CHAPTER 3 DERIVING THE IMPROVED TWO GENERATOR MODEL

3.1 REQUIREMENT FOR AN IMPROVED TWO GENERATOR MODEL

This dissertation shows that when a real network is reduced into a classical two generator model that includes the network shunts (i.e. improved classical two generator model) the obtained model predicts the locus traced in the real network with great accuracy.

When the network shunts are removed, the obtained classical two generator model does not predict the locus traced in the real network with great accuracy.

The real network this dissertation considers is the Mpumalanga-to-Western-Cape network. This network can be reduced into a two generator model because when the Western-Cape is slipping poles with respect to Mpumalanga, the network behaves like two generators ($E_{MPU}$ and $E_{KRG}$) slipping poles.

This dissertation also shows that not all busbars trace a classical locus. This is due to the affect the shunts have on the network impedance and the source voltage ratio.

The classical two generator model is helpful in understanding the results obtained with complex networks. In the same way, the improved two generator model is helpful in showing that shunts make the swing seen at Hydra, when looking into the feeder that links Hydra to Mpumalanga, trace a non-classical locus. The improved two generator model is also helpful in showing that the non-classical behaviour is due to the affect shunts have on the network impedance and source voltage ratio.
3.2 BASIC DECRPIPTION OF THE IMPROVED TWO GENERATOR MODEL

The improved two generator model is the classical two generator model with the network shunts included.

Hence, the improved two generator model should use the constant voltage behind transient reactance generator model.

The explanation why the network shunts could greatly affect the impedance locus that the swing traces, is based on the equation Clarke uses to compute the seen impedance. This equation is obtained from her paper “Impedances Seen by Relays During Power Swings With and Without Faults” [23].

The mentioned Clarke paper uses a constant impedance load model. Hence, the improved two generator model should model the network shunts to be constant impedance elements and the load to be constant impedance.

3.3 NETWORK OUT-OF-STEP EVENT

This dissertation uses the failure of a 400 kV bus-section breaker at the Hydra substation as the cause of the Mpumalanga-to-Western-Cape network slipping poles.

The busbar layout of the 400 kV feeders integrating Hydra into the Eskom network is shown in figure 3.1.
FIGURE 3.1 Layout of the 400 kV Hydra busbar. “SC” is an abbreviation for Series Compensated; “BC” is an abbreviation for Bus Coupler and “BS” is an abbreviation for Bus Section

The sequence of events used to model the failure of the bus section breaker BS 4 in PSS/E is a three phase fault that occurs on busbar B; the three phase fault causes a bus strip (i.e. breakers BC 2, BS 3 and the feeders shown dotted in figure 3.1 trip) and the fault is cleared within 80 milliseconds.

With regard to the fault clearance time, we note typical bus zone schemes release the trip signal within 20 to 60 milliseconds. The breaker contacts should open in about 40 milliseconds. Hence, the worst clearance time will be 100 milliseconds – we use 80 milliseconds.
3.4 TOPOLOGY OF THE IMPROVED TWO GENERATOR MODEL

The improved two generator model is made up of three networks (figure 3.2). These networks are the Mpumalanga generation pool, the Western Cape generation pool and the tie line linking Mpumalanga to the Western Cape.

With regard to figure 3.2 we note, \( Z_{\text{TIE}} \) is the impedance of the Mpumalanga-to-Western-Cape tie line when all shunts are omitted; \( X_{\text{TRFR MPU}} \) is the reactance of the generator transformer linking the Mpumalanga generator to the network; \( X_{\text{TRFR KBG}} \) is the reactance of the generator transformer linking the Koeberg generator to the network; \( x'_{\text{MPU}} \) is the transient reactance of the Mpumalanga generator; \( x'_{\text{KBG}} \) is the transient reactance of the Koeberg generator; \( Z_{\text{LINK}} \) represents the network linking the Mpumalanga generator to the busbar \( V_{\text{MPU}} \); \( \delta_{\text{POOL}} \) is the power angle that develops between \( V_{\text{MPU}} \) and \( V_{\text{KOEBG}} \); \( P_{\text{EXP}} \) is the active power the Perseus/Beta busbar (section 3.5.3.3) exports to the Western Cape; \( a Z_{\text{TIE}} \) is the electrical distance between the Perseus busbar and \( V_{\text{MPU}} \); \( b Z_{\text{TIE}} \) is the electrical distance between the Muldersvlei busbar and \( V_{\text{KOEBG}} \); \( c Z_{\text{TIE}} \) is the electrical distance between the Hydra busbar and \( V_{\text{MPU}} \); \( P \) is the MW demand of a given load centre (e.g. \( P_P \) represents the MW demand of the Perseus load centre); \( Q \) is the MVAR demand of a given load centre. \( Q \) includes line charging (e.g. \( Q_P \) represents the MVAR demand of the Perseus load centre as well as the line charging seen at Perseus); \( E'_{\text{MPU}} \) is the voltage behind transient reactance of the Mpumalanga generator and \( E'_{\text{KBG}} \) is the voltage behind transient reactance of the Western Cape generator.
FIGURE 3.2 Topology of the improved two generator model

The 400 kV Grootvlei/Atlas/Tutuka busbar, $V_{MPU}$, forms the southern border of the Mpumalanga generation pool (section 3.5.1). The 400 kV Koeberg busbar, $V_{KOEKG4}$, forms the northern border of the Western Cape generation pool (section 3.5.1).

The network between the 400 kV Grootvlei/Atlas/Tutuka busbar and the 400 kV Koeberg busbar constitutes the tie line.

The load flow of the tie line is shown in figure 3.3. This load flow model is the most accurate of all the models this dissertation uses to model the tie line.

The improved two generator model models the tie line using the circuit linking $V_{MPU}$ to $V_{KOEKG4}$ in figure 3.2.
FIGURE 3.3 Steady state load flow of the Mpumalanga-to-Western-Cape network. The stub shown at Aurora represents the network linking Aurora to Kronos. The phrase “detailed network model” refers to this network.

Section of the network where all equipment/feeders are not shown.
3.5 NORMAL OPERATION OF THE TIE LINE

The bulk of the power requirements of the Eskom network are generated in Mpumalanga. Situated 1 200 km from this major source of generation is the Koeberg nuclear power station - located in the Western Cape (figure 3.4).

FIGURE 3.4 Main transmission network of Eskom

The power generated in the Western Cape is smaller than the Western Cape load. To meet the Western Cape load, power is imported from Mpumalanga. The Mpumalanga-to-Western-Cape tie line forms the means of transmission.

3.5.1 Northern- and southern borders of the tie line

In the improved two generator model (figure 3.2) the voltage phasor, $E_A^\prime$, of generator A leads the voltage phasor, $E_B^\prime$, of generator B.

In the detailed network model (figure 3.3), Mpumalanga is exporting power to the Western Cape. Hence, generator A represents the Mpumalanga generation pool and generator B represents the Western Cape generation pool.
Geographically generator A and generator B are separated by approximately 1,200 km. The tie line interconnects generator A and generator B.

In the improved two generator model $V_{MPU}$ is the northern border of the tie line and $V_{KOEBG4}$ is the southern border of the tie line.

To define the electrical location of $V_{KOEBG4}$, we note this dissertation considers the case where Koeberg is the only source of generation in the Western Cape. Hence, the Koeberg generator represents generator B. Therefore, $V_{KOEBG4}$ is the 400 kV busbar at Koeberg.

In the improved two generator model a single line links $V_{MPU}$ to the Perseus/Beta busbar, $V_{PB}$ - Perseus and Beta are considered to be the same busbar due to the voltages and power angles being approximately equal (figure 3.3).

In reality two transmission networks feed into Perseus/Beta. They are the Tutuka-to-Perseus/Beta network and the Grootvlei/Atlas-to-Perseus/Beta network (figure 3.3). Each network is associated with a generation pool (section 3.8.1).

To obtain a single line linking these two networks to the Perseus/Beta busbar a point of common coupling must be found.

The power Mpumalanga exports to the Western Cape passes through Hydra. Consequently, Hydra could be the point of common coupling. Figure 3.5 shows large excursions in voltage, frequency and angle occur at Hydra during the first slip cycle – Appendix A shows the first slip cycle is completed within 1.474 seconds.

The power angles shown in figure 3.5 and figure 3.7 are the angles that develop between the synchronous rotating reference and the busbar voltage.
When moving closer (i.e. north) to the centre of the Mpumalanga generation pool, say to Atlas, the excursions in voltage, frequency and power angle are less.

This behaviour of Atlas matches the behaviour expected of the high voltage busbar of the Mpumalanga power pool. To clarify, we note the bulk of the Eskom generation takes place in Mpumalanga – i.e. the reactance of this generation pool is small and the inertia is large.

Figure 3.6 shows that the Grootvlei, Atlas and Tutuka busbars behave coherently. Grootvlei, Atlas and Tutuka can, therefore, be used to represent the southern border of the Mpumalanga generation pool. The geographical location of Grootvlei is shown in figure 3.4.

**FIGURE 3.5** Voltage, frequency and power angle measured at Atlas and Hydra

FIGURE 3.5 Voltage, frequency and power angle measured at Atlas and Hydra
FIGURE 3.6 Voltage and frequency measured at Grootvlei, Atlas and Tutuka

FIGURE 3.7 Power angle that develops between the synchronous rotating reference and the Grootvlei, Atlas and Tutuka busbar voltage
3.5.2 Tie line loads and SVCs

Static Var Compensators (SVC) are installed at Perseus, Hydra and Muldersvlei. All these SVCs have a large dynamic range – e.g. the dynamic range of the Hydra SVC is 250 MVAR reactive to 250 MVAR capacitive. In addition, a large MW demand exists at each of Perseus, Hydra and Muldersvlei (figure 3.3).

Hence, expressed in MVA, a large portion of the shunts present in the Mpumalanga-to-Western-Cape tie line are situated at Perseus, Hydra and Muldersvlei. Therefore, it is reasonable to model the Mpumalanga-to-Western-Cape tie line by a circuit that has shunts at only Perseus, Hydra and Muldersvlei.

The admittance of these shunts can be computed using:

\[ Y = \frac{S}{|V|^2} = \frac{(P - jQ)}{|V|^2} \]  

Where \( |V| \) is the voltage magnitude; \( P \) is the MW demand and \( Q \) the MVAR demand of the load situated at \( V \).

PSS/E defines shunts in terms of their MW and MVAR demand at nominal voltage.

This section obtains the MW and MVAR demand of the Perseus, Hydra and Muldersvlei shunts.

With regard to the MW demand of the Perseus, Hydra and Muldersvlei shunts, we note these demands are obtained when adding the MW demands of the remaining tie line loads to the existing Perseus, Hydra and Muldersvlei loads. The decision
where a specific load is lumped, i.e. at Perseus or Hydra or Muldersvlei, is based on which of the mentioned busbars is electrically the closest to the specific load.

To clarify, consider the Bacchus load shown in figure 3.3. The Hydra-Bacchus power angle, $\delta_{HB}$, has the value $\delta_{HB} = 30.7^\circ = (-57.8^\circ + 88.5^\circ)$; the Bacchus-Muldersvlei power angle, $\delta_{BM}$, has the value $\delta_{BM} = 8.7^\circ = (-88.5^\circ + 97.2^\circ)$; 8.7° is much less than 30.7°. Consequently, the Bacchus load is assigned to Muldersvlei.

Table 3.1 lists the loads making up the Perseus, Hydra and Muldersvlei load centres. Also listed is the MW demand, expressed as a demand at 400 kV when the busbar voltage is 1.0 per unit.

To obtain the reactive portion of the shunts at Perseus, Hydra and Muldersvlei all line capacitance, voltage regulating shunts and loads are removed from the detailed network model. The tie line loads are then reassigned to Perseus, Hydra or Muldersvlei according to table 3.1. The SVCs at Perseus, Hydra and Muldersvlei are then used to regulate their respective 400 kV voltages to achieve the magnitudes shown in the detailed network model (figure 3.3) – i.e. $|V_{PERSS}| = 1.04$ p.u., $|V_{HYDRA}| = 1.036$ p.u. and $|V_{MULDRA}| = 1.003$ p.u. Additional shunt capacitors are then installed at Grootvlei, Atlas, Tutuka and Koeberg so that Koeberg and Mpumalanga inject the same amount of VARS into the tie line as is the case for the detailed network model.

To clarify the above, we consider the Atlas busbar. We note, in the detailed network model (figure 3.3) Atlas imports 85.5 MVAR from Grootvlei and exports 29.4 MVAR to Matla. In the simplified network model (figure 3.8) Atlas imports 87.1 MVAR from Grootvlei and exports 23.2 MVAR to Matla.

The MW and MVAR demand of each load centre is listed in table 3.2.
With regard to table 3.1, we note Grootvlei, Atlas and Tutuka form the southern border of the Mpumalanga generation pool. This explains why the Atlas load is not included in the Perseus load centre. The Aurora load includes the Northern Cape ring up to Kronos. The Droërivier load includes the two line reactors at Droërivier (figure 3.3). The Perseus load includes the line reactor at Beta (figure 3.3).

Table 3.1 MW and MVAR demand of the loads constituting the Perseus, Hydra and Muldersvlei load centres

<table>
<thead>
<tr>
<th>LOAD CENTRE</th>
<th>BUSBARS MAKING UP THE CENTRE</th>
<th>POWER ANGLE</th>
<th>EQUIVALENT LOAD AT 400 kV</th>
</tr>
</thead>
<tbody>
<tr>
<td>• WESTERN-CAPE (SVC at Muldersvlei)</td>
<td>Koeberg</td>
<td>( \delta = -98.0 )</td>
<td>99.0</td>
</tr>
<tr>
<td></td>
<td>Aurora</td>
<td>( \delta = -99.5 )</td>
<td>299.1</td>
</tr>
<tr>
<td></td>
<td>Acacia</td>
<td>( \delta = -99.5 )</td>
<td>856.1</td>
</tr>
<tr>
<td></td>
<td>Muldersvlei</td>
<td>( \delta = -97.2 )</td>
<td>855.8</td>
</tr>
<tr>
<td></td>
<td>Bacchus</td>
<td>( \delta = -88.5 )</td>
<td>248.0</td>
</tr>
<tr>
<td></td>
<td>Proteus</td>
<td>( \delta = -82.6 )</td>
<td>252.0</td>
</tr>
<tr>
<td></td>
<td>Total load</td>
<td></td>
<td>2610.0</td>
</tr>
<tr>
<td>Muindersvlei SVC</td>
<td></td>
<td></td>
<td>-206.0</td>
</tr>
<tr>
<td>• CENTRAL (SVC at Hydra)</td>
<td>Hydra: 132 kV</td>
<td>( \delta = -57.8 )</td>
<td>111.0</td>
</tr>
<tr>
<td></td>
<td>Hydra: 220 kV</td>
<td>( \delta = -57.8 )</td>
<td>96.0</td>
</tr>
<tr>
<td></td>
<td>Poseidon</td>
<td>( \delta = -57.8 )</td>
<td>1235.8</td>
</tr>
<tr>
<td></td>
<td>Droërivier</td>
<td>( \delta = -71.3 )</td>
<td>21.0</td>
</tr>
<tr>
<td></td>
<td>Total load</td>
<td></td>
<td>1463.8</td>
</tr>
<tr>
<td>Hydra SVC</td>
<td></td>
<td></td>
<td>-512.7</td>
</tr>
<tr>
<td>• NORTHERN (SVC at Perseus)</td>
<td>Beta-Perseus</td>
<td>( \approx -36.25 )</td>
<td>466.0</td>
</tr>
<tr>
<td></td>
<td>Theseus</td>
<td>( \delta = -26.6 )</td>
<td>379.7</td>
</tr>
<tr>
<td></td>
<td>Leander</td>
<td>( \delta = -27.3 )</td>
<td>242.4</td>
</tr>
<tr>
<td></td>
<td>Total load</td>
<td></td>
<td>1088.1</td>
</tr>
<tr>
<td>Persues SVC</td>
<td></td>
<td></td>
<td>-280.4</td>
</tr>
</tbody>
</table>
FIGURE 3.8 The simplified Mpumalanga-to-Western-Cape network. The line charging and SVC support present in the detailed network model (figure 3.3) are represented by the VAR injections taking place at Grootvlei, Atlas, Tutuka, Perseus, Hydra, Muldersvlei and Koeberg. The MW and MVAR demands of the Perseus, Hydra and Muldersvlei shunts are according to table 3.2. The shunt capacitors located at Grootvlei and Atlas ensure that the voltages at these two busbars match the voltages shown in figure 3.3. The shunts capacitors installed at Tutuka and Koeberg ensure that the VARs flowing from Tutuka and Koeberg are as is shown in figure 3.3. The phrase “simplified network model” refers to this network.
## Table 3.2 MW and MVAR demand of the Perseus, Hydra and Muldersvlei load centres

<table>
<thead>
<tr>
<th>LOAD CENTRE</th>
<th>DESCRIPTION</th>
<th>EQUIVALENT LOAD AT 400 kV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MW demand as per table 3.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MVAR support as per figure 3.8</td>
<td></td>
</tr>
<tr>
<td>WESTERN-CAPE (SVC at Muldersvlei)</td>
<td>Total load</td>
<td>2 610.0</td>
</tr>
<tr>
<td>CENTRAL (SVC at Hydra)</td>
<td>Total load</td>
<td>1 463.8</td>
</tr>
<tr>
<td>NORTHERN (SVC at Perseus)</td>
<td>Total load</td>
<td>1088.1</td>
</tr>
</tbody>
</table>

With regard to table 3.2, we note the Western Cape load is lumped at Muldersvlei. The load of the Central area is lumped at Hydra. The load of the Northern area is lumped at Perseus.

To show how the MVAR demands listed in table 3.2 are obtained, we compute the MVAR demand, $Q_{MULD}$, of the Muldersvlei load centre.

$Q_{MULD}$ is obtained by summating the VARs flowing on the lines linking Muldersvlei to the network.

Using figure 3.8 it follows $Q_{MULD}$ is made up of the MVAR flow of the Muldersvlei-Droërivier feeder ($-663$ MVAR); the MVAR flow of the Muldersvlei-Bacchus feeder ($-557$ MVAR); the MVAR flow of the Muldersvlei-Koeberg 1 feeder ($9.9$ MVAR); the MVAR flow of the Muldersvlei-Koeberg 2 feeder ($10.4$ MVAR) and the MVAR flow of the Muldersvlei-Acacia feeder ($4.9$ MVAR).

The sum of the above flows is $-1 194.8$ MVAR. This is the MVAR demand at Muldersvlei when the Muldersvlei busbar voltage is 1.003 p.u. (figure 3.8).
express this demand as a demand at nominal voltage, we need to multiply by 
\[0.994 = \left(\frac{1}{1.003}\right)^2\]. Therefore, the MVAR demand at Muldersvlei is 
\[-1.187.7 = -1.194.8 \times 0.994\] MVAR.

To clarify the above equation we note, admittance, \(Y\), is independent of voltage. Hence,

\[
Y = \frac{S_1^*}{|V_1|^2}
\]

\[
= \frac{S_2^*}{|V_2|^2}
\]

\[
\frac{S_2^*}{|V_2|^2} = \frac{S_1^*}{|V_1|^2}
\]

\[
S_2^* = \frac{|V_2|^2}{|V_1|^2}S_1^*
\]

\[
S_2 = \frac{1}{|V_1|^2}S_1
\]

(3.2)

Where \(|V_1|\) is the busbar voltage; \(S_1\) is the demand when the voltage is \(|V_1|\); \(S_1^*\) is the complex conjugate of \(S_1\); \(S_2\) is the demand when the voltage is \(|V_2| = 1.0\) p.u. and \(S_2^*\) is the complex conjugate of \(S_2\).

3.5.3 Tie line impedance

This section obtains the impedance, \(Z_{TIE}\), of the tie line linking \(V_{MPU}\) to \(V_{KOEBG4}\) (figure 3.2).
The $Z_{TIE}$ obtained should ensure the power, $P_{EXP}$, Perseus/Beta exports to Hydra in figure 3.2 matches the same power export measured in the simplified network model (figure 3.8).

When the detailed network model (figure 3.3) is re-arranged to have shunts where the improved two generator model (figure 3.2) has shunts the simplified network model is obtained.

$$R_{TIE} = (3/2) \times 0.07463 X_{TIE}$$  (Appendix B). Therefore, to obtain $Z_{TIE}$ we need to obtain $X_{TIE}$ only.

The equation relating $X_{TIE}$ to $P_{EXP}$ is:

$$P_{EXP} = \text{Real}(V_p I_{EXP}^* ) = \text{Real} \left[ \left( E_A^{TH} - I_{EXP} Z_A^{TH} \right) I_{EXP}^* \right]$$  (3.3)

Where:

$$I_{EXP} = E_A^{TH} \frac{Z_2 / Z_3 + 1 - E_B^{TH}}{Z_1 Z_2 / Z_3 + Z_1 + Z_2}$$  (3.4-a)

$$Z_1 = Z_A^{TH} + (c - a) Z_{TIE}$$  (3.4-b)

$$Z_2 = Z_B^{TH} + (1 - c - b) Z_{TIE}$$  (3.4-c)

$$Z_3 = \frac{1}{Y_H}$$  (3.4-d)

For the case where the network between $V_{MPS}$ and $V_{KOEG}$ is considered, we have:

$$Z_A^{TH} = Z_p // (a Z_{TIE})$$  (3.5-a)

$$E_A^{TH} = V_{MPS} Z_p / (Z_p + a Z_{TIE})$$  (3.5-b)

$$Z_M^{TH} = Z_M // (b Z_{TIE})$$  (3.5-c)
\[
E_B^{TH} = V_{KOEBG4} Z_M / (Z_M + b \ Z_{TIE})
\]  
(3.5-d)

For the case where \(X_{TIE}\) is solved for the input parameters to the equation relating \(X_{TIE}\) to \(P_{EXP}\) will be \(V_{MPU}\) and \(V_{KOEBG4}\) (i.e. \(|V_{MPU}|\), \(|V_{KOEBG4}|\) and \(\delta_{POOL}\)); the electrical distances \(a\), \(b\) and \(c\); and the shunt admittances \(Y_p = 1/Z_p\), \(Y_H = 1/Z_H\) and \(Y_M = 1/Y_M\) that respectively represent the Perseus, Hydra and Muldersvlei load centres.

Appendix C computes the electrical distances. They are:

\[
a = 0.15 \quad (3.6-a)
\]
\[
b = 0.06 \quad (3.6-b)
\]
\[
c = 0.33 \quad (3.6-c)
\]

To compel the equation relating \(X_{TIE}\) to \(P_{EXP}\) (equation 3.3), to model the simplified network (figure 3.8) \(|V_{MPU}|\), \(|V_{KOEBG4}|\), \(\delta_{POOL}\), \(Y_p\), \(Y_H\) and \(Y_M\) are obtained from the load flow shown in figure 3.8.

3.5.3.1 Calculating \(\delta_{POOL}\), \(|V_{MPU}|\) and \(|V_{KOEBG4}|\)

\(\delta_{POOL}\) can be computed using (equation C.5):

\[
\delta_{POOL} = \left( \frac{\delta_{GRTVLA} + \delta_{ATLAS4} + \delta_{TUTUK4}}{3} \right) - \delta_{KOEBG4}
\]  
(3.7)

When using the load flow shown in figure 3.8:

\[
\delta_{POOL} = \left( \frac{\delta_{GRTVLA} + \delta_{ATLAS4} + \delta_{TUTUK4}}{3} \right) - \delta_{KOEBG4}
\]
\[
\frac{(-14.4^\circ - 15.2^\circ - 11.6^\circ)}{3} = (-108.5^\circ)
\]

\[
= 94.77^\circ
\]

(3.8)

\[|V_{MPU}| \text{ can be computed using (equation C.1):}
\]

\[
|V_{MPU}| = \frac{|V_{GRTVL4}| + |V_{ATLAS4}| + |V_{TUTUK4}|}{3}
\]

(3.9)

When using the load flow shown in figure 3.8:

\[
|V_{MPU}| = \frac{|V_{GRTVL4}| + |V_{ATLAS4}| + |V_{TUTUK4}|}{3}
\]

\[
= \frac{(1.001 + 0.993 + 1.009)}{3}
\]

\[
= 1.001 \text{ p.u.}
\]

(3.10)

When using the load flow shown in figure 3.8:

\[|V_{KOEBG4}| = 1.005 \text{ p.u.}
\]

(3.11)

3.5.3.2 Calculating \(Y_p\), \(Y_H\) and \(Y_M\)

To clarify how the admittances of the different load centres are obtained, we note that admittance can be computed using:

\[
Y = \frac{S^*}{|V|^2}
\]

\[
= (P - jQ)/|V|^2
\]

(3.12)
Where \( |V| \) is the voltage magnitude; \( P \) is the MW demand and \( Q \) the MVAR demand of the load located at \( V \).

When \( |V| = 1.0 \) p.u. then \( Y = (P - jQ) \).

The MW and MVAR demand, expressed as a demand at nominal voltage, of each of the load centres modelled in figure 3.2 are (table 3.2):

\[
P_p = 1088.1 \text{ MW}; \quad Q_p = -2188.9 \text{ MVAR} \quad (3.13-a)
\]

\[
P_H = 1463.8 \text{ MW}; \quad Q_H = -1915.6 \text{ MVAR} \quad (3.13-b)
\]

\[
P_M = 2610.0 \text{ MW}; \quad Q_M = -1187.7 \text{ MVAR} \quad (3.13-c)
\]

Where \( P_p \) and \( Q_p \) represent the Perseus load centre, \( P_H \) and \( Q_H \) represents the Hydra load centre and \( P_M \) and \( Q_M \) represent the Muldersvlei load centre.

### 3.5.3.3 Calculating \( P_{\text{EXP}} \)

To clarify how the numerical value of \( P_{\text{EXP}} \) is obtained we note that electrically Perseus and Beta are the same busbar. Hence, \( P_{\text{EXP}} \) is the power the Perseus/Beta busbar exports to the Western Cape.

#### Table 3.3 The active power Perseus/Beta exports to the Western Cape

<table>
<thead>
<tr>
<th>LINE MEASURED</th>
<th>MEASURE AT</th>
<th>ACTIVE POWER [MW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perseus – Hydra : Feeder 1</td>
<td>Perseus</td>
<td>1271.0</td>
</tr>
<tr>
<td>Perseus – Hydra : Feeder 2</td>
<td>Perseus</td>
<td>1305.0</td>
</tr>
<tr>
<td>Beta – Hydra : Feeder 1</td>
<td>Beta</td>
<td>760.0</td>
</tr>
<tr>
<td>Beta – Hydra : Feeder 2</td>
<td>Beta</td>
<td>850.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>4186.0</strong></td>
</tr>
</tbody>
</table>

Table 3.3 lists the line flows making up \( P_{\text{EXP}} \). These line flows are obtained from the load flow shown in figure 3.8.
3.5.3.4 Calculating $X_{TIE}$

Figure 3.9 shows the Excel spread sheet serving as the user interface to the Excel routine used to obtain $Z_{TIE}$.

$R_{TIE} = 0.07463 \times (3/2) \times X_{TIE}$

<p>| VALUES COMPUTED BY EXCEL IS IN RED |</p>
<table>
<thead>
<tr>
<th>REAL</th>
<th>IMAGINARY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{TIE}$</td>
<td>0.00585</td>
</tr>
<tr>
<td>$Z_{TIE}$</td>
<td>0.05225</td>
</tr>
</tbody>
</table>

% of $X_{TIE}$, Perseus is from Mpumalanga

$A = 0.15$

% of $X_{TIE}$, Muldersvlei is from Koeberg

$B = 0.06$

% of $X_{TIE}$, Hydra is from Mpumalanga

$C = 0.33$

Perseus load at 1 p.u. voltage

$P = MW, Q = MVAR$

$P_{PERSS} = 1088.1$

$Q_{PERSS} = -2188.9$

$Z_{PERSS} = 0.01821 -0.03663$

Muldersvlei load at 1 p.u. voltage

$P = MW, Q = MVAR$

$P_{MULD} = 2610$

$Q_{MULD} = -1187.7$

$Z_{MULD} = 0.03174 -0.01444$

Hydra load at 1 p.u. voltage

$P = MW, Q = MVAR$

$P_{HYD} = 1463.8$

$Q_{HYD} = -1915.6$

$Z_{HYD} = 0.02518 -0.03296$

Mpumalanga transformer reactance

$X_{mpu} = 1E-099i$

Koeberg transformer reactance

$X_{kbg} = 1E-099i$

Mpumalanga voltage (reference)

$\phi_{mpu} = 1.001$

Koeberg voltage

$\phi_{kbg} = 1.005$

Pool power angle

$\phi = -94.77$

FIGURE 3.9 User interface to the Excel routine used to solve $Z_{TIE}$

$Z_{TIE}$ is obtained by solving the equation relating $X_{TIE}$ to $P_{EXP}$ (equation 3.3) using numerical iteration.

The solution the Excel routine finds for $Z_{TIE}$ is:

$$Z_{TIE} = 0.00585 + j \ 0.05225 \ \text{p.u; } S_{BASE} = 100.0 \ \text{MVA} \quad (3.14)$$
3.5.4 Atlas and Koeberg load

3.5.4.1 MW and MVAR demand of the Atlas load

The load flow of the detailed network model (figure 3.3) shows that the MW demand, $P_{ATL-L}$, and MVAR demand, $Q_{ATL-L}$, of the Atlas load are:

$$P_{ATL-L} = -162 \text{ MW.} \quad (3.15-a)$$

$$Q_{ATL-L} = 230 \text{ MVAR} \quad (3.15-b)$$

To model $P_{ATL-L}$ and $Q_{ATL-L}$ in PSS/E we note PSS/E defines the system loads in terms of the MW and MVAR demand at nominal voltage.

The demand of the Atlas load, expressed as a demand at nominal voltage, is:

$$P_{ATL-L} = -162 \frac{1}{(0.993)^2} \text{ MW.}$$

$$\approx -164.3 \text{ MW.} \quad (3.16-a)$$

$$Q_{ATL-L} = 230 \frac{1}{(0.993)^2} \text{ MVAR.}$$

$$\approx 233.3 \text{ MVAR.} \quad (3.16-b)$$

Section 3.5.2 explains the formula used. 0.993 p.u. is the magnitude of the voltage at Atlas (figure 3.3).

3.5.4.2 MW and MVAR demand of the Koeberg load

According to Table 3.1, the Aurora, Koeberg and Acacia loads are lumped as a single load off Muldersvlei. When line charging (section 3.5.5) is considered the single load off Muldersvlei is no longer accurate.
To increase the accuracy of the load flow the Koeberg load, the Aurora load, and that portion of the Acacia load fed from Koeberg are moved to Koeberg.

To obtain the portion of the Acacia load fed from Koeberg, we note the active power flowing from Koeberg to Acacia in the detailed network model is 408 MW (figure 3.3). This is approximately half the Acacia load (table 3.1). Koeberg supplies 75.54% = 100×93/(93 + 30.1) of the VARS required at Acacia (figure 3.3). Therefore, expressed as a load at nominal voltage, Koeberg supplies 428.05 = 856.1/2 MW and 93.45 = 0.7554×123.7 MVAR of the ACAC4 load.

In addition to the above, Koeberg supplies a local load of 100 MW and 26.2 MVAR and supplies Aurora with 308 = (148 + 160) MW and −196.2 = −(91.2 + 105) MVAR.

The 400 kV Koeberg voltage obtains a magnitude of 1.005 p.u. in the detailed network model. Hence, expressed as a load at nominal voltage the load, $P_K$ and $Q_K$, off the 400 kV busbar at Koeberg is:

$$P_K = \frac{(308 + 100)}{(1.005)^2} + 428.05 \text{ MW.}$$

$$= 832 \text{ MW.} \quad (3.17-a)$$

$$Q_K = \frac{(-196.2 + 26.2)}{(1.005)^2} + 93.45 \text{ MVAR.}$$

$$= -74.86 \text{ MVAR.} \quad (3.17-b)$$
3.5.4.3 MW and MVAR demand of the Muldersvlei load

Moving load from Muldersvlei to Koeberg (section 3.5.4.2) implies the load listed in table 3.1 for the Muldersvlei load centre is no longer valid. The load off the Muldersvlei busbar should be changed to:

\[
P_M = \left( \frac{856.1}{2} + 855.8 + 248.0 + 252.0 \right) \text{ MW.}
\]

\[= 1783.85 \text{ MW.} \quad (3.18-a)
\]

\[
Q_M = \left( 1 - 0.7554 \right) 123.7 - 52.3 + 23.4 + 35.0 \text{ MVAR.}
\]

\[= 36.36 \text{ MVAR.} \quad (3.18-b)
\]

3.5.5 Line charging shunts of tie line

The tie line shown in the improved two generator model (figure 5.1) is divided into the MPU to PERS/BET, PERS/BET to HYDRA, HYDRA to MULDR and the MULDR to KOEBG4 sections. The line charging of each section, computed for the detailed network model, is listed below.

The line charging of the MPU to PERSS/BET section:

\[2760.03 \text{ MVAR} \quad (3.19-a)
\]

The line charging of the PERSS/BET to HYDRA section:

\[699.8 \text{ MVAR} \quad (3.19-b)
\]

The line charging of the HYDRA to MULDR section:

\[1288.33 \text{ MVAR.} \quad (3.19-c)
\]
The line charging of the MULDR to KOEBG section:

\[ 93.57 \text{ MVAR.} \quad (3.19-d) \]

The transmission line of each of the sections listed in equation 3.19, except the MULDR to KOEBG section, is modelled using a double \( \Pi \) circuit. A single \( \Pi \) circuit is used to model the transmission line linking MULDR to KOEBG. The shunts of the \( \Pi \) model represent the line charging.

To clarify, the parameters of the double \( \Pi \) circuit used to represent the MPU to PERS/BET section of the tie line are computed. Figure 3.10 illustrates. The line charging of the MPU to PERS/BET section is 2 760.03 MVAR.

\[ B = \frac{2 \times 760.03}{4} \text{ MVAR} \quad B = 2 \times \left( \frac{2 \times 760.03}{4} \right) \text{ MVAR} \quad B = \frac{2 \times 760.03}{4} \text{ MVAR} \]

**FIGURE 3.10** The double \( \Pi \) model (charging only) used to model the line charging of the MPU to PERS/BET section

Figure 3.11 shows the load flow of the tie line obtained for the case where the line charging is modelled according to equation 3.19 and the loads and voltage regulating shunts are modelled according to table 3.1.

In figure 3.11 generators are connected to the 400 kV busbars at Koeberg and Mpumalanga. The support these generators provide matches the support they provide in the detailed network model (figure 3.3).
FIGURE 3.11 Load flow of the tie line when the line charging is modelled
according to equation 3.19

With regard to obtaining the voltage set points of the Koeberg and Mpumalanga
generators, we note Grootvlei, Atlas and Tutuka jointly form the southern border
of the Mpumalanga power pool. This southern border obtains the following
voltage magnitude in the detailed network model:

\[
|V_{MPU}| = \frac{|V_{GRTVL}| + |V_{ATLAS}| + |V_{TUTUK}|}{3}
\]

\[
|V_{MPU}| = \frac{(1.001 + 0.993 + 1.009)}{3} \text{ p.u.}
\]

\[
= 1.001 \text{ p.u.}
\]  

\( (|V_{MPU}| \text{ obtains the same value in the simplified network model – figure 3.8) } \)
In the detailed network model, the 400 kV busbar at Koeberg obtains the voltage:

\[ |V_{KOEB} | = 1.005 \text{ p.u.} \quad (3.20-b) \]

In figure 3.11, Koeberg generates 450 MW. This generation matches the power Koeberg generates in the detailed model. The Mpumalanga generator is the network swing bus.

The line charging shunt shown in figure 3.11 off the 400 kV busbar at Koeberg is tuned so that the Koeberg generator injects 16.2 MVAR into the network - i.e. supports the network voltage according to the load flow shown for the detailed network model.

The voltage set points of the VAR generators at the Perseus/Beta, Hydra and Muldersvlei busbars are set to match the respective voltage each of these busbars obtain in the detailed network model – i.e. \[ |V_{HYDRA} | \approx 1.036 \text{ p.u} \text{ and } |V_{MULDR} | = 1.003 \text{ p.u.} \]

The voltage of the Perseus/Beta busbar can be computed using:

\[
|V_{PB}| = \frac{|V_{PERSS}| + |V_{BETAS}|}{2} \quad (3.21-a)
\]

In the detailed network model \[ |V_{PB}| \] is:

\[
|V_{PB}| = \frac{(1.04 + 1.044)}{2} \text{ p.u.} \\
\approx 1.042 \text{ p.u.} \quad (3.21-b)
\]

Figure 3.11 shows when the VAR generators at Perseus/Beta, Hydra and Muldersvlei are set to control their own busbar voltage (i.e. \[ |V_{PB}| \approx 1.042 \text{ p.u}; \]
\[ |V_{HYDRA}| \approx 1.036 \text{ p.u.} \text{ and } |V_{MULDR}| \approx 1.003 \text{ p.u.} \]

The mentioned VAR generators inject VARs into the network. This means the MVAR value of the line charging is too low and should be increased.

\[
\begin{align*}
B &= \frac{2 \times 760.03}{4} \\
B &= 2 \times \left( \frac{2 \times 760.03}{4} \right) \\
B &= 2 \times 760.03 \\
417 \times \frac{1}{4} \times \frac{1}{1.042^2} + 2 \times \left( 417 \times \frac{1}{4} \times \frac{1}{1.042^2} \right) + 417 \times \frac{1}{1.042^2}\\n&= 786.02 \text{ MVAR} \\
&= 1572.05 \text{ MVAR} \\
&= 786.02 \text{ MVAR}
\end{align*}
\]

**FIGURE 3.12** An illustration of how the MVAR rating of the shunts used to model the line charging is changed

To show how the MVAR value of the line charging is increased, figure 3.12 recalculates the MVAR values of the double \( \Pi \) model used to model the line charging of the MPU to PER/BETA section. The change in MVAR is based on the observation that the VAR generator feeding into the PERS/BET busbar injects 417 MVAR into the network (figure 3.11)

To ensure the Koeberg generator provides only 16.2 MVAR reactive support when generating 450 MW while maintaining its 400 kV voltage at 1.005 p.u. (figure 3.3), the reactive component of the 400 kV load off Koeberg is changed from \(-74.86\) MVAR to \(-65.6\) MVAR.
The line charging shunts are tuned until the output of the VAR generators is zero.

Before commencing with changing the line charging, the Koeberg and Muldersvlei loads are changed as is discussed in section 3.5.4.2 and section 3.5.4.3.

![Figure 3.13](image)

FIGURE 3.13 Load flow of the tie line for the case where the tuning of the line charging shunts make the VAR generators to generate approximately zero VARS

When the line charging is modelled according to table 3.4 the VAR generators generate approximately zero VARS (figure 3.13). Figure 3.14 shows the load flow when the VAR generators are switched off. The sections to follow refer to this network using the phrase “tie line model”.

Table 3.4 MVAR rating of the line charging present in the tie line

<table>
<thead>
<tr>
<th>TIE LINE SECTION</th>
<th>LINE CHARGING SHUNT [MVAR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPU-TO-PRESS/BET</td>
<td>3 272</td>
</tr>
<tr>
<td>PERS/SS/BET-TO-HYDRA</td>
<td>936</td>
</tr>
<tr>
<td>HYDRA-TO-MULDR</td>
<td>1 329.2</td>
</tr>
<tr>
<td>MULDR-TO-KOEBG</td>
<td>93.4</td>
</tr>
</tbody>
</table>
3.5.6 Accuracy of the tie line model

To show the tie line model (figure 3.14) is an accurate representation of the tie line in the detailed network model (figure 3.3) the voltage, active- and reactive powers at selected locations in the former and latter networks are compared.

Table 3.5 Comparing the tie line model and detailed network model with regard to voltages

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE OBTAINED IN TIE LINE MODEL [p.u]</th>
<th>VALUE OBTAINED IN DETAILED MODEL [p.u]</th>
<th>ACCURACY [%(A/B)\times100]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{MPU}$</td>
<td>A 1.001</td>
<td>B 1.001</td>
<td>100.000</td>
</tr>
<tr>
<td>$V_{PB}$</td>
<td>1.042</td>
<td>1.042</td>
<td>100.000</td>
</tr>
<tr>
<td>$V_{HYDRA}$</td>
<td>1.035</td>
<td>1.036</td>
<td>99.903</td>
</tr>
<tr>
<td>$V_{MULDR}$</td>
<td>1.003</td>
<td>1.003</td>
<td>100.000</td>
</tr>
<tr>
<td>$V_{KOE}$</td>
<td>1.005</td>
<td>1.005</td>
<td>100.000</td>
</tr>
</tbody>
</table>
In the tie line model $|V_{MPU}| = 1.001$ p.u; $|V_{PB}| = 1.042$ p.u; $|V_{HYDRA}| = 1.035$ p.u; $|V_{MULDA}| = 1.003$ p.u. and $|V_{KOE}^{4}| = 1.005$ p.u.

In the detailed network model $|V_{MPU}| = 1.001$ p.u, $|V_{PB}| = 1.042$ p.u, $|V_{HYDRA}| = 1.036$ p.u, $|V_{MULDA}| = 1.003$ p.u, and $|V_{KOE}^{4}| = 1.005$ p.u.

Table 3.5 shows the tie line model and the detailed network model match each other with regard to voltage magnitude.

In the tie line model the active power, $P_{K(IV→LV)}$, and the reactive power, $Q_{K(IV→LV)}$, flowing from the 400 kV busbar at Koeberg into the Koeberg generator are equal to $P_{K(IV→LV)} = -450$ MW and $Q_{K(IV→LV)} = 13.5$ MVAR; the active power, $P_{H→P/B}$, and the reactive power, $Q_{H→P/B}$, flowing from Hydra towards the Perseus/Beta busbar are equal to $P_{H→P/B} = -3941$ MW and $Q_{H→P/B} = 841$ MVAR and the active power, $P_{MPU→P/B}$, and the reactive power, $Q_{MPU→P/B}$, flowing from Mpumalanga towards Perseus/Beta are equal to $P_{MPU→P/B} = 5539$ MW and $Q_{MPU→P/B} = -818$ MVAR.

In the detailed network model $P_{K(IV→LV)} = -450$ MW and $Q_{K(IV→LV)} = 16.2$ MVAR; $P_{H→P/B} = (-1179-1209-734-820) = -3942$ MW and $Q_{H→P/B} = (277+291+96.5+76.2) = 740.7$ MVAR.

To obtain $P_{MPU→P/B}$ and $Q_{MPU→P/B}$ we note, Grootvlei, Atlas and Tutuka jointly form the southern border of the Mpumalanga power pool. The flows on the lines linking the mentioned busbars to the Western Cape are $P_{MPU→P/B} = (864+865+1156+1259+1237) = 5381$ MW and $Q_{MPU→P/B} = (-382.0-119.0-174.0-314.0-307.0) = -1296.0$ MVAR.
Table 3.6 Comparing the tie line model and detailed network model with regard to active power flows

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE OBTAINED IN TIE LINE MODEL [MW]</th>
<th>VALUE OBTAINED IN DETAILED MODEL [MW]</th>
<th>ACCURACY [%] (A/B)×100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{K(HV \rightarrow LV)}$</td>
<td>-450.0</td>
<td>-450</td>
<td>100.0</td>
</tr>
<tr>
<td>$P_{H \rightarrow P/B}$</td>
<td>-3941.0</td>
<td>-3942.0</td>
<td>100.0</td>
</tr>
<tr>
<td>$P_{MPU \rightarrow P/B}$</td>
<td>5539.0</td>
<td>5381</td>
<td>102.9</td>
</tr>
</tbody>
</table>

Table 3.6 shows the tie line model and the detailed network model match each other with regard to active power flows.

Except for the VARs, $Q_{MPU \rightarrow P/B}$, flowing from $V_{MPU}$ to $V_{P/B}$, the tie line model and the detailed network model match each other with regard to reactive power flows (Table 3.7). The modelling error with regard to $Q_{MPU \rightarrow P/B}$ does not affect the accuracy by which the improved two generator model, in the post contingency state, represents the impedance locus that the swing traces in the detailed network model (Chapter 4, section 4.2.2.2).

Table 3.7 Comparing the tie line model and detailed network model with regard to reactive power flows

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>VALUE OBTAINED IN TIE LINE MODEL [MVAR]</th>
<th>VALUE OBTAINED IN DETAILED MODEL [MVAR]</th>
<th>ACCURACY [%] (A/B)×100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{K(HV \rightarrow LV)}$</td>
<td>13.5</td>
<td>16.2</td>
<td>83.3</td>
</tr>
<tr>
<td>$Q_{H \rightarrow P/B}$</td>
<td>841</td>
<td>740.7</td>
<td>113.5</td>
</tr>
<tr>
<td>$Q_{MPU \rightarrow P/B}$</td>
<td>-818</td>
<td>-1296</td>
<td>63.1</td>
</tr>
</tbody>
</table>
3.6 POST-CONTINGENCY TIE LINE

Section 3.3 discusses the network event used to make the Mpumalanga-to-Western-Cape network slipping poles.

The end stage of the network event is a bus strip operation leaving Hydra with two feeders connected – the uncompensated Beta feeder and the uncompensated Droërivier feeder. These feeders are shown in figure 3.3.

The tie line model represents the above change in network topology by tripping feeder A1 and feeder B1 and then closing feeder A2 and feeder B2 (figure 3.15). The busbar shown in the middle of feeder B2 represents the Droërivier busbar.

The tie line model, changed according to the above, is referred to using the phrase “post contingency tie line model”.

\[\text{FIGURE 3.15 Defining the feeders A1, B1, A2 and B2}\]
3.6.1 Clearing the fault

The per unit resistance \( R_{BH} \), reactance \( X_{BH} \) and susceptance \( B_{BH} \) of the uncompensated Beta-Hydra line are:

\[
R_{BH} = 0.00221 \quad (3.22-a)
\]
\[
X_{BH} = 0.04854 \quad (3.22-b)
\]
\[
B_{BH} = 1.911 \quad (3.22-c)
\]

The per unit resistance \( R_{HD} \), reactance \( X_{HD} \) and susceptance \( B_{HD} \) of the uncompensated Hydra-Droërivier line are:

\[
R_{HD} = 0.0038 \quad (3.23-a)
\]
\[
X_{HD} = 0.04799 \quad (3.23-b)
\]
\[
B_{HD} = 1.618 \quad (3.23-c)
\]

To clarify how the resistance, reactance and susceptance of the Droërivier-Muldersvlei section of the tie line is obtained, we note the Droërivier-Muldersvlei section of the post-contingency network and the Droërivier-Muldersvlei section of the simplified network model (figure 3.8) are the same networks.

This is because in the improved two generator model (figure 3.2) the Hydra-Droërivier-Muldersvlei section is modelled to have load only at Hydra and Muldersvlei - the same modelling applies in the simplified network model. In addition, the network event used to make the Mpumalanga-to-Western-Cape network slipping poles does not trip lines between Droërivier and Muldersvlei.

To obtain the expression used to compute the per unit resistance, \( R \), and reactance, \( X \), of the Droërivier-Muldersvlei section we define \( |V_A| = V_A \) to be the
magnitude of the busbar voltage at Droërivier; $|V_B| = V_B$ to be the magnitude of the busbar voltage at Muldersvlei; $P_A$ to be the active power flowing from Droërivier to Muldersvlei; $Q_A$ to be the reactive power flowing from Droërivier to Muldersvlei and $\delta$ to be the power angle with which $V_A$ leads $V_B$.

The apparent power, $S_A$, flowing at $V_A$ can be computed using:

$$S_A = V_A I_A^*$$  \hspace{1cm} (3.24)$$

$V_A$ is the reference vector. Hence, equation 3.24 can be expanded into:

$$S_A = \left| V_A \right| \left( \frac{|V_A| - |V_B| e^{-j\delta}}{Z} \right)^*$$

$$= \left| V_A \right| \left( \frac{|V_A| - |V_B| e^{-j\delta}}{|Z|} \right) (R + j X)$$

$$= \left| V_A \right| \left( \frac{|V_A| - |V_B| (\cos \delta + j \sin \delta)}{|Z|} \right) (R + j X) \hspace{1cm} (3.25)$$

From equation 3.25 it can be shown:

$$P_A = \frac{RV_A^2 - RV_AV_B \cos \delta + XV_AV_B \sin \delta}{R^2 + X^2} \hspace{1cm} (3.26-a)$$

$$Q_A = \frac{XV_A^2 - RV_AV_B \sin \delta - XV_AV_B \cos \delta}{R^2 + X^2} \hspace{1cm} (3.26-b)$$

Therefore:

$$\frac{P_A}{Q_A} = \frac{RV_A^2 - RV_AV_B \cos \delta + XV_AV_B \sin \delta}{XV_A^2 - RV_AV_B \sin \delta - XV_AV_B \cos \delta}$$

$$- RQ_A V_A^2 + RQ_AV_A V_B \cos \delta - RP_AV_A V_B \sin \delta$$
When substituting equation 3.27 into equation 3.26-a, we obtain:

\[
P_A = \frac{dXV_A^2 - dXV_A V_B \cos \delta + XV_B V_A \sin \delta}{(dX)^2 + X^2}
\]

Therefore:

\[
X^2 (P_A \cdot d^2 + P_A) - X (dV_A^2 - dV_A V_B \cos \delta + V_A V_B \sin \delta) = 0
\]

\[
X = \frac{(dV_A^2 - dV_A V_B \cos \delta + V_A V_B \sin \delta)}{(P_A \cdot d^2 + P_A)} \quad (3.27-c)
\]

From the load flow of the simplified network model (figure 3.8), it follows

\[
|V_A| = 0.953 \text{ p.u.} \quad |V_B| = 1.003 \text{ p.u.} \quad P_A = 2363 = (1263 + 369 + 731) \text{ MW},
\]

\[
Q_A = 183.1 = (85.9 + 26.2 + 71.0) \text{ MVAR} \quad \text{and} \quad \delta = 33.7 = (109.3 - 75.6) \text{ degrees.}
\]

The substitution of the above values into equation 3.27 yields an \( X \) and \( R \), expressed in per unit on a 100 MVA base, that are:

\[
X = 0.02268 \text{ p.u.} \quad (3.28-a)
\]

\[
R = 0.00302 \text{ p.u.} \quad (3.28-b)
\]

The line charging present in the Droërivier-Muldersvlei section of the tie line, expressed in MVAR at nominal voltage, is 841.32 MVAR.

Figure 3.16 summarises the electrical parameters of feeder A2 and feeder B2.
3.6.2 Operating reactance of the tie line SVCs

The slip cycle is characterised by large excursions in voltage of those busbars situated close to the electrical centre.

During the slip cycle the SVCs attempt to maintain their busbar voltage. This makes the MVAR contribution from the SVCs vary.

The improved two generator model uses a constant impedance load model. Hence, the change in SVC operating reactance is modelled using a constant impedance shunt – i.e. an average reactance should be obtained for the SVCs.

This dissertation uses the average VAR contribution the SVC makes during the slip cycle to calculate the MVAR rating of the constant shunt used to model a specific SVC.
Figure 3.17 shows the average VAR contribution each of the tie line SVCs make during the first slip cycle.

**FIGURE 3.17** Average VAR contribution the tie line SVCs make during the first slip cycle

The behaviour shown in figure 3.17 is obtained for the case where the detailed network model is weakened as is discussed in section 3.3, the load is constant impedance and the constant voltage behind transient reactance generator model is used.

To clarify the results shown in figure 3.17 we note, at each time step the average VAR contribution is computed – i.e. the average VAR contribution for those SVCs not operating continuously saturated, e.g. the Perseus SVC, will change between consecutive time steps; the steady state power angle between the Mpumalanga and Koeberg generators is $\delta = 95.47^\circ$ - i.e. first slip cycle
completed when $\delta = 455.47^\circ = 360^\circ + 95.47^\circ$; and the computation of the average VAR contribution starts after clearing the fault - i.e. starts at $\delta = 100^\circ$.

Table 3.8 lists the average VAR contributions shown in figure 3.17. Note, two SVCs are installed at Hydra and Perseus.

To illustrate the effect using constant impedance SVCs have, two dynamic studies are conducted.

Study 1, applies the network event discussed in section 3.3; allows the SVCs to control their busbar voltages; uses constant impedance loads and uses the constant voltage behind transient reactance generator model.

The result obtained is shown in figure 3.17.

Table 3.8 Average VAR contribution the tie line SVCs make during the first slip cycle

<table>
<thead>
<tr>
<th>STATIC VAR COMPENSATOR</th>
<th>AVERAGE VAR CONTRIBUTION THE SVC MAKE DURING THE FIRST SLIP CYCLE [MVAR]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perseus</td>
<td>$-1.4 \times 2$</td>
</tr>
<tr>
<td>Hydra</td>
<td>$-256.4 \times 2$</td>
</tr>
<tr>
<td>Muldersvlei</td>
<td>$-205.9$</td>
</tr>
</tbody>
</table>

**FIGURE 3.18** The time it takes Study 2 to complete the first slip cycle
Study 2, applies the same network event but uses constant impedance shunts to model the tie line SVCs (MVAR rating according to table 3.8); uses constant impedance loads; and uses the constant voltage behind transient reactance generator model.

The result obtained is plotted in figure 3.18.

Study 1 takes \((1.3 - 0.1)\) seconds to complete the first slip cycle. Study 2 takes \(1.493 = (1.593 - 0.1)\) seconds to complete the first slip cycle. Hence, it takes \(24.4\% = \left(1 - \frac{1.493}{1.2}\right)\times 100\) longer to complete the slip cycle when modelling the SVCs using constant impedance shunts.

24.4\% is noticeable, however, the aim is not to obtain an improved two generator model that completes the slip cycle in the same time as the detailed network model does. The aim is to obtain an improved two generator model that traces an impedance locus that matches the mentioned locus traced in the detailed network model. Chapter 4, section 4.2.2.2 shows this aim is achieved.

### 3.7 WESTERN CAPE GENERATION POOL

This section obtains the inertia constant, generator reactance and generator transformer reactance of the Western Cape generator.

This dissertation considers the case where the only source of generation in the Western Cape is one on-line unit at Koeberg.

Past experience showed the probability of losing angular stability is high when Koeberg is operating at a quarter of its output – i.e. generating \((2 \times 900)/4 = 450\) MW. Hence, this dissertation considers the case where Koeberg is generating 450 MW.
The per unit generator transient reactance, expressed on a 1074 MVA base, of a typical 1074 MVA generator is:

D-axis reactance: \( x_d' = 0.348 \text{ p.u.} \) \hspace{1cm} (3.29-a)

Q-axis reactance: \( x_q' = 0.6887 \text{ p.u.} \) \hspace{1cm} (3.29-b)

When magnetic saliency is not modelled, the transient reactance should be computed using (Appendix D, section D.2):

\[
x_i' = \frac{x_d' + x_q'}{2} = \frac{0.395 + 0.6887}{2} = 0.5419 \text{ p.u;} \ S_{BASE} = 1074 \text{ MVA.} \quad (3.30)
\]
PSS/E expresses the generator reactance and inertia as per unit values on the generator MVA base. The generator transformer reactance could be expressed as a per unit value on a 100 MVA base or the generator MVA base.

To allow comparing the impedances of the Western Cape generation pool with the transmission network impedance the former impedance is converted to a 100 MVA base.

To clarify how to convert to a 100 MVA base we note, the per unit impedance can be computed using:

\[
Z_{pu} = \frac{Z_{act}}{Z_{base}} \quad (3.31-a)
\]

\[
Z_{act} = Z_{1}^{pu} Z_{1}^{base}
\]

\[
= Z_{1}^{pu} \left( \frac{V_{r_{base}}}{S_{1}^{base}} \right)^{2} \quad (3.31-b)
\]

Changing the base does not change the ohmic value of the impedance, hence:

\[
Z_{2}^{pu} \left( \frac{V_{2}^{r_{base}}}{S_{2}^{base}} \right)^{2} = Z_{1}^{pu} \left( \frac{V_{r_{base}}}{S_{1}^{base}} \right)^{2}
\]

\[
Z_{2}^{pu} = Z_{1}^{pu} \left( \frac{V_{1}^{r_{base}}}{V_{2}^{r_{base}}} \right)^{2} \frac{S_{2}^{base}}{S_{1}^{base}} \quad (3.31-c)
\]

Therefore, to convert from a 1 074 MVA base to a 100 MVA base we need to multiply by \( 0.09311 = \frac{100}{1 074} \) (equation 3.31-c). Hence, the transient reactance of the generator representing the Western-Cape is:

\[
x_{i}^{'} = 0.5419 \times 0.09311
\]

\[
= 0.05046 \text{ p.u}; \ S_{BASE} = 100 \text{ MVA.} \quad (3.32)
\]
The per unit reactance, $X_{TRFR}^{KBG}$, of a typical 1 074 MVA generator transformer is:

$$X_{TRFR}^{KBG} = 14.5 \% ; \ S_{BASE} = 1 \ 074 \ \text{MVA.} \quad (3.33-a)$$

Converting $X_{TRFR}^{KBG}$ to a base MVA of 100, yields:

$$X_{TRFR}^{KBG} = 0.145 \times 0.09311$$

$$= 0.01350 \ \text{p.u; } S_{BASE} = 100 \ \text{MVA} \quad (3.33-b)$$

The inertia constant, $H_{1 \ 074}$, of a typical 1 074 MVA generator is:

$$H_{1 \ 074} = 5.61 \ \text{MW.s/MVA} \quad (3.34)$$

The reactance of the generator transformer, generator and the generator inertia are obtained from the Eskom Dynamics Database.

### 3.8 MPUMALANGA GENERATION POOL

Figure 3.20 shows the equivalent electric circuit of the Mpumalanga generator.

$E_{MPU}$ is the voltage behind transient reactance of the Mpumalanga generator;

$x_{MPU}^{'}$ is the reactance of the Mpumalanga generator; $X_{TRFR}^{MPU}$ is the reactance of the generator transformer linking the Mpumalanga generator to $Y_{PP-L}$, i.e. the Mpumalanga load; and $X_{LINK}^{'}$ is the reactance linking $Y_{PP-L}$ to the southern border, i.e. busbar $V_{MPU}$, of the Mpumalanga generation pool.

The remainder of this section shows how to reduce the Mpumalanga generation pool into the single generator shown in figure 3.20.
3.8.1 Defining the Mpumalanga generation pool

The Tutuka-to-Perseus/Beta network and the Grootvlei/Atlas-to-Perseus/Beta network (figure 3.3) are the networks transferring power from the Mpumalanga generation pool to the Perseus/Beta busbar. Different sections of the Mpumalanga generation pool supply the former and latter networks.

To identify the section of the Mpumalanga generation pool supplying the Tutuka-to-Perseus/Beta network, we note that the Tutuka-to-Perseus/Beta network exports 2,616.2 MW (1,320.2 + 1,296 MW – refer to figure 3.8) to Perseus/Beta. Tutuka power station supplies 1,740 MW (580 * 3 MW) of this 2,616.2 MW. Hence, a deficit of 876.2 MW (2,616.2 – 1,740 MW) exists in the generation. This shows Tutuka is not the only generation pool supplying the Tutuka-to-Perseus/Beta network.

To identify the generators supplying the 876.2 MW deficit, we note, these generators and Tutuka should form a single generator – i.e. these generators behave coherently during the swing.
Generators that are electrically close, behave coherently. The steady state value of the angle, $\delta_n$, that develops between the voltage behind transient reactance and the synchronous rotating reference is used to identify the generators that are electrically close to Tutuka. The subscript, e.g., $n = \text{TUT}$, refers to the generator for which $\delta_n$ is computed. Table 3.9 lists the magnitude $\delta_n$ obtains at each of the generators situated in the Mpumalanga generation pool.

**Table 3.9 Angle between the voltage behind transient reactance and the synchronously rotating reference**

<table>
<thead>
<tr>
<th>POWER STATION</th>
<th>ANGLE [Degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Matla</td>
<td>24.12°</td>
</tr>
<tr>
<td>Hendrina</td>
<td>24.1°</td>
</tr>
<tr>
<td>Duvha</td>
<td>24.0°</td>
</tr>
<tr>
<td>Kriel</td>
<td>21.7°</td>
</tr>
<tr>
<td>Majuba</td>
<td>19.3°</td>
</tr>
<tr>
<td>Tutuka</td>
<td>17.6°</td>
</tr>
<tr>
<td>Kendal</td>
<td>16.4°</td>
</tr>
<tr>
<td>Arnot</td>
<td>16.3°</td>
</tr>
</tbody>
</table>

$\delta_n$, computed for Tutuka, i.e. $\delta_n = \delta_{\text{TUT}}$, is equal to 17.6°. $\delta_n$ obtains similar values at Arnot, $\delta_{\text{ARN}} = 16.3°$, Majuba, $\delta_{\text{MAJ}} = 19.3°$, and Kendal, $\delta_{\text{KEN}} = 16.4°$.

Therefore, the power stations that could supply the deficit of 876.2 MW are Arnot, Majuba and Kendal.

Figure 3.21 shows Tutuka imports 985.0 MW from Kendal. Consequently, Kendal provides the 876.2 MW generation shortage. Hence, Tutuka and Kendal jointly supply the Tutuka-to-Perseus/Beta network.
FIGURE 3.21 Power Tutuka imports from Kendal in the simplified network model shown in figure 3.8

Therefore, all the power stations listed in table 3.9, except Tutuka and Kendal, supply the Grootvlei/Atlas-to-Perseus/Beta network.

When Tutuka and Kendal are excluded, $\delta_n$ reaches its maximum at Matla and reaches its minimum at Arnot. Consequently, Matla is the northern border and Arnot the southern border of the generation pool supplying the Grootvlei/Atlas-to-Perseus/Beta network.

The generation capacity of the Tutuka/Kendal generation pool is significantly less than the generation capacity of the Matla/Arnot generation pool.
The phrase “Primary Pool”, or PP, refers to the Matla/Arnot generation pool and the phrase “Secondary Pool”, or SP, refers to the “Tutuka/Kendal” generation pool.

The Secondary Pool is connected to Grootvlei/Atlas/Tutuka via the reactance \( x'_{SP} + X^{TRFR}_{SP} \) and the Primary Pool is connected to Grootvlei/Atlas/Tutuka via the reactance \( x'_{PP} + X^{TRFR}_{PP} + X_{LINK} \) (figure 3.22).

The Secondary Pool is connected as is shown because in the detailed network model the Tutuka generator is linked to the Tutuka 400 kV busbar via a generator transformer reactance and generator reactance. In addition, the Kendal generator is electrically close to Tutuka (table 3.9). Consequently, the Tutuka/Kendal generation pool can be obtained by lumping the Kendal generator at Tutuka and linking the obtained Tutuka/Kendal generator to the 400 kV Tutuka busbar using a generator transformer.

FIGURE 3.22 Parameters and vectors used to describe the two generator model of the Mpumalanga generation pool

\( E'_{PP} \) is the voltage behind transient reactance of the Primary Pool; \( x'_{PP} \) is the transient reactance of the Primary Pool; \( X^{TRFR}_{PP} \) is the reactance of the generator transformer linking \( E'_{PP} \) to \( Y_{PP-L} \); \( X_{LINK} \) is the reactance of the network linking
\[ Y_{PP-L} \] to the Grootvlei/Atlas/Tutuka busbar; \( E_{SP} \) is the voltage behind transient reactance of the Secondary Pool; \( x'_{SP} \) is the transient reactance of the Secondary Pool; \( X_{SP}^{TRFR} \) is the reactance of the generator transformer linking \( E_{SP} \) to the Grootvlei/Atlas/Tutuka busbar; and \( V_{MPU} \) is the southern border of the Mpumalanga generation pool - made up of the Grootvlei, Atlas and Tutuka busbars.

To clarify \( X_{LINK} \), we note that the 400 kV Matla and Arnot busbars maintain their voltage during the first slip cycle (figure 3.23). The 400 kV Grootvlei, Atlas and Tutuka busbars do not maintain their voltage as well as Matla and Arnot do (figure 3.6). \( X_{LINK} \) allows the Matla/Arnot generator to maintain its voltage while the Grootvlei/Atlas/Tutuka busbar does not.

**FIGURE 3.23** Voltage and frequency measured at Matla and Arnot during the first slip cycle

\( X_{LINK} \) is a pure reactance. Therefore, the active power losses in this section of the Primary Pool is zero.
3.8.2 Electric circuit of the Mpumalanga generation pool

This dissertation uses the improved two generator model of the Mpumalanga-to-Western-Cape network to illustrate the affect that the network shunts could have on the swing locus.

Therefore, the two generator model of the Mpumalanga generation pool shown in figure 3.22 must be reduced into a single generator model.

This section obtains the parameters of the electric circuit shown in figure 3.20.

The Thevenin impedance, $Z_{TH}^{MPU}$, seen at $V_{MPU}$ when looking into the Mpumalanga generation pool and the Thevenin voltage, $E_{TH}^{MPU}$, experienced at busbar $V_{MPU}$ when the network is run open circuited between the busbars $V_{MPU}$ and $V_{PB}$ can be used to construct the electric circuit of the Mpumalanga generation pool.

The dissertation, however, does not model the Mpumalanga generator using $Z_{TH}^{MPU}$ and $E_{TH}^{MPU}$. The reasons are: $E_{TH}^{MPU}$ differs greatly from 1.0 p.u; the series impedance $Z_{TH}^{MPU}$ makes the MW demand of the Mpumalanga load governed by the current flowing through $Z_{TH}^{MPU}$ and not the voltage across $Y_{PP-L}$; and the power generated by the Mpumalanga generator drops to a value that is less than the expected 26 118 MW operating point (Appendix E, section E.2.1.1).

3.8.2.1 Calculating $x_{MPU}^T$, $x_{MPU}^{TREF}$ and $H_{MPU}$

PSS/E expresses the generator’s parameters on the generator base MVA.

The base MVA, $S_{BASE}^{MPU}$, of the Mpumalanga generator can be computed using:
Where, $S_{pp}^{BASE}$ is the base MVA of the Primary Pool ($S_{pp}^{BASE} = 27$ 000 MVA - equation D.6) and $S_{sp}^{BASE}$ is the base MVA of the Secondary Pool ($S_{sp}^{BASE} = 3$ 000 MVA - equation D.7).

Hence, $S_{MPU}^{BASE}$ is:

$$S_{MPU}^{BASE} = S_{pp}^{BASE} + S_{sp}^{BASE}$$

$$= 27000 + 3000$$

$$= 30000$$ MVA  \hfill (3.36)$$

Base transformation is used to adapt the parameters of a typical 750 MVA generator and associated generator transformer to realise the Mpumalanga generation Pool.

A typical 750 MVA generator is used since the majority of generators found in the Mpumalanga generation pool are 750 MVA. The parameters of the typical 750 MVA generator and associated generator transformer are obtained from the Eskom database.

With regard to changing the parameters of a typical 750 MVA generator to realise the 30 000 MVA generator we note, the per unit reactance of the 750 MVA generator is the per unit reactance of the 30 000 MVA generator (Appendix E, section E.2.1.1).

To allow comparing (figure 3.25) the impedances of the Mpumalanga power pool impedances with the transmission network impedances the former impedances are converted to a 100 MVA base.
The transient reactance of a typical 750 MVA generator is 0.336 p.u; $S_{BASE} = 750$ MVA. Therefore, the transient reactance, $x'_{MPU}$, of the Mpumalanga generator, expressed on a 100 MVA base, is:

$$x'_{MPU} = 0.336 \times \frac{100}{30\ 000}$$

$$= 0.00112 \text{ p.u; } S_{BASE} = 100 \text{ MVA.} \quad (3.37)$$

The reactance of a typical 750 MVA generator transformer is 0.125 p.u; $S_{BASE} = 750$ MVA. Therefore, the reactance, $x^{TRFR}_{MPU}$, of the Mpumalanga generator transformer, expressed on a 100 MVA base, is:

$$x^{TRFR}_{MPU} = 0.125 \times \frac{100}{30\ 000}$$

$$= 0.00042 \text{ p.u; } S_{BASE} = 100 \text{ MVA.} \quad (3.38)$$

The per unit inertia of the 750 MVA generator is the per unit inertia of the 30 000 MVA generator. To clarify, we note the inertia constant, $H_{750}$, of a typical 750 MVA generator can be computed using:

$$H_{750} = \frac{\text{Kinetic energy of all rotating parts at synchronous speed}}{\text{Generator three phase nameplate MVA rating}}$$

$$= \frac{1}{J_{SYNC}^2} \frac{J_{SYNC}}{S_{3\phi}} \text{ MWs/MVA.} \quad (3.39)$$

Where $J$ is the mass polar moment of inertia of all rotating parts, i.e. rotors of generator and turbine, of the 750 MVA generator; $w_{SYNC}$ is the rated angular
velocity (mechanical rad/s) of the 750 MVA generator; and $S_{3\phi}$ is the MVA rating of the 750 MVA generator.

When $A$ times 750 MVA generators are operating in parallel, we have:

Kinetic energy: \[ A\left( \frac{1}{2}Jw_{\text{SYNC}}^2 \right) \] (3.40-a)

MVA rating: \[ AS_{3\phi} \] (3.40-b)

The inertia constant, $H_{750}$, of the generator used to model $A$ times 750 MVA generators operating in parallel is:

\[
H_{750} = \frac{1}{2}(A \times J)w_{\text{SYNC}}^2
\]

\[
= \frac{(A \times S_{3\phi})}{(A \times S_{3\phi})}
\]

\[= H_{750} \text{ MWs/MVA.} \] (3.41)

The inertia constant, $H_{750}$, of a typical 750 MVA generator is:

\[H_{750} = 3.02 \text{ MWs/MVA.} \] (3.42)

3.8.2.2 $X_{\text{LINK}}$ of the Mpumalanga generation pool

This section calculates the value of $X_{\text{LINK}}$ shown in figure 3.20.

To clarify how $X_{\text{LINK}}$ is obtained, we note it takes the detailed network model 1.593 seconds to complete the first slip cycle (figure 3.18). This result is obtained for the case where the network external to the generators is modelled to match the improved two generator model – i.e. constant impedance shunts (constant
impedance load and constant impedance shunts for the SVCs – refer to section 3.6.2) and constant voltage behind reactance generator model.

\( X_{\text{LINK}} \) is tuned so that the improved two generator model completes the first slip cycle in 1.593 seconds when subjected to the network event discussed in section 3.3.

**FIGURE 3.24** Time it takes the improved two generator model to complete one slip cycle

When \( X_{\text{LINK}} = 0.0028 \) p.u, expressed on a 100 MVA base, the improved two generator model completes the first slip cycle in 1.608 seconds (figure 3.24). Note: the steady state power angle, \( \delta \), that develops between Mpumalanga and Koeberg is 100.07 degrees. Hence, a slip cycle is completed when \( \delta = 460.07 \) degrees.

### 3.9 LOAD FLOW OF THE IMPROVED TWO GENERATOR MODEL

When the Mpumalanga generator and the Western Cape generator are connected to the tie line the improved two generator model is obtained. Figure 3.25 shows the improved two generator model.
FIGURE 3.25 Steady state load flow of the improved two generator model used to model the Mpumalanga-to-Western-Cape network