APPENDIX F  THE VOLTAGE BEHIND TRANSIENT REACTANCE

This appendix obtains a mathematical expression for the voltage, \( E' \), behind transient reactance and shows that \( E' \) will be initialised correct when the steady state operating point of the generator is modelled correct.

F.1  COMPUTING \( E' \)

\( E' \) can be computed from the terminal voltage, \( V = V_d + j V_q \), using (figure F.1):

\[
E' = (V_d - x'_q I_q) + j (V_q + x'_d I_d)
\]  \hspace{1cm} (F.1)

FIGURE F.1 Vector diagram of a salient pole generator in the transient state [45, p76]

Where \( x'_d \) and \( x'_q \) are the generator d- and q-axis transient reactances; \( I_d \) and \( I_q \) are the d- and q- axis components of the stator current and \( V_d \) and \( V_q \) are the d- and q- axis components of the terminal voltage.
To decompose $V$ into d-axis, $V_d$, and q-axis, $V_q$, components the angle, $\delta$, that develops between the generator q-axis and $V$ must be known (figure F.1). From the steady state generator vector diagram (figure F.2) it follows:

\begin{align}
I_d &= I \sin(\delta + \theta) \tag{F.2-a} \\
I_q &= I \cos(\delta + \theta) \tag{F.2-b} \\
\sin(\delta) &= \frac{I_q x_q}{V} \tag{F.2-c}
\end{align}

By substituting equation F.2-b into equation F.2-c we obtain:

\begin{equation}
\sin(\delta) = \frac{I \cos(\delta + \theta) x_q}{V} \tag{F.3}
\end{equation}

But:

\begin{equation}
\cos(\delta + \theta) = \cos(\delta) \cos(\theta) - \sin(\delta) \sin(\theta) \tag{F.4}
\end{equation}

By substituting equation F.4 into equation F.3 we obtain:
\[
\sin(\delta) = \frac{I_x \cos(\delta) \cos(\theta) - \sin(\delta) \sin(\theta)}{V}
\]

\[
V = I_x \left[ \cot(\delta) \cos(\theta) - \sin(\theta) \right]
\]

\[
I = \frac{V}{x_q \left[ \cot(\delta) \cos(\theta) - \sin(\theta) \right]}
\]

\[
\cot(\delta) = \frac{V + I_x \sin(\theta)}{I_x \cos(\theta)}
\]

\[
\tan(\delta) = \frac{I_x \cos(\theta)}{V + I_x \sin(\theta)}
\]

Expressing the phase angle, \( \theta \), in terms of the apparent power, \( S \), the active power, \( P \), and the reactive power, \( Q \), gives:

\[
\cos(\theta) = \frac{P}{S}
\]

\[
\sin(\theta) = \frac{Q}{S}
\]

\[
\tan(\theta) = \frac{Q}{P}
\]

By substituting equation F.6-a and equation F.6-b into equation F.5 we obtain:

\[
\tan(\delta) = \frac{x_q \frac{P}{S}}{V/1 + x_q \frac{Q}{S}}
\]

\[
S = VI
\]

But:

\[
\tan(\delta) = \frac{x_q P}{V^2 + x_q Q}
\]

Therefore:

\[
\delta = \tan^{-1} \left( \frac{x_q P}{V^2 + x_q Q} \right)
\]

(F.7)
Therefore, \( V_d \) and \( V_q \) can be computed using:

\[
V_d = V \sin(\delta) \tag{F.8-a}
\]

\[
V_q = V \cos(\delta) \tag{F.8-b}
\]

Equation F.8 can be used to compute \( V_d \) and \( V_q \). It then follows from equation F.1 that \( I_d \) and \( I_q \) need to be solved before we can compute \( E' \).

To show how to obtain the equations for \( I_d \) and \( I_q \) we note:

\[
S = V I^* \]

\[
= (V_d + j V_q) (I_d - j I_q) \]

\[
= (V_d I_d + V_q I_q) + j (V_q I_d - V_d I_q) \]

\[
= P + j Q \]

\[
P = V_q I_d + V_d I_q \tag{F.9-a}
\]

\[
Q = V_q I_d - V_d I_q \tag{F.9-b}
\]

In matrix format:

\[
\begin{bmatrix}
V_d & V_q \\
V_q & -V_d
\end{bmatrix}
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix}
=
\begin{bmatrix}
P \\
Q
\end{bmatrix} \tag{F.10}
\]

Cramer’s rule is used to solve the above matrix. For this, we write the matrix as:

\[
[A_1; A_2] x = b \tag{F.11}
\]
According to Cramer entry \( x_i \) of vector \( \mathbf{x} \) can be solved using:

\[
    x_i = \frac{\det(B_i)}{\det(A)}
\]

(F.12)

Where:

\[
    B_1 = [b; A_2]
\]

(F.13-a)

\[
    B_2 = [A_1; b]
\]

(F.13-b)

Hence:

\[
    B_1 = [b; A_2] = \begin{bmatrix} P & V_q \\ Q & -V_d \end{bmatrix}
\]

Therefore:

\[
    \det(B_1) = -PV_d - QV_q
\]

(F.14-a)

Hence:

\[
    B_2 = [A_1; b] = \begin{bmatrix} V_d & P \\ V_q & Q \end{bmatrix}
\]

Therefore:

\[
    \det(B_2) = QV_d - PV_q
\]

(F.14-b)

And:

\[
    \det(A) = -V_d^2 - V_q^2
\]

(F.15)

When applying equation F.12 we obtain:

\[
    I_d = \frac{-PV_d - QV_q}{-V_d^2 - V_q^2}
\]

(F.16-a)

\[
    I_q = \frac{QV_d - PV_q}{-V_d^2 - V_q^2}
\]

(F.16-b)

The above equations show the generator transient voltage, \( E' \), can be expressed in terms of the steady state operating point of the generator. To clarify, we note equation F.1 shows \( E' \) can be expressed in terms of \( V_d \); \( V_q \); \( I_d \) and \( I_q \).

However, \( I_d \) and \( I_q \) can be expressed in terms of \( P \), \( Q \), \( V_d \) and \( V_q \) (equation F.16) - i.e. the steady state operating point of the generator; \( V_d \) and \( V_q \) can be
expressed in terms of $\delta$ (equation F.8) and $\delta$ can be written in terms of $P$ ; $Q$ and $V$ (equation F.7).