasing

The problem is most important if errors are to be avoided. It is best illustrated by the Sampling Theorem which will be explained below.

Obviously any frequency of a signal that has been digitised must have at least two points per cycle for that frequency to be observed. This is illustrated in the figure below.
The dotted signal cannot be identified at the sampling rate in the diagram. Unfortunately, when one transforms into the frequency domain (Fourier transformation) these higher frequencies reappear as a frequency in the range being considered. Consider the continuous function $x(t)$ which can be Fourier transformed

$$x(t) = \int_{-\infty}^{\infty} A(f)e^{2\pi ift}df$$

If the signal is now sampled, giving $N$ points, time $h$ apart, the times of sampling are $kh$ where $k = 0, 1, 2, \ldots, N$. Then

$$X(kh) = \int_{-\infty}^{\infty} A(f)e^{2\pi ifh}df$$

Equation (2-3) can be written as

$$X(kh) = \sum_{n=-M}^{M} A_n e^{2\pi ikh}$$

where

$$F = \frac{1}{h} = \text{sampling frequency}$$

$F$, the sampling frequency is twice the Nyquist frequency. This is the highest frequency that can be distinguished at the particular sampling frequency.
Since \( m \) goes from \( +\pi \) to \( -\pi \), the integral (on summing) remains from \( +\pi \) to \( -\pi \).

The exponential \( e^{2\pi i m n} \) is periodic with period \( F \). Since each integration in the summation is over a period \( F \), one can write

\[
X(\chi) = \sum_{m=-\infty}^{\infty} \int_{0}^{F} A_n(n + mF)e^{2\pi i k n} \, dn
\]

where \( e^{2\pi i m n} = e^{2\pi i k (n + mF)} \)

therefore

\[
X(\chi) = \int_{0}^{F} \left( \sum_{m=-\infty}^{\infty} A_n(n + mF)e^{2\pi i k n} \right) \, dn
\]

Let

\[
A_n^P = A_n + A_{n+F} + A_{n+2F} + \ldots
\]

Therefore

\[
X(\chi) = \int_{0}^{F} A_n^P e^{2\pi i k n} \, dn \quad (2-4)
\]

Equation (2-4) is the version of (2-3) when one has a discrete time series and not a continuous series. It is important to notice that the integration is now over a fixed frequency range \( 0 \) to \( F \) whereas it was from \( +\pi \) to \( -\pi \). The Fourier coefficients in (2-4) are the aliased Fourier coefficients since they contain the contribution due to frequencies outside the range being considered.

To eliminate aliasing one must ensure that the signal is band limited so that the terms \( A_{(n+F)} \), \( A_{(n+2F)} \) are zero.
Assume that one has a signal containing frequencies of up to 520 Hz. The signal is sampled at a rate of 512 Hz. The Nyquist frequency at this sampling rate is 256 Hz. When one transforms into the frequency domain, where will the effect of the 520 Hz frequency be found?

\[
\begin{align*}
A_C^p &= A_n + A_{n+256} + \cdots \\
A_B^p &= A_0 + A_{8+256} + \cdots \\
\text{therefore} \quad A_B^p &= A_0 + A_{520} + \cdots
\end{align*}
\]

Therefore the 520 Hz frequencies will contribute to the 8 Hz frequency.

Aliasing is a result of not knowing the frequency content of the signal being analysed. To eliminate aliasing one should band-limit the signal by analogue filtering before digitising. Once the signal has been digitised the aliasing errors are present.

Graphically, aliasing can be illustrated by the following figure from United Geophysical Corporation (1965).

![Graph](image.png)

**Fig. (2-2) Frequency Folding.**

The folding occurs at the Nyquist frequency 256 Hz. A frequency of 520 Hz (input) is equivalent to 8 Hz (output) because of the folding.
2.4 Leakage

This problem results from the fact that we are dealing with a discrete sample and not an infinite one. The infinite sample has been windowed by a window that is zero everywhere, except during the discrete sample where it is 1. Let the length of the sample be $T$.

If one Fourier analyses a time series of length $T$, Fourier coefficients at the discrete frequencies

$$W_n = \frac{2\pi n}{T}, \quad n = 0, 1 \ldots \frac{N}{2}$$

are obtained, where $N$ is the number of points.

Consider the Fourier analysis of a single sine wave, the frequency of which is one of the calculated $W_n$ above. The only frequency coefficient obtained will be that for the frequency $W_n$.

If on the other hand the frequency of the sine wave is not one of the $W_n$, Fourier coefficients for all the $W_n$ are obtained. This effect is obtained when the sample length $T$ is not a multiple of the period of the sine wave being analysed. The Fourier coefficients peak near (on the low frequency side) the frequency $W$ and then fall off as

$$\frac{1}{|W - W_n|}$$

When the signal contains many frequencies and the frequency resolution is not small enough, the accumulated leakage from each frequency can distort the power spectrum.
By multiplying the infinite series by a window that gradually changes from 0 to 1 instead of a step change the leakage can be reduced. Multiplying by a Hannin window reduces the effect to

\[ \frac{1}{|W - W_0|^2} \]  

(Maling, Morrey and Lang (1967))

When one uses a window to reduce leakage two other problems are introduced.

1) the relative amplitudes of the frequency coefficients are altered

2) the statistical stability of the spectrum is reduced as the data has been altered.

The choice of data window used is a compromise between the leakage reduction and the resulting loss in statistical stability.

Durrani and Nightingale (1972) give a discussion on the effects of data windows and also a comprehensive description of the effects of various different windows. Windows are discussed by Blackman and Tukey (1959).

In the frequency domain the Hannin window consists of multiplying the Fourier coefficients (sine and cosine terms) by a moving average with factors 1/4, 1/2 and 1/4. For an analysis where the Fourier coefficients are centred about N/2,

\[
\begin{align*}
a_1 &= \frac{1}{2} \left( a_1 + a_2 \right) \\
a_n &= \frac{1}{4} a_{n-1} + \frac{1}{2} a_n + \frac{1}{4} a_{n+1} \\
a_N &= \frac{1}{2} a_{N-1} + \frac{1}{2} a_N 
\end{align*}
\]

Similarly for the sine coefficients

\[
\begin{align*}
b_1 &= \frac{1}{2} \left( b_1 + b_2 \right) \\
b_n &= \frac{1}{4} b_{n-1} + \frac{1}{2} b_n + \frac{1}{4} b_{n+1} \\
b_N &= \frac{1}{2} \left( b_{N-1} + b_N \right)
\end{align*}
\]
It is important to note that the Fourier coefficients must be Hanned before they are combined to give the periodogram. Hanning the periodogram has no noticeable effect on reducing the leakage.

It can be shown (Villassenor (1968)) that the time domain equivalent of the Hanning window is the function

\[ x(kh) = \frac{1}{2} \left( 1 - \cos \frac{2\pi k}{N} \right) x(kh) \]

\[ k = 0, 1, ..., N-1 \]

\[ h = \text{time interval between samples} \]

It may save computer time to multiply in the time domain rather than convolve in the frequency domain.

Using a window changes the input series. According to Bingham, Godfrey et al (1967) a reasonable compromise (between changing the data and improving the resolution) for data stretching from \( t = 0 \) to \( t = T \) would consist of two cosine bells, each extending for \( 1/10 \)th of the signal. The centre \( 8/10 \)th of the signal is left untouched. The equations representing the above in the time domain would be:

\[ x(t) = x(t) \frac{3}{2} \left( 1 - \cos \frac{5}{2} \right) \quad 1 \leq t < 0.1T \]

\[ x(t) = x(t) \quad 0.1T \leq t < 0.9T \]

\[ x(t) = x(t) \left( 1 - \cos \frac{7}{2} \right) \quad 0.9T \leq t < T \]

When using the above windows it is important to remove any linear trends in the data. A varying mean does not allow a smooth transition at the ends of the data.
To reduce leakage effects as long a sample length as possible should be used (as many cycles of the lowest frequency as possible). Sample lengths large compared to the low frequencies in the signal are analysed in this investigation and so the leakage effects should be small. Although the option of using the Hanning window is available in the programs it was seldom used.

At this stage it is possible to represent the turbulent velocity signal by a discrete Fourier series. Some problems encountered in obtaining this representation have been discussed. The next problem is to calculate

i) The periodogram

ii) The autocorrelation of a single series, and

i) The cross spectrum

ii) The cross correlation of two series.

2.6 The Wiener-Khintchine Relationship

The above indicates the relationship between the Fourier coefficients of two random stationary processes and their cross correlation. A brief explanation will be given here, while a more complete derivation can be found in Bendat and Piersol (1966).

Assume we have two continuous time series \( x(t) \) and \( y(t) \). Since the series are real we can represent them by their complex conjugates.

Using complex Fourier representation

\[
\begin{align*}
  y(t) &= \int_{-\infty}^{\infty} A(g)e^{2\pi gt}dg \\
  x(t) &= \int_{-\infty}^{\infty} A^*(f)e^{2\pi ft}df
\end{align*}
\]
where the asterisk represents the complex conjugate.

The covariance is defined as

\[ \text{Cov}(t) = E[X(t)Y(t+\tau)] \]

Hence from definitions above:

\[ R_{xy}(\tau) = \int_{-\infty}^{\infty} E[A(t)B^*(t)]e^{i2\pi(t-g)}e^{i2\pi\tau} \text{d}g \]

It can be shown, (Bendat and Pairsol (1966)) that for a stationary random process the following relationship holds:

\[ R_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)\delta(f-g)e^{i2\pi(t-g)}e^{i2\pi\tau} \text{d}g \]

where \( S_{xy}(f) \) is the two-sided cross-spectral density function (two-sided since the limits are from \(-\infty\) to \(+\infty\)). \( \delta \) is Dirac delta function.

Integrating with respect to \( g \), the integral is zero except when \( g = f \). When \( g = f \), \( e^{i2\pi(t-g)} = 1 \).

Hence

\[ R_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)e^{i2\pi\tau} \text{d}f \]  \hspace{1cm} (2-5)

where \( \tau \) is the time delay.

From equation (2-5) we see that \( R_{xy}(\tau) \) and \( S_{xy}(f) \) are Fourier transform pairs. We therefore have the two relationships

\[ X(t) = \int_{-\infty}^{\infty} A(f)e^{i2\pi ft} \text{d}f \]

and

\[ R_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)e^{i2\pi ft} \text{d}f. \]

From the expression

\[ E[A(t)B^*(t)] = S_{xy}(f)\delta(f-g) \]
when \( f = g \)

\[
E[A(f)A^*(f)] = S_{xy}(f)
\]  

From equation (2-6) we see that by multiplying the Fourier series coefficients of a series by the complex conjugate of the Fourier coefficients of another series we get the cross-spectral density function of the two signals.

Using the cross-spectral density in equation (2-5) one is able to obtain the cross-correlation between the two signals. Obviously when \( X = Y \) one obtains the periodogram and the autocorrelation.

When using analogue methods to measure the power spectral density one only measures positive frequencies. In that case the physically realisable one-sided power-spectral density function \( G_{xy}(f) \) is obtained where \( G_{xy}(f) = 2S_{xy}(f) \).

![Fig. (2-3) Physically Realisable One-sided Power-spectral Density Function](image)

The next section describes the method of analysis used for the analysis of a single sample.
2.6 Method of Analysis

2.6.1 Single Series Analysis

Assuming that the signal is random and stationary, the following procedure is followed:

1) The analogue signal is band limited so that aliasing errors are eliminated. This is done using analogue filters.

2) The signal is then digitised at a rate that will enable reliable information concerning the frequencies of interest to be obtained. There is not much literature on the subject but it seems that sampling at least five times the highest frequencies of interest is adequate. Of course the sampling rate must also be high enough to prevent aliasing.

3) The series is then Fast Fourier transformed using a digital computer to obtain the Fourier coefficients \( A(f) (a(f) + ib(f)) \).

4) The Fourier coefficients \( A(f) \) can be windowed to reduce leakage effects.

5) An estimate of the periodogram \( S_{xx}(f) \) is obtained as follows

\[
S_{xx}(f) = X(f)X^*(f)
\]

\[
S_{xx}(f) = (a(f) + ib(f))(a(f) - ib(f))
\]

\[
S_{xx}(f) = a^2(f) + b^2(f)
\]

(2.7)

When the Fourier coefficients are windowed before being combined to form the periodogram, the periodogram so formed is known as the modified periodogram. If the Fourier coefficients are not windowed it is called a raw periodogram. From 2.6 we see that the periodogram is a real function. Obviously the autocorrelation will be an even function as from 2.5.1 it has only cosine terms in its Fourier series.
vi) The autocorrelation is obtained by inverse Fourier transforming equation (2-7) according to equation (2-5)

\[ R_{xx}(\tau) = \int s_{xx}(f)e^{i2\pi \tau f} df \]

The figure below describes graphically the above procedure:

- **X(t)** time series
- **Fast Fourier transform**
- **a(f) + ib(f)** Fourier coefficients
- **Hanning window (optional)**
- **a(f) + ib(f)** modified Fourier coefficients
- **s^2(f) + b^2(f)** modified or raw periodogram
- **Inverse Fast Fourier transform**
- **\( R_{xx}(\tau) \)** autocorrelation

**Fig. (2-4): Single Series Method of Analysis**

2.6.2 Two Series Analysis

The cross-spectrum and cross correlation of two signals \( X(t) \) and \( V(t) \) are obtained in exactly the same way as for the single series analysis except that the Fourier coefficients of the \( X \) and \( Y \) series must be combined. Assuming the two signals to be random and stationary the procedure is shown in the figure (2-5).

The cross-spectral density function is a complex function.

The real part \( a^x_{by} + b^y_{bx} \) is called the co-spectrum while the imaginary part \( a^y_{bx} - a^x_{by} \) is called the quadrature-spectrum.

Since the cross-spectral density function has both sine and cosine terms, the cross correlation is not necessarily a symmetrical function.
Zone Averages

The conventional hot-wire anemometer signal is tape-recorded and digitised as described in 2.5.1. The signal is then processed to determine which points are in the turbulent zone and which points are in the non-turbulent zone. This was performed using digital programs developed by Dreyer (1973). Briefly, a range of frequencies containing the turbulent fluctuations is chosen and a cut-off value for the amplitude then determined experimentally.

From the intermittency programs one obtains an intermittency signal consisting of a series of one's and zero's, depending on whether the associated point in the conventional signal is in a turbulent zone or non-turbulent zone. The next step is to form a signal containing velocity fluctuations only from the turbulent zone. In this analysis all the velocity values associated with the one's were added one after the other. The magnitude of the errors introduced will be dependent on the length of the individual turbulent bursts. This is discussed further.
Using the digital system available, the turbulent and non-turbulent zones were analyzed according to 2.6.1. For comparison purposes, the analysis was also performed on the signal from which the turbulent and non-turbulent zone signals were obtained.

2.7 Averaging over Samples

The quantity of data obtained in the digitizing process is far too large to analyze in one pass through the digital computer. Generally the data is divided into sections, the size being dependent on the computer available, and the results of the sections averaged to give a final answer.

In this analysis sections of 2048 data points were used and quantities such as the periodogram and autocorrelation calculated for each section. At the end of the analysis an average value is obtained.

The averaging of the periodogram in this manner is discussed by Welch (1967) who shows that the statistical accuracy of the results (for a stationary process) depends directly on the sample size and the number of samples analyzed. An idea of the accuracy in this analysis was obtained by performing the following statistical test:

i) For each sample analyzed the standard deviation is calculated \( \sigma_i \).

ii) After each sample has been analyzed the accumulated mean standard deviation is calculated

\[
\bar{X} = \frac{1}{m} \sum_{i=1}^{m} \sigma_i
\]

where \( m \) = number of sections (2048 points).

iii) Assuming the signal to be a random stationary series the quantity \( \bar{X} \) should follow the student "t" distribution (a result of the central limit theorem).
iv) The variance of the standard deviation is calculated

\[ \text{var}(x) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]

v) The standard deviation of the series of sample standard deviations is then calculated

\[ SD = \sqrt{\text{var}(x)} \]

vi) The following confidence interval for the mean value of the sample standard deviation can be set up

\[ \bar{x} - \frac{t \cdot SD}{\sqrt{m}} < X < \bar{x} + \frac{t \cdot SD}{\sqrt{m}} \]

where

\[ SD = \text{standard deviation} \]
\[ t = \text{students 't' coefficient} \]
\[ m = \text{number of sections (2048 points)} \]

if the confidence interval is narrow enough the calculation can be terminated.

The theory and possible errors involved in analysing a long section of data by dividing it into manageable sections is discussed in detail by Krause et al (1970). Two points should be considered.

An error (subdivision error) results when the signals being analysed have a mean value and the statistic being calculated cannot be obtained directly from the signals actually recorded. As an example consider the cross correlation of two signals \( x(t) \) and \( y(t) \) with mean values \( \bar{x} \) and \( \bar{y} \). The cross correlation \( R_{xy}(t) \) is defined as
\[ r_{xy}(\tau) = \frac{1}{T} \int_{0}^{T} (x(t) - \bar{x})(y(t + \tau) - \bar{y})dt - \bar{x}\bar{y} \]

\[ = \frac{1}{T} \int_{0}^{T} x(t)y(t + \tau)dt - \bar{x}\bar{y} \] 

(2-8)

If the total time \( T \) is divided into \( m \) sections of \( \Delta t \) and the cross correlation for each section calculated and then averaged.

\[ r_{xy}(\tau) = \frac{1}{m} \sum_{j=1}^{m} \left[ \frac{1}{(1-\Delta t) \Delta t} \int_{(j-1)\Delta t}^{j\Delta t} x(t)y(t + \tau)dt - \bar{x}\bar{y} \right] \]

(2-9)

The subdivision error will then be (2-9) - (2-8)

\[ = \frac{1}{m} \sum_{i=1}^{m} \frac{x_i y_i}{ \bar{x}\bar{y}} \]

For \( m = 1 \) the subdivision error is zero as expected.

Another error can result in calculating autocorrelations and cross correlations from segmented data. The statistical accuracy of coefficients with large time lags (relative to the segment length) will be less than those of smaller lags if the calculations move in only one direction.

To reduce the error, extra data points must be added to the segment when large time lags are being calculated. A second method is to continue the segment periodically. The Fast Fourier transform uses the latter method.
The applicability of Taylor's hypothesis is discussed by Lin (1953).

In this case

$$p_x(r) = f(r)$$

giving

$$x_f = r_E^U$$

and

$$x_f = U_E$$

(3-10)

The importance of equation (3-10) is that $U_E$ and $x_f$ are measured using one hot-wire anemometer, while the measurement of $x_f$ and $x_f$ requires $x_f$. Assuming Taylor's hypothesis $x_f$ and $x_f$ can be obtained using a single hot-wire anemometer.

The experimental apparatus required for measuring the above time and length scales is described by Rasmussen (1966) and DISA (1968).

3.3.4 The Periodogram (Power spectrum)

If the discrete series of velocity fluctuations is represented as

$$X(kh) \quad h = \frac{T}{N}$$

where $T$ = total time of the series. Then the complex Fourier coefficients can be obtained from

$$A_n = \frac{1}{N} \sum_{k=1}^{N} X(kh)e^{-2\pi i kh}$$

Note: The series $X(kh)$ is written as a complex function although in reality the imaginary part is zero.

The periodogram is defined as

$$S(f) = |A(f)|^2$$
This chapter will deal with the measurements used to characterize a turbulent field. Firstly, the response equations of the hot-wire anemometer are discussed and then correlation analysis will be introduced. Lastly, the various measurements made are mentioned.

3.2 The Hot-Wire Anemometer

The hot-wire anemometer is the most common instrument used in measuring turbulent velocity fluctuations. The structure and characteristics of the equipment used are described by Hinze (1959). In this section, a description of what the hot-wire anemometer measures is given.

\[
\begin{align*}
U &\quad V \\
\end{align*}
\]

Fig. (3-1) Single Hot-wire Sensor

Figure (3-1) shows a hot wire in a turbulent flow, the wire being perpendicular to the mean velocity \( U (V = W = 0) \). Velocity fluctuations \( u, v, w \) are superimposed on the stream. Effectively, the velocity measured by the hot wire is

\[
U_{\text{eff}} = \sqrt{(U + u)^2 + v^2 + w^2}
\]

for \( u, v, w \) small compared to \( U \) (i.e., \( \text{han} 15\% \))

\[
U_{\text{eff}} = \sqrt{(U + u)^2} = U + u
\]
The wire essentially measures only the longitudinal fluctuations. If the hot wire is placed at an angle to the mean velocity, then the effective velocity has components in both the \( u \) and \( v \) directions and hence the effective cooling velocity is a function of both the \( u \) and \( v \) velocity fluctuations (Fig. (3-2)).

Two hot-wire probes at a point in the stream, symmetrically displaced in an \( X \) array about the mean flow direction are affected equally by the \( u \) fluctuations, but oppositely by the \( v \) fluctuations. Using two wires as described above it is possible to separate the \( u \) and \( v \) fluctuating components (Fig. (3-3)).

Next one must obtain the relationship between the effective velocity and the \( u \) and \( v \) velocity components. Experiment has shown (Webster (1962), Champagne (1965)) that the tangential component of the velocity along the inclined wire does in fact effect the heat transfer from the hot wire. In general the cosine law is assumed which means that the amount of heat transfer depends only on the flow field through the normal component of the velocity across the hot wire.

i.e. \( U_{eff}(\theta) = U(0)\cos \theta \)

\[ \frac{U}{U(0)} \]

Fig. (3-2) Inclined Hot-wire Sensor

\( U(0) = \) measured velocity at \( \theta = 0 \).
A modified relationship suggested by Champagne (1965)

\[ \nu_{\text{eff}}(s)^2 = \nu(0)^2(\cos^2\theta + k^2\sin^2\theta) \]

where

\[ \nu_{\text{eff}}(s) = \text{velocity measured at } s = \theta \]
\[ \nu(0) = \text{velocity measured at } s = 0 \]
\[ k = \text{sensitivity coefficient} \]

It has been found (Champagne (1965) and Jorgenson (1971)) that the value of \( k \) depends on:

1) the length to diameter ratio of the wire
2) the mean velocity

Generally the value of \( k \) varies between 0.1 and 0.3. The work by Jorgenson (1971) describes the errors and values of \( k \) involved when using the above relationship on probes manufactured by the DISA Company. It seems that for values of \( \theta = 45^\circ \) it is advisable to use a sensitivity coefficient.

Having a well-substantiated relationship for the effective mean velocity, one requires a similar relationship for the fluctuating velocity components.

A complete derivation can be found in Appendix I. If second order velocity products (\( u^2, v^2, w^2, uv \)) are neglected then the following relationship is obtained:

\[ \frac{c_l}{\nu_{\text{eff}}} = \frac{u + v \sin \alpha \cos \alpha (k^2 - 1)}{u \sin \alpha + k \cos \alpha} \]

(3-1)

\( c_l = \text{instantaneous velocity fluctuation} \).
When using this relationship the value of $E$ (average measured voltage) is underestimated by approximately

$$\frac{\nu^2 + \nu^2}{2U^2} \frac{1}{(s_1\sin^2\alpha + k_2\cos^2\alpha)}$$

Note, $E = KU_{\text{eff}}$.

For this equation the effective cooling velocity and mean velocity are related by

$$U_{\text{eff}}^2 = U(0)^2(s_1\sin^2\alpha + k_2\cos^2\alpha) \quad (3-2)$$

Considering the $X$ array in Fig. (3-3) one obtains an equation like (3-1) for each of the wires

\begin{align*}
\frac{c_1}{U_{\text{eff}}} &= \frac{U + \nu\sin\alpha\cos\alpha (k_1^2 - 1)}{U(s_1\sin^2\alpha + k_1^2\cos^2\alpha)} \quad \text{for wire (1)} \\
\frac{c_2}{U_{\text{eff}}} &= \frac{U - \nu\sin\alpha\cos\alpha (k_2^2 - 1)}{U(s_1\sin^2\alpha + k_2^2\cos^2\alpha)} \quad \text{for wire (2)}
\end{align*} \quad (3-3)

Using equation (3-2) we have

\begin{align*}
c_1 &= \frac{s_1\sin^2\alpha + k_1^2\cos^2\alpha}{s_1\sin^2\alpha + k_1^2\cos^2\alpha} \\
c_2 &= \frac{s_1\sin^2\alpha + k_2^2\cos^2\alpha}{s_1\sin^2\alpha + k_2^2\cos^2\alpha}
\end{align*} \quad (3-4)
The equations (3-4) are suitable for use in digital analysis. Records of \( c_1 \) and \( c_2 \) can be made at identical times on magnetic tape. These are then processed to give the \( u \) and \( v \) fluctuations. The sensitivities of the two wires are generally different and this fact can be incorporated into the solution of equation (3-4).

When using analogue methods for obtaining turbulent intensities and Reynolds shear stresses, equations (3-4) are not used. Champagne (1965) derived correction factors (a function of \( k \) and \( \alpha \)) by which the quantities measured, assuming the cosine law, must be multiplied to take into account the directional sensitivity of the probe. It is also required that \( k_1 \) and \( k_2 \) be the same.

3.3 Correlation Analysis

3.3.1 Introduction

Usually the turbulent velocity is assumed to consist of a mean velocity and a superimposed fluctuating component whose average is zero. When this is used in the Navier-Stokes equation and the equation averaged, additional stresses, Reynolds stresses, are introduced, e.g. the \( xy \)-component of the Reynolds stress is defined by

\[
\tau_{xy} = -\rho uv.
\]

These stresses represent the mean rate of transfer of momentum across a surface due to the velocity fluctuations.

To solve the resulting equations of motion a relationship between the Reynolds stress and the mean velocity is required. Another approach would be to obtain equations describing the behaviour of the Reynolds stresses. Any attempt to do this using the equations of motion results in the introduction of triple velocity correlations, and velocity pressure correlations. The latter can be eliminated but in so doing velocity correlations at different points are introduced.
To try and terminate the process of continually introducing higher order velocity correlations, workers such as Frenk et al. and Klebanoff (1971) and van Atta and Chen (1969) have tried to find relationships relating, say, the two- and four-velocity correlations. The correlation functions between the derivatives of velocity fluctuations have been studied by Frenk et al. and Klebanoff (1971).

The turbulent field can be characterised by length scales obtained from velocity correlations. They are also used to study the spatial configuration of the turbulent field. If one assumes homogeneity and isotropy of the turbulent field, theory can be applied to the form of the velocity correlations, and useful information concerning eddy sizes obtained.

Mathematically, correlation analysis is well covered in the literature - see Bendat and Piersol (1966), but only recently has it become possible to measure the correlation functions simply. A discussion of various methods available is given by Jenson (1970). Correlation analysis as applied to turbulence is discussed by Hinze (1959) and Townsend (1956).

3.3.2 Longitudinal and Lateral Length Scales

The scales are obtained from the longitudinal and lateral spatial correlations of the velocity at two different points. The normalised lateral spatial correlation is defined as

$$g(r) = \frac{u(x)u(x + r)}{u(x)^2u(x + r)^2}$$

and the normalised longitudinal correlation as

$$f(r) = \frac{u(x)u(x + r)}{u(x)^2u(x + r)^2}$$

where the velocities $u$ and separation $r$ are shown in figure (3-4).
Figure (3-6) Longitudinal and Lateral Length Scales

The way in which the correlation behaves as $r \to \infty$ depends on the turbulent field (periodicities, etc.). Assuming isotropy the shape of the correlation function in the region $r = 0$ can be described in terms of the velocity and its derivatives.

Considering the longitudinal spatial correlation we have (from a Taylor expansion)

$$f(r) = 1 - \frac{r^2}{2 \lambda_f^2} \quad (3-5)$$

where

$$\frac{1}{\lambda_f^2} = \frac{1}{(u' \lambda_f)^2} = \frac{1}{u'^2} \quad (3-6)$$

Since this defines a parabola with its vertex at $r = 0$, $f(r) = 1$ where it osculates the correlation curve. $\lambda_f$ is found from the intersection of the parabola with the $r$ axis. $\lambda_f$ is known as the Taylor microscale.
The Taylor dissipative length scale \( \lambda_T \) is associated with the smaller eddies of the turbulence but is not the length scale associated with the small eddies responsible for viscous dissipation of energy. These eddies are associated with the Kolmogorov length scale which for isotropic turbulence is related to the Taylor length scale.

For homogeneous isotropic turbulence the rate of viscous dissipation is given by

\[
\epsilon = 15\nu \left( \frac{\partial u}{\partial y} \right)^2
\]  

\( \nu \) = viscosity (kinematic)

The Kolmogorov length scale associated with the dissipating eddies is defined by

\[
n = \left( \frac{\nu^3}{\epsilon} \right)^{1/4}
\]  

Using equations (3-6), (3-7) and (3-8) one obtains

\[
\frac{\lambda_T^2}{\lambda_n^2} = \frac{15\nu^2}{\epsilon^2}
\]  

\( \lambda_T \) is generally larger than \( \lambda_n \) and at high Reynolds numbers it corresponds to an eddy size containing a negligible part of the total energy and is responsible for a negligible part of the total dissipation. It is a measure of the ratio of the total energy to the rate of energy loss and can therefore be regarded as a "reaction time" of the turbulence. A more complete discussion is given by Townsend (1956).

In general the lateral correlation will give a microscale \( \lambda_g \) where \( \lambda_T \neq \lambda_g \). For homogeneous isotropic turbulence the relationship

\[
\lambda_T = \sqrt{2}\lambda_g
\]

exists.
Another length scale, giving the length of the largest correlation between two velocities can be measured. This length would be \( r^* \)

where

\[
f(r) = 0 \quad \text{for} \quad r > r^*
\]

In fact a slightly different length scale is defined

\[
A_f = \int_0^\infty f(r)dr
\]

where \( A_f \) = macro or integral scale of turbulence.

If \( f(r) \) is a rectangular function then \( A_f = r^* \). Periodicities or randomness change the shape of \( f(r) \) and hence \( A_f \) can vary considerably from \( r^* \).

3.3.3 Autocorrelation

The autocorrelation is defined as

\[
R_x(t) = \frac{1}{T} \int_0^T u(t) u(t + \tau)dt
\]

\( \tau = \) time delay

\( T = \) total length of sample

and describes the general dependence of the values of the data at one time on the values at another time. As \( T \to \infty \) the exact autocorrelation is approached. Useful properties of the autocorrelation are:

1) \( R_x(0) = \frac{u(t)^2}{\infty} \geq |R_x(\tau)| \)

2) \( R_x(\tau) = R_x(-\tau) \)

Usually one deals with the normalised autocorrelation

\[
r_x(\tau) = \frac{R_x(\tau)}{R_x(0)}
\]
The main application of autocorrelations (although it does not strictly apply to turbulence studies) is in finding deterministic data that has been disguised in a random background. Deterministic data (also periodic) have autocorrelations over all time displacements while random data has an autocorrelation that diminishes to zero for large displacements (Kendall and Pickles 1966).

The normalised autocorrelation of the fluctuations

\[ \rho_X(t) = \frac{\langle u(t) u(t+\tau) \rangle}{\sigma_u^2} \]

can, assuming homogeneity be represented by the parabola

\[ \rho_X(t) = 1 - \frac{\tau}{2\tau_E} \]

even near the origin \( \tau = 0 \).

\[ \frac{1}{\tau_E} = \frac{1}{\sigma_u} \left( \frac{\partial u}{\partial t} \right)^2 \bigg|_{t=0} \]

\( \tau_E \) is a microscale, while a macroscale \( \tau_E \), where

\[ \tau_E = \int_0^t \rho_X(t) dt \]

can also be defined. The procedure just followed is identical to that for obtaining the Taylor microscale from the longitudinal spatial correlation.

In the case where the turbulent field has a constant mean velocity \( U \) in the \( x \) direction and the turbulence level \( \frac{\sigma_u^2}{U} \)

is low, Taylor's hypothesis may be assumed. 

i.e. \[ \frac{d}{dt} \sigma_u^2 = - U \frac{\partial}{\partial x} \sigma_u \]
where $^\star$ indicates the complex conjugate.

i.e., \[ S(f) = (A_r(f))^2 + (A_i(f))^2 \]

where

- $A_r(f) = \text{coefficient of the cosine term of the Fourier series}$
  (real part of complex coefficient)

- $A_i(f) = \text{coefficient of the sine term of the Fourier series}$
  (imaginary part of the complex coefficient)

When one calculates the Fourier coefficients using the Fast Fourier transform, the set of harmonics for which the periodogram is calculated are dependent on the total length of the sample analysed.

\[ f = \frac{k}{T}, \quad k = 1, 2, 3 \ldots N/2 \]

The periodogram was first introduced by Schuster (1898). Unfortunately, it could not be fully exploited because of the large quantity of calculation involved. Jones (1965) indicated that for moderate sample sizes (1,000 points) the modern computer made calculation of the periodogram feasible. In the same year the Fast Fourier transform appeared and made the periodogram a definite proposition.

Briefly, the periodogram describes the relative energy distribution amongst the frequencies of the turbulent fluctuations.

![Log-log Plot of Periodogram](image.png)

Fig. (3-5) Log-log Plot of Periodogram
Generally, the periodogram is plotted on a log-log scale and the form shown in figure (3-5) is usually obtained.

In the case of a homogeneous, isotropic turbulent field, the shape of the periodogram, also called the Taylor one-dimensional energy spectrum, can be discussed theoretically (Lin and Reid 1963). The name is such as only one component of the velocity is considered. Kolmogorov predicted that at sufficiently high Reynolds numbers of the turbulence (> 74), the energy spectrum function takes on the universal form

\[ S(f) \propto f^{-5/3} \]

The Reynolds number used is defined in terms of the Taylor microscale and root mean square of the velocity fluctuations.

On the log-log plot, one would therefore expect the slope of the periodogram to be approximately -5/3. This fact can be used to check the assumption of isotropy, even when the Reynolds number of the turbulence is not too high (Bradshaw 1967). The Taylor micro and macro length scales can be obtained from the periodogram.

\[ \tau_T = \frac{U}{2\pi} \left( \int_0^\infty f^2 S(f) df \right)^{-1/2} \]

\[ A_T = \frac{U \lim}{f \to 0} f S(f) \]

This is possible as the autocorrelation and periodogram are Fourier transform pairs.

In general, it is easier to calculate the Taylor length scales using the autocorrelation.

Besides describing the turbulent field, the periodogram can sometimes be used to check if any vibrations due to external sources are present (fan speed, 50 cycle hum).
3.3.5 Moving Axis Autocorrelation

To apply Phillips (1957) theory to the Reynolds shear stress profile one requires the convected integral time scale. In his theory Phillips indicates that the rate of change of Reynolds shear stress in a shear layer is proportional to the convected integral time scale.

For the hot-wire configuration in figure (3-6)

\[
\begin{align*}
\text{the normalised cross correlation of the two } u \text{ velocity fluctuations is defined as } \\
\rho_{uu}(r, \tau) = \frac{\int \bar{u}(x, t) u(x+r, t+\tau) \, dy}{\int \bar{u}^2(x, t) \, dy} \int \bar{u}^2(x+r, t+\tau) \, dy}
\end{align*}
\]

For a turbulent field a curve similar to figure (3-7) is usually obtained.

![Fig. (3-7) Cross Correlation $R_{uu}(r, \tau)$](image-url)
If the measurements are now made for various values of \( r \) the following series of curves is obtained.

![Fig. (3-8) Moving Axis Autocorrelation](image)

Fig. (3-8) Moving Axis Autocorrelation

Obviously the maximum values of each curve occur at the time delay coinciding with the 'convection velocity' of the turbulence. The convection velocity should be the same for each wire separation. A discussion on the convection velocities is given by Mills (1964) and a possibly better convection velocity defined.

The dotted line, passing through the maximum of each curve, defines the autocorrelation of the turbulence in a frame of reference moving with the convection velocity of the turbulence.

According to Davies, Garrett et al. (1963) the shape of the moving axis autocorrelation gives an indication of how fast the turbulence pattern is changing. If the turbulent field is regarded as frozen, then the maximum of each curve \( (R_{uu}(r,\tau) \text{ vs } \tau) \) should be the same. The amount by which it decreases shows the amount by which the field has changed while being convected downstream.

The integral under the moving axis autocorrelation is defined as the convected integral time scale. It is an indication of the time it takes for the particular turbulence pattern to disappear.
3.3.5 The Cross Spectrum

The cross spectrum of two signals is a complex function. The real part is called the co-spectrum while the imaginary part is called the quadrature-spectrum.

Thus far the cross spectrum has not been used much in turbulence analysis, its physical interpretation being obscure, but is (in this analysis) calculated so that the cross correlation, which has a physical interpretation, can be obtained (Townsend (1956)).

Consider two random, stationary, complex series

\[ X(kh) \] and \[ Y(kh) \]

where \( k = 1, 2, ..., N \).

One can calculate the Fourier series coefficients \( A(f) \) and \( B(f) \) and obtain the cross spectrum

\[ S_{xy}(f) = A(f)B^*(f) \]

Where \( * \) means complex conjugate.

Thus

co-spectrum \( = A^*B + A^*B^f \)

and

quadrature-spectrum \( = A^*B - A^*B^f \)

\( A^f \) = imaginary part of the Fourier series coefficient of \( X \).

In this analysis the cross spectrum of the \( u \) and \( v \) velocity fluctuations is calculated.

3.3.7 The Cross Correlation

The cross correlation function indicates the dependence of one set of data on another set of data displaced by a time delay \( \tau \). For two continuous series \( x(t) \) and \( y(t) \), the cross correlation is defined as

\[ R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt \]
The cross correlation does not necessarily have a maximum at $\tau = 0$ and is also not an even function like the autocorrelation. Some useful properties of the cross correlation are listed below:

1) $R_{xy}(\tau) = R_{yx}(-\tau)$
2) $|R_{xy}(\tau)|^2 \leq R_x(0)R_y(0)$
3) $|R_{xy}(\tau)| \leq \frac{1}{2}[R_x(0) + R_y(0)]$

A normalised cross correlation function may be defined as

$$p_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_x(0)R_y(0)}}$$

The above only applies when the two series have zero mean values. When $R_{xy}(\tau) = 0$ for all time displacements then the two series are statistically independent. If $x(t)$ and $y(t)$ were statistically independent and had mean values of $\bar{x}$ and $\bar{y}$ then the cross correlation would be $\bar{x}\bar{y}$ for all time displacements.

In turbulence studies cross correlations of signals at different points in the flow are used to obtain information on eddy sizes and movement. They give a "picture" of the turbulent flow. Cross correlations of the various velocity components give an idea of the dependence of the fluctuations on one another, while the value at $\tau = 0$ gives values of the Reynolds shear stresses.
3.4 The Measurements Taken

3.4.1 Single Wire Measurements

Using a single hot wire the following quantities were measured in the turbulent shear layer described in Chapter V.

1) Mean velocity

2) Turbulent intensity of the $u$ velocity fluctuations

$$\text{Intensity} = \frac{u'^2}{U}$$

where $u'^2$ = root mean square of fluctuations

$U$ = mean velocity

3) The periodogram of the $u$ velocity fluctuations.

4) The autocorrelation of the $u$ velocity fluctuations. From these the Taylor micro and macro scales were obtained and hence the Kolmogorov length scale.

3.4.2 Two Wire Measurements

Using an $X$-array probe the following quantities were measured:

1) Mean velocity

2) Turbulent intensities of the $u$ and $v$ velocity fluctuations

3) The periodograms for the $u$ and $v$ velocity components

4) The autocorrelations for the $u$ and $v$ velocity components

5) The Reynolds shear stress $\tau_{xy} = -uv$

6) The cross spectrum of the $u$ and $v$ components

7) The cross correlation of the $u$ and $v$ components

Using two hot-wire probes as described in 3.3.5 one obtained:

8) The moving axis autocorrelation

9) The convected wire scale

3.4.3 Conditional Measurements

Using intermittency programs developed by Dreyer (1973), single-wire data was separated into turbulent and non-turbulent zones. The programs developed here were then used to calculate the quantities in 3.4.1 for each zone.
CHAPTER IV

4. THE COMPUTER PROGRAMS

4.1 General

A brief description of the available computer programs as well as the procedure for the analysis of data obtained from a single hot wire and data obtained from two hot wires is given. The general arrangement of the digital side of the analysis is described. More detailed descriptions are to be found in Appendix (II). The analogue side of the setup is described in Chapter V.

The system is not designed for on-line digital analysis. During the experiment, useful data is stored as an analogue signal on the instrumentation tape recorder. Having an analogue copy of the signal allows for versatility in the digitising process as the data can be redigitised at a different sampling rate if necessary. Recording a clock signal onto an extra channel of the tape recorder makes it possible to return to a particular section of data. This is necessary when two signals are to be cross correlated and they must therefore start at the same instant in time.

All the programs have been written in FORTRAN IV, to be processed by the I.B.M. 360/50 digital computer at the University of the Witwatersrand Computer Centre. Since the programs require disc space, magnetic tapes, private Libraries, and the Calcomp plotter, the Job Control Language can become fairly complex. Special 'In-line Procedures' have been written to overcome this as well as to make executing the same program a number of times in the same job, but with different data, a simple matter.
The majority of the programs and subroutines are stored on disc. In case of the Library being damaged they are all also available on punched cards.

4.2 Data

The main means of supplying data to the programs is from magnetic tape as the digitiser writes the turbulent velocity samples onto magnetic tape in Integer *2 form. Besides being used as an input device, magnetic tapes are also used to store the data.

Other data has to be supplied by means of punched cards, while the program NLBOSE writes information onto temporary disc space. This can be used as data by subsequent programs.

4.3 Results

The results from the various programs are either printed on the line printer, plotted using the Calcotip plotter, written onto temporary disc space or else written onto magnetic tapes. A special program NLSTORE has been written to transfer results calculated by NLBAUTO or NLBCROSS (which calculate autocorrelations and cross correlations respectively) onto magnetic tape. Temporary disc space is space available only for the duration of the Job.

4.4 The Programs

All the programs and subroutines mentioned in this section are stored in the Library ACM.NLB.LIB. Programs not used regularly are not on the Library, but it is no problem to put them there if necessary. The name by which each program or subroutine is referenced in the following sections is the same as the name of the module in ACM.NLB.LIB, that contains it.


**STATSS**
Subroutine STATSS calculates various statistics (mean, standard deviation, minimum, maximum, total) of an input series.

**CHECK**
Subroutine CHECK performs a Students "t" test on the mean of a set of values.

**NLBONE**
This is a large general purpose program, executed by NLBCROSS or NLBAUTO. It is discussed further later in this section.

**PGRAM**
Subroutine PGRAM calculates the periodogram of an input series.

**STORF**
Program NLBSTORE transfers data written onto disc by NLBONE, NLBAUTO or NLBCROSS onto magnetic tape.

**NLBCNV**
Program NLBCNV converts Integer *2 values supplied by the digitiser into Real *4 values.

**CHECK**
Program NLBCHEK prints on the line printer the first block of any file on a tape containing Integer *2 values.

**HANNIN**
Subroutine HANNIN windows an input array using the Hamming coefficients.

**HANNIN**
Subroutine HANNIN windows an input array using the Hanning coefficients.

**GRAPH**
Subroutine GRAPHE plots a single graph using the Calcomp plotter.
**SMOOTH**

Subroutine SMOOTH is used to smooth an input series using a triangular smoothing function.

**REMOVE**

Subroutine REMOVE calculates the mean and any linear trend in an input sample and then removes them if they are larger than a predetermined value.

**AUTOC**

Subroutine AUTOC calculates the autocorrelation of a time series, using the output from subroutine PREAD as its input.

**READS**

Subroutine READS is used to read arrays larger than 1024 samples when the data being read is blocked in blocks of 1024.

**RHARI**

Subroutine RHARI performs a Fast Fourier transformation (or inverse transformation) on a real time series.

**NBHIST**

Subroutine NBHIST is used to plot a histogram using the Calcomp plotter.

**MOMENT**

Subroutine MOMENT calculates some higher order skewness factors for a time series.

### 4.5 Method of Analysis

Generally the following procedure is used to analyze data on an integer *2* tape. See Appendix II for program descriptions.

1. The integer *2* tape is checked by running NLSCHECK. One can see whether the values are reasonable. By digitizing a sine wave one can check if the digitizer is functioning normally.
Using NLBCONV the files that are to be analysed are converted into Real *4 form and then stored on another magnetic tape.

The data on this tape can be examined by using either program NLBANY or NLBANYG.

NLBSMP is used on a few samples to check if the sampling rate is sufficient. If not, the analogue signal could be digitised again at a faster rate. See Appendix II.

If the data consists of two simultaneous records, e.g., from a X wire, then it is possible to separate the two signals into the u and v velocity fluctuations. This is achieved using NLBSEP. The u and v signals are then stored on tape.

The signals are then in a form that can be analysed by one of the general programs NLBAUTO or NLBCROSS. NLBAUTO is used for analysis of single series, while NLBCROSS is used when cross spectra and cross correlations are required. Both programs in fact execute the program NLBONE. The difference is that NLBCROSS analyses two single series simultaneously, calculating cross correlations as well as the autocorrelations for each series.

NLSTORE can then be used to transfer the results from the temporary disc space used by NLBAUTO or NLBCROSS onto magnetic tape. They can then be analysed further at a later stage.

The results from the magnetic tape can then be plotted on the Calcomp plotter. Graphs produced by these programs can be found in Chapter VI.

NLBONE produces two signals, one in the turbulent zone and one in the non-turbulent zone, from the original signal and an intermittency signal.
4.6 The General Program NLBONE

The total signal (a complete file on the magnetic tape) is analysed using samples of 1024, 2048, 4096 or 8192 values. The sample size is specified by the user. If NLBCROSS is executed one of the signals is transferred to temporary disc space. This makes reading alternately from the two signals more efficient. The program then reads a sample from one of the signals and proceeds to analyse it.

4.6.1 Quantities Calculated

i) Various statistics are obtained using subroutine STATSS. To avoid having to inspect the complete signal to find the minimum and maximum values, 4 times the standard deviation of the first sample is used as the range in amplitude of the signal.

ii) The mean and any linear trend is calculated and removed if necessary.

iii) The periodogram is calculated. It is flattened if so required by the user.

iv) The autocorrelation is calculated using the output from subroutine - PGRAM used above in iii).

v) When more than one sample has been analysed, a Students "t" test is performed on the standard deviations of each sample. If the confidence interval is narrow enough, calculations cease.

vi) If NLBCROSS is being executed, the equivalent sample from the other signal is read and if) - vi) performed.

vii) The cross spectrum of the two samples is then calculated.

viii) The cross correlation of the two samples is then calculated.

ix) The various quantities calculated are added to those of the previous samples and then stored on a temporary disc data set.
x) The program then reads the next sample from the file until the
specified number of samples have been analysed.

4.5.2 Output of NIBONE

Output for each sample analysed is produced as well as overall and averaged
properties of the complete signal.

For each sample the following is printed on the line printer:

i) Total
ii) Mean
iii) Standard deviation
iv) Minimum value
v) Maximum value
vi) Sample r.m.s. value
vii) Sample slope
viii) The number of samples outside the range considered for the
      amplitude probability density function.

The above quantities are useful for checking that the data is correct.
It is easy to read 'garbage' from a magnetic tape or disc and
occasionally the digitiser writes incorrect data. Improbable results
can usually be picked out by following the minimum and maximum values
of each sample.

Averaged values of the following quantities over the whole signal
analysed are also printed out

i) Overall total
ii) Overall mean
iii) Averaged standard deviation
iv) Overall minimum
v) Overall maximum
vi) Overall amplitude probability density function
vii) Averaged periodogram
viii) Averaged autocorrelation
ix) Overall r.m.s. value
x) The confidence interval for 90% significance Student "t" test.
The quantities (vii), (vi) and (viii) are also written onto disc.
If NLBCROSS is executed, the above quantities are produced for each of
the signals as well as the following:
 i) Averaged co-spectrum
 ii) Averaged Quadrature-spectrum
 iii) Averaged cross correlation
These quantities are also written onto disc and can therefore be
manipulated further by executing another program immediately after
NLBAUTD or NLBCROSS.
CHAPTER V
5. EXPERIMENTAL APPARATUS AND PROCEDURE

This chapter is in three sections, one dealing with the turbulent shear layer being studied, the second describes the measuring equipment while the third deals with the experimental procedure.

5.1 The Turbulent Shear Layer

A wind tunnel, discussed in detail by Drayer (1973), was constructed so as to produce two streams of air with different but uniform velocities at the exit. These two streams then mixed in a perspex test section containing port holes to allow the hot-wire probes to be inserted. Fig. (5-1) shows the wind tunnel and indicates the physical dimensions of the shear layer produced.

Mean velocity and turbulence intensity profiles can be found in Chapter VI. The turbulence intensities indicate that it is possible to use the anemometers in the constant temperature mode.

More information concerning the shear layer is given below:

1) At the entrance to the test section the fast side velocity is 17.5 m/sec and is flat to within 3%.

2) The slow side velocity is 7 m/sec and is flat to within 1.5%.

3) The layer was two dimensional to within 0.5% on the slow side, 0.6% on the fast side.

4) Down the length of the test section (1 metre) the mean velocity increased 8% in the slow region and 3% in the fast region. The increase is due to the expanding mixing region and the growth of boundary layers.
Fig. (5-1) The Wind Tunnel
v) The layer obtains similarity less than 30 cms downstream of the splitter plate.

v') The approximate width of the shear layer is given in the table below:

<table>
<thead>
<tr>
<th>Distance (cms)</th>
<th>16,9</th>
<th>31,9</th>
<th>45,9</th>
<th>59,9</th>
<th>73,9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (mm)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

vii) The traversing mechanism is accurate to within 0,5 mm.

5.2 The Analogue Apparatus

The analogue equipment used consisted mainly of DISA equipment. Because of its complexity, the setup is best described using figure (5-2). Basically the system consists of two hot-wire anemometers and their peripherals. An instrumentation tape recorder is used to record signal that is to be analysed. The convected integral time scale was measured using the analogue correlator as it was not intended to take intermittency into account.

The various units are discussed below, the reference numbers relating to figure (5-2).

1. All the hot-wire probes used were supplied by DISA. The models used in this work were:

i) 55F11 Gold plated probe with a straight sensor. The platinum plated tungsten sensor is approximately 3 mm. in length and 5 μm. in diameter. The ends are gold plated leaving a sensing length of 1,25 mm. The widely spaced prongs ensure a low level of interference between the prongs and sensor (≤2%).

ii) 55A38 X-array probes with the sensor plane parallel to the probe axis were used to measure u and v fluctuating components as well as the shear stress. The platinum plated tungsten probes...
are 1.2 mm long and 5 µm in diameter. The two sensors are approximately 1 mm apart and the thermal wake interference is reduced to a negligible level (Jerume, Guftton and Patel (1971)).

2. The hot-wire anemometers used are DISA 55D01, operated in the constant-temperature mode. The frequency response is dependant on the probe type and probe operating temperature. For probes similar to those used for this work the frequency response curve is flat up to 10 KHz.

3. Two DISA 55D10 lineariser units are used to linearise the voltage versus velocity signal from the anemometers. The frequency response curves of the instruments are flat up to 10 KHz.

4. The digital voltmeter consisted of a Hewlett Packard 2212A voltage to frequency converter connected to a Hewlett Packard digital counter. Generally the counting time was 1 second although periods of up to 10 seconds are possible. The average value is displayed visually. The voltmeter could also be connected to a printer.

5. The root-mean-square voltmeter used was a DISA model 55035. Voltages proportional to the mean square, root mean square and squared value of the input signal are available at terminals. The input voltage range and integrator time constant can be varied.

6. DISA 55025 auxiliary units are used to band limit the signals. The high pass filter removed the dc component while the low pass filters were set at 10 KHz, the frequency limit of the tape recorder. The unit can be used to invert a signal or find the difference between two signals. The analogue filter roll-off is 6 dB/octave.
The instrumentation tape recorder used is a Philips Ana-log 7 with four channels plus a voice track. Signals can be recorded at one speed and played back at another without any distortion, the greatest speed being 76 cm/sec. Signals can be recorded either directly or on frequency modulation. The F.M. frequency response at the various speeds is:

- 76 cm/sec: 0 - 10 KHz
- 38 cm/sec: 0 - 5 KHz
- 9.5 cm/sec: 0 - 1.25 KHz
- 2.375 cm/sec: 0 - 312 Hz

The input voltage range can be varied but the output range is constant at ±1 volt.

8. A DISA 55075 time delay unit in conjunction with the analogue correlator are used to measure the cross correlation between the signals from the two anemometers. Its operation is based on the principle that correlation measurements do not require use of the whole analogue signal. It and the correlator are especially constructed to work together. Frequencies of up to 10 KHz can be delayed for periods of up to 30 msec.

9. The DISA 55070 analogue correlator is used to measure cross correlation functions or normalised cross correlations of the hot-wire anemometer signals. For frequencies between 4 Hz and 10 KHz the frequency response curve is flat. The input can be two signals (with or without a time delay between them) or else it can be connected to the time delay unit. The integration time for the time mean value amplifiers can be varied. Terminals on the correlator make it possible to obtain voltages proportional to the following functions:
i) The correlation function
ii) Sum of amplified input signals
iii) Difference of amplified input signals
iv) The instantaneous squared value of the sum of the amplified input signals
v) Instantaneous squared values of the difference between the amplified input signals
vi) The average value of iv)
vii) The average value of v)

10. The digitiser used to convert the analogue input onto a digital tape that can be read by computer is situated at the Bernard Price Institute for Geophysical Research.

The input range has a maximum of ±5 volts and the digitised values range between ±2048. The output is on magnetic tape in an I.B.M. compatible Integer *2 form. The data is recorded in blocks of 1024 values. Sampling rates of up to 10 KHz can be obtained. Generally a slower playback speed on the analogue tape deck was used and the digitiser operated at about 4 KHz.

By recording a clock signal on one of the channels on the analogue tape, it is possible to start the digitising process at a particular time. This is used when signals from two different channels are to be cross correlated.
Fig. (5-2) The Analogue Equipment

1. Probe
2. Anemometer
3. Linearizer
4. DVM
5. RMS Voltmeter
6. Auxiliary Unit
7. Time Delay Unit
8. Correlator
9. Tape Recorder
5.3 The Experimental Procedure

Before doing an experimental run the analogue equipment must be allowed to warm up for a couple of hours. Generally the procedure then followed was:

1) The probes to be used are calibrated using the calibration rig. The results are then fitted to King's law using a least squares technique and the necessary quantities for the lineariser calculated.

ii) If an X-array probe is to be used it is calibrated in the position that it is to be operated in. This allows the true mean velocity to be obtained.

iii) The linearisers are adjusted. If both anemometers are being utilised the voltage ranges for the two linearisers are made equal. This should give the two calibration curves similar slopes.

iv) A series of voltages and velocities over the velocity range expected are read and the linearity of the calibration curves checked.

v) The probes are then transferred to the traversing mechanism and lined up with the standard markings on the test section walls. A zero value, corresponding to the centre of the test section is read from the Vernier scale on the traversing mechanism.

vi) The probes are connected to their corresponding anemometers, linearisers, auxiliary units, etc. One of the hot wires is monitored by the digital voltmeter and after the high pass filter, the root-mean-square voltmeter. The other channel is connected to the oscilloscope so that a visual check on the signal being measured can be made.
vii) The probe is moved to the first position of the traverse and the instrument tape recorder started. The cassette contains 11 minutes of tape when recording at 76 cms/sec.

viii) Using the voice track the start of the first reading is indicated. The digital voltmeter and root-mean-square voltmeter values are noted. On the voice track are noted the position in the shear layer, the tape recorder input level, digital voltmeter values and any other relevant information. This is invaluable later when checking to see if the correct reading is being digitised. After about 20 seconds the termination of the reading is indicated on the voice track.

ix) The traversing mechanism is immediately moved to the next position and the second reading taken, noting the details on the voice track. The tape recorder is not stopped during a traverse to avoid unnecessary stopping and starting which wastes tape space.

x) It is important to note down which probe is connected to which anemometer and lineariser. The input level on the tape recorder should be noted as well as the channel being used. These are important when trying to obtain velocities from digitised values. When using a X-array probe one should further note the configuration of the wire with respect to the mean velocity and which channel is recording which probe signal. Without the above information the separation of the two X-array signals into velocity components is difficult.
xi) When measuring the moving axis autocorrelation two anemometers connected to 55F11 probes are used. One probe is inserted into the traversing mechanism. The other is in a holder that can slide down the length of the test section. The position across the stream is determined by inserting perspex spacers between the side of the test section and the movable holder. The two probes are then lined up as close as possible (± 1 mm) and the position across stream read off the Vernier scale.

xii) The downstream probe is then moved a known distance downstream and the cross correlation between the two signals measured for various time delays between them. The probes are then moved further apart and the procedure repeated.

xiii) Another spacer is then inserted, the position across stream noted and then the readings in (xii) repeated.

xiv) Once all the necessary readings have been taken, another set of velocity voltage readings are taken and the linearity of the calibration curves checked again.

xv) The temperature is continually checked to see that it does not vary more than about 2°C.
CHAPTER VI

6. EXPERIMENTAL RESULTS

6.1 Calibration of the Hot-wire Anemometer Probes

The voltage vs. velocity relationship of the hot-wire probe generally follows King's law (Hinze (1959));

\[ V^2 = A + BU^n \]

where

- \( V \) = voltage
- \( U \) = velocity
- \( A, B, n \) are constants

Linearization of the hot-wire signals is performed using an analogue lineariser as applying King's law in the digital programs would be uneconomical. The linearizer used required a knowledge of the constants \( A, B \) and \( n \) and these were obtained by fitting the experimental velocity and voltage values to King's law using the least squares method.

Typical calibration curves for the single-wire and cross-wire probes used in this analysis are shown in figures (6-1) and (6-2).

From the curves the following proportionality constants are obtained:

- Single wires: 4.09 m/s
  volt
- Cross wire: 6.74 m/s
  volt

The hot-wire anemometers were checked for linearity before and after each experimental run.

5.2 Directional Sensitivity of the Hot-wire X-Array Probe

The directional sensitivity of the sensors on the X array probe was measured at various mean-stream velocities for the angle to the mean stream at which the sensor was to be used. To obtain the angle between the sensors and the mean-stream velocity the following procedure was followed.
**Diagram 1:** (f-1) Calibration Curve for Cross-wire Probe

**Diagram 2:** (f-2) Calibration Curve for Single Wire Probe
The sensor was rotated to obtain the maximum voltage on one of its wires and then rotated in the opposite direction to find the maximum voltage on the other wire. Assuming the wires to be symmetrical about the mean flow, the required angle is half that between the two sensors. The directional sensitivity coefficient is then obtained from the equation:

$$U^2(\alpha) = U(0)^2 \left( \sin^2 \alpha + k^2 \cos^2 \alpha \right)$$

where

- $U(\alpha)$ = velocity with wire at angle $\alpha$
- $U(0)$ = velocity with wire perpendicular to the mean flow
- $k$ = directional sensitivity coefficient
- $\alpha$ = angle between the sensor and the mean-velocity direction

The values of $k$ obtained were unsatisfactory as the consistency was poor when one experiment was compared to the next. A short analysis was conducted to see the effect of various angles $\alpha$ and coefficients $k$ on the root-mean-square values of the velocity fluctuations and also on the values of the Reynolds shear stress.

A particular section of tape-recorded data was chosen and various values of $k$ and $\alpha$ inserted into the above equation. Root-mean-square values of the resulting $u$ and $v$ velocity fluctuations as well as the Reynolds shear stress were calculated. The results are shown in Table (6-1).

The results show the following trends:

1) varying only $\alpha$ does not introduce much error.

2) varying the values of $k$ for the two sensors (i.e., different values for each sensor) introduces large variations in the values of the velocity fluctuations. The Reynolds shear stress is not affected to the same degree.
Because the values of $k_1$ and $k_2$ depend directly on the values of $a_1$ and $a_2$, it is essential to measure $a_1$ and $a_2$ accurately. The experimental method used was not accurate enough and hence the cosine law was assumed when separating the $u$ and $v$ velocity fluctuations.

<table>
<thead>
<tr>
<th>$a_1^2$</th>
<th>$a_2^2$</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$\bar{u}^2$</th>
<th>$\bar{v}^2$</th>
<th>$\bar{uv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>47</td>
<td>47</td>
<td>0.00</td>
<td>0.00</td>
<td>38</td>
<td>25</td>
<td>-15</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>0.00</td>
<td>0.00</td>
<td>38</td>
<td>23</td>
<td>-15</td>
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<tr>
<td>55</td>
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<td>0.00</td>
<td>0.00</td>
<td>40</td>
<td>21</td>
<td>-13</td>
</tr>
<tr>
<td>47</td>
<td>47</td>
<td>0.32</td>
<td>0.37</td>
<td>32</td>
<td>36</td>
<td>-16</td>
</tr>
<tr>
<td>45</td>
<td>45</td>
<td>0.32</td>
<td>0.37</td>
<td>34</td>
<td>34</td>
<td>-15</td>
</tr>
</tbody>
</table>

Table (6-1) Results of Directional Sensitivity Analysis

6.3 Measurements in the Free Shear Mixing Layer

6.3.1 General

The results are presented in three sections:

1) Results of a single-wire traverse 30 cm. downstream of the splitter plate.

2) Results of an $x$-array traverse across the same section.

3) Zone results obtained by separating the conventional single-wire into turbulent and non-turbulent zones and then analyzing each zone separately.

As an initial check, a sine wave was generated using a sine wave generator. It was tape recorded, digitised and processed using the digital system available. The frequency of the sine wave was not a harmonic of the total sample length.
Fig. (6-3) shows the periodogram which gives good resolution although smaller peaks are evident due to leakage. Fig. (6-4) shows the amplitude probability density function which is of the correct form.

In Fig. (6-5) the first few cycles of the autocorrelation are plotted. The irregular peaks are a result of the frequency not being a harmonic of the total sample length, and that the Calcomp plotter joins the points on the graph by straight lines.

Only eight samples of 2048 points each were required to satisfy the program statistical check. At that stage the students "t" 90% confidence interval for the mean standard deviation was only 0.02% either side of the mean standard deviation. The signal to noise ratio obtained from the periodogram is approximately 60:1.

The results were written onto magnetic tape and later re-read and plotted using the Calcomp plotter. After smoothing, the periodograms and autocorrelations were decimated and 512 points plotted. This procedure gave adequate detail in the final results.

One concludes that the digital system gives good results on a single sine wave, and performs as it was been intended to work.

6.3.2 Single Hot-wire Traverse 30cms Downstream of the Splitter Plate

At each position marked in Fig. (6-6) a section of hot-wire anemometer signal was tape recorded and digitised at about 32 500 samples per second. Thirty sections each consisting of 2048 points were analysed for each position. The main analysis required 14 minutes of computer time. After averaging over the 30 samples, the Students "t" test on the standard deviation of the samples indicated that the 90% confidence interval was about 5% either side of the mean standard deviation.
Fig. (6-3) Periodogram of the Analogous Sine Wave
Fig. (6-1) Amplitude Probability Density Function of First Tumor

Fig. (6-5) Autocorrelation of Analogous Data Wave
The position across the shear layer was rendered dimensionless by using the variable \( y \) where \( y = \frac{Ax}{30} \) \( y = 0 \) where the mean velocity is half the value of the two mean-stream velocities. (Ax in cms and 30 represents 30 cms downstream of the splitter plate).

Analogue measurements of the mean velocity and r.m.s. values of the longitudinal velocity fluctuations are shown in Table (6-2). Superimposed on the r.m.s. profile of Fig. (6-6) are scaled values of the r.m.s. values calculated from the digital records on magnetic tape. The two curves are close enough to show that there has been no change in the anemometer signal during the tape recording and digitising process.

From the recorded signals and using the digital methods previously described, the periodograms at various points across the shear layer were calculated. These are shown in Fig. (6-7). Interesting points to note are:

i) On the edges of the mixing layer, particularly on the low velocity side, there are peaks in the periodograms occurring at frequencies of about 5.5 KHz, 6.7 KHz and 10 KHz. Peaks at these high frequencies are most unlikely in this turbulent flow. Using a wave analyser and a microphone to check for fan noise or vibration did not help in identifying the frequencies. The peaks were found to persist in the cross-wire results and hence are probably frequencies introduced by the digitiser. They are of a low intensity and only appear on the low intensity edges of the turbulent mixing layer. They could be removed by digital filtering techniques.

ii) The periodograms calculated in the region where the mean velocity is flat but the r.m.s. values are still changing have a low
Fig. (6-6) Mean Velocity Profile and R.M.S. Values of the u Fluctuations (10 cm downstream)
Table (6-2)
R.M.S. and Mean Velocity Profiles 30 cm Downstream

<table>
<thead>
<tr>
<th>Position</th>
<th>File No.</th>
<th>$\frac{u^2}{A}$ Arbitrary</th>
<th>$\sqrt{\frac{u^2}{A}}$ Units</th>
<th>$\sqrt{\frac{u^2}{A}}$ Volts (Scaled)</th>
<th>$\sqrt{\frac{u^2}{A}}$ Analogues Volts</th>
<th>Mean Volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.080</td>
<td>18</td>
<td>$9.1 \times 10^2$</td>
<td>30.1</td>
<td>0.033</td>
<td>0.031</td>
<td>4.82</td>
</tr>
<tr>
<td>-0.046</td>
<td>19</td>
<td>$22.3 \times 10^2$</td>
<td>47.2</td>
<td>0.052</td>
<td>0.053</td>
<td>4.62</td>
</tr>
<tr>
<td>-0.033</td>
<td>20</td>
<td>$237.3 \times 10^2$</td>
<td>154.0</td>
<td>0.171</td>
<td>0.167</td>
<td>4.69</td>
</tr>
<tr>
<td>+0.020</td>
<td>21</td>
<td>$10.1 \times 10^4$</td>
<td>317.2</td>
<td>0.352</td>
<td>0.352</td>
<td>3.72</td>
</tr>
<tr>
<td>+0.053</td>
<td>22</td>
<td>$8.3 \times 10^4$</td>
<td>251.2</td>
<td>0.279</td>
<td>0.271</td>
<td>2.67</td>
</tr>
<tr>
<td>+0.087</td>
<td>23</td>
<td>$33.8 \times 10^4$</td>
<td>88.1</td>
<td>0.065</td>
<td>0.062</td>
<td>2.22</td>
</tr>
<tr>
<td>+0.158</td>
<td>24</td>
<td>$8.1 \times 10^2$</td>
<td>22.0</td>
<td>0.024</td>
<td>0.022</td>
<td>2.20</td>
</tr>
<tr>
<td>+0.153</td>
<td>25</td>
<td>$4.8 \times 10^2$</td>
<td>22.0</td>
<td>0.024</td>
<td>0.022</td>
<td>2.20</td>
</tr>
</tbody>
</table>
Frequency bulge. This is in the most intermittent region of the mixing layer and could give an indication of the frequency of the turbulent bursts passing the probe.

iii) The periodograms all flatten out at high frequencies. This is the noise level introduced by the digitisation process and the computational error in the computer programs. The signal was filtered at 10kHz using analogue methods, and playing the signal onto an oscilloscope showed very little frequency content above 4kHz.

The autocorrelations calculated as the inverse Fourier transform of the periodograms are shown in Fig. (6-8). A parabola of the form

\[ y = 1 - \frac{x^2}{2\lambda^2} \]

where \( y \) = correlation coefficient
\( x \) = time delay

was fitted to the first few points and the Taylor's microscale at the various positions calculated (see Table (6-3) and Fig. (6-9)). The Taylor's microscale at the centre of the shear layer compares favourably with that obtained by Hygnanski and Fiedler (1970).

Values for the Kolmogorov length scale were estimated using equation (3-9) and the values of the Taylor's microscale already calculated. These are shown in Table (6-3).

The Reynolds number based on the longitudinal microscale and the r.m.s. values of the \( v \) velocity fluctuations are shown in Table (5-3).

From the shapes of the autocorrelation curves the flow on the edges of the mixing region has a different structure to that in the mixing region. The values of \( Af \times v \) in Table (6-3) show a reasonably
**Fig. (5-3)** Taylor's Microscale Profile (30 cm downstream)

**Fig. (6-10)** Skewness and Kurtosis Profiles (30 cm downstream)
<table>
<thead>
<tr>
<th>Position y</th>
<th>λ</th>
<th>Mean Velocity U m/sec</th>
<th>u'</th>
<th>Af</th>
<th>n</th>
<th>Re_λ</th>
<th>Af^U/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.080</td>
<td>0.140</td>
<td>19.7</td>
<td>2.76</td>
<td>0.699</td>
<td>0.210</td>
<td>44</td>
<td>13.8</td>
</tr>
<tr>
<td>-0.046</td>
<td>0.155</td>
<td>19.7</td>
<td>3.05</td>
<td>0.632</td>
<td>0.180</td>
<td>76</td>
<td>12.5</td>
</tr>
<tr>
<td>-0.013</td>
<td>0.155</td>
<td>19.2</td>
<td>2.98</td>
<td>0.655</td>
<td>0.087</td>
<td>245</td>
<td>8.7</td>
</tr>
<tr>
<td>+0.020</td>
<td>0.185</td>
<td>15.2</td>
<td>2.92</td>
<td>0.578</td>
<td>0.067</td>
<td>494</td>
<td>8.8</td>
</tr>
<tr>
<td>+0.053</td>
<td>0.217</td>
<td>10.9</td>
<td>2.56</td>
<td>0.717</td>
<td>0.068</td>
<td>317</td>
<td>8.8</td>
</tr>
<tr>
<td>+0.087</td>
<td>0.248</td>
<td>9.1</td>
<td>2.26</td>
<td>0.751</td>
<td>0.140</td>
<td>71</td>
<td>6.9</td>
</tr>
<tr>
<td>+0.120</td>
<td>0.124</td>
<td>9.0</td>
<td>1.67</td>
<td>1.033</td>
<td>0.170</td>
<td>25</td>
<td>9.3</td>
</tr>
<tr>
<td>+0.153</td>
<td>0.155</td>
<td>9.0</td>
<td>1.12</td>
<td>1.241</td>
<td>0.160</td>
<td>13</td>
<td>11.2</td>
</tr>
</tbody>
</table>

Table (6-3)
Table of the Following Turbulence Measurements:

λ = Taylor's microscale
Af = Taylor's macroscale
n = Kolmogorov length scale
constant value in the mixing region with those on the edges increasing. A similar structural difference was observed by Wygnanski and Fiedler (1970). A similar profile was observed in the cross-wire traverse (Fig. (6-18) and Fig. (6-19)).

From the recorded signals, amplitude probability density functions were obtained at the various points. The profile is shown in Fig. (6-11). Using this data the skewness and kurtosis profiles across the shear layer were calculated, where

\[
\text{skewness} = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^3 f_i}{\left(\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^2 f_i\right)^{3/2}}
\]

and

\[
\text{kurtosis} = \frac{\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^4 f_i}{\left(\frac{1}{N} \sum_{i=1}^{N} (x_i - \overline{x})^2 f_i\right)^2}
\]

and

\[N = \text{number of intervals.}\]
\[x_i = \text{value of the midpoint of the interval.}\]
\[\overline{x} = \text{mean value.}\]
\[f_i = \text{number of values in a particular range.}\]

In this analysis \( N = 35 \). The various profiles are shown in Fig. (6-10). In the shear layer studied the kurtosis profile exhibited a similar shape to that obtained by Wygnanski and Fiedler (1970). The different flow conditions at the edge of the shear layer probably account for the differences. Unusually large values of the skewness are obtained on each side of the shear layer where the mean-velocity profile flattens out. The profile is different to that of Wygnanski and Fiedler (1970).
<table>
<thead>
<tr>
<th>Position y</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.080</td>
<td>5.7</td>
<td>4.0</td>
</tr>
<tr>
<td>-0.046</td>
<td>16.1</td>
<td>3.7</td>
</tr>
<tr>
<td>-0.013</td>
<td>181.6</td>
<td>5.0</td>
</tr>
<tr>
<td>+0.020</td>
<td>34.2</td>
<td>2.8</td>
</tr>
<tr>
<td>+0.053</td>
<td>-174.4</td>
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</tr>
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<td>+0.087</td>
<td>17.0</td>
<td>3.7</td>
</tr>
<tr>
<td>+0.120</td>
<td>2.5</td>
<td>3.2</td>
</tr>
<tr>
<td>+0.153</td>
<td>1.4</td>
<td>3.3</td>
</tr>
<tr>
<td>+0.187</td>
<td>5.0</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table (6-41)  
Table of Skewness and Kurtosis Values
6.3.3 Cross-wire Traverse 30 cm Downstream of the Splitter Plate

The signals from both wires were tape recorded and digitised at 32.5 KHz.

It was possible to obtain sections of both signals starting at the same time by means of tape recording a clock signal, that activated the digitiser onto the third channel of the tape recorder.

It was found that the digitiser would sometimes write incorrectly onto the magnetic tape. Once the fault began it continued until the digitising stopped. By monitoring the r.m.s. values of the digitised samples it was possible to find 20 consecutive blocks of unaffected data at only four of the measured positions. With so few points available it was not possible to obtain a good profile across the shear layer.

Figure (6-12) compares the r.m.s. values of the two anemometer signals obtained by analogue measurement and those from the digital analysis. The scaling constants for the two signals differ by only 2%.

(Anemometer 1 = 1185, Anemometer 2 = 1214.) These results are also shown in tables (6-5) and (6-6). From these results it can be deduced that the signals considered were not part of a faulty digitising run.

Using a computer program, which separates the u- and v- velocity components from the cross-wire signals according to the equation (3-4), the signals were processed assuming the cosine law i.e.

\[ k_1 = k_2 = D \quad \text{(directional sensitivity)} \]
\[ \theta_1 = \theta_2 = 45^\circ \quad \text{(angle the probe makes with the mean-stream velocity)} \]
Fig. (6-12). R.M.S. Values of Signals from Two Wires of X-Array Probe.
<table>
<thead>
<tr>
<th>Position ( y )</th>
<th>r.m.s. Anem. 1 ( \text{volts} )</th>
<th>r.m.s. Anem. 2 ( \text{volts} )</th>
<th>Mean Vel. ( \text{Volts} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.201</td>
<td>0.026</td>
<td>0.034</td>
<td>2.617</td>
</tr>
<tr>
<td>-0.135</td>
<td>0.028</td>
<td>0.035</td>
<td>2.658</td>
</tr>
<tr>
<td>-0.101</td>
<td>0.033</td>
<td></td>
<td>2.655</td>
</tr>
<tr>
<td>-0.068</td>
<td>0.046</td>
<td>0.053</td>
<td>2.670</td>
</tr>
<tr>
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<td>0.092</td>
<td>0.090</td>
<td>2.692</td>
</tr>
<tr>
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<td>0.215</td>
<td>0.150</td>
<td>2.670</td>
</tr>
<tr>
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<td>0.420</td>
<td>0.250</td>
<td>2.610</td>
</tr>
<tr>
<td>+0.015</td>
<td>0.550</td>
<td>0.320</td>
<td>2.250</td>
</tr>
<tr>
<td>+0.031</td>
<td>0.580</td>
<td>0.340</td>
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</tr>
<tr>
<td>+0.049</td>
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<td>1.420</td>
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<td>1.320</td>
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<td>0.080</td>
<td>0.071</td>
<td>1.310</td>
</tr>
<tr>
<td>+0.132</td>
<td>0.049</td>
<td></td>
<td>1.296</td>
</tr>
<tr>
<td>+0.165</td>
<td>0.039</td>
<td>0.032</td>
<td>1.295</td>
</tr>
<tr>
<td>+0.199</td>
<td>0.032</td>
<td></td>
<td>1.295</td>
</tr>
<tr>
<td>+0.232</td>
<td>0.026</td>
<td>0.027</td>
<td>1.297</td>
</tr>
</tbody>
</table>

Table (6-5)

Analogue Measurements Using X-Array Probe
An attempt was made to measure the $u$- and $v$-velocity fluctuations using the simple analogue circuit described in Appendix III. Table (5-7) gives the analogue measurements of the $u$- and $v$-velocity fluctuations while Table (6-8) shows the results obtained from the digital analysis. The very different scaling factors for the $u$ and $v$ fluctuations and the resulting bad fit of the $v$ fluctuations would indicate a fault in the analogue measurement of the $v$ fluctuations. The results are plotted in Fig. (6-13).

Figs. (6-14) and (6-15) show the periodograms of the $u$- and $v$-velocity fluctuations at the four points considered. Peaks at frequencies similar to those found in the single-wire measurements are also noticeable here. The curves show a higher low frequency content for the $u$ fluctuations.

The Reynolds shear stress was calculated from the digital records and the values obtained used to check Phillips' (1967) theory with satisfactory results. Shear stress values are given in Table (6-9).

### Table (6-6)

**Analogue and Digital r.m.s. Measurements**

<table>
<thead>
<tr>
<th>Position Y</th>
<th>r.m.s. Anom. 1 volts</th>
<th>Digital r.m.s. value volts</th>
<th>r.m.s. Anom. 2 volts</th>
<th>Digital r.m.s. value volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.018</td>
<td>0.215</td>
<td>0.214</td>
<td>0.158</td>
<td>0.156</td>
</tr>
<tr>
<td>+0.015</td>
<td>0.550</td>
<td>0.551</td>
<td>0.318</td>
<td>0.319</td>
</tr>
<tr>
<td>+0.039</td>
<td>0.510</td>
<td>0.515</td>
<td>0.298</td>
<td>0.295</td>
</tr>
<tr>
<td>+0.099</td>
<td>0.060</td>
<td>0.060</td>
<td>0.071</td>
<td>0.071</td>
</tr>
</tbody>
</table>
### Table 2 (5-7)

**Analogue Measurements of u- and v-Velocity Fluctuations.**

<table>
<thead>
<tr>
<th>Position ( y )</th>
<th>Analogue ( \sqrt{\overline{u'^2}} ) volts</th>
<th>Analogue ( \sqrt{\overline{v'^2}} ) volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.214</td>
<td>0.023</td>
<td>0.013</td>
</tr>
<tr>
<td>-0.147</td>
<td>0.025</td>
<td>0.021</td>
</tr>
<tr>
<td>-0.081</td>
<td>0.033</td>
<td>0.025</td>
</tr>
<tr>
<td>-0.047</td>
<td>0.047</td>
<td>0.037</td>
</tr>
<tr>
<td>-0.310</td>
<td>0.064</td>
<td>0.050</td>
</tr>
<tr>
<td>-0.120</td>
<td>0.150</td>
<td>0.082</td>
</tr>
<tr>
<td>+0.003</td>
<td>0.290</td>
<td>0.141</td>
</tr>
<tr>
<td>+0.016</td>
<td>0.360</td>
<td>0.220</td>
</tr>
<tr>
<td>+0.036</td>
<td>0.370</td>
<td>0.290</td>
</tr>
<tr>
<td>+0.053</td>
<td>0.320</td>
<td>0.260</td>
</tr>
<tr>
<td>+0.069</td>
<td>0.200</td>
<td>0.190</td>
</tr>
<tr>
<td>+0.086</td>
<td>0.089</td>
<td>0.104</td>
</tr>
<tr>
<td>+0.103</td>
<td>0.049</td>
<td>0.062</td>
</tr>
<tr>
<td>+0.135</td>
<td>0.029</td>
<td>0.036</td>
</tr>
<tr>
<td>+0.219</td>
<td>0.019</td>
<td>0.024</td>
</tr>
<tr>
<td>+0.286</td>
<td>0.017</td>
<td>0.022</td>
</tr>
<tr>
<td>+0.319</td>
<td>0.016</td>
<td>0.020</td>
</tr>
</tbody>
</table>
### Table (6-8)
Comparison of Analogue and Digital Values of the $u$- and $v$-Velocity Fluctuations

<table>
<thead>
<tr>
<th>Position $y$</th>
<th>File No.</th>
<th>Digital $\frac{u'^2}{\sqrt{\tau}}$ volts</th>
<th>Analogue $\frac{u'^2}{\sqrt{\tau}}$ volts</th>
<th>Digital $\frac{v'^2}{\sqrt{\tau}}$ volts</th>
<th>Analogue $\frac{v'^2}{\sqrt{\tau}}$ volts</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.018</td>
<td>5 &amp; 6</td>
<td>0.130</td>
<td>0.131</td>
<td>0.151</td>
<td>0.075</td>
</tr>
<tr>
<td>+0.015</td>
<td>9 &amp; 10</td>
<td>0.338</td>
<td>0.332</td>
<td>0.334</td>
<td>0.209</td>
</tr>
<tr>
<td>+0.039</td>
<td>13 &amp; 14</td>
<td>0.332</td>
<td>0.332</td>
<td>0.299</td>
<td>0.199</td>
</tr>
<tr>
<td>+0.099</td>
<td>19 &amp; 20</td>
<td>0.058</td>
<td>0.057</td>
<td>0.054</td>
<td>0.079</td>
</tr>
</tbody>
</table>

### Table (6-9)
Digital Values of the Reynolds Shear Stress

<table>
<thead>
<tr>
<th>Position $y$</th>
<th>UV arbitrary units</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.018</td>
<td>$-1.4 \times 10^4$</td>
</tr>
<tr>
<td>+0.015</td>
<td>$-1.4 \times 10^5$</td>
</tr>
<tr>
<td>+0.039</td>
<td>$-1.2 \times 10^5$</td>
</tr>
<tr>
<td>+0.099</td>
<td>$-6.0 \times 10^2$</td>
</tr>
</tbody>
</table>
Fig. (6-11) Calculated and Analogous Values of $u$ and $v$ Velocity Fluctuations
Fig. (6-14) Periodogram of the $u$ Velocity Fluctuations (Log-Log)

Fig. (6-15) Periodogram of the $v$ Velocity Fluctuations (Log-Log)
The amplitude probability density functions are shown in Figs. (6-16) and (6-17). A similar profile was obtained for the u fluctuations using the single-wire probe.

In Figs. (6-16) and (6-19) the first section of the autocorrelation curves for the u- and v-velocity fluctuations is shown. The different shapes of the curves across the shear layer correspond to those obtained for the single-wire traverse (Fig. (6-9)). The v-velocity fluctuations reach zero correlation before the u-velocity fluctuations and also become negative. Both correlations have the form for turbulence consisting of two eddy ranges (Townsend (1956)). This form is also noticeable in the single-wire measurements.

The cross-correlation between the u- and v-velocity fluctuations is of interest as $R_{uv}(t = 0)$ is an estimate of the Reynolds shear stress. Comparing this value with that obtained by multiplying the fluctuations shows good agreement. See Table (7-5). The cross correlation, quadrature spectrum and co-spectrum at the four points considered are shown in Fig. (6-20), Fig. (6-21), Fig. (6-22) and Fig. (6-23).

The cross correlation is zero for time delays of 0.001 s except for the high degree of periodicity at $y = +0.099$ on the low velocity side of the shear layer. The periodicity of approximately 180 Hz is of the same order as the low frequency bulges seen in all the calculated periodograms.

The co-spectrum and quadrature spectrum functions are not generally calculated in turbulence analysis and it is interesting to note the definite manner in which the correlation peaks at 180 Hz are shown.

In general the single-wire and X-array probe results compare favourably, indicating a reliable analysis system.
Fig. (6-17) Amplitude Probability Density Function of \( \nu \) Fluctuations
Fig. (6-18) Autocorrelations of the u-Velocity Fluctuations

Fig. (6-19) Autocorrelations of the w-Velocity Fluctuations
6.3.4 Zone Properties of the Turbulent Shear Layer 30cms Downstream of the Splitter Plate

A section of the single-wire probe signal measured in the intermittent region of the shear layer was tape-recorded and digitised at 20 KHz. The intermittency signal associated with the recorded velocity fluctuations was obtained using a slightly modified version of the intermittency program developed by Dreyer (1973). Basically the intermittency decision is made by considering both the frequency and amplitude of the velocity fluctuations. After deciding which points are turbulent, a data sequence is formed by adding the turbulent points one after the other.

The errors introduced will be a function of the number of points in a turbulent burst and the digitising rate used. In this analysis turbulent bursts ranged from two points to one hundred points. A reasonable average is twenty sample points. At the sampling rate of 20 KHz, this would give reasonable indications of the frequencies above 1 KHz, i.e. 3 on the log-log plot.

Comparing the periodograms to those from the single-wire traverse indicates that the general shape is probably correct. From Fig. (6-7) the slope of the periodogram does not increase above $-\frac{5}{3}$ in the non-turbulent zone but does in the centre of the shear layer. The turbulent zone and non-turbulent zone results show the same trend. A better result could be obtained by selecting only long turbulent bursts or by smoothing the intermittency signal, i.e. when turbulent points are separated by only a few non-turbulent points, regard all the points as turbulent.
To obtain the 10 blocks of turbulent data it was necessary to process 44 blocks of the original signal. The program analyses 4 blocks of data at a time and gives the calculated intermittency factor for each set. The values (11 of them) varied between 0.17 and 0.28 with an average value of 0.23. Approximately 22 minutes of execution time on the computer were required to analyse the 44 blocks of data. This seems rather expensive as it is only 2.2 seconds of data real time.

The 10 blocks (making 5 samples of 2048 points) of turbulent data and the same quantity of non-turbulent data were analysed and compared with the results of the analysis of 10 sections (20 blocks) of the original signal.

A short section of the measured signal with the associated intermittency is shown in Fig. (6-24). It would seem that the intermittency signal leaves some turbulent sections in the non-turbulent zone.

In Fig. (6-25), Fig. (6-26) and Fig. (6-27) periodograms of the conventional signal, turbulent zone and non-turbulent zone are shown. The apparent large low frequency content in the non-turbulent zone and the almost isotropic form in the turbulent zone are the forms expected for the periodograms. All the periodograms pass through the Kolmogorov slope of $-5/3$ when plotted as a log-log plot. The high frequency content of the non-turbulent zone is partially due to the slightly incorrect intermittency determination.

The autocorrelation curves shown in Fig. (6-28), Fig. (6-29) and Fig. (6-30) also indicate some interesting results. That of the conventional signal has the classic form of two superimposed ranges of eddies. Separating the velocity fluctuations gives two curves showing only one range of eddies (Townsend (1956)). As expected, the large eddy scale is associated with the non-turbulent region while the
Fig. 13-21: Velocity Signal with Superimposed Intermittency Signal
Fig. (6-26) Turbulent Zone Periodogram of the u Fluctuations

Fig. (6-25) Conventional Periodogram of u Fluctuations
Fig. (6-27) Non-turbulent zone Paa (edogram of the $u$ fluctuations)

Fig. (6-28) Conventional Autocorrelation of the $u$ fluctuations
Fig. 6.29 Turbulent Zone Autocorrelation of the $u^2$ fluctuations

Fig. 6.30 Non-turbulent Zone Autocorrelation of the $u$ Fluctuations
Fig. (6-11) Conventional Amplitude Probability Density Function

Fig. (6-12) Turbulent Zone Amplitude Probability Density Function
Fig. 10.3.1 Non-turbulent Zone Area It's Probability Density Function
small eddy scale is associated with the turbulent region.

Studying the amplitude probability density functions in the various regions (Fig. (6-31), Fig. (6-32) and Fig. (6-33)) shows some interesting results. The fluctuations seem normally distributed in the conventional signal while on separating into zones, the turbulent zone velocity fluctuations are very skew with the large velocity fluctuations in the direction of mean flow. This agrees also with Fig. (6-24) which shows "turbulence" for the positive velocity fluctuations more than the negative fluctuations.

Although only a few samples were analysed, the results show the power of the analysis system and also verify conclusions drawn from the conventional measurements.

5.4 Check of Phillips Theory on the Maintenance of Reynolds Shear Stress

Phillips (1967) obtained the relationship

$$\frac{\partial \tau}{\partial x} = v_e \frac{a^2 u(x)}{2}\frac{\partial^2 u(x)}{\partial x^2}$$

where $v_e = \frac{4A}{\pi} = "eddy"$ viscosity

$A = constant$

If the Reynolds stress vanishes at some point $x_0$ where the mean velocity gradient is zero, then the above expression can be integrated by parts.

$$\tau(x) = v_e \frac{a^2 u(x)\partial u}{\partial x} - \int_{x_0}^{x} \frac{\partial u(x)}{\partial x} \frac{a^2 u}{\partial x} dx$$
Assuming the second term to be small compared to the first:

\[ \tau(x) = \frac{v_a(x) A}{2} \]

\[ \tau(x) = \frac{A o}{2} \frac{d u}{d x} \]

\[ \frac{A o}{2} = \frac{\tau(x)}{\frac{d u}{d x}} \]

The convected integral time scale was obtained in the following manner. Using two hot-wire probes the cross correlation between the longitudinal fluctuations for various time delays and wire separations were measured. The maxima of the correlation curves at each point across the stream were used to fit an exponential of the form

\[ y = e^{-ax} + e^{-bx} \]

where

\[ y = \text{correlation coefficient} \]
\[ x = \text{time delay} \]

Table (6-10) shows the following calculated quantities:

1) Value of \( a \) in the exponential function.
2) Value of \( b \) in the exponential function.
3) Value of the time delay axis used as the limit in the integration of the fitted exponential function. \((X)\)
4) The value of the correlation at the integration limit. \((Y)\)
5) The calculated convected integral time scale.

Fig. (6-34) shows a plot of the convected integral time scale and the mean velocity. Fig. (5-35) shows the shape of the cross correlations at one point in the shear layer. The slope of the velocity
<table>
<thead>
<tr>
<th>Position Y</th>
<th>a</th>
<th>b</th>
<th>X mBeC</th>
<th>Y mBeC</th>
<th>θ mBeC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.071</td>
<td>0.09</td>
<td>0.41</td>
<td>58</td>
<td>0.0034</td>
<td>7.0</td>
</tr>
<tr>
<td>0.054</td>
<td>0.10</td>
<td>0.40</td>
<td>50</td>
<td>0.0034</td>
<td>6.2</td>
</tr>
<tr>
<td>0.039</td>
<td>0.04</td>
<td>0.60</td>
<td>125</td>
<td>0.0034</td>
<td>13.3</td>
</tr>
<tr>
<td>0.055</td>
<td>0.08</td>
<td>0.41</td>
<td>79</td>
<td>0.0035</td>
<td>9.1</td>
</tr>
<tr>
<td>0.005</td>
<td>0.07</td>
<td>0.41</td>
<td>73</td>
<td>0.0035</td>
<td>8.5</td>
</tr>
<tr>
<td>-0.012</td>
<td>0.08</td>
<td>0.59</td>
<td>67</td>
<td>0.0033</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Table (6-10)

Calculations for the Convected Integral Time Scale

<table>
<thead>
<tr>
<th>Position Y</th>
<th>Actual Volts</th>
<th>Calculated Volts</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.065</td>
<td>2.517</td>
<td>2.507</td>
<td>0.010</td>
</tr>
<tr>
<td>-0.048</td>
<td>2.519</td>
<td>2.547</td>
<td>0.028</td>
</tr>
<tr>
<td>-0.032</td>
<td>2.516</td>
<td>2.529</td>
<td>0.004</td>
</tr>
<tr>
<td>-0.015</td>
<td>2.474</td>
<td>2.441</td>
<td>0.033</td>
</tr>
<tr>
<td>+0.002</td>
<td>2.344</td>
<td>2.315</td>
<td>0.019</td>
</tr>
<tr>
<td>+0.018</td>
<td>2.164</td>
<td>2.181</td>
<td>0.007</td>
</tr>
<tr>
<td>+0.035</td>
<td>2.025</td>
<td>2.040</td>
<td>0.018</td>
</tr>
<tr>
<td>+0.052</td>
<td>1.885</td>
<td>1.908</td>
<td>0.023</td>
</tr>
<tr>
<td>+0.068</td>
<td>1.763</td>
<td>1.798</td>
<td>0.015</td>
</tr>
<tr>
<td>+0.085</td>
<td>1.753</td>
<td>1.727</td>
<td>0.025</td>
</tr>
<tr>
<td>+0.102</td>
<td>1.745</td>
<td>1.710</td>
<td>0.034</td>
</tr>
<tr>
<td>+0.118</td>
<td>1.738</td>
<td>1.765</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

Table (6-11)

Least Squares Fit for the Mean Velocity Profile
Fig. (6-34) Graph of Convection Integral Time Scale and Mean Velocity (30 cm Downstream)
Profile was obtained by fitting an equation of the form

\[ y = a + bx + cx^2 + dx^3 \]

to the experimental mean velocity using the least squares method. The following equations resulted

\[ y = 2.23 -0.02x -0.021x^2 +0.02x^3 \]
\[ \frac{\partial y}{\partial x} = -0.28 -0.04x +0.06x^2 \]
\[ \frac{\partial^2 y}{\partial x^2} = -0.04 +0.12x \]

The point of inflection occurs at

\[ x = 0.03 \text{ cm} \quad \text{i.e.} \quad y = 0.015 \]

The results are shown in Table (6-11). As the transverse-velocity fluctuations were not measured, the lateral fluctuations \((y^2)\) were used. Table (6-12) shows the values used in the check. The value of the constant \(A\) obtained by Wygnanski and Fiedler (1970) was 0.2, which shows good agreement with the average value of 0.23 obtained in this analysis.

Previously Phillips (1963) applied the theory to the data of Davies et al (1963) for the mixing zone of a jet and obtained a value of \(A = 0.17\). Belkin and Haberstrom (1968) calculated \(A\) for fully developed pipe flow at five different Reynolds numbers and obtained an average value of \(A = 0.33\). Recently Atesman (1971) applied this theory to data from a nearly homogeneous shear flow and obtained \(A = 0.55\).
<table>
<thead>
<tr>
<th>Position y</th>
<th>$\tau_{uv}$ Arbitrary Units</th>
<th>Slope of mean Velocity</th>
<th>$\tau^2$ Arbitrary Units</th>
<th>$\Theta$ msec</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.018</td>
<td>$-1.4 \times 10^4$</td>
<td>-0.19</td>
<td>$5.2 \times 10^4$</td>
<td>7.2</td>
<td>0.20</td>
</tr>
<tr>
<td>+0.015</td>
<td>$-1.4 \times 10^5$</td>
<td>-0.28</td>
<td>$2.6 \times 10^5$</td>
<td>9.0</td>
<td>0.21</td>
</tr>
<tr>
<td>+0.039</td>
<td>$-1.2 \times 10^5$</td>
<td>-0.26</td>
<td>$1.9 \times 10^5$</td>
<td>9.2</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Table (6-12)
Calculations for Check on Phillips (1957) Theory
CHAPTER VII

7. DISCUSSION ON THE ERRORS

The errors involved result from the following steps in the analysis:

1) The analogue instruments.
2) The digitising process.
3) Sampling of a continuous signal.
4) Analysing a limited time of signal.
5) Computational errors.
6) Statistical errors.

An error in all the experimental measurements resulted from the fact that the mean-velocity direction was not perpendicular to the hot-wire probes. This was due to the expanding mixing region.

7.1 The Analogue System

All the analogue equipment is standard equipment and hence the limitations are specified. For all the units the frequency response curves are flat up to 10 KHz. The harmonic distortion of the analogue tape recorder used is less than 0.1%.

Some error will be introduced by the analogue filters in the auxiliary units. Both high pass and low pass filters have a roll-off of 6dB/octave. Some frequencies higher than 10 KHz will therefore be present.

7.2 The Digitising Process

The errors due to quantisation are discussed briefly by Bendat and Piersol (1966) and from a design point of view by Glisson, Black and Sage (1970).

Assuming the quantisation errors have a uniform probability density function over one scale unit, it can be shown that the errors will have a zero mean value and a standard deviation of approximately 0.29 units.
This can be regarded as the quantisation noise in the desired signal.

The digitiser used digitises an input voltage of ±5 volts into ±2048 units. Assuming only half the input voltage is utilized, the signal to noise ratio due to quantification will be

\[
\text{SNR} = \frac{2048}{0.29} = 7070
\]

\[= 20 \log_{10} 7070 \text{ dB}
\]

\[= 18 \text{ dB}
\]

7.3 Sampling Errors

It can be shown (Gissson, Black and Sage (1970)) that if the signal has been sampled at twice the Nyquist Frequency and the signal is band limited, no errors are introduced. Aliasing errors, discussed in Section 2.3 result if these conditions are not met.

In this analysis the analogue filters were set at 70 KHz. Due to the digitising rate of about 32 KHz and the filter roll-off of 6 dB/octave, there will be small aliasing errors present. Filtering the signal and feeding it to an oscilloscope showed very little signal above 4 KHz. The aliasing errors should be minimal.

A sample of data was taken from the cross-wire analysis at \(y = +0.039\) and processed by program NLBSNP (see Appendix II). The results

(a) = 305
(b) = 91
(c) = 102
(d) = 204

show that no aliasing should be present. Only \(y = 0.039\) was chosen as it is in the centre of the shear layer and will contain the highest frequencies. All the signals were digitised at the same rate.
7.4 Analysing a Limited Time of Signal

As a result of truncating the signal length the resolution of the resulting spectrum is limited. If a total time of $T$ seconds is analysed, the maximum resolution that can be obtained is $\frac{1}{T}$ Hz. The errors due to leakage (section 2.4) are proportional to the spectral resolution.

It can be shown (Glisson, Black and Sage (1970)) that when a sinusoid is not one of the frequencies calculated in the associated periodogram, the signal to noise ratio can drop by about 4 dB. This decrease must be taken into account when calculating the minimum input signal to noise ratio of the digitiser.

7.5 Computational Errors

The largest computational errors will be in the Fast Fourier transform routine and only this aspect will be considered. Various workers, Welch (1969), Glisson, Black and Sage (1970) and Graefe (1970) have discussed the computational errors obtained when using fixed point arithmetic in a digital computer. This aspect is important in the design of special purpose computers.

Using the lower error bound (i.e. no overflow occurs during the calculation) gives a result that increases at the rate observed for floating point calculations.

\[
\frac{\text{r.m.s. (result)}}{\text{r.m.s. (error)}} = 0.5^M (2L + 1)^{1/2}
\]

where

\[
M = \text{power of 2 i.e. } N = 2^M
\]

\[
L = \text{number of bits used by the computer to represent the values i.e. } B \text{ bits plus a sign.}
\]
The digitizer used for this analysis represents the data with 11 bits plus one for a sign. Assuming the above relationship, a pessimistic signal to noise ratio is obtained

\[ \text{SNR} = \frac{2048}{0.3 \times \sqrt{0.5}} \]

\[ = 2345 \quad \text{dB} \]

\[ = 20 \log_{10} 2345 \text{ dB} \]

\[ = 55 \text{ dB} \]

It has been shown, Oppenheim and Weinstein (1969) that the errors in the Fourier transform and its inverse are not independent and hence a smaller ratio would be expected for the correlation functions. The computer uses floating point arithmetic with \( B = 15 \) and hence the errors should be considerably smaller than those calculated above.

### 7.6 Characterising the Digital System

Regarding the digitiser and digital analysis programs as a spectrum analyser one can characterise it by specifying the following quantities:

#### 7.6.1 Resolution

The resolution is calculated as \( \frac{1}{T} \text{HZ} \) where \( T \) is the sample length (in seconds) analysed. In all the analyses performed a sample size of 2048 points was used. For the single- and cross-wire analyses the sampling rate was 32.5 KHz giving

\[ \text{Resolution} = \frac{32500}{2048} = 15.65 \text{ HZ} \]

The signal used for zone averages was digitised at 20 KHz giving a resolution of 9.75 HZ.
7.6.2 Dynamic Range

The dynamic range is a rough measure of the ability of the digitiser to pass relative amplitude information contained in the signal. It can be related to the number of binary bits used by the digitiser

\[ DR = 6i + 4 \, \text{dB} \]

where

\[ i - 1 \] = number of binary bits used by the digitiser to represent the data.

For this system

\[ DR = 6 \times 11 + 4 = 70 \, \text{dB} \]

7.6.3 The Minimum Detectable Signal (MDS)

The MDS is the smallest signal to noise ratio at the input that leads to a reliable detection at the analyser output. Generally one requires an output signal to noise ratio of about 10 dB (Gisson, Black and Sage (1970)).

Due to leakage, the input signal to noise ratio will drop by a small amount (+4 dB) and this must be accounted for.

The processing gain (PG) of the analyser gives an indication of the signal enhancement obtained in the analyser

\[ PG = 20 \log \left( \frac{\text{SNR out}}{\text{SNR in}} \right) \, \text{dB} \]

and it can be related to the signal bandwidth and length of the sample processed.

\[ PG = 10 \log_{10} \left( \frac{T}{B} \right) \]

where \( T \) = total sample time

\( B \) = bandwidth of the signal
In this analysis

\[ P_G = 10 \log_{10} \left( 10^8 \times \frac{1}{15.29} \right) \]

\[ P_G = 28 \text{ dB} \]

Assuming the minimum output of 10 dB, the minimum possible signal to noise ratio at the input is

\[ P_G = 20 \log_{10} \left[ \text{SNR out} \right] - 20 \log_{10} \left[ \text{SNR in} \right] \]
\[ P_G = 10 + 4 - \text{MDS} \]

(10 = minimum output, 4 to compensate for leakage)

\[ \therefore \text{MDS} = -14 \text{ dB} \]

A rough check is obtained by considering the sine wave analysis (Section 6.3.1). The signal to noise ratio there is about 60 i.e. 35 dB. The corresponding MDS would be 11 dB which is much larger than the minimum of -14 dB.

If the input noise is white noise, the dynamic range and processing gain should be about the same. If the noise is 1/f noise, then the dynamic range should be two or three times the processing gain. In this case

\[ \text{DR} = 70 \text{ dB} \]
\[ P_G = 28 \text{ dB} \]

7.7 The Statistical Accuracy of the Results

The computational accuracy of the results has been dealt with in the previous section. An indication of the statistical accuracy i.e. the accuracy due to using a limited section of signal and then averaging a finite number of sections, should also be obtained.
As a result of the central limit theorem, the mean value of the standard deviation of each section (2048 points) analysed, should follow the Students "t" distribution and hence this was used to obtain a confidence interval for the mean value. A more detailed description of the test can be found in Section (2.7).

Table (7-1) shows the results of the test for the single-wire traverse. In this analysis the 90% confidence interval was used, so that Table (7-1) and Table (7-2) show the final value of the standard deviation, the percentage each side of the mean that the confidence interval extends and the value of the confidence interval. The 90% confidence interval is less than 5% of the mean value at the centre of the shear layer and increases to about 10% at the low intensity edges of the layer. For the present analysis this accuracy is acceptable.

Table (7-2) shows the results obtained for the X-array probe traverse across the shear layer. The percentages are slightly higher than those for the single-wire analysis but this is probably due to the fact that only 40 instead of 60 blocks of data were analysed. The digitiser was not operating properly when the records at $y = -0.001$, $y = +0.032$, $y = +0.065$ and $y = +0.082$ were digitised. This is shown up well by the far higher percentages i.e. 38% and 44% for the $u$ fluctuations and 39.7% and 45.4% for the $v$-velocity fluctuations. The other percentages give an acceptable statistical accuracy.

Another check was made on the results by using the following relationships involving the autocorrelation and cross-correlation:

1) $R_u(r = 0) = \bar{u}^2$
2) $R_{uv}(r = 0) = \bar{uv}$

See Tables (7-3) to (7-5).
<table>
<thead>
<tr>
<th>$y$</th>
<th>% of $u$ mean</th>
<th>90% Confidence Interval</th>
<th>$u$ velocity fluctuations</th>
<th>% of $u$ mean</th>
<th>90% Confidence Interval</th>
<th>$v$ velocity fluctuations</th>
<th>% of $u$ mean</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.080</td>
<td>4.3</td>
<td>27.4</td>
<td>1.2</td>
<td>-0.035</td>
<td>5.6</td>
<td>81.4</td>
<td>4.7</td>
<td>135.9</td>
</tr>
<tr>
<td>-0.047</td>
<td>3.3</td>
<td>45.6</td>
<td>1.5</td>
<td>-0.018</td>
<td>8.1</td>
<td>216.9</td>
<td>17.6</td>
<td>224.7</td>
</tr>
<tr>
<td>-0.010</td>
<td>4.8</td>
<td>152.3</td>
<td>7.4</td>
<td>-0.001</td>
<td>8.1</td>
<td>6323</td>
<td>2412</td>
<td>6281</td>
</tr>
<tr>
<td>+0.020</td>
<td>2.4</td>
<td>315.6</td>
<td>7.4</td>
<td>+0.015</td>
<td>3.6</td>
<td>570.4</td>
<td>20.7</td>
<td>491.4</td>
</tr>
<tr>
<td>+0.053</td>
<td>2.9</td>
<td>250.1</td>
<td>7.2</td>
<td>+0.087</td>
<td>4.6</td>
<td>570.0</td>
<td>2.6</td>
<td>519.4</td>
</tr>
<tr>
<td>+0.120</td>
<td>5.8</td>
<td>26.3</td>
<td>1.5</td>
<td>+0.153</td>
<td>8.8</td>
<td>19.7</td>
<td>1.7</td>
<td>434.6</td>
</tr>
<tr>
<td>+0.187</td>
<td>8.8</td>
<td>19.7</td>
<td>1.7</td>
<td>+0.187</td>
<td>8.8</td>
<td>19.7</td>
<td>1.7</td>
<td>598.4</td>
</tr>
</tbody>
</table>

Table (7-1)

Results of Statistical Test on the Data from the Single-Wire Traverse

Number of blocks analysed = 60

<table>
<thead>
<tr>
<th>$y$</th>
<th>Mean $u$ St. Deviation</th>
<th>90% Confidence Interval</th>
<th>% of $u$ mean</th>
<th>Mean $v$ St. Deviation</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.035</td>
<td>5.6</td>
<td>81.4</td>
<td>4.7</td>
<td>5.7</td>
<td>135.9</td>
</tr>
<tr>
<td>-0.018</td>
<td>8.1</td>
<td>216.9</td>
<td>17.6</td>
<td>3.6</td>
<td>224.7</td>
</tr>
<tr>
<td>-0.001</td>
<td>8.1</td>
<td>6323</td>
<td>2412</td>
<td>38.7</td>
<td>6281</td>
</tr>
<tr>
<td>+0.015</td>
<td>3.6</td>
<td>570.4</td>
<td>20.7</td>
<td>2.57</td>
<td>491.4</td>
</tr>
<tr>
<td>+0.053</td>
<td>2.9</td>
<td>570.4</td>
<td>19.7</td>
<td>2.3</td>
<td>434.6</td>
</tr>
<tr>
<td>+0.087</td>
<td>3.6</td>
<td>551.9</td>
<td>19.3</td>
<td>2.3</td>
<td>434.6</td>
</tr>
<tr>
<td>+0.120</td>
<td>5.8</td>
<td>551.9</td>
<td>19.3</td>
<td>2.3</td>
<td>434.6</td>
</tr>
<tr>
<td>+0.153</td>
<td>5.8</td>
<td>551.9</td>
<td>19.3</td>
<td>2.3</td>
<td>434.6</td>
</tr>
</tbody>
</table>

Table (7-2)

Results of the Statistical Test on the Cross-wire Data

Number of blocks analysed = 40
### Table (7-3)
**Comparison for u velocity component**

| Y     | A          | B          | C          | |---|---|---|
|-------|------------|------------|------------| |---|---|---|
|       | $R_u(t = 0)$ | $u_2^{(60) \text{Blocks}}$ | $u_2^{(60) \text{Blocks}}$ | | | | |
| -0.018 | $0.49 \times 10^5$ | $0.51 \times 10^5$ | $0.47 \times 10^5$ | | 3.2 | 2.5 | 4.2 |
| +0.015 | $0.33 \times 10^5$ | $0.33 \times 10^5$ | $0.32 \times 10^5$ | | 0.0 | 3.1 | 3.1 |
| +0.049 | $0.31 \times 10^5$ | $0.31 \times 10^5$ | $0.31 \times 10^5$ | | 0.0 | 0.0 | 0.0 |
| +0.099 | $0.34 \times 10^5$ | $0.34 \times 10^4$ | $0.33 \times 10^4$ | | 0.0 | 1.8 | 1.8 |

### Table (7-4)
**Comparison for v velocity component**

| Y     | A          | B          | C          | |---|---|---|
|-------|------------|------------|------------| |---|---|---|
|       | $R_{uv}(t = 0)$ | $u_2^{(60) \text{Blocks}}$ | $u_2^{(60) \text{Blocks}}$ | | | | |
| -0.018 | $0.51 \times 10^5$ | $0.52 \times 10^5$ | $0.52 \times 10^5$ | | 7.9 | 0.0 | 1.9 |
| +0.015 | $0.25 \times 10^6$ | $0.25 \times 10^6$ | $0.26 \times 10^6$ | | 4.0 | 3.9 | 7.7 |
| +0.049 | $0.19 \times 10^6$ | $0.19 \times 10^6$ | $0.19 \times 10^6$ | | 0.0 | 0.0 | 0.0 |
| +0.099 | $0.80 \times 10^6$ | $0.82 \times 10^4$ | $0.68 \times 10^4$ | | 2.4 | 20.0 | 18.0 |

### Table (7-5)
**Comparison for the Reynolds shear stress**

| Y     | A          | B          | C          | |---|---|---|
|-------|------------|------------|------------| |---|---|---|
|       | $R_{uv}(t = 0)$ | $u_2^{(60) \text{Blocks}}$ | $u_2^{(60) \text{Blocks}}$ | | | | |
| -0.018 | $-0.17 \times 10^5$ | $-0.17 \times 10^5$ | $-0.17 \times 10^5$ | | - | - | 21.0 |
| +0.015 | $-0.14 \times 10^6$ | Not | $-0.14 \times 10^6$ | | - | - | 0.0 |
| +0.049 | $-0.12 \times 10^6$ | Calculated | $-0.12 \times 10^6$ | | - | - | 0.0 |
| +0.099 | $-0.66 \times 10^3$ | $-0.66 \times 10^3$ | | | - | - | 8.3 |
The root-mean-square values of the signals are calculated before the data is modified, while the autocorrelation and cross correlation are the last quantities calculated. When calculating the auto- and cross- correlations, values for each section are averaged to give a final answer, while this is not the case when calculating the r.m.s. values. Comparing the two results will give an idea of the effects of averaging as well as the accuracy of the analysis procedure adopted.

It is interesting to note that the difference between the $R_y, R_y(t=0)$ and the $u^2$ and $v^2$ values does not exceed 5% (column $A-E$). The column $E-C$ shows that in some cases analysing 40 blocks of data is sufficient, whereas in others the r.m.s. value can change by 20%.

At $y = +0.099$ one sees that analysing 40 blocks of data was sufficient for the $u$ velocity fluctuations but not for the $v$ fluctuations.

The results show that the statistical accuracy in the turbulent shear layer is good but decreases as one moves into the low intensity edges of the layer.
CHAPTER VIII
3. CONCLUSIONS
3.1 The Analysis System

I) A system for the digitising and digital analysis of analogue turbulent data has been completed. The digital programs are written in Fortran IV and are processed by the IBM 360/50 computer at the University of the Witwatersrand Computer Centre.

II) A single time series can be analysed providing estimates of:
   a) The periodogram,
   b) The amplitude probability density function,
   c) The autocorrelation.

III) Two signals can be analysed simultaneously to give the above mentioned properties for each time series as well as:
   a) The cross spectrum.
   b) The cross correlation.

Samples of either 1024, 2048, 4096 or 8192 values can be analysed and any number of samples averaged to provide the final answer.

IV) It is possible to separate the two signals from an X-array probe into the two velocity components. It is possible to produce turbulent and non-turbulent zone signals from the signal and its associated intermittency signals.

V) Comparing the root-mean-square values of the fluctuations and the value of the autocorrelation at \( \tau = 0 \) \( (\mathcal{R}_x(\tau = 0)) \) gives values differing by 5%. The values of \( \mathcal{R}_x(\tau = 0) \) are obtained by averaging over 20 samples of 2048 points each.
vi) Calculating the root mean square values of 30 samples of data and comparing it to $R_x(\tau = 0)$ calculated over 20 samples gives values differing by up to 20%. The Student's $t$ test on the mean value of the standard deviations of the samples showed a 90% confidence interval of at worst 9%, but generally 5% of the final mean value.

vii) A short sensitivity analysis indicated the importance of correctly measuring the angles $\alpha_1$ and $\alpha_2$ between the hot-wire sensors and the mean-flow direction. Different values of the directional sensitivity coefficients $k_1$ and $k_2$ change the relative root-mean-square values of the $u$ and $v$ fluctuations considerably. The accuracy of the values of $k_1$ and $k_2$ depend primarily on the values of $\alpha_1$ and $\alpha_2$.

3.2 Turbulence Results

i) Using digital Fourier analysis it is possible to obtain turbulence quantities which are very difficult to measure using analogue methods. In this investigation estimates of the periodogram and autocorrelation of the turbulent and non-turbulent zones in the intermittent region of a free mixing layer were obtained.

ii) The power spectra are of the expected form. The turbulent-zone periodogram is the classic turbulent-flow periodogram which passes through the Kolmogorov slope of $-\frac{5}{3}$. Indications are that the low frequency content is higher in the non-turbulent zone than in the turbulent zone.

iii) The autocorrelation of the mixture (intermittent signal) indicates isotropic turbulence with two distinct eddy sizes (Townsend (1956)). From the zone calculations one sees that the large eddies are in the non-turbulent zone while the smaller eddy size is associated with the turbulent region. Both autocorrelations tail off as
expected for $u$ velocity fluctuations.

iv) The amplitude probability density function of the mixture shows the velocity fluctuations to be nearly normal. On separation there are two distinct distributions. Both are skew with their tails in the direction of mean flow.

v) From the conventional measurements performed across the shear layer 30 cm downstream of the splitter plate, the following conclusions can be drawn:

a) Values of the Taylor microscale measured using the auto-correlation are similar to those obtained by Wygnanski and Fiedler (1970). The average value across the shear layer is 1.4 times larger than that obtained by Leipman and Laufer (1947).

b) The single-wire traverse results show different flow structures as observed by Wygnanski and Fiedler (1970). The different regions could also be deduced from the autocorrelation of the $v$-velocity fluctuations.

c) No linear relationship between the convected integral time scale and slope of the mean velocity profile was found. This supports the results of Wygnanski and Fiedler (1970) but not those of Davies et al. (1963).

d) Applying the available data to Phillips (1967) theory on the maintenance of Reynolds shear stresses gave an average value of the constant $A$ to be 0.23. This compares favourably with the value of 0.20 obtained by Wygnanski and Fiedler (1970).

e) The periodograms at all the points considered follow Kolmogorov's universal equilibrium theory over a short range of frequencies.
The Reynolds number based on the Taylor microscale and the root-mean-square value of the $u$-velocity fluctuations varies between 20 and 500. The results tend to support those of Bradshaw (1967).

8.3 Suggestions for Further Work

There are two important aspects of the work which should be investigated further.

i) The accuracy of the result as a function of sample length analysed and the number of samples averaged to give the final result.

ii) The method of separating the intermittent signal into turbulent and non-turbulent zones must be improved to give better accuracy in the low frequency regions.
NOMENCLATURE

\( A \) Constant in Phillips' theory and King's law.
\( A_n \) Digital form of the complex Fourier coefficient.
\( A^i \) Imaginary part of the Fourier coefficient.
\( A^r \) Real part of the Fourier coefficient.

\( A(f), B(f) \ldots \) Continuous form of the complex Fourier coefficient.
\( A^*(f), B^*(f) \) Complex conjugate of \( A(f) \) and \( B(f) \).
\( B \) Bandwidth of the signal and King's law constant.

\( DR \) Dynamic range.
\( F \) Sampling frequency (twice the Nyquist frequency).

\( MDS \) Minimum detectable signal.
\( M \) Sample size.

\( PG \) Processing gain.

\( R_{xx}(r) = R_x(r) \) Autocorrelation
\( R_{xy}(r) \) Cross-correlation function of the \( x \) and \( y \) series.

\( S_{xx}(f) = S_x(f) \) Spectral density function.
\( S_{xy}(f) \) Two-sided cross-spectral density function of the \( x \) and \( y \) series.

\( SNR \) Signal to noise ratio.
\( T \) Period of the Fourier series.
\( U, V, W \) Mean velocity components.

\( U_{eff} \) Effective mean velocity measured by the anemometer.

\( X(t) \) Complex form of the continuous time series.
\( X(kn) \) Complex form of the digitised time series.

\( a_n^x, b_n^y, \text{ etc.} \) Sine and cosine terms of the \( n \)th harmonic in Fourier series of \( x \) and \( y \) signals.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a(f), b(f))</td>
<td>Cosine and Sine terms in the Fourier series (continuous notation).</td>
</tr>
<tr>
<td>(c, c_1, c_2)</td>
<td>Instantaneous effective velocity fluctuations.</td>
</tr>
<tr>
<td>(f(r))</td>
<td>Normalised longitudinal correlation.</td>
</tr>
<tr>
<td>(g(r))</td>
<td>Normalised lateral correlation.</td>
</tr>
<tr>
<td>(h)</td>
<td>Time between samples.</td>
</tr>
<tr>
<td>(i)</td>
<td>(\sqrt{T})</td>
</tr>
<tr>
<td>(k, k_1, k_2)</td>
<td>Directional sensitivity coefficients of the hot-wire sensors.</td>
</tr>
<tr>
<td>(n)</td>
<td>Number of bits used by the digitiser.</td>
</tr>
<tr>
<td>(n)</td>
<td>Harmonics in the digital Fourier series (n = 0, 1, 2 \ldots \infty).</td>
</tr>
<tr>
<td>(r)</td>
<td>Separation between two sensors.</td>
</tr>
<tr>
<td>(t)</td>
<td>Student's &quot;(t)&quot; statistic.</td>
</tr>
<tr>
<td>(u, v, w)</td>
<td>Fluctuating velocity components.</td>
</tr>
<tr>
<td>(x(t))</td>
<td>Continuous real time domain function.</td>
</tr>
<tr>
<td>(x(kh))</td>
<td>Digitised real time domain function.</td>
</tr>
<tr>
<td>(y = \frac{4\pi}{\lambda_0})</td>
<td>Dimensionless position across the shear layer, (y = 0) where (U = \frac{U_{\text{fast}} + U_{\text{slow}}}{2}).</td>
</tr>
<tr>
<td>(\alpha, \alpha_1, \alpha_2)</td>
<td>Angle between the hot-wire sensor and the mean stream velocity.</td>
</tr>
<tr>
<td>(\theta)</td>
<td>90°-a.</td>
</tr>
<tr>
<td>(\tau)</td>
<td>Time delay.</td>
</tr>
<tr>
<td>(\lambda_F)</td>
<td>Taylor dissipative length scale (microscale).</td>
</tr>
<tr>
<td>(\lambda_g)</td>
<td>Microscale obtained from the lateral correlation.</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Rate of viscous dissipation.</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Kinematic viscosity.</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Kolmogorov length scale.</td>
</tr>
<tr>
<td>(\Lambda, \Lambda_F, \Lambda_g)</td>
<td>Macro or integral scale of turbulence.</td>
</tr>
</tbody>
</table>
\( \rho_X(\tau) \)  
Normalised autocorrelation.

\( \tau_E \)  
Microscale obtained from the autocorrelation.

\( J_E \)  
Macroscale obtained from the autocorrelation.

\( \rho_{XY}(\tau) \)  
Normalised cross-correlation.

\( \Theta \)  
Convected integral time scale.

\( dB \)  
Decibels \( 20 \log_{10} \) SNR (SNR in volts)

\( \overline{\cdot} \)  
Average value of quantity in brackets.
REFERENCES


IEEE TRANSACTIONS ON AUDIO AND ELECTROACoustics, Special Issue on the
Fast Fourier transform. IEEE Transactions on Audio and Electro-

JENSON, J.O., 'A Survey of Methods for Measuring Correlation Functions'.
DISA Information, 10: 10-14, 1970.

JEROME, F.E., GUITTON, D.E. and PATEL, R.P., 'Experimental Study of
the Thermal Wake Interference between Closely Spaced Wires of an

JONES, R.H., 'A Re-appraisal of the Periodogram in Spectral Analysis'.
Technometrics, 7 no.4: 531-542, 1965.

JORGENSEN, F.E., 'Directional Sensitivity of Wire and Fiber-film Probes'.

KAPLAN, R.E. and LAUFER, J., 'The Intermittently Turbulent Region of
the Boundary Layer'. Proceedings of the 12th International
Congress of Applied Mechanics, ed. HETENYI, M. and VINCENTI, W.B.,

KIBENS, V., 'The Intermittent Region of a Turbulent Boundary Layer'.

Analysis of Random Data Records by Piecewise Accumulation of
Time Averages'. NASA Technical Note, NASA TN D6073:
1970.

LIEPMAN, H.H. and LAUFER, J., 'Investigations of Free Turbulent Mixing'.
National Advisory Committee for Aeronautics, Technical Note 1257,
1947.

LIEPMAN, H.H. and LAUFER, J., 'Investigations of Free Turbulent Mixing'.
National Advisory Committee for Aeronautics, Technical Note 1257,
1947.

LIN, C.D., 'On Taylor's Hypothesis and the Acceleration Terms in the
Navier-Stokes Equations'. Quarterly of Applied Mathematics.
LIN, C.C. and REID, W.H., 'Turbulent Flow, Theoretical Aspects'.
Handbuch der Physik VIII/2, ed. FLUEGGE, S.,

MALLING, G.C., MORREY, W.T.(Jr), and LANG, N.W. 'Digital Determination
of Third-Octave and Full-Octave Spectra of Acoustical Noise'.
IEEE Transactions on Audio and Electroacoustics, AU-15 no.2:

MOLLO-CHRISTENSEN, E.L., 'Physics of Turbulent Flow'. AIAA Journal,
9 no. 7: 1217-1228, 1971.

OPPENHEIM, A.V. and WEINSTEIN, C., 'A Bound on the Output of a Circular
Convolution with Application to Digital Filtering'. IEEE
Transactions on Audio and Electroacoustics, AU-17 no.2:
120-124, 1969.

PAPULIS, A., 'The Fourier Integral and its Applications'. Electronic

PHILLIPS, O.M., 'The Maintenance of Reynolds Stress in Turbulent

PHILLIPS, O.M., 'Shear-Flow Turbulence'. Annual Review of Fluid

RASMUSSEN, C.G., 'The Measurement of Turbulence Characteristics'.

SCHESTER, A., 'On the Investigation of Hidden Periodicities with
Application to a Supposed 26-day Period of Meteorological

SINGLETON, R.C. and POULTER, T.C., 'Spectral Analysis of the Call of
the Male K III or Whale'. IEEE Transactions on Audio and Electro-


APPENDIX I

The Hot-Wire Response Equations

A hot-wire is assumed to be in a plane perpendicular to the $Z$ direction. Its direction is given by the unit vector $\hat{s}$.

![Diagram of hot-wire setup]

Figure A.1

The mean velocity is assumed to be in the $xy$ plane, and without loss of generality can be assumed in the $x$-direction. The effective cooling velocity is assumed to be the velocity perpendicular to the wire plus a fraction $k$ of the velocity parallel to the wire.

$$U_e = V_{\text{perp}} + kV_{\text{parr}} \quad (I-1)$$

Now

$$W = \text{instantaneous velocity vector} = \begin{bmatrix} U + u \\ v \\ v \end{bmatrix}$$

also

$$V_{\text{perp}} = W \cdot (I - \hat{s}\hat{s})$$

$$V_{\text{parr}} = (W \cdot \hat{s})\hat{s}$$
where the direction vector \( \hat{\vec{d}} \) is written as

\[
\hat{\vec{d}} = \hat{i} \cos \alpha + \hat{j} \sin \alpha
\]

\[
= \hat{c} + \hat{s}
\]

Working in terms of components

\[
(U_e)_{ki} = N_{ij}V_j
\]

where from (1-1)

\[
N_{ij} = I_{ij} - s_i s_j + k s_i s_j
\] (1-2)

\[
s_1 = c = \cos \alpha
\]

\[
s_2 = s = \sin \alpha
\]

\[
s_3 = 0
\]

For constant temperature, linearised mode of operation, the Hot-wire voltage \( E \) is proportional to the magnitude of the effective cooling velocity.

\[
E = K|U_e| \tag{1-3}
\]

\[
iU_e = \sqrt{\frac{E}{c}}
\]

From (1-1)

\[
V_e = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{align*}
U_e &= \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
s_1 & s_2 & s_3 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
U + u \\
v \\
w \\
\end{bmatrix}
\end{align*}
\]

Adding the first two matrices and using the relationship \( s^2 + c^2 = 1 \)

\[
V_e = \begin{bmatrix}
s_1 & s_2 & s_3 \\
\end{bmatrix}
\begin{bmatrix}
U + u \\
v \\
w \\
\end{bmatrix}
\]

Multiplying the two matrices and simplifying one obtains

\[
V_e = \begin{bmatrix}
(U + u)(k + s^2(1-k)) + c(s^2k-1) \\
(U + u)c(k - 1) + v(s^2(k-1) - 1) \\
w \\
\end{bmatrix}
\]
Now the magnitude of the effective velocity vector $\mathbf{v}_e$, 

$$|\mathbf{v}_e|^2 = \mathbf{v}_e \cdot \mathbf{v}_e = v_{e1}^2 + v_{e2}^2 + v_{e3}^2$$

$$= [(U + u)(k + \gamma^2(1 - k)) + av(k - 1)]^2$$

$$+ [(U + u) \cos (k - 1) + v(\gamma^2(k - 1) + 1)]^2 + v^2$$

Simplifying

$$|\mathbf{v}_e|^2 = (U^2 + 2Uu + u^2)(\gamma^2 + k^2c^2) + v^2(\gamma^2 + c^2)$$

$$+ 2(U + u)\gamma \cos (k - 1) + v^2$$

$$|\mathbf{v}_e|^2 = U^2(\gamma^2 + k^2c^2) + 2Uv + \frac{2v^2}{U}(\gamma^2 + 1)$$

$$+ \frac{v^2}{U^2}(\gamma^2 + k^2c^2) + \frac{2uv(\gamma^2 - 1)}{U^2}$$

To obtain $|\mathbf{v}_e|$, use is made of the Binomial expansion

$$(1 + a)^m = 1 + ma + \frac{m(m - 1)a^2}{2} + \ldots$$

$$|\mathbf{v}_e| = U(\gamma^2 + k^2c^2) + Uv + \frac{Uv(\gamma^2 - 1)c}{U(\gamma^2 + k^2c^2)} + \frac{v^2}{U^2}(\gamma^2 + k^2c^2)$$

$$+ \frac{av(\gamma^2 - 1)c}{U^2(\gamma^2 + k^2c^2)} + \ldots \text{ higher order velocity terms}]$$

$$|\mathbf{v}_e| = U(\gamma^2 + k^2c^2) + Uv + \frac{Uv(\gamma^2 - 1)c}{U(\gamma^2 + k^2c^2)} + \frac{v^2}{U^2}(\gamma^2 + k^2c^2)$$

$$+ \frac{w^2}{2U^2(\gamma^2 + k^2c^2)} + \ldots \quad (1-4)$$
Neglecting the contributions from second order terms in the velocities $(u^2, v^2, uv)$ if $E = \bar{E} + e$ and using (1-3), where $E$ is the instantaneous measured voltage. On averaging (1-4)

$$\bar{E} = KU^2_0 s^2 + k^2 c^2$$

and

$$\bar{e} = KU^2_0 s^2 + k^2 c^2 \left[ \frac{u}{v} + \frac{v(k^2 - 1)}{v^2 + k^2 c^2} \right]$$

The largest terms neglected in the mean velocity are

$$\frac{u^2 + v^2}{2U^2(s^2 + k^2 c^2)}$$

If these terms are not $\ll 1$ then the above approximation is invalid.
APPENDIX II

THE COMPUTER PROGRAMS

The various programs available for the analysis of digital turbulence data are described in this section. Rather than give the actual program a description is given (where necessary) of how the various quantities are calculated. A general description of the software is given as well as instructions on how to use each program or subroutine.

Primarily the object was not to write one enormous program calculating everything, but to write subroutines performing the various calculations which would later be combined to form more complex programs. This means that the user can write programs satisfying his own particular needs. There have also been a number of fairly general programs written.

The General Layout

The first time that computer programs are required is once the analogue signal has been digitised. It is informative to know what the digitised tape actually looks like.

```
File | Block | Interblock
MARK (1024 samples)  gap
```

The digitiser samples the analogue signal and then converts it into an Integer *2 number. The samples are then written in blocks of 1024 onto the magnetic tape. Integer *2 means that each value is represented as an integer using two BYTES. Each BYTE consists of 4 BITS. A bit is the basic on/off element used for binary representation. The magnetic tapes available can handle 1600 BPI (bits per inch) although we operate at 800 BPI. When a particular section of data has been
digitised and written onto magnetic tapes, the FILE can be terminated by using a TAPE MARK. This is produced by the digitiser itself.

Summarising we have:

i) FILES - The size of which we determine.

ii) BLOCKS - A group of 1024 values (2048 Bytes).

The magnitude of an Integer *2 variable is limited and so the data is converted to the Real *4 representation so that too larger values are not encountered in the calculations. After checking and then converting the data we can proceed to calculate turbulence characteristics.

Generally when debugging a program one only requires a small quantity of data. A catalogued data set on disc is available and programs to transfer about 4096 points have been written. Using this data has two advantages:

i) Faster turnaround time of the programs

ii) Less wear and tear on the magnetic tapes.

A Library has been formed on disc which contains all the subroutines and the programs that are executed by "In Line Procedures". This removes the need for large decks of cards and allows the same subroutines to be used in different programs at the same time. It is very simple to add more subroutines to the Library. All programs and subroutines are also on punched cards in case the Library is damaged.

The general programs are written in a manner that makes it possible to store the results on magnetic tape. This is most useful if one intends at a later stage to compare results or try different methods of plotting them.

Each of the programs and subroutines available will be discussed separately, and where necessary the theory discussed. Because of its importance the Fast Fourier Transformation will be discussed first.
SUBROUTINES

SUBROUTINE HAR1

This subroutine performs a Fast Fourier Transformation or Inverse
Fourier Transformation on real data. The FFT algorithm (Cooley & Tukey 1965)
is used so ensuring fast execution times compared to conventional methods
e.g. Goertzel. SUBROUTINE HAR1 converts the real data into a three
dimensional complex array which is then processed by SUBROUTINE HARM.
The output of HARM is then manipulated to give the Fourier Coefficients
of the original time series. The subroutine can only handle samples
that are a power of 2.

NAME

SUBROUTINE HAR1 (A, M, INV, S, IFERR, MN)

Purpose

Performs Fourier Transformations on real, one dimensional data.

Description of Parameters

A : As input contains the data to be transformed

1) Time Domain to Frequency Domain

The input consists 2**(M+1) time samples. The output consists
of 2**(M+1) + 2 values where:

A(1) = D.C. value (mean)
A(2) = 0.0
A(3) to A(2**(M+1) + 2) contain the cosine and sine terms
of the Fourier series

A(3) = Cosine term of 1st harmonic
A(4) = Sine term of 1st harmonic etc.
if) Frequency Domain to Time Domain

The input A contains A(2**(M+1)+2) frequency domain values

\[ A(1) = \text{D.C.} \]
\[ A(2) = 0.0 \]
\[ A(3) = \text{Cosine term} \]
\[ A(4) = \text{Sine term, etc.} \]

M: Integer constant which determines the size of A in time domain

\[ = 2**(M+1)) \]

INV: Work area, must be dimensioned to 1/3 size of A

S: Work area, to be dimensioned as for INV

IFERR: Error indicator

i) If \( NN = 0 \pm 1 \), -1 then if IFERR = 1 the value of \( M \) is \( 20 < M < 3 \).

ii) If \( NN = +2, -2 \) then IFERR = 1 means that the Sine and Inverse Tables are not large enough or have not been computed.

NN: This is an option parameter which may have the following values

i) 0 - Set up Sine and Inverse Tables only

ii) 1 - Set up the Sine and Inverse Tables and calculate the Fourier Transform

\[ X(j) = \sum_{k=0}^{N-1} A(k) e^{- \frac{2\pi i j k}{N}} \]

iii) -1 - Set up the Sine and Inverse Tables and calculate the Inverse Fourier Transform

\[ A(k) = \frac{1}{N} \sum_{j=0}^{N} X(j) e^{\frac{2\pi i j k}{N}} \]
iv) 2 - Calculate the Fourier Transforms only (assume that the Sine and Inverse Tables exist).

v) 2 - Calculate the Inverse Fourier Transform only. Assume Sine and Inverse Tables exist.

Remarks

N is limited to \( 3 < N < 20 \)

Subroutines and Functions Required

HARM

Some timing information is given below:

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Approx. Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>2.8</td>
</tr>
<tr>
<td>2048</td>
<td>5</td>
</tr>
<tr>
<td>4096</td>
<td>13.8</td>
</tr>
</tbody>
</table>
SUBROUTINE STATSS

This subroutine calculates various statistical properties of the sample. It is a simplified and modified S.S.P. routine. Calculation of the Amplitude Probability Density is optional.

If the input array is \( A \), the following quantities are calculated:

1) The maximum value of \( A \).
2) The minimum value of \( A \).
3) The total \( \sum A \).
4) \( \sum A^2 \).
5) The mean \( \frac{1}{N} \sum A \).
6) The standard deviation \( \sqrt{\frac{1}{N-1} \sum (A - \text{mean})^2} \).
7) The number of values outside range used for A.P.D.F.

Note:

\[
(x - \text{mean})^2 = (x^2 - 2x\text{mean} + \text{mean}^2)
\]

But mean = \( \frac{\sum x}{N} \)

\[
= \frac{\sum x^2 - 2\text{mean} \sum x + \text{mean}^2}{N - 1}
\]

\[
= \frac{\sum x^2 - 2\text{mean} \sum x + \text{mean}^2}{N - 1}
\]

Above is the quantity actually calculated

NAME

SUBROUTINE STATSS (A, NO, DBG, FREQ, PCT, STATS, YES)

PURPOSE

Calculates various statistics and the Amplitude Probability Density function of a sample.
DESCRIPTION OF PARAMETERS

A: The input array. It is not altered during execution of the subroutine.

NO: Number of points. The dimension of A.

UBO: An array of dimension 3
   i) UBO(2) is the number of intervals required for the Amplitude Probability Density Function.
   ii) If UBO(1) = UBO(3) then the subroutine calculates the minimum and maximum values of the array.
       Otherwise:
       UBO(1) = minimum value.
       UBO(3) = maximum value.

FREQ: Output array containing the Amplitude Probability Density Function
       Dimension UBO(2).

PCT: FREQ/NO*100

STATS: Output array of dimension 7 containing the following:
       STATS(1) = Total.
       STATS(2) = Mean.
       STATS(3) = Standard deviation.
       STATS(4) = Minimum value.
       STATS(5) = Maximum value.
       STATS(6) = Sum of A*A.
       STATS(7) = No of points outside range.

HYES: Option parameter:
   i) HYES = 1 Causes Amplitude Probability Density Function to be calculated.
   ii) HYES ≠ 1 Indicates that the A.P.D.F. is not to be calculated.
Remarks
None

Subroutines and Functions Used
None
SUBROUTINE REMOVE

This subroutine checks the sample to see if there is a mean or linear trend. Although the D.C. is filtered out of the signal, means and linear trends in each sample can still be found.

The method used is that described by Bendat & Pierson (1966). It only uses 2/3 of the data points and hence saves computer time.

Let the original signal be represented as

\[ u(t) = \bar{u} + \bar{a}(t - \frac{T}{2}) + x(t) \quad 0 < t < T \]

where

\( \bar{u} = \) sample mean
\( \bar{a} = \) average slope of \( u(t) \)
\( T = \) total time

Graphically

\[ \begin{aligned}
&\text{Graphically} \\
&\text{We need the quantities:} \\
&1) \bar{u} \\
&2) \bar{a} \\
&\int_{t=0}^{T/3} u(t) \, dt = \int_{t=0}^{T/3} \bar{u} \, dt + \int_{t=0}^{T/3} \bar{a}(t - \frac{T}{2}) \, dt + \int_{t=0}^{T/3} x(t) \, dt \quad - I \\
&\int_{t=T/3}^{2T/3} u(t) \, dt = \int_{t=T/3}^{2T/3} \bar{u} \, dt + \int_{t=T/3}^{2T/3} \bar{a}(t - \frac{T}{2}) \, dt + \int_{t=T/3}^{2T/3} x(t) \, dt \quad - II
\end{aligned} \]
Obviously
\[ \int_{0}^{T/3} U(t) dt = \int_{0}^{T} U(t) dt \]

Subtracting \( I \) from \( T \)
\[ \int_{2T/3}^{T} U(t) dt = \int_{0}^{T/3} U(t) dt = \int_{2T/3}^{T} [\frac{T}{2} (t - \frac{T}{2})] dt + \int_{2T/3}^{T} x(t) dt - \int_{0}^{T/3} \frac{T}{2} [t - \frac{T}{2}] dt - x(t) dt \]

The series \( x(t) \) has no linear trend and hence
\[ \int_{0}^{T/3} x(t) dt = \int_{0}^{T} x(t) dt \]

Hence we have
\[ \int_{0}^{T} U(t) dt - \int_{2T/3}^{T} U(t) dt = \int_{0}^{T/3} \frac{T}{2} (t - \frac{T}{2}) dt - \int_{0}^{T/3} \frac{T}{2} (t - \frac{T}{2}) dt \]

Now
\[ \int_{2T/3}^{T} [\frac{T}{2} (t - \frac{T}{2})] dt = \frac{8T^2}{2} \int_{2T/3}^{T} \frac{T}{2} - \frac{8T^2}{2} \int_{2T/3}^{T} \frac{T}{2} = \frac{1}{2} \pi^2 \]
\[ \int_{0}^{T/3} \frac{T}{2} (t - \frac{T}{2}) dt = \frac{8T^2}{2} \int_{0}^{T/3} \frac{T}{2} - \frac{8T^2}{2} \int_{0}^{T/3} \frac{T}{2} = -\frac{1}{2} \pi^2 \]

Therefore
\[ \int_{2T/3}^{T} U(t) dt - \int_{0}^{T/3} U(t) dt = \frac{3}{2} \pi^2 \]

Therefore
\[ \frac{1}{\sigma^2} \left( \int_{2T/3}^{T} U(t) dt - \int_{0}^{T/3} U(t) dt \right) \]
The corresponding version of III for use with discrete data would be:

\[ U(t) \text{ is represented as} \]

\[ U(nh), \ n = 1, 2, \ldots, N \]

\[ t = nh \quad h = \text{time interval between samples} \]

\[ \bar{u} = \frac{1}{2Nh^2} \sum_{n=1}^{N} U(n) - \frac{N}{3} U(n) \]

Let \( \nu = \frac{N}{3} \)

Therefore

\[ N - \nu = N - \nu \frac{N}{3} = \frac{2N}{3} = 2\nu \]

\[ \frac{1}{2Nh^2} \sum_{n=1}^{N} [ U(n) - \frac{N}{3} U(n) ] \]

Finally,

\[ \bar{u} = \frac{1}{h^2 \nu(N - \nu)} \sum_{n=1}^{N} U(n) - \frac{\nu}{N} U(n) \]

Since we use large samples it is generally sufficient to take \( \nu \) as the integer nearest \( N/3 \).

The series \( x(t) \) can be calculated as follows:

\[ x(nh) = U(nh) - \bar{U} - \bar{u}(n - \frac{N}{3}), \ n = 1, \ldots, N \]

**Name**

SUBROUTINE REMOVE (X, H, N, RESULT)

**Purpose**

The subroutine calculates the mean and slope in a sample and then removes one, both or neither depending on the magnitude of them.
Description of Parameters

X: Input array of dimension N. As output it consists of the modified signal.

H: Time between samples of X

RESULT: Contains the mean and slope of the sample. Dimension in 2.

RESULT (1) = mean
RESULT (2) = slope

Remarks

By changing the values in the IF statements in the subroutine the critical values of the mean value and slope can be varied.

Functions and Subroutines Used

None.

For the size of sample used the routine gives reasonably accurate results as shown in the table below:

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Actual Slope</th>
<th>Calculated Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.2</td>
<td>0.22</td>
</tr>
<tr>
<td>200</td>
<td>0.2</td>
<td>0.21</td>
</tr>
<tr>
<td>500</td>
<td>0.2</td>
<td>0.20</td>
</tr>
</tbody>
</table>
SUBROUTINE HAMM

The subroutine HAMMS the input signal. In effect it is a moving average, the coefficients being 0.46; 1.08; 0.45.

Name

SUBROUTINE HAMM (A, N)

Purpose

The subroutine Hamm the input array.

Description of Parameters

A: Input array. As output it is the Hammed array.
N: Number of points in the sample.

Remarks

None.

Subroutines and Functions Used

None.
SUBROUTINE HANNIN

The subroutine HANNIN the sample. In effect it is a moving average, the coefficients being 0.25; 0.5; 0.25.

Name

SUBROUTINE HANNIN (A, N)

Purpose

The subroutine Hanns the input array A.

Description of Parameters

A: Input array. As output it is the Hanned array.

N: Number of points in the sample.

Remarks

None
SUBROUTINE PGRAM

The subroutine calculates:

1) The Fourier series coefficients $a(f), b(f)$.
2) The Periodogram coefficients $a(f)^2 + b(f)^2$
3) The Phase angles $\tan(b(f)/a(f))$ of the input signal. The routine uses the Fast Fourier Transform Algorithm.

Name

SUBROUTINE PGRAM (DATA, N, AMP, D, NYES, INV, S)

Purpose

The subroutine calculates the Fourier coefficients of the input array using the Fast Fourier Transform. The coefficients can be Hanned if required. They are then combined to give the Periodogram (Power Spectrum). The Phase information is also calculated.

Description of Parameters

DATA: As input DATA is the Real array of $N$ data points. Dimension $N + 4$. The output array is of size $N + 2$ and consists of:
- DATA (1) = mean value
- DATA (2) = 0.0
- DATA (3) = 1st Cosine term
- DATA (4) = 1st Sine term
  ... and etc.

AMP: Array of amplitude squared terms $(a^2 + b^2)$. Dimension $N/2$. The O.C. term is not included.

N: Number of points in the input. It must be a power of 2.

D: Array of Phase Information. Dimension $N/2$.

NYES: is an option parameter. If NYES = 1 then the Fourier coefficients are Hanned. If NYES # 1 then the coefficients are not Hanned.


Remarks:
The input array is destroyed on passing through the subroutine.

Functions and Subroutines Used
RHAR1.
SUBROUTINE AUTOC

The subroutine rearranges the output of subroutine PGRAM and then Inverse Fast Fourier Transforms it. It should be used in conjunction with PGRAM. The mean value of the array to be IFFT is made equal to zero. The Cosine terms are equal to the Periodogram coefficients while the Sine terms are all zero. The output consists of the autocorrelation coefficients with time lags being multiples of the time interval between samples. Because the autocorrelation function is an even function only half of the calculated coefficients are unique.

NAME

SUBROUTINE AUTOC (DATA, AMP, N, INV, S)

Purpose

The subroutine rearranges the Periodogram coefficients and then IFFT them to give the autocorrelation function. In general the Periodogram coefficients are obtained using PGRAM.

Description of Parameters

DATA: is the output array of autocorrelation coefficients, dimension N + 4.

AMP: Input array of Periodogram coefficients. Dimension N/2.

N: Number of points in the autocorrelation (same as original time series).

INV: Work area for subroutine RHAR1, dimension N/8.

S: Work area for subroutine RHAR1, dimension N/8.

Remarks

Generally used in conjunction with subroutine PGRAM.

Functions and Subroutines Used

RHAR1.
SUBROUTINE READS

The data on the magnetic tapes is blocked in groups of 1024 samples. The maximum number of points any READ statement can read is one block i.e. 1024 values. If a sample of say 4096 points is to be read, each block that is read must be put into different areas of the sample array. If this is not done then the first 1024 points will be over-written by those from the next READ. The subroutine reads any sample size (a multiple of 1024) greater than 1024.

Name
SUBROUTINE READS (N, NA, DATA)

Purpose
To read an array >1024 points when the data is blocked in blocks of 1024.

Description of Parameters
N: Size of sample to be read.
NA: The data set reference number of the data to be used.
DATA: Output array of data, dimension N.

Remarks
Samples of less than 1024 points cannot be read.

Functions and Subroutines Used
None
SUBROUTINE SMOOTH

The subroutine takes the input array and smooths it using a moving average technique. The width of the triangular weighting function is variable. It is a useful method of smoothing the periodogram (Singleton and Poulter 1967). An optional check is available to safeguard very sharp peaks which would otherwise be removed. It is a fairly time consuming process but useful for final "tailoring" of the results.

If the span of the smoothing function is $S$, then;

$$X(i) = \frac{i\cdot X(i-2) + 2\cdot X(i-1) + 3\cdot X(i) + 2\cdot X(i+1) + i\cdot X(i+2)}{\sum_{j=0}^{n} i-j}$$

where

$$M = \frac{S+1}{3} = 3$$

when using a span of 3, the effect is identical to Hanning the sample.

Name

SUBROUTINE SMOOTH (X, IS, N, IT)

Purpose

The subroutine smooths an input array using a triangular smoothing function the span of which is variable.

Description of Parameters

X: Input array of dimension $N$. As output it contains the smoothed data.

N: Number of points.

IS: Span of the smoothing function.

IT: IT = 0, then sharp peaks are left.
    IT = 1, then sharp peaks are smoothed.
Remarks

The maximum span that can be used is 19.

Functions and Subroutines Used:

None.

Some time information is shown in the table below:

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>Span</th>
<th>Time (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1024</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>1024</td>
<td>7</td>
<td>19</td>
</tr>
<tr>
<td>2024</td>
<td>15</td>
<td>38</td>
</tr>
</tbody>
</table>
SUBROUTINE MOMENT

The subroutine calculates Higher Order Skewness factors of the input array. It is written so that if the sample is too large to be handled in one pass, smaller samples can be taken and the results averaged. The moments calculated are defined by,

\[ S(NP) = \frac{\langle [u(x, t + h) - u(x, t)]^{NP} \rangle}{\langle [\{u(x, t + h) - u(x, t)\}^{2}]^{NP/2} \rangle} \]

where the overbar implies averaging and where \( u(x, t), u(x, t + h) \) are velocities at the same point in space, but different instants in time. The two quantities

\[ \langle [u(x, t + h) - u(x, t)]^{NP} \rangle \]

and

\[ \langle [u(x, t + h) - u(x, t)]^{2} \rangle \]

are the outputs since they can be averaged over successive samples. Only one value of \( NP \) can be calculated at any time, but as many different values of the time delay as required can be calculated. The time delay must be multiples of the time interval between samples.

NOTE: this is not the optimum way to calculate the moments above.
The F.F.T. can be used as described by Van Atta and Chan (1969).

NAME

SUBROUTINE MOMENT (X, N, NT, NT, NP, TEMP, BEMP)

PURPOSE

The subroutine calculates higher order skewness factors for a particular value of \( NP \) over as many time intervals as required.
PROGRAMS

The previous section described most of the subroutines available. They can be used by any program written by the programmer. This section describes some of the programs available to perform particular calculations.

One general program that calculates cross correlations and autocorrelations has been written. Most programs are available in two forms, the difference being the form of the Job Control Language processing it.

The first form is the standard I.B.M. Job Control Language. In the second form In-line procedures have been utilised. The necessary Job Control Language for the program is formed into a standard procedure. By using substitution Job Control Language it is possible to change any particular parameter in the Job Control Language by over-riding it in either the PROC or EXEC statements.

To use In-line procedures the program must be compiled and Linkage Edited and stored in a Partitioned Data Set, whereas for standard Job Control Language the program can be in card form. The biggest advantage of the In-line procedure mode of execution is that the same program can be executed a number of times in the same JOB using the minimum of Job Control Language. Each time the routine is executed different Job Control Language parameters or data can be used. Graphically a JOB consisting of an In-line procedure is shown below:
Beginning of Job

Formation of Procedure PROC1. It is JCL to read off Tape.

Execute PROC1 with
Name = NBDQI
File = 2
Time = 5

Execute FORTRAN Program (cards) to plot results.

Execute PROC1
Name = NBDQI
File = 6
Time = 10

Execute a different Plotting Routine.

END

The Parameters that can be Varied

1) Name of tape (Name = ).
2) File to be read (File = ).
3) Execution time (Time = ).

The actual program to be executed is stored on disc and is found by PROC1.

There is no reason why an In-line procedure cannot be executed alternately with various smaller Fortran programs which output results. If the same plotting program is used each time it would be advantageous to form another In-line procedure that executes the plotting routine.

Define File Statements have been used to transfer data from one step in the program to another. The disc data set formed by a Define File statement is only erased at the end of a JOB. Small programs doing specific calculations therefore be executed after a large general program, the data being read off the data set that the general program wrote onto.

The first programs mentioned are for checking the data on the tapes and converting the data into a convenient form.
PROGRAM NLBCHECK

The program is used to check the data on the digitised tape containing Integer *2 data. It is possible to print the first block of any file on the tape to see if the values are those expected. The program reads right through the file and any error interrupting the reading routine will cause a message to be written. The program is available in two forms, one using standard Job Control Language; the other the In-line procedure NLBPROC1. The program is stored as the module CHECK on the Library ACM.NLL.LIB. In general it is easier to use the form using the In-line procedure.

The following deck of cards is required to execute the program:

//NLBCHECK JOB
//NLBPROC1 PROC formation of
/ //NLBPROC1 PEND In-line
//NLBPROC1 PEND Procedure

//ONE EXEC NLBPROC1, NAME = , TIME = etc.
data cards for step ONE
//TWO EXEC NLBPROC1, NAME = , TIME = etc.
data cards for step TWO

These are two parameters that can be changed:
i) NAME = : name of tape to be read.
ii) FROM = : file number to be read.

If all the steps use the same tape then the NAME = parameter can be specified on the PROC statement

//NLBPROC1 PROC, NAME = NLB60

iii) One data card giving the file number of the file
to be read must be supplied for each EXEC statement
PROGRAM NLECNY

The object is to convert the Integer *2 results on the digitiser tape into Real *4 results. The standard representation is Real *4 as the allowable magnitude of Real *4 is higher than that of Integer *2 variables.

Depending on the analysis to be performed it might be advantageous to have certain files of information next to one another. Files that were terminated because the digitiser had a Parity check can be neglected completely. The program reads a particular file, converts it to Real *4, counts the number of blocks and then writes the complete file onto another magnetic tape.

The program is available with standard Job Control Language or using the In-line procedure NLECNY.

The variable parameters are:
i) TAPE = : Name of Integer *2 tape.
ii) ONTO = : Name of Real *4 tape.
iii) FROM = : Number of the file to be read.
iv) TO = : Number of the file to be written.
v) TT = : Execution time required.

vi) Integer *2 tape is AMLABEL.

vii) Real *4 tape is LABELLED.

viii) Each EXEC statement requires a data card. Any information can be written onto the first 20 characters. If no information is required place a blank card as the data card.

ix) Job Co. - Language messages are on SISOUT = 5 while program message SISOUT = A.
Description of Parameters

X: Input array.

N: Number of points.

NT: An array, the members being the time delays to be calculated (multiples of the time interval between samples).

NI: Dimension of NT. It is the number of time intervals to be calculated.

NP: Power of the skewness factor.

TEMP: Array of dimension NT containing the values of

\[ \left[ u(x, t + h) - u(x, t) \right]^{NP} \]

for each time delay.

BEMP: Array of dimension NI, containing the values of

\[ \left[ u(x, t + h) - u(x, t) \right]^{2} \]

for each time delay.

Remarks

A reference indicating the importance of the above type of skewness factors is (Frenkel and Klebanoff (1965)).

Functions and Subroutines Used

None.
SUBROUTINE NBHIST

The subroutine plots a Histogram on the Calcomp Plotter. The user must supply the frequencies to be plotted, the number of intervals as well as the minimum and maximum values of the array. The last two quantities are not essential as they are just used to calculate the ranges of each interval.

Name

SUBROUTINE NBHIST (XX, YY, YL, NUM, K)

Purpose

The subroutine plots a Histogram on the Calcomp Plotter.

Description of Parameters

XX: X coordinate of the left hand side of the Histogram (inches).
YY: Y coordinate of the left hand corner of the Histogram (inches).
XL: Width of the Histogram (inches).
YL: Maximum height of the Histogram (inches).
NUM: Input array of dimension K containing the frequencies to be plotted

NUM(K+1) = minimum value of array
NUM(K+2) = maximum value of array.

K: Number of frequencies to be plotted.

Remarks

At the end of the routine a new origin is defined, 2 inches past the end of the Histogram and at y = 0,5 inches.

Functions and Subroutines Used

The plotter subroutines are stored on the System Library SYS1.LOAD.
SUBROUTINE GRAPH

Given two arrays, this subroutine plots them on the Calcomp Plotter (rectangular co-ordinates). It is advisable to read the write up on the Calcomp Plotter supplied by the Computing Centre to be able to make full use of the routine.

Name
SUBROUTINE GRAPH (X, Y, XL, YL, NP, LT, INT, INC)

Purpose
The arrays X and Y are plotted on rectangular axes on the Calcomp Plotter.

Description of Parameters
X: Input array (X axis). Dimension (NP+2).
Y: Input array (Y axis). Dimension (NP+2).
XL: Length of X axis (inches).
YL: Length of the Y axis (inches).
NP: Number of points in the X and Y arrays.
LT: Equivalent to LINREF in the Calcomp write-up. It describes the type of line to be drawn while the magnitude determines the frequency of the plotted points.
INT: Equivalent to INTEQ in the Calcomp write-up. Its magnitude determines what type of symbol is to be plotted.
INC: Equivalent to INC in the Calcomp write-up. It is used to select the data to be plotted. Sometimes only every Nth point is to be plotted.
Remarks

Provision has been made for headings along each of the axes and the writing of a title. They are supplied as data on punched cards.

The headings can be up to 20 characters in length. The data must be supplied as follows:

i) X axis heading.

ii) Y axis heading.

iii) title.

iv) 3FIO.2 characters in the following order:

   XC - X co-ordinate of the title.

   YC - Y co-ordinate of the title.

   SZ - height of the characters to be printed (in the title).

Functions and Subroutines Used

The plotter subroutines are stored on the System Library SVCLLOAD.
The tape being read must be a NO LABEL tape.

All Job Control Language messages are printed on SYSOUT = B while any messages written by the program itself are on SYSOUT = A. This keeps the two separate.

Example: Read the first block of files 6 - 10 on tape NLB00

//NLBCHKE JOB
//NLBPROC1 PROC NAME = NLB00

procedure
//NLBPROC1 PEND
// EXEC NLBPROC1, FROM = 6
06001
// EXEC NLBPROC1, FROM = 7
07001
// EXEC NLBPROC1, FROM = 8
08001
// EXEC NLBPROC1, FROM = 9
09001
// EXEC NLBPROC1, FROM = 10
10001
//
Ex/ File number 14 on NLBO is to be converted to Real *4 and written as File number 1 on NLBO. The estimated time required is 2 minutes, then File 23 is to be written as File number 2.

//NLBCOPY JOB -
//NLBPROC PROG, TAPE = NLBO , ONTO = N B01, TT = 2

//NLBPROC PEND

EXEC NLBPROC, FROM = 14, TO = 1
FILE 14 TO FILE 1

EXEC NLBPROC, FROM = 23, TO = 2
FILE 23 TO FILE 2

Obviously File number 2 of NLBO cannot be written before File number 1 has been written. The Real *4 tape must therefore be written in sequence. The program is stored as NLBCON on the Library ACK.NLB.LIB.
PROGRAM NLBTEST

This program is used to check each digitised file for digitising errors and to calculate the mean value of the file. This is necessary when calculating the r.m.s. values of the file using program NLBSEP.

The In-line procedure NLBTEST executes the module NLOOK which is stored on the library ACM.NLB.LIB.

Parameters to be supplied

1) Name of the magnetic tape to be used
   TAPE =

2) Execution time required
   T =

3) File number to be processed
   FILE =

   One data card is required to supply the file number. The format is I2.
PROGRAM NLBSNP

This program reads a sample from the Real 4 tape and then checks if the sampling rate is at least four times the highest frequencies present. The method was suggested by Villasenor (1968).

The user determines from where the sample is read. Sample size (a power of 2) is supplied as data. Briefly, the following procedure is followed:

i) The sample $X_k$ is read where $k = 1, 2, ..., N$.

ii) The sample is divided into two series, one consisting of the odd values of $X_k$ and the other consisting of the even values of $X_k$.

\[
\begin{align*}
T &= \text{total time} \\
X_k &
\end{align*}
\]

\[
\begin{align*}
Y_k &
\end{align*}
\]

\[
\begin{align*}
Z_k &
\end{align*}
\]

iii) The Fourier series coefficients $X_k(f)$, $Y_k(f)$, $Z_k(f)$ are then calculated for the series $X_k$, $Y_k$, $Z_k$ using the Fast Fourier Transform algorithm.

iv) The following quantities are calculated and returned by the program:

\[
\begin{align*}
(a) \text{ ODD-EVEN} & : \sum_{k=1}^{N/4} \left( \frac{|Y_k(f)|^2 + |Z_k(f)|^2}{|X_k(f)|^2} \right) \\
(b) \text{ ODD-BOTH} & : \sum_{k=1}^{N/4} \left( \frac{|Y_k(f)|^2}{|X_k(f)|^2} \right) \\
(c) \text{ EVEN-BOTH} & : \sum_{k=1}^{N/4} \left( \frac{|Z_k(f)|^2}{|X_k(f)|^2} \right) \\
(d) \text{ HARMONICS} & : \sum_{k=1}^{N/4} \left( \frac{|Y_{N/2-k+1}(f)|^2}{|X_k(f)|^2} \right)
\end{align*}
\]
The \( Y_k \) and \( Z_k \) series extend (less one time interval) for the same total time \( T \) as \( X_k \), so that the sampling rates for these series is \( 1/2 \) that for \( X_k \). If \( X_k \) has \( N \) points in the series then there are \( N/2 \) harmonics calculated. The \( Y_k \) and \( Z_k \) series have \( N/2 \) points and hence \( N/4 \) harmonics. Since all the samples extend over the same total time \( T \), the first \( N/4 \) harmonics of each should be identical.

We now assume that there are no frequencies higher than the Nyquist frequencies of \( Y_k \) and \( Z_k \) (that is \( 1/2 \) the Nyquist frequency of \( X_k \)).

(a) is the difference between the Fourier coefficients of similar series displaced by one time interval. The magnitude should give an estimate of allowable errors and can be used as a maximum possible value of the others.

Any frequency that is aliased will be added onto different frequencies in the \( Y_k \) series compared to the \( Y_k \) and \( Z_k \) series. If this occurs, the sums (b) and (c) should be relatively large. A harmonic that will be aliased in the \( Y_k \) and \( Z_k \) series but not the \( X_k \) series will give values of (b) and (c) half as great as before, but its effect will be shown on (d) as well. In this case (b), (c) and (d) should all have approximately the same value.

The program is executed and the returned values of (a), (b), (c) and (d) examined to see if any of (b), (c) or (d) are larger than (a). If so, the sampling rate is not four times the highest frequencies present in the signal.
PROGRAMS NLBPLOT1 AND NLBPLOT2

These two programs plot the results of NLBAUTO and NLCROSS respectively. The results must be in the form written by NLBSTORE.

No data cards, besides those required by subroutine GRAPH are required as all the necessary information is on the data sets produced by NLBSTORE. The job control statement

//SO.FT02F..1 DO ... 

specifies from where the results are to be read.

The size of the axis, symbols printed, etc., are varied by changing the parameters in subroutine GRAPH.
(a) Information on each sample

The following is printed out for each sample analysed:

i) Total.
ii) Mean.
iii) Standard deviation.
iv) Minimum value.
v) Maximum value.
v) Sample R.M.S.
vii) Number of samples outside the range used for the Amplitude Probability Density function. As the overall range is not known before the analysis, the first sample is analysed and ±4 STANDARD DEVIATION of Mean is taken as the range for the A.P.D.F. The A.P.D.F. for the first sample is not used. A check is made on the number of values outside this range.
viii) Sample size.
ix) Sample slope.

(b) Information on all samples

The following overall properties are printed:

i) Overall average.
ii) Overall minimum.
iii) Overall maximum.
iv) Overall R.M.S.
v) Overall Standard deviation.
v) Minimum value used for A.P.D.F.
vii) Maximum value used for A.P.D.F.
APPENDIX III

The analogue values of the \( u \) and \( v \) velocity components were measured using the circuit shown in Fig. (III-1).

![Circuit Diagram](image)

**Fig. (III-1) Circuit for Obtaining the \( u \) and \( v \) Velocity Fluctuations and the Reynolds Shear Stress**

A hot-wire transducer placed in a flow velocity field is primarily sensitive to the velocity component normal to the axis of the hot wire. It therefore only indicates the longitudinal component of flow velocity fluctuations when the wire axis is placed perpendicular to the mean flow direction. If the hot-wire axis is placed at an angle of 45° with respect to the mean flow velocity, then the transducer has equal sensitivity to the longitudinal component of flow velocity fluctuations \( u \) and the transverse component \( v \). For measurement of the transverse component an X-probe is used. An X-probe consists of two mutually perpendicular wires placed at an angle of 45° with respect to the direction...
viii) Total number of points in the series that were outside the above range.

ix) The Student "t" 90% confidence interval for the mean value of the standard deviation.

x) The percentage of the mean value that the above interval constitutes.

xi) The overall Amplitude Probability Density function is printed as the number of the interval versus the frequency.

xii) Either the Hannan or Unhannad Periodogram is printed. Values of 999 indicate that the particular value was not calculated.

xiii) The normalised autocorrelation is printed. Once again any values of 999 indicate that the value was not calculated.

If the analysis was only a single series analysis above would constitute all the output. When a two-series analysis is being performed each series is treated separately as above. After the output for each series is printed the combined output, consisting of the Cross-spectrum and Cross-correlation is printed.

i) Co-spectrum is printed.

ii) Quadrature-spectrum is printed.

iii) Cross-correlation is printed.

(c) Information stored on temporary disc space

Two temporary disc data sets are defined using a DEFINE FILE statement, essentially one for each series to be analysed.

Each data set consists of 7 records, each of which can be up to
The following cards are required when using the program:

```
//NLBSHP JOB -
// EXEC FORTECOLS -
// FORT.SYSIN DD *
program
//LKED.SYSLIB DD *

//SD.SYSIN DD *
Data card give sample size (Integer, Cols. 1 - 4)
//SD.FT03FI01 DD -
```

The DD card for FT03FI01 must direct the program to the sample to be read. As the program will not be used frequently it is only used with standard Job Control Language.

A sample of 4096 points takes approximately \(\frac{7}{2}\) minutes to process.
The hot wires are connected to two constant-temperature anemometers as shown in Fig. (III-1).

The output signals from the anemometers are fed to the circuits which compute the sum and difference of input signals. The sum signal corresponds to twice the instantaneous value of the longitudinal component of the turbulence whilst the difference signal corresponds to twice the instantaneous value of the transverse component. This will be seen from the following:

Assuming the Cosine Law for the wires the output of hot wires 1 and 2

\[ e_1 = (au + bv) \]
\[ e_2 = (au - bv) \]

where \( a \) and \( b \) are sensitivity coefficients. The two anemometers should be adjusted to have equal sensitivities. The sum and difference signals are then:

\[ e_s = e_1 + e_2 = 2au \]
\[ e_d = e_1 - e_2 = 2bv \]

The r.m.s. value of these signals is proportional to the longitudinal and transverse components of the turbulence.
4100 words long (1 word = 4 bytes = 1 Real *4 value). The average values of the correlations and Spectra are stored on these data sets.

When the analysis ends, the following quantities reside on each data set.

1) Single series analysis

The data set defined by DEFINE FILE 7(7,4100, U, IT) contains:

Record Number
1 : Contains the first N/2 values of the Fourier coefficients.
2 : Contains the last N/2 - 2 coefficients i.e. from N/2 + 1 to N + 2 where N is the sample size. These coefficients are divided into two so that sample sizes of 8192 can be analysed. The coefficients of each sample are written over those of the previous sample.
3 : Contains the N/2 Periodogram coefficients. The values for all the samples are added -

\[ \text{value on disc} = \sum_{j=1}^{J} \left( a^2 + b^2 \right) \]

\[ J = \text{number of samples analysed}. \]

4 : Contains the frequencies for the Amplitude Probability Density function as well as other information. If \( K_1 \) is the number of frequencies in the A.P.D.F. the locations

\( 1 + K_1 \) contain the frequencies.

\( K_1 + 1 \) contains 9999 (to be used as a marker),
PROGRAMS ALBANY AND NLBANYG

If the user is interested in any particular section of a file on the Real *4 tape these programs can be used to print it, transfer it to the disc data set ACM.NLB.NELLT or plot it.

NLBANY prints the required blocks and writes them onto an On-line disc data set. It is standard Fortran IV and requires one data card with two integer values

Columns 1 3 6 9
N   N

N = The number of the block before the first one to be read.
N = The number of blocks to be printed.

NLBANYG performs the same function except that the section read is plotted on the Calcomp Plotter. For space considerations only 8 blocks (maximum) of 1024 points can be processed. The number must also be an integer multiple of 1024. One data card with two integer values is required

Columns 1 3 6
N   N

N = The number of the block before the first one to be read.
N = The number of blocks to be read.

As written the program reads off NLB01 but this can be changed easily. The data set written onto is catalogued and called ACM.NLB.NELLT.
K1 + 2 contains N (sample size),
K1 + 3 contains K1 (number of intervals),
K1 + 4 contains DT (time interval between samples),
K1 + 5 contains DVM (the mean value of signal fed in as data),
K1 + 6 contains JA(1) (File number of X series to be analysed),
K1 + 7 contains JA(2) (File number of Y series to be analysed),
K1 + 8 contains JA(3) (which indicates whether a single or two series analysis is performed).
5 : Contains the autocorrelation coefficients (totalled over each sample). There are N/2 + 1 coefficients.
6 & 7 : Are not used in the single series analysis.

11) Two series analysis.
There are two disc data sets defined by
DEFINE FILE 7 (7, 4100, U, IT)
and DEFINE FILE 8 (7, 4100, U, IT)
Records 1 to 5 are the same as for the single series analysis,
DEFINE FILE 7 containing the information on the X series and
DEFINE FILE 8 containing the information on the Y series. The records 6 and 7 of each data set contain the following information.
DEFINE FILE 7
Record
6 : Contains the N/2 Co-spectrum coefficients.
PROGRAM NLBLZONE

This program processes the output of NLBLINT. It reads 4096 velocity values, the associated intermittency signal, and separates the velocity fluctuations into two signals, one containing the turbulent points and one containing the non-turbulent points. These two signals are written onto an output device. Execution is terminated when the end of the data file is reached. The output is written in blocks of 1024 values.

No data cards are required but the following Job Control Statements must be specified:

i) //GO.FT03F001 DD ...

is the unit from which the data is read.

ii) //SYSUT2 DD in STEP 2 is the unit onto which the turbulent signal must be written.

iii) //SYSUT2 DD in STEP 3 is the unit onto which the non-turbulent points must be written.

The program writes the separated signals onto temporary disc space and the system program IEGENER is then used to write from the disc onto the units specified by the user.
7: Contains the $N/2$ Quadrature-spectrum coefficients.

**DEFINE FILE 6.**

**Record**

6: Contains the first $N/2 + 1$ Cross-correlation coefficient.

7: Contains the last $N/2 + 1$ to $N + 2$ coefficients.

**Data**

Two data cards are required, providing the following information:

a) **CARD 1**

i) Sample size $N$.

ii) Number of samples (maximum) to be analysed, $NS$.

iii) Number of intervals for the Amplitude Probability Density Function, $K1$.

iv) Indicator as to whether Fourier coefficients must be hanned or not, $NH$.

$NH = 1$ Yes

$NH 
eq 1$ No

v) The mean value of the signals (of use when analysing turbulence fluctuation only), $DVH$.

vi) The time interval (in sec) between values of the signal, $DT$.

b) **CARD 2**

i) File number for $x$ series $JA(1)$.

ii) File number for $y$ series $JA(2)$.
Once the frequency range and amplitude cut-off have been decided upon, the program is used to obtain an intermittency signal. This is a modified version of a program written by Drayer (1973) and the particular write-up should be consulted.

Briefly, the program reads 4096 values of the recorded signal (//FT03F001) and then writes the section of signal, followed by its associated intermittency signal, onto an output device (//FT04F001).

The calculation is terminated in two ways:

1) No more data is available to be read.
2) Enough turbulent and non-turbulent points have been calculated.

If one is only interested in obtaining, say 1024 turbulent points and 10240 non-turbulent points, then the IF statements in the main routine must be altered.

E.g. IF(NTURB.GT.1024) turbulent points
     IF(NNTUB.GT.10240) non-turbulent points.

The .GT. is necessary as the program continues until the next multiple of 1024 points has been reached. The output is written in blocks of 1024 values.

The graph section has been bypassed in this version as so many points are analyzed. No data cards are required for the program, but the following job control cards must be specified:

1) //GO.FT03F001 DD ...
   is the unit from where the data is read
2) //GO.FT04F001 DD ...
   is the unit onto which the signal and its associated intermittency signal are written.
111) Indicator showing whether a single or two series analysis is to be performed:

\[ JA(3) = 0 \] Single series,
\[ JA(3) = 1 \] Two series.

The figure below indicates how the data is to be arranged on the data cards:

<table>
<thead>
<tr>
<th>Column</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>30</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>Card 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>N</td>
<td>NS</td>
<td>KI</td>
<td>NH</td>
<td>OVM</td>
<td>DT</td>
<td></td>
</tr>
<tr>
<td>Format</td>
<td>16</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>F10.2</td>
<td>F10.2</td>
<td></td>
</tr>
<tr>
<td>Column</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Card 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantity</td>
<td>JA(1)</td>
<td>JA(2)</td>
<td>JA(3)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Format</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure Giving Format of Data Cards

NOTE: Integer numbers must be right justified.

The input to NLBONE has been discussed as has its output and the path of analysis. It is now only necessary to describe the in-line procedures required to execute the program. The program NLBONE contains all the Job Control Language to run the program in the conventional way.
PROGRAM NLASEP

This program is used to separate the signals from the two wires of an X-array probe into the instantaneous values of the u and v velocity fluctuations. The user must supply the angles each wire makes with the mean-stream velocity as well as the directional sensitivity coefficient of each wire. Quantities calculated are:

i) Number of blocks to be manipulated.
ii) r.m.s. value of the u component.
iii) r.m.s. value of the v component.
iv) The Reynolds Shear Stress.
v) r.m.s. value of the 1st signal before separation into components.
vi) r.m.s. value of the 2nd signal before separation into components.

The instantaneous u and v fluctuation signals are written onto magnetic tape for later reference.

The program consists of 5 steps. To save time the program first transfers the blocks of data to be manipulated onto a temporary disc data set. Efficient system programs are used to perform this. The data are then read and separated into the u and v velocity components. These are written onto another temporary disc data set and then finally transferred to magnetic tape for storage purposes. The program assumes that the data read off magnetic tape is REAL*4 and blocked in blocks of 1024 values.

There is no In-line procedure for this routine and hence job control cards must be changed when analysing different sets of data.

Theory

The program reads 2 blocks of data, one from file 1 and the corresponding block from file 2. Equations (3-4) are then solved giving the instantaneous values u and v for the values of \( c_1 \) and \( c_2 \) used.
PROGRAM NLBAUTO

NLBAUTO is the name of the program as well as the In-line
procedure used to execute NLEONE when a single series analysis is to be
performed.

Parameters to be supplied:

1) Name of Real *4 tape containing data.
   TAPE =

2) Name of program to be executed.
   FDS = NLEONE (in general)

3) File number to be used.
   N =

4) Execution time required.
   TT =

5) Disposition of the temporary data sets (DEFINE FILE).
   GF = NEW or OLD

NOTE:

v) Applies when the program is to be executed a number of times in the
same JOB. The first time either NLBAUTO or NLCROSS is executed
GF = NEW. On all subsequent occasions GF = OLD.

The program uses 275 K of storage.

Ex/ Autocorrelations and Periodograms of the files 3 and 6 on NLBO1
are to be calculated. Sample sizes are 2048, and the maximum number of
samples to be analysed is 20. The execution time for each step is
estimated at 15 minutes.

1) 3 is to be named. 6 must not.

2) Number of intervals is 30 in each case.
Values of \( k_1, k_2, \alpha_1 \) and \( \alpha_2 \) must be supplied by the user. The r.m.s. quantities are calculated from sample to sample but the taking of square roots is only done at the very end of the calculations. A parameter in the program is varied depending on the number of blocks to be analysed.

**Job Control Statements**

Steps ONE, TWO, FOUR and FIVE all execute the system program IDEGENR which transfers a file of data from one unit to another. The statement

```
//SYSUT1 DD ...
```

indicates the unit from which the data is to be read and

```
//SYSUT2 DD ...
```

indicates the unit onto which it must be written. In steps ONE and TWO the user must specify the correct parameters for //SYSUT1 and in steps FOUR and FIVE the parameters for //SYSUT2 DD ... must be specified.

The only parameter in step THREE that could need specification is the TIME.GO = parameter which depends on the number of blocks being analysed.

**Data Cards**

Only two data cards are required. The first contains 4 parameters in 4F10.2 format. They are:

1) \( \text{ALPHA1} \) = Angle between the hot-wire sensor and the mean-flow direction for channel 1 (in degrees).
2) \( \text{ALPHA2} \) = same as above, but for channel 2.
3) \( \text{SENS1} \) = Directional sensitivity coefficient for channel 1. It is defined in equation (3-2).
4) \( \text{SENS2} \) = same as above, but for channel 2.
iii) For 3 DVM = 1.7
DT = 0.002
iv) For 5 DVM = 1.6
DT = 0.002

The JOB set-up would be as follows:

//NLBAUTO JOB -
//NLBAUTO PROC 9DS = NLBONE, TAPE = NLB01, TT = 15
//NLBAUTO PENJ
//ONE EXEC NLBAUTO, N = 03, DF = NEW
//SYSIN DD*
2048 20 30 1 1.7,002
030300
//TWO EXEC NLBAUTO, N = 05, DF = OLD
//SYSIN DD*
2048 20 30 2 1.6,002
060600
The second card contains 2 values in 2E12.5 format

\[ \text{AVE1} = \text{Average value of channel 1.} \]
\[ \text{AVE2} = \text{Average value of channel 2.} \]
PROGRAM NLBCROSS

NLBCROSS is the name of the program as well as the In-line procedure used to execute NLBONE when performing a two series analysis.

Parameters to be Supplied

1) Name of Real *4 tape with both series.
   DATA =

11) Name of program to be executed.
   PDS =

111) Execution time required.
   TT =

1v) File number of X array,
   X =

1v) File number of Y array,
   Y =

11v) Disposition of the temporary data sets.
   DF = OLD or NEW (see NLBAUTO)

The data for NLBONE is as described in the section for NLBONE.

Ex/ Cross-correlations and Cross-spectra are required of files 3 and 6 on NLBO1. Both must be Hanned, otherwise the information is the same as the previous example. The JOB setup would be:

//NLBCROSS JOB -
//NLBCROSS PROC DATA = NLBO1, PDS = NLBONE

//NLBCROSS PEND

// EXEC NLBCROSS, TT = 15, X = 03, Y = 06, DF = NEW
//SYSIN DD *

2049 20 30 1 1,7 0,02
030601
//
PROGRAM NLNLD:

This program is the only general routine written. It is fairly complex and will be discussed in some detail. The routine makes use of the subroutines already discussed and hence the theory behind them is not shown. The following functions are calculated:

1) Various statistics of the samples.
2) The Periodogram.
3) The autocorrelation.
4) The Co-Spectra of two samples.
5) The Quadrature-Spectra of two samples.
6) The Cross-correlation.

The routine can be used to analyse a single time series in which case the quantities (iv), (v) and (vi) are not calculated. Briefly, the program reads a sample and calculates the quantities (i) - (vii) for the sample.

The results are then averaged with the results of previous samples and stored on a temporary disc data set. Depending on the number of samples already analysed or the result of the Student "t" test another sample is read or the program proceeds to print the results.

The results on disc can be read and stored on magnetic tape by NLBSTORE. NLBPLT1 will plot single series analyses data (read off tape), while NLBPLT2 will plot data from a two series calculation. The programs are discussed later.

The program is used in conjunction with two different Ir-line procedures, depending on whether one is doing a single series or two series manipulation. The single series procedure is called NLBAUTO
NOTE:

The program assumes that all the data applies to both signals
i.e., both have the same mean value.

The program uses 275K of storage.

Timing information:

<table>
<thead>
<tr>
<th>Sample Size</th>
<th>No. of Samples</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>2048</td>
<td>4</td>
<td>5 mins.</td>
</tr>
</tbody>
</table>
while that for two series calculations is called NLBCROSS. The program
can also be executed using standard Job Control Language, a set of which
is available.

Sample sizes are decided on by the user and can be either 1024, 2048, 4096 or 8192. The present form of the program cannot take
samples larger than 8192 due to hardware limitations on the computer.
Besides the final averaged results, some results for each sample are
returned by the program. When calculating two series quantities, both
signals must be on the same tape (but different files). Depending on
the number of samples to be analysed, an appropriate quantity of the
second file is transferred to disc to facilitate easy reading of the
two files.

A simplified block diagram of the program is given below. All
the necessary subroutines are stored in the library ACM.NLB.LIB, as
is the program NLBONE.

NOTE: Cross-correlations are given as $R_{XY}(s)$ where

$X$ is the first series

$Y$ is the second series
PROGRAM NLSTORE

This program is designed to read the results on the temporary disc data sets and then write them onto magnetic tape. The results can then be manipulated or plotted at a later stage.

The in-line procedure NLSTORE is used to execute the program. No data is necessary, as the program reads it off the disc. The Define File records of 4100 are read and depending on the sample size, divided into varying numbers of blocks of 1024. For convenience each record is divided into the same number of blocks, so that some of the blocks written contain nothing.

Ex/ NLBCROSS executed with N = 4096.

i) JA(3) = 1 and hence two files to be read.

ii) 2048 Periodogram coefficients.

iii) Therefore 2 blocks of 1024 (sometimes 1 or 2 values in next block are neglected).

iv) 2 blocks from each record on DEFINE FILE 7 ( ) are written onto magnetic tape, and then the same amount of DEFINE FILE 8 ( ) written onto tape.

v) If JA(3) = 0 i.e. NLBAUTO then DEFINE FILE 8 ( ) is neglected and records (6) and (7) on DEFINE FILE 7 ( ) are not read since they would contain information from NLBCROSS.

The parameters to be supplied in NLSTORE are:

i) Name of labelled tape,

TAPE =

ii) File onto which the results must be written.

FILE =

iii) Time of 2 minutes is specified.

As an example to illustrate the use of NLBAUTO, NLBCROSS and NLSTORE,
Start.

Read in program constants.

If two series analysis, then transfer the required amount of 2nd file to disc and return to 1st file.

Read sample.

Calculate min., max., st.dev., total, amp.prob.dens.function (on 2nd sample use 4 x st.dev. of 1st sample as range).

Average amplitude probability density function (1st sample neglected).

Remove mean and linear trends

Calculate periodogram.

Average periodogram coefficients and store Fourier coefficients for cross-correlation.

Calculate autocorrelation.

Average the autocorrelation

Call check on mean of sample st.dev.

It is an autocorrelation and more samples must be analysed.

It is an autocorrelation and all the samples have been analysed (or results accurate enough)

The last sample analysed was from 1st file (X).

Calculate cross-spectrum of last X and Y samples.

Average cross-spectrum.

Calculate cross-correlation of last X and Y samples.

Average cross-correlation.

Have all the samples been analysed or are results accurate enough?

Change back to X.

Is this a two series calculation?
Change do loop parameter.

Output for each sample of X series and for Y series if do loop returns.

Output overall averaged results for X series and for Y series if do loop returns.

Is this a two series analysis?

Output cross-correlation and cross-spectrum

End.

(1) Theory

The theory behind all the steps besides the decision making has been described elsewhere. To try and minimise the computer time used, a statistical check on the mean of the sample Standard Deviation is performed. For a more complete explanation of the theory and problems involved see F.R. Krause et al (1970). A maximum number of samples to be analysed is specified, but if the statistical test is satisfied before that stage then execution of the program is terminated.

In this program only the Standard Deviation of each sample is checked. The ideal situation would be where each of the quantities calculated has its own statistical test. As each test is satisfied so that quantity is not calculated anymore. This would then give the lowest execution time.

(15) Output

The program prints out information pertaining to each sample as well as overall properties. Various parameters as well as quantities such as the autocorrelation, Amplitude Probability Density function are stored on a temporary disc data set and can therefore be used by a program executed after NLDONE (as long as it is still in the same job). Firstly the information printed out will be described.
perform the example for NLBAUTO, writing answers of FILE 6 onto first file of RESULT. As the next step perform the example for NLBCROSS and write the results onto file 2 of RESULT.

The job setup would be:

```plaintext
//NLBALL JOB -
//NLBAUTO PROC PDS = NLDONE, TAPE = NLDONE, TT = 15

//NLBAUTO PEND
//NLBCROSS PROC DATA = NLDONE, PDS = NLDONE, TT = 15

//NLBCROSS PEND
//NLSTOR C PROC TAPE = RESULT

//NLSTOR C PEND
//ONE EXEC NLBAUTO, N = 03, DF = NEW
//SYSIN DD *
2048 20 30 1 1,7 ,002
030300

//TWO EXEC NLBAUTO, N = 06, DF = OLD
//SYSIN DD *
2048 20 30 2 1,7 ,002
060601

//TIV EXEC NLBCROSS, FILE = 01
//SIX EXEC NLBCROSS, X = 03, Y = 06, TT = 15, DF = NEW
//SYSIN DF *
2048 20 30 1 1,7 0,002
030601

//SEV EXEC NLSTOR C, FILE = 02
```

**NOTE:** NLBCROSS calculates the autocorrelation of 03 and 06 anyway.
Author: Burnton Neil Lorraine
Name of thesis: The analysis of turbulent flows using a digital computer, with special reference to the plane mixing layer.
1973

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