Telling and illustrating additive relations stories ANNEXURES

A classroom-based design experiment on young children's use of narrative in mathematics

This document provides a collation of Annexures 1-10 which are referred to in the PhD thesis Roberts (2015) 'Telling and illustrating additive relations word problems'.

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Contents

Annexure 1: The case of Mpho

In this Annexure I present an analysis of one of the learners – Mpho – who represents the learner who had the highest learning gains from the lower attainment ranges from the pre-test to the post-test. This Annexure opens with an analysis of evidence that his learning shifted which is drawn from the pre- and post-test and interview data. This is organised in terms of the learning goals:

1. Solve, explain and report on solutions for range of additive relation word problems (LG 1: Problem solving)
2. Illustrate solutions to additive relation task, flexibly moving between representations (LG 2: Representations)
3. Narrate stories to explain and pose additive relation word problems (LG 3: Stories)

Inferences are made between the evidence of shifts in learning (comparing before the intervention to after it), and Mpho's experiences during the lesson intervention. These inferences are intended to answer the following questions:

> What evidence of shifts in Mpho's learning, if any, are seen as a result of the teaching intervention:
>
> - What evidence of learning to solve and pose additive relation word problems (LG 1) is seen during and following the intervention?
> - How does Mpho make his thinking visible by using narrative (LG 3) and diagrams (LG 2) in this intervention?

As such the second and third learning goals are enabling goals to support the learners in solving and explaining their solutions to word problems.

Mpho's context within the class

Mpho had Sesotho as his home language. He had repeated Grade 1, and so was to be automatically promoted to Grade 3 (as he was already on year older than his age cohort). He was allocated to the support group in the class as his attainment in the pre-test using the simple marking framework was 0%.

Quantitative evidence of learning gains

Figure 1: Pre-test, post-test and delayed post-test correct solutions (Mpho)

Marks	Q1 Change result unknown 1	Q2 Compare (matching) 1	Q3 Collection 1	Q4 Compare (disjoint set) 1	Q5: Partition problem 1	Q6: Bare calculation 21-6 = 1	Q7: Bare calculation 23-18 = 1	7.0
Mpho pre test	0	0	0	0	0	0	0	0
Mpho post test	1	1	0	0	0	1	0	3
Mpho delayed post-test	1	0	1	1	1	1	0	5

Qualitative evidence of learning gains

In this section I present the evidence of learning gleaned from Mpho's written tests (the pre-test, post-test and delayed post test scripts). In analysing his responses to each question in the test I present the quantitative gains as measured by marks, (which were awarded using a marking scheme similar to those adopted in standardised assessments. The quantitative marks (presented above) primarily measure the extent to which Mpho was able to correctly solve and record his solution to a word problem, or a calculation (LG 1: Solving problems). By examining his written scripts in detail it is possible to reflect on what he did in each question, and what his working reveals about his thinking. In addition, I reflect on how his thinking is made visible using representations in these written assessments. In so doing I make use of the coding framework developed for analysing learners' representations and comment on what these representations reveals about Mpo's calculation strategies (LG 2: Representations).

Mpho was in a post-test assessment group where there were several distractions. Children requested to go to the toilet, and perhaps as several of them were not able to answer the questions posed in the test, a few children were out of their seats. I had to intervene to ensure that the children remained focused on the test. I felt that this atmosphere may have disturbed children in this group, and therefore offered them a further opportunity to solve problems that they had answered incorrectly, during the individual interview session. Mpho was not shown his previously incorrect response to this question,he was simply asked to try some problem again in the one to one interview situation. The question was read to him (as was the case in the small group assessment) and then he worked on his own.

The extent to which Mpho made use of narratives to explain and pose word problem situations, is not evident from written tests,however he made use of stories during the intervention (evident from his interactions in the videos of the lessons, as well as from his work in his book). He was also expected to interpret and use narrative in the pre and post interviews. I therefore present evidence of how

Mpho made his thinking visible using representations (LG 2: Representations) and narrative (LG 3: Stories) during the interviews in the next section.

Learning goal 1: Solving additive relations word problems

Six additive relation word problem types were included in the intervention: Change problem; Compare (Reach a target) problem; Collection problem; Compare (Matching) problem; Compare (Disjoint sets) problem; and Partition problem. For each word problem type, evidence from Mpho of his being able to solve these problems was collected.

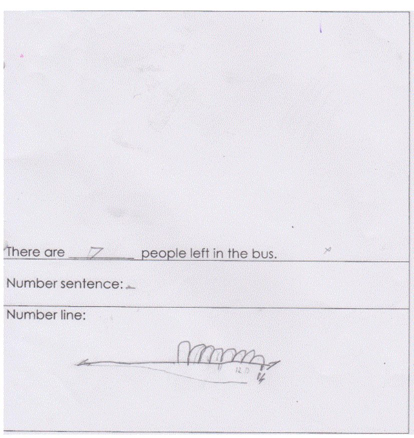
Overview of solving word problems

With teacher support, Mpho expanded his example space from not being able to solve any word problems, to correctly solving change and compare (matching) problems. He made some progress with regard to working on the partition problem although his solution to this was not complete. He was not yet able to solve collection and compare (disjoint sets) problems by the post-test. However 7 months after the intervention, in the delayed post-test, Mpho has able to solve these word problem types.

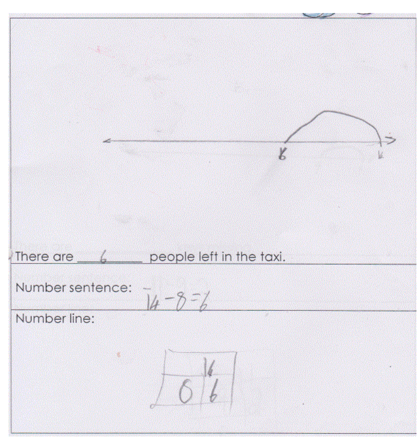
Change word problems

Figure 2: Mpho's change problems pre-test and post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



There are 14 people in a taxi. 8 people get out of the taxi. How many people are left in the taxi?



In the pre-test Mpho did not show his working, and just wrote that there were 7 people left in the bus (incorrect solution to 14 subtract 8). He did not attempt to write a number sentence. He sketched a number line marking 14, and showing 8 hops backwards from 14. He started to label his hops backwards counting back, marking 13, 12 and 11 (but 11 was incorrectly positioned as 4 hops back from 14). This indicated that Mpho seemed to be aware of the need for a counting back (in ones) strategy from 14. However crossing the 10 in a backwards count, did not yet seem secure for him. It is likely that he arrived at the solution of 7 people by either incorrectly counting back 8 in ones when bridging the ten, or by losing track of his backwards counts (so only counting back 7 counts instead of 8).

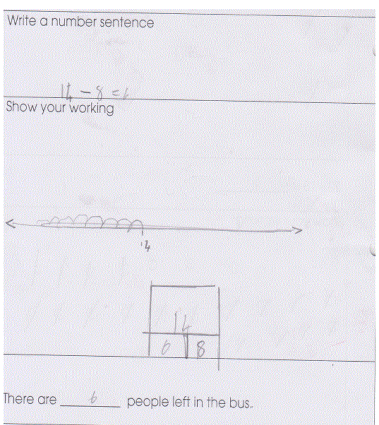
In the post-test Mpho correctly answered that there were ‘6’ people left in a taxi. He was able to write an appropriate number sentence: ‘14 – 8 = 6’. He made use of two diagrammatic representations to support this. He drew a whole-part-part diagram, where he correctly partitioned 14 into two parts and labelled the bigger part as 8. For his number line representation he drew 14 and showed a jump to 6. He did not label the size of his jump, nor did he indicate whether his jump was back from 14, or forwards from 6 to reach 14.

Contrasting Mpho’s responses from the pre-test to the post-test it appears that he may have been less reliant on a counting by ones calculations strategy, by the post-test. In the post-test neither of his representations depicted ones and he was accurately able to solve the problem. He may have used a count by ones strategy on his fingers, however this was not made visible in his written work.

By the delayed post-test assessment Mpho seemed to be securely answering the change word problem:

Figure 3: Mpho’s change problem delayed post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



Mpho correctly calculated the solution to this word problem, wrote an appropriate number sentence (Syntax model, type 9a) depicted the situation on a number line using a take-way strategy depicting hops in ones backwards from 14 (line model, Type 5a), drew a whole-part part diagram partitioning 14 (Type 7a) and completed the answer sentence by writing the number symbol,‘6’. He did not label the jumps or the landing numbers for his number line, although his drawing suggested that he continued to use a count back in ones strategy to solve this problem.

Change (reach a target) word problems

This problem type was not assessed in the written tests.

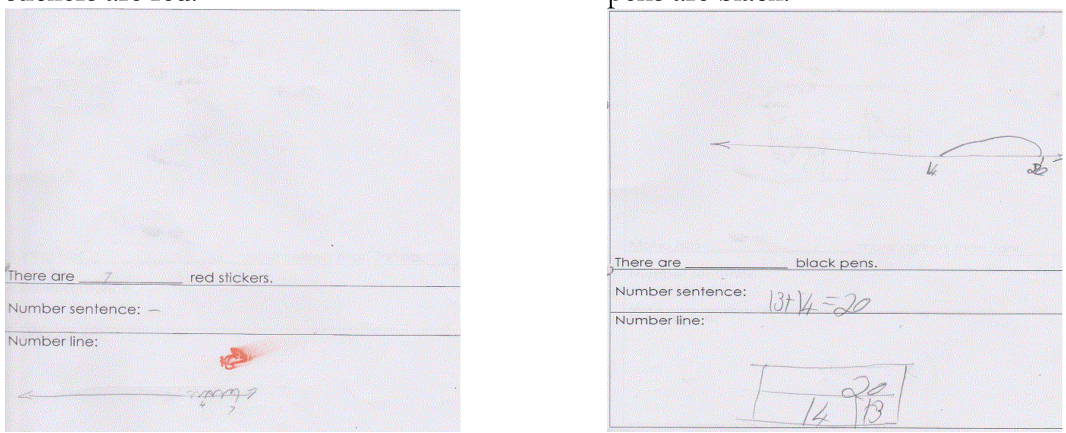
Collection word problems

The following collection problems were posed in the written tests:

Figure 4: Mpho's collection problem pre-test and post-test

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?

The teacher has 13 pens. Some are black and some are red. 4 of the pens are red. How many pens are black?



In the pre-test Mpho did not use the space provided for working, and simply wrote that there were '7' red stickers (incorrect answer). He did not write down a number sentence. He sketched an empty number line showing 7 with 4 hops in ones to reach 4.

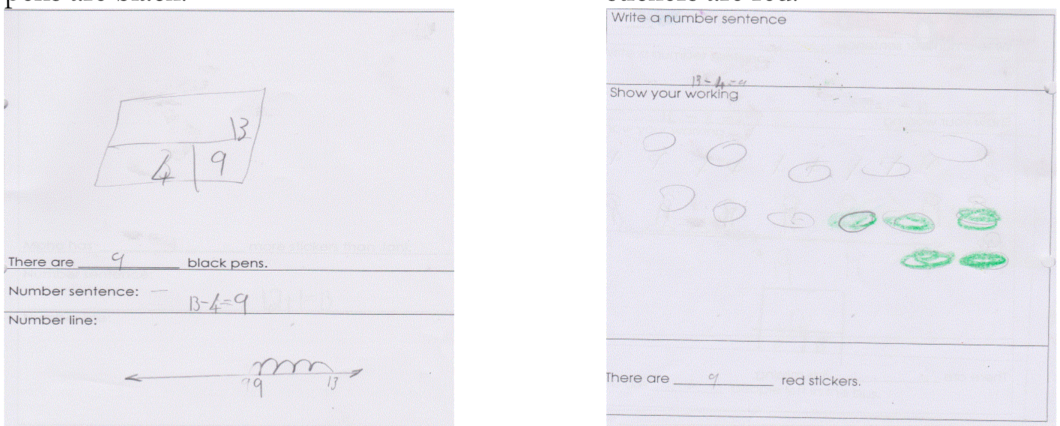
In the post-test Mpho was not able to correctly answer the question. He made use of 13, and seemed to incorrectly record the 4 as 14. He chose to add the 13 and 14, which he incorrectly calculated to be 20. It seemed that he did not understand the problem context, and/or chose the incorrect mathematical model for this problem. His representations in this problem are interesting, as the representations all correctly depicted his incorrect number sentence of 13 + 14 = 20. He used an empty number line to show a large jump from 20 to 14 (which may be assumed to be 13, although he does not label this). He does not show the direction of his jump, although the position of the number one the far right of his page, suggested he may have been working from the right. He drew a whole-part-part diagram, where he partitioned 20 into two parts, labelling the larger part 13. The representations were coherently applied to his incorrect mathematical model, and none of them appeared to create a conflict for him. Mpho remained unable to interpret and solve a collection type word problem, although he demonstrated flexible movement between three symbolic syntactical representations: number sentence, empty number line and whole-part-part diagram.

With this repeat opportunity, Mpho demonstrated that he understood the problem context and was able to solve it:

Figure 5: Mpho's collection problem post interview and delayed post test

The teacher has 13 pens. Some are black and some are red. 4 of the pens are red. How many pens are black?

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?



In the repeat of the post-test task (in a 1:1 situation), Mpho spontaneously modelled the problem situation making use of a whole-part-part diagram (Type 7a). He correctly positioned the 13 as the whole, and partitioned 13 into 9 and 4. He incorrectly placed 4 in the larger partition, than the 9 partition. He completed the answer sentence, writing '9' into the 'There are 9 black pens' answer sentence. He correctly wrote a number sentence for this situation: '13 – 4 = 9' (Type 8a). He also correctly sketched this problem situation on an empty number line (Type 5a). He first drew the 13, and then positioned the 9 to left of the 13. He drew in four hops, starting at 13 and hopping back to reach 9. It was not clear during this observation whether his count was backwards: '12, 11, 10, 9' or forwards (keeping track of the counts): '1, 2, 3, 4' as he did this mentally. This interaction in the follow up interview seemed to confirm that Mpho had been distracted during the post-test assessment session in the small group. In the quiet and calm space of a 1:1 interview, he was fluently able to correctly responded to this question and record his response following the written (and teacher read) prompts. While this repeat question was not taken to be indicative of a mark adjustment (the previous attempt was used in all the collated class data), this interaction revealed the possible disjuncture between what learners is able to do in a group assessment situation, and what they can do when sitting alone next to an encouraging adult.

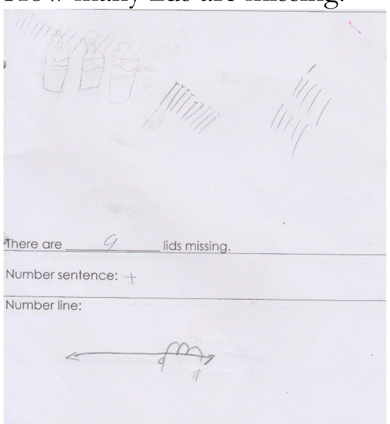
In the delayed post-test assessment of this task, Mpho correctly modelled the problem using an indexical representation (Type 3a). He drew 14 circles to denote the stickers, coloured in 5 of these stickers, and self-corrected by attempting to erase the fifth coloured circle. He wrote an appropriate symbolic syntactical number sentence (Type 8a): '13 – 4 = 9', and completed the answer sentence writing there are '9' red stickers.

Compare (matching) word problems

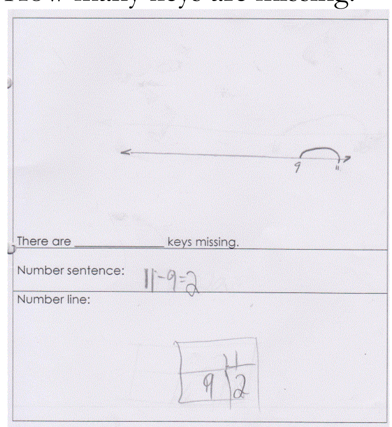
The following compare (matching) problems were posed in the written tests:

Figure 6: Mpho's compare (matching) problem pre-test and post-test

There are 11 bottles but only 9 lids.
How many lids are missing?



There are 11 locks but only 9 keys.
How many keys are missing?



In the pre-test Mpho started draw an iconic representation (Type 2) of the problem context, sketching three bottles. He drew an indexical representation (Type 3a)using tally marks next to this, and then attempted his solution again (he tried to erase this prior work, although the markings remain clear enough to see on his script). He had two other indexical representations of his work. He sketched a group of 11 lines, where he used a group by five structure (Type 3b as he groups 11 into 5 + 5 + 1). He sketched another group of 9 lines, each which had a dot drawn above it (Type 3a). It is assumed that the lines represented bottles, and the dots represented lids. He drew two lines below the nine lines with their dots. These two bottles did not have dots above them. From this drawing, it seems as if Mpho understood the problem context, and was able to solve it diagrammatically using indexical representations (Type 3). However when writing this down symbolically, he completed the answer sentence by writing '9': There are 9 lids missing. This may indicate that Mpho did not understand the meaning of the word 'missing', as it would be correct to answer: 'There are 9 lids', which may be considered sensible if this question was considered to be a reading comprehension question. Mpho did not write a number sentence, and wrote only the plus sign: '+'. For his number line he depicted 11 on an empty number line, and showed 3 hops backwards to 9. He was not awarded any marks for this word problem as he did not write the answer symbol, or the number sentence and while he had one correct representation (the iconic sketch of lines and dots), this was contradicted by an incorrect representation (the number line showing 3 hops from 11 to 9).

In the post-test Mpho was able to correctly solve this problem. Although he did not write the symbolic answer to complete the answer sentence, he depicted his solution using three symbolic representations: An empty number line where he drew a jump from 9 to 11 (or vice versa) (Type 5b);

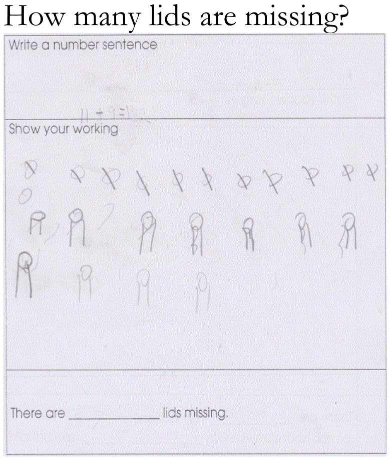
a whole-part-part diagram where he correctly partitioned 11 into 9 and 2 and labelled the larger part 9 (Type 7b), and a symbolic syntactical number sentence: '11 – 9 = 2' (Type 8a).

The learning gain for Mpho evident in this pre- and post-test comparison did not seem to be his improved problem solving (from the pre-test he seemed able to solve the problem from his indexical representation); but rather his improved ability to record his solutions symbolically. By the post-test he was able to make use of the empty number line with showing hops in ones, write a number sentence, and represent this as a whole-part-part diagram. In this compare (matching) question Mpho could model the problem situation appropriately (in contrast to the previous collections problem where his model was incorrect, although his recording of this incorrect model was coherent).

In the delayed post-test Mpho made an error when working on this problem, although he modelled it appropriately:

Figure 7: Mpho's compare (matching) problem (delayed post-test)

There are 11 bottles but only 9 lids.
How many lids are missing?

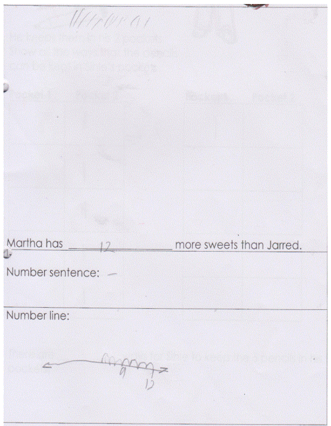


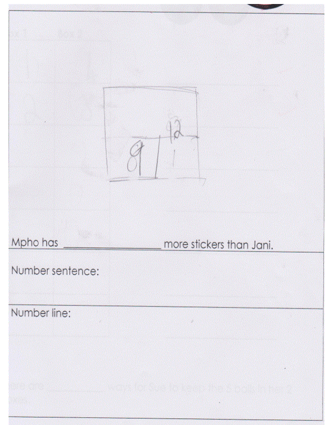
Mpho used an indexical representation (Type 3a or 3b) for this problem. He may have been attempting to use a 10-wise grouped arrangement (with the top row being 10, and the bottom row being 1 to depict 11). However he drew 11 circles in the top row and 1 in the bottom row (giving a total of 12). He then crossed out 11 of these circles. He repeated this process using more iconic representations, drawing 11 bottles . He draw curved lines through 8 of the bottles. He offered a number sentence of '11 – 9 = 1' (and there was a faint '2' written to the right of the '1')). It was clear than Mpho had erased '+' and replaced it with '-'when writing this number sentence. He was awarded a partial mark for his take-away strategy and his partially correct number sentence '11 – 9 = …' He was not awarded full marks as he did not offer the correct answer, and his strategy was inaccurate.

Compare (disjoint set) word problems

The following compare problems were posed in the written tests:

Figure 8: Mpho's compare (disjoint set) problems pre-test and post-test





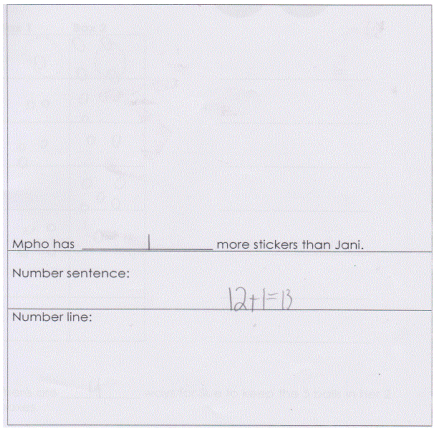
In the pre-test Mpho drew 12 lines using an indexical representation (Type 3a) in his working space. He answered incorrectly that Martha has '12' more sweets than Jarred. From this it may be inferred that Mpho did not yet understanding the meaning of 'more sweets than'. It would make sense for him to answer '12', if the question was 'how many sweets does Jarred have?' which is the type of question posed for reading comprehension tasks. This response was typical of incorrect responses in the class. Mpho did not write a number sentence, but drew a symbol which appeared to be a minus sign. Mpho sketched an empty number line where we depicted 13 to the right of 9. He incorrectly showed 4 hops between 12 and 9.

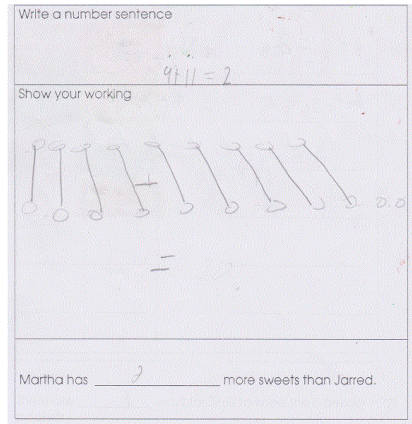
In the post test Mpho correctly modelled the problem situation using a whole-part-part diagram . Considering the markings he erased from this diagram he first labelled the whole part as 9, and partitioned this into a 6 part, and an unknown part. He erased this version, and correctly labelled the whole as 12, with a partition of 9. He seemed to calculate that the missing part was '1', and wrote this into the whole-part-part diagram, however he erased this. He did not respond to the prompts to complete the answer sentence, write a number sentence or draw a number line. Comparing the pre-test to post-test responses on this question for Mpho, it appeared as if there were no learning gains in relation to solving the problem. Although he was able to model the situation using the whole-part diagram, he did not complete this and was unable to find a solution to the problem.

When a compare disjoint set question was posed to Mpho again in the post interview session, there was still no evidence of him correctly solving this compare (disjoint set) problem type:

Figure 9: Mpho's compare (disjoint set) problem post-interview and delayed post-test

Jani has 9 stickers. Mpho has 12 stickers. How many more stickers does Mpho have than Jani?	Jarred has 9 sweets. Martha has 11 sweets. How many more sweets does Martha have than Jarred?





Mpho did not show any working for his compare (disjoint set) problem which was reposed in the post-interview. This may have suggested that he did not yet have a visual way of representing this problem, or that he did not yet understand the problem context. He offered '1' as the solution to the problem and the number sentence '12 + 1 = 13'. His number sentence was relevant to his suggested answer, but did not use either of the numbers offered in the problem situation (9 or 11). This work was taken to confirm that Mpho did not yet understand the compare (disjoint set) word problem. His reasoning was not probed orally during the pre-interview, as this had been reposed to establish whether he had been distracted by the others in the group. Discussing this problem with him would have shifted this task into something other than a repeat attempt at the written test item in a 1:1 situation. His reasoning about compare (disjoint set) problems was probed through other questions and task items during the interview (which are discussed in the section that follows this one).

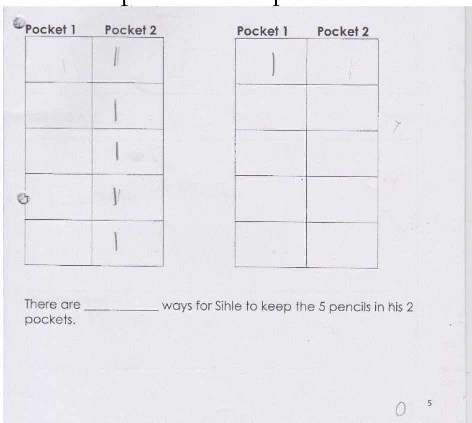
Mpho made some progress with regard to compare (disjoint set) problem in the delayed post-test however. In his response he was able to show a 1:1 matching indexical image of the 9 sweets and the 11 sweets. he used lines to show the relationship or matching action from one set to the other, making the '2' unmatched sweets (his answer) clearly visible. Mpho offered an incorrect number sentence for this writing '9 + 11 = 2'. He was awarded the marks for this problem as he showed his working (correctly) and answered (correctly), which is how mark allocations are made in standardised assessments (where there is no expectation that the learner would write a number sentence to solve a problem). Mpho's solution on the delayed post-test problem revealed that he now had an understanding of the compare (disjoint set) problem, and could use a 1:1 matching action on an indexical representation to solve it, He could not yet represent this using a symbolic syntactical number sentence.

Partition word problems

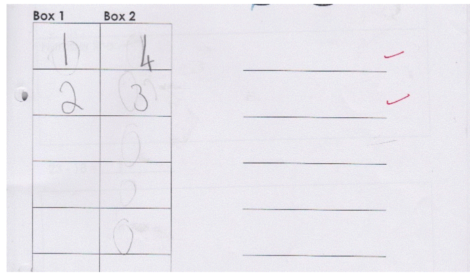
The following partition problems were posed in the written tests:

Figure 10: Mpho's partition problems pre-test and post-test

Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be kept in Sihle's pockets.



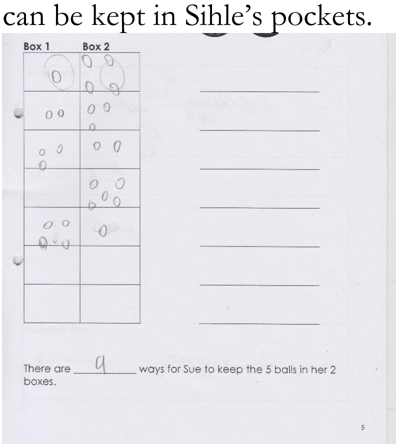
Sue has 5 balls. She keeps them in two boxes. Show all the ways that the balls can be kept in Sue's boxes.



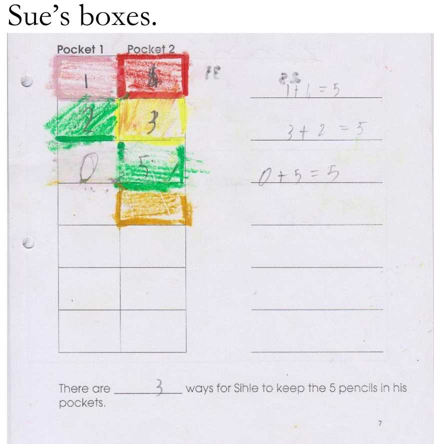
In the pre-test drew 5 lines (depicting 5 pencils) in pocket 2, and 1 line pocket 2. He used an indexical representation for an incorrect partition (5-1). In the post-test, Mpho made use of number symbols and created two correct partitions: 1-4 and 2-3. He did not write down any number sentences.

Figure 11: Mpho's partition problems post interview and delayed post-test

Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be kept in Sihle's pockets.



Sue has 5 balls. She keeps them in two boxes. Show all the ways that the balls can be kept in Sue's boxes.



In the post interview Mpho was able to identify 5 correct partitions. He made use of indexical representations, which made use of a group pattern (using dice patterns). He incorrectly answered the question of how many ways there were, by counting his non-zero partitions. He did not write any number sentences (syntax model) to support his working. In the delayed post-test he identified 3 correct partitions and supported these with number sentences. This time he correctly answered how many ways he had found (indicating 3).

Learning goal 2: Use of representations

To reflect on learning gains relating to the use of representations I draw on the written pre and post-tests.

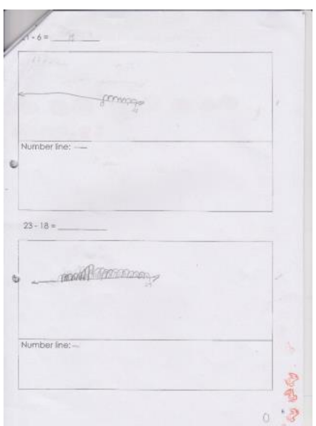
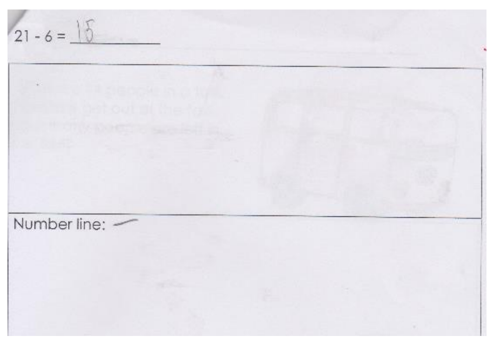
Overview of representations

Mpho made use of 21 representations in the pre-test, and decreased this to 16 in the post-test. However by the post-test he made more use of group-wise actions, and greater use of syntax model representations. In the pre-test Mpho used 4 group model representations and in one of these he depicted a group wise arrangement in fives. He did not use any group model representations in the post-test. Mpho made use of number lines in both written tests. In the pre-test he once used a structured number line with reference to '0', and by the post test he was no longer using a reference number, working on an empty number line. He shifted his actions on number lines from counting in ones in the pre-test, to group-wise actions in the post test. He commonly made use of take-away strategy, although in both tests he provided examples of difference strategies on the number line.

Mpho did not make use of any syntax model representations in the pre-test. By the post-test 7 of his representations were syntax models. At times he used whole-part-part diagrams where the parts were accurate and at times he used whole-part-part diagrams where the parts were roughly equal.

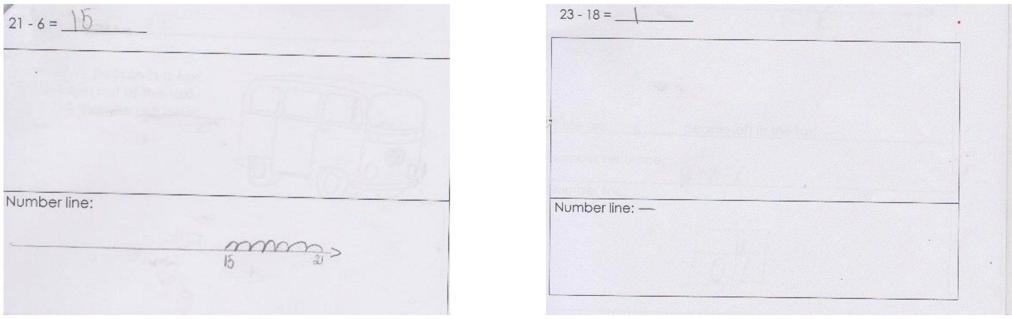
Use of representations for bare calculations

Figure 12 Mpho's use of representations for bare calculations 21 – 6 pre-test and post-test

In the pre-test Mpho incorrectly recorded the answer to '21 – 6' as '13'. He made use of an empty number line, starting at 21 and depicting 7 hops back, showing that he landed on 13. So it appears that he made two errors: Firstly he counted back 7 instead of 6, and he then his counting back was not correct. In the post-test he correctly answered that '21-6' was '15', but did not show any working for how he obtained this solution.

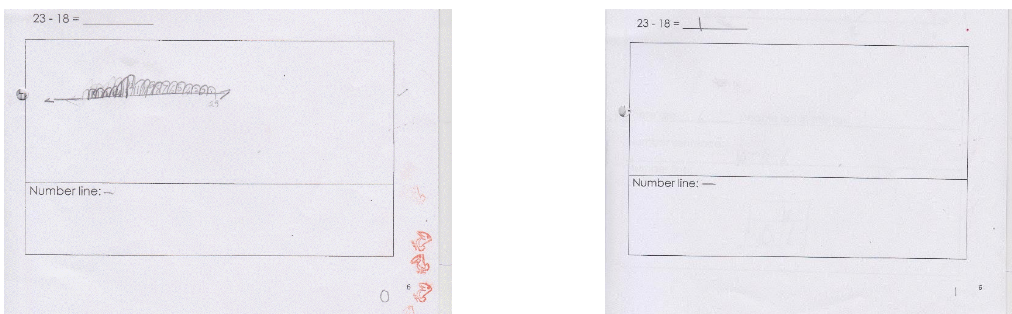
Figure 13 Mpho's use of representations for bare calculations 21 – 6 post-interview and delayed post-test



This question was repeated in the post interview and Mpho correctly answered it. He made use of a take-away strategy showing 6 hops back from 21 to reach 15. In the delayed post-test Mpho again correctly answered this question. He made use of a group model drawing indexical arrangements of groups of 5 (in a dice pattern) and crossing out 6 of these ones. His take-away actions depicted crossing out actions for each one (and not crossing out a group of 5).

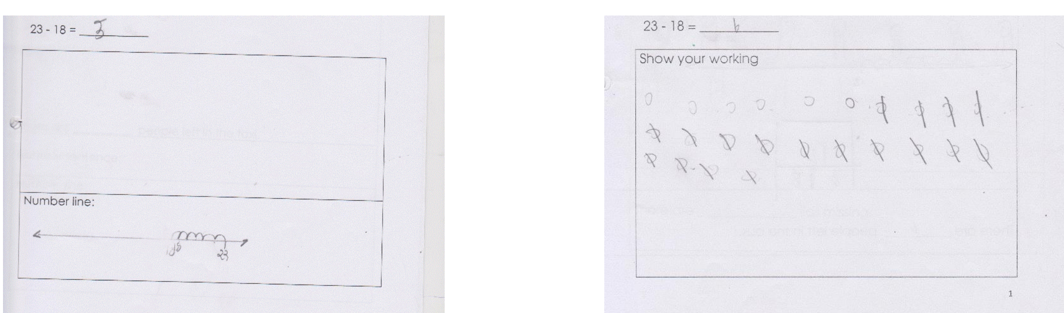
In the written test a calculation where a difference strategy was more efficient was also posed.

Figure 14 Mpho's use of representations for bare calculations 23 – 18 pre-test and post-test



In the pre-test Mpho correctly depicted his take-away calculation strategy, starting at 23 and counting back 18. However he did not write down where he landed. This may indicate that he was insecure with counting back 18 counts from 23 (a more efficient strategy would have been counting up from 18 to reach 23). In the post-test Mpho correctly calculated that '21 – 6' was '15' and wrote this down using symbolic number symbols. He did not show how he worked this out.

Figure 15 Mpho's use of representations for bare calculations 23 – 18 post-interview and delayed post-test



In the post-interview Mpho correctly calculated '23-18=5' making use of an empty number line, starting at 23 and counting back 5 hops to reach 18. In the delayed post-test Mpho made use of a group model (drawing two horizontal rows of 10 circles each, followed by a row of 4), but he incorrectly recorded 24 instead of 23. He used a take-way image, crossing out 18 circles. He concluded that the solution was 6 (which was correct for his depiction of 24 – 6). He was not awarded the mark for this.

Vignette: Mpho's activity on syntax model fluencies

Mpho's activity on the syntax model fluency task is a telling case. It reveals that in the pre-interview Mpho was not secure with the subtraction sign, and could not correctly answer basic subtraction calculations with the start unknown. He was able to complete all of the other number sentences. He paired number sentences as going together, paying attention to the order of the numbers in the number sentences.

In the pre-interview Mpho fluently answered 3 + □ = 5 and □ + 3 = 5 without using any representations or gestures. When he got to 5 – 3 = □, he asked if he could change the minus sign to 'a cross'? When I said it had to stay like that and read the number sentence: 'five minus 3 equals', Mpho held up 5 fingers on his left hand (in a single action). He raised three fingers on his right hand (in a single action). He seemed to know that minus meant he had to bend his fingers. He bent each finger on his left hand saying '1,23' as he bent each one down. He now had no fingers raised on his left hand and concluded that the answer was 0. He wrote 0. I asked him to check saying 'but you had five to start with'. He then raised 5 fingers on his left hand. He bent 3 fingers on his left hand saying '1, 2, 3' as he bent them down one at a time. He concluded that the answer was 2 and erased the 0 to replace it with 2. He answered □ = 5 – 3, with 0 (not showing any representations or gestures). For □ – 3 = 5, he took out 3 bottle-top counters, moved each one counting '1, 2, 3' and concluded that the answer was 0. He seems to focus only on the '-3' component of the number sentence, and enacted an action of removing 3, however he ignored the '=5' component of the number sentence. He completed the other number sentences fluently and correctly without any gestures or representations. Mpho took 1 minute 56 seconds to complete this task. He answered the two start unknown subtraction problems incorrectly.

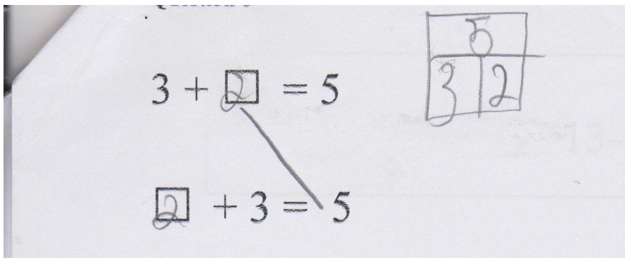
> He paired 3 + 2 = 5 with 2 + 3 = 5.
> He paired 5 – 3 = 2 with 5 – 2 = 3.
> He paired 0 = 5 – 3 with 0 – 3 = 5.
> He paired 5 = 3 + 2 with 5 = 2 + 3

When asked why those went together, he said that 'they equalled the same number'. He was attending to numbers in the boxes, and pairing them if they had the same result. He worked systematically from the top of the list, looking for the next number sentence in the list that shared the same answers.

In the post-interview, Mpho expected that all of the answers in the number sentence would be the same. He quickly completed all of them, writing and saying 2 in every box. He no longer struggled to interpret the minus sign, and he did not ask to change it to a plus sign. He worked accurately and quickly on all the number sentences.

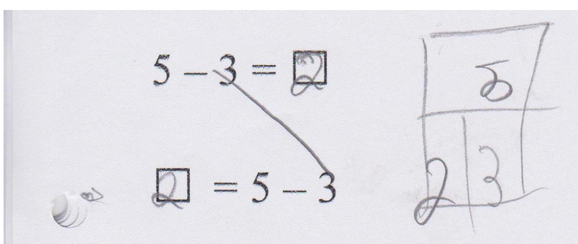
When Mpho was drawing the whole-part-part diagram for each of his pairs of number sentences, he (like Gavril, but unlike Retabile) was attending the position of each part in the whole-part-part diagrams.

Figure 16 Mpho's whole-part-part diagram for 3 + 2 = 5 and 2 + 3 = 5



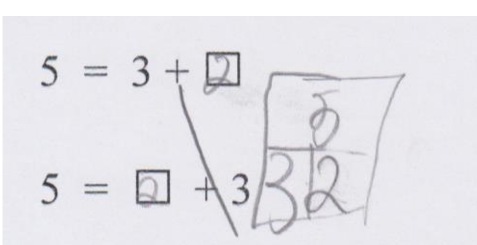
Despite the change in the order of the numbers, he considered the same whole-part-part diagram to be relevant for both number sentences. This was different to Gavril, who wanted the 2+3 =5 number sentence to be represented with the 2 part on the left, and the 3 part on the right. Nevertheless Mpho found the position of the parts to be important as he reversed the order of the parts for his next pair of number sentences:

Figure 17 Mpho's whole-part-part diagram for 5 – 3 = 2 and 2 = 5 - 3



He re-drew the same whole-part-part diagram (5-2-3) for his number sentences 2 – 3 = 5 and 5 – 2 = 3. The reasons for his decisions of how to position each part were not clear. Perhaps Mpho imagined subtract 3 as a take-away action from 5 (which would put the 3 on the right), and he did not consider a comparison 1:1 matching action (which would put the 3 on the left).

Figure 18 Mpho's whole-part-part diagram for 5 = 3 + 2 and 5 = 2 + 3



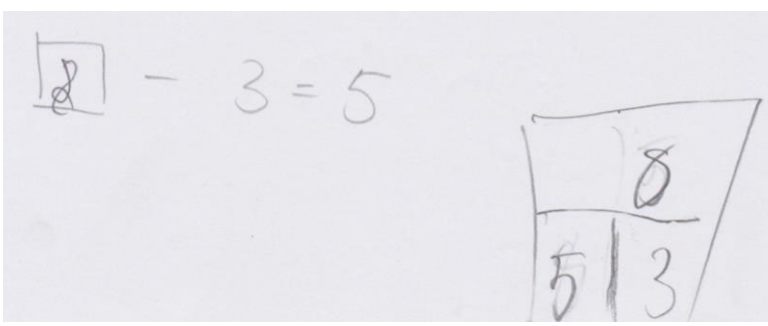
Perhaps with addition, Mpho was aware of the commutative property, and so the order of the parts was not significant for these two number sentences.

Mpho's work on two of the number sentences was of interest, as he continued to show some misconceptions about subtraction. Mpho seemed to expect that a bigger number subtracted from a smaller number would result in 0. This became evident when I probed Mpho about his reasoning for 2 – 3 = 5.

> T: I just want to check this one. 2 minus 3 gives me 5. I've got 2 [T raises 3 fingers], and I take away 3: 1 [bends 1 finger], 2, [bends the other finger] 3 [nothing is done]
> Mpho: Oh [erases answer]
> T: So I've got 2 minus 3 it should give me 0 or less than 0. Have a look at the number sentence here. So this 2 is not right, you need to check that 2.
> Mpho: It's 5
> T: Okay, let's see. 5 minus 3 is
> Mpho: 2
> T: 2. so something's going wrong here…If I've got 2 minus 3 I must get nought or less than nought.
> Mpho: Less than nought
> T: Ja, even less than nought. So here let's see what number must I take away 3 to give me 5?
> Mpho: Is zero. Because 2 is not like 5. Because if here's 2 [Mpho raises 2 fingers] …1 [He bends 1 finger], 2 [he bends the other finger].
> T: Then it's gone.
> Mpho: Mmmm
> T: So if I had 2 take away 2, I would get to zero. But I want to know what take away 3 gives me 5?
> Mpho: Is…7…[Mpho raises 7 fingers, bends 3 fingers, and is left with 4 fingers raised]
> T: Oo, 7 is quite close it gives you 4.
> (Mpho post interview)

Mpho continued to try and solve ☐ – 3 = 5 using trial and error. He first tried 7, then 9 and then 8. Each time he used his fingers and a take-way strategy bending each raised finger one at a time. He concluded that the answer was 8. He wrote 8 into the open number sentence. When prompted to draw a whole-part-part diagram for this number sentence, he first positioned the 5 as the whole, and 8 and 3 as the parts. When he was asked 'Is the 5 bigger than the 8?' he self-corrected and drew the following:

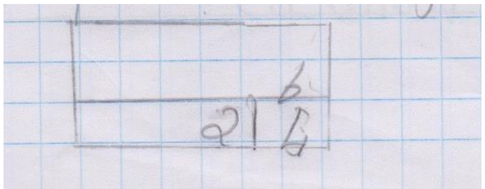
Figure 19 Mpho's whole-part-part diagram for 8 – 3 = 5

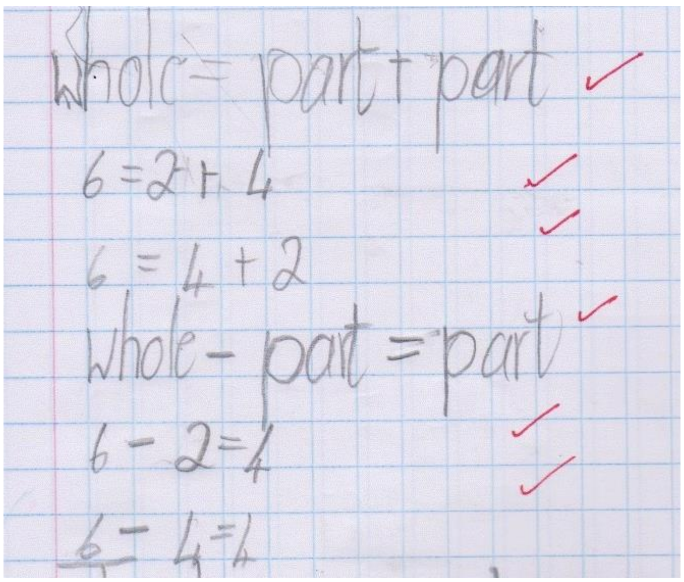


Learning goal 3: Story telling

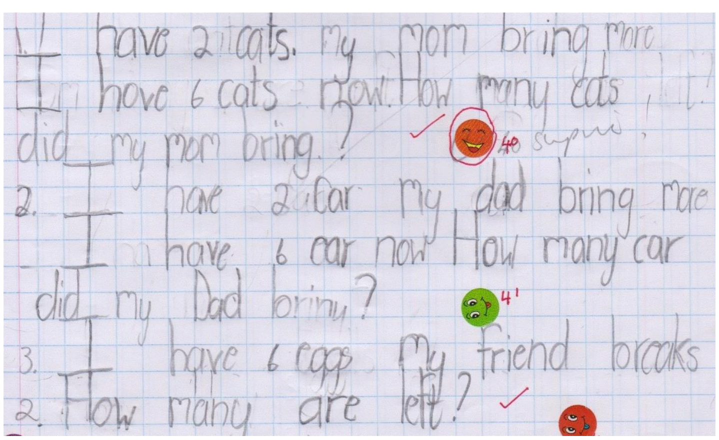
Overview of word problem stories

Mpho's activity with *Main task 6: Learners' generating examples* is revealing of his personal example space for additive relation word problems. He worked with the number triple 6-4-2 and correctly specialised a whole-part part diagram and related family of number sentences for this.





Mpho's whole-part-part diagram reveals that he was not yet aware of the significance of size the two parts in the diagram, as he labelled the larger part 2, and the smaller part 4. His work on the family of equivalent number sentences was however accurate. He told the following as his three stories for additive relation:



I have 2 cars. My mom bring more. I have 6 cats now. How many cats did my mom bring?
I have 2 car. My dad bring more. I have 6 cars now. How many car did my mom bring?
I have 6 eggs. My friend breaks 2. How many are left?

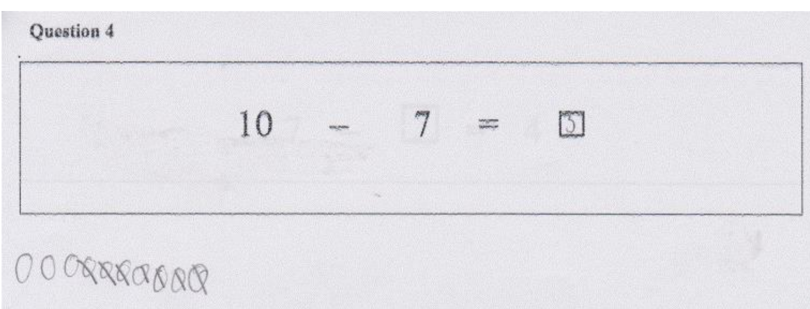
As directed, Mpho kept the numbers invariant across his three stories. He varied the characters (cats, cars, and eggs) and varied the verb from the first two stories (where the verb used was 'bring') for the third story (where he used the verb 'break'). He followed the instruction to include an example that used the term 'more'. However he did not use the comparative language of 'than',he chose to make use of change contexts for all three of his stories. His use of more related to an action of increasing (bringing more). It seemed that for Mpho the change story type was dominant. However he was able to vary the stories by posing two change increasing stories, and one change decrease story.

In the interview, as Mpho had not successfully solved the compare type problem in the written test, nor had he solved it correctly in the post-interview session. As such the questions probing for narrations involving compare (disjoint set) problems were omitted. The support group had not experienced the compare (disjoint set) problems during the small group sessions, and Mpho's concept of this comparison concept was not yet secure. There were no notable shifts in his example space for additive relations when comparing the pre-interview to the post-interview. In both settings Mpho made use of change type contexts.

Explaining missing subtrahend problems

In the pre-interview, it was clear that Mpho conceptualised subtraction using a change context, with a take-away model. When asked to tell a story for 10 – 7 = …:, he chose to first calculate the answer to this by drawing 10 lines, crossing out 7 of the lines, and writing down '3' as his solution.

Figure 20: Mpho's drawing to calculate 10 – 7 = …(pre-interview)



When prompted again to tell a story for this number sentence he offered: 'ten minus seven equals three'. His 'story' was simply a reading of the number sentence which he had not solved. I assumed that Mpho had never had to pose word problems before, and so offered him an example of a story, where he was able to choose one of two story contexts, before asking his to tell his own story:

T: Let me give you an example of a story and then maybe you can also tell one. Aaaah, what do you like? Do you like balls, or cars?
Mpho: Balls
T: Balls, OK, then there were 10 balls. And something happened to them. 7 of the balls went away. Okay, so

shall we kick them over the fence? 7 of the balls were kicked over the fence. How many were left?
Mpho: [Inaudible]
T: So we had 10 balls, we've worked it out. There were 10 balls, 7 of them were kicked over the fence…
Mpho: 3
T: 3 are left.

Here is it apparent that for Mpho when faced with an open number sentence, his inclination was to solve the problem. While I was trying to bring to his attention the story structure: that there was a starting value (10), then some change (something happened) resulting in a new state; his attention seems to be on the remaining balls. However when asked to tell his own story he was able to keep the numbers invariant (working with 10 and 7), and choose a new context:

T: Okay now can you tell me a story? I told you a story about balls.
Mpho: Mmmm. There was 10 cars. 7 was broken. How much is left?

He selected the alternative context (cars not balls) which I had offered to him previously, and introduced an action or change in state of being 'broken'. He was able to pose a question, using a similar phrasing to what I had used, but changing this slightly to: 'how much is left' reflecting a common grammatical error for second language speakers of English (in the appropriate use of the verb to be (is/was/were) and the distinction between much/many). This revealed that he could change the problem context, although he mixed two problem types in his phrases. His story was a mixture of the collection type problem where two states are in focus: broken and not broken, however his question referred to a change type problem context where there had a change which meant some things remained or where left. Notwithstanding these errors, Mpho was on task in terms of creating his own story appropriate for the number sentence 10 – 7 =…

In the subsequent line of questioning for Mpho to tell harder stories, it became apparent that the numbers in the story were they key dimension of possible variation for his personal example space:

T: Can you tell me another story? Maybe a bit harder one.
Mpho: There was 20 cars 10 broke-ted, there was 10 left.
T: Well done, there were 20 cars, 10 were broken, um, and there were 10 left. That's great.
(*Mpho pre-interview)*

In his retelling of the story, Mpho kept the context invariant (sticking with broken cars) but changed the numbers, to bigger whole numbers (which he seemed to consider to harder). He chose numbers which he probably knew as a known fact (10 plus 10 is 20, or 20 minus 10 is 10), as he did not need to calculate the solution using either fingers or drawings (which he had required previously for 10 minus 7). The solution to his problem (10 cars) seemed to come to mind spontaneously for him. He neglected to pose the question, leaving this implicit, and rather answered it. Again the difficulties Mpho has as a second language speaker of English was evident with the past tense of an irregular verb, where an auxiliary verb was missing, and the verb was treated as if it was a regular verb ('broke-ted' instead of 'were broken'). When I again constrained the numbers for Mpho, he was able to tell a new story for 10 – 7 = …, and again made use of a change context:

T: Any other stories this time using 10 minus 7?
Mpho: There was 10 hearts, 7…10 hearts, 10 heart, 4 choc…10 chocolates, 7 was eaten. Is 3.
(Mpho pre-interview)

Mpho's story was not completely coherent, but the overall meaning of what he was trying to communicate was evident. He seemed to imagine 10 chocolate hearts where 7 were eaten. This was confirmed when I restated his story and modelled the process of posing a question, and he nodded in agreement:

T: That's a nice one. So we've eaten the hearts, they're chocolate hearts?
Mpho:[nods]
T: So we've got 10 chocolate hearts…10 chocolate hearts. 7 were eaten. How many were left?.
T: [smiles and nods]
(Mpho pre-interview)

Again Mpho neglected to pose the question, but offered the solutioT: 'is 3'. This revealed that he could vary the change problem context from broken cars to eaten chocolates. He did not offer another problem type. All his variations were change (with a small hint of including collection problem types). This suggested that his personal example space may have been limited to change and collection problem types.

In the post-interview Mpho was more fluent in being able to offer stories for a symbolic number sentence, and he no longer required an example and teacher prompting to support his narration:

T: Now I want to see if you can tell me a story for 10 minus 7.
Mpho: Mpho have 10 sweets. My friend take aways 7 sweets. How many Mpho have?
T: Lovely! So Mpho has 10 sweets, my friend take away 7, how many sweets does Mpho have? That's a good one.
(Mpho post-interview)

Mpho spontaneously used a change problem context involving a friend who took sweets away from him. By the post-test he was still showing difficulties with the verbs in English grammar ('take aways' not 'takes away', 'have' not 'has', 'have' not 'does have'), however his story was mathematically coherent and he posed a question rather than providing the solution. When asked to tell another one that a bit harder using 10 minus 7, he again chose to vary the numbers used in the problem:

T: Can you tell me another one that's a bit harder using 10 minus 7?
Mpho: Mmm My friend have 70 sweets,
T: Oh dear, we're going to use 10 and 7. Okay, not different numbers.
Mpho: My friend have 14…13 sweets, he take away 7 sweets, 15, 16 sweets[reaches for structured number line, counting back in ones from 16] , is 1, 2, 3, 4, 5, 6. My friend have 17 sweets, he take away 7, [using structured number line, counting back in ones from 17] 1, 2, 3, 4, 5, 6, 7
T: And you get to?
Mpho: 10
T: well done.
(Mpho post-interview)

Mpho continued to associate 'harder' with bigger numbers. He maintained the context of sweets taken away by a friend, and increased the number range considerably (again making use of change problem context).

As a result I directed him to constrain the numbers to 10 and 7. With the direction to make the problem harder, he changed '10 – 7 = …' to a missing minuend[1] number sentence: '… - 7 = 10'. However this was not a known fact calculation for Mpho (and this was perhaps why he considered this to be a 'harder' story). He seemed to be trying to find a solution to this by trial and error. He tried 14, then 13 and needed a structured number line to guide his calculation. At first he just looked at the structured number line, and later he touched it showing jumps backwards in ones, to confirm his calculation process. He tried 15, then 16 before settling on 17 as his starting number. His strategy of using a number line, making use of a forwards count (1,2,3,4) while moving his finger backwards along the structured number line lead to the correct solution of 17 – 7 = 10. This interaction revealed that what was in focus for Mpho was the structure of the number sentence. He varied the story (maintain the re-prompted constraint to only use 10 and 7) by changing the number sentence. In so doing he did not reveal whether his personal example space included problem types other than change type problems (as the question intended). However he did reveal his calculations strategy, which seemed to have shifted from a counting back in ones, where backwards counting was less secure for him (evident in his pre-test), to using a counting forwards approach, where he kept track of the backwards movement using a structured number line.

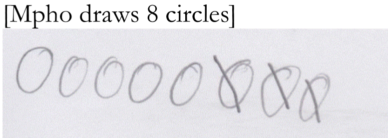
Telling compare (disjoint set) stories

In the interviews Mpho did not spontaneously tell any compare (matching) stories. Mpho did not recount any compare (reach a target stories) during the interviews.

Generalized problem situations: contrasting change and compare situations

In the interviews I opposed this question to Mpho: 'So I have some apples, and you take some of my apples. How can we work out how many I have left?' In the pre-interview he was unsure of how to specialise this general context, and asked me how many apples I had.

Mpho: How many apples do you have?
T: How many do I have? I just have some. Do you want to say how many I have?
Mpho: 1, 2, 3, 4, 5, 6, 7, 8
[Mpho draws 8 circles]



Mpho: 1, 2, 3
[Mpho crosses out 3 circles]
T: okay,
Mpho: equals…5
T: Well done. So I have 8 apples in your story, you took away 3 and I was left with 5.
Great.
()Mpho, pre-interview

[1] In the number sentence a – b = c, a is referred to as the minuend, b is the subtrahend, and d is the difference.

He chose to count and draw the number of apples which he said I had (8 circles). His dominant take-away action was then used to cross out three circles. He announced that this resulted in 5. However he was reluctant to speak in the pre-interview and I told the story for him, using the numbers he had introduced.

In the post-interview, Mpho responded to this question verbally, and did not draw anything. He specialised making use of known fact number triple of 20-10-10:

> Mpho: You have 10…ah….20 apples [picks up bead string]
> T: Okay, I've got 20 apples. Ja.
> Mpho: I take 10 away, is…is…10. [He moves a group of 10 beads]
> T: 10 left, great, that's a good one. So if I have 20 and you take away 10 then there're 10 left.

This activity with the bead string also revealed that Mpho acted on the bead string using a group wise structure. He knew that there were 20 beads on the string, and made use of the 5-5-5-5 structure to quickly located 10 and move these in one movement, acting on the group of 10.

Vignette: Mpho's activity on the generalised problem tasks

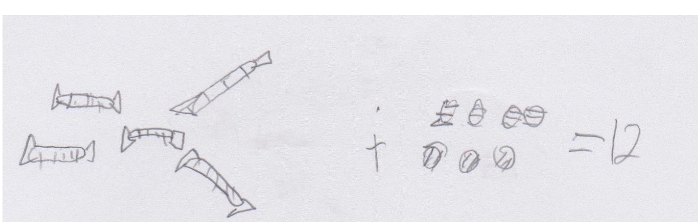
The interviews included two generalised additive relation tasks, designed to see how engaged with the contrasting structures of change (decrease) and compare (disjoint sets) stories.

When Mpho was asked how he would solve a general change situation: 'So I have some apples, and you take some of my apples. How can we work out how many I have left?' he specialised using 8 and 5, and drew a take-away image, counting forwards and acting in ones to show that 3 apples were left. He also specialised in the post-interview choosing 20 and 10 and concluding there were 10 left. 20-10=10 was a known fact for Mpho.

Mpho is a telling case as he was unable to model a process to work on the generalised compare (disjoint set) problem at either the pre-interview or the post-interview stage. No learning gains were evident for him at these two points. However with teacher support during the post-interview he was able to solve similar problems immediately following the teacher modelling. By the time he wrote the delayed post-test Mpho was invoking a 1:1 matching action, and successfully solved the compare (disjoint set) problem.

In the pre-interview when Mpho was asked a generic compare problem he chose to draw and not speak:

Figure 21 Mpho's drawing of I have 5 and you have 7



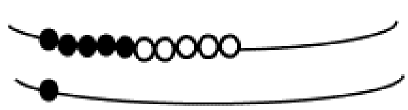
[Mpho draws 5 iconic sweets arranged in a dice pattern]
T: Those are my sweets. Okay how many have I got?
Mpho: 5
T: 5, okay.
[Mpho draws 7 iconic sweets in a pair-wise arrangement]
T: Okay, have you got some sweets? How many have you got?
Mpho: 6
T: You've got 6.
Mpho: And teacher has 5.
[Mpho writes + and = 12]
T: Mmmm
Mpho: 5 plus 6 equals 12.
T: 5 plus 6 is 12. Okay, so must we always add the sweets together when we're looking for 'how many more?' Is that what you usually do?
Mpho nods
T: Ok
(Mpho pre-interview)

From this activity it seems that Mpho specialised the compare problem to give me 5 sweets, and himself 7 sweets, indicating he understood the problem context of 'I have some sweets, you have some sweets. You have more than me'. He specialised and chose numbers where he had more than me. He then incorrectly announced that he had six sweets (possibly as a result of 2 of his sweets being drawn so close together that they touched). He seemed confident that what was expected of him was to add the two amounts, which he depicted using a + sign. He then offered the solution as 12 sweets. This provided evidence that Mpho did not know to make use of a 1:1 matching action to compare the two groups of sweets. He did not seem to understand the question: 'how many more …?' and thought that addition was required.

In the post-interview the idea of comparison was still not secure for Mpho. When posed the same general compare problem, he correctly specialised the problem giving himself 10, and me 1 sweet:

T: Okay, you've got 10. How many have I got?
Mpho: 1
T: I've only got 1. Oh, poor me, I've only got 1. How many more have you got than me? I've got 1.
Mpho: 2 more
T: Okay, but you've got how many?
Mpho: 1 less
T: You've got…how many sweets?
Mpho: 10
T: 10 sweets. You've got 10. Okay, where're your 10 sweets? Show me your 10.

Figure 22: Depiction of bead strings comparing 10 to 1



[Gavril shows a row if 10 beads on the string]
T: Look here are your 10 [one side of bead string] and here is my 1 [other side of bead string]. How many more have you got than me?

Mpho: 1
(Mpho post-interview)

From this it seemed as if Mpho did not understand the question. He seemed to answer with how many I had. This was a typical response from learners in the design experiment, who seemed to ignore the comparative aspect in the question. He simply reported how many beads he had, as if the question had been: 'How many do you have?' and not 'How many more do you have than me'.

In the post interview I moved one of Mpho beads to touch my bead (a 1:1 matching action). He then self-corrected his answer:

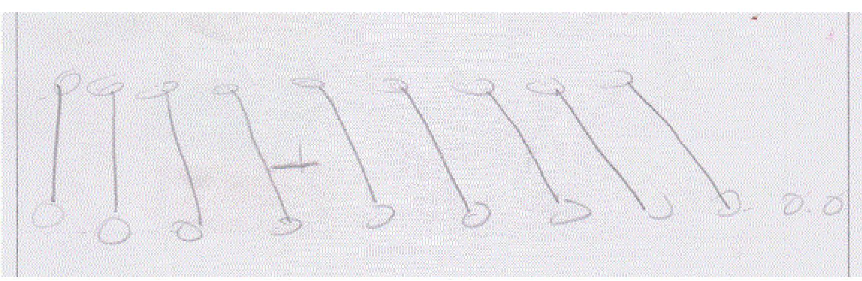
T: I've got 1. You've got a lot more. You've got 1, and then you've got all of these [T gestures to 9 beads not touching mine].
Mpho: 9
T: You've got 9 more.
(Mpho post-interview)

As his initial response has been incorrect, I varied the numbers to confirm that he was able to work with a 1:1 matching action.

T: Okay. Let's try another one. If I've got…3…I've got 3 sweets. And you've got 10. How more have you got?
[Mpho takes the bead string and count out 3, then counts out 10. Matches them and counts again in ones]
Mpho: 7
(Mpho post-interview)

When Mpho was prompted with a visual clue of two disjoint sets arranged in parallel to each other (using bead strings), then he was able to respond to questions of comparison. Without this teacher prompt, he did not seem to know to invoke a 1:1 matching context. It is interesting to note that by the delayed post-test, Mpho was invoking a 1:1 matching action when presented with a compare type problem, as his self- initiated drawing for the compare problem reveals:

Figure 23: Mpho's drawing for compare problem in delayed post-test (11 compared to 9)



Mpho's enabling tasks

Mpho was a hard working member of the support group, completing 52 independent work cards in the 10 day intervention.

Table 1: Mpho's enabling tasks

Enabling task	**Cards completed**
Enabling task 7: Vocabulary of more than and less than	14
Enabling task 8: Group model fluencies	7
Enabling task 9: Line model fluencies	1
Enabling task 10: Syntax model fluencies	10
Enabling task 11: Basic number facts and bridging the tens	12
Enabling task 12: Word problem fluencies	8
Total	52

Annexure 2: The case of Retabile

In this Annexure I present an analysis of one of the learners – Retabile- before, during and after the intervention. This Annexure opens with an analysis of evidence that her learning shifted which is drawn from the pre- and post-test and interview data. This is organised in terms of the direct objects of learning: additive relation word problems, which are distinguished from the indirect objects of learning: additive relation calculation strategies. This is followed by a chronological account of the evidence collected from Retabile during the intervention, in Retabile's classwork book.

Finally inferences are made between the evidence of shifts in learning (comparing before the intervention to after it), and Retabile's experiences during the lesson intervention. These inferences are intended to answer the following questions:

> What evidence of shifts in Retabile's learning, if any, are seen as a result of the teaching intervention:
>
> - What evidence of learning to identify, pose and solve additive relation word problems is seen during and following the intervention?
> - How does Retabile make her thinking visible by using narrative and diagrams in this intervention?

Retabile's context within the class

Retabile had isiXhosa as her home language. She had not repeated Grade 1 or Grade 2 and was the correct age for her cohort. She was allocated to the core group in the class as her attainment in the pre-test was 43%. She attained 100% for the post-test, which was the highest shift in the class:

Quantitative evidence of learning gains

Figure 24: Pre-test, post-test and delayed post-test correct solutions (Retabile)

	Q1 Change result unknown	Q2 Compare (matching)	Q3 Collect ion	Q4 Compare (disjoint set)	Q5: Partition problem	Q6: Bare calcul ation 21-6 =	Q7: Bare calculati on 23-18 =	
Marks	1	1	1	1	1	1	1	7
Retabile pre-test	1	1	1	0	0	0	0	3
Retabile post-test	1	1	1	1	1	1	1	7
Retabile delayed post-test	1	1	1	1	1	1	1	7

This quantitative depiction shows how Retabile's attainment shifted by question (using the simple marking framework). Retabile was already able to solve matching and collection word problems. She showed attainment improvements for the change and compare word problems. She was able to work systematically and to show a complete solution for the partitions problem. Her bare calculations improved. By the delayed post-test, Retabile maintained her excellent results, again obtaining 100% for the written test. This quantitative depiction does not consider the diagrammatic representations and strategies that she used to engage with the problem.

I now analyse the evidence of her shifts in learning by considering the quantitative shifts in attainment for each question in the pre- and post-test and including a qualitative analysis of what she wrote down in these tests; as well as a qualitative analysis of her responses to the interview questions. In so doing I focus on each of the learning goals: solving additive relation word problems, use of representations and use of storytelling.

Qualitative evidence of learning gains

Learning goal 1: Additive relation word problems

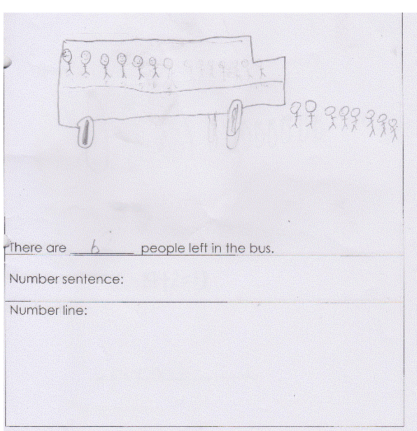
I consider the evidence of shifts relating to the direct objects of learning for six additive relation word problem types: Change problem; Change (reach a target) problem; Collection problem; Compare (matching) problem; Compare (Disjoint sets) problem and Partition problem. For each one I collate and discuss evidence from Retabile of her being able to solve these problems

With teacher support, Retabile expanded her example space from take-away contexts only, to take away contexts as well as a compare (reach a target) problem, and two compare (disjoint set) problems: one using the comparative language of 'more than' and another using the comparative language of 'less than'. Although she was able to solve a compare problem, when posing compare problems the comparative language was however not yet secure for Retabile and she required teacher prompting initially get be able to re-voice these compare stories.

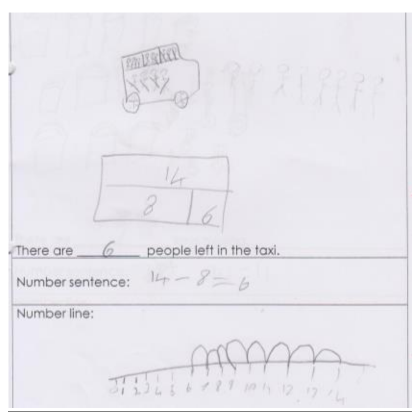
Change word problems

Figure 25: Retabile's change problem pre-test and post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



There are 14 people in a taxi. 8 people get out of the taxi. How many people are left in the taxi?

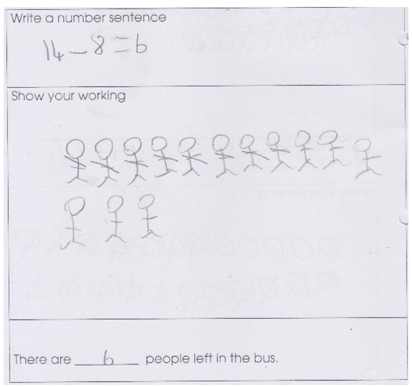


In the pre-test Retabile drew an illustration of the problem situation context (a bus). She first showed 14 people in the bus. She erased 8 of these people, and then redrew them outside of the bus. She did not write a number sentence, or draw a number line solution.

In the post-test Retabile first drew the same kind of illustration of the problem context (a taxi). She supported this with a whole-part-part diagram where the 8 part was shown as bigger than the 6 part. She was able to write a number sentence choosing 14 – 8 to depict this. She showed her solution on a number line using a take-away strategy counting back in ones. Her number lines started at 0, and showed each whole number up to 14.

Figure 26: Retabile's change problem delayed post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



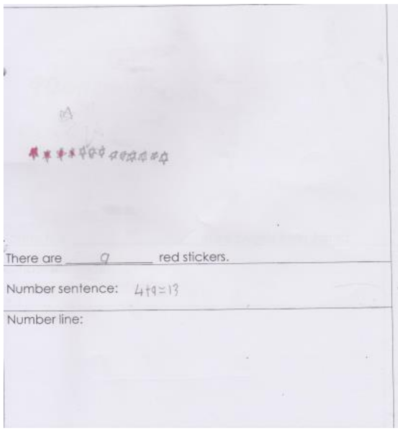
In the delayed post-test Retabile again solved the problem. However she reverted to an iconic representation showing 14 people, with a crossing out action on each person. She correctly wrote a number sentence and completed the answer sentence.

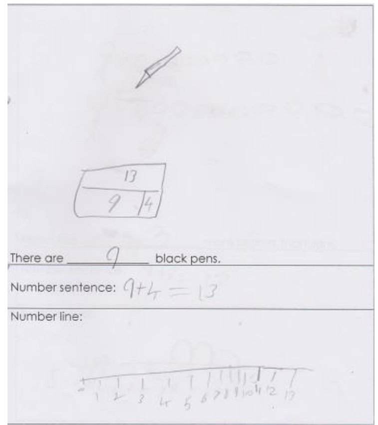
Collection word problems

The following collection problems were posed in the written tests:

Figure 27: Retabile's collection problem pre-test and post-test

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?	The teacher has 13 pens. Some are black and some are red. 4 of the pens are red. How many pens are black?



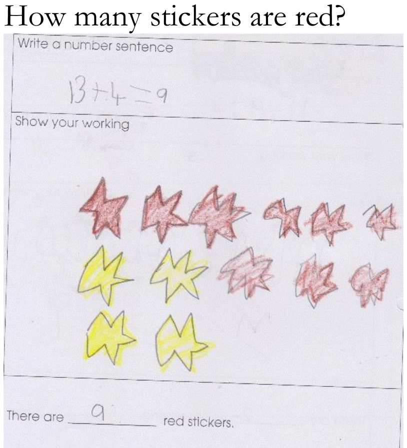


In the pre-test Retabile again drew an illustration of the collection problem situation (showing 13 stars and colouring in 4 of these red). She was able to write a number sentence choosing 4 + 9 = 13. She did not attempt a number line representation.

In the post-test Retabile only drew one pen to depict the collection problem situation. She then moved into the more symbolic representations of a whole-part-part diagram. This diagram showed the 9 part as bigger than the 4 part. She chose 9 + 4 = 13 as her number sentence. She attempted the number line representation by drawing a line and labelling the points from 1 to 13. She did not show any action on the number (for either a compare or a take-way strategy).

Figure 28: Retabile's collection problem delayed post-test

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?



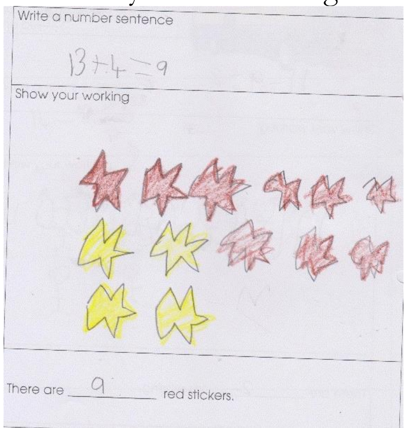
In the delayed post-test Retabile correctly solved the collection problem. She wrote a number of sentences and completed the answer sentence. She made use of iconic representations, where she coloured in 4 of the stickers' gold, to reveal the 9 red stickers.

Compare (matching) word problems

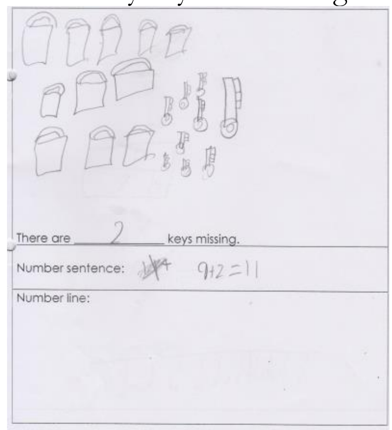
The following compare (matching) problems were posed in the written tests:

Figure 29: Retabile's compare (matching) problem pre-test and post-test

There are 11 bottles but only 9 lids.
How many lids are missing?



There are 11 locks but only 9 keys.
How many keys are missing?

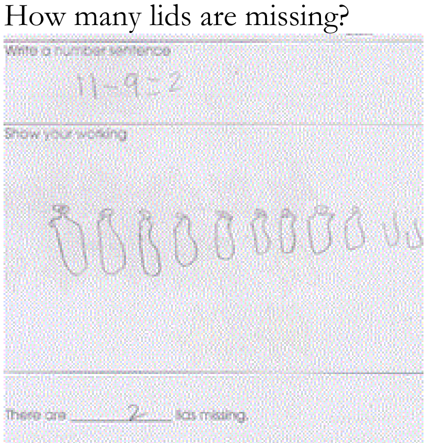


In the pre-test Retabile drew an illustration of the problem context showing 11 bottles, of which 9 had lids. She was able to write a number sentence choosing 9 + 2 = 11. She attempted a number line solution where she indicated 7 hops to the right of 2. She erased this from her work.

In the post-test Retabile again drew an illustration of the problem context showing 11 locks and 9 keys. This time there was no clear matching or relationship between the locks and the keys. She chose the same number sentence to depict this: 9 + 2 = 11. She did not attempt a number line representation.

Figure 30: Retabile's compare (matching) delayed post-test

There are 11 bottles but only 9 lids.
How many lids are missing?



In the delayed post-test Retabile correctly solved the word problem completing a number sentence and the answer sentence. She made use of an iconic representation (with no grouping) where the lids were arranged touching each bottle.

Compare (reaching a target) word problems

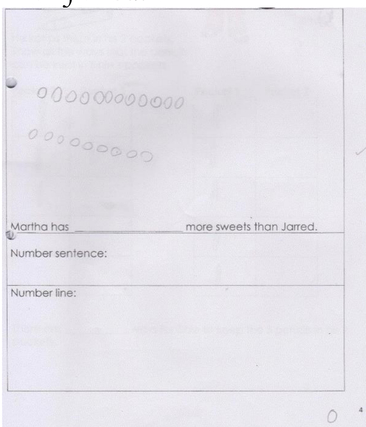
This problem type was not assessed in the written tests. However Retabile solved these problems during the lesson intervention.

Compare word (disjoint set) problems

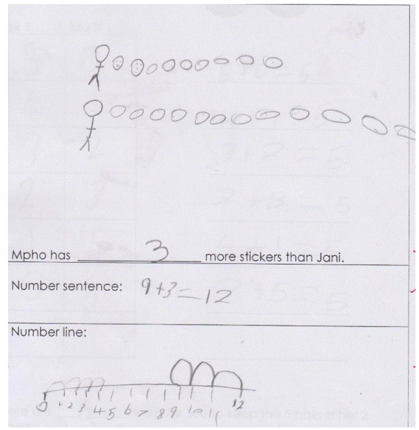
The following compare problems were posed in the written tests:

Figure 31: Retabile's compare (disjoint set) pre-test and post-test

Jarred has 9 sweets. Martha has 12 sweets. How many more sweets does Martha have than Jarred?



Janie has 9 stickers. Mpho has 12 stickers. How many more stickers does Mpho have than Jani?

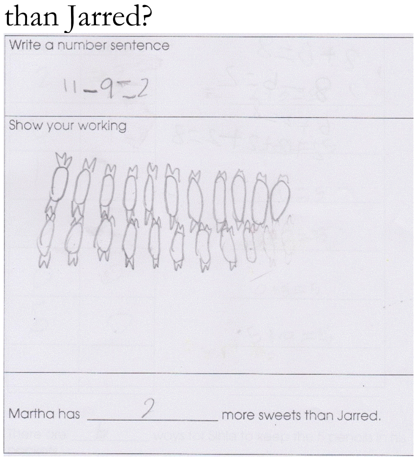


In the pre-test Retabile drew circles to denote the sweets in the compare problem situation. She drew a line of 12 sweets (for Martha) and 9 sweets (for Jarred). She did not seem to know what do with these drawings, as she did not write an answer, or write a numbers sentence or draw a number line.

In the post-test Retabile used circles to denote the stickers in the compare problem situation. She sketched stick figures for Mpho and Jani. She gave no visual clue of her process of working with this illustration, but she was able to answer correctly and provide a number sentences: 9 + 3 = 12. She sketched a number line with the numbers marked from 1 to 12, and used a compare strategy to show the 3 hops between 9 and 12.

Figure 32: Retabile's compare (disjoint set) delayed post-test

Jarred has 9 sweets. Martha has 11 sweets. How many more sweets does Martha have than Jarred?

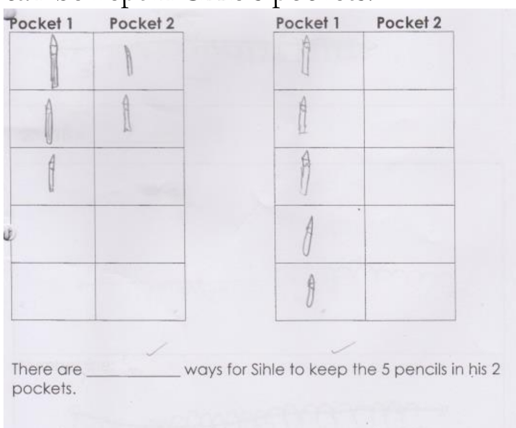


Partition word problems

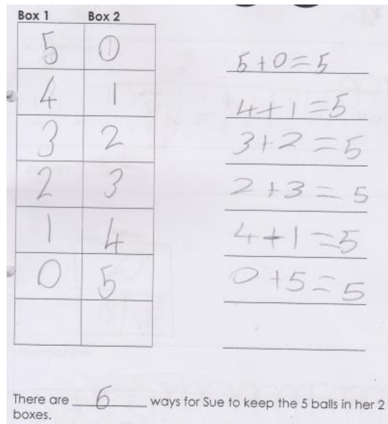
The following partition problems were posed in the written tests:

Figure 33: Retabile's partition problem: pre-test and post-test

Partition problem: pre-test
Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be kept in Sihle's pockets.



Partition problem: post test
Sue has 5 balls. She keeps them in two boxes. Show all the ways that the balls can be kept in Sue's boxes.

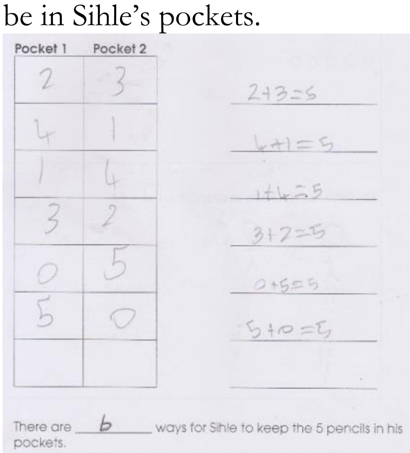


In the pre-test Retabile draw illustrations of pencils to depict the partition problem situation. She seemed to provide two possible options: 3 pencils in pocket 1 and 2 pencils in pocket 2; and 5 pencils in pocket 1 and 0 in pocket 2. She did not answer how many ways there were in total. The layout of the question seemed to confuse her, and she did not establish a complete solution.

In the post-test Retabile worked with number symbols to depict the partition problem situation. She provided a full set of options, working systematically from 5 to 0. She also wrote accompanying number sentences (which was encouraged by the slightly adapted layout of the question). She correctly answered that there were 6 possible ways to arrange the balls.

Figure 34 Retabile's partition problem: Delayed post test

Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be in Sihle's pockets.



Retabile was able to answer the partition problem in the delayed post-test, working systematically and offering a complete solution. She made use of number sentences.

Learning goal 2: Use of representations

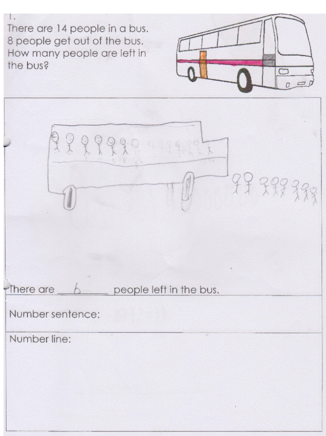
To reflect on learning gains relating to the use of representations I draw on the written pre and post-tests.

Overview of representations for additive relations

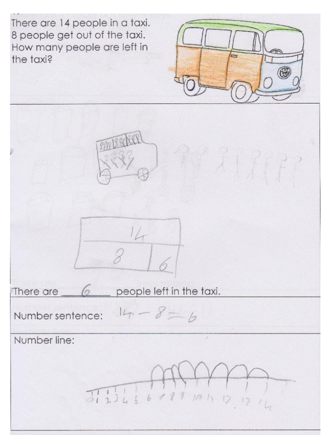
Retabile increased the number of representations which she used to explain her thinking in the written tests from 9 in the pre-test to 16 in the post-test. The range of representation types which she used also shifted. Retabile made use of iconic and indexical representations in both the pre-test and the post-test. In all of these representations she made use of a counting in ones structure, she did not depict group-wise arrangements or actions. In the pre-test Retabile made use of a structured number line using 0 as a reference point. She depicted an efficient take-away strategy and made use of counting in ones actions. By the post-test Retabile continued to use number lines which showed the numbers starting at 0 or 1. She was not yet able to make use of empty number lines starting with the numbers involved in the calculations. However she also made use of an efficient difference strategy for the compare problem. In the pre-test Retabile made use of a number sentence (without depicting the unknown). In the post-test she repeated this and also made use of whole-part-part diagrams (2 where the parts were accurate, and 2 where the parts were no accurate). She increased her use of syntax models by the post-text.

The evidence that she increased the type of diagrammatic representations available to her is starkly evident in her response to the change word problem:

Change problem: pre-test



Change problem: post-test

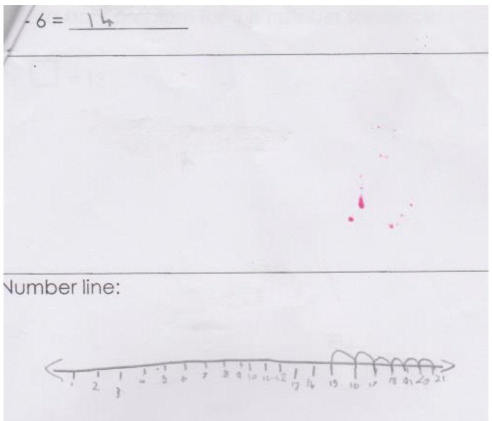


In both tests she provided the correct answer using a number symbol, and an iconic depiction of the problem situation using ones (Type 2a). In the post-test she also wrote an appropriate number sentence (Type 9). She made use of a number line representation (Type 4) on which she required 0 as reference point (Type 4a), depicted hops back in ones (Type 5a) and made use of a take away calculation strategy (Type 6a).

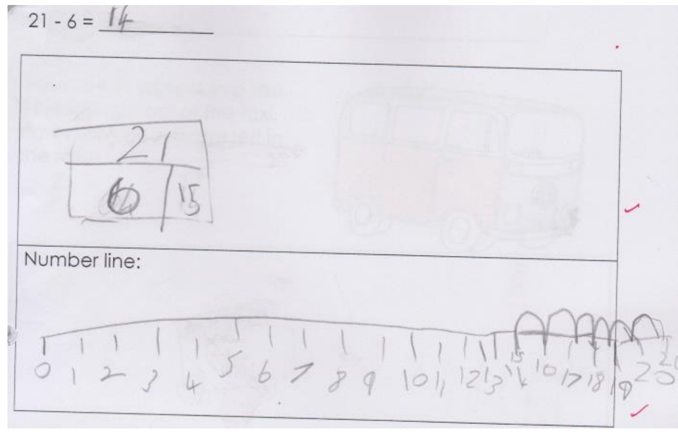
Use of representations for bare calculations

Two bare calculation questions were posed in the written test, to see how children made use of representations in their calculations.

Pre-test calculate: 21 – 6 = …



Post-test calculate: 21 – 6 = …



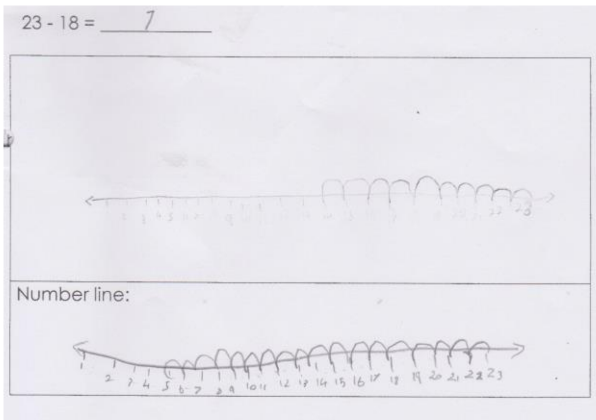
In the pre-test Retabile incorrectly recorded the answer to 21 – 6 as 14. However as her number line representation shows, she drew a number line starting at 1 marked in ones to 21, depicting 6 hops back from 21 to land on 15. This was a take-away strategy for this calculation.

In the post-test Retabile again incorrectly answered that 21 – 6 was 14. However she showed in two ways how she arrived at a solution with a whole-part-part image of 21, 9 and 15. She originally partitioned 21 into 6 and 14 (where part 6 was shown as smaller than part 14), but then erased this and corrected it to 6 and 15. In this case her part 6 was smaller than her part 15. In her number line depiction she made use of a take-away strategy. She numbered her line from 0 to 21, labelling each one, and almost ran out of space to reach 21. She hopped back 6 from 21 to land on 15. As she showed that her intended answer was 15 (in both of her depictions of the problem) she was awarded the mark for this calculation.

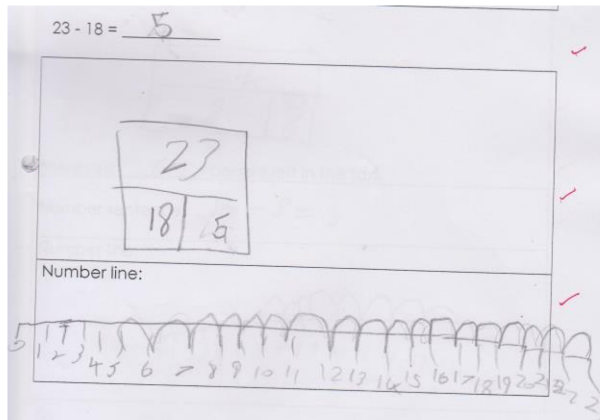
Delayed post-test

In the delayed post-test, Retabile correctly solved 21-6, making use of an indexical representation of a take-away strategy. She did not make use of any grouping, and her take-away action of crossing out was on each one.

Pre-test: Calculate 23 – 18 = …



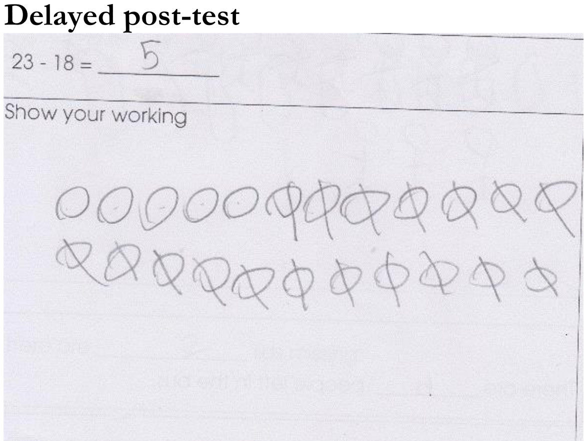
Post-test: calculate 23 – 18 = …



In the pre-test Retabile incorrect calculated 23 – 18 to be 7. She depicted a correct take-away strategy for this using a number line drawn from 1 to 23, with counting back in hops from 23 and landing on 5. She did not use the more efficient compare strategy for these numbers.

In the post test Retabile correctly calculated 23 – 18 to be 5. She showed both a whole-part-part diagram (where the 5 part was bigger than the 18) and a take away strategy on a number lines. Again her number line started at 0 and extended to 23 (where she nearly ran out of space for the 23).

Delayed post-test



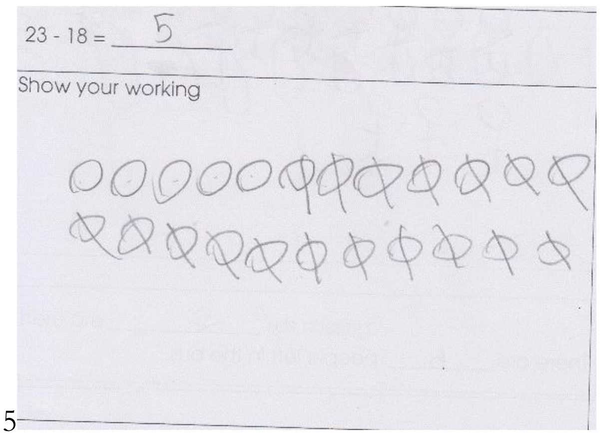
In the delayed post-test Retabile correctly answered the question, making use of an indexical representation, with no grouping. She used a take-away strategy acting on each one.

Learning goal 3: Story telling

Overview of word problem example space

Retabile's activity with *Main task 6: Learners' generating examples* is revealing of her personal example space for additive relation word problems. She worked with the number triple 10-7-3 and correctly specialised a whole-part part diagram and related family of number sentences for this

Figure 35 Retabil specialising a whole-part diagram and related family of equivalent number sentences



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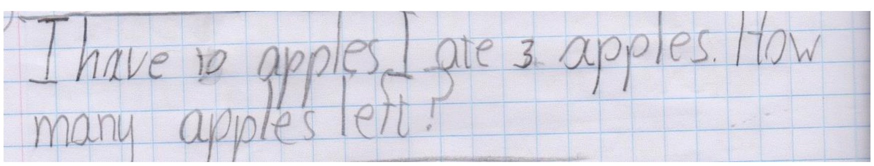
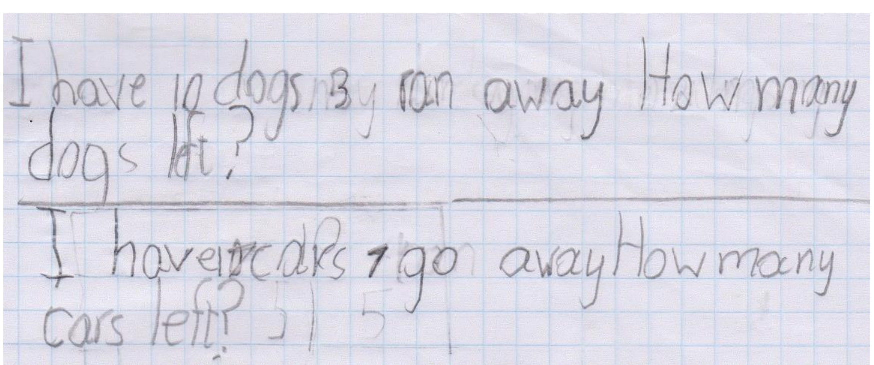
All three of her stories invoked a take-away action for this number triple:





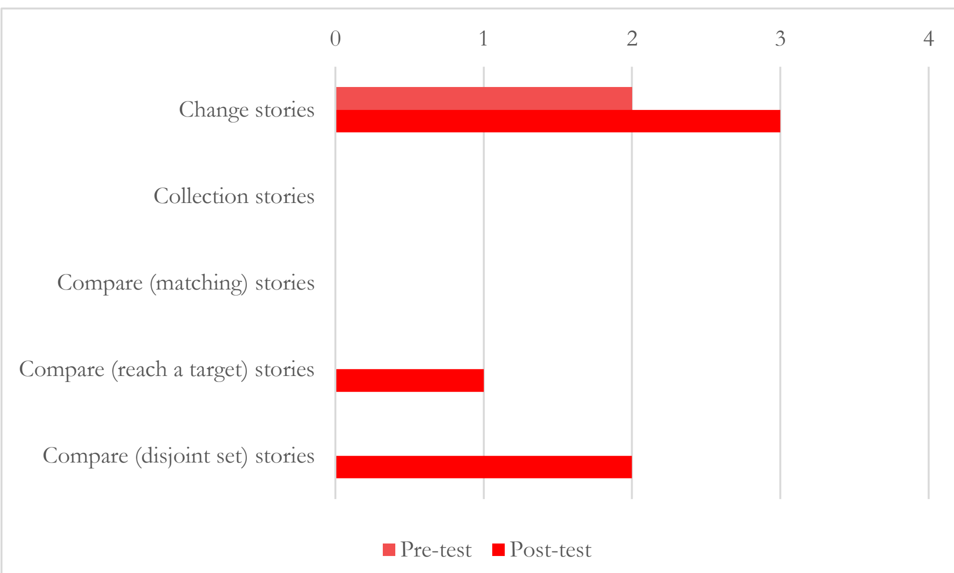
I have 10 apples. I ate 3 apples. How many apples left?

I have 10 dogs. 3 ran away. How many dogs left?
I have 10 cars. 7 go away. How many cars left?

As directed she kept the numbers invariant. She varied the characters in her story and the verbs relating to removal (apples being eaten, dogs running away, and cars going away). Her question was kept invariant with the structure 'How many 'characters' left?' She did not follow the instruction to make use of the words more and than, in one of her stories. It seems that for Retabil the take-away model was dominant for her in stories. She did vary the numbers slightly from the second story to the third, which seems to make use of the known facto relationship that if 10 – 3 = 7, then 10 – 7 = 3. From her activity on these tasks it seemed as if compare problems did not come to mind easily for Retabile.

However considering the stories that Retabile narrated during the interviews, she increased her example space of word problems from telling two stories which were both change problem situations, to telling six stories which included three change situations, one compare (reach a target) and two compare (disjoint set) situations.

Figure 36: Frequency of narratives in interviews (Retabile)



When asked for a story to explain 10 – 7 = …:, in the pre-interview Retabile offered the following:

R: Ten people are in the bus. And seven people come out. And then three people is left.
(Retabile pre-interview)

This seemed to be modelled on the word problem posed in the pre-test as it involved the same problem situation of a bus and people leaving it. She adapted this story appropriately for the new numbers. She then attempted to recall another word problem from the pre-test, referring to the matching problem involving bottles and missing tops:

> R: There are (pause) there are ten (pause) There are ten tops…And bottles…There are ten bottles.
> *(Retabile pre-interview)*

However she could not recall the problem situation for this word problem. With my prompting of what happened to the bottles (which perhaps lead her towards a change story) she turned this story into a change problem:

> R: There were ten bottles somebody took them…Seven…They took seven…There were three left.
> *(Retabile pre-interview)*

There were several stories which Retabile told in her post-interview, which were all 'change' type problems, but where the problem situation was varied:

> R: I have 10 cars. 7 go away. How many cars are left?
> R: I got 10 marbles. 7 marbles. 7 marbles… I gave my brother 7 marbles. I…How many marbles do I have?
> R: I have 10 apples. I eat 7 apples. How many apples I have left?
> *(Retabile post-interview)*

It is significant that Retabile did not spontaneously offer stories that were not in a change context. She was aware of these stories, but they did not come to mind for her without teacher prompting. As Retabile did not volunteer additive relation stories which were not 'change problems', I prompted her to see if she could recount a 'sticker story' using 10 and 7

> T: Can you tell me a sticker problem with 10 and 7?
> R: Ten and 7. I have 7 stickers. How many more stickers do I need to get 10 stickers?
> *(Retabile post-interview)*

When prompted, she was able to fluently tell a compare (reach a target) story, using the term 'more' appropriately.

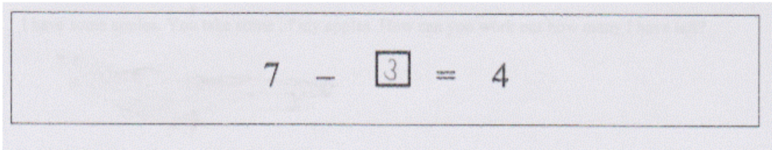
Explaining missing subtrahend problems

Retabile's understanding of additive relations and her ability to imagine a problem context that suited a number sentence was also explored in the interviews. She was asked to help a friend to understand this missing subtrahend number sentence: 7 - … = 4.

In the pre-interview she solved this calculation, and explained it using a counting back strategy:

> R: **Seven…Seven minus**
> [She writes 3 in the box]

Figure 37: Retabile's completing 7 - ... = 4 (pre-interviews)



T: So she [your friend] says how do you know it is 3?
R: **I counted. 6,5,4. And then it was 3**

In the post interview she also had to first establish that the unknown was 3:

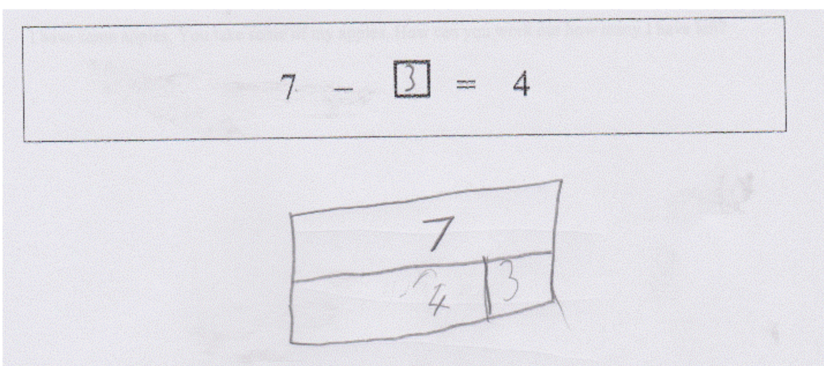
Seven minus three is equal to four.
(Retabile post-interview)

She did this mentally, just voicing what she was thinking about, and writing the 3 in the box for the unknown. I then prompted her to try and explain this to a friend.

T: Have you got a picture that you could draw or a story you could tell or diagram that would help someone to understand that?
(Retabile post-interview)

In response to this prompt Retabile drew a whole-part-part diagram, correctly depicting the 4 as bigger than the 3:

Figure 38: Retabile's wpp diagram for 7 - ...=4



I them prompted Retabile to tell a story to help to explain this

T: Is there a story that you could tell, for that, to help your friend
R: Yes
T: Ok. What story?
R: I saw 7 butterflies. Three fly away.
T: Good I saw 3 butterflies 3 fly away.
R: How many… how many butterflies…how many butterflies are left?

Retabile used a 'change story' for this calculation making use of butterflies flying away as her problem situation. However she was not able to refer to the known as being the change, and inserted the three (which was meant to be the unknown) into her story. She recounted a change story for 10 – 3 = …, and not a change story for 10 - … = 3.

I suspected that she did not have the vocabulary to label and refer to the unknown in her story. I attempted to model this for her and see if she could re-voice it:

> T: I have seven. Something happens to *some* of them and I have got four left. How can you tell a story about that? It can still be butterflies if you want.
> R: Seven…. Seven sweets…I gave my friend three sweets.

Despite my prompting, Retabile was not able to label the unknown as 'some'. She still conceptualised the calculation as 7 – 3 = …, and did not seem to be aware that I was focusing on 7 - … = 4. I tried once more to direct her attention to the position of the unknown, and wrote down 7 – 3 = …, and 7 - … = 4 to try and make her aware of this distinction:

> T: Ok remember 3 isn't meant to be in your story. We have to give the answer is three.
> R: I have 7 sweets…I give … I give away…[long pause]
> T: Do you want to give *some* to your friend? Can I help you a bit?. How about I have 7 sweets. I give *some* to my friend. I have four left.
> R: Yes
> T: How many sweets did I give to my friend?[long pause] Does that work? I have seven. I give some sweets to my friend and I have four left. How many sweets did I give my friend?
> R: Three
> T: Mmm. Can you tell that story?
> R: I have seven sweets. I give some to my friend. I have … I have four sweets left.
> T: And then what question are you going to ask?
> R: Three
> T: Three. That's the answer not the question
> R: I have seven … I give three… I have four left

This provided me with evidence that Retabile was able to re-voice the story, but only immediately after I had narrated it. She was not yet able to securely label and refer to the unknown in a missing subtrahend calculation. She seemed to think about this calculation in terms of the triple: 7, 3 and 4 (which she depicted using a whole-part-part diagram). For her the dominant way of thinking about this seemed to be as 7 – 4 = … She was thinking of 3 as the unknown, and this was what should be being asked about. She was not yet able to work with 7 - … = 4, and tell stories or questions which asked for the value of the unknown.

It was of interest that all the problem situations Retabile invoked (butterflies and sweets) made use of a 'change problem situation', which seemed to be her dominant model for subtraction. Varying the position of the unknown in this way, and labelling and referring to an unknown in a story context had not been a focus in the teaching intervention. This engagement with Retabile however pointed to the need for this to be an explicit focus in future interventions.

Telling compare (disjoint set) stories

Later in the interview I probed to see whether she was able to tell a compare (disjoint set) story where she made use of the words 'more than'. She was less fluent in recounting this and required some teacher support to invoke a context of two disjoint sets:

> T: I want to see if you can use 'more than' [in a story].
> R: More than
> T: What if you have 10 and I have seven?
> R: I have 10 cars. Teacher Nicky have 7 cars. [… long pause][2]
> T: Now you have got to ask the question. It is quite a tricky question hey? [… long pause]. Let's tell your story again (pointing at the wpp diagram). You have 10 cars. Teacher Nicky has seven cars…
> R: Yes…
> T: What question can we ask?
> R: You have ten and I have seven [Hides her face in her hands…long pause]
> T: How…
> R: How many…
> T: Good. How many…
> R: Cars.
> T: Mmm
> R: How many more do I have than Teacher Nicky?
> T: Beautiful.
> R: How many more cars do I have than Teacher Nicky?
> *(Retabile post-interview)*

This reveals that recounting a 'compare story' was not yet fluent for Retabile. She required teacher prompting to imagine that the comparison was between two disjoint sets (her and teacher Nicky). But she then spontaneously introduced a problem context of cars being compared in the two sets. It was difficult for her to pose the question, and needed a teacher prompt to start her off. Once I had suggested 'how …' she then was slowly able to formulate the appropriate question with phrases being re-voiced for her by the teacher, which seemed to reassure her to continue. Finally she was able to restate the question and introduce the problem situation of cars, which she had introduced (and then neglected) as she tried to formulate the question. Difficulties with posing compare questions was consistently evident within the intervention experience, where this language seemed new to learners. Many learners needed prompts (at times spoken, and at times written) to articulate the compare questions.

Retabile gained confidence through this post-interview process, and when prompted to tell another story like 'her car story', was able to do so fluently:

> T: Can we tell another one like that, that is not about cars?
> R: I have seven dogs. Teacher Nicky have ten dogs. How many more does Teacher Nicky have than me?
> *(Retabile post-interview)*

[2] I note the grammatically incorrect use of 'have' for 'has'. However as this is plain English grammar, and not specific to the mathematics I have not attended to this kind of error (which is common amongst second language speakers of English).

This gave me some evidence that Retabile was now able to tell compare stories, and I wondered if this extended to comparative situations where the words 'less than' were in focus:

> T: That's a good one. And if I was going to say 'how many less?'. Can we make one that's got a question: 'How many less?' [I write 'less' on a piece of paper for her to see the word]
> R: I have seven cats. Teacher Nicky have ten cats. How many …how many less do I have than teacher Nicky?
> *(Retabile post-interview)*

She did this fluently, and spontaneously changed the problem context from dogs to cats. She correctly reversed the word order in the sentence for herself and teacher Nicky, for the 'less than' comparison to make sense. This provided evidence that although Retabile was unsure at first, she was then able to demonstrate her ability to tell compare stories in the post-interview situation.

Generalized problem situations: contrasting change and compare situations

In both interviews Retabile spontaneously specialised the generalized problems and introduced whole numbers which she could work with to replace the unknowns. There were no shifts in learning evident for the generalised change problem. In both cases she specialized and modelled a process of solving the change problem.

There was some evidence of learning gains for the generalised compare (disjoint set) problem situation however. This is Retabile's response to the pre-interview task of 'I have some sweets. You have some sweets. You have more sweets than me. How can you work out how many more sweets you have?';

> R: I have got 3. You have got 7.
> T: Ok so you have got 3 and I have got 7.
> [Long pause…]
> So how many more have I got?
> [Long pause…]
> Will it help if we have a look? [T reaches for counters] You said you have got?
> R: Three [T gives her 3 counters]
> T: And I have got?
> R: Seven
> T: Seven [T takes out 7 counters]. How many more have I got?
> [Long pause…] Do you know how to work out 'how many more'?
> R: No. [She shakes her head]

This shows that Retabile knew that was happening in the problem situation (that she had 3, and I had 7), but she did not know how to compare these amounts and respond to the question: 'How many more do I have?'

By the post-interview the process of comparison seems no longer to be a problem for her. She had been able to solve a compare word problem in the post-test. She correctly re-voiced the generalised question, specialising it to involve 10 and 6 sweets, posed an appropriate question, and quickly provided an answer:

R: I have 10 sweets. You have 6 sweets.
T: OK
R: How …many… more.. do I have…?
T: Good. How many more do you have?
R: Two
T: Two more?...Can you show me a diagram?
R: Yes [Draws wpp with 10 as whole, and 6 as part and 2 as part]
[Long pause, appears concerned]
T: What's gone wrong there?
[Long silence then she corrects 2, and replaces with 4].

Retabile was able to provide an answer to the questions 'How many more do I have?', and conceptualised this diagrammatically as a whole-part-part diagram. However she was incorrect in her calculation that 10 compared to 6, was 2. She corrected this only after teacher intervention for her to find the error. However I think this provides evidence that Retabile shifted in her knowledge of how to compare disjoint sets (she correctly positioned and labelled the one set as a whole, and the other as part as the unknown), although she calculated the unknown incorrectly and had to revisit this calculation.

Retabile's enabling tasks

Retabile completed 25 independent work cards, with most of these being word problem fluency cards.

Table 2: Retabile's enabling tasks

Enabling task	Cards completed
Enabling task 7: Vocabulary of more than and less than	1
Enabling task 8: Group model fluencies	1
Enabling task 9: Line model fluencies	2
Enabling task 10: Syntax model fluencies	5
Enabling task 11: Basic number facts and bridging the tens	5
Enabling task 12: Word problem fluencies	11
Total	25

Annexure 3: The case of Gavril

In this Annexure I present an analysis of one of the learners – Gavril- before, during and after the intervention. This Annexure opens with an analysis of evidence that her learning shifted which is drawn from the pre- and post-test and interview data. This is organised in terms of the direct objects of learning: additive relation word problems, which are distinguished from the indirect objects of learning: additive relation calculation strategies. Finally inferences are made between the evidence of shifts in learning (comparing before the intervention to after it), and Gavril's experiences during the lesson intervention. These inferences are intended to answer the following questions:

What evidence of shifts in Gavril's learning, if any, are seen as a result of the teaching intervention:

- What evidence of learning to identify, pose and solve additive relation word problems is seen during and following the intervention?
- How does Gavril make her thinking visible by using narrative and diagrams in this intervention?

Gavril's context within the class

Gavril had a local language from Ruanda as his home language. He had not repeated Grade 1 or Grade 2 and was the correct age for his cohort. He was allocated to the extension group in the class as his attainment in the pre-test was 86%. He attained 100% for the post-test. So Gavril represents an example (from the 12 selected learners for interviews) of the smallest shift in attainment from the pre-test to the post-test, in the upper attainment levels.

Quantitative evidence of learning gains

The following presents a visual depiction of Gavril's shifts in attainment evident from the pre-test to the post-test and then to delayed post-test.

Figure 39: Pre-test, post-test and delayed post-test correct solutions (Gavril)

	Q1 Change result unknown	Q2 Compare (matching)	Q3 Collection	Q4 Compare (disjoint set)	Q5: Partition problem	Q6: Bare calculation 21-6 =	Q7: Bare calculation 23-18 =	Marks
Marks	1	1	1	1	1	1	1	7
Gavril pre-test	1	1	1	1	1	0	1	6
Gavril post-test	1	1	1	1	1	1	1	7
Gavril delayed post-test	1	0	1	1	1	0	1	5

Gavril was already able to solve all of the word problems. In the post-test he repeated this attainment, and showed further improvement by correctly solving the bare calculation to obtain 100%. By the delayed post-test, Gavril made calculation errors with of the matching word problem and in one of the bare calculation questions. As such his marks declined from all 7 questions being correct, to 5 correct solutions in the delayed post-test.

The evidence of his shifts in learning by considering the quantitative shifts in attainment for each question in the pre- and post-test include a qualitative analysis of what he wrote down in these tests. In addition a qualitative analysis of his responses to the interview questions is now analysed.

I analyse the evidence of his shifts in learning by considering the quantitative shifts in attainment for each question in the pre-test, post-test and delayed post-test and includea qualitative analysis of what he wrote down in these tests, as well as a qualitative analysis of his responses to the interview questions. In so doing I focus on each of the learning goals: solving additive relation word problems, use of representations and use of storytelling. It is worth noting that Gavril reported that he was very tired in the post-interview session (he yawned repeatedly and indicated that he had only gone to sleep at midnight the previous night). For the delayed post-test Gavril rushed to finish his script, was the first to submit, but could not be motivated to check his work (despite teacher encouragement).

Qualitative evidence of learning gains

Learning goal 1: Additive relation word problems

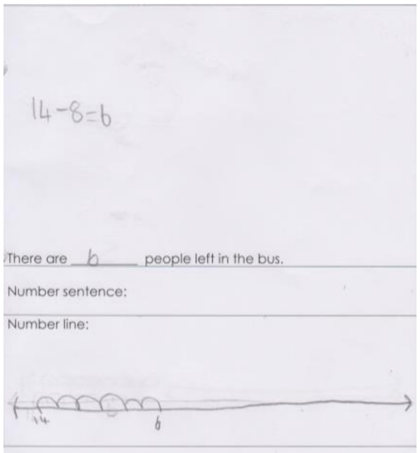
I consider the evidence of shifts relating to the direct objects of learning for six additive relation word problem types: Change problem; Change (reach a target) problem; Collection problem; Compare (matching) problem; Compare (Disjoint sets) problem and Partition problem. For each one I collate and discuss evidence from Gavril of his being able to solve these problems

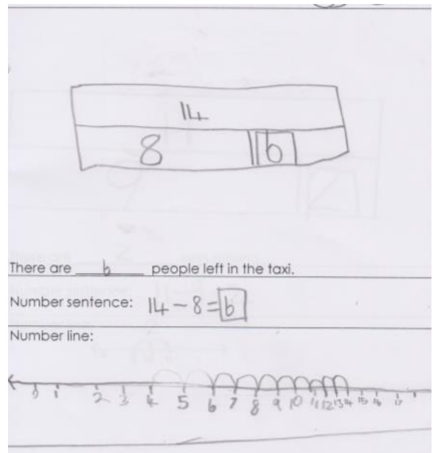
Gavril came into the intervention able to solve all of the word problems posed in the written tests. His errors related to calculation errors, and not to how to interpret and solve the problems

Change word problems

Figure 40: Gavril's change problem pre-test and post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?	There are 14 people in a taxi. 8 people get out of the taxi. How many people are left in the taxi?



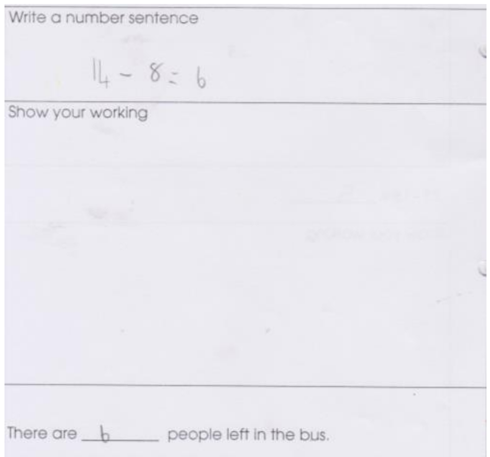


In the pre-test Gavril wrote a number sentence (syntax model) in the space for working. He correctly answered that there were 6 people left, and drew an empty number line which revealed that this line model representation was not yet secure for him. He positioned the larger number (14) to the left of the smaller number (6). He also depicted 6 (not 8) hops back from 14 to land on 6. It seemed that he was not yet distinguishing the number of hops from the landing position.

In the post-test Gavril made use of a whole-part-part diagram (syntax model) where the rectangle for 8 was longer than the rectangle for 6. He also revealed awareness of the unknown in the problem, by sketching a box around the unknown (6). He repeated this box notation for the unknown in his number sentence. This time he made use of a structure number line where he included 0 as a reference point. His numbers were arranged with the smallest on the left and biggest on the right. He correctly showed 8 hops back from 14 landing on 6. He self-corrected his original error where he had hopped back 10, by erasing the extra two hops. This denotes three learning gains in relation to two qualitative aspects which were not rewarded in the quantitative mark allocation: Firstly he made more secure use of the number line arranging numbers from left to right in ascending order and distinguishing the number of hops from the landing point; secondly he showed awareness of the unknown making use of box notation for this; and thirdly he represented the additive relationship using a whole-part-part diagram. By the post-test Gavril was moving flexibly between syntax and line model representations.

Figure 41: Gavril's change problem delayed post-test

There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



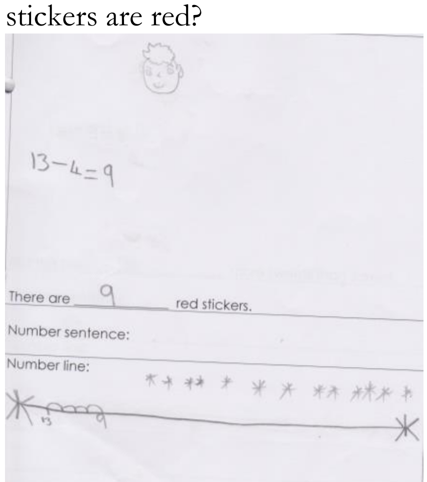
In the delayed post-test Gavril answered correctly, choosing only to make use of a number sentence (syntax model). He did not indicate the unknown in this number sentence.

Collection word problems

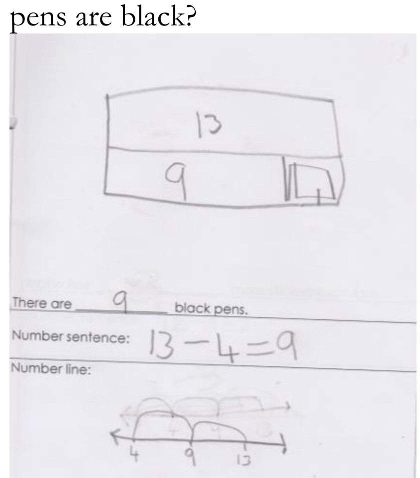
The following collection problems were posed in the written tests:

Figure 42: Gavril's collection problem pre-test and post-test

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?



The teacher has 13 pens. Some are black and some are red. 4 of the pens are red. How many pens are black?

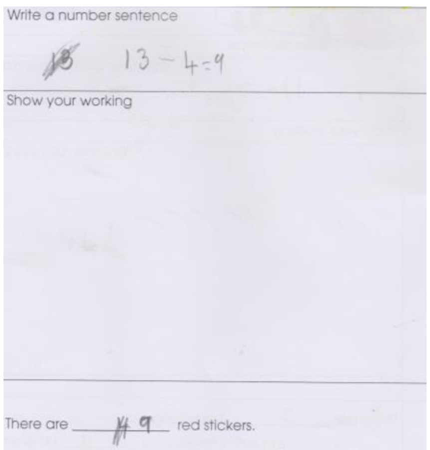


In the pre-test Gavril used a syntax model writing the number sentences 13 – 4 = 9 for his working. He completed the answer sentences. He sketched 13 iconic stars although he did not physically act on these, and it is not clear whether he used these to help him solve the problem. Gavril also made use of a line model. His line model reversed the numbers (with 13 placed to the left of the 9). He correctly depicted 4 hops back from 13 to reach 9.

In the post-test Gavril drew a whole-part-part diagram with the rectangle for 9, longer than the rectangle for 4. He completed the answer sentence correctly and provided a number sentence. His line model was revealing of some insecurity on these use of this representation. He correctly arranged the smaller numbers on the left, and bigger number son the right. However he did not show the relationship between 9 and 13 as being a jump of 4. He marked the jump from 4 to 9 and from 9 to 13, but did not label these. He seemed unable to relate the number triple 4-9-13 on the number line, showing all three members of the triple as points on the line.

Figure 43: Gavril's collection problem delayed post-test

Sue has 13 stickers. Some are gold and some are red. 4 of the stickers are gold. How many stickers are red?



In the delayed post-test Gavril correctly solved the collection problem. He wrote a number sentence (syntax model) and self-corrected his initial error in the answer sentence. His representations did not reveal his calculation strategy for this calculation.

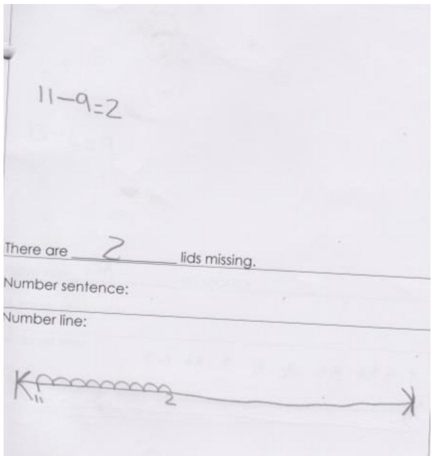
Compare (matching) word problems

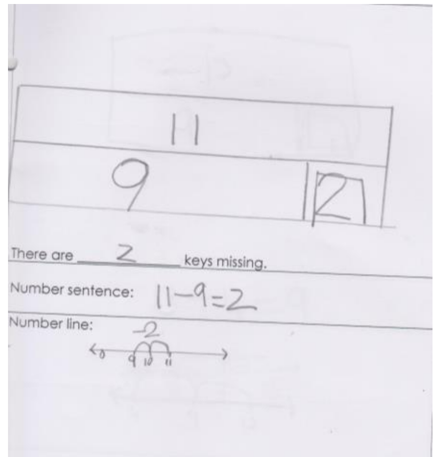
The following compare (matching) problems were posed in the written tests:

Figure 44: Gavril's compare (matching) problem pre-test and post-test

There are 11 bottles but only 9 lids. How many lids are missing?

There are 11 locks but only 9 keys. How many keys are missing?





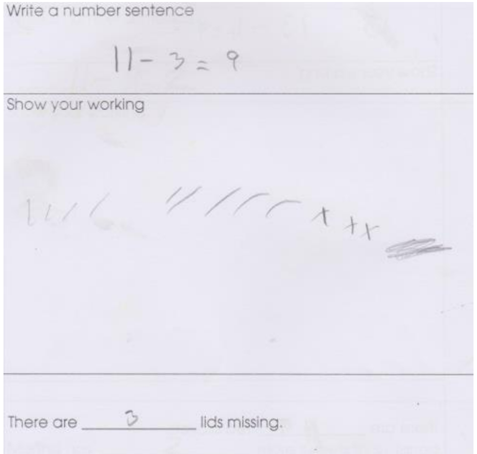
In the pre-test Gavril again made use of a number sentence (syntax model) and correctly answered the question. He drew an empty number line and again reversed the numbers (with the larger number (11) placed on the left, and the smaller number (2) on the right. This time he correctly depicted a take-away strategy showing 9 hops back from 11 to reach 2. However given the numbers in this problem, this strategy was not efficient.

In the post-test Gavril took care to draw a whole-part-part diagram (syntax model) using a ruler, and depicting the rectangle for the 9 as bigger than the rectangle for the 2. Once again he revealed his awareness of the unknown, drawing a box around the 2. He completed the answer sentence correctly and provided a number sentence. When working on a line model representation, he made use of an empty number line, with an efficient strategy of two hops back from 11 to reach 9. He labelled his hop back action using a label '-2'.

For this question although there were no learning gains evident in the quantitative marking of the questions, there were learning gains evident in the qualitative analysis: Gavril indicated his awareness of the unknown, he made use of a whole-part-part diagram, and he shifted from an inefficient take-away strategy on the line model, to an efficient difference strategy.

Figure 45: Gavril's compare (matching) delayed post-test

There are 11 bottles but only 9 lids.
How many lids are missing?



In the delayed post-test Gavril used an indexical model (which was ungrouped) and incorrectly sketched 12 lines (not 11). He used a take-away action by crossing out 3 lines to be left with 9. His related number sentence (syntax model) depicted this calculation error where he wrote 11-3 = 9, and the error was carried through into his answer. For this question there was a regression in learning from the pre-test and the post-test. This was reflected in the quantitative marking, and evident in the qualitative analysis.

Compare (reaching a target) word problems

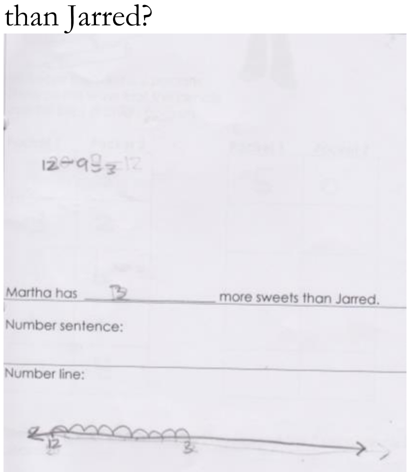
This problem type was not assessed in the written tests. However Gavril solved these problems during the lesson intervention.

Compare word (disjoint set) problems

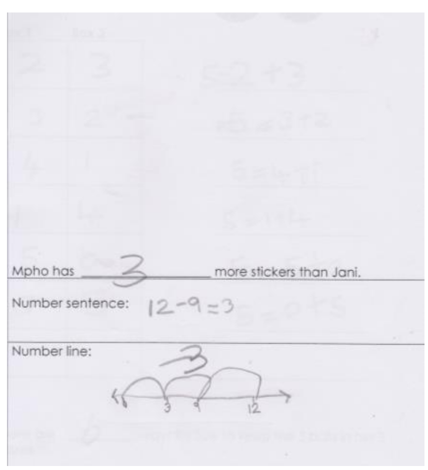
The following compare problems were posed in the written tests:

Figure 46: Gavril's compare (disjoint set) pre-test and post-test

Jarred has 9 sweets. Martha has 12 sweets. How many more sweets does Martha have than Jarred?



Janie has 9 stickers. Mpho has 12 stickers. How many more stickers does Mpho have than Jani?

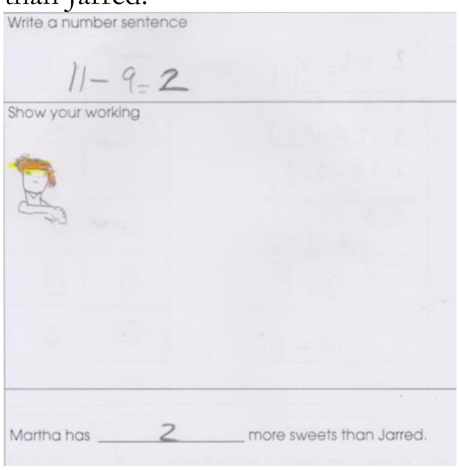


In the pre-test Gavril chose to use a syntax model for his working, writing a number sentence. He correctly completed the answer sentence. He sketched an empty number line showing 9 hops back from 12 to reach 3, although he reversed the numbers (by positioning 12 on the left of 3).

In the post-test Gavril correctly answered the question and chose not to show any working. He wrote a number sentence and completed the answer sentence. His confusion with regard to the number line model was again revealed when he correctly depicted 3, 9 and 12 as points on the line and arranged these from smallest to biggest, from left to right. He was aware that he needed to denote a -3 action, and created 3 jumps: from 0 to 3, from 3 to 9 and from 9 to 12 labelling these as -3. It was clear that he was not yet sure of whether to depict the 3 in the number triple, as 3 jumps, or place the 3 as a number on the line. These conceptual difficulties with the line model were only evident in the qualitative analysis, and were not evident in the quantitative mark allocation.

Figure 47: Gavril's compare (disjoint set) delayed post-test

Jarred has 9 sweets. Martha has 11 sweets. How many more sweets does Martha have than Jarred?



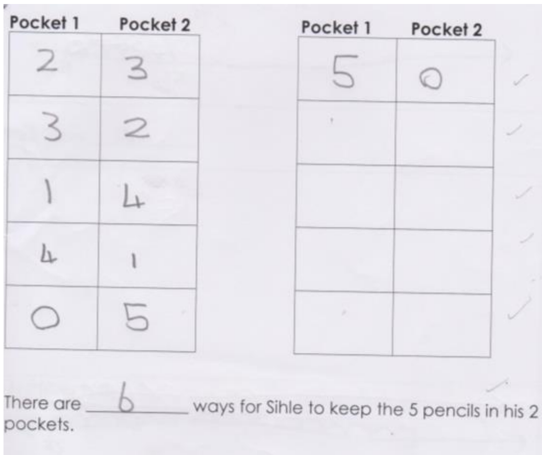
Gavril correctly answered the compare (disjoint set) problem in the delayed post-test. He chose not to show any working, and used a number sentence (syntax model) in response to the prompt.

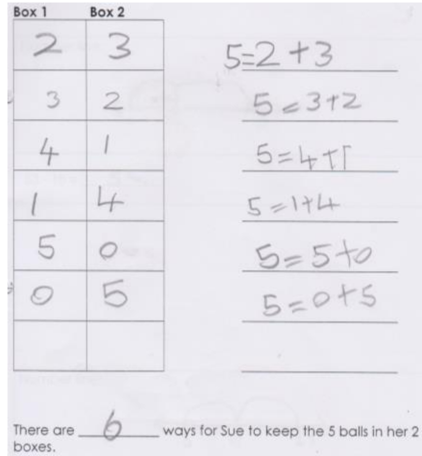
Partition word problems

The following partition problems were posed in the written tests:

Figure 48: Gavril's partition problem pre-test and post-test

Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be kept in Sihle's pockets.	Sue has 5 balls. She keeps them in two boxes. Show all the ways that the balls can be kept in Sue's boxes.



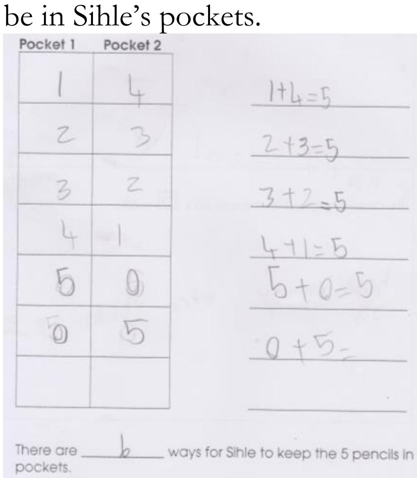


In the pre-test Gavril correctly worked out the partitions problems. He worked using number symbols and worked in a systematics way by reversing each partition (partition a-b was followed by partition b-a). He correctly concluded that there were 6 ways to arrange the pencils.

In the post-test Gavril again solved this problem without any errors. His solution was systematic and complete. His answer sentences were written in the form whole= part + part.

Figure 49 Gavril's partition problem: Delayed post test

Sihle has 5 pencils. He keeps them in his two pockets. Show all the ways that the pencils can be in Sihle's pockets.



Gavril was able to answer the partition problem in the delayed post-test, working systematically and offering a complete solution. He made use of number sentences in the form part + part = whole.

Learning goal 2: Use of representations

To reflect on learning gains relating to the use of representations I draw on the written pre and post-tests.

Overview of representations for additive relations

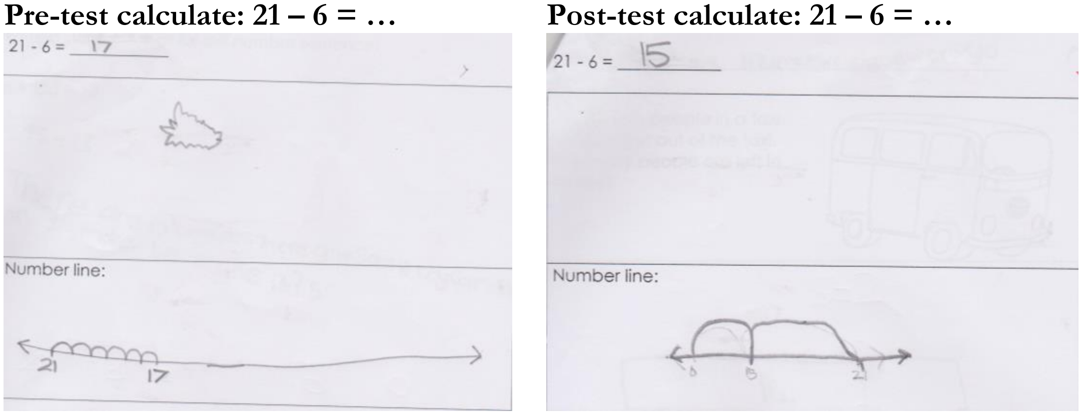
Gavril increased the number of representations which he used to explain his thinking in the written tests from 11 in the pre-test to 13 in the post-test. The range of representation types which he used also shifted. Gavril made use of one indexical representation depicting a counting in ones structure in the pre-test. He did use these representations in the post-test, however his line model representations marked a shift to group-wise actions on the lines. In the pre-test Gavril commonly made use of counting in ones action on an empty number line andhe only used a take-away strategy. He frequently drew a number line with the numbers arranged with the bigger number on the left and the smaller number on the right. In the post-test, Gavril used a group wise action and frequently included a reference number of 0 or 1 in his number lines. By the post-test he consistently sketched the smaller numbers on the left and bigger numbers on the right. But he was no sure of how to depict a number triple on a number line – commonly positing all three numbers on the line (and not placing one the members of the triple as the size of the jump). He did however include both take-away and difference number line strategies. In the pre-test Gavril was able to write number sentences. By the post-test Gavril was aware of the unknown in a number sentence and depicted the unknown value with a box around the number. He also made use of accurate whole-part-part diagrams where he indicated the unknown value with a box.

Use of representations for bare calculations

Two bare calculation questions were posed in the written test, to see how children made use of representations in their calculations.

Figure 50: Gavril's bare calculation 21 – 6 pre-test and post-test

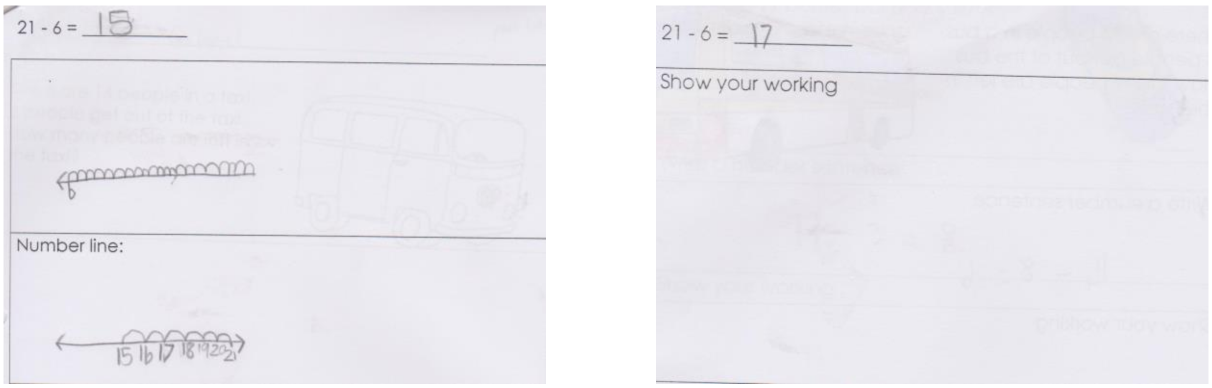
Pre-test calculate: 21 – 6 = … **Post-test calculate: 21 – 6 = …**



In the pre-test Gavril incorrectly calculated 21 – 6 as 17. He depicted this on an empty number line showing 6 hops back from 21, landing on 17. He reversed the numbers putting the bigger number (21) on the left of the smaller number (17). It is not clear how his error arose, it may have been a result of incorrect backward counting when bridging the twenty.

In the post-test Gavril correctly calculated 21 – 6 to be 15. He depicted an empty number line marking the points 6, 15 and 21 onto the line. He showed group wise jumps from 6 to 15 and then from 15 to 21. He seemed to confuse the position of the 6 in the additive relation to be a point on the line, and not a size of the jump from 15 to 21. He did not label the size of his jumps.

Figure 51: Gavril's bare calculation 21 – 6 post interview and delayed post-test



In the post-interview Gavril was shown his classwork book and particularly the section where he worked on a quick way to calculate (using a difference strategy) on the number line. His responses in the post-interview were recorded on video, and as such his calculation strategy was evident. Once again Gavril calculated 21 – 6 to be 17.

Gavril said twenty-one and held up his hands showing 10 fingers. He held his hands still and then moved them forward and back into the same position. He repeated 'twenty one' and bent one of his fingers on his right hand. He then bent down the other 4 fingers on his right hand one at a time saying 'one, two, three, four'. He moved onto his right hand bending down two more fingers while saying 'five, six'. He now had 3 fingers raised and 7 fingers down. He looked at these fingers and wrote down 17 as the answer.

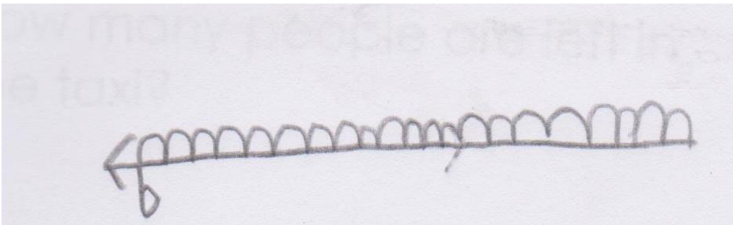
Step 1: 21 depicted as two hands of 10 fingers and 1 finger bent down (incorrectly working with 19)
Step 2: Subtract 6 by bending down 6 fingers (from the 19 to get 13 remaining).
Step 3: Interpreting the result of 7 fingers down, and 3 raised fingers to concluding that the answer was 17.

It seems Gavril did not consistently apply meaning to the action of raising or bending his fingers. First he bent a finger to indicate 1 more (than 20), then he bent a further 6 fingers (in effect adding on 6 to get 27). Looking at the 7 bent fingers he concluded that the answer was 17 (and not 27).

Having written 17 Gavril spontaneously started drawing a number line. He drew the line and then labelled 6 below the line. He sketched 21 hops, counting in ones with each hop: '1,2,3,4,5… When he got to 13 he had run out of space and needed to extend his number line. He checked that there were

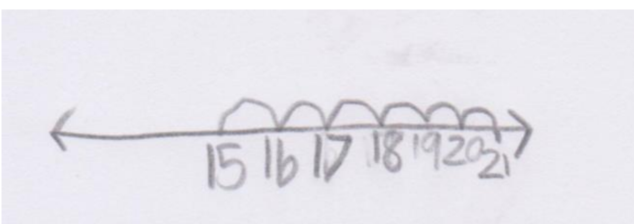
21 hops saying 1,2,3,4 , …, 21 with each depiction of a hop. He has made an error was he has labelled the starting point 6, but has counted out 21 from 0. His line depicts 21 hops from 6 landing on 27. Gavril then self-corrected his error, and begins counting on from the label 6, saying '7,8,9,10,.., 21'. When he said 21 he was 15 hops away from 6. There are 6 more hops to reach the end of his line (which extended to 27, as he counted on 21 from 6).

Figure 52: Gavril's sketch of 21 hops on from 6



He counted the remaining 6 hops, using his finger to mark the 21 position. He seems concerned. He then recounted from his finger, counting 5 hops. He therefore concluded 'the answer was 5'. When I re-voiced: 'The answer is 5', he pointed to this initial answer (17) and said 'or it's 17'. I said 'we can't have 2 answers, hey?' and he nodded. I suggested he tried to work on structured number line. He took the 0-20 structured number line and positioned his pencil just to the right of the 20. He moved his pencil back 6 jumps and landed on 15, and announced: 'Oh 15!' He erased his initial answer of 17 and replaced it with 15. He was then able to depict this on an empty number line.

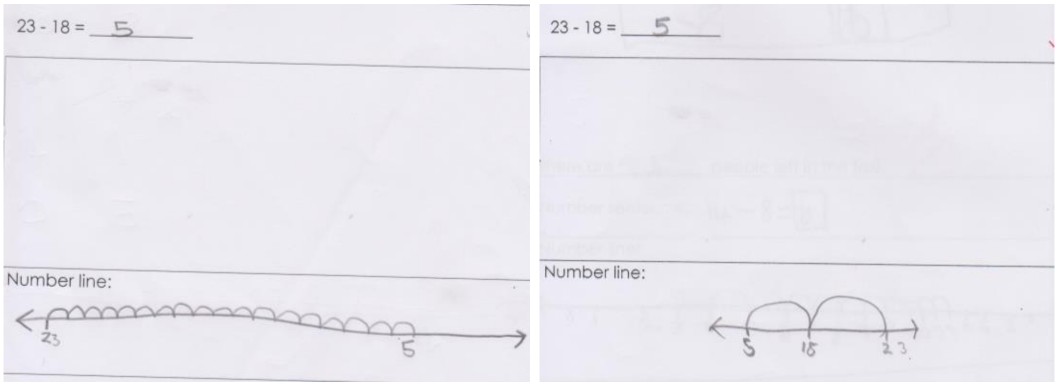
Figure 53: Gavril's sketch of 6 hops back from 21



He drew a line and starting on the right marked 6 hops. He then labelled each landing spot counting back in ones from 21. After 18 he wrote 16. I asked him to be careful and check if he had missed one. He self-corrected and wrote in 17 then 16 then 15.

In the delayed post-test, Gavril repeated the calculation error he made in the pre-test. As he did not show his working with any representations, it is not known how this error arose. This may suggest a consistent error with backward counting when bridging twenty.

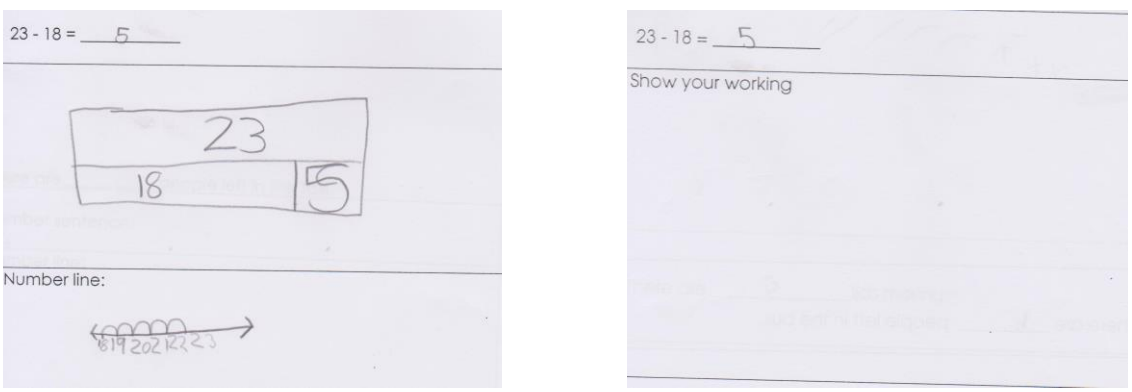
Figure 54: Gavril's bare calculation 23 – 18 pre-test and post-test



In the pre-test Gavril correctly calculated 23 – 18 to be 5. He depicted a correct take-away strategy for this using an empty number line showing 18 hops back from 23 to reach 5. He reversed the numbers on the line.

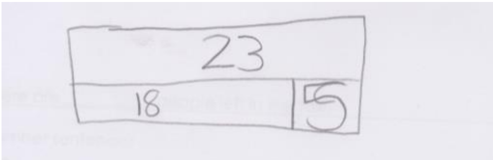
In the post-test Gavril again calculated 23 – 18 correctly. He depicted this on an empty number line, positioning the numbers in the correct order and again positioning each number in the number triple onto the line. He shows group-wise jumps from 5 to 18 and from 18 to 23 and did not label the size of his jumps.

Figure 55: Gavril's bare calculation 23 – 18 post-interview and delayed post-test



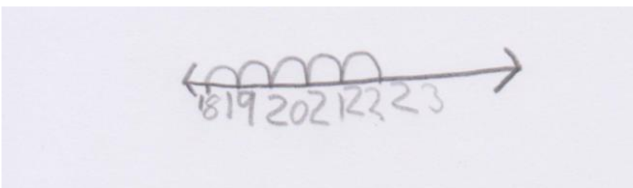
In the post-interview Gavril was asked to work on the bare calculation again. He chose to work on the structured number line first. He positioned his pencil to the right of the 20 and gestured three hops back to reach 20. He repeated this counting 1,2,3,4,5,6…17, 18' He said 'number 5'. He wrote 5 as the solution. He had used an inefficient take-away calculation strategy. I said 'That was quite a long way. You started at 23 and jumped back 18. Is there a quicker way to do it?'. He sat quietly for some time. I then asked if he could draw a whole-part-part diagram for that calculation. Gavril drew the following:

Figure 56 Gavril's whole-part-part diagram for 23-18-5



I gestured at the right of the 18 rectangle and asked: Can you start at the 18, and see how many more to get to 23?' (Gesturing to the right of the 23 rectangle). Gavril reached for the structured 1-20 number line. He started at 18, and gestured 2 hops to reach twenty, and continued with 3 more hops as he counted to 23. Gavril repeated this to show me the process: starting at 18 and gesturing with 5 hops (as he counted 1-5). He then drew an empty number line to depict this 'quicker' process.

Figure 57 Gavril's empty number line for 23 – 18 = 5



With teacher support Gavril was able to depict the quicker counting up strategy. However it was notable that he required the use of the structured number line and when he sketched this he labelled each landing point. The distinction between the number of hops, and positioning the number triple on the number line seemed clearer (but only in the 1:1 context with an encouraging adult).

In the delayed post-test Gavril correctly answered the question, choosing not to show any working for this calculation.

Syntax model fluency task in the interviews

The interviews included a task to assess learners' fluency in working with syntax model representations for the same additive relation. Learners were asked to engage in this task.

> Look at these number sentences. Which ones go together? Which ones are odd ones out? Why?
>
> 1. 3 + ☐ = 5
> 2. ☐ + 3 = 5
> 3. 5 – 3 = ☐
> 4. ☐ = 5 – 3
> 5. ☐ – 3 = 5
> 6. 5 – ☐ = 3
> 7. 5 = 3 + ☐

8. $5 = \square + 3$

In the pre-interview Retabile took 3 minutes and 1 second to complete all of the number sentences. Retabile required some support with start unknown problems in the pre-test and she worked on □ - 5 = 3 using a trial and error method. In the post-interview, Retabile took 49 seconds to complete all of the number sentences. She incorrectly completed □ - 5 = 3 as 2 – 5 = 3. She said that all of the number sentences were the same, and that they all related to the same whole-part-part diagram (which she drew as 5-3-2). When she was prompted to check □ - 5 = 3, she self-corrected using trial and error. She then drew a new whole-part-part diagram for this number sentence (8-5-3) and identified this number sentence as the odd one out. Retabile's responses to the syntax model fluency task were similar to Gavril's, however she did not consider the order of the parts in the whole-part-part diagram to be a significant factor. Retabile engaged with the syntax model tasks (Enabling task 11) during whole class sessions on Day 4 to Day 9. She also completed 5 individual work cards on the fluency models.

Both Gavril and Mpho's activity with this syntax fluency task were of interest as both learners demonstrated learning gains in that by the post-interview they had become fluent with completing families of equivalent number sentences, and that they could draw whole-part-part diagrams to represent the number sentences. However the application of the commutative property to addition and subtraction was not yet secure for these learners. Gavril still considered the order of numbers when adding to be a significant feature of the whole-part-part diagram; and Mpho had conceptual difficulties when (incorrectly) trying to apply the commutative property to subtraction. This suggested that additional tasks which focused explicitly on commutativity for both addition and subtraction (and the implications on whole-part-part diagrams) would be a useful addition to the intervention design in future design cycles.

I use Gavril's activity with this task as a best case (this was similar to the way in which Retabile engaged with the task).

Vignette: Gavril's activity on syntax model fluencies

In the pre-interview Gavril's engagement with the equivalent number sentence task revealed that he was not secure with the concept of equals as 'the same as'. In earlier questions in the interview he had read the equal sign as 'equals', 'makes' and 'gives'. Start unknown bare number sentences presented a conceptual difficulty for Gavril in the pre-interview. I briefly describe how evidence of this conflict emerged for him and how I clarified the meaning of the equal sign and introduced vocabulary for unknowns to him.

In the pre-interview Gavril worked quickly to correctly complete the first three number sentences. However at the fourth number sentence it became clear that he was not familiar with $\square = 5 - 3$.

He stopped and gestured to write a 2 into the box, but stopped again as he raised five fingers in a single action on his left hand. He then raised 2 fingers on his right hand, and then raised another finger on his right hand. He touched the fifth number sentence:

☐ - 5 = 3

He touched the fourth number sentence, raised his right hand to his mouth a said 'ooo'.

He raised 5 fingers (in a single action) on his left hand, and raised 3 fingers in a single action on his right hand. He mouthed the word 'eight'. He bent three fingers in a single action and mouthed the word 'five'. He seemed unsure of where to write his answer.

Table 3: Gavril struggling with start unknown number sentences

	Teacher and learner talk	**Gesture/ writing**
T	I think you were working on this one	Pointing to □ - 5 = 3
Gavril	This one and this one is the same, neh? (isn't it)	Pointing to □ = 5 – 3 □ - 5 = 3
T	Are they the same?...All of them look nearly the same but I don't think they are the same.	
Gavril	Oh 'cause here is a plus and here is a equals. So must I take away? Take away? Is this taking away 3?	Gavril pointed at – sign in □ = 5 – 3 Gavril pointed at equals sign in □ - 5 = 3 Gavril pointed at □ = 5 – 3
T	Mmm. Equals means the same as. Something equals 5 take-away 3. What is the same as 5 take away 3?	Teacher pointed at □ = 5 – 3
Gavril	 Something take away 3 gives 5	Gavril raised 5 fingers and bent three fingers. He wrote down 2 in the box for: □ = 5 – 3. Gavril pointed at □ - 5 = 3 Gavril wrote 8 in the box.

In the pre-interview Gavril took 3 minutes 26 seconds to complete his work on these number sentences. He then identified the 5th number sentence (□ - 5 = 3) as the odd one out.

In the post-interview Gavril quickly wrote down the answers for the first two number sentences. For the third number sentence he held up all his fingers (5 on each hand), looked at them and then wrote down 2 for 5 – 3 = □, and followed this immediately be writing 2 for □ = 5 – 3. Start unknown number sentences were no longer a problem for Gavril.

When working on □ - 5 = 3, Gavril raised all 10 fingers. He bent 3 fingers on his right hand (so 7 fingers were raised). He then raised 8 fingers, and wrote 8. He seemed to use a trial and error method, first trying 7 as his solution and then adjusting this to 8. He completed the other number sentences quickly taking 1 minutes 10 seconds for all of the number sentences (this was much faster than his 3 minutes 26 seconds in the pre-interview). The shorter time was taken as evidence of greater fluency with equivalent number sentences as Gavril now expected that the number sentences would be related.

When asked which number sentences went together, Gavril paired the number sentences as follows.

> He considered that 3 + □ = 5 went with □ + 3 = 5.
> He paired 5 – 3 = □ with □ = 5 – 3
> He indicated that 5 – □ = 3, 5 = 3 + □ and 5 = □ + 3 all belonged together
> He identified □ – 3 = 5 as the odd on out.

When prompted, Gavril drew a whole-part-part diagram for 3 + 2 = 5. He said that 2 + 3 = 5 had a different whole-part-part diagram to this, and drew the distinction:

Figure 58: Gavril's whole-part-part diagram for 3 + 2 = 5.

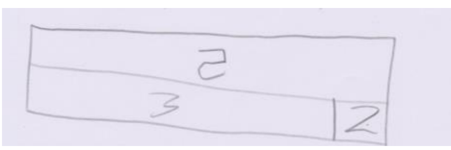
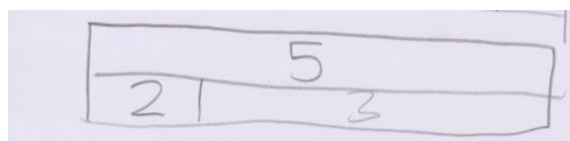


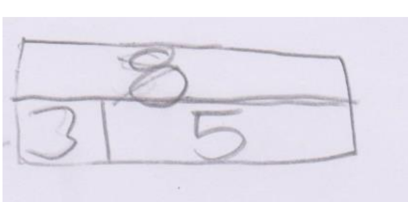
Figure 59: Gavril's whole-part-part diagram for 2 + 3 = 5.



For Gavril the sequence of numbers in the number sentence was significant and influenced how he imagined the whole-part-part diagram. The first number in the number sentence had to appear as the left part of the diagram. This perception is probably a result of the learner activity on *Main task 2: Partition problem* where the partitions of five monkeys in two trees were distinct ways: 5-3-2 and 5-2-3. In future design cycles the commutative property of addition and the choice to consider number triples and distinct or similar (depending on the context) should be discussed.

When imagining a whole-part-part diagram for 5 – 3 = 2, Gavril thought that the whole-part-part diagram would again be different. But as he drew the same whole-part-part diagram as for 3 + 2 = 5 he realised independently that this was not different, but the same as the one he had previous drawn. He said that the whole-part-part diagram for 2 = 5 – 3 would also be the same. But when reflecting on 8 – 3 = 5 he said. 'it doesn't go with any' he drew the following whole-part-part diagram:

Figure 60: Gavril's whole-part-part diagram for 8 – 3 = 5.



At first he labelled this as 7-3-5 and then corrected this to 8-3-5. For each of the other number sentences he identified that they all had the same whole-part structure as his drawings for either 3-2-5 or 2-3-5.

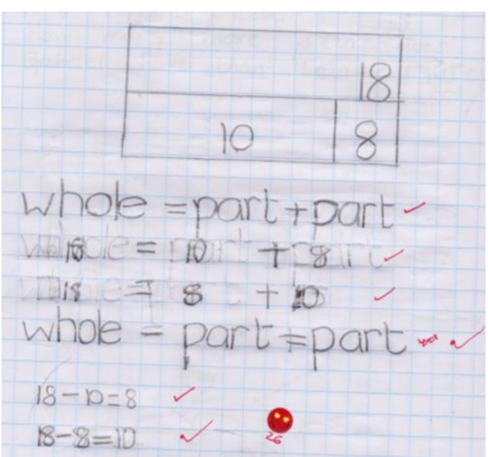
This vignette reveals several learning gains for Gavril in relation to the syntax model. Firstly he no longer struggled to complete start unknown number sentences, for an additive relation. Secondly he expected similar number sentences (where the two of the numbers in the additive relation were kept invariant, but the position of the unknown was varied) to be related. Thirdly he was able to complete these number sentenced more quickly – which possibly indicated his use of one number sentence as a known fact which could then be applied to the other number sentences. Finally he was able to visualise the additive relations using a whole-part-part diagram where he noticed the size of the whole in relation to the two parts, and considered the order of the parts in the number sentence to be reflected in the diagram. This is attributed to learner activity on the syntax model which was facilitated through *enabling task 9: Syntax model fluencies* which was in focus in the whole class sessions each day from Day 4 to Day 9, and where opportunities to practice these fluencies were provided through the individual work cards. For Gavril most of this learning is attributed to his engagement in the whole class activities, as he only completed two syntax model fluency cards independently during the intervention.

Learning goal 3: Story telling

Overview of word problem example space

Gavril's activity with *Main task 6: Learners' generating examples* is revealing of her personal example space for additive relation word problems. He worked with the number triple 18-10-8 and correctly specialised a whole-part part diagram. He made some errors when writing the related family of number sentences for this.

Figure 61 Gavril specialising a whole-part diagram and related family of equivalent number sentences

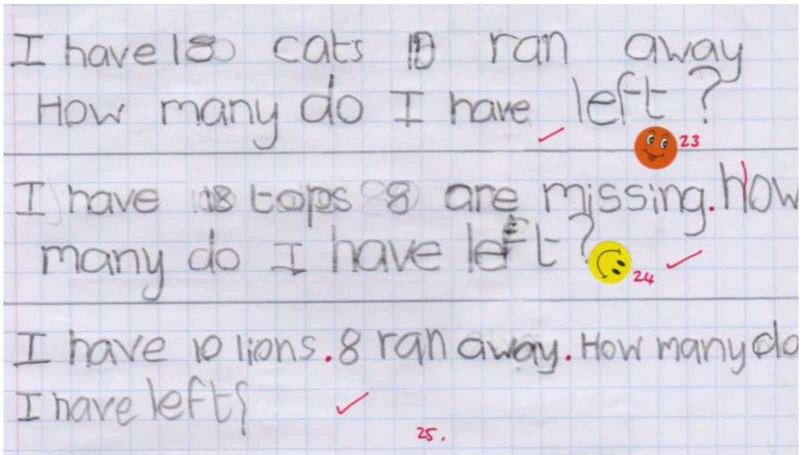


Gavril showed a tendency to rush his work both during class and in assessment tasks. He wrote

‘whole = part + part’ and repeated this was ‘whole = part + part’ (instead of writing ‘whole – part = part’). When this was pointed out to him, he corrected this on his own.

All three of his stories invoked a take-away action for this number triple:

Figure 62 Gavril’s learner generated examples for 18-10-8

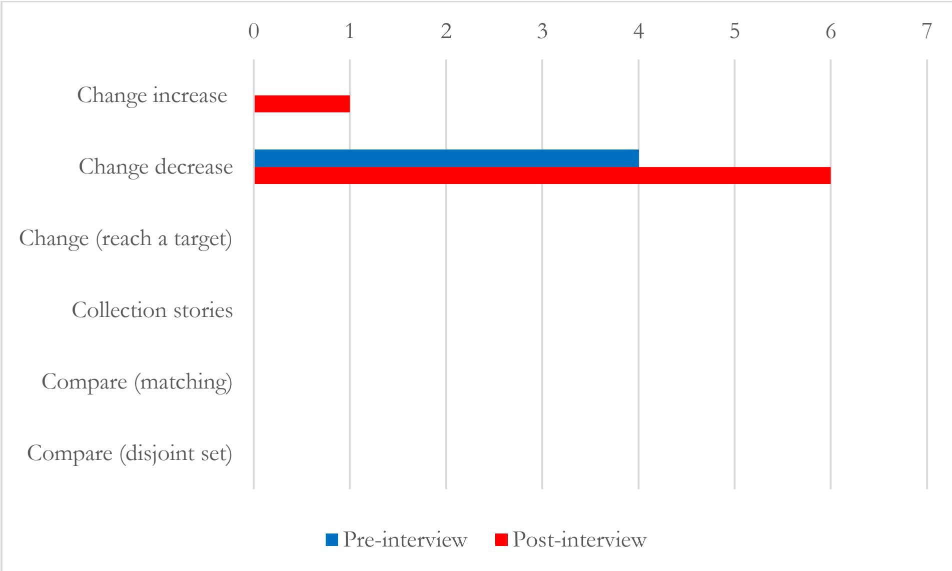


I have 18 cats. 10 ran away. How many do I have left?
I have 18 tops. 8 are missing. How many do I have left?
I have 10 lions. 8 ran away. How many do I have left?

As directed he kept the numbers invariant. He varied the characters in his stories and the verbs relating to removal (cats and lions running away, and tops being missing). His questions were kept invariant with the structure ‘How many do I have left?’ He did not follow the instruction to make use of the words more and than in one of his stories. It seems that for Gavril the take-away model was dominant for him in stories. He did vary the numbers slightly in the second story, which seems to make use of the known fact relationship that if 18 – 8 = 10, then 18 – 10 = 8. From his activity on these tasks it seemed as if compare problems did not come to mind easily for Gavril.

Considering the stories that Gavril narrated during the interviews, his example space for word problem remained dominated by change actions. He was only able to bring change type stories to mind, and despite being offered direct prompts to shift away from these situations, his concept of subtraction remained fixed with a ‘take-away’ action.

Figure 63: Frequency of narratives in interviews (Gavril)



When asked for a story to explain 10 – 7 = …: in the pre-interview, Gavril offered the following:

> Gavril: There are 10 toy cars and 7 toy cars. 10 take away 7 equals question mark. Answer is 3…
> T: What happened to the cars?
> Gavril: The cars went lost. So 10 took away the 7 cars
> (Gavril pre-interview)

In the post-interview Gavril fluently recounted a change decrease problem

> Gavril: I have 10 cars. 7 ran away. How many do I have left?
> (Gavril post-interview)

When asked for a harder problem using 10 minus 7, Gavril struggled to constrain his response to the number sentence that was given. He first offered a combine/join situation which we referred to altogether. When directed to use the number sentence as given, he referred to take-away without any characters in the story. When directed to ask a question, he associated subtraction with the question 'How many are left?'

> Gavril: I got 7 cars. Tim have 10 cars. We mix it up. Then I take 10 he takes 7…yo!...yo. …And then altogether makes 17.
> T: Okay, altogether makes 17. But if I want to ask for this number sentence [Gestures to 10 – 7 =…] what question would I ask?
> Gavril: 17 take away 10 gives a 7.
> T: Okay, so that's 17 take away 10 is 7. I want 10 take away 7.
> Gavril: 10 take away 7.
> T: Can we have a story about 10 take away 7 if you've got 10 cars and Tim's got 7 cars?
> Gavril: Oh, equals 3.
> T: Hmmm is does equal three. So can we ask a question where someone needs to answer three?
> Gavril: I would ask…mmmm. I would ask….How many do they have left?
> (Gavril post interview)

Explaining missing subtrahend problems

Gavril's understanding of additive relations and his ability to imagine a problem context that suited a number sentence was also explored in the interviews. He was asked to help a friend to understand this missing subtrahend number sentence: 7 - … = 4.

In the pre-interview, Gavril responded as follos

> T: Gavril, can you please help me, what does this mean.
> [T gestures to 7 - … = 4]
> T: Can you explain what's happening here?
> Gavril: I need 7 to take away something to make 4.
> T: Ok. So 7 take away something must make 4?
> [Gavril raises 7 fingers, bends working out answer on fingers. Writes down answer of 3]
> (Gavril pre-interview)

In the post interview Gavril refers to the unknown as 'mmm' and demonstrates he uses a trial and error method, but 'putting a number' and trying it. He also reveals his awareness of the equivalence of the two number sentences: 7 - … = 4 and 7 – 4 = ….

> T: Your friend says to you, Gavril I don't understand this, what's going on here. How would you explain that to him…to them?
> Gavril: Uh…7 take away mmm equals 4.
> T: Okay. And how do you find out what mmm is?
> Gavril: By putting a number or…or….by putting…um… a number…
> [Gavril looks at his left hand, then raises two fingers on his right hand. He bends the two fingers on his right, and one on his left hand].
> Gavril: Number 3.
> T: Number 3. How did you find number 3?
> Gavril: Cause I had 7 {shows 7 raised fingers], I took away 3 [lowers 3 fingers, showing 4 remaining].
> T: Ok, and then you were left with?
> Gavril: 4
> (Gavril post interview)

Telling compare (disjoint set) stories

Despite prompts and attempts to shift Gavril away from change word problem contexts towards using comparative language, he remained stuck with change contexts

I then prompted Gavril to tell a sticker story, hoping that this would provoke him to use the comparative language of 'more'. He ignored the suggestion to use stickers, and chose to refer to cars, which he then changed back to stickers. He imposed a change decrease action onto this story:

> T: Can you tell me a sticker story?
> Gavril: I have 7…mmm…ja…I have 10 cars…hmm…10 stickers…I give Anele 7. How many do I have left?
> (Gavril post-interview)

It is notable that all the stories Gavril used to explain the subtraction number sentence made use of change actions.

Later in the post-interview I again tried to get Gavril to bring comparison stories to mind:

> T: So Gavril I want to see, can you tell me a story where the question is how many more do you have than me? Can you tell me a story about that? How many more do you have than me? You don't have to write it you can just tell me.
> Gavril: Oh!…you have 30 stickers, I have…Oh, yo…ja…you have 30 stickers I take 20. How many do….hoe!....how many you got?
> (Gavril post-interview)

Although Gavril could solve compare (matching) and compare (disjoint set) problems, he was not yet able to bring these to mind and retell these stories.

Generalized problem situations: contrasting change and compare situations

This is Gavril's response to the pre-interview task of a generalised change problem:

> T: I have some apples, you take some of my apples. So you can imagine, I have some apples and you take them away. How can you work out how many apples I have left?
> Gavril: I can maybe imagine…I can maybe imagine 5.
> T: Okay
> Gavril: Cause you take, you buy mos [about/like] 10, I take 5, you take 5.
> (Gavril pre-interview)

This shows that Gavril was able to specialise, and chose a known fact additive relation (10-5-5) and he invoked a change decrease action of removal (take-away).

In the post-interview Gavril did not immediately specialise, and only did this with a teacher prompt:

> T: So I have some apples and take some of my apples. How can you work out how many I have left?
> Gavril: You can't work out because they didn't…there's no number.
> T: Ok, do you want to put some numbers?
> Gavril: You got 10, I take 7. Um how many do I have left? Okay…oh…I have left, okay, I have 3 left.
> T: You've got three left. Okay, great job.
> (Gavril post-interview)

In the pre-interview it was clear that Gavril was not sure of how to solve a compare (disjoint set) problem. He was able to specialise, making use of 20 and 10, and then he asked whether he needed to 'take-away' or 'plus'.

> T: I've got some sweets you've got some but you've got more than me. How do you work out how many more than me you have?
> Gavril: I have 20, and you've got 10, then, whooo, must I take away or must I plus?
> T: Mmm, it's quite tricky hey? Think about it. I've got…you said you've got 20 and I've got 10, you've got more than me right? How many more have you got than me?
> Gavril: Yo!
> T: Is that idea of "more" a bit tricky? Shall we try it with smaller number first?
> (Gavril pre-interview)

I introduced concrete counters, giving Gavril 6 bottles tops and keeping 4 bottle tops in front of me. He said that he had six, and that I had four. I arranged the bottle tops to make a 1:1 matching arrangement, and gestured to the two more. With this intervention Gavril was then able to return to his calculation involving 20 and 10:

> Gavril: I got 20, you got 10, you need 10 more.

T: Well done, so I need 10 more to be the same as you. Okay, good job. And with the 4 and the 6, how many more was it?
Gavril: Two
T: Two. Well done.
(Gavril pre-interview)

In the post-interview Gavril was able to respond to this question fluently. He once again specialised using a known fact relationship of 20-10-10:

T: I have some sweets and you have some sweets. You have more sweets than me. How do we work out how many sweets you have? How many more sweets you have?
Gavril: Ok…I have 10 and…mmm…I have 20 and you have 10. Oooh. So…
T: So how many more have you got than me?
Gavril: Ooooo 10.
T: You've got 10? Oh, how did you work that out?
Gavril: Because 10 plus 10 is 20. And take away another 10 is still 10.
(Gavril post-interview)

This interaction demonstrated his awareness of the equivalence of 'part+part=whole', and 'whole-part=part'. He was no longer concerned as to whether it was 'plus' or 'take away' as he knew: '10 plus 10 is 20. And take away another 10 is still 10'.

Gavril's enabling tasks

Gavril completed 18 independent work cards:

Table 4: Gavril's enabling tasks

Enabling task	Cards completed
Enabling task 7: Vocabulary of more than and less than	1
Enabling task 8: Group model fluencies	5
Enabling task 9: Line model fluencies	2
Enabling task 10: Syntax model fluencies	2
Enabling task 11: Basic number facts and bridging the tens	3
Enabling task 12: Word problem fluencies	5
Total	18

Annexure 4: The implementation of the third cycle intervention

Chapter 9 focuses attention on the implementation of the third cycle intervention. It answers the question: '*How did the third cycle intervention play out in this local context?*' For each of the main tasks vignettes from the lesson transcripts, together with evidence from learner activity is presented. The activity relating to enabling tasks is briefly described. Finally those theoretical features that do not figure in the description of tasks are exemplified with vignettes from the lesson transcripts. Together these provide a descriptive account of the third cycle intervention with reference to each of the theoretical features which informed its design.

I open this section with a short description of the chronology of events over the ten day intervention and then describe how each of the main tasks unfolded. These related to the problem solving activities, where both storytelling and representations were used as a means of supporting learnings.

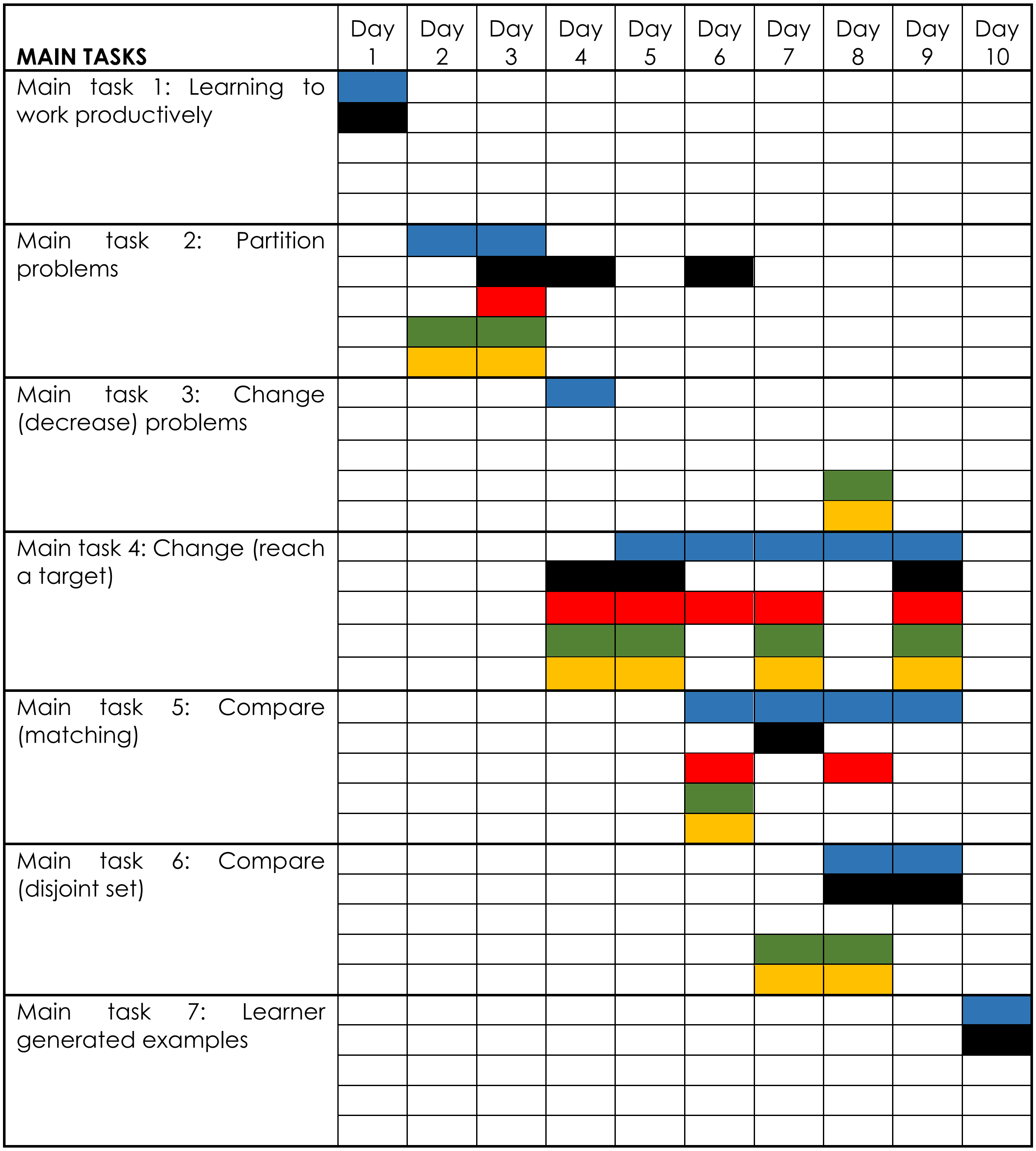
Chronology of cycle 3 tasks

In this section I provide a summary account of the third cycle intervention with reference to the main and enabling tasks. This broad overview is supported by a detailed chronological account of the third cycle intervention, which includes critical self and peer reflection, in Annexure 4.

Main tasks

The lessons were structured using both whole class and small group interactions *(Feature 9.1: Teaching the whole class as well as in small groups)*. The following table maps the main tasks to each day of the intervention, with reference to the teaching format (whole class, or group work).

Table 5: Mapping of main tasks to days in the third cycle intervention



Key

Hypothesised trajectory

Whole class

Support group

Core group

Extension group

In terms of the selection and sequencing of tasks, the actual learning trajectory (from the planned teaching perspective) closely matched the hypothesised learning trajectory for the extension and core groups. There were also a few differences (as is to expected in any design experiment) which are worth highlighting. However the support group did not follow the hypothesised learning trajectory closely for the main tasks. There were several changes in the actual learning trajectory for the support group, implemented as contingency responses to the observed learner activity with the support group.

Main tasks for core and extension groups

For the core and extension groups there was a slight change in the sequencing of the main tasks as intended in the hypothesised learning trajectory. Working on *Main task 2: Partition problems* spanned three days (Days 2-4). A slower introduction to the symbolic whole-part-part diagram (in comparison to Cycle 2), meant that an indexical whole-part-part diagram was used first. The process of creating the indexical whole-part-part diagrams by tearing five strips, as well as shifting from working randomly to working systematically on this task took longer than expected. But this task was considered to be a foundational task which set up the structural approach to additive relations. As such this additional time was through to be justified. In response to the need for additional time on this task, *Main Task 3: Change word problems* was omitted. Components of this Main task 3 were reinserted on Day 8 for core and extension learners and on Day 10 with the whole class.

Main tasks for support group

The assumed starting point for some of the support group learners was below what was expected. The differences may be a result (at least in part) of the fact that the Cycle 2 intervention had been undertaken near the end of the academic year (November) while the Cycle 3 took place near the beginning of the academic year (April). The learners in Cycle 3 were therefore less mature than those in Cycle 2. In addition, the support group for Cycle 3 included four learners with substantial developmental delays and one learner who had very limited English (he was a recent immigrant with French as a home language).

On the third day, through discussion with Vanessa, the support group of eight learners were split into two groups of four and additional group work time was arranged for these two groups. Despite this organisational change, the support group children were not provided with the same opportunities to engage with the *Main task 2 Partition problem* in the same way as the core and support groups were. They were not provided with a second group work session to repeat the partition problem solving process. They were not given time to work randomly, before being supported to work systematically, nor were they encouraged to act out the story, or to retell the story with numbers other than 5 monkeys. They did not work on variations of the partition problem using other number of monkeys. These learners were present in the class when the partition problem was revisited during whole class sessions, but without focused small group work on this (coupled with the lack of focus evident from most support groups learners) it was unlikely that their engagements with these whole class sessions resulted in learning. This support group did not work on *Task 6 Compare (disjoint set)* in a small group setting.

They were present in the class when *Task 6 Compare (disjoint set)* was the focus a whole class sessions, and they observed other class members engaging with this problem.

There were several assumed starting points relating to LG 2: Representations to interpret calculation strategies' which were not yet secure of these learners. Concerning calculation strategies 'Count all calculation strategies involving counting in ones' and a take away calculations strategy were not yet secure for many of the support group learners. In terms of the line model representation 'action of hops forwards making more or backwards making less' and 'orientation of smaller on left and bigger on right' were not yet secure for these learners.

Enabling tasks

The following tables maps the enabling tasks to each day of the intervention, with reference to the teaching format (whole class, group work or independent seat work).

Table 6: Mapping of enabling tasks to days in the third cycle intervention

ENABLING TASKS	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Enabling task 8: Vocabulary of more than and less than										
Enabling task 9: Line model fluencies										
Enabling task 10: Group model fluencies										
Enabling task 11: Syntax model fluencies										
Enabling task 12: Basic number facts and bridging tens calculations										

ENABLING TASKS	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
Enabling task 13: Word problem fluencies										

Key

Hypothesised trajectory
Whole class
Support group
Core group
Extension group
Independent seat work

Enabling tasks for the core and extension groups

For the core and extension groups, all of the enabling tasks featured in the whole class and small group sessions of the Cycle 3 intervention as planned in the hypothesised learning trajectory.

Enabling tasks for the support group

Most learners in the support group were not yet secure with Enabling task 8: Vocabulary of 'more than' and 'less than' or with Enabling task 11: Line model fluencies. As such the teaching time prioritised developing these fluencies. Based on engagement with the support group, an additional line model enabling task was developed during Cycle 3. This made use of an 'embodied number line' and will be described in detail below. The support group was encouraged to start to use line and group model and syntax models but the emphasis was on a shift from counting all or counting on use group actions. Unlike the core and extension groups, the support group did not work on syntax model fluencies where the difference strategy was compared to the take-away calculation strategy.

Main tasks

In this section I describe how the main tasks unfolded in the third cycle intervention. This done by drawing on extracts from chronological description of the lesson intervention presented in Annexure 4. For each task I report on the task with a particular focus on the teaching as the means to support the intended learning and draw on best case examples of learner activity on the main tasks, as an indication of the potential learning afforded through the task. I use telling cases to illustrate some of the variation in learning experiences. The variation in learning experiences is however described in relation to the case study learners in Chapter 11.

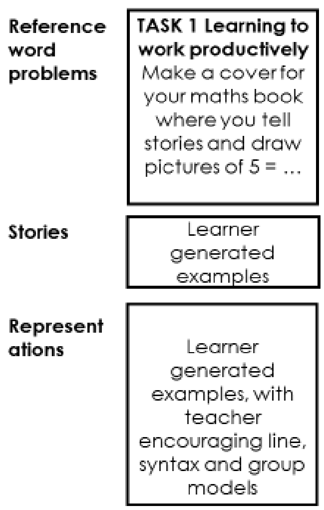
For each main and enabling task I make connections with the relevant features of the theoretical framework. In so doing I draw on orientating theories, mathematical design features of the intervention (domain specific features of the intervention) and frameworks for action, as presented in Chapter 3. I do not refer to constraints imposed on the intervention (also domain specific features of the intervention) as these were maintained in the cycle intervention exactly as planned.

There are some features of the framework for action, which do not figure in the accounts of main and enabling tasks. Where this is the case, I exemplify these features separately to provide evidence of how these featured figured in the third cycle intervention.

Main task 1: Learning to work productively

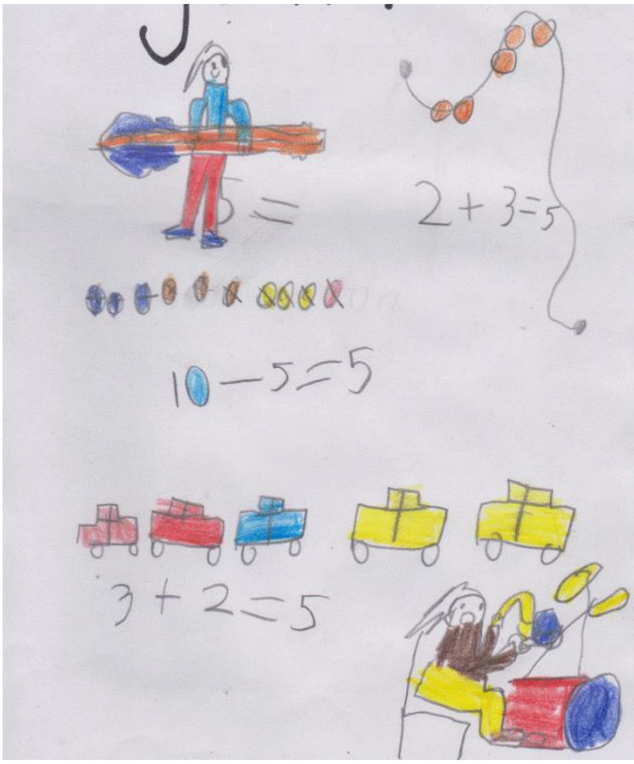
Main task 1 was the focus during the first lesson of the intervention:

Figure 64 Main task 1: Learning to work productively



This task was introduced during a whole class teaching session through teacher lead discussion where children closed their eyes and imagined five in different ways. Learners were encouraged to use numbers, drawings or tell a story about 5s. There was then a bit of discussion on images of five, stories about five as provided by the learners. Some of the learner suggestions were drawn onto the black board. Learners then worked on their book covers for five equals during independent seat work time. This task draw on Feature 3.1 Learning as educating awareness and harnessing natural powers and Feature 3.6 Seeking to gain insight into learners personal potential example spaces, through learners' generating examples. The following is a best case example of learner activity on this task (Feature 3.4 Learning as engaging in 'tasks' where a learning trajectory is inferred from 'learner activity' with these tasks):

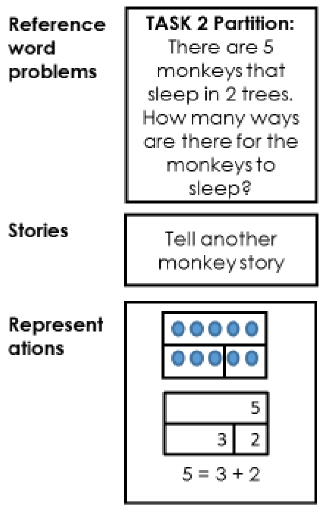
Figure 65: Best case book cover for 5 equals (Main task 1)



This was considered a best case example as it depicted several different elements which had been in focus during the class discussion, and process of imagining five. It showed five things (cars), five beads on a string, five in a number sentences (with both addition and subtraction number sentences included). The number sentence 10 – 5 = 5 was shown visually using an iconic representation of a take-away image. This child also drew a drum kit (where there seemed to be five parts), and depiction of a person holding something (it was not clear what this represented). The child took care in the process and used colour to add interest to their cover.

Main task 2: Partition problems

Figure 66: Main task 2 Partition problems



Main task 2 marks the beginning of the tasks all of which focus on additive relations word problems. These tasks all draw on Feature 7.1 Paradigmatic knowing/logical-scientific knowing and narrative knowing / thinking were viewed as necessarily and simultaneously present in any mathematics word problem. In providing accounts of how these tasks unfolded for the cycle 3 intervention I include accounts of story-telling (distinguishing teacher telling from learner telling), problem solving and use of representations. These map to the learning goals of the intervention and also show the entangled nature of logical-scientific and narrative knowing. The partition problem is described in more detail to provide a rich description of how narrative was used in the intervention. The other tasks are offer more condensed descriptions (additional detail is available in Annexure 4)

The partition problem as introduced in small groups on Day 2 and 3 of the intervention. The problem context of the partition problem was monkeys in trees, and as such it was referred to by the classroom community as the 'monkey' story. The partition problem was revisited on Day 4. On Day Day 6 of the intervention the problem context was changed from monkeys to golden books. I describe the partition problem tasks chronologically as learner activity with this task unfolded over several days. For the core and extension groups the problem was first introduced in small groups and was then revisited during whole class teaching session on several occasions. For the support group the problem was first introduced in a whole class session and only one small group session focused on this task.

Teacher telling of the monkey story (small groups, Day 2&3)

Teacher telling of stories relates to Feature 7.2 Story telling was used as a pedagogic strategy to motivate learning and encourage sense making. The following account from the extension group session describes how the partition story was told. After this account I briefly describe the main differences with core and support groups.

For the extension group, at the outset of the partitions problem I introduced a syntax model representation to introduce the vocabulary of a 'whole' and a 'part' (Feature 6.4 A structural approach to additive relations was foregrounded making use of a whole-part-part-part diagram and Feature 8.5: A few key representations were carefully selected and included line, group and syntax models). From previous intervention cycles I was aware that a common misconception was that the two parts where drawn as equal in size, or that the relative size of the parts was not noticed as being dependent on the numbers involved. I therefore showed learners two examples of generalised whole-part-part diagrams:

Figure 67: Whole-part-part posters for wall display

By varying the relative size of the two parts I hoped to draw attention to the partition between the two parts being dynamic (moving depending on the size of the two parts) (Feature 2.2 Discernment as contrast and Feature 3.5 Varying tasks along prioritised dimension of variation, while keeping the critical features invariant and these should be experienced in rapid succession). I asked learners to repeat the words 'whole part part' back to me. I then provided examples which demonstrated the meaning of the words whole and part. Once I felt learners had some sense the vocabulary of 'whole' and 'part', I then introduced the partitions problem as a story:

> T: We are going to work on lots of stories while I am teaching you. And the story we are going to do today is a story about some monkeys. Monkeys.
> L: Uh uh uh uh uh! [Learner makes a monkey noise and scratches under his arm]
> T: Who can make a monkey noise? I heard one there.
> [Some learners make monkey noises, and T encourages a quiet girl to try making a monkey noise. She declines. A boy bangs his chest and T points out he is making a gorilla noise]
> *(Day 2 Lesson transcript, extension group)*

With the story context of monkeys grounded and enacted by some children, I drew a tall tree on the white board. I draw a shorter tree next to it. From prior lesson intervention cycles I was aware that being able to refer to each tree was useful (as when there were just two trees we struggled to discuss which tree was in focus). I labelled the trees 'tall' and 'short', saying the words and writing them. I continued the story as follows:

> T: So I have two trees in my story….And there is a group. A group of monkeys. And there are five monkeys in the group. There are only five monkeys in the group. And these five monkeys sleep in these two trees. So when it is day time they are running around [monkey noises] running around doing all funny things. But at night time they climb up into the two trees. Now you are a scientist. And you want to know: where are the monkeys sleeping? So each night you come out into the place where the two trees are to come and see. How many monkeys are in this tree…?.. And how many monkeys are in that tree? … And you want to find out how many different ways can the monkeys be in the two trees.

(Day 2 Lesson transcript, extension group)

The teacher telling of the monkey story was much the same for both the core group on day 2, and the support group on day 3.

Solving the monkey story problem I (small groups, Day 2&3)

I continue the description of the extension group session to provide evidence of how learners were supported to solve the monkey story. Descriptions of the problem solving process draw on Feature 1.1: Learners make sense of problems for themselves while conforming to agreed social practices. This includes the descriptions of how representations were used in this process. The similarities and differences with the core and support group sessions are then provided.

I invited the extension group learners to draw their own trees on their white boards and put the five monkeys into their trees. Once all the leaners had positioned their 5 monkeys in the 2 trees and shown their white boards I coached them through an example of how this could be represented using a whole-part-part diagram that was made of paper strips. I made use of the number triple 4-1-5 as this was the example I had drawn on my board. I then instructed learners to draw a whole-part-part diagram for the arrangement of monkeys in their trees. This is a best case example of a child's drawing of the five monkeys in the trees:

Figure 68: Best case example of learner drawing of 5 monkeys in two trees

Notice that children were first introduced to the general case of whole and two parts. They then had to specialise this relationship to an arrangement of their own choosing (Feature 3.3 Facilitating shifts in attention from the particular to the general (generalising) and from the general to the specific).

I wanted to encourage flexible movement between representations (Feature 8.1) to ensure that symbolic whole-part-part diagram was fully understood. I handed out 5-strips to each learner. I modelled a process of tearing the whole 5 into two parts to match my 4-1 arrangement of monkeys drawn into my trees, and my 5-4-1 symbolic whole-part-part diagram. Learners then repeated this for their arrangement of monkeys. I encouraged the learners to find more ways and handed out more green 5-strips as required. Learners worked on tearing the cards and finding new arrangements of monkeys.

Table 7: Partitioning 5-strips to find how many ways

As they worked on finding new ways, I asked learners to check that they had not got repeats (the same way again) and at times pointed out repeats where I saw them.

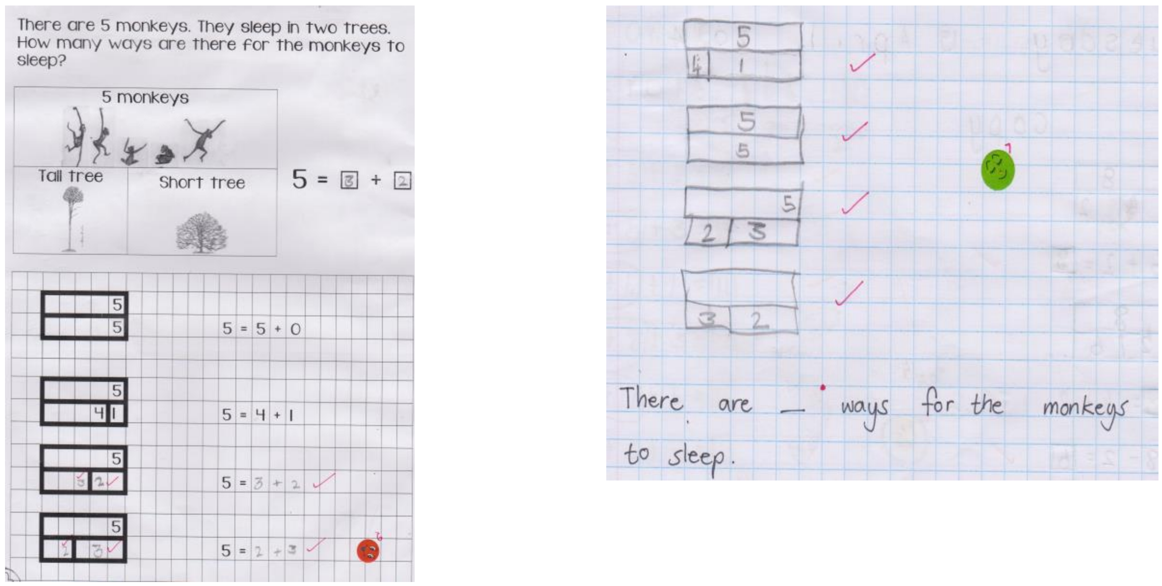
I intervened with a learner when I saw he had torn his strip in two places making three partitions by drawing his focus back to the problem situation (Feature 10.2 Providing specialised and explicit feedback and paying attention to learner errors):

> T: You have got a problem as you made three trees. Are there three trees?
> Learner: No
> T: No I think you better give me this one back [T picks up the three pieces]. That would be three trees we only have two trees. [T gives him another strip to tear once to create two parts].
> (Day 2 Lesson transcript, extension group)

The children continued trying to find new ways and remove repeats. My intension was to encourage them to start working systematically. However noticing that the lesson was nearly over I opted to rather model a process of working systematically and recording the ways using number sentences. I drew a situation where all five monkeys are in the tall tree (iconic representation of 5-0 monkeys), and ask learners what happens if the monkeys jump across to the small tree one at a time. I changed my iconic drawing to 4-1 monkeys in the trees. By the third arrangement of a monkey jumping across to the tall tree, learners volunteered what numbers must be written into the whole-part-part diagram and said the corresponding number sentence, which I then wrote down. We now had a systematic set of six options which was drawn on the white board. I pointed out the pattern of what was happening to the tall tree: 5, 4, 3, 2, 1, 0 and learners chorused the pattern, and then chorused the pattern for the small tree: 0, 1, 2, 3, 4, 5 . I ended the lesson with learners counting how many ways were recorded. They conclude that there are six ways.

I then directed the learners to rearrange their two paper strip parts to make a systematic pattern. Once a learners' work was arranged systematically and they had checked that they had six ways, I gave them a worksheet to record their ways. This denoted a shift from using paper strips – where each 'monkey' was visible thereby forming an indexical whole-part-part diagram, to making use of a symbolic whole-part-part diagram (Feature 8.4 Increasingly structured representations).

Figure 69: Telling case example of 5 monkeys worksheet and book work (Extension group)



The worksheet was designed to model a process of using a whole-part-part diagram and a number sentence to record each option. It encouraged a systematic process, and then shifted the recording of the fifth and sixth option to the learners who were expected to follow a similar recording for the other options but in their workbooks. As the telling case example reveals this transition from worksheet to book work was difficult for most of the children. Most of the children in the extension group only completed what was made available in the structured layout of the worksheet, and did not shift to the workbooks. Notice too, the movements between iconic (drawings of monkeys and trees), indexical (5 strip partitions), symbolic and symbolic syntactical representations.

A similar process was followed with the core group. There were two main differences in comparison to the extension group teaching sequence. Firstly I omitted the task for children to draw one case of the monkeys in the trees. Secondly as a learner started working systematically on her own, her work (and not mine) was used to model a systematic process. With the support group the introduction to the whole-part-part diagram and the story telling was approached in similar way to the extension and core group. However the support learners did not record their work using a worksheet. I modelled a process for the learners to work systematically breaking up the 5-strip cards from the outset. With each card, I directed learners to stick the card into their book and write a number sentence. I modelled this process for the first three cards, and learners copied this process. There was a substantial amount of time spent trying to get this group to focus and stay on task, with many learners fidgeting and disturbing each other. They managed to follow my directions for the first three cards, and by the fourth card they were partitioning the cards, and saying which number sentence to write. Once each learner had completed tearing, sticking in the paper strips, and writing a number sentence, and I had checked their work, I allowed them to return to their seats.

Through my daily reflection on learner activity (Feature 3.4 Learning as engaging in tasks where a learning trajectory is inferred from the 'learner activity' with these tasks), I was aware that the support group required additional intervention. I felt that the group work session with the support group on Day 3 had been difficult as I found there were frequent interruptions and some conflict between particular members of this group. I thought my work on the partitions problem with this group was very procedural and teacher directed, and discussed this with Vanessa. We agreed to split this support group into two smaller sub-groups so that more focused work could be conducted with these learners. Vanessa arranged that I could work with these smaller sub-groups in an available venue when she was busy with reading group work. I also designed additional enabling tasks relating to fluencies which I notices were absent for this group.

Re-telling the monkey story (whole class, Day 3)

The next day (Day 3) I wanted to see if children could recall the partition problem:

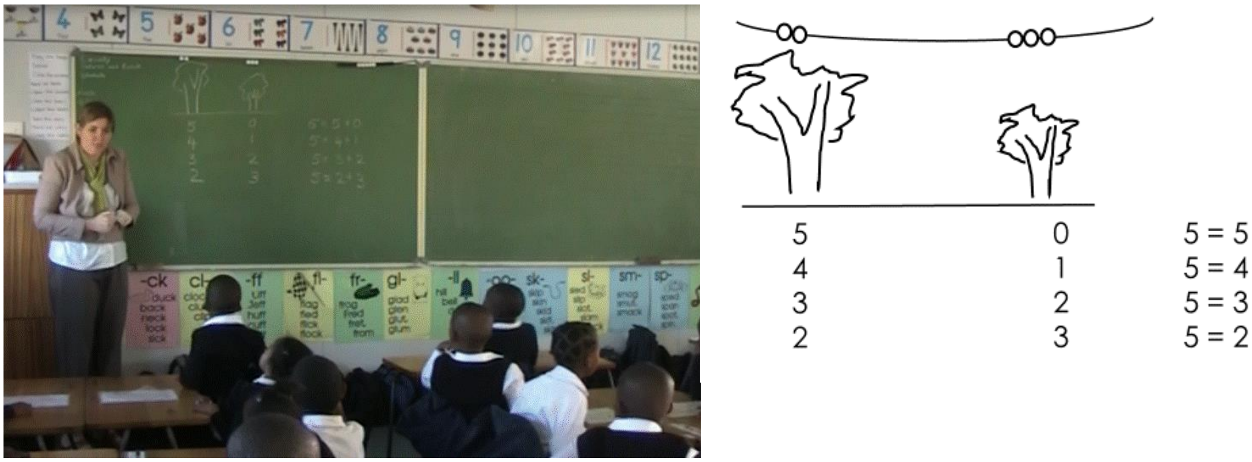
> T: Can someone remind me what that problem was by showing me you want to talk?
> [Core learner raises her hand].
> Learner: What was the problem we worked on?
> Learner: Monkeys sleeping in trees.
> T: How many monkeys were there? Gavril?
> Gavril: 5
> T: There were five monkeys. ..We had two trees. What was different about the two trees or were they exactly the same?
> Ls: No, one was short and one was tall.
> T: Ok so we had a tall tree and we had a short tree.
> [T draws a short and tall tree on the board, just below the bead string].
> (Day 3 lesson transcript, whole class session)

So some of the children who had worked on the problem recalled the problem context and some of its contextual detail. My expectation that children would retell stories previous told by the teacher drew on Feature 7.3 Story telling was viewed as a cognitive strategy which draws on the human powers of imagining and expressing. I then re-told the monkey story (for the benefit of the blue group, and to remind the whole class). This denoted a shift back to Feature 7.2 Storytelling as pedagogic strategy.

Teacher use of representations to solve the monkey problem (whole class, Day 3)

I then modelled a systematic process of having all the monkeys starting in the short tree, and moving one monkey across the tall tree. At each stage I moved one bead from the bead string above the short tree, to be positioned above the tall tree. I wrote and said the number sentence for each one. As this progressed I paused and asked learners who many monkeys were in each tree, and asked the learners to tell me what to write for the number sentence.

Figure 70: Teacher modelling five monkeys in 2 trees



Once all six options were written down, I reminded the class of the need to find out how many ways there were. A learner volunteered that there were six ways and came up to the board to show how he counted the number sentences (touching each number sentence and saying the number).

I then repeated this process of modelling a systematic solution to this problem but this time I asked children to make their hands be the trees, and their fingers the monkeys *(Feature 8.1 Flexible movement between representations)*.

Learners retelling and varying the monkey stories (small groups Day 3&4)

I describe the core group session to exemplify how the monkey story was retold by learners. I then offer the main differences evident from group sessions with extension and support groups.

On the third day during the group work session with the core group took place in a separate venue. I allowed the boys to act out the 5 monkeys' story, using two chairs (Feature 7.4 Tasks that demanded storytelling and modelling in English). We worked systematically, starting with all the boys in one tree and a boy moving one at a time to the other chairs.

Figure 71: Core group boys acting out 5 monkeys' problem (4 and 1 option)

As the boys jumped one at a time from the one tree (chair) to the other, the girls had to provide the number sentence, which I wrote down on the board. At first the children offered the number sentences in the form 'part + part = whole', however I consistently reversed this and wrote 'whole = part + part' and talked about these two options being the same. By the end of story the girls were offering the number sentences in the form 'whole = part + part'.

This process was then repeated for the four girls. The slightly changed story (there were now only 4 monkeys) was retold, and the four girls acted this out again working systematically. The boys offered the number sentences with each jump. They were able to say the number sentences in the form 'whole = part + part'. They concluded that there were five ways.

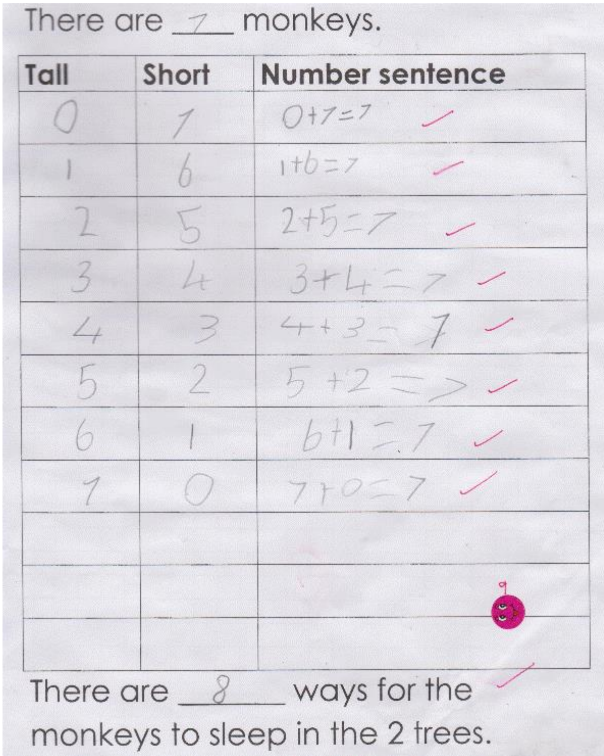
The tasks for the extension group proceeded in the same way as the core group. The support group was not given an opportunity to retell or vary the monkey story.

Learner use of representations to solve the monkey story (Small groups, Day 3&4)

I continue the description of learner activity with the core group session, and again highlight the key differences with the extension and support group sessions.

The core group children then worked on a worksheet where each child was given a different number of monkeys for their story (Feature 9.3 While they had similar tasks they each worked on unique problems, and Feature 3.5 Varying tasks along prioritised dimensions of variation). Two learners were not able to work on their stories without concrete modelling support. For each of them I brought them two white boards and counters to support them to work concretely to directly model the situation using counters (each board was used to depict a tree, and the counters depicted the monkeys). They physically moved the counters systematically from one board to the next. I supported them in this process for the first one or two jumps. They then worked independently but continued to need to move the counters before recording each response.

Figure 72: Best case example of completion of a monkey's problem worksheet



There are 7 monkeys.

Tall	Short	Number sentence
0	7	0+7=7 ✓
1	6	1+6=7 ✓
2	5	2+5=7 ✓
3	4	3+4=7 ✓
4	3	4+3=7 ✓
5	2	5+2=7 ✓
6	1	6+1=7 ✓
7	0	7+0=7 ✓

There are 8 ways for the monkeys to sleep in the 2 trees. ✓

Each learner successfully completed at least two versions of the monkey problem.

The extension group proceeded in much the same way as the core group. Once again two children required direct modelling support. Some children made use a bead string to partition the groups and move the monkeys systematically from one group to another. The support group did not revisit the monkey story problem, and as such the retelling varying and recording of solutions for the partition problem was omitted for this group.

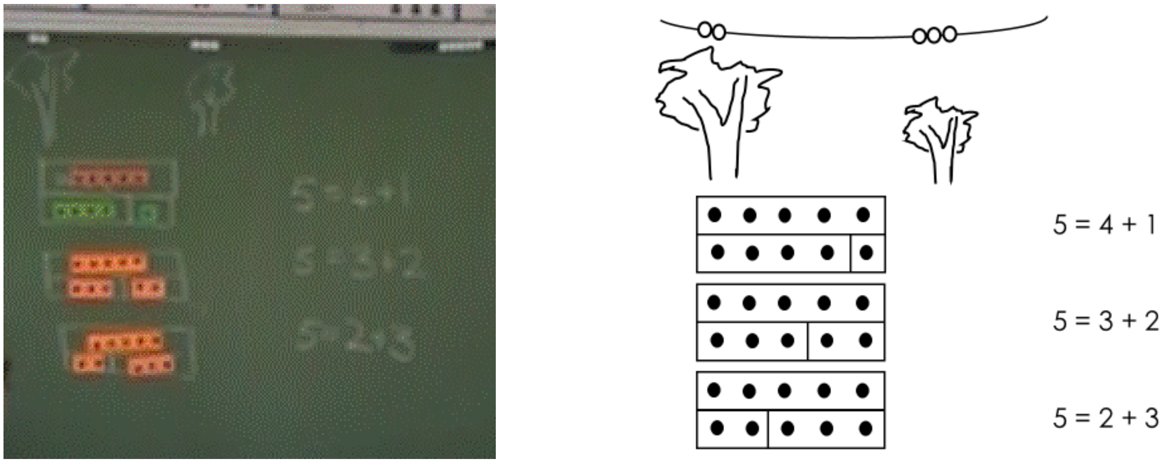
Collectively re-telling and representing the monkey story (whole class Day 4)

The partition problem was revisited in a whole class session on the fourth day. The learners recalled that the story was about monkeys and two trees. I then modelled a process of working with the 5 strips of card to create whole-part-part diagrams and number sentences for each option (Feature 8.1 Flexible movement between representations). I worked systematically starting with 5 monkeys in the tall tree. At each step I asked for learner engagement in the story (Feature 7.4 Tasks that demanded storytelling and modelling in English). The following was a typical interaction:

> T: What's the next part of the story?
> L: One more monkeys jumped into the short tree
> T: One more monkey jumped into the short tree. So now we need the number sentence. Who can give me the number sentence?
> L: 5 equals 2 plus 3
> T: 5 equals 2 plus 3. We need to make that. Can you make that for me? [T hands volunteer learners a 5-strip]. I need a 2 and a 3. [T draws another whole-part-part diagram on the board] So we started off with the whole – all

5 monkeys – and we made two parts. [T takes 2 and 3 parts from learner] Who can help me? Must I put the 2 here? Or which way around must it go? The two must go there?
Ls: No
T: The two must go there?
Ls: Yes
T. Ok the two goes here. We can call it left. Two goes on the left. Think. What's the next step in the story?
(Day 4 lesson transcript, whole class)

Figure 73: Modelling the 5 monkeys' problem making whole part part diagrams from 5-strips



Once I had worked through all the options of the monkey problem, with the interaction from children, telling me the next step in the story, making each partition and offering the number sentence, I returned to the main problem of how many ways were there. Some learners volunteered that there were six ways. A learner came up to board and showed how he knew it was six (he touched and counted each number sentence). As in the previous lesson I point out the systematic working and asked for descriptions of the patterns. The learners then modelled the monkeys in the tall tree with their fingers: '5; 4; 3; 2; 1; 0' and the monkeys in the small tree on their other hand: '0; 1; 2; 3; 4; 5'.

Teacher variation of the monkey story (Whole class, Day 5)

In the opening of the lesson on the fifth day I asked volunteer children to retell the monkey story, and drew attention to the 2 trees and 5 monkeys. I then shifted to a 'new story' about golden books. I wanted a scenario where systematic working was required as part of the story (Feature 3.5 Varying tasks along prioritised dimension of variation, while keeping the critical features invariant, and this should be experienced in rapid succession). I therefore worked with golden books, which I pretended were very heavy and had to be moved from one box to another – one at a time. I narrated and acted out this story using 5 departmental workbooks, and with two boys holding a red and a blue box. After each movement of a book from the red to the blue tub, I asked a learner to make the appropriate partition, by tearing a 5-strip. This was stuck onto the board on the form of an indexical whole-part-part diagram. I selected another learner to provide a number sentence. It was noticeable that by now, learners were offering the number sentences in the form whole = part + part. They were no longer using the form part + part = whole (which had previously been dominant). At the 5-3-2 partition I stopped the process, to ask whether this was really a new story:

> T: Is this story a new story?
> L: No…yes…[various answers called out by learners]
> T: Is it a new story? What's different about this story and the monkey story?
> L: There are golden books but in the other story there are monkeys
> T: OK, so on the one story there are monkeys and trees. And in the other story there are golden books and boxes. But is our picture looking the same?
> Ls: Yes…no… yes it's the same… It's the same…no
> T: It's the same kind of story
> (Day 5 lesson transcript, whole class)

I had deliberately varied the problem context, and kept the numbers invariant. I wanted to draw the learners' attention to the patterns in the systematic partitioning process. I pointed out the numbers and patterns in the golden books story and compared this to the monkey story. Learners agreed that this was the same as the monkey story and I concluded: 'So the things in the story have changed, but the numbers have stayed the same' (Feature 2.2 Discernment as contrast, with learning opportunity creating by using invariance in the midst of change).

Learners re-telling stories and teacher talk about story telling (Whole class, Day 6)

The Day 6 lesson opened with revision and comparison of the three stories that had been introduced to this point in the intervention with the whole class: the partition problem about monkeys in trees; the partition problem about golden books in boxes; and the compare (reach a target) problem about stickers in the learner's books.

Volunteer learners told me about these stories and I wrote monkeys, stickers and golden books onto the blackboard (Feature 7.3 Story telling as a cognitive strategy and Feature 7.4 Tasks that demanded storytelling and modelling in English). I then spent some time explaining the parts of a whole story and drawing a whole-part-part diagram of the components of a story:

Figure 74: Whole-part-part image of stories

Story			
What	Where	Action	Problem

I exemplified the ideas of what (the characters), where (the setting), the action (plot) and some kind of problem (conflict) by telling and acting out short story segments.

I then asked the learners to re-tell the monkey story and used the bead string, and two sketches of trees to model the systematic process. After the second change in the plot (another monkey jumped to the short tree) I asked learners what numbers to put into the whole-part-part diagram. Learner volunteers offered that 5 monkeys were the whole, and that the two parts were 4 and 1. A learner then offered a number sentence: 5 = 4 + 1. This story was fluently retold by volunteer learners, with other learners supporting the recording of number sentences and symbolic whole-part-part diagrams

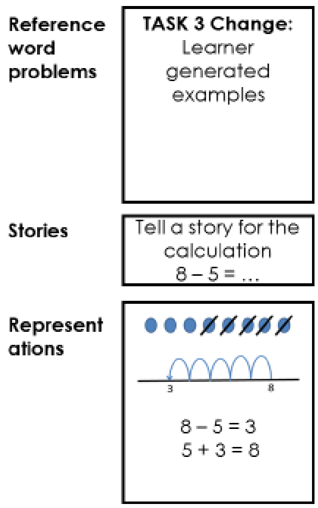
(Feature 6.4 A structural approach to additive relations was foregrounded making use of a whole-part-part diagram and families of equivalent number sentences).

Reflecting back on how Main Task 2 unfolded over several days, it worth considering which dimensions of variation were being constrained (kept invariant), while other were allowed freedom. At first the focus was on varying the representations used to depict the problem situations (Feature 8.1 Flexible movement between representations where sense making was primary). Multiple representations were used to support the telling of the same story, and the same problem solving process: bead strings, 5-strips, number sentences, indexical whole-part-part diagrams, symbolic whole-part-part diagrams, children and chairs, fingers on two hands. Familiarity with the story (the story stayed the same over four days), created familiarity which allowed more and more learners to be secure when participating in its narration. Gradually the representations became more structured (Feature 8.4 A learning-teaching trajectory from counting to calculating which made reference to increasingly structured representations). From the outset, symbolic number sentences were used alongside these indexical representations. The indexical whole-part-part diagram was slowly shifted to be symbolic whole-part-part diagram (Feature 8.2 Secure use of a particular representation takes time, and over time representations should be reified to become cognitive tools).

The story was at first told orally and in detail (Feature 7.2 Story telling was used as pedagogical strategy to motivate learning and encourage sense making). The same story was also retold with different learners narrating different parts of the story, and other learners supporting the recording of each partition (Feature 7.3 Story telling was viewed as pedagogic strategy to motivate learning and encourage sense making). Through this collective and repeated narration the story became part of taken-as-shared reference example. Besides the variation in representations discussed above, the partition problem story was varied in two additional ways (Feature 3.5 Varying tasks along prioritised dimension of variation while keeping the critical features invariant, and this should be experienced in rapid succession). Firstly the core and extension learners retold and worked on telling and solving stories where the number of monkeys was varied. In this process there was no discussion relating to generalising this problem (to observe that if there are n monkeys, then there are $n + 1$ ways), as this was not the task focus during the intervention. Such generalisation was planned for work beyond the intervention. Secondly, the problem situation was varied: from monkeys in trees to heavy golden books, and was noted by learners as being 'the same' as the monkey story.

Main task 3: Change word problems

Figure 75 Main task 3: Change word problems

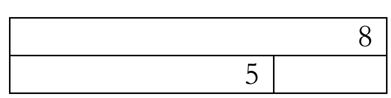


This task was not included in the cycle 3 intervention, for the reasons elaborated on in the previous chapter. Omitting this task meant that early on in the intervention cycle little attention was paid to Feature 6.2 A take away calculation strategy was contrasted to a difference strategy with consideration for the efficacy in the choice of strategy depending on the numbers involved. This feature was however the focus of attention on Day 8 of the intervention, when the core and extension groups were working on line model and syntax model fluencies. I provide a description of the extension group work, which was similar to that of the core group. The support group did not engage with this task.

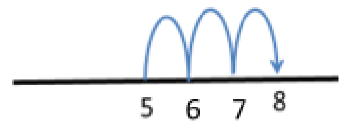
I expected the extension group learners to write down a subtraction number sentence, draw the relevant whole-part-part diagram and show this as a 'count on' process on the number line. This was intended to draw attention the equivalence of the number sentences: 'whole – part = part' and 'part + part = whole' and to connect this with the 'counting up' strategy on a number line. I started with 8 – 5 = … which I wrote on the white board. The learners were quickly able to work with this and identified 8 as the whole, 5 as a part, and 3 as the other part. I asked learners to help me draw a whole-part-part diagram for this number sentence. I sketched the diagram, and volunteer learners told me where to write the 5 and the 8

Learner and teacher talk
T: I want to say 8 minus 5
[T gestures to 8 – 5 = …].
I could also have my part plus my part is equal to eight.
[T gestures to 5 and the blank part and then to the whole 8]
5 plus what gets me to 8?
[no response from learners]
[T sketches empty number line and marks on 5]
T: I start at 5. If I add one 1 get to..

Black board
8 – 5 = …



Ls: 6
[Teacher shows hop of 1 to the right and labels it 6]
T: If I add 1 I go to …
Ls: 7
[T sketches hop to 7]
T: If I add 1 I go to …
Ls: 8
[Teacher sketches hop to 8]
T: So how many have I jumped?
Ls: 8…3…3
T: So I have started with my part, and I have seen: How far must I jump to get to the 8?
Ls: 3



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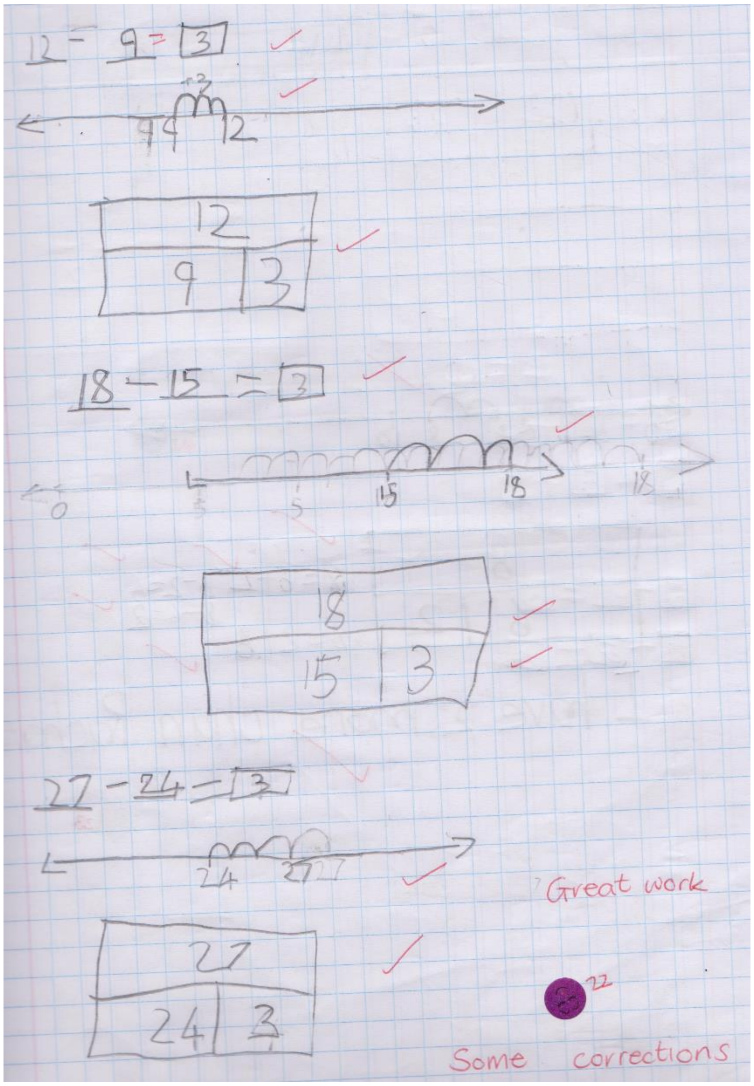
In a similar way I then worked with a harder one of 18 – 14 = …Learners directed me to draw a whole-part-part diagram and show hops on a number line from 14 to reach 18. Learners then worked in their books to start with a subtraction number sentence, draw a whole-part-part diagram and then show this on a number line. I varied the number sentences given to each child, making sure that the numbers lent themselves to a different strategy. The numbers were less than 20, and close together. As learners finished I gave them further number sentences to work with (and each learner worked on their own number sentence). The following is a best case example of this work:

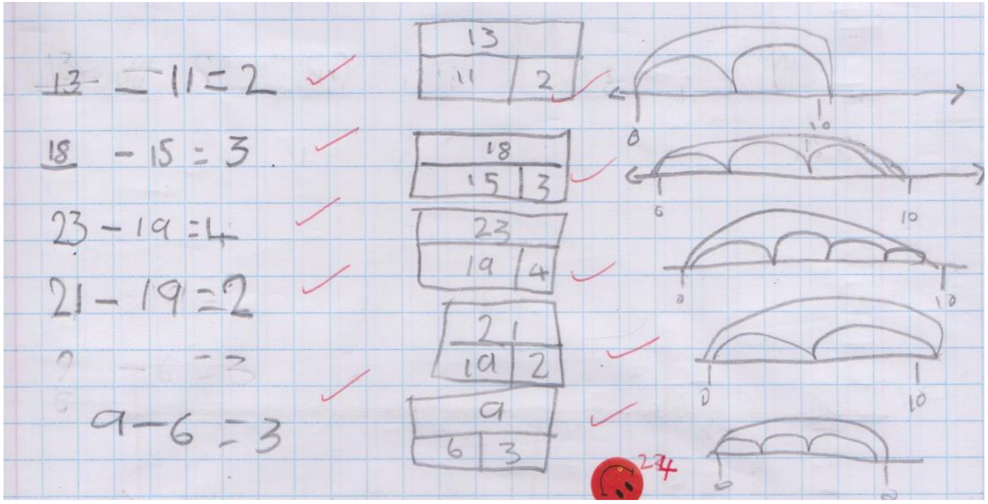
Figure 76 Best case example of learner activity (Episode 8.11)



This learner worked on 12 – 9 = …; 18 – 15 = … and 27 – 24 = … It is clear that for 18 – 15 = … this child first attempted a take-away strategy showing counting back 15 from 18. However, following a teacher prompts to start with the part and count up to the whole, she erased this and depicted the more efficient counting up strategy. She repeated this efficient strategy for 27 – 24 = …

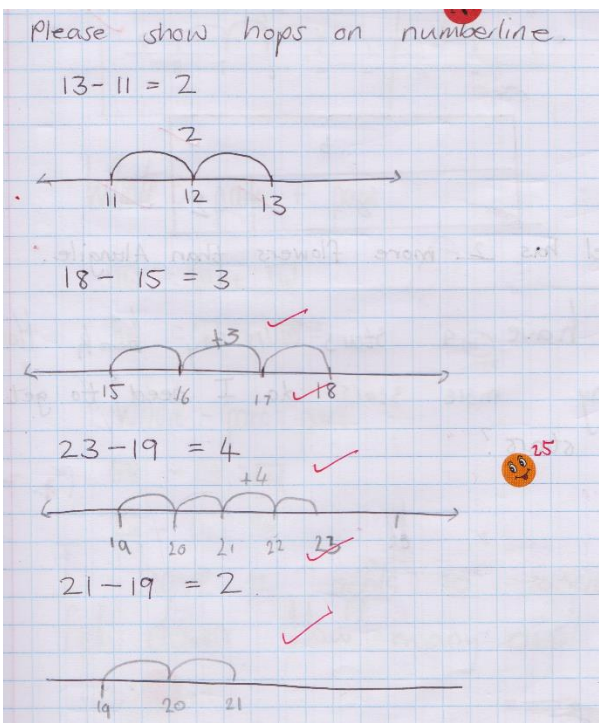
The following is a telling case example of a child who was able to solve the number sentences mentally, and depict a whole-part-part diagram accurately. However she was not yet clear on how to connect this with the number line representation.

Figure 77 Telling case example of learner activity in the extension group (Episode 8.11)



This learner was depicting a 0 – 10 number line and showing the correct number of hops between the two numbers. She showed 2 hops for 13 – 11, and 3 hops for 18 – 15, and 4 hops for 23 – 19. However she did not label the starting and ending points appropriately, using 0 and 10 for all her examples. I supported this learner to notice the significance of the labels on the number line by providing a task in her book which I wrote when I marked her work and noticed this error:

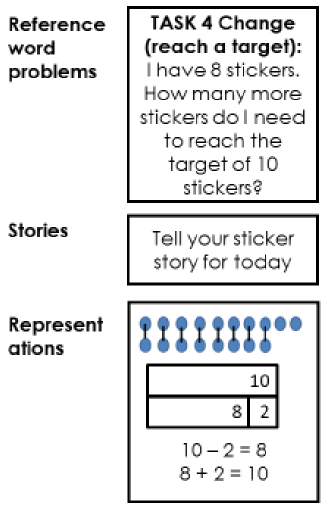
Figure 78 Telling case example of learner activity extension group (Episode 8.11)



I repeated her number sentence examples. For the first case I provided the labels and the hops. In the second case I provided only the labels and she drew in the hops. In the third case I provided only the first label. In the final example I provided only a line, with no labels. She was then able to complete this task appropriately *(Feature 10.2 Providing specialised and explicit feedback and paying attention to learner errors).*

Main task 4: Change (reach a target) problems

Figure 79 Main task 4: Change (reach a target) problems



The problem context of the change (reach a target) problem was the stickers in the learners' books, and as such it was referred to by the class room community as the 'sticker' story. This task was present in both the whole class and the small group sessions from Day 4 to Day 10. It was formally discussed on Days 4, Day 5 and Day 6 and thereafter solving this problem was daily individual occurrence for each learner. This built on *Feature 9.4 Providing immediate extrinsic recognition of effort which accumulated into a reward (and tied into the mathematics).*

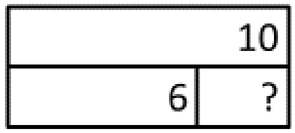
To provide an account of this task I include a description of how the problem was introduced by the teacher during a whole class session, and then select an exemplar episode from a group work session to provide the typical kind of learner activity which resulted from this task.

Teacher telling of the sticker story (whole class, Day 4)

On Day 4 learners received their books with my marking in it from the previous day's work and as usual were counting how many stickers that they had. It was noticeable that despite my numbering their stickers (with number symbols) the learners chose to count all of their stickers from one *(Feature 6.1 A counting based conception of early number development was adopted)*. I introduced the sticker story which was a compare (reach a target) problem type:

> T: I have managed to give out some stickers as I saw some very nice work. We are aiming to get ten [stickers]. Some of you have already got ten. Some of you have got a bit less than ten. We are aiming for ten. [T draws rectangle (whole) with 10 written in it on the board] Our whole that we are aiming for is ten. Maybe you have got 6 [stickers]. And you are aiming for ten. [T draws two parts and labels larger one 6]

Figure 80: Depiction of teacher whole-part-part image drawn on the board



Which part is missing? How many more stickers do you need?
[Some learners' hands go up]
Learner: Four
T: Four [Teacher writes 4 into the other part]. We know with our whole-part-part diagram that that 10 must be the same as 6 plus 4. [Teacher writes number sentence 10 = 6 + 4]. Your whole is 10 that you are aiming for. You might have less than ten. Then you know which part you are aiming for. Over the next couple of days I am sure you are going to reach ten.
(Day 4 transcript, whole class)

The 'sticker story' was introduced by teacher telling *(Feature 7.2 Storytelling as a pedagogic strategy to motivate learning and encourage sense making)* How the individual learners' sticker stories *(Feature 7.3 Story telling as a cognitive strategy)* unfolded during the intervention is evident mostly clearly from the small group interactions. Across these small group sessions it is clear that I made use of different representations to support the process: bead strings (exemplified by an extract from a Support A group session), indexical whole-part-part diagrams (exemplified by an extract from a Core group session); and symbolic whole-part-part (exemplified by an extract from an extension group session).

Teacher use of representations to solve sticker stories (small group, Day 4,5&6)

In this section I aim to show how the teacher use of representations progressed over time as learners engaged with their sticker stories. This is relevant to *Feature 8.1 Flexible movement between representations* and provides some evidence of how, over time, these representations became increasingly structured *(Feature 8.4 A learning teaching trajectory was adopted that made use of increasingly structured representations)*

I start off with presenting three episodes from the support A group, which reflects shifts from concrete to indexical to symbolic models of representation, *(Feature 8.3 While modes of representation and useful categories for reflecting on a child's representations, they do not map neatly to a child's calculation strategy)* and denotes the use of line, group and syntax models for this group *(Feature 8.5 A few key representations were carefully selected and included line, group and syntax models)*. I then provide an extract from an episode with the core group, where the syntax model of the whole-part-part diagram was shifted from an indexical diagram to an indexical diagram including number symbols. I finally offer an episode from the extension group session where learners were expected to work in general terms, making use of a general whole-part-part diagram and a general number sentence, to then specialise this general form to solve their particular sticker problems *(Feature 3.3 Facilitating shifts in attention from the particular to the general (generalising) and from the general to the specific (specialising)*. From these extracts it should be clear that work with each group differed – depending on the learner activity observed. If learners were not secure with counting, then bead strings were introduced. If learners were secure with resultative counting, they were expected to work indexically and then move to symbols. Learners who could tell their own sticker story and solve it mentally, were expected to shift this to a general representation, and then apply this general form to their sticker story (specialise).

Support A group

I draw on a three extracts from the Change (reach target) episodes with the Support A group to exemplify how the shifts from a bead string, to indexical representations of bead strings, to indexical and symbolic whole-part-part syntax models to working with a semi structured number line was facilitated when working on this problem

I opened the Support A group session in Day 4 (Change –reach a target) with learners counting the stickers in their books *(Feature 6.1 A counting-based conception of early number development was adopted)*. It was clear from this interaction that counting in ones was not yet secure for two of these children. Michelle counted her stickers twice to reach 9 in both cases, but when asked how many she had, she said 6. Paul counted his stickers and announced 12, this was recounted and he announced 10 and when this was checked with teacher support it was established that he had 7 stickers. Lydon and Mpho could count their stickers, and were aware of how many they had counted when asked. I worked through the sticker problem slowly for each child. In each case I asked the child to show me their number of stickers on the bead string. I encouraged group-wise counting on from 5. I drew an indexical whole-part-part image of their sticker story additive relation. Learners had to check if I needed to draw more stickers for them. The interaction was slow revealing conceptual difficulties with reliable counting, as this extract from the third sticker story (Paul's story) demonstrates:

> T: How many stickers have you got Paul?
> Learner: I have 12
> T: You have 12 stickers! Paul show me 12 on the bead string
> Mpho: You don't have 12
> …
> Paul: [Counts all again in ones] I have 11.
> T: Let's see.
> Paul: 1,2,3 [counting with no evidence of stickers on a page]
> T: Don't count ones that are not there yet. Let's count: 1,2..
> Paul 1,2,3,4,5,6,7 [turns through each page and touches each sticker]
> T: Seven. Ok so how many do you have Paul?
> Learner: 7

This interaction also revealed difficulties with the ideas of 'more', 'less', 'smaller' and 'bigger':

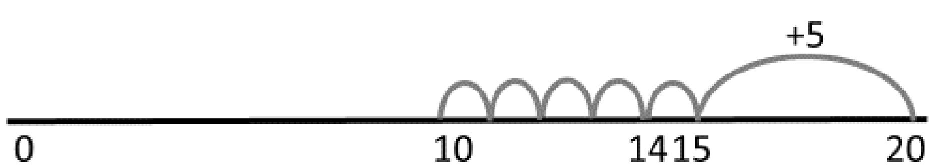
> T: Show me 7 on the bead string
> [Paul moves a group of 5 in one motion, and then moves 2 more on the bead string]
> T: Does Paul have more or less than 5?
> Paul: Less
> T: He has 7. Is 7 bigger than 5 or smaller than 5?
> Learner 2: Smaller than 5
> T: 7 is smaller than 5?
> Mpho: Big
> Paul: It's big
> Mpho : Bigger
> T: It's bigger. .. How many more than 5 has he got?
> Paul: 2 more [shows the 2 red beads on the bead string]
> T: You have got 7. Is that how many Paul has got? He has 5 [points to the blue group of 5 beads], 6, 7 [points to each red bead]
> Ls: Yes

Each learner worked through their sticker story with support and prompts from me. I tried to shift these children to start to 'count on' from 5 *(Feature 6.1 A counting-based conception of early number development was adopted)*. I asked whether they had more or less than 5 and shifted the diagrams I drew of their numbers, from indexical to symbolic representations.

On Day 5 the Support A group again worked on their sticker stories of how many more stickers they needed to reach their target. This time I asked learners to position their number of stickers on a semi-structured number line. I drew the semi-structured number line onto the board, by holding up the bead string and marking the position of 0, 5,10, 15 and 20 onto the board. The learners provided the numbers which I then wrote down. I asked learners to position their number of stickers on this line and tell me whether their number was more or less than 10. Learners then each came up to the board and wrote their number in the correct place. If they had more than 10 stickers, I asked them how many stickers they needed to reach 20. If they had less than 10 stickers I asked them how many more stickers they needed to reach 10. For some learners this resulted in a lot of discussion and needing to refer to the bead string to count all from 1, to find whether they had more or less than 10 stickers. For others they knew whether they had more or less than 10, but then were not sure whether to place their number on the left or right of the ten marked on the number line.

This process was repeated with the Support A group on Day 6. Each learner counted their stickers, positioned their number of stickers on a semi-structured number line on the board (with 0, 10 and 20 marked onto it) calculated how many more stickers they needed to reach 20. This process was now more secure for the majority of learners and they were encouraged to show their actions on the semi-structured number line using hops (in ones) and jumps (of more than one). One of the learners had 14 stickers and drew 4 hops from 10 to reach 14, which he labelled. He then announced he needed 6 more stickers to reach 20, and was prompted to show a big jump of 6 to reach 20. Another learner had 15 stickers. He drew one hop from 14 to 15 and marked on the 15. He then said he needed '5 more' and after teaching prompting to show that as big jump, he sketched the jump from 15 to 20 and was prompted to label it as +5.

Figure 81: Support group number line sketches on the board



Core group

With the core group on Day 5 learners were asked how many stickers they had in their books. The bead string was not used, but I drew an indexical whole-part-part depiction making use of the 5-5 partition from the bead strings. The following provides a typical interaction on a core group learners' sticker story:

T: We are aiming to get a whole group of stickers. A whole group of 5 and another 5 [gestures to 2 groups of 5 circles]. How many are there altogether?
Ls: Ten
T: So a whole group of ten. Let's see how many [stickers] [learner's name] needs. [Learner name] how many did you say you have?
Learner: Six
T: So [learner's name] Ayanda has got six: 1,2,3,4,5 6 [T draws 6 circles below the row of 10, using the 5 and 1 structure].

Figure 82: Core learner's sticker story

That's the whole and here is the part [T gestures to whole 10, and the part which is 6]. How many does [learner name] still need? [T gestures to the missing part]
Ls: Four.
T: If one whole is 10 and she only has 6, she needs 4 more [T then writes symbols 10, 6 and 4 onto the whole part part diagram]

Extension group

With the extension group on Day 5 the introduction to the compare (reach a target) problem for the extension group was adapted as the majority of them could read. I did not first model a process of drawing a whole-part-part diagram. Instead I gave out a two parts of a puzzle to each pair of learners. The puzzle pieces gave a compare (reach a target) story: Part 1: Sihle has 8 stickers, Part 2: How many more stickers does Sihle need to get 10 stickers?. (The characters and numbers used differed for each pair, following *Feature 9.3 Expecting learners to work independently and ensuring that whole they had similar tasks, they each worked on a unique problem)*. Each child in the pair had a part which they had to read and then collaborate to solve the problem *(Feature 1.1 Learners make sense of problems for themselves while conforming to agreed social practices)*.

Learner retelling of sticker stories (Change – reach a target)

Learners retold their sticker stories on a daily basis (Feature 7.3 *Story telling as a cognitive strategy* and *Feature 7.4 Task which demanded storytelling and modelling in English)*. At times this re-telling involved responding to teacher prompts: How many stickers do you have? How many more stickers do you need to reach ten? Over time this was shifted to a more general teacher prompt of 'Tell me your sticker story', where a typical response was in the form: 'I have 17 stickers. I need 3 more to reach 20'. Later in the intervention learners were encouraged to tell their sticker story and pose a question in this story. They could not answer the question they were expected to ask it. Most core and extension learners reached this point by Day 6, as is evident from this extract from the descriptive account of a 6 whole class session:

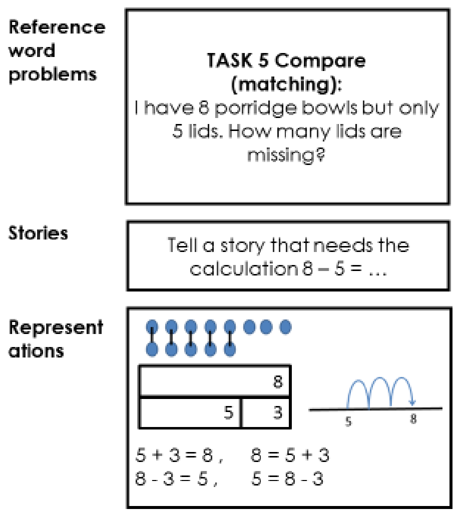
By day 6 learners were able to fluently retell sticker stories. Some learners began to vary critical aspects of the story as this sticker story by a core learner reveals:

> *Core learner: I have 7 stickers. How many more do I need to get to 11 stickers?*
> *(Day 6 lesson transcript, whole class)*

It was of interest that this learner varied the target number in his story (in all the previous telling of the sticker stories, the target was either 10 or 20). Here he chose to vary the target number to 11.

Main task 5: Compare (matching) problems

Figure 83 Main task 5: Compare (matching) problems



The problem context of the compare (matching) problems was plastic porridge bowls and matching lids, as such it was referred to by the class room community as the 'bowls and lids' story. The compare (matching) problems were introduced on Day 6 to all three groups. I provide an extract from the descriptive account of core group, as Main task 5 was introduced in a similar way for all three groups. I then include a short description of how Main task 5 was revisited with the support group on Day 8.

Teacher telling of the bowls and lids story (Day 6 group work)

To ground or enter the new problem I then asked the core learners who had asked for porridge that morning, and talked about and held up one of their plastic porridge bowl with its lids. I wrote a new story on the white board: 'There are ____ bowls but only ___ lids. How many lids are missing?' and said our new story was about bowls [pointing at the word bowls] and lids [pointing at the word lids]. In this case, the teacher story telling *(Feature 7.2 story telling as a pedagogic strategy)* was in the form of a written text.

Learner re-telling of the bowls and lids story (Day 6 group work)

Learners then chant read the story as I pointed to each word, I encouraged them to say ‘mmm’ for the unknown numbers. Once they had read through the story twice I invited learners to give me some numbers for this story. After a few suggestions they agreed on 7 bowls and with the teacher prompt that there must be fewer lids, they chose 6 lids. By allowing the children to choose the numbers, I hoped to make clear that structure of the story could remain the same, but that the numbers could vary *(Feature 3.5 Varying tasks along prioritised dimensions of variation, while keeping the critical features invariant)*.

Teacher use of representations for the bowls and lids story (Day 6 group work)

Learners then guided me through creating a whole-part-part diagram for this problem (Feature 6.4 A structural approach to additive relations was foregrounded making use of a whole-part-part diagram and families of equivalent number sentences). I drew an empty whole-part-part diagram. Learners told me which numbers to write in the whole and each part, placing the 7 bowls as the whole, the 6 lids as a part, and concluding that there was 1 lid missing. I then invited learners to change the numbers in the story, and they chose 10 bowls and 8 lids. I changed the numbers in the story text, and they again guided me to write the 10, 8 and 2 into the whole-part-part diagram. (Feature 3.5 Varying tasks along prioritised dimensions of variation, while keeping the critical features invariant).

I handed each learner a card with similar stories on it (the numbers varied and the context was either bowls and lids, or bowls and spoons) (Feature 9.3 Expecting learners to work independently and ensuring that they all had similar tasks, they each worked on a unique problem).

Learner use of representations for the bowls and lids story (Day 6 group work)

Learners were directed to work on their own problems, first copying out the story then drawing a whole-part-part diagram for their situation. Some learners were able to work directly with the symbolic whole-part-part diagram. Other learners opted to first draw an iconic picture of the number of bowls with their lids.

Figure 84: Telling case

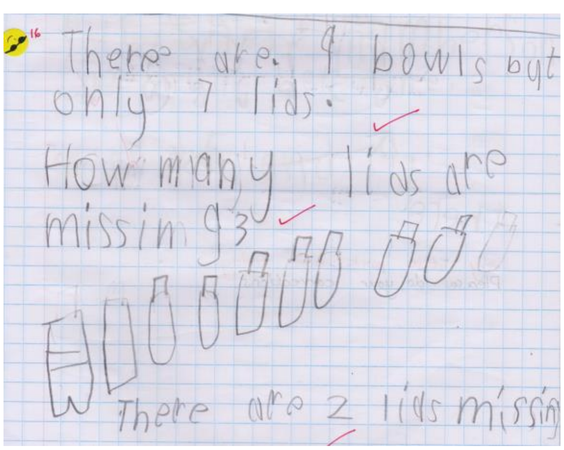
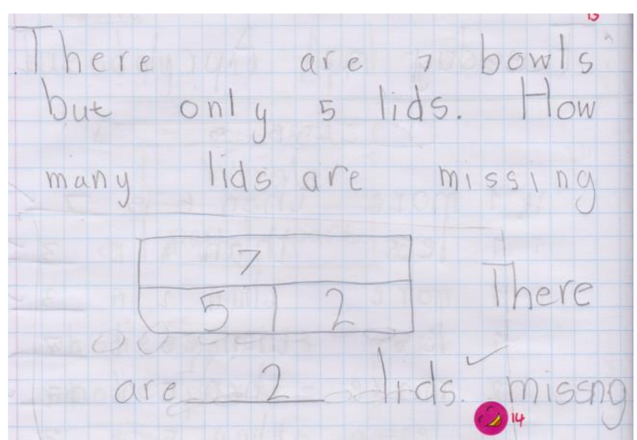


Figure 85 Best case of bowls and lids story in work book



The telling case activity reveals that this learner drew the 7 lids next to, or touching the bowls. He used this to solve the problem and did not draw a whole-part-part diagram or write a number sentence *(Feature 1.1 Learners make sense of problems for themselves while conforming to agreed social practices)*. The best case reveals the learner using the syntax model with a symbolic whole-part-part diagram which is specialised for the numbers in her story (*Feature 8.4 Increasingly structured representations* and *Feature 3.3 facilitating shifts in attention from the general to the specific (specialising))*.

Most learners needed support to remember to write and complete an answer sentence: 'There were ___ lids missing.' As learners completed their problems in their books I checked them *(Feature 10.2 Providing specialised and explicit feedback and paying attention to learner errors)* and when any errors were corrected they then returned to continue independent seat work.

Revisiting the compare (matching) problem (Day 8, support group)

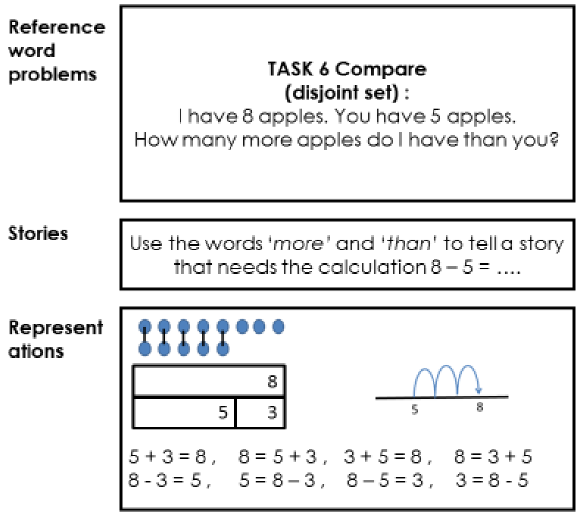
The first group session with the support group proceeded much as the core group description above. In the first attempt at this story support learners needed to support to draw a picture, or make a 1:1 matching action to solve this problem. I therefore repeated this process with a new context, and providing 5 strips, which I thought would help facilitate a 1:1 matching action. I introduced a compare (matching) story for this group using a context of locks and keys. I drew an iconic padlock and key on the white board. I then handed out orange and green five strips:

> T: The green ones are going to be the locks. I am first going to give you the locks
> L: The orange are keys.
> T: You get 5 locks, you get 5 locks, you get 5 locks and you get 5 locks. [Hands out a whole 5 strip to each child]. You get 1 more than 5, you get 3 more than 5…[Hands out various parts of 5 strip]

Each child was given a different number of locks and keys *(Feature 3.5 Varying task along prioritised* dimensions of variation, while keeping the critical features invariant and Feature 9.3 Expecting learners to work independently, and ensuring that while they had similar tasks they each worked on unique problems). I then asked each child 'how many more than five?' they had. I directed them to put their locks in front of them and to turn over the whole 5-strip as 'we know it is a 5' and to write 5 on the back of this strip. This marked a shift in representation from indexical to iconic (Feature 8.4 increasingly structured representations). I repeated this process handing out the orange keys, and posed the question: How many keys are missing? The children then worked through this problem with their varying numbers of locks and keys. I helped individual children to match the groups of five, and then continued with a 1:1 matching. In several cases I encouraged learners to turn over the 5 strip, so they could see and match each lock to each key. They drew a whole-part-part diagram for their problem situation and concluded with an answer sentence: 'There are ___ keys missing'.

Main task 6: Compare (disjoint set) problems

Figure 86 Main task 6: Compare (disjoint set) problems



The problem context of the compare (disjoint set) problems was varied. For the extension group they chose to focus on gold treasure, for the core group the story focused on shells on a beach, in the whole class sessions a story comparing stickers and a story comparing the numbers of children of two people were told. By encouraging learners to choose a story context I hoped to draw attention to the possible variability in problem context, while keeping the story structure invariant (Feature 3.5 Varying tasks along prioritised dimensions of variation, while keeping critical features invariant) Collectively the stories involving disjoint sets were referred to as 'how many more' stories.

The compare (disjoint sets) problem was introduced in small groups for the core and extension groups. This task was omitted for the support group. The task was revisited in whole class sessions on Day 8 and again on Day 9.

I provide a description of the work in small groups on this task with the core group. The extension group work proceeded in a similar fashion. While the core group were told a story about shells on a beach, the extension group chose to have their story about gold pieces (also on a beach).

Teacher telling the 'how many more shells' story (Day 7, core group)

To introduce the compare (disjoint sets) story to the core group I made two of the learners in the group characters in the story. I split the question into two parts: 'Who has more?' and 'How many more?' then I wrote down the questions on the white board. The questions were available to refer back to in written format when the learners were retelling and explaining their stories. My introduction of the compare (disjoint sets) problems was as follows:

> T: I have got a story that is about Rebecca [Learner's name in the group]. So Rebecca is on the beach. Rebecca is looking for shells. She is looking to see if she can find shells. And Rebecca collects 7 shells. Show me 7 shells Rebecca

[T hands Rebecca the bead string].
T: And Sasha [Learner's name in the group] is also on the beach. And Sasha collects some shells. She collects 9 shells. Can you show me the 9 shells?
[T hands Sasha a bead string].
T: So we have got Sasha
[T writes S and white board]
and how many have you got?
Ls (including Sasha): 9
[T writes S = 9]
T: Nine shells. And Rebecca how many have you got?
Ls (including Rebecca) 7
T: Seven
[T writes R = 7]
Is it Ok if I don't write the whole name? I just write S for Sasha and R for Rebecca?
Ls: [nods] mmm/yes
T: And they have collected shells. I have two different questions. My first question is who has more?
[T writes: Who has more?]
Ls: Sasha has more
T: So we are looking for a person's name when we say: 'who has more?'. Now I have another question: How many more does Sasha have?
[T writes: How many more does Sasha have? Ls read this sentence as the T points to each word].

Teacher use of representations to solve the 'how many more shells' problem (Day 7, core group)

I then modelled how the whole-part-part diagram could be used to help to solve this problem (Feature 6.4 A structural approach to additive relations was foregrounded making sue of a whole-part-part diagram and families *of equivalent number sentences)*. I used gestures to depict a big amount, and something that was 'more' shown with hands far apart, and something that was 'less' with hands closer together.

T: We know that Sasha has 9
[T gestures a big amount by raising her arms].
Has Rebecca got more?
[T gestures making her arms further apart]
or less
[T gestures bringing her hands closer together]
than Sasha?
Ls: Rebecca has less
T: So Sasha has 9 and Rebecca's got less
[Gestures with arms for Sasha, and shows below this a smaller section – as for the whole-part-part diagram image]

I drew a whole-part-part diagram on the white board and volunteer learners told me where to write the 9 (as the whole) and where to write the 7 (as the bigger part).

T: Can you write a number sentence?
…
Rachel: 9 equals 7 plus 2
[T writes 9 = 7 + 2]
T: In the problem there was a number I was looking for, which was the number I was looking for?
[No response]
T: In the story I knew that Sasha has 9 and I know that Rebecca has 7. Which number was I looking for?
Ls: The 2
T: I was looking for the 2. So here is my number sentence [T draws box around the 2]

T: So how do we answer these questions? [T gestures to previous writing of the questions] Who has more?
Ls: Sasha
T: Who has more?
Ls: Sasha
T: How many more does Sasha have? [T gestures to this written question]
Ls: Sasha has 2 more

Learners solving 'how many more' problems (Day 7, core group)

Having worked through the problem with me modelling a process, I then gave each pair of learners a similar problem to solve using the learner pairs as the characters. The problem was given verbally. The learners managed to solve their problems, create whole-part-part diagrams to depict the problem situation and a few different options for possible number sentences.

Learners retelling 'how many more' stories (Day 7, core group)

Although all learners had managed to solve the problem in their pairs, the extent of how insecure their meaningful engagement with the problem was revealed in their attempts to re-tell and explain the problems (Feature 7.2 Story telling as a cognitive strategy and Feature 7.4 Task that demanded storytelling and modelling in English). The learner struggled to articulate their stories, showing particular difficulties in posing the questions (the same was true for the extension group learners).

T: The last thing that we are going to do is I want you to tell a story. So I want you to tell me: what was it that you had? My story was on a beach about shells, and Sasha and Rebecca were the characters. I want the two of you to make a story about those numbers. One of you had this much [T gestures big, for the whole] and the other one had that much [T gestures bigger, for the bigger part] and we needed to know what's missing? [T gestures smaller, for the smaller part]. How many more? So I want you to be able to ask: how many more? So I want you to practice a story, and then you are going to ask the question: How many more?

The following two best case examples of pairs recounting their story, reveals that for at least three learners they had managed to find some meaning from the episode:

T: Girls are you ready?
Sasha: One day me and Rachel went to the beach… then Rachel got 9 shells and I got 5. She took [inaudible] and she saw a rabbit, and I took and I saw cards.
T: And what question are you going to ask for that story?
Sasha: How many do Rachel have more than me?
T: Good How many does Rachel have more than me?

Sasha added in some creative elements to their story (which were not completely intelligible) and notably required prompting to pose a question, which she was then able to do fluently. The following is a best case example of from a pair of boys:

T: Sipho and Lee are you ready to tell your story? OK let's listen
Sipho: Once Sipho and me and Liso we go to buy a book.
Lee: Clothes
Sipho: Clothes and … teacher
Less: [inaudible] and he have 6 clothes [pointing at Sipho] and I have 10 clothes.
T: Ok Sipho has 6 clothes and you have 10 clothes. And what's your question?
Sipho: How… How many more Lee have?
T: How many more does Lee have than…

Lee: You [pointing at Sipho]
T: Than you …than Sipho. Try that whole sentence again
Sipho: How many more do Lee have …
T: than
Sipho: Than me
T: Well done

The following telling case example, reveals how for the majority of the learners this episode had not been meaningful for them:

> Andile's story: I have um… I have 5 apples…um 5 apples… And [inaudible] I make 2. Now my mommy call me to go to the.[inaudible] .Now we go. My mommy send me to to to…tooo …the shop … we come with my friend [inaudible]..and then he did go to [inaudible] to … we got that in phase 1 [inaudible]… and come to our house.
> T: Ok and what is your question? We have got to have a problem and a question.
> Andile: umm umm …a question. Mmmm [no further response]

The difficulties experienced by another pair of learners (Rebecca and Retabile) is reported on in the case of Retabile. Aware that most of learners had not benefitted from this episode I shifted to them revisiting and solving the change (reach a target) problem. They counted their stickers and told me whether they had more or less than 10 stickers, and how many more stickers they needed to reach their target of 10 or 20 stickers.

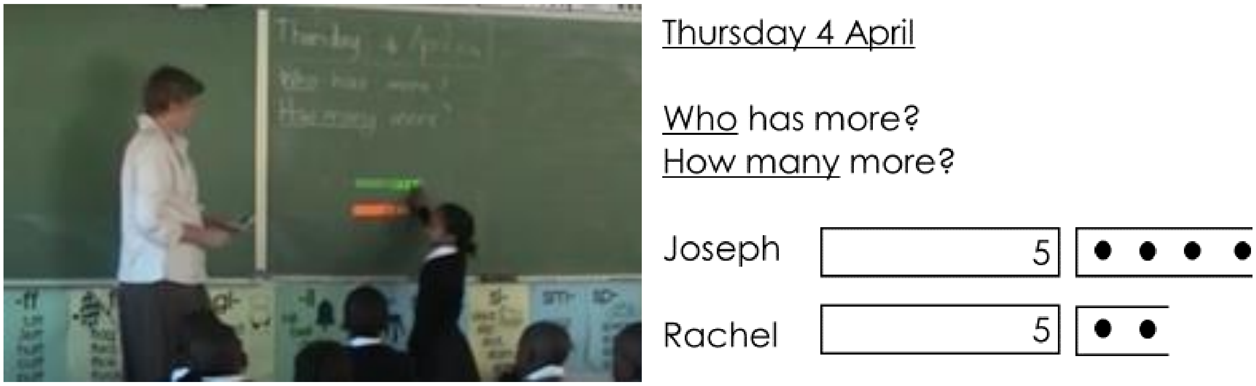
Teacher re-telling a 'how many more stickers' problem (Day 8, whole class)

During the whole class session on Day 8 I invited a learner to choose what our story for this lesson would be about. A learner suggested 'stickers'. I then used two volunteer learners to model a compare (disjoint set) comparison using the context of how many stickers they each had. Two learners (Joseph and Rachel) came to the front of the class and I handed them each five strips, announcing these were their stickers, and asking each of them how many stickers they had.

> T: And these are Joseph's stickers. [T hands Joseph some a 5 strip and then a torn five strip with 4 dots on it] Joseph, can you tell me how many stickers you got?
> Joseph: [counts in ones] Nine
> T: You have got 9 stickers. Hold them up so everyone can see there are nine stickers. And there is somebody else who is in the story. Rachel…she is in the story. Come and see. These are Rachel's stickers [give her five strips in orange]. How many stickers have you got Rachel?

It was established that Joseph has 9 stickers and Rachel has 7 stickers. They held them up for the class to see. I explained that we were asking two questions: 'Who has more?' And 'How many more?' and I wrote these questions on the board. Learners called out that Joseph had more, and I wrote 'Joseph' on the board and directed Joseph to attach his stickers onto the board 'to make a bar graph'. Joseph used prestick to make a row of his nine stickers. I wrote 'Rachel' below the word Joseph and Rachel attached her seven stickers onto the board.

Figure 87: Rachel attaching her sticker to the board



Teacher use of representations for the 'how many more stickers' story (Day 8, whole class)

I referred back to the monkey story where we used five strips, and I reminded learners that we know a whole five-strip has five stickers on it.

> T: So I am not going to count all of them, because I know that there are 5. [T turns whole five strip card over and writes 5 on it] I am just going to write 5 here. You happy with that?
> Ls: Yes
> T: Can I do the same thing here?
> Ls: Yes
> T: We know that there are 5, then we don't have to keep counting in ones. Those are little hops that the Grade 1s use. In Grade 2 we need to be jumping so we are going to have a jump of 5.

I then encouraged a process of counting on from five:

> T: Hands up, who can tell me how many Joseph has?
> L: Four
> T: Has Joseph got 4? He has got 4 here (pointing to 4 visible dots) … but he has got 4 more than 5. Who can tell me how many 4 more than 5 is? Put the 5 in your head [T gestures to touch her head, and show counting on using her fingers] Ok how many has Joseph got?
> L: 9
> T: Well done. So he has got 5. Put the 5 in your head [gestures to touch her head, points at each visible dot]
> Ls: 6,7,8,9
> T: Great

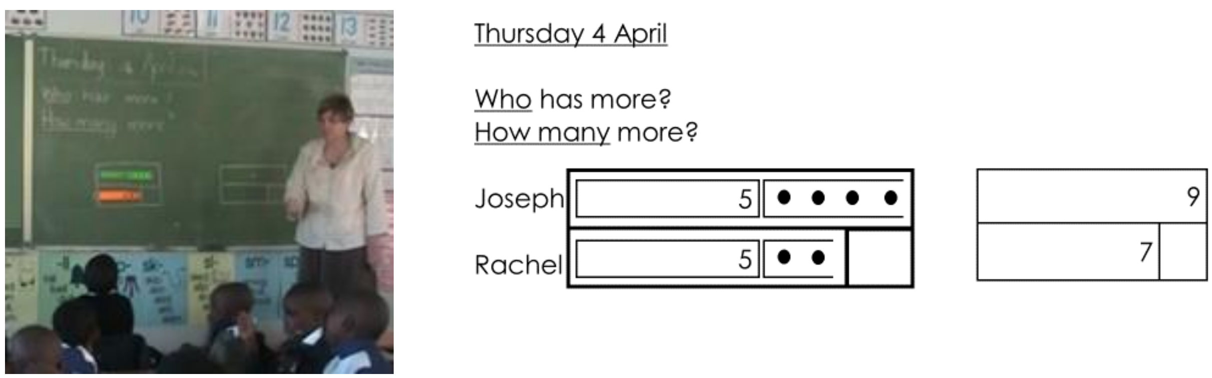
A similar process was repeated for Rachel (counting on from 5 to reach 7). I then returned to the questions and the learners confirmed that Joseph had more. When responding to 'How many more does Joseph have than Rachel?' the difficulties many learners have with this concept were still evident, and they needed to be supported to initiate a 1:1 matching action:

> T: How many more does Joseph have than Rachel? Joseph?
> Joseph: 3
> T: Do you have 3 more? OK let's check. We know Joseph has got 5 [Gestures to Joseph's 5 strip]. Has Rachel got 5? [Gestures to Rachel's 5 strip]

Ls: Yes
T: There is 6 (T points to Joseph's 6th sticker) and there is 6 [Gestures to Rachel's 6th sticker]. There is 7 there is 7. So that's the same up to here. Up to there [gesturing at 7 with a vertical line] they have got the same. We are now looking for how many more has Joseph got than Rachel
Chesnay: 2 more
T: 2 more. Can you come and show me the 2 more than Joseph has? Which are the 2 more that Joseph has?
Chesnay: Those are the 2 more [Chesnay points at the 2 more, pointing to the 8th and 9th stickers in Joseph's row]
T: Great thank you

I then drew a rectangle around Joseph's stickers, and around Rachel's stickers to create a whole-part-part diagram, where the '2 more for Rachel to have the same as Joseph' was left empty. I replicated this w-p-p diagram on the board, and invited 3 learners to come and each fill in the whole and the two parts. Volunteer learners were able to write 9 for the whole, 7 for the bigger part, and 2 for the smaller part:

Figure 88: Learners completing a whole-part-part diagram



I then invited learners to give me number sentences for this whole-part-part diagram and wrote down what learners offered:

L: 9 equals 7 plus 2 [T writes 9 = 7 + 2]
L: 9 equals 2 plus 7 [T writes 9 = 2 + 7]
T: That's my whole is equal to the part plus the part. Can somebody give me minus?
L: Nine minus 7 equals 2 [T writes 9 – 7 = 2]
T: And another one
L: Nine minus 2 equals 7 [T writes 9 – 2 = 7]
T: Fantastic

I then attempted to draw learners' attention back to the questions, and to bring the question (the unknown) into focus within the symbolic syntactical representations. To do so I introduced a symbolic notation of using a box (or a number in a box) to depict an unknown. I drew boxes around all the twos (the unknown in the story):

9 = 7 + [2]
9 = [2] + 7

9 – 7 = [2]
9 – [2] = 7

I modelled finishing a problem with an answer sentence:

> T: What were our questions? What were our questions Denise?
> Denise: [no response]
> T: Gavril what was the question?
> Gavril: How many more does Joseph have than Rachel?
> T: Lovely: How many more does Joseph have than Rachel? Here was Joseph here was Rachel [gestures to w-p-p diagram] And what didn't we know? What number didn't we know?
> Ls: 2
> T: Which number didn't we know?
> Ls: 2
> T: So the two was the number we had to find. We didn't know what this was [covers the 2 in the w-p-p diagram] and we had to try and work it out. So in our number sentence, I am showing that that's the one I didn't know [T draws squares around the 2s in all the number sentences]. That was what we were trying to find. So to finish off our word problem we need to say: Rachel had mmm more stickers [Writes: Rachel had ____ more stickers]

After noticing the error (through the learners expressions, and comments) this closing statement is then changed to 'Joseph had _2__ more stickers than Rachel'.

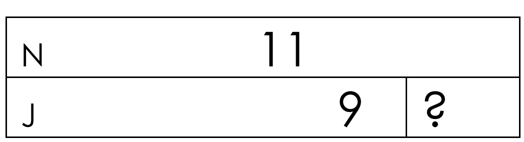
Teacher re-telling a 'how many more' story about children (whole class, Day 9)

I introduced a compare (disjoint set) problem which I referred to as a 'how many more' problem. This this time I told a story involving myself and the teaching assistant (Mrs J) as the characters in the story, and we were comparing how many children we each had. The children contributed that Mrs J had 9 children, and I announced that I had 11 children. The story was: Mrs J has 9 children. Teacher Nicky has 11 children. How many more children does Mrs J have than Teacher Nicky?

Teacher use of representations to solve the 'how many more' children problem (whole class, Day 9)

This problem solved by comparing 9 and 11 on two bead strings through a 1:1 matching process. This was then extended to consider a symbolic whole-part-part diagram for this additive relation.

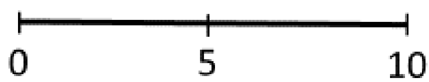
Figure 89: Depiction of Teacher drawing of whole-part-part diagram



Learners offered 11 = 9 + [2] as a possible number sentence for this additive relation. When asked for 'another way to write the number sentence', A learner offered ' 9 minus', and I asked 'Are we going to start with the part, or do we start with the whole?', to which he answered 'whole' and corrected to offer '11 take away 2', which another learners completed to be 'equals 9'. I wrote down '11 – [2] = 9'. A similar process was followed with learners offering '11 minus 9 equals 2' which I wrote down as 11 – 9 = [2]'.

The lesson then shifted to consider this problem on a number line. I built on the work done on embodying the number line, which was included in the group work sessions on Day 8. A volunteer learner was asked to come to the front of the class. With his back to the class he stretched both arms out. The class identified his left hand has 0, his right hand as 10 and his head as 5. I drew this as a number line onto the board.

Figure 90: Depiction of teacher drawing of 0-10 number line on the black board



A second volunteer then outstretched his arms with his left hand touching the right hand of the first learner. Learners counted on in ones from 10 as I gestured to points along the child's arms: 0, 1, 2, 3, 4, [along his left arm] 5, [his head], 6, 7, 8, 9, 10 [along his right arm].

Figure 91: Embodied number lines: gesturing to 18 with class chorusing '18'

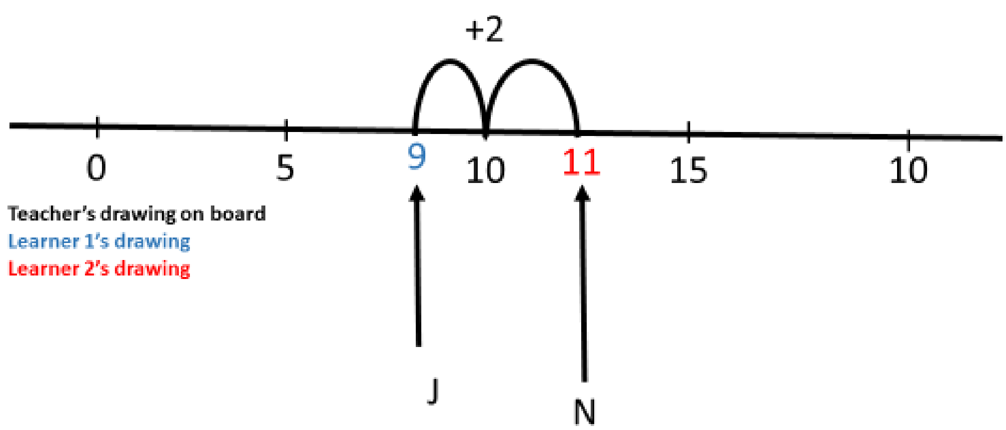
I repeated this process with the second volunteer learner, gesturing to his left hand (10), his right hand (20) and asking the class to identify the number on his head.

> T: We start at 10 [gestures to the second learner's left hand]. We go all the way to 20 [gestures to the second learner's right hand], and what's in the middle? [Gestures to the second learner's head]
> Ls: 15, 15

I then extended the number line sketch on the board to include the points marked 15 and 20. I asked 'I want somebody to show me where is Mrs Js children on the number line.' I then redirected the learners mark the number of children onto a semi-structured number line and to consider the problem posed in the story:

> T: I want to know: How many more does Teacher Nicky have than Mrs J? I need to be able to show that on the line [Erases sketches of jump from 5 to 9, and hop from 10 to 11]. How many more? You can tell me [learner's name]? How many more?
> Learner: 2 more
> T: Mrs J had 9 children, but then 10 and 11 [Sketches two hops - from 9 to 10 and from 10 to 11 - on the number line]

Figure 92: Depiction of semi-structured number line sketch of 2 more than 9 is 11

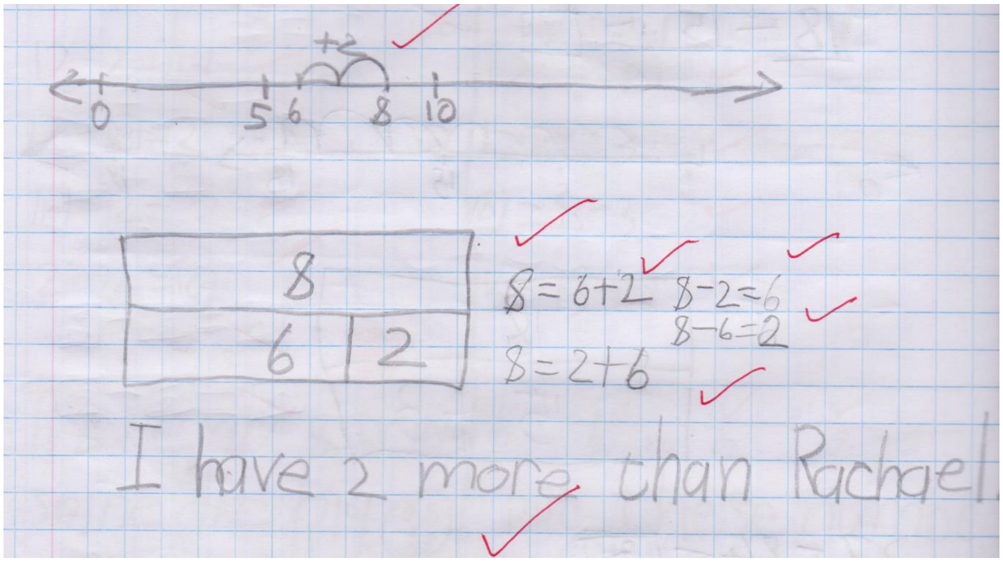


Learners' solving 'how many more' problems (Day 9, whole class)

The learners were then directed to work on their own 'How many more' problem. I had posed individual compare (disjoint set) problems, which I had stuck into their books on the previous day. Each child worked on a unique problem as the names of the children, what was being compared, and the number used varied from problem to problem. The normal teacher, teaching assistant and myself walk around helping individual children with this task. In particular we directed learners who had found a solution to the problem, to explain their work using a number line, whole-part-part diagram and number sentences. We expected learners to write an answer sentence which related their solution to the problem context.

The following is a best case example of a learner's work on this task: 'Kate has 8 pencils. Rachael has 6 pencils. How many more pencils does Kate have than Rachael?'

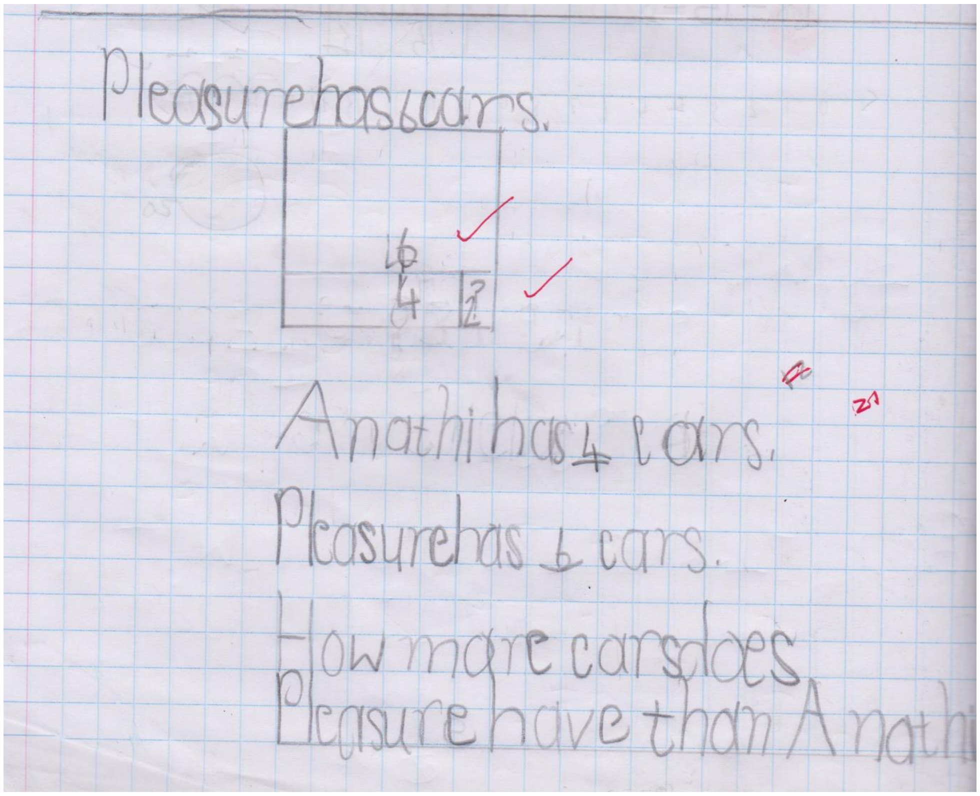
Figure 93: Best case example of learner work on a compare (disjoint set) problem

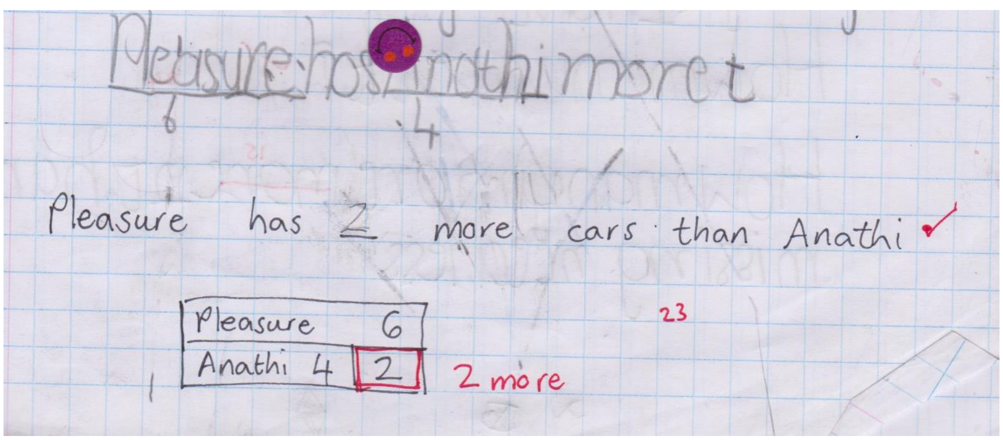


This learner was able to answer the question. She depicted her work on an empty number line, using 0 and 10 as reference points. She depicted this using a whole-part-part diagram, and wrote four equivalent number sentences for this problem situation. Finally she completed her work by writing an answer sentence.

This telling case example reveals that not all the learners were able to make use of as many representations to depict their working on the compare (disjoint set) problem:

Figure 94: Telling case example of learner work on a compare (disjoint set) problem

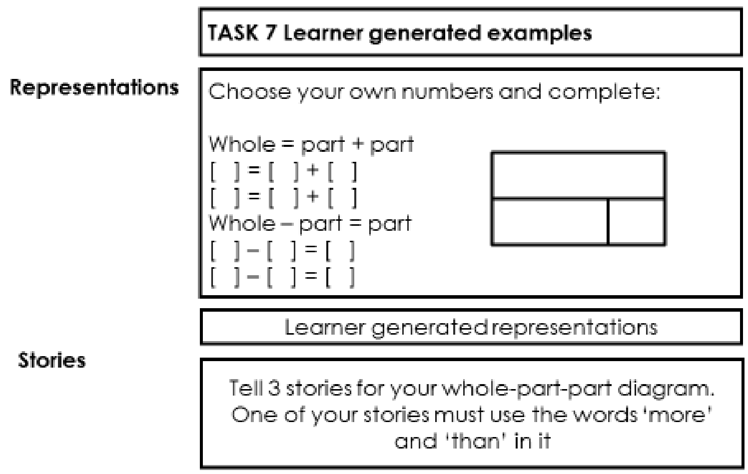




This learner was able to solve the problem using a whole-part-part diagram, in which he wrote the correct solution ‘2’. However he was not able to depict this on the number line, using number sentences or write an answer sentence. His attempt at an answer sentence reveals his difficulty with the comparative language: ‘Paul has Anathi more t’. I responded to this learner by writing the answer sentence for the learner, where he was then expected to write in the answer.

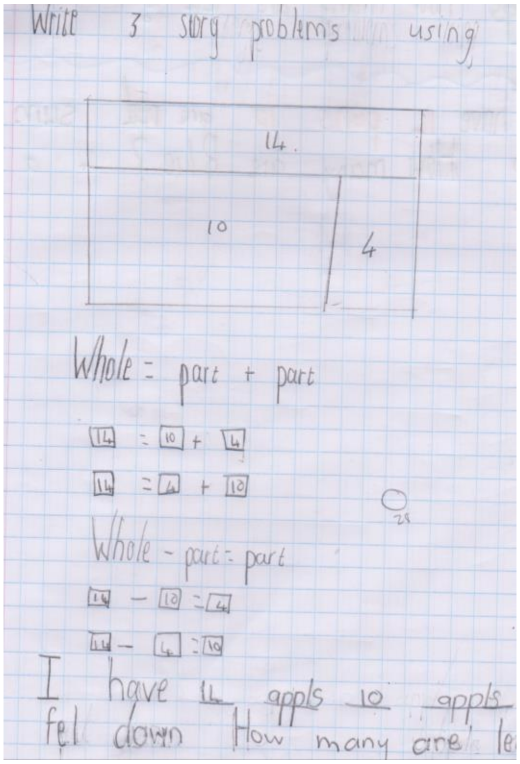
Main task 7: Learners' generating examples

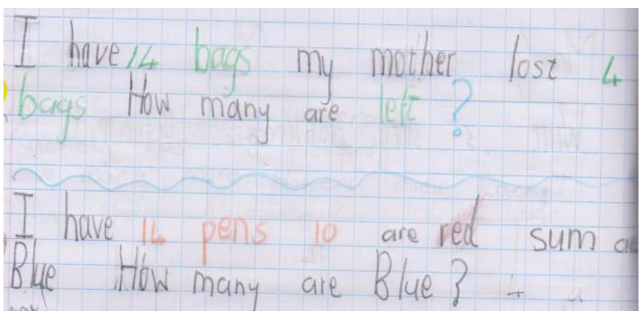
Figure 95 Main task 7: Learner generated examples



The main focus of Day 10 of the intervention was on learner writing and illustrating additive relation stories. Main task 6 was presented using a generic whole-part-part diagram, where they had to specialise this diagram by introducing numbers of their choosing into the diagram. The learners had to write the family of number sentences for this image. With this in place, they then wrote three different story problems for the situation depicted by their whole-part-part diagram and families of number sentences. I encouraged learners to make use of the word 'more' or the phrase 'more than' in their stories.

Figure 96 Best case example for writing stories for 14-10-4

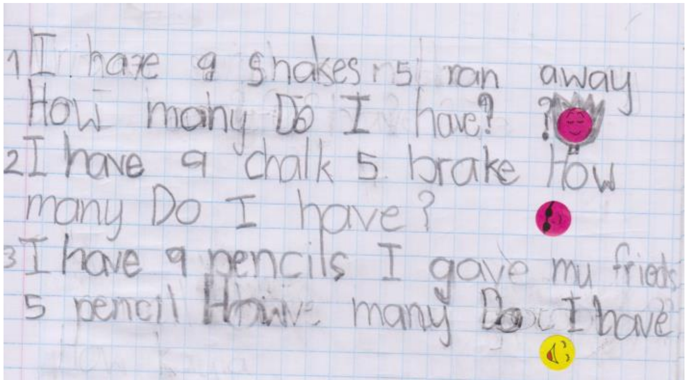




This learner correctly drew a whole-part-part diagram using the numbers 14 and 10 (which I provided to her). She then correctly found the difference between 14 and 10, writing in 4. She copied down the generic statements of 'whole = part + part' and 'whole – part = part' from the black board. She correctly specialised these number sentences for her number triple of 14, 10 and 4. When writing stories to depict this situation she first offered two change decrease stories: one involving herself as a character and apples falling down; and her then her mother losing bags. As her final story she offered a collection problem type, involving red and blue pens. None of her stories made use of the term 'more' or the phrase 'more than'.

The following is telling case of learner who was able to complete the whole-part-part diagram and related number sentences for 9-5-4. However all three of his stories were of the change problem type:

Figure 97: Telling case of writing stories for 9-5-4



This learner kept the numbers invariant, always using 9 and 5 (which was the task instruction). His story structure was always one of change decrease. He was always the character in all three stories, but the objects and actions varied. Firstly he referred to snakes running away, then he chose chalk breaking, and then giving pencils to a friend. He was not able to vary the problem type within three examples.

Enabling tasks

As planned, learners worked on enabling tasks as the introduction to whole class and group work sessions and learners also worked independently on cards designed to provide practice one this task (Feature 10.3 Facilitating opportunities for learners to practice and receive feedback on prerequisite fluencies).The enabling tasks figure as background activities that were not central learning goals for the intervention. As such, the descriptive accounts of how these enabling tasks were approached during the cycle 3 intervention, is provided in Annexure 11.

To illustrate the enabling tasks, I report only on the inclusion of new line model enabling task where children made use of their bodies. This was a task that was developed as contingency response to the learner activity in the support group. I refer to this as embodied number line task.

Embodied number line task

On Day 8 of the intervention, I repeated '2 more/less than' oral activities using a structured number line for Support B. However for this group, the counting on and back in twos using comparative language was still not secure. I therefore switched to '1 more/less than' and used their bodies as a number lines. I directed a child to hold out his arms with his back to the group, I encouraged the children to imagine 0 in the child's left hand and 10 in their right hand. When I asked what number would be on his head, the group established that 5 was in the middle. I modelled a process of moving along the child's arms from left to right saying: '1 more than 0 is 1', '1 more than 1 is 2', etc. The children joined in. The action of moving our hands in an arc (as we depicted hops on a number line) and landing further along the child's outstretched arms was used. Then in pairs (one child being the number line, the other doing the count and moving along the number line) counted '1 more than ___ is ___' moving right, and '1 less than ___ is ___' moving left. This task was repeated with the other support group and revisited in small groups. It was intended to support learners with three interrelated ideas: the arrangement of numbers in a line model with smaller on the left and bigger on the right, the direction of movement along this line for making bigger or 'more than', and for making smaller 'less than', and the using the vocabulary of 'more than' and 'less then' in a meaningful context.

On Day 9 during the mental maths starter two learners were invited to the front of the class to become parts of an embodied number line. They outstretched their arms, and their left hand was 0, their head was 5, and their right hand was 10. This semi-structured empty number line (with markings in multiple of five) was used to depict the compare (disjoint set story). In this case the difference between two numbers, as the hops from one number to the next, was depicted. This whole class activity is reported on as part of the account on Main task 6 Compare (disjoint set).

Frameworks for action

There are some features of the theoretical framework which arise as a result of the broader view of variation theory (Feature 2.1 Contrasting a classic view on variation theory to a broader view on variation, this study is located in the broader view). These result from relaxing the constraint on learning in relation to mathematics, to consider learning more broadly (Feature 3.2 Learning as transforming the human psyche (awareness, emotion and behaviour)).

Most of the features of relating to Feature 9 Training behaviour and harnessing emptions and Feature 10 Principles of general pedagogic style could be exemplified in relation to particular tasks in the cycle 3 intervention. However Feature 9.2 Encouraging a growing brains mind-set and Feature 10.1 Adopting and mathematical thinking questioning style, listening to and exploring suggestions from learners, and making use of wait time were not described in relation to a particular task.

Managing behaviour and encouraging a growing brains mindset

I opened the first lesson with some discussion about how children can grow their brains through working hard. I repeated this idea throughout the intervention encouraging the learners to think hard in order to exercise and so grow their brains. Time was spent on discussing the classroom rules and expectations in terms of learner behaviour. Consequences for inappropriate behaviour were agreed ('staying in at break time or after school to write lines' was suggested by learners and then agreed to) and recognition of appropriate and good behaviour (collecting stickers). This was followed by some discussion on how to work independently: what to do if learners needed stationery, or could not do the card, or needed help. The children were coached as to where to collect stationery to borrow and what to do if they were stuck: copy out the card, choose another card. The children were coached to put their completed cards into the plastic tub for their group and take another card from this tub. This was intended to give the children some independence and to keep them on task during seatwork.

The purpose of the mathematics lesson on the second day was to further encourage a classroom routine of working independently during seatwork and reflecting constructively on feedback obtained from my teacher marking of individual learner work. As such emphasis was placed on celebrating errors as opportunities for learning, and which facilitate a growing brain. On this day (as for the first day) my primary focus was on establishing a productive learning environment for mathematics.

There was an episode in the whole class teaching in the second lesson which was designed to motivate learners and encourage them to view mistakes as opportunities for learning. Having returned the learners books marked, I asked children to raise their hands if they had any mistakes in their books. At first no one raised their hands, and then two children indicated that they had some mistakes. I congratulated these children and gave them each a gift wrapped eraser. More children then raised their hands and went around the class verify that they had mistakes that could be fixed and giving them erasers. I then asked if there were any children who did not have any mistakes. Gavril and Cassidy raised their hands. I apologised to them:

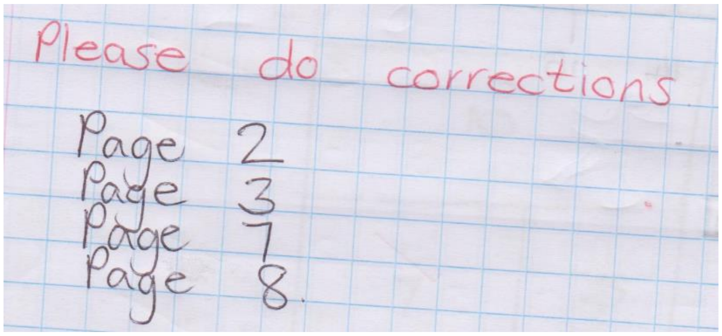
> T: Mistakes are fantastic. Mistakes mean we have something to learn. Is there anybody who had no mistakes?
> Ls: Two learners raise their hands.
> T: I am sorry those who had no mistakes. You know what: that's my fault. I gave you work that was too easy. You already knew how to do that. If you already knew how to do it, you didn't grow your brain. And that's my fault because I gave you work that was too easy. I am sure that there will be harder work where you will have a few mistakes. So I am going to make sure that everybody gets an eraser.
> *(Day 2 video transcript)*

I then gave each learner an eraser and encouraged them to find their mistakes and fix them. I explained that they could find mistakes by looking for a dot, which was waiting to become a tick when I next looked at their books. I emphasised that we would be doing corrections in every maths lesson.

Learner corrections was a recurrent part of the seatwork sessions, and stickers were withheld until corrects had been completed (Feature 9.4 Providing immediate extrinsic reward for effort). I provided specific feedback on all seatwork tasks by marking correct response with a tick and errors with the dot (that was waiting to become a tick, after being corrected). This connects with Feature 10.1 Providing specialised and explicit and paying attention to learner errors. A short extract from the account of lesson 9, reveals the way in which I communicated to learners in their books, but also kept my own notes of individuals requiring support on correcting errors.

In lesson 9 individual seat work learners completed corrections, and worked on their cards. In my journal I also noted down which learners required teacher support to review their corrections, and listed where they needed to focus in their books

Figure 98: Teacher instruction to review corrections (written in learner book)



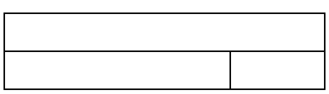
During this episode I went to each learner whom I had noted down in my journal to ensure they focused on correcting work which they had not previously reviewed.

Seven month after the intervention, when I asked learners what they remembered about the intervention several learners commented that 'they grew their brains', others referred to 'you made us fix our mistakes' and 'we got erasers'. In the cards that learner wrote to me to thank me for the intervention, there was also mention of 'growing brains'.

Mathematical thinking questioning style

I have chosen a short extract (from minute 3 to minute 8) in lesson 9. This has been selected as illustrative of the way in which I conducted teacher lead whole class discussions. I use this to exemplify my enactment of some of the intended teacher roles. This exemplar episode is the beginning of the ninth lesson with the whole class. I asked a volunteer learner to draw a big whole-part-part diagram on the board:

Figure 99: Depiction of learner's drawing of a whole-part-part diagram



When I asked for "a number sentence number sentence that shows how the whole and the part and the part fit together?" A learner offered a particular number: '7'. In this case I rejected the learner's suggestion saying "I don't want numbers, I wanted the whole, part and part". I rejected an offer of the particular, and requested a general articulation. I consider this to have been a missed opportunity which was in conflict with the hoped for theoretical *Feature 10.2 Listening to and exploring suggestions from learners*. I could have listened to the learner's suggestion of the particular number, and shifted to the general from his suggestion (however I did not notice this opportunity in the teaching moment). Misunderstanding my request for the number sentence, the learner directed me to label the whole-part-part diagram.

Figure 100: Labelling the whole-part-part diagram

Whole	
Part	Part

I then again asked for a volunteer to "give me a number sentence of how the whole, the part and the part fit together". Another learner offered a general articulation: "whole equals part plus part", which I wrote down on the blackboard as "whole = part + part".

I asked the class for other ways to write the number sentence (whole = part + part):

Learner and teacher talk	**Black board**
T: Lovely…It shows me how they fit together in the picture. The whole is equal to the part plus a part. I need another one.	Whole Part | Part
L: I know. I know.	
T: Without calling out, so try again [learner name]. Without calling out have you got another way to write that? What is it [learner name]?	whole = part + part
L: Part	
T: Do I start with the part, or do I start with the whole?	
[T gestures to the part and then to the whole in the whole-part-part diagram]	
Ls: Whole	
L: Whole	
T: Whole, [T writes: whole] and then what happens?	
L: Minus [T writes: -]	whole - part = part
T: Lovely	
L: A part [T writes: part] equals a part	
T: A whole minus a part equals a part. [T writes: = part]	

This provides an example of how I involved learners in the whole class discussion, by calling on volunteer learners to either write on the board or show parts of a process such as drawing the w-p-p diagram or asking volunteer learners to offer the numbers we work with. Although I was doing most of the drawing and writing on the board, the learners were involved in how these representations came to be on display.

There was then another learner interjection immediate following the above, which interrupted the flow of the teacher lead whole class discussion. This time I was able to listen to the learner and take his suggestion seriously, offering it up to the class for consideration.

Learner and teacher talk	**Black board**
[A learner has his hand up]	
T: What do you want to say? Is there another one?	Whole Part | Part
[L puts his hand on his head and looks at T]	
T: OK?	
L: A whole plus a [inaudible]…a part	**whole + part = part**
T: Ok let's try this one. A whole plus [T writes: whole +]	
L: A part	whole = part + part
T: Ja	
L: Equals a part	whole - part = part
T: Equals a part [T writes: = part]. Ok lets test this one. This is a new	

one. I haven't seen this one before. Shall we test it with some numbers?
Ls: Yes

To review the offer of 'whole + part = part' from the learner, I asked another learner to give me 'a whole' using the bead string. In this way I shifted from the general to the particular (Feature 3.3 Facilitating shifts in attention from the general to the specific (specialising). However by offering the choice of which whole (or which number to specify) to a volunteer learner to pick, I hoped to at least hint at the randomness of this choice.

Holding up the string of 10 beads for the class to see, the learners identified the ten beads as 10, which I wrote into the whole of the w-p-p diagram. I took the ten beads and split it into two parts (six and four). The learners identified and chorused the two parts as six and four. I wrote '6' and '4' into the parts of the whole-part-part diagram. I used this opportunity to treat all three of the possible statements involving 10, 6 and 4 as conjectures which required testing.

I guided a process of testing the 'whole = part + part' number sentence. Each time I ask learners to tell me what was the whole, what was the part, and which operational symbols to use, until I had written 10 = 6 + 4. I did not accept this as necessarily true and again I offered this up for review to the class.

Learner and teacher talk	**Black board**
T Ok so we are going have 6 is the bigger one [T writes 6 in the bigger part] Ls: six T: and ? Ls: and four T: and 4 is little one [T writes 4 into the smaller part] Ls: Six, four T: Ok let's test these. We are going to test these ones that we have done. [T gestures to the whole in 'whole = part + part'] Ls: Whole T: The whole is 10 [T writes 10 =] is equal to…? Ls: part T: the part which is ..?[T writes 6] Ls: 6 T: 6 [T writes plus] plus L: plus four T: Four. Is that true? Ls: Yes T: Are we sure? I have got six [T shows 6 beads on the string] , and I have got 4 [shows 4 beads, then joins the two groups] Does it give me ten? Ls: Yes T: So this one we are happy with? [T puts ticks next to 'whole = part + part' and '10 = 6 + 4']. That works.	W hole 10 / Part 6 / Part 4 whole + part = part whole = part + part ✓ 10 = 6 + 4 ✓ whole - part = part

This extract shows a questioning style where statements made are presented for review and judged to be true or false.

Several different learners involved and contributing to the whole class discussion. There were various points at which learners called out and chorused answers (frequently with some contesting responses being corrected by the group). I do not claim that all the learners in class were actively engaged in this discussion (there was clear evidence of a few learners being distracted, or off task in the video and the normal classroom teacher removing the distraction from them). However this extract reveals that in general, the whole class discussion included adopting a mathematical thinking questioning style, listening and exploring suggestions from learners.

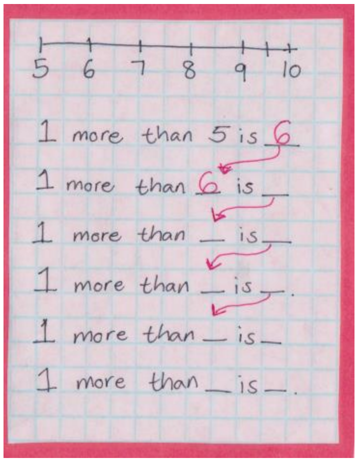
Annexure 5: Independent work cards

Each learner worked through the cards individually, and each card was distinct. This was a deliberate design decision to minimise learner copying when working on identical tasks. I included various kinds of cards designed for different levels of difficulty. If, when marking a child's book, I noticed errors, I noted down the learner's name in my journal and aimed to spend a few minutes with them on the particular card.

Depending on learners' work in their books I allocated cards to them as I felt appropriate for their next step. The cards were deliberately designed to include a variety of formats (different coloured cards, distinct types of paper, some handwritten, some typed). The formats were varied so that children could recognise and easily find cards of a similar type. If they wanted to do a card 'like' the one they just completed, this was easily achieved. For each card type I had developed approximately 10 variants, where the numbers were changed, and where the characters and problem situations were varied for word problems.

Enabling task 7: Vocabulary of more than and less than

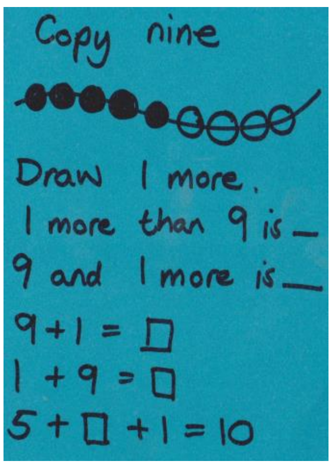
Figure 101: Easy card task – Number line with 1 more than pattern



The card was intended to consolidate the phrase '1 more than'.

It made use of a structured number line starting at 5. Mental maths activities were designed for this phrase to be vocalised by each learner. This card was intended to consolidate that oral work, in written form.

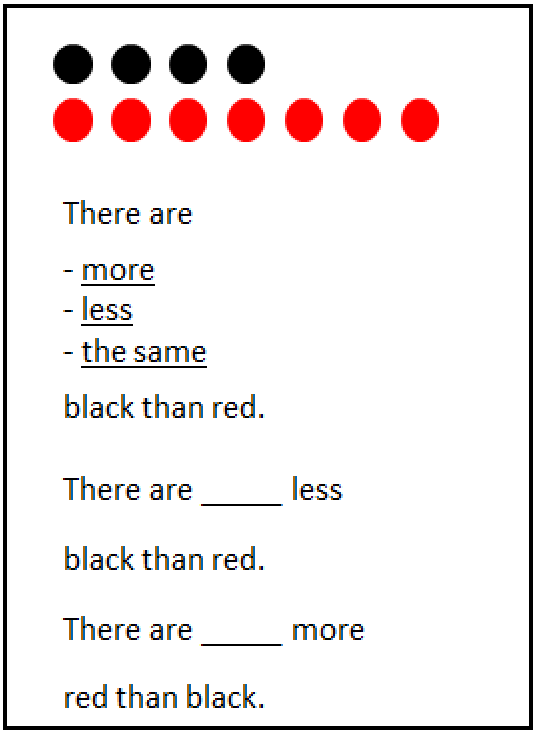
Figure 102: Medium card task – coping a number of beads and drawing 1 more, number sentences



This card was designed to reinforce work done on the bead strings. It imposed a 5-wise structure. It made use of the term 'more' as an action 'draw 1 more'.

The related numbers sentences all made use of addition and the bead string structure. Equivalent English sentences including the phrase 'more than' were included.

Figure 103: Difficult card task – matching comparison



This card was designed to support comparison in a matching context. It made use of the terms 'more', 'less' and 'the same'.
The last two sentences aimed to draw attention to the idea of: 'how many more' and 'how many less', in English sentences, with the related change in word order.

Enabling task 8: Group model fluencies

Figure 104: Medium card task – draw indexical representations of numbers enclosing groups of 10

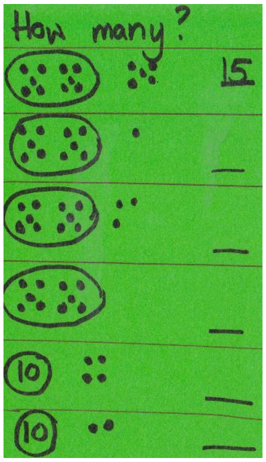
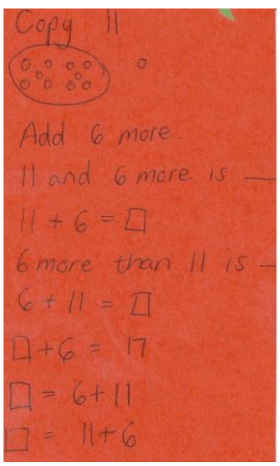


Figure 105: Medium card task – drawing a number, enclosing the tens and adding more



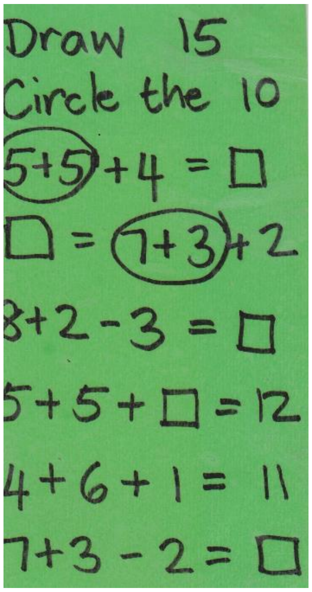
This card was designed to help to shift learners' 'counting in ones' indexical representations of numbers to impose a five-wise and ten-wise structure onto the numbers.

The dice pattern, or pair-wise depiction of the group of five were encouraged as a quick way to see how many there are (without counting in ones).
The last two tasks were intended to help shift learners from enclosing a group of ten ones, to enclosing a symbolic number 10.

This card also made use of the indexical representation of enclosing a group of ten. Learners were expected to copy this representation and then act on it with the instruction 'add 6 more'.

The sentences which were provided for learners to complete mixed English phrases with symbolic representations of this situation. The number sentences generated were intended to reveal the equivalence of number sentences: part + part = whole, and whole = part + part; where the order of the parts was not significance (commutativity of addition).
At first children worked with each number sentence as a new calculation. However as they worked on similar cards this became routinized and they anticipated the same result for each number sentence.

Figure 106: Medium card task – drawing a number, enclosing the tens

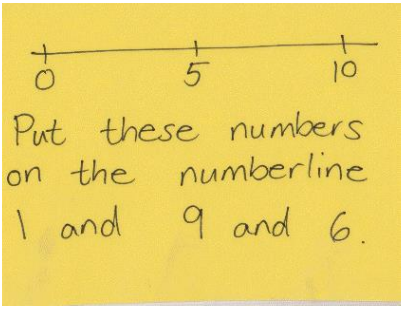


This card was designed to follow on from the previous one, as this time learners were asked to draw a number and circle the ten. An indexical representation where 5-wise or 'counting in ones' arrangements was expected.

The card then shifted to the symbolic syntactical mode of representation where finding the tens strategy was encouraged through enclosing pairs of number symbols making ten.
The position of the unknown was deliberately varied.

Enabling task 9: Line model fluencies

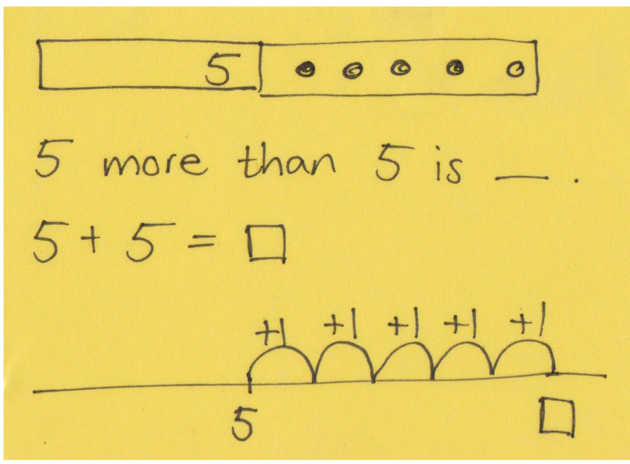
Figure 107: Easy card task – Sketch and place numbers on a number line



This type of card was designed to develop learner's fluency in using an empty number line representation.

The structured number line, showing 5-wise groups from 0 to 5 to 10, was copied.
Then learners positioned three numbers onto this structured number line.

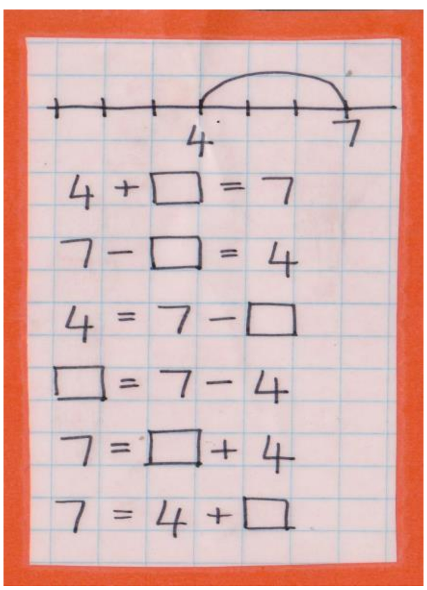
Figure 108: Easy card task – 5 strip with more than and number line



This card built on work done using 5-strip cards. It was intended to encourage a 'counting on from 5' strategy. It made use of the terms 'more than' and included a number sentence. It imposed a 5-wise structure.

The number line representation was designed to encourage use of an unstructured number line (starting at 5) and hopping on in ones.

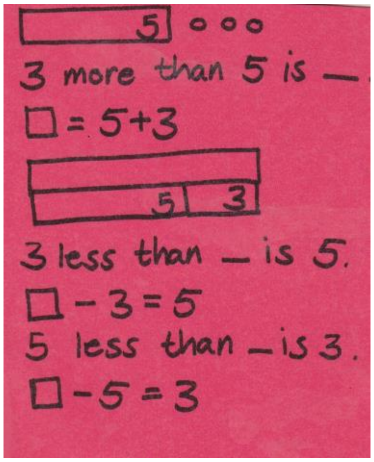
Figure 109: Difficult – number line with group-wise jump and family of number sentences



This card made use of an empty number line representation. However the grid lines imposed a structure in ones. The number line did not include 0 or 1 as a reference point. A group-wise jump was reflected (not a hop in ones).
The related family of number sentences (experienced on other cards as relating to the whole-part-part diagram) was presented here. The position of the unknown was varied accordingly.

Enabling task 10: Syntax model fluencies

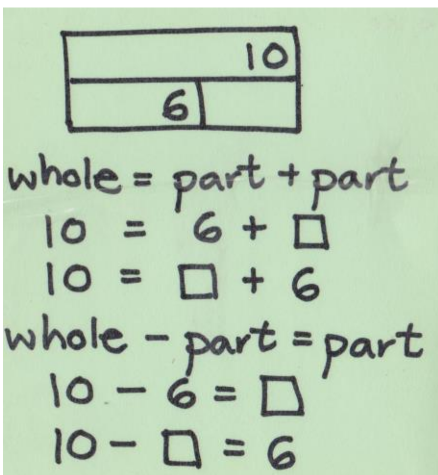
Figure 110: Medium card task – drawing a 5 strip and more ones, w-p-p diagram and number sentences



This card was designed to focus on the vocabulary of 'more than' and 'less than'. It imposed a five-wise structure.

Learners were expected to copy the image of the five strip, and 3 ones. The design was intended to encourage the learners to count on from 5.
This card made use of the whole-part-part image and related families of number sentences.
The number sentences were also written using English phrases 'more than' and 'less than'.

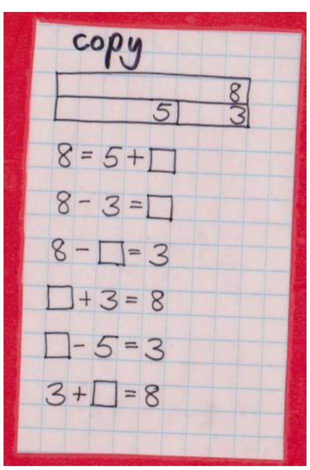
Figure 111: w-p-p diagram and 4 number sentences



This card made use of a free-hand sketch of the whole-part-part diagram.

The four equivalent number sentences in the form whole = part + part, and whole – part = part were included. The position of the unknown was varied accordingly.

Figure 112: Medium card task – w-p-p diagram and 8 number sentences

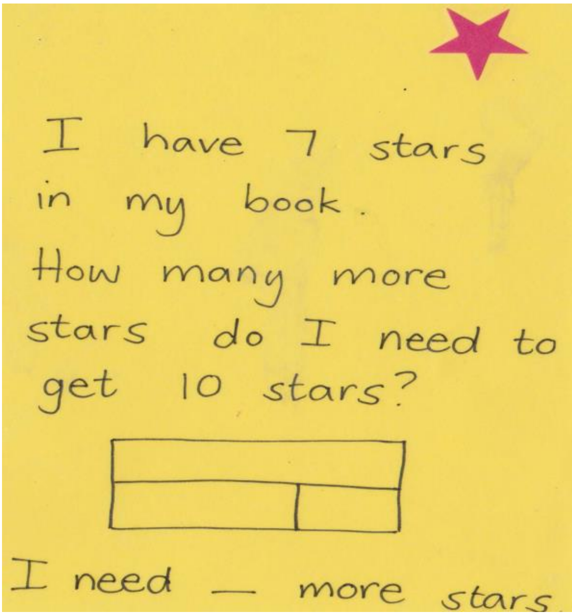


This card was intended to consolidate the whole-part-part diagram and its related family of 8 equivalent number sentences.

The grid paper was used with each block representing one unit in the whole-part-part image.

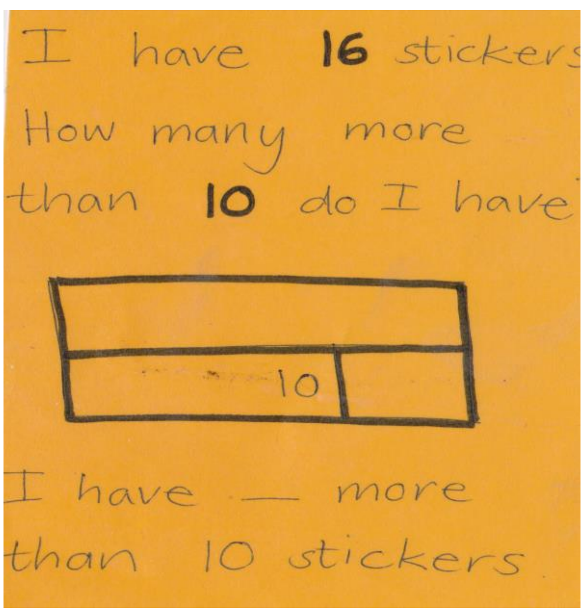
Enabling task 11: Word problem fluencies

Figure 113: Difficult card task – Word problem: compare (reach a target)



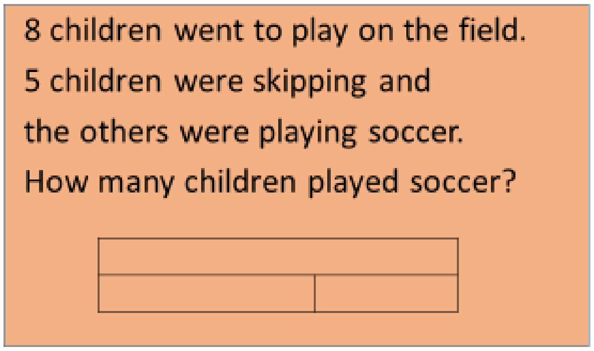
This was one of several cards where one step additive relation problems were posed.
This additive relation word problem was of the compare (reach a target) type. The card included an empty whole-part-part diagram which children were expected to copy and complete. They were also prompted to write an answer sentence with the units.

Figure 114: Difficult card task – Word problem: compare (disjoint set)



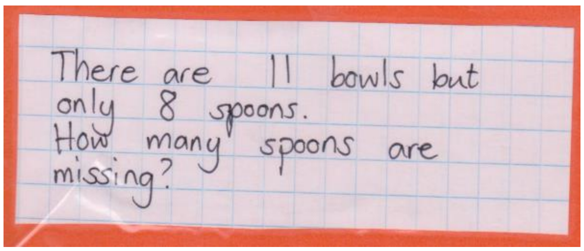
This was an additive relation story of the compare (disjoint sets) type. A partially completed whole-part-part diagram was included. Learners were prompted to copy and complete an answer sentence with units.

Figure 115: Difficult card task – Word problem: collection



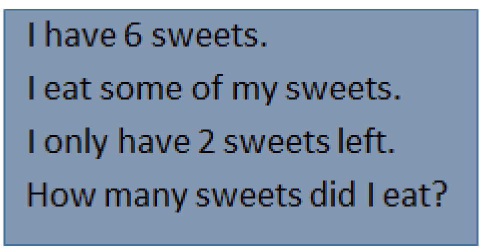
This was an additive relation story of the collection type.
A blank whole-part-part diagram was included to encourage the use of this representation.

Figure 116: Difficult card task – Word problem: compare (matching)



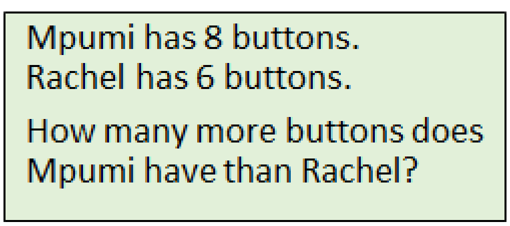
This was an additive relation story of the compare (matching) type.
The children in this school are given porridge daily, and the distribution of plastic bowls and spoons was a morning ritual.

Figure 117: Difficult card task – Word problem: change, change unknown



This was an additive relation story of the change type with the change unknown.

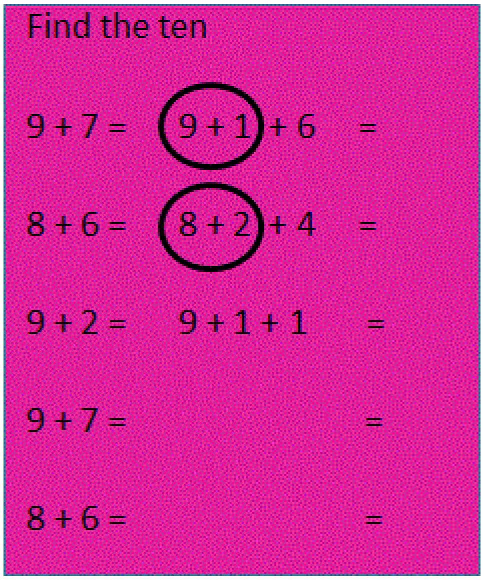
Figure 118: Difficult card task – Word problem: compare (disjoint set)



This was an additive relation story of the compare (disjoint set) type.
These questions were set to include the names of children in this class.

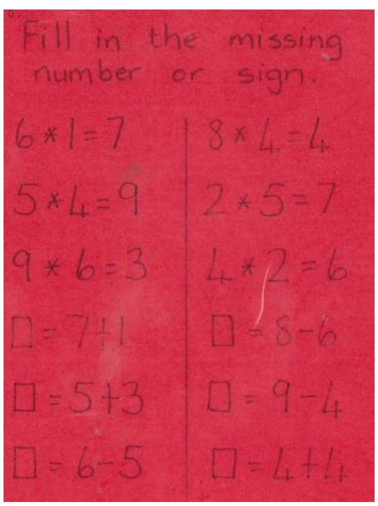
Enabling task 12: Basic number facts and bridging the tens strategies

Figure 119: Difficult card task – Find the ten



This card was designed to encourage a 'find the ten', or 'bridge through ten' calculation strategy.
It built on the indexical representations of enclosing 10 ones to make 1 group of ten.

Figure 120: Medium card task – fill in the missing number or sign



This card was designed for learners to become fluent in reading and completing number sentences. They had to replace the * symbol with either + or – for the number sentence to make sense.

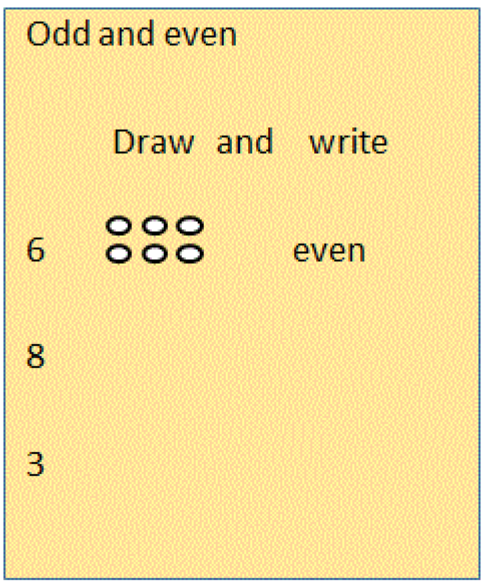
The second half of the card was designed to draw attention to the meaning of the equals sign. It presented number sentences in the form whole = part + part or part = whole – part.
These were included as it was anticipated that learners would be more familiar with number sentences in the form whole - part = part or part + part = whole

Figure 121: Medium card task – two step bare calculations (single digits)

[ ] = 9 − 1 − 2
3 + 4 + 2 = [ ]
6 + 2 − 5 = [ ]
3 + 2 + [ ] = 7
9 − 4 − 3 = [ ]
7 + 2 − 6 = [ ]
8 − 2 − 3 = [ ]
2 = 4 + 5 − [ ]

These cards include two step addition and subtraction calculations involving single digit numbers. The position of the unknown was deliberately varied.

Figure 122: Easy card task – Drawing single digit numbers odd/even



For this card learners were expected to draw a number of dots, and write whether the number was odd or even.

Annexure 6: WCED permission

Western Cape Government
Education

Directorate: Research

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Private Bag x9114, Cape Town, 8000
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REFERENCE: 20140403-27648
ENQUIRIES: Dr A T Wyngaard

Ms Nicolette Roberts
6 Hamilton Avenue
Craighall Park
2196

Dear Ms Nicolette Roberts

RESEARCH PROPOSAL: TELLING AND ILLUSTRATING ADDITIVE RELATION STORIES: A CLASSROOM-BASED DESIGN EXPERIMENT ON YOUNG CHILDREN'S USE OF NARRATIVES IN MATHEMATICS

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **10 April 2014 till 30 September 2015**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:
    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 04 April 2014

Lower Parliament Street, Cape Town, 8001
tel: +27 21 467 9272 fax: 0865902282
Safe Schools: 0800 45 46 47
Private Bag X9114, Cape Town, 8000
Employment and salary enquiries: 0861 92 33 22
www.westerncape.gov.za

Annexure 7: University of Witwatersrand ethical clearance

Wits School of Education

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Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

Student Number:
Protocol Number: 2013ECE107D

Date: 30 October 2013

Dear Nicky Roberts

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

Telling and illustrating additive relation stories: A classroom-based design experiment on young children's use of narrative in mathematics.

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely

M Mabeta

Matsie Mabeta
Wits School of Education

011 717 3416
Cc Supervisor: Prof. H Venkatakrishnan

Annexure 8: Parent permission form

27th of February 2013

Dear Grade 2 Parent

Re: Grade 2 maths study: Parent information

My name is Nicky Roberts I am a PhD student in the School of Education at the University of the Witwatersrand. I am doing research on adding and subtracting word problems in Grade 2.

I would like to work with Mrs McDonald and your child's class in April and May 2014. Your child will:

- write a pre-test
- participate in a 20-minute pre-individual interview (only 16 learners)
- participate in 10 mathematics lessons to be taught in the normal lesson time
- participate 20-minute post-interview (only 12 learners)
- write a post-test

The reason why I have chosen Capricorn Primary is because of our existing partnership in the Focus on Primary Maths project. Your child will not be disadvantaged in any way. There are no foreseeable risks in participating in this study. This study can in no way influence whether you child can attend Capricorn Primary.

I will not use your child's name in the study. I will take photographs of children's mathematics work. I will video record the interviews and lessons. The photographs and videos will only be used for the analysis of the research. All research data will be destroyed 5 years after completion of the project.

Please let me know if you require any further information.

Yours sincerely

SIGNATURE:

NAME : N. Roberts

ADDRESS : 6 Hamilton Avenue, Craighall Park

EMAIL : nickyroberts@icon.co.za

TELEPHONE NUMBERS : 071 525 8389

Please could support this research by giving permission for your child to be part of this study.

Please complete this reply slip:

Your Name: ____

Your Child's Name: ____

My child can participate in the study (write the tests, and participate in the lessons)	Yes / No
My child's mathematics work can be photographed	Yes / No
My child can be video-recorded during class and in the interviews	Yes / No

Your signature: ______________________________Date: ____________________________

Annexure 9: Learner consent form

The following is an example of a learner permission form.

My name is: ______________________

Yes, Teacher Nicky can use my maths work in her research.

Yes, Teacher Nicky can take photographs of my maths work.

Teacher Nicky will not use my real name when she writes about me.

Date: Tuesday 6 May 2014

My favourite story is stickers

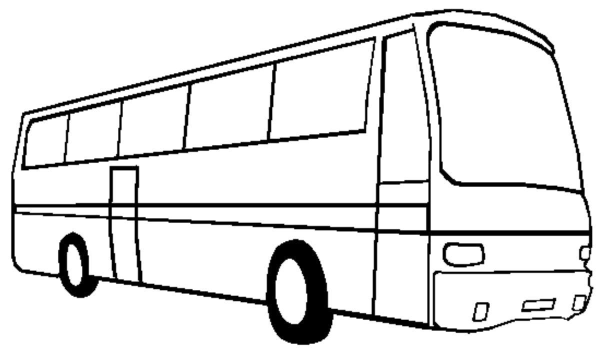
I learnt about making 5 sums

Annexure 10: Written tests

Pre-test

Name: ____

1.
There are 14 people in a bus. 8 people get out of the bus. How many people are left in the bus?



There are ___________ people left in the bus.

Number sentence:

Number line:

2.

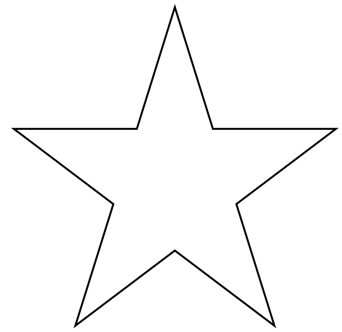
There are 11 bottles but only 9 lids. How many lids are missing?

There are ________________ lids missing.

Number sentence:

Number line:

3.
Sue has 13 stickers.
Some are gold and some are red.
4 of the stickers are gold.
How many stickers are red?



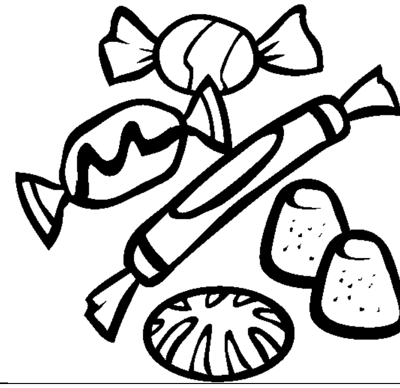
There are ________________ red stickers.

Number sentence:

Number line:

4.

Jarred has 9 sweets.
Martha has 12 sweets.
How many more sweets does Martha have than Jarred?



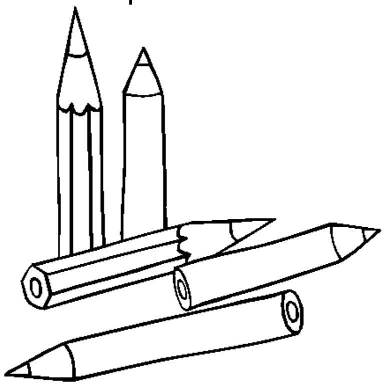
Martha has ______________________ more sweets than Jarred.

Number sentence:

Number line:

5.

Sihle has 5 pencils.



He keeps them in his 2 pockets.
Show all the ways that the pencils can be kept in Sihle's pockets

Pocket 1	**Pocket 2**

Pocket 1	**Pocket 2**

There are _____________ ways for Sihle to keep the 5 pencils in his 2 pockets.

21 - 6 = ___________

Number line:

23 - 18 = ___________

Number line:

Post-test

Name: ____

1. Calculate

21 - 6 = ____________

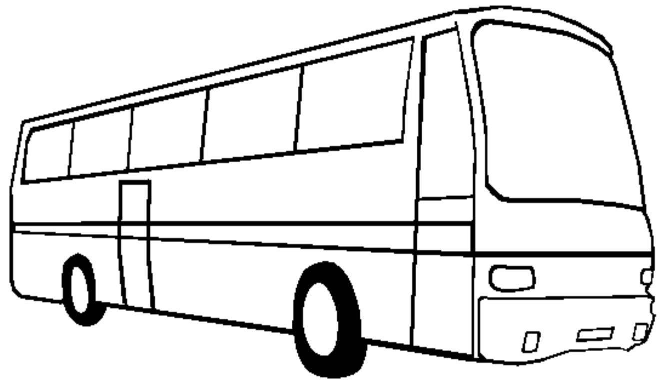
Show your working

23 - 18 = ____________

Show your working

Solve these word problems

2.
There are 14 people in a bus.
8 people get out of the bus.
How many people are left in the bus?



Write a number sentence
Show your working
There are ___________ people left in the bus.

3.

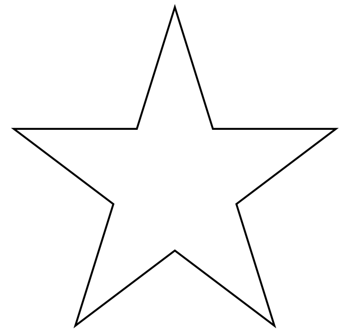
There are 11 bottles but only 9 lids.
How many lids are missing?

Write a number sentence

Show your working

There are ________________ lids missing.

4.



Sue has 13 stickers.
Some are gold and some are red.
4 of the stickers are gold.
How many stickers are red?

Write a number sentence
Show your working
There are ______________ red stickers.

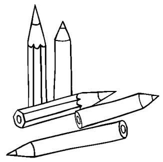
5.

Jarred has 9 sweets.
Martha has 11 sweets.

How many more sweets does Martha have than Jarred?

Write a number sentence
Show your working
Martha has ____________________ more sweets than Jarred.

7. Sihle has 5 pencils.



He keeps them in his 2 pockets.

Show all the ways that the pencils can be in Sihle's pockets

Pocket 1	**Pocket 2**

____

____

____

____

____

____

____

There are ____________ ways for Sihle to keep the 5 pencils in his pockets.

Delayed post–test

Name: ____

1. Calculate

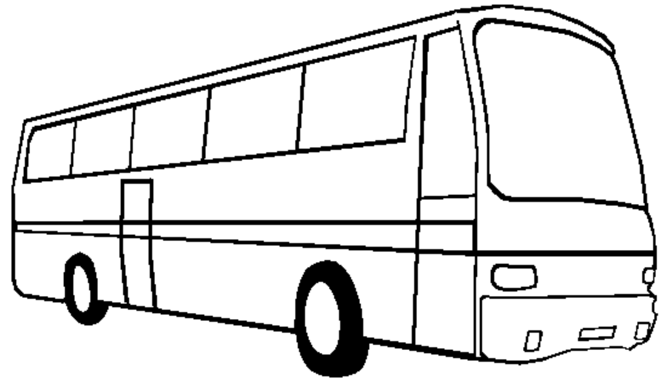
21 - 6 = ____________

Show your working

23 - 18 = ____________

Show your working

2.
There are 14 people in a bus.
8 people get out of the bus.
How many people are left in the bus?



Write a number sentence
Show your working
There are ___________ people left in the bus.

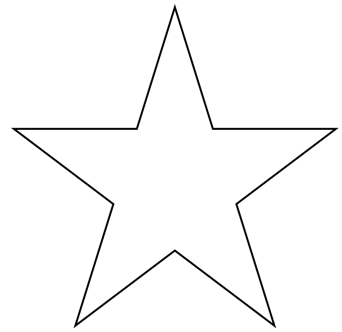
3.

There are 11 bottles but only 9 lids.
How many lids are missing?

Write a number sentence
Show your working
There are ________________ lids missing.

4.

Sue has 13 stickers.
Some are gold and some are red.
4 of the stickers are gold.
How many stickers are red?



Write a number sentence

Show your working

There are ______________ red stickers.

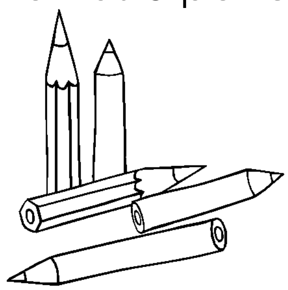
5.

Jarred has 9 sweets.
Martha has 11 sweets.

How many more sweets does Martha have than Jarred?

Write a number sentence
Show your working
Martha has ____________________ more sweets than Jarred.

6. Sihle has 5 pencils. He keeps them in his 2 pockets.



Show all the ways that the pencils can be in Sihle's pockets

Pocket 1	**Pocket 2**	**Number sentence**

There are _____________ ways for Sihle to keep the 5 pencils in his pockets.